TESTS OF INDEPENDENCE FOR NOMINAL AND ORDINAL DATA; COMPARISON AND APPLICATION

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"Bismillah Hir Rahman Nir Rahim"

In the name of Allah, the most gracious the most merciful

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ABSTRACT

This study aims to analyze the performance of tests of independence for categorical data which may further be classified as nominal and ordinal data. Tests of independence are one of the most frequently used statistical tools in econometrics. Researchers are often interested in the independence of variables summarized in Contingency Tables (CTs). Many tests are available in the literature to test independence in CTs. However, there is no clarity about the choice of tests that are incapable to provide a comparison of a large number of tests.

A central problem and question facing researchers is to decide which tests of independence are most stringent for the data in hand. Most of the studies make pairwise comparisons of tests and such studies are unable to guide optimal tests among a wide set of tests. Furthermore, such studies used different conventional statistical techniques to find an optimal test of independence for nominal and ordinal data.

This study used Monte Carlo Simulations (MCS) to evaluate the performance of a large number of tests of independence for nominal and ordinal data.

The study compares eleven tests of independence for nominal data namely, Pearson's Chi-Square (χ^2) test, Log Likelihood Ratio (G^2) test, Fisher Exact Test (FES), Freeman and Tuckey Test (FTS), Cressie and Read Test (CRS), Kulber and Liaber test (KLS), Neyman Modified Chi-Square Test (NMCS), Modular Test (MDS), D Square (D^2), BP Test, and Logarithmic Minimum Square Test (LMS). We were able to calculate the most stringent test and it turned out that Logarithmic Minimum Square (LMS) is the most stringent test for nominal data in $w \times k$ CTs.

Similarly, seven popular tests of independence for ordinal data are compared namely, Spearman ρ coefficient of correlation, Kendall's$\tau - a$, Kendall's$\tau - b$, Kendall's $\tau - c$ coefficient, Goodman and Kruskal γ, Sumer's D and Novel Phi_k (ϕ_k). Since the likelihood function is not found in the literature; for ordinal data, the stringency criteria cannot be applied to compare tests. Therefore, the comparison was made based on power and our MCS concludes based on solid estimations using the power criteria that the most powerful test is Novel ϕ_k in $w \times k$ CTs for ordinal data.

Keywords: Tests of Independence, Size of test, Power of test, Stringency Criteria, Contingency Table. Nominal data, Ordinal data

Jel Classification: C1, C12, C14, C15, C46, C63

DEDICATION

This piece of work is dedicated to my father

Mr. Muhammad Rasool Khan (Late)

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Mr. Shakeel Shahzad

Table of Contents

LIST OF TABLES

LIST OF FIGURES

LIST OF MAJOR SYMBOLS

W, K	Dimension of CT
i, j "i"	shows the number of cells in rows, and "j" is the number of cells in column
a,b,c,d	The counts of the cells in CTs
d_*	Somers coefficient
D^2	D square test statistics
e_*	Expected cell counts in the CTs
H_0	Null hypothesis
H_1	Alternative hypothesis
ln	Natural log
N	Grand Total
SS	Sample Size
n_*	Observed cell counts in the CTs
s^2	Variance
s	Standard deviation
df	Degree of freedom
α	Significance level alpha
γ	Goodman – Kruskal Gamma Coefficient
λ_*	Goodman- Kruskal Lambda Coefficient
N_c	Ordered pairs consistent concerning X and Y variables;
N_d	Ordered pairs inconsistent concerning X and Y variables;
T_x	Pairs related to X but ordered to Y;
T_y	Pairs related to Y but ordered to X;
T_{xy}	Pairs related due to both variables X and Y.
τ_*	Goodman- Kruskal Tau coefficient
+	"+" shows the power of tests as it increases shows the most powerful test)
χ^2	Chi-square statistics

LIST OF ABBREVIATIONS

ACV	Asymptotic Critical Values
CPI	Corruption Perception Index
CRS	Cressie and Read Statistics
CTs	Contingency Tables
DGP	Data Generating Process
FTS	Freeman and Tuckey Statistics
FES	Fisher Exact Statistics
FSCV	Finite Sample Critical Values
KLS	Kullback - Leibler Statistics
LMS	Logarithmic Minimum Square Test
LIC / MIC	Low-Income Countries / Middle-Income Countries
MCS	Monte Carlo Simulations
MDT	Modular Test
MoU	Measure of Untruthfulness
NMCS	Nyman Modified Chi-Square Statistics
PoT	Power of Test
PDS	Power of Divergence Statistics
SC	Stringency Criteria
SD	Size Distortion
SoC	Strength of Correlation
TT	Theoretical Table

CHAPTER 1

INTRODUCTION

1.1. Background of the Study

Tests of independence are one of the most used statistical tools in econometrics. Contingency Tables (CTs) are cross-classified tables of frequency counts which provide a wide range of information. The study of CTs is one of the most appealing and active topics in statistics because of its applications and importance in the social and biological sciences.

Numerous studies use CTs and focus on determining and testing the independence of variables, such as Haberman (1981), Berry and Mielke Jr (1988), Lawal and Uptong (1990), Mature and Elsayigh, (2010), Yenigün, Székely et al. (2011), Assad, (2012), Lipsitz, Fitzmaurice et al. (2015), Sulewski (2017), Sulewski (2019) and Islam & Rizwan, (2020). Even though there are several powerful tests available in the literature; for CTs there is little clarity about the relative merits of tests of independence. It is not known which of the tests is most optimal for the available data set.

The data in the CTs is also known as categorical data, which can be further divided into two types such as nominal and ordinal. There are certain tests designed for nominal data e.g., the Chi-square test (χ^2), Log-likelihood ratio test (G^2), Fisher exact test statistics, Freeman and Tucky test statistics, Kullback - Libeler test statistics, Cressie - Read test statistics, etc. There are some other tests of independence designed for ordinal data such as Spearman correlation coefficient (ρ), Kendall's tau, Goodman and Kruskal γ, Sumer's D among others; and some of the above tests which are being used for both type of data e.g. Chi-square test (χ^2). It's not clear what would happen if the tests designed for one type of data are used for the other type of data.

Most of the tests make use of Asymptotic Critical Values (ACV) and can be studied for large samples. Since, large sample tests sometimes fail to behave well in small samples. Therefore, we tested the size distortion of tests using asymptotic critical values. Since numerous tests are based on asymptotic critical values; sometime asymptotic critical values may not work robust in finite samples. Consequently, there is a need to obtain Finite Sample Critical Values (FSCV) that work robust even with small samples. For this reason, we focused on finite sample critical values in this study. The nominal critical value of each independence test is already given in the literature, e.g., the critical value for the chi-square test of independence is the value at which the area of the chi-square distribution with $(\mathrm{w}-1)(\mathrm{k}-1)$ degree of freedom is greater than 95%. To keep the size of the test constant at the nominal level (α) at 1%, 5%, and 10%; if size distortion exists, then finite simulated critical values for each test of independence are required to be computed for power computation.

There are several independence tests for both nominal and ordinal data in the literature and this always leads to confusion when it is applied on real datasets. Choosing the most stringent and powerful test in the literature is a key issue. Most researchers have compared the performance of independence tests, but they have carried out pairwise comparisons instead of comparisons for large numbers of tests. The variation in Data Generating Process (DGP) is often ignored by early researchers and the finite sample properties are not analyzed.

In this context, this study is aimed to evaluate the performance of the independence test for a variety of DGP for categorical data in CTs. We compared tests of independence for nominal data based on the Stringency Criteria (SC) which are computed from the power envelop, and provide an opportunity to compare large numbers of tests. We used Power Criteria (PC) for tests of independence for ordinal data. In addition, this study analyzes the size distortion of w × k CTs under different DGP using Asymptotic Critical Values (ACV) and Simulated Critical Values (SCV).

1.2 Problem Statement

Many statistical articles discuss the comparison of tests of independence for categorical data. Likewise, many statistical tests for nominal and ordinal data have been modified and developed over time.

As stated earlier these studies make pairwise comparisons of tests which are insufficient for the selection of tests among a large class of available tests. Thus, the literature is silent and no consensus has been developed on the most stringent test expected in the literature in w × k CTs. Consider the data types described in Table 1.1.

Table 1. 1: Typical W × K Contingency Table for Variable X and Y

Variable X	Variable Y				**Total**
	Y_1	Y_2	...	Y_K	
X_1	n_{11}	n_{12}	...	n_{1k}	$n_{1.}$
X_2	n_{21}	n_{22}	...	n_{2k}	$n_{2.}$
..	...	...	...	...	...
X_W	n_{w1}	n_{w2}	...	n_{wk}	$n_{w.}$
Total	$n_{.1}$	$n_{.1}$	...	$n_{.k}$	**N**

Table 1.1 elaborates that if we have "n" draws having two variables 'X' and 'Y'; each character is having certain categories. n_{11}, is the number of draws having category 1 in the 'X' variable and category 1 in the 'Y' variable. Similarly, n_{wk} is the number of draws having category 'w' in the 'X' variable and category 'k' for the 'Y' variable such that the categories of 'X' and 'Y' can have a natural ordering and in that case,

data would be termed as ordinal data. While sometimes the categories do not have any order and are known is nominal data.

The marginal sums in the w × k CT would be:

$$n_{1.} = \sum_{j=1}^{k} n_{1j},\ n_{2.} = \sum_{j=1}^{k} n_{2j}, \dots ,\ \ n_{w.} = \sum_{j=1}^{k} n_{wj} \tag{1.1}$$

$$n_{.1} = \sum_{i=1}^{w} n_{i1},\ n_{.2} = \sum_{i=1}^{w} n_{i2}, \dots ,\ \ n_{.k} = \sum_{i=1}^{w} n_{ik} \tag{1.2}$$

The value "N" is the sum of all the counts of the w × k CT,

$$N = \sum_{i=1}^{w} n_{i.} = \sum_{j=1}^{k} n_{.j} = \sum_{i=1}^{w}\sum_{j=1}^{k} n_{ij} \tag{1.3}$$

Suppose each entry of table 1.1 is divided by 'n' such that the Table 1.2 takes following forms.

Table 1. 2: W × K Contingency Table for Observed Frequencies Variable X and Y

Variable X	**Variable Y**				**Total**
	Y_1	**Y_2**	**...**	**Y_j**	
X_1	$n_{11/n} = p_{11}$	$n_{12/n} = p_{12}$	...	$n_{1j/n} = p_{1j}$	$p_{1.}$
X_2	$n_{21/n} = p_{21}$	$n_{22/n} = p_{22}$	...	p_{2j}	$p_{2.}$
...	...	...	...	...	...
X_i	$n_{i1/n} = p_{w1}$	$n_{i2/n} = p_{w2}$	...	p_{ij}	$p_{i.}$
Total	$p_{.1}$	$p_{.2}$	...	$p_{.j}$	**1**

Table 1.2 shows that p_{11} represents proportions of draws having category 1 in 'X' and category 1 in 'Y'. Whereas, p_{ij} represents category 'i' in variable 'X' and category 'j' in variable 'Y' and the term is as follows;

$$\sum_{i=1}^{w}\sum_{j=1}^{k} p_{ij} = 1 \qquad (1.4)$$

Table 1.2 are random draw; the actual probability could be different from the observed proportions. The actual probabilities are shown in table 1.3 as follows:

Table 1. 3: Theoretical Distribution of Variable X and Y in $W \times$ K CTs

	'Y' Variable				**Marginal Probability**
'X' Variable	(π_{11})	(π_{12})	-	$(\pi_{2\mathrm{j}})$	$\pi_{1.}$
	(π_{21})	(π_{22})	-	$(\pi_{2\mathrm{j}})$	$\pi_{2.}$
	-	-	-	-	-
	-	-	-	-	-
	$(\pi_{\mathrm{i}1})$	$(\pi_{\mathrm{i}2})$	…	(π_{ij})	$\pi_{\mathrm{i}.}$
Marginal Probability	$\pi_{1\bullet}$	$\pi_{2\bullet}$	… …	$\pi_{\mathrm{j}\bullet}$	**1**

If, $\frac{n_{ij}}{n}$ in Table 1.2 gives observed frequency for a particular cell. Then π_{ij} would be the theoretical probabilities associated with the cell. Researchers are mostly interested in the variables, especially when looking at the CTs to see if there is a relationship between variables, whether they are independent or not.

Therefore, the condition for independence can be written as follows: Suppose we have equation 1.5.

$$\pi_{.\mathrm{j}} = \sum_{\mathrm{i}=1}^{\mathrm{w}} \pi_{\mathrm{ij}} \qquad (1.5)$$

Thus, the condition for independence is the probability of any cell in the CTs equal to the product of the row and column probability of the concerned cell. This condition transforms into equation 1.7.

$$\pi_{.i.j} = \pi_{.i} \times \pi_{.j} \quad \text{For all i, j.} \tag{1.6}$$

This gives us;

$$\sum_{i=1}^{w} \pi_{ij} = \sum_{j=1}^{k} (\pi_{i.}\pi_{j.}) \tag{1.7}$$

Thus, the null and alternative hypothesis for independence becomes,

$$Null\ Hypothesis : \sum_{i=1}^{w} \sum_{j=1}^{k} (\pi_{ij} - \pi_{i.}\pi_{j.})^{n} = 0 \tag{1.8}$$

$$Alternative\ Hypothesis: \sum_{i=1}^{w} \sum_{j=1}^{k} (\pi_{ij} - \pi_{i.}\pi_{j.})^{n} \neq 0 \tag{1.9}$$

$$For\ i = 1 \dots\dots\dots\ w, \qquad j = 1 \dots\dots\dots\ k.$$

1.3 Research Objectives

The core objective is to evaluate the performance of tests of independence for a variety of DGPs for nominal and ordinal data. This study aims to expand the literature on the following topics.

A. Given the Nominal Data Organized in W × K CTs:

a) To calculate size distortion (SD) for a test having asymptotic critical value (ACV) in the finite sample using Monte Carlo simulations (MCS).

b) To calculate finite sample critical value (FSCV) for the tests with no asymptotic critical value (ACV) and the tests with size distortion (SD).

c) To compare the power of tests of independence using stringency criteria (Based on power envelope) to evaluate the most stringent test of independence for nominal data

B. Given the Ordinal Data Organized in W × K CTs:

a) To calculate size distortion (SD) for a test having asymptotic critical value (ACV) in the finite sample using Monte Carlo simulations (MCS).

b) To calculate finite sample critical value (FSCV) for the tests with no asymptotic critical value (ACV) and the tests with size distortion (SD).

c) To compare the power of tests of independence using power criteria (PC) to evaluate the most powerful test of independence for ordinal data.

C. Application of the Most Stringent/Powerful Test on Real Data Sets:

a) Application of the most stringent test for nominal data on the relationship between girls' enrollment in education across provinces in Pakistan.

b) Application of the most powerful test of independence of ordinal data on corruption perception index and countries categorized by per capita income.

1.4 Research Outline

This dissertation consists of ten chapters. Chapter one explains the study background, the problem statement, and the research goals. Chapter two contains a discussion of the systematic and critical literature review for existing comparisons of tests of independence for nominal and ordinal data in CTs. Chapter three contains a brief discussion of various proposed tests of independence tests for nominal and ordinal data used in this study.

Chapter four discusses the methodology to achieve specific and central goals, which consists of simulation design, DGP, computation of Size Distortion (SD), FSCV, and power analysis. In addition, this chapter explains the evaluation of the most stringent techniques i.e., SC and PC for tests of independence for nominal and ordinal data. Chapter five provides a brief discussion of the results of SD and FSCV based on solid estimations of Monte Carlo Simulations (MCS) for nominal and ordinal data.

Chapter six demonstrates a discussion on MCS results of the most stringent test for nominal data (Power analysis of different scenarios in $w \times k$ CTs, evaluation of most stringent test of independence obtained by using SC). Chapter Seven discusses the results of the most powerful test of independence for ordinal data using power criteria (PC) in $w \times k$ CTs. Chapter eight and nine explains the applications of the most stringent test of independence on real nominal data along with the application of the most powerful test on a real ordinal data set. Last chapter explains conclusion, recommendations, and future directions.

CHAPTER 2

LITERATURE REVIEW ON TESTS OF INDEPENDENCE

This chapter discusses a comprehensive and critical examination of the literature on tests of independence for nominal and ordinal data. Section 2.1 explains early development of tests of independence for nominal and ordinal data, their comparison, and critical analysis of numerous approaches for comparing tests of independence for nominal and ordinal data. Section 2.2 describes summary and research gap in literature in CTs.

2.1 Brief Literature Review

The concept of correlation was created by Francis Galton's Brooks in 1887, and he was the pioneer to utilize its significance in the social and biological sciences. His contributions to the development of regression and correlation are most notable in the literature of econometrics. Pearson has penned numerous essays and focused significant emphasis on the development of correlation (Stigler 1986). In his book "On the theory of contingency and its relevance to association and correlation," Karl Pearson coined the phrase "Contingency Table". Theoretical debates, concerns, and issues surrounding the testing of independence in CTs have a long history and were first investigated in 1800s. The chi-square test was produced by Pearson's renowned goodness of fit test when a 2 x 2 CTs was analyzed (Pearson, 1900; 1904). By examining the equality of two independent binomial proportions of a single dichotomous factor, Yule (1911) developed the first association test. Fisher (1934) used the extended hyper geometric distribution to describe the combinatorial randomization of two-factor association, which gave rise to his exact test.

By 1920s the philosophy of hypothesis testing had been well established by Fisher (1925, 1935), and Neyman and Pearson (1928), among others. It also initiated the long debate concerning the two approaches: significance testing for Fisher and hypothesis testing for Neyman and Pearson. Testing independence for a 2 x 2 CTs was a notable example in these arguments. While the debate was focused on the notions of inductive inference, significance level, and decision theory for testing hypotheses, the importance of power evaluation was accepted e.g., (Fisher, 1946) with the adoption of the idea of identifying appropriate critical regions for constructing more sensitive tests. For example, in testing the equality of two binomial parameters by Yule's test, the 'p' values and the power at alternatives can be computed from either the normal approximation or the exact distribution. However, unified power analysis has not been fully developed for Pearson's chi-square or Fisher's exact test for assessing independence in a 2 x 2 CTs.

In the 1960s, the invention, development, and modification of tests of independence drew the attention of econometricians and statisticians. During the period from 1950 to 1970, a rapid improvements were made in various areas of statistics and econometrics, including the CTs for categorical data analysis.

Meanwhile, a controversial issue arises when using the exact test, due to its discrete nature; with the limited sample space defined by fixed row and column margins, it yielded a conservative test when the sample size is not large. The criticism of the conservativeness of Fisher's exact test reached a climax when Berkson, (1978) dispraised Fisher's exact test using arguments based on Yule's test for the equality of two independent binomial proportions. Since then, Yule's test has been discussed most widely exact as unconditional test in the literature. Yates, (1984) gave supporting arguments for Fisher's exact test, noting that "Tests for independence in a 2 x 2 CTs must be conditioned on both margins". Most discussants on Yates' paper agreed with his assertion. However, this remains a debated issue in the literature, primarily due to the lack of unified power analysis for both Pearson's chi-square test and Fisher's exact test. (Cheng, P. E., Liou, M., Aston, J. A., & Tsai, A. C. (2008).

We have found in the literature that many tests for independence in CTs have been compared recently, with a wide range of findings such as Assad, (2012) Lin, Chang et al. (2015), Amiri and Modarres (2017), (Sulewski (2013), Sulewski (2017) etc. The modification in the test of χ^2 proposed by Lawal and Upton (1984) bring it closest to the nominal level alpha (α). There are numerous studies on the CTs and χ^2 test of independence in the literature e.g. Meng and Chapman (1966), Diaconis and Efron (1985), Albert (1990), and Andrés and Tejedor (1995), Where there are various ways to interpret the test of χ^2 statistics. Extensive information on the approximation of chi-square (χ^2) and the Likelihood ratio test (G^2) provided by several studies e.g., Cochran (1954), (Koehler and Larntz 1980, Cressie and Read 1989). Henceforth, the tests of χ^2 and G^2 are consistent and asymptotically unbiased independence tests (Haberman 1981). According to Cressie and Read (1989), these tests belong to the family of power divergence statistics (PDS[1]).

Irwin independently created the Fisher-Irwin test in 1935, which is also known as the Fisher-Irwin test and is a well-known and extensively investigated test (Fisher, 1935). Campbell (2007) suggests the application of χ^2 test for large sample size and Fisher Irwin test for small sample size. Basically, some scholars claim that the actual rejection rate of Fisher's exact test under H_0 is lower than nominal level of significance (Liddell 1976, Douglas, Fienberg et al. 1990).

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[1] Cressie and Read (1984) proposed the power divergence statistics (PDS). PDS family consists numerous tests of independence namely, χ^2, G^2, Modified G^2, FTS, NMCS and CRS. Sulewski, P. (2017)

In addition, Haberman (1981) compared the power of the two-tailed Fisher - Irwin test to six non-randomized unconditional exact tests. The D^2 test which is a modification of the χ^2 test, was proposed by (Zelterman 1987). Furthermore, the study of Lawal and Uptong (1990) attempts to compare the modified χ^2 test statistics, (Lawal and Upton 1984) to the power of divergence statistics in terms of statistical power.

A simulation study by Yenigün, Székely et al. (2011) to perceive the empirical power performance of maximal correlation tests and compare it with tests of χ^2 and G^2. This study highlights some cases for which the maximal correlation tests seems to have more power when considered continuous variable are dependent and uncorrelated. Assad, (2012) described and compared four independence tests in his dissertation and found that Fisher's exact independence test is robust to all four tests used in his study, i.e., Pearson's product moment coefficient correlation; Goodman and Kruskal's measure of correlation, fisher exact test and chi-square test of independence This study used few independence tests for categorical data. There is confusion among the data as it has not been segregated as nominal and ordinal. Additionally, the study has been conducted on the evaluation of optimal tests for a small contingency table that is 2×2 contingency table.

Sulewski, (2013), proposed a modular test that represents the modification in χ^2 test for two-way and higher-order contingency tables. The study compared Shan and Wilding (2015) modify the extension of the unconditional approach based on maximization and estimation to fixed sum designs. This method is based on χ^2 , G^2, Yates corrected test statistics are evaluated with respect to the actual type 1 error, power, and rates.

Lipsitz, Fitzmaurice, et al. (2015) proposes forest and score test statistics for independence. The proposed Wald and Score test statistics, unlike the Rao-Scott test statistics, exist without restriction. Comparing the power of the Rao-Scott test statistic, the Score statistic, and the Wald test statistic, it was found that the Wald test statistic has the maximum power.

The technique of bootstrap is a crucial technique for statistical hypotheses testing. Bootstrapping procedure approximates the sampling distribution of statistics based on the null or the alternative hypothesis by using re-sampling. The non-parametric bootstrap approach is more effective than χ^2 statistic, the χ^2 statistic with a Yates' correction and the Fisher exact test (Amiri and von Rosen 2011).

Lin, Chang, et al. (2015) applied an extensive simulation to identify the accuracy of the χ^2 and G^2 tests, and then recommend techniques of bootstrapping that tends to perform better than the asymptotic tests in term of adhering to the nominal level for small to large sample sizes and extreme cell frequencies. The proposed method of bootstrapping is criticized for being a conventional method. Moreover, Amiri and Modarres (2017) define a test statistic for bootstrapping that deliver more precise results in term of inference in the case of small sample size in a contingency table.

Piotr Sulewski has many recent research contributions in literature-related development and comparison of tests of independence such as (Sulewski (2013), Sulewski (2017), Sulewski (2019), and Sulewski (2020) in which comparison is carried out of tests of independence in CTs. These studies are worthy but still, there is a lack of clarity for the evaluation of an optimal test for nominal data for a large number of tests as well as for various types of data-generating processes such as in one of his study comparisons of modular test is carried out with the PDS for selected larger size of contingency table other than 2×2 concerning their size of power. The study used power criteria (PC) and still, there is a lack of confusion in the case of several types of data sets which one is the optimal test that can be applied to all types of data. In most scenarios, the power of each test is extremely useful for comparing different tests especially when comparing tests of independence. However, in some scenarios, this approach does not provide a satisfactory conclusion.

When two categorical variables are both naturally ordered, a variety of effect size measures have been proposed for such ordinal data, including spearman's ρ, Gamma coefficient, Kendall's tau-b, Kendall's tau-c, and Somers'd (Garson, 2008). The correlation coefficient"ρ" is a summary measure that describes the degree of the statistical link between two interval or ratio-level variables. The correlation coefficient is scaled so that it is always between -1 and +1. When is close to 0 this means that there is little or no link between the variables. There is extremely limited literature exists on comparisons of experiments of independence for ordinal data in contingency tables. Mardia (1969) studied the performance of some tests of independence for ordinal data. They found Kendall's coefficient and a certain other measure of correlations are asymptotically equivalent that tests based on Spearman's rank correlation coefficient. The asymptotic relative efficiency of spearman's rank test was found greater than or equal to 1.

There is limited literature studying comparisons of tests of independence for categorical data particularly concerning ordinal data. From 1900 to the present day, several tests have been invented, and modified criticism has been leveled at times due to data assumptions, the nature of dimensions, statistical techniques, and the nature of variable types in the contingency table. Accordingly, over time one hand several tests have been developed' On one side; on the other hand, the question remains as to which test is the stringent test and which test should be used for a particular type of nominal and ordinal data. In response to this specific question numerous studies have been conducted, e.g. B. Haberman (1981), Lawal and Uptong (1990), Yenigün, Székely et al. (2011), Berry and Mielke Jr (1988), Mature and Elsayigh, (2010), Assad, (2012), Lipsitz, Fitzmaurice et al. (2015), Sulewski (2017) and Islam, & Rizwan, M. (2020). There is still no consensus on the stringent tests for categorical data and the studies have been criticized for developing of new modified tests and performance methods. Mature and Elsayigh use standard error criteria for performance in their study, while some other researchers used size and power analysis techniques (Sulewski 2017).

There is limited literature on the tests of independence for ordinal data; However, most of these are limited in scope and do not come to the precise conclusion of finding the optimal test of independence. Charles H, (1961) discussed in his book the relative efficiency of four measures of correlation Pearson product-moment correlation, Kendal τ_a, τ_b and τ_c compared and found that Kendal Tau is more reliable. Selecting the most powerful test from the literature is a central problem in the social sciences and the primary question facing researchers is figuring out which test to use for available data.

2.2 Literature Summary and Research Gap

There are several tests of independence for nominal data in the literature which always lead to confusion when applied to real data sets. In the literature the study is associated with comparing tests of independence; many studies are found using pairwise comparisons but there is no consensus for universal comparisons of tests of independence for nominal and ordinal data using special techniques such as stringency criteria (SC). Let’s assume that there are two tests T_1 and T_2 for comparison of independence in CTs. For some alternatives, T_1 may perform better than T_2 and for some other alternative scenarios, T_2 the test may perform better than T_1. To solve this type of puzzle, Maxwell L King developed a technique, (1985) and further popularized by Zaman, (1996) to compare different tests and solved the above scenario problem known as Stringency Criteria (SC). The literature is silent about using a comparison of a large number of tests of independence for numerous DGP in $w \times k$ CTs.

In summary of the literature, it is noted that researchers have compared tests of independence for nominal and ordinal data for specific data-generating process rather than taking a variety of DGP. This study takes into consideration a variety of DGP in 2×2 and $w \times k$ order CTs. This study also contributes in literature related to ordinal data as limited literature is available comparing the tests of independence on ordinal data. Subsequently, this study examines the size and power properties of different tests of independence and evaluate the most stringent test for nominal data as well as the most powerful test for ordinal data.

CHAPTER 3

COMPUTATIONAL DETAILS OF TEST OF INDEPENDENCE FOR CATEGORICAL DATA

This chapter describes an overview of several tests of independence for nominal data in 2×2 and $w \times k$ CTs in section 3.1. Section 3.2 and 3.3 describes preliminaries of ordinal data and explains computational details of popular tests of independence/measure of correlation used in $w \times k$ CTs.

3.1 Tests of Independence for Nominal Data for $w \times k$ CTs

Statistical science has been enriched by many tests proposed in different periods as tests of independence in $w \times k$ CTs. We describe notations and formulas of some of the recent and well-known popular tests of independence concerning $w \times k$ CTs below.

3.2.1 Chi-Square (χ^2) Test Statistics

The chi-square statistics to examine the independence for X and Y has the following forms

$$\chi^2_{XY} = \sum_{i=1}^{w} \sum_{j=1}^{k} \frac{(n_{ij} - e_{ij})^2}{e_{ij}} \tag{3.17}$$

Where n_{ij}, is observed counts, e_{ij} is expected counts and the sign $\sum$ denotes sum over a row or a column. The statistics have an asymptotically (i.e. sample size $\rightarrow\infty$) follows chi-square distribution with $df = (w-1)(k-1)$ provided that the hypothesis H_o of the independence of X and Y is true.

3.1.2 Likelihood Ratio(G^2) Test Statistics

The likelihood-ratio test is an alternative to the Pearson chi-square test for testing the independence of row and column classifications in unordered CTs. The likelihood ratio test examines the independence for X and Y has the form for $w \times k$ CTs. [Sokal & Rohlf, 2012]

$$G_{XY}^2 = 2\sum_{i=1}^{w}\sum_{j=1}^{k} n_{ij} \ln(\frac{n_{ij}}{e_{ij}}) \tag{3.18}$$

Where n_{ij} are observed in the i_{th} row and j_{th} column. e_{ij} is the expected number of the i_{th} row and the j_{th} column. When the null hypothesis of the independence of X, Y variable is accepted. The statistics follow an asymptotic non-central chi-square distribution with $(k-1)(w-1)$ degree of freedom.

3.1.3 Fisher Exact Test Statistics (FES)

The Fisher exact test (Fisher, 1922) is also popular, independently developed by Irwin (1935), and known as the Fisher-Irwin (FI) test. The FI test is most applied to 2×2 CTs because it can be computationally time-consuming for tables bigger than 2×2. According to a study by Yates (1934) and shier, (2004); the test χ^2, Pearson is used when $e_{ij} \geq 1$ for each $= 1, \dots .w; j = 1, \dots, k,$, and when no more than 20% of the expected counts are less than 5. If the above-mentioned condition is not met, then the Fisher-Yates test can be used.

An extension of the Fisher-yates test for the tables $w \times k$ was proposed by Freeman and Halton (1951). If the null hypothesis $[H_0]$, the independence of X and Y variables is true, is the probability of a specific distribution of numbers in the table $w \times k$, for the determined marginal numbers and the symbols adopted in the 2×2 CTs, It is given by the formula (Kang 1999).

$$FES_{xy} = \frac{\prod_{i=1}^{w} n_{i.} \cdot \prod_{j=1}^{k} n_{.j}}{n!. \prod_{i=1}^{w} \prod_{j=1}^{k} n_{ij}} \quad (3.19)$$

Where $n_{ij} = i = 1,2 \ldots \ldots w, j = 1,2 \ldots \ldots k)$, We generate each table that is compatible with the given marginal totals, and calculate the exact probability "p" of each table, through the formula of Fisher, (1934) formula. The subroutine increment generates each of the possible CTs by computation of its simple exact probability, which is based on studies of Yates, (1934) and Fisher, (1934). If n_1, n_2, n_3 ..., are the totals of the row and m_1, m_2, m_3 ..., are the totals of the column, anda_1, a_2, a_3 ..., are the array's elements, and "G" represents the total, then

$$FES_{xy} = \frac{n_1!\, n_2!\, n_3!\, n_4! \ldots \ldots \ldots.\ \ m_1!\, m_2!\, m_3!\, m_4! \ldots \ldots.}{G!\, a_1!\, a_2!\, a_3!} \quad (3.21)$$

Apart from the challenge of logical design, there is also the challenge of lengthy computation. In fact, the original papers give simulated critical values (SCV) instead of the critical values (CV) based on any standard distribution. Therefore, it is assumed that there is no standard distribution of fisher exact test.

3.1.4 Neyman Modified Chi-Square Test Statistics (NMCS)

The Neyman modified Chi-Square statistics for $w \times k$ CTs has the following computational form; [Neyman 1949]

$$NMCS_{XY} = \sum_{i=1}^{w} \sum_{j=1}^{k} \frac{(n_{ij} - e_{ij})^2}{n_{ij}} \quad (3.22)$$

Where n_{ij} are observed counts in the i_{th} row and j_{th} column. e_{ij} is expected counts of the i_{th} row and the j_{th} column. When the null hypothesis of the independence of X, Y variable is true. The test statistics are nonparametric and do not follow any standard or known distribution (Sulewski, P., & Motyka, R. 2015)

3.1.5 Kullback and Liabler Test Statistics (KLS)

The Kulback and Liaber test statistics for $w \times k$ CTs for two variables X and Y have the following computational form; [Kullback 1959]

$$KLS_{XY} = 2\sum_{i=1}^{w}\sum_{j=1}^{k} e_{ij}(\frac{e_{ij}}{n_{ij}}) \tag{3.23}$$

Where n_{ij} are observed counts in the i_{th} row and j_{th} column. e_{ij} is expected counts of the i_{th} row and the j_{th} column. When the null hypothesis of the independence of X, Y variable is true. The test statistics are nonparametric and do not follow any standard or known distribution. Sulewski, P., & Motyka, R. (2015)

3.1.6 Freeman and Tuckey Test Statistics (FTS)

The Freeman and Tuckey test for higher order CTs for two variables X and Y has the following computational form: [Freeman, Tukey 1950]

$$\mathrm{FTS_{XY}} = 4\sum_{i=1}^{W}\sum_{j=1}^{K}(\sqrt{n_{ij}} - \sqrt{e_{ij}})^2 \tag{3.24}$$

Where n_{ij} are observed counts in the i_{th} row and j_{th} column. e_{ij} is expected counts of the i_{th} row and the j_{th} column. When the null hypothesis of the independence of X, Y variable is true. The statistics have an asymptotic non-central chi-square distribution with $(w-1)(k-1)$ degree of freedom.

3.1.7 Cressie and Read Test Statistics (CRS)

The computational form of Cressie and Read (CR) test for two variables X and Y are stated below; [Cressie, Read 1984]

$$\mathrm{CRS_{XY}} = \frac{9}{5}\sum_{i=1}^{w}\sum_{j=1}^{k} n_{ij}[\left(\frac{n_{ij}}{e_{ij}}\right)^2 - 1] \tag{3.25}$$

Where n_{ij} are observed counts in the i_{th} row and j_{th} column. e_{ij} is expected counts of the i_{th} row and the j_{th} column. When the null hypothesis of the independence of X, Y variable is true. The statistics follow an asymptotic non-central chi-square distribution with $(w-1)(k-1)$ degree of freedom.

3.1.8 D Square (D^2) Test Statistics (DST)

The D - Squared (D^2) test which has been developed by Zelterman, (1987) has the following computational form for $w \times k$ CTs are sated below.

$$D^2{}_{xy} = \sum_{i=1}^{w}\sum_{j=1}^{k}\frac{(n_{ij}^* - e_{ij}^*)^2 - n_{ij}^*)}{e_{ij}^*} \tag{3.26}$$

Where, n_{ij}^* are observed in the i_{th} row and j_{th} column. n_{ij}^* are expected numbers of i_{th} row and j_{th} column. When the null hypothesis H_0 about the independence of X and Y is accepted then $D^2{}_{xy}$ has an asymptotic non-central chi-square distribution with $df = (w-1)(k-1)$.

3.1.9 Modular Test Statistics |χ| (MDS)

Sulawesi, (2013) proposed $|\chi|$ test which is the modification of chi-square tests and is given by.

$$MDS|\chi|_{XY} = \sum_{i=1}^{w}\sum_{j=1}^{k}\left|\frac{n_{ij} - e_{ij}}{e_{ij}}\right| \tag{3.27}$$

The test statistics follow the chi-square distribution. Where, n_{ij} are observed in the i_{th} row and j_{th} column. e_{ij} are expected numbers of i_{th} row and j_{th} column. When the null hypothesis ‘ H_0’ about the independence of X and Y is true then $|\chi|_{XY}$ has an asymptotic non-central chi-square distribution with a degree of freedom $= (\mathrm{w}-1)(\mathrm{k}-1)$.

3.1.10 Logarithmic Minimum Square Test (LMS)

LMS tests have the following computational form for $\mathrm{w} \times \mathrm{k}$ CTs are sated below.

$$\mathrm{LMS_{XY}} = -\sum_{i=1}^{W}\sum_{j=1}^{K} \ln\left[\frac{\min(\mathrm{n_{ij}}, \mathrm{e_{ij}})}{\max(\mathrm{n_{ij}}, \mathrm{e_{ij}})}\right] \tag{3.28}$$

The above LMS formula shows that $n_{ij} \neq 0$ and $e_{ij} \neq 0$ for each $\mathrm{i} = 1 \ldots \ldots \mathrm{w}$: $\mathrm{j} = 1 \ldots \ldots \mathrm{k}$, as a result, the size of the sample cannot be too narrow to calculate the power of the test for different scenarios stated in Chapter 4. Since it is known that resampling must reflect the null hypothesis. This is important to resample the CT, if $p_{ij} = p_{i+}p_{+j}$ holds. When testing the independence for two categorical variables, Amiri and von Rosen (2011) and Lin et al. (2015) used the expectations of cells under the null hypothesis: H_o: $e_{ij} = \frac{n_{i+}n_{+j}}{n}$.

We can convert the cell counts of the $\mathrm{w} \times \mathrm{k}$ CTs $(n_{11,} \ldots \ldots n_{1k,}, \ldots \ldots;\ n_{w1} \ldots, n_{wk})$ to $(n_{1,}n_{2,}, \ldots \ldots, n_N)$ where $n_{u,}$ are the n_{ij} values indexed row by row. A new variable for each cell is $Z = \left(n_{1,}n_{2,} \ldots \ldots z_{N,}\right)^t$ and the associated probabilities are $p = \left(p_{1,}p_{2,} \ldots \ldots p_{N,}\right)^t$, for a given CT, the variable Z and probabilities p, we can write this as $Z \sim Multi\ (n, p)$.

Let $Z = (z_1 = n_1, z_2 = n_2 \ldots \ldots z_N)$ be a multinomial sample with $\sum_{i=1}^{N} n_1$, estimates of the sample proportions are $\breve{p}$= $\breve{p}_1, \ldots \ldots \ldots \ldots \breve{p}_N)$, where $\breve{p}_j = \frac{n_j}{n}$, the bootstraps resample is defined as sampling with replacement with elements of z with size n. the bootstraps estimates of the proportions are $\hat{p}$= $\hat{p}_1, \ldots \ldots \ldots \ldots . \hat{p}_N$), where $\hat{p}_i$= $\frac{\hat{n}_1}{n}$. A modified Logarithmic minimum square test (LMS_m) is to be used in a case if there is zero in the cells. Sulewski, P. (2019).

$$\mathrm{LMS_m} = -\sum_{i=1}^{W}\sum_{j=1}^{K} \ln\left[\frac{\min\left(n_{ij}, e_{ij}\right)}{\max\left(n_{ij}, e_{ij}\right)} + 0.00001\right] \tag{3.29}$$

3.1.11 BP Tests Statistics (BPS)

Amiri and Modarres, (2017) proposed the BP test of independence using a test statistic for the bootstrap sample defined as

$$\mathrm{BPT_{XY}} = n(\hat{p} - p_0)^t A(\hat{p} - p_0) \tag{3.30}$$

Where P_o is calculated under H_o: $p_{ij,} = p_{i+,}p_{j+}$,$\sum p = Diag(p) = P^t p$, $A = \sum p^{-1}$ $and\ p\ is$ the vector of observed proportions. Since the inverse of $\sum p$ does not exist (det ($\sum p) = 0$)), therefore, we used the Moore – Penrose[2] generalized inverse which has been used previously in literature. Sulewski, P. (2019).

3.2 Independence in Ordinal data

Ordinal data can take different forms; For example, one can measure students' height and weight and calculate the correlation between pairs of measurements. Both height and weight are continuous variables and do not fall under the category of categorical variables. However, researchers often measure these variables in different intervals. One can ask for the range instead of the exact height such as (taking the most appropriate height i.e., 50-55, 55-60, 65-70, etc.,). Such intervals have natural ordering

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[2] Moore-Penrose is a linear algebra technique used to approximate the inverse of non-invertible matrices. This technique can approximate the inverse of any matrix, regardless of whether the matrix is square or not. In short, Pseudo-inverse exists for all matrices. If a matrix has an inverse, its pseudo-inverse equals its inverse.

and may be taken as discrete variables having a proper rank. The results obtained can be replaced by ranks and the correlation between pairs of ranks can be computed.

Rank is the sequence number of the statistical observation in the sample after the observations have been ordered by the value of one of the variables. Usually, an ascending ordering and numbering from 1 are used. Replacing a variable with its ranks is an operation called ranking. In the case of observations with an equal value of the ranked variable (so-called linked ranks), usually, all these observations are assigned the same rank, which is the average of their sequential numbers. Therefore, ranks cannot have integer values. If $x_1 = 2;\ x_2 = 1;\ x_3 = 4;\ x_4 = 4,$ it's after sorting ; $x_2 = 1; x_1 = 2;\ x_3 = 4;\ x_4 = 4,$ Then the rank has the form ; $r_2 = 1; r_1 = 2;\ r_3 = 3{,}5;\ r_4 = 3,5,$ and after restoring the original order, $r_1 = 2;\ r_2 = 1;\ r_3 = 3{,}5;\ r_4 = 3,5.$

In many situations where ranks are used, it is not possible to obtain numerical measures (e.g., ranking students in terms of their degree of social adoption). Ranks have been used in correlation studies for many years, but they are also used for many other purposes, such as in tests that compare two correlated or independent samples. Ranks are expressed in terms of natural numbers 1, 2 …. N and identify with symbols $X_1, X_2, \dots \dots .. X_n$. The sum of these numbers and the sum of their squares is written as follows:

$$\sum_{i=1}^{N} X_w = \frac{N(N+1)}{2} \tag{3.31}$$

$$\sum_{i=1}^{N} X_w^2 = \frac{N(N+1)(2N+1)}{6} \tag{3.32}$$

Average and variance of numbers $1,2, \dots \dots . N$ is.

$$\overline{X} = \frac{N+1}{2}, \qquad s^2 = \frac{(N-1)(N+1)}{2.6} \tag{3.33}$$

For, r_x , r_y condition of independence is $\pi_{wk} = \pi_w \pi_k$ implies that Cov (r_x , r_y) =0.

3.2.1 Inversion Factors

N' units are ranked by X and Y traits. X ranks are denoted by $X_1, X_2, X_3 \ldots \ldots X_N$ and the ranks in the range of Y are denoted by $Y_1, Y_2, Y_3 \ldots \ldots Y_N$. One of the inversion factors is the sum of the squared differences between the pairs of ranks.

$$d^2 = \sum_{i=1}^{N}(X_i - Y_i)^2 \tag{3.34}$$

This quantity takes on a minimum value of zero if the items in the range of both variables are in the same order. If the pairs of ranks are in the reverse order, then the measure takes the maximum value equal to

$$d_{max}^2 = \frac{N(N^2-1)}{3} \tag{3.35}$$

Another inversion factor is the 'S' statistic. If the ranks for variable X are in ascending order, then the ranks for variable Y show some degree of inversion concerning X. To compute the 'S' statistic, each rank for variable Y compares with all other ranks. If the pair of ranks is in ascending order, the value of the S statistic increases by 1. If the pair of ranks is in decreasing order, the value of the S statistic decreases by 1. This statistic is the sum of such with $N(N-1)/2$ comparisons. When sets of ranks are arranged in ascending order, the measure S takes the maximum value equal to

$$S_{max} = \frac{N(N-1)}{2} \tag{3.36}$$

Rearranging data into descending will not change the value of S.

3.3 Tests of Independence for Ordinal Data in 'W × K CTs

We have taken popular tests of independence[3] ordinal data in $w \times k$ CTs which are described below.

3.3.1 Spearman's Rank Correlation Test ($\boldsymbol{\rho}$)

Spearman's rank correlation test is a non-parametric test/measure of strength and direction of association that exists between two variables measured as an ordinal scale. It is denoted by a symbol r_s and Greek letter ρ. The inversion measure is presented in 3.3.1 in the definition of spearman's ρ coefficient. It is identical to the Pearson correlation coefficient calculated for ranks. Which is used to describe the strength of the correlation of two variables, especially when they are qualitative. When the number of observations is small, it can be used to examine the relationship between quantitative variables by prior ranking. Spearman's ρ is described by the formula.

$$\rho = r_s = 1 - \frac{6d^2}{N(N^2 - 1)} \tag{3.37}$$

Where, d^2 is the measure of inversion, and N - is the number of observations (sample size).

Based on the "n" of a sample taken from the population, the null hypothesis is that the spearman ρ coefficient is zero i.e., $(H_0 : \rho_s = 0)$ against the alternative hypothesis $H_1 : \rho_s \neq 0 \vee H_1 : \rho_s > 0 \vee H_1 : \rho_s < 0$.

[3] Tests of independence are same as measure of correlations for ordinal data. The study of seven well known tests of independence/measure of correlations for ordinal data is carried out in this study.

3.3.2 The Kendall τ-a coefficient

The τ-a Kendall coefficient is used only in cases where the so-called tied (related) ranks. Linked pairs occur when not all observations have the same values and respondents cannot be strictly ordered by the value of a given variable. A pair is said to be linked if the same value (rank) is observed for one or both variables. The relation can be due to variable X, variable Y, or both. For two-way tables, all cases falling into the same category of one variable (row or column) are related to each other.

There are five types of pairs:

$$N_c + N_d + T_x + T_y + T_{xy} = \frac{n(n-1)}{2} \tag{3.38}$$

If the difference $N_c - N_d$ is divided by the number of all pairs in the 'N' element set, the coefficient proposed by Kendall in the form Kendall, (1938).

$$\tau_a = \frac{N_c - N_d}{\binom{n}{2}} = \frac{2(N_c - N_d)}{n(n-1)} \tag{3.39}$$

If the empirical data is written in the form of $\mathrm{w} \times \mathrm{k}$ CT and the categories of this table are ordered, then.

$$N_c = \sum_{i=1}^{w} \sum_{j=1}^{k} n_{ij} C_{ij} \tag{3.40}$$

$$C_{ij} = \sum_{a=1}^{i-1} \sum_{b=1}^{j-1} n_{ab} + \sum_{a=i+1}^{w} \sum_{b=j+1}^{k} n_{ab} \tag{3.41}$$

$$N_d = \sum_{i=1}^{w} \sum_{j=1}^{k} n_{ij} D_{ij} \tag{3.42}$$

$$D_{ij} = \sum_{a=1}^{i-1} \sum_{b=j+1}^{k} n_{ab} + \sum_{a=i+1}^{w} \sum_{b=1}^{j-1} n_{ab} \tag{3.43}$$

If the numbers of matched pairs are denoted by N_c and unmatched pairs by N_d ; then the difference between $N_c - N_d$ is the difference between the matched pairs and unmatched pairs. In case of $N_c - N_d > 0$, the relationship is positive while in the case of $N_c - N_d < 0$ the relationship is negative.

3.3.3 The Kendall coefficient of τ-b

The τ-b coefficient proposed by Kendall has the following computational form.

$$\tau_b = \frac{N_c - N_d}{\sqrt{(N_c - N_d + T_x)((N_c - N_d + T_y)}} \tag{3.44}$$

Its popularity is because it takes on values close to Pearson's linear correlation coefficient, especially when the number of categories for each of the analyzed variables is not less than 5. The τ-b coefficient is symmetric, takes values from the interval, but takes extreme values only for square tables. It is the geometric mean of the two asymmetric Somer’s D coefficients.

$$\tau_b = \pm\sqrt{d_{yx} - d_{xy}} \tag{3.45}$$

If the empirical data is written in higher order CT, then the following formula is used for computation;

$$\tau_b = \frac{N_c - N_d}{\sqrt{D_w - D_k}} \tag{3.46}$$

3.3.4 Kendall Stuart τ-c coefficient

The Kendall Stuart τ-c coefficient proposed by Kendall and Stuart for CTs has the following computational form stated by Kendall and Stuart, (1973)

$$\tau_c = \frac{2m(N_c - N_d)}{n^2(m-1)} = m = \min(w,k) \qquad (3.47)$$

It was designed specifically for tables and can formally take values from $(-1\ to + 1)$. Interpreting its size is difficult as it is strongly dependent on the size of the table. The τ-c coefficient is symmetrical and has no proportion reduction error (PRE)[4] interpretation.

3.3.5 Goodman – Kruskal Gamma (γ)

The γ coefficient proposed by Goodman and Kruskal does not consider bonded pairs, it can be computed from the following formula, Goodman and Kruskal, (1954).

$$\gamma = \frac{N_c - N_d}{N_c + N_d} \qquad (3.48)$$

This coefficient is symmetric and takes on values from the range. Values close to zero indicate that there is no or only a weak relationship between the variables, values close to | 1 | mean a strong dependence. Gamma can be used as a test of independence using a Z score where the null hypothesis is H_1 = no association against the alternative hypothesis of $H_1 =$ there is an association amongst the variables.

3.3.6 Sommers's coefficient

The Somers Delta or "d" coefficient proposed by summer's taking into account bonded pairs has the form; (Somers 1962).

(Y - Dependent variable)

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[4] Proportion Reduction Error (PRE) is predicting the ordering of unrelated pairs with respect to the independent variable in CTs.

$$d_{y\,|x} = \frac{N_c - N_d}{N_c + N_d + T_y} \tag{3.49}$$

(X - Dependent variable)

$$d_{x\,|y} = \frac{N_c - N_d}{N_c + N_d + T_x} \tag{3.50}$$

The "d" coefficient is asymmetric, its size depends on which variable is dependent. Comparing it with the coefficient γ, it was found that it does not reach an absolute value greater than γ. It takes values from the interval. If the number of columns is greater than the number of rows, it does not get the value 1, because then there are connections due to Y. Likewise (X is a dependent variable) it does not get the value 1 when the number of rows is greater than the number of columns.

Somer's also used the following formula for symmetrical variants

$$d_s = \frac{N_c - N_d}{N_c + N_d + 0{,}5\left(T_x + T_y\right)} \tag{3.51}$$

(Y - Dependent variable)

$$d_{y|x} = \frac{N_c - N_d}{D_w} \tag{3.52}$$

(X - Dependent variable)

$$d_{x|y} = \frac{N_c - N_d}{D_k} \quad , \quad d_s = \frac{N_c - N_d}{0{,}5\left(D_w + D_k\right)} \tag{3.53}$$

$$D_w = n^2 - \sum_{i=1}^{w} n_{i.}^2 \tag{3.54}$$

$$D_k = n^2 - \sum_{j=1}^{k} n_{\bullet j}^2 \ , \tag{3.55}$$

D_w, D_k are the coefficients, Somers's coefficient also has an interpretation in terms of PRE and is analogous to the interpretation of coefficient γ. The difference is that proportional error reduction is about predicting the ordering of unrelated pairs with respect to the independent variable. The coefficient factor d_{yx} can be interpreted as the probability that random observation 'j' ranks higher/lower and variable 'Y' when it ranks higher on variable 'X.'

3.3.7 Novel Phi_k (Φ_k) Correlation

The Novel ϕ_k correlation is useful for assessing the association between nominal, ordinal, ratio, and interval variables. This has the specialty that it does not only capture the linear association but nonlinear association as well in CTs.

The calculation of correlation coefficients between paired data variables is a standard tool of analysis for every data analyst. Pearson's correlation coefficient is a de facto standard in most fields, but by construction only works for interval variables (sometimes called continuous variables). Pearson is unsuitable for data sets with mixed variable types, e.g., where some variables are ordinal or categorical.

While many correlation coefficients exist, each with distinctive features, we have not been able to find a correlation coefficient with Pearson-like characteristics and a sound statistical interpretation that works for interval, ordinal, and categorical variable types alike.

The correlation coefficient ϕ_k follows a uniform treatment for interval, ordinal and categorical variables, captures non-linear dependencies, and is like Pearson's correlation coefficient in the case of a bivariate normal input distribution.

Visualizing the dependency between variables can be tricky, especially when dealing with (unordered) categorical variables. To help interpret any variable relationship found, we provide a method for the detection of significant excesses or deficits of records with respect to the expected values in a contingency table, so-called outliers, using a statistically independent evaluation for the expected frequency of records, accounting for the uncertainty on the expectation. We evaluate the significance of each outlier frequency in a table and normalize and visualize these accordingly. The resulting plots we find to be valuable to help interpret variable dependencies and work alike for interval, ordinal and categorical variables.

The Novel ϕ_k The correlation estimator is computed as:

Step 1	→	A w × k CT is created, filling of the CT for ordinal data or chosen variable pair, which contains N records, has w rows.
Step 2	→	Evaluate the χ^2 contingency test using Pearson's χ^2 test statistic
Step 3	→	Interpret the χ^2 value as coming from results and if $\chi^2 < \chi^2_{pre}$, set $\rho = 0$.
Step 4	→	Else, with fixed N, w, k, χ^2 invert the function and solve numerically for the rho value. The solution for ρ defines the correlation coefficient Novel Φ_k.

CHAPTER 4

METHODOLOGY

This chapter presents the methodology and procedure used to compare tests of independence for nominal and ordinal data. Section 4.1 explains the simulation design, Data Generating Process (DGP), Computation of size distortion (SD), Computation of finite sample critical values (FSCV), Power envelope, Maximum likelihood, Power analysis, and SC for selection of most stringent test of independence for nominal data. Section 4.2 discusses the methodology for the most stringent test of independence/measure of correlation for ordinal data using PC.

4.1 Methodology and Procedure of Tests of Independence for Nominal Data

The methodology for tests of independence in $w \times k$ CTs is discussed below:

4.1.1 Simulation Design

The core objective is to assess the performance of tests of independence for nominal data by comparing the power of tests using the stringency criteria (based on the power envelope). To achieve these objectives, the study focuses on Monte Carlo Simulations (MCS). We have analyzed the performance of tests of independence using algorithms based on MCS and through SC select the most stringent tests of independence for nominal in $w \times k$ CTs.

The proposed methodology consists of the following three steps:

a. Data generating process (DGP)
b. Calculation of finite sample critical values (FSCV)
c. Power curve, power envelope, and stringency criteria (SC)

The sub-objective of the simulation experiment is to find out the size and power properties of tests of independence for nominal data. Therefore, we need several DGP in 2×2 and $w \times k$ dimensions[5] CTs. The selection of the DGP for the MCS study is particularly important mostly in the comparative analysis. The tests or approaches can be compared in the same framework to recommend the superiority of one test or the weakness of another test. Fig 4.1 shows the simulation design for the present study.

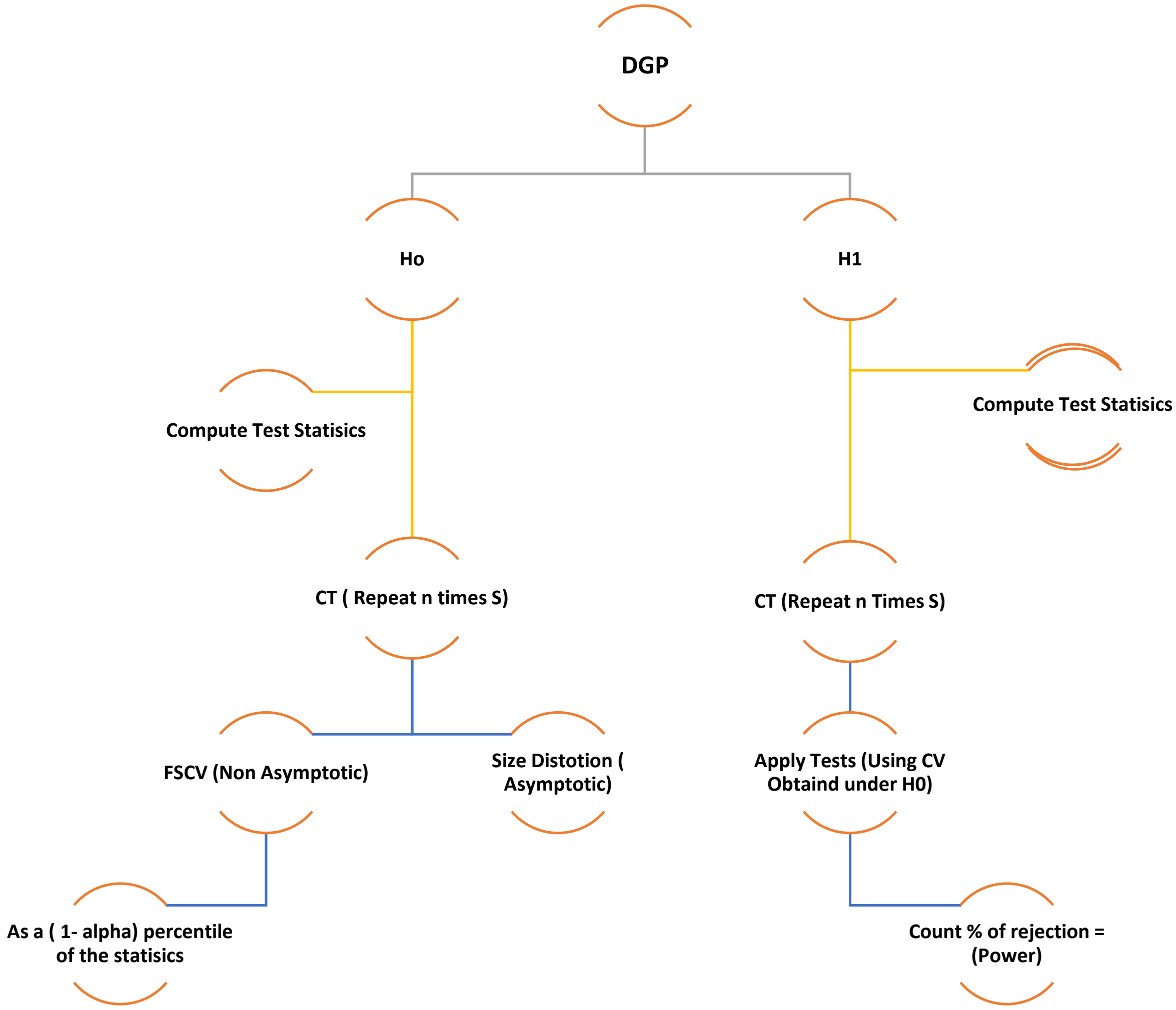

Figure 4.1: Simulation design for categorical data

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For simulation purposes, we construct a sample of nominal random variables that cover conditions of independence. Suppose we want to generate a random number having theoretical distribution shown in Table 1.3. The row probabilities of each random number

[5] The dimensions chosen by this dissertation covers the dimension i.e., 2×2 CT and for 3×3 CT, used by earlier studies. In addition to that we have added some new dimensions i.e., 2×3 CT , 3×2 CT , 4×4 CT , 5×5 CT 6×6 CT and 12×12 CTs which provides sufficient space for GENERALIZATION.

are given by $\pi_{1.}, \pi_{2.} \dots \dots \dots . \pi_{w.}$ whereas column probabilities are given by $\pi_{.1}, \pi_{.2} \dots \dots \dots \dots \pi_{.k}$. The procedure is as follows, Take a contingency table of W*K having all zeroes.

a) Let's Generate "X" such that X ~ U [0, 1] and define X' as follows.

Take $x' = 1$ if $x \leq \pi_{1.}$

$x' = 2$ if $x > \pi_{1.}$ and $x < \pi_{1.} + \pi_{2.}$

: : : :

: : : :

: : : :

$x' = W$ if $x > \pi_{1.} + \pi_{2.} \dots \dots . . \pi_{w-1.}$

b. Similarly, generate "Y" such that Y ~ U [0, 1]

Take $y' = 1$ if $y \leq \pi_{.1}$

$y' = 2$ if $y > \pi_{.1}$ *and* $< \pi_{.1} + \pi_{.2}$

: : : :

: : : :

: : : :

: : : :

$y' = K$ if $y > \pi_{.1} + \pi_{.2} + \dots \dots . . \pi_{.k-1}$

c. Adding 1 to row x' and column y' .

d. Repeating step 1 n times to get a contingency table with n data points.

4.1.2 Computation of Size Distortion in CTs

The following steps are involved in calculating size distortion.

a) Generate data under H_0.

b) Arrange data in 2×2 or $w \times k$ contingency tables.

c) Calculate test statistics (selected one of eleven tests taken under this study).

d) Use asymptotic critical values (ACV) to Accept/Reject.

e) Repeat 20,000 (MCS)[6] times, Count% rejection probability, distortion is $(p - \alpha)$ where, "p" is actual rejection probability and "α" is a nominal size. If size distortion is greater than 0, calculate 95%, percentile.

4.1.3 Computation of Finite Sample Critical Values in CTs

The following steps are involved in the calculation of finite sample critical values.

a) Generate data under H_0

b) Arrange data in 2×2 or $w \times k$ contingency tables.

c) Calculate test statistics (Selected one of eleven tests taken under this study).

d) Repeat 20,000 times (MCS).

e) Critical Value is$(1 - \alpha)$ percentile of the tests statistics obtained.

4.1.4 Computation of Power in CTs

a) Generate data under H_1 with pre-specified MoU.

b) Arrange data in 2×2 or $w \times k$ contingency tables.

c) Calculate test statistics, and decide acceptance/rejection using ACV / FSCV.

d) Repeat steps a, b, and c "20,000" times (MCS) and calculate power = % of rejections.

[6] The replications involves so many regressions and millions of calculations are needed just to complete one replication. There are so many scenarios presented in dissertations for 18 tests of independence used for nominal and ordinal data. Since for each scenario I needed such calculations. The total arithmetic's needed to do the analysis becomes in billions, therefore even with heavy duty computers, it is problematic to increase MCSS.

4.1.5 Computation of Maximum Likelihood Ratio Test

Maximum likelihood estimation is a systematic technique for estimating parameters in a probability model from a data sample. Suppose a sample $x_1, x_2, x_3 \dots\dots\dots.. x_n$ has been obtained from a probability model specified by a contingency table; then the maximum likelihood estimate is produced as follows for higher order contingency table.

Let we have observed the table under H_0 shown in equation 4.1.

$$Z = \begin{pmatrix} n_{11} & n_{12} \dots\dots\dots\dots & n_{1w} \\ n_{21} & n_{21} \dots\dots\dots\dots & n_{2w} \\ n_{31} & n_{32} \dots\dots\dots\dots & n_{3w} \end{pmatrix} \tag{4.1}$$

Then the general n × k becomes

$$n = \sum n_{wk} \tag{4.2}$$

Then the probabilities under H_0 becomes i.e., the Theoretical table shown in equation 4.4

$$Z_1 = Z/_n \tag{4.3}$$

$$Z_1 = \begin{pmatrix} \pi_{11} & \pi_{12} \dots\dots\dots\dots & \pi_{1w} \\ \pi_{21} & \pi_{22} \dots\dots\dots\dots & \pi_{2w} \\ \pi_{31} & \pi_{32} \dots\dots\dots\dots & \pi_{3w} \end{pmatrix} \tag{4.4}$$

Suppose we have theoretical probabilities as defined in Table 1.3. The likelihood under H_1 can be written as below:

$$\text{Liklihood} = \frac{\binom{n}{n_{11}}\binom{n}{n_{12}}\dots\dots\dots\dots\binom{n}{n_{wk}}\ (\pi_{11})^{n_{11}}\ (\pi_{12})^{n_{12}}}{(\pi_{wk})^{n_{wk}}} \tag{4.5}$$

$$Liklihood = \prod_{i=1}^{W}\prod_{j=1}^{K}\binom{n}{n_{ij}}(\pi_{ij})^{n_{ij}} \tag{4.7}$$

$$Log\ Liklihood = \sum_{i=1}^{W}\sum_{j=1}^{K} Log\ \binom{n}{n_{ij}} + \sum_{i=1}^{W}\sum_{j=1}^{K} Log\ (\pi_{ij})^{n_{ij}} \tag{4.8}$$

Using equation 4.8 we calculated the maximum likelihood under H_0.

4.1.6 Measurement of Untruthfulness (MoU)

MoU is the measure of deviation from the condition of independence and is denoted by the symbol $'\theta'$ in this dissertation. Sulewski, (2017) proposed MoU for W*K CTs defined as:

$$MoU = \sum_{i=1}^{W}\sum_{j=1}^{K} | \pi_{ij} - \pi_{i+}\pi_{+j} | = \theta \quad (4.9)$$

Replacing theoretical probabilities with empirical ones we obtain MoU as

$$\text{MoU} = 1/n \sum_{i=1}^{W}\sum_{j=1}^{K} | n_{ij}^{*} - \frac{n_{i+}^{*}n_{j+}^{*}}{n} | = 1/n \sum_{i=1}^{W}\sum_{j=1}^{K} | n_{wij}^{*} - e_{ij}^{*} \quad (4.10)$$

The MoU takes values in (0, 2), and is applied in Monte Carlo Simulation.

4.1.7 Power Envelope Curve and Stringency Criteria (SC)

The power curve is the graph of power plotted against the measure of untruthfulness (MoU). For each test of independence when we calculate critical values and draw the power curve taking different alternatives θ_i on the X-axis and power of point optimal test on the y-axis that is the plot of (θ_i, T^m) where,(T_θ^m) is the maximum power that is attained by the approximate point optimal test. Then we calculate the shortcomings of the numerous tests of independence through stringency criteria.

Consider tests $T^1, T^2, T^3 \dots, T^M$ with power function (T^m, θ), m = 1, 2,…., M, that depends on θ, the degree to which the null hypothesis is violated. At each value of θ, find out the test with maximum power to produce the envelope function

$$\text{S}(\theta) = \max_m \{P\left(T^m, \theta\right), \ m = 1,2, \dots, \text{M}, \} \quad (4.11)$$

For each test, find the largest “Shortcoming” defined as

$$\text{D}(T^m) = \max_0 \{S(\theta) - P(T^m, \theta)\} \qquad m = 1,2, \dots, \text{M}, \quad (4.12)$$

The most stringent test T^* is that which minimizes the maximum shortcoming. That is,

$$T^* = arg\min_{T^m}\{D(T^m),\ \ \mathrm{m}\ =\ 1,2,\ldots,\mathrm{M},\}$$

We identified the test with minimum shortcomings which is the most stringent test for nominal data and with maximum shortcomings are considered the poorest tests for nominal data.

4.1.8 Construction of Scenarios in W×K CT

Let X and Y be two variables of the same object having levels X_1, X_2, X_3 and Y_1, Y_2, Y_3. Testing for independence of these two variables with suitably arranged in two way and higher order CTs with different scenarios[7] are presented in Table 4.1 and 4.2. If row 2 is scalar multiple of row 1, then we have independence. The dependency in CTs can be drawn by adding / subtracting same scale to a row so that $\mathrm{r}_2 = \mathrm{ar}_1$. “a” is chosen such that MoU becomes at desired level in W×K CT.

Table 4. 1: Scenario of 2×2 Contingency Table

Scenario I			Scenario – II		
	Y1	Y2		Y1	Y2
X1	π_{11}	$\pi_{12} - a/2$	X1	$\pi_{11} - a$	π_{12}
X2	$\pi_{21} + a$	$\pi_{22} - a/2$	X2	π_{21}	$\pi_{22} + a$
Scenario – III			**Scenario – IV**		
	Y1	Y2		Y1	Y2
X1	$\pi_{11} + a$	$\pi_{12} - a/2$	X1	$\pi_{11} - a$	π_{12}
X2	π_{21}	$\pi_{22} - a/2$	X2	$\pi_{21} + a$	π_{22}
Scenario – V					
	Y1	Y2			
X1	$\pi_{11} - a$	$\pi_{12} + a$			
X2	$\pi_{21} + a$	$\pi_{21} - a$			

(Author’s Source)

[7] We created many scenarios in the above procedure for 2×2 CT , 2×3 CT , 3×2 CT , 3×3 CT , 4×4 CT , 5×5 CT 6×6 CT and 12×12 CTs. Table 4.1 and 4.2 describes scenarios for 2×2 CT and 3×3 CT.

Table 4. 2: Scenario of 3×3 Contingency Table

Scenario – I				Scenario – II			
	Y1	Y2	Y3		Y1	Y2	Y3
X1	$\pi_{11} - a$	$\pi_{12} - a/2$	π_{13}	X1	$\pi_{11} - a$	π_{12}	$\pi_{13} + a$
X2	$\pi_{21} - a/2$	π_{22}	$\pi_{23} + a/2$	X2	$\pi_{21} - a/2$	π_{22}	$\pi_{23} + a/2$
X3	π_{31}	π_{32}+ a /2	$\pi_{33} + a$	X3	$\pi_{31} - a$	π_{32}	$\pi_{33} + a$
	Scenario – III			Scenario – IV			
	Y1	Y2	Y3		Y1	Y2	Y3
X1	$\pi_{11} - a$	π_{12}	$\pi_{13} + a$	X1	π_{11}	$\pi_{12} - a/2$	π_{13}
X2	π_{32}+ $a/2$	π_{22}	π_{32}- $a/2$	X2	$\pi_{21} - a$	π_{22}	$\pi_{23} + a$
X3	$\pi_{31} + a$	π_{32}	$\pi_{33} - a$	X3	$\pi_{31} + a$	π_{32}+ $a/2$	$\pi_{33} - a$

Scenario – V			
	Y1	Y2	Y3
X1	$\pi_{11} - a$	π_{12}	$\pi_{13} - a/2$
X2	π_{21}	π_{22}	π_{23}
X3	$\pi_{31} + a/2$	π_{32}	$\pi_{33} + a$

(Author's Source)

4.2 Methodology for Power analysis of Tests of independence for Ordinal data

In addition, above to achieve most stringent test of independence / measure of correlation for ordinal data. This study seven popular tests of independence/measures of correlations based on Power.

Let X be a random variable which can be ordered into K categories. The variable can be generated as

$$x1 \sim\ U(0, k)$$

Then X becomes categorical random variable

$$\text{X} = \text{round}\ (x_1) \qquad (4.13)$$

Let Y is another variable which is dependent on X be generated as

$$Z_1 =\ U(0, k2) \qquad (4.14)$$

Where k_2 is numbers of categories in Y.

Suppose $\qquad y = ax_1 + bz_1 \qquad$ where a+b = 1, $\qquad$ (4.15)

Then $\qquad$ Y = round (y_1)

The equation 4.15 can give us perfectly correlated variables when a = 1 and b= 0 and perfectly independent when a = 0 and b=0.

Thus, correlation is determined by a, b (a+b=1) where a=1 and b=1 then there is perfect correlation and are independent.

4.2.2 Finite Sample Critical Values (FSCV) and Power

All the tests / Measure of correlations for ordinal data are non-parametric and critical values are calculated by simulations. Therefore, it is useless to calculate size distortion. However, power shall be calculated as described below.

4.2.3 Computation of Finite Sample Critical Values in CTs

The following steps are involved in calculation of finite sample critical values.

f) Generate data under H_0

g) Arrange data in w × k contingency tables.

h) Calculate tests statistics (Selected one of seven tests taken under this study).

i) Repeat 20,000' times (MCS).

j) Critical Value is$(1-\alpha)$ percentile of the tests statistics obtained.

4.2.4 Computation of Power in CTs

e) Generate data under H_1 with pre-specified MoU.

f) Arrange data in w × k contingency tables.

g) Calculate test statistics, decide acceptance / rejection using FSCV.

h) Repeat step a, b, and c "20,000" times (MCS) and calculate power = % of rejections.

CHAPTER 5

ANALYSIS OF SIZE OF TESTS FOR CATAGORICAL DATA

The core objective of the study is to evaluate the most stringent and powerful test of independence for nominal and ordinal data in $w \times k$ order CTs. To achieve sub objectives; section 5.1 describes size distortion. Section 5.2 and 5.3 explains computation of size distortion in 2×2 and $w \times k$ order of CTs for numerous sample sizes for nominal data. Moreover, section 5.4 discusses computation of Finite Sample Critical Values (FSCV) for tests of independence for nominal and ordinal data that do not follow any standard or known distribution. Finally, the chapter covers FSCV for tests of independence for ordinal data discussed in section 5.5.

5.1 Size Distortion as Measure of Performance

It is well well-known that powers of econometric tests are comparable if the size remain same, and so is the case with the selected eight mentioned below tests of independence for nominal data. Usually, when tests are to be compared, the process starts by finding out the critical values with fixed size, say nominal level (α) at 1%, 5% or 10%. These critical values are then applied to calculate power curves. Alternatively, we use ACV and SCV for asymptotic tests[8] to measure size distortion where the size of entire procedure can be calculated fixing the size at each single step that is at nominal level (α) at 5%. The test with minimum size distortion would be the optimal test. The best performance would be considered as of the procedure having minimum size

[8] Large sample tests often fails to behave well in small samples. However, we tested the size distortion of asymptotic tests and found very small distortion.

distortion and highest power. Finally, using SC we evaluate the most stringent test of independence for nominal data.

Let's alpha (α) be the size of a test then,

$$\alpha = P(Reject\ H_0/H_0\ is\ True)$$

In our case, the null hypothesis H_0: nominal variable is independent "x and y" and for calculation of size, the data is generated such that H_o is true against the alternative hypothesis $i.e.,$ "H_1". We also assumed that the size of complete process will be 1%, 5% & 10%. At the end, the difference between empirical size and the nominal size (1%, 5% and 10%) can be referred as size distortion. The results for various orders of CTs for tests of independence are given below:

5.2 Computation of Size Distortion (SD) and Simulated Critical Values (SCV) for Nominal Data in 2×2 CTs.

Through simulation and procedure adopted in chapter 4, empirical values/ (SCV) are computed for different level of $\alpha = 1\%$, 5% & 10% for various 2×2 and $w \times k$ CTs at different sample size (Small, Medium, and Large).

5.2.1 Computation of Finite Sample Critical Values for 2×2 CTs

Simulated critical values are produced for tests of independence for power computation when a test does not follow a standard or known distribution. The tests of independence namely, Fisher exact Statistics (FES), Neyman modified chi squared statistics (NMCS) and Kullback - Leibler Statistics (KLS) falls in category of not following any standard or known distribution. Therefore, simulated critical values (SCV) have been computed at various level of $\alpha = 0.01$, $\alpha = 0.05$ and $\alpha = 0.10$ for 2×2 CTs at different sample size (SS: small, medium, and large). The results are shown in Table 5.2.

Table 5. 1: SCV of Tests of independence for 2 × 2 CTs

Tests Name	**FES**					**NMCS**					**KLS**				
(α)	**α = 5%**					**α = 5%**					**α = 5%**				
Sample Size	25	50	100	200	400	25	50	100	200	400	25	50	100	200	400
CT 2 ×2	-.057	-.056	-.053	-.045	-.049	.030	.032	.058	.056	.042	.036	.037	.043	.044	.045

The last row in above Table 5.1 gives us simulated FSCV for Fisher exact Statistics (FES), Neyman modified chi squared statistics (NMCS) and Kullback - Leibler Statistics (KLS). The results show computations of SCV which are further used in computation of power to evaluate optimal tests of independence for nominal data.

5.2.2 Computation of Empirical Size of Tests of Independence for 2×2 CTs

We calculated empirical values for selected tests of independence at nominal level α = 0.05 for 2 × 2 CTs and found negligible size distortion at different sample size (SS: 25, 50,100, 200 and 400).

The results of panel – I indicates when nominal size is 1 % then χ^2 test, D^2 test and BPS have empirical value of .018 at sample size 25. , G^2 test and Modular test have empirical value .017 at sample size 25. FTS and CRS have empirical value i.e., .016 at sample size of 25. However, LMS test has empirical value .014 at sample size 25. The results further shows that when nominal size is 1% then χ^2 test, D^2 test and Modular test have empirical value .017 at sample size 50. LMS has empirical value .014 at sample size 50. Moreover, when nominal size is 1% then χ^2 test has empirical size .016, .015 and .012 at sample size of 100,200 and 400, respectively.

Panel - II indicates when nominal size is 5% then χ^2 test has empirical size .038, .051, .054, .047 and .052 at (SS: 25, 50,100, 200, 400). LMS test shows when nominal size is 5% then empirical size .06, .057, .04, .042 and .052 at (SS: 25, 50,100, 200,400).

Panel – III indicates when nominal size is 10% then χ^2 test has empirical size .107, .093, .104, .103 and .101 at sample size (SS: 25, 50,100, 200,400).The results of BPS indicates that when nominal size is 10% then BPS has empirical size of .122, .12, .09, .114 and .11 at (SS: 25, 50,100, 200,400).

We observed from Panel I-II and III that as the sample size increase the difference between nominal and empirical (simulated) critical value decreases or in order words the size reduces with sample size and same is true for others below mentioned tests of independence in the given tables for nominal data.

Table 5.2: Empirical size of tests[9] of independence of Nominal Data for 2×2 CTs

Tests \ Sample Size	Panel – I					Panel – II					Panel - III				
	Nominal Level (α = 0.01)					α = 0.05					α = 0.10				
	25	50	100	200	400	25	50	100	200	400	25	50	100	200	400
χ^2 Statistics	.018	.017	.016	.015	.012	.038	.051	.054	.047	.052	.107	.093	.104	.103	.101
G^2 $Statisics$	.017	.016	.014	.013	.011	.062	.058	.057	.047	.051	.112	.112	.114	.110	.110
D^2 Statistics	.018	.017	.016	.016	.013	.064	.056	.055	.054	.052	.116	.116	.115	.112	.110
MDS $\|\chi\|$	.017	.017	.015	.016	.012	.062	.060	.058	.052	.052	.121	.120	.118	.116	.114
FTS	.016	.016	.016	.015	.012	.062	.058	.046	.046	.048	.122	.122	.123	.111	.110
LMS	.014	.014	.013	.013	.012	.060	.057	.040	.042	.052	.120	.120	.119	.115	.110
CRS	.016	.016	.015	.015	.013	.062	.058	.056	.046	.054	.122	.124	.123	.118	.111
BPS	.018	.019	.013	.012	.011	.058	.058	.056	.048	.051	.122	.120	.090	.114	.110

[9] Eight of the selected tests of independence out of eleven tests have been analyzed for empirical values and the results are shown in Table 5.1. FES, KLS and NMCS does not follow any standard or known distribution therefore, simulated critical values (SCV) have been computed and are shown in Table 5.2.

5.3 Computation of Empirical Size of Tests of Independence for w × k CTs

We calculated empirical sizes for selected eight tests of independence at nominal level (α = 0.01, α = 0.05 and α = 0.10) for w × k CTs presented in Table 5.3, 5.4 and 5.5.

The results of panel – I indicates when nominal size is 1 % then χ^2 test has empirical size of .014 at sample size 25. G^2 Test, CRS, FTS, LMS, BPS and Modular statistics have empirical size 0.02, 0.21, 0.21, 0.22 and 0.21 at sample size 25. Panel - II indicates when nominal size is 5% then χ^2 test has empirical size .043, .055, .051, .046 and .051 at (Small, Medium, and Large). LMS test shows when nominal size is 5% then empirical size .072, .077, .055, .054 and .053 at (Small, Medium, and Large). Panel – III indicates when nominal size is 10% then χ^2 test has empirical size .106, .107, .113, .112 and .091 at sample size (Small, Medium, and Large).

The results of size distortion of BPS indicates that when nominal size is 10% then BPT has empirical size of .134, .124, .121, .121 and .101 at (Small, Medium, and Large). Moreover, as the sample size increase the difference between nominal and empirical (SCV) size reduces with sample size and same is true for others tests of independence for nominal data. Looking to the empirical and nominal values in the above tables 5.1 and 5.3 drawn for 2×2 and w × k CTs, this can be concluded that size distortion is negligible for tests of independence for nominal data.

Table 5. 3: Empirical size of test of independence for nominal data for 2 x 3 and 3 x 3 CTs

Panel		Panel – I					Panel – II					Panel - III				
		α = 0.01					α = 0.05					α = 0.10				
	Tests Name \ [10]Sample size	25	50	100	200	400	25	50	100	200	400	25	50	100	200	400
Size Distortion 2 x 3 CT	χ^2Statistic	0.014	0.013	0.012	0.011	0.011	0.043	0.055	0.051	0.046	0.051	0.106	0.107	0.113	0.112	0.099
	G^2 $Statisics$	0.020	0.019	0.011	0.008	0.012	0.078	0.072	0.055	0.053	0.057	0.134	0.124	0.122	0.121	0.101
	D^2Statistics	0.022	0.014	0.017	0.007	0.011	0.043	0.054	0.059	0.046	0.053	0.133	0.121	0.116	0.116	0.091
	MDS $\|\chi\|$	0.021	0.016	0.018	0.012	0.011	0.073	0.061	0.059	0.054	0.052	0.451	0.141	0.123	0.123	0.103
	FTS	0.021	0.019	0.019	0.018	0.014	0.075	0.063	0.055	0.054	0.052	0.134	0.124	0.122	0.121	0.113
	CRS	0.021	0.029	0.024	0.014	0.017	0.055	0.055	0.005	0.055	0.051	0.151	0.137	0.129	0.125	0.115
	LMS	0.022	0.029	0.024	0.018	0.015	0.072	0.077	0.055	0.054	0.053	0.134	0.124	0.122	0.121	0.101
	BPS	0.021	0.029	0.028	0.018	0.014	0.078	0.074	0.055	0.052	0.051	0.134	0.124	0.121	0.121	0.101
		α = 0.01					α = 0.05					α = 0.10				
	Tests Name \ Sample Size	25	50	100	200	400	25	50	100	200	400	25	50	100	200	400
Size Distortion 3 x 3 CT	χ^2 Statistics	0.016	0.012	0.014	0.011	0.010	0.038	0.046	0.054	0.046	0.044	0.157	0.173	0.127	0.137	0.127
	G^2 $Statisics$	0.018	0.016	0.014	0.015	0.012	0.040	0.045	0.052	0.049	0.048	0.126	0.136	0.138	0.140	0.113
	D^2 Statistics	0.013	0.012	0.012	0.011	0.010	0.040	0.045	0.053	0.049	0.047	0.126	0.127	0.128	0.149	0.122
	MDS$\|\chi\|$	0.016	0.015	0.014	0.014	0.012	0.047	0.045	0.057	0.049	0.049	0.126	0.126	0.137	0.146	0.108
	FTS	0.016	0.15	0.013	0.014	0.014	0.045	0.046	0.057	0.044	0.048	0.124	0.121	0.122	0.122	0.112
	LMS	0.019	0.015	0.012	0.013	0.012	0.053	0.043	0.055	0.043	0.046	0.132	0.152	0.112	0.162	0.122
	CRS	0.012	0.015	0.014	0.015	0.015	0.072	0.048	0.053	0.043	0.048	0.141	0.181	0.151	0.151	0.131
	BPS	0.012	0.013	0.014	0.014	0.014	0.031	0.044	0.058	0.042	0.046	0.141	0.141	0.133	0.131	0.121

[10] As we have small size contingency table so accordingly the same sample size which was used for 2 x 2 contingency table is also used for 2 x 3 and 3 x 3 CT. The sample size (SS: 25,50,100,200,400)

Table 5.4: Empirical size of test of independence for nominal data for 4 x 4 and 5 x 5 CTs

	[11]Sample size / Tests Name	α = 0.01					α = 0.05					α = 0.10				
		50	100	200	400	800	50	100	200	400	800	50	100	200	400	800
Size Distortion 4 x 4 CT	χ^2 Statistics	0.012	0.014	0.017	0.007	0.011	0.043	0.054	0.059	0.046	0.053	0.003	0.121	0.116	0.116	0.091
	G^2 $Statisics$	0.011	0.016	0.018	0.012	0.015	0.073	0.081	0.053	0.089	0.052	0.001	0.141	0.123	0.123	0.103
	D^2 Statistics	0.011	0.019	0.011	0.008	0.016	0.065	0.093	0.055	0.054	0.057	0.134	0.123	0.128	0.122	0.123
	MDS |χ|	0.018	0.016	0.014	0.015	0.012	0.040	0.045	0.052	0.049	0.048	0.121	0.136	0.138	0.140	0.113
	FTS	0.013	0.012	0.012	0.011	0.010	0.040	0.045	0.053	0.049	0.047	0.126	0.122	0.123	0.149	0.122
	LMS	0.013	0.012	0.012	0.011	0.010	0.040	0.045	0.053	0.049	0.047	0.123	0.121	0.125	0.149	0.122
	CRS	0.016	0.015	0.014	0.014	0.012	0.047	0.045	0.057	0.049	0.049	0.126	0.126	0.137	0.146	0.108
	BPS	0.016	0.15	0.013	0.014	0.014	0.045	0.046	0.057	0.044	0.048	0.124	0.121	0.122	0.122	0.112

	Sample Size / Tests Name	α = 0.01					α = 0.05					α = 0.10				
		75	150	300	600	1200	75	150	300	600	1200	75	150	300	600	1200
Size Distortion 5 x 5 CT	χ^2 Statistics	0.016	0.019	0.018	0.015	0.012	0.038	0.035	0.031	0.048	0.052	0.182	0.132	0.082	0.032	0.118
	G^2 $Statisics$	0.019	0.14	0.016	0.012	0.013	0.033	0.065	0.079	0.034	0.048	0.222	0.192	0.162	0.132	0.102
	D^2 Statistics	0.020	0.024	0.062	0.019	0.015	0.034	0.065	0.059	0.054	0.049	0.183	0.163	0.143	0.123	0.103
	MDS |χ|	0.026	0.018	0.014	0.069	0.013	0.040	0.065	0.038	0.063	0.052	0.163	0.153	0.143	0.133	0.123
	FTS	0.016	0.029	0.019	0.018	0.019	0.034	0.057	0.062	0.055	0.047	0.181	0.176	0.171	0.166	0.122
	LMS	0.016	0.014	0.016	0.019	0.010	0.044	0.065	0.039	0.054	0.048	0.192	0.172	0.152	0.132	0.112
	CRS	0.020	0.017	0.016	0.015	0.012	0.063	0.065	0.028	0.073	0.047	0.182	0.163	0.144	0.125	0.106
	BPS	0.016	0.014	0.015	0.012	0.013	0.072	0.057	0.072	0.055	0.052	0.182	0.132	0.162	0.132	0.118

[11] As the size of the contingency table increases, accordingly the size of sample size increases. Thus for 4 x 4 and 5 x 5 CT the sample size (SS: 50,100,200,400,800) (SS: 75,150,300,600,1200)

Table 5. 5: Empirical Size of Test of independence for nominal data for 6 x 6 and 12 x 12 CTs

	Sample Size[12] / Tests Name	α = 0.01					α = 0.05					α = 0.10				
		100	200	400	800	1600	100	200	400	800	1600	100	200	400	800	1600
Size Distortion 6 x 6 CT	χ^2 Statistics	0.013	0.012	0.012	0.011	0.010	0.040	0.045	0.053	0.049	0.047	0.123	0.121	0.125	0.149	0.122
	G^2 *Statisics*	0.016	0.015	0.014	0.014	0.012	0.047	0.045	0.057	0.049	0.049	0.126	0.126	0.137	0.146	0.108
	D^2 Statistics	0.018	0.016	0.014	0.015	0.012	0.073	.0649	0.037	0.075	0.054	0.182	0.148	0.132	0.129	0.110
	MDS $\lvert\chi\rvert$	0.013	0.012	0.012	0.011	0.010	0.053	.0439	0.034	0.054	0.053	0.198	0.188	0.131	0.123	0.108
	FTS	0.013	0.012	0.012	0.011	0.010	0.064	.0429	0.021	0.041	0.048	0.207	0.187	0.178	0.160	0.123
	LMS	0.013	0.012	0.012	0.011	0.010	0.073	.0489	0.034	0.058	0.047	0.218	0.208	0.161	0.133	0.118
	CRS	0.016	0.015	0.014	0.014	0.012	0.047	0.045	0.057	0.049	0.049	0.126	0.126	0.137	0.146	0.108
	BPS	0.020	0.017	0.016	0.015	0.012	0.063	0.065	0.028	0.073	0.047	0.182	0.163	0.144	0.125	0.106
	Sample Size / Tests Name	α = 0.01					α = 0.05					α = 0.10				
		400	800	1600	3200	6400	400	800	1600	3200	6400	400	800	1600	3200	6400
Size Distortion 12 x 12 CT	χ^2 Statistics	0.011	0.016	0.018	0.012	0.015	0.043	.0814	0.079	0.030	0.055	0.156	0.186	0.122	0.103	0.101
	G^2 *Statisics*	0.011	0.019	0.011	0.008	0.016	0.086	.0225	0.047	0.064	0.051	0.153	0.183	0.167	0.114	0.106
	D^2 Statistics	0.018	0.016	0.014	0.015	0.012	0.073	.0649	0.037	0.075	0.054	0.182	0.148	0.132	0.129	0.110
	MDS $\lvert\chi\rvert$	0.013	0.012	0.012	0.011	0.010	0.053	.0439	0.034	0.054	0.053	0.198	0.188	0.131	0.123	0.108
	FTS	0.013	0.012	0.012	0.011	0.010	0.064	.0429	0.021	0.041	0.048	0.207	0.187	0.178	0.160	0.123
	LMS	0.013	0.012	0.012	0.011	0.010	0.073	.0489	0.034	0.058	0.047	0.218	0.208	0.161	0.133	0.118
	CRS	0.013	0.012	0.012	0.011	0.010	0.073	.0669	0.034	0.057	0.047	0.118	0.148	0.131	0.143	0.128
	BPS	0.016	0.015	0.014	0.014	0.012	0.064	.0560	0.055	0.054	0.056	.1522	0.130	0.129	0.120	0.112

[12] As the size of the CTs increases, accordingly the size of sample size increases. Thus for 6 x 6 and 12 x 12 CT the sample size (SS: 50,100,200,400,800) (SS: 100,200,400,800,1600) and (SS: 400,800,1600,3200,6400).

5.3.1 Computation of Finite Sample Critical Values in $w \times k$ CTs

Fisher exact test statistics (FES), Neyman modified chi squared test (NMCS) and Kullback - Leibler test (KLS) do not follow any standard or known distribution. Therefore, SCV are computed for power comparison. The tests of independence for nominal data which are selected in this study consists of eleven tests of independence among which three tests do not follow any distributions namely, Fisher exact test statistics (FES), Neyman modified chi squared test (NMCS) and Kullback-Leibler test (KLS). SCV have been computed at various level of $\alpha = 0.01$, $\alpha = 0.05$ and $\alpha = 0.10$ in $w \times k$ CTs at different sample size (Small, Medium, and Large). The results at $\alpha = 0.01$ and $\alpha = 0.10$ are shown in appendix - I while at nominal level $\alpha = 0.05$ results are shown in Table 5.6 and 5.7.

We computed FSCV for three tests of independence at $\alpha = 0.01$, $\alpha = 0.05$ and $\alpha = 0.10$ for $w \times k$ CTs. We took a variety of DGP in different specification of CTs in $w \times k$ and found that there is no size distortion at different sample size (Small, Medium, and Large) in $w \times k$ CTs. Moreover, as the sample size increase so empirical size converges to the nominal size i.e., size distortion reduces which are shown in table 5.6 and 5.7.

These values are used in computation of power analysis. Analogously, we computed empirical size for 2 × 3, 3 × 3, 4 × 4, 5 × 5, 6 × 6, 1 2 × 12 CTs.

Table 5.6: Finite Sample Critical Values of Test of Independence for Nominal Data in W × K CTs.

		FES					NMCT					KLS				
SCV 2 X 3 CT	Test	α = 5%					α = 5%					α = 5%				
	SS	25	50	100	200	400	25	50	100	200	400	25	50	100	200	400
	CT 2 × 3	.031	.062	.037	.062	.057	.035	.037	.059	.041	.044	.031	.052	.056	.036	.048
	CT3 × 3	.062	.036	.040	.061	.057	.057	.053	.054	.043	.045	.068	.034	.063	.067	.057
		FES					NMCT					KLS				
SCV 4 X 4 CT	Test	α = 5%					α = 5%					α = 5%				
	SS[13]	50	100	200	400	800	50	100	200	400	800	50	100	200	400	800
	CT 4 × 4	.033	.039	.046	.041	.055	.087	.027	.029	.035	.042	.094	.082	.081	.076	.066
		FES					NMCT					KLS				
SCV 5 X 5 CT	Test	α = 5%					α = 5%					α = 5%				
	SS	75	150	300	600	1200	75	150	300	600	1200	75	150	300	600	1200
	CT 5 × 5	.082	.081	.076	.075	.072	.100	.097	.094	.092	.066	.126	.121	.105	.103	.092

[13] As the size of the contingency table increases, accordingly the size of sample size increases. Thus for 4 x 4 and 5 x 5 CT the sample size is (S: 50,100,200,400,800) and (S: 75,150,300,600,1200)

Table 5. 7: Finite Sample Critical Values of Test of Independence for Nominal Data in W × K CTs.

SCV 6 X 6 CT		Fisher Exact Test					NMCT					KL Statistics				
	NS	α = 5%					α = 5%					α = 5%				
	SS[14]	100	200	400	800	1600	100	200	400	800	1600	100	200	400	800	1600
	CT 6 × 6	.096	.082	.071	.072	.062	.058	.051	.058	.051	.049	.131	.123	.118	.112	.098

SCV 12 X 12 CT		Fisher Exact Test					NMCT					KL Statistics				
	NS	α = 5%					α = 5%					α = 5%				
	SS	400	800	1600	3200	6400	400	800	1600	3200	6400	400	800	1600	3200	6400
	CT 12 × 12	.058	.057	.055	.053	.051	.059	.059	.057	.052	.051	.139	.135	.127	.102	.097

[14] As the size of the contingency table increases, accordingly the size of sample size increases. Thus for 6 x 6 and 12 x 12 CT the sample size is (S: 100,200,400,800,1600) and (S: 400,800,1600,3200,6400)

5.4 Simulated Critical Values for Tests of Independence in $\mathbf{w \times k}$ CTs for Ordinal Data

As non-parametric tests of independence do not follow any standard or known distributions. Therefore, for power comparison of tests of independence, simulated critical values are needed.

Finite sample critical values (FSCV) are computed for seven tests namely Spearman ρ coefficient of correlation, Kendall's$\tau - a,$ Kendall's$\tau - b$, Kendall's$\tau - c$ coefficient , Goodman and Kruskal γ, Sumer's D and Novel $\emptyset_k$ tests of independence for ordinal data at various level (α = 0.01, α = 0.05, α = 0.10) at different sample sizes (small, medium, and large) for w × k CTs are shown in Table 5.8 - 5.11.

Table 5. 8: Simulated Critical Values of Tests of Independence for 2 × 3 CTs for Ordinal Data

	Sample Size[15] / Tests	α = 0.01					α = 0.05					α = 0.10				
		25	50	100	200	400	25	50	100	200	400	25	50	100	200	400
SCV 2 X 3 CT	Spearman ρ	0.006	0.01	0.009	0.007	0.01	0.043	0.055	0.051	0.046	0.056	0.106	0.107	0.183	0.112	0.099
	Kendall τ-a	0.011	0.017	0.013	0.009	0.04	0.072	0.078	0.059	0.059	0.057	0.144	0.124	0.123	0.141	0.101
	Kendall τ-b	0.012	0.015	0.015	0.007	0.02	0.043	0.055	0.052	0.048	0.047	0.132	0.161	0.133	0.112	0.169
	Kendall τ-c	0.011	0.016	0.013	0.022	0.01	0.078	0.072	0.057	0.059	0.053	0.121	0.141	0.122	0.121	0.171
	Gd -Kruskal γ	0.011	0.019	0.012	0.008	0.01	0.072	0.071	0.051	0.052	0.051	0.114	0.104	0.162	0.132	0.162
	Somers'd	0.012	0.012	0.013	0.007	0.02	0.043	0.052	0.054	0.049	0.059	0.153	0.151	0.123	0.122	0.112
	Spearman ρ	0.011	0.019	0.012	0.008	0.01	0.062	0.079	0.055	0.051	0.056	0.134	0.124	0.112	0.127	0.101
	$Novel\ \emptyset_k$	0.011	0.019	0.013	0.009	0.02	0.05	0.072	0.055	0.052	0.052	0.104	0.144	0.132	0.129	0.114

[15] As we have small size 2 x 3 and 3 x 3 order of CTs. The sample size is used according to statistical calculation (SS: 25, 50,100,200,400).

Table 5. 9: Simulated Critical Values of Test of Independence for Ordinal Data for 2 × 3 CT for Ordinal Data

	Tests Name \ Sample Size	α = 0.01					α = 0.05					α = 0.10				
		25	50	100	200	400	25	50	100	200	400	25	50	100	200	400
SCV 3 X 3CT	**Spearman ρ**	.0240	0.016	0.014	0.012	0.011	0.038	0.046	0.054	0.046	0.044	0.245	0.210	0.173	0.143	0.103
	Kendall τ-a	.0210	0.018	0.014	0.015	0.011	0.04	0.045	0.058	0.049	0.046	0.212	0.187	0.163	0.132	0.113
	Kendall τ-b	0.017	0.011	0.010	0.008	0.006	0.04	0.045	0.058	0.049	0.046	0.185	0.166	0.142	0.139	0.105
	Kendall τ-c	0.015	0.009	0.011	0.011	0.010	0.04	0.045	0.058	0.049	0.046	0.153	0.143	0.129	0.133	0.109
	Gd - Kruskal γ	0.013	0.008	0.011	0.012	0.014	0.04	0.045	0.058	0.049	0.046	0.123	0.121	0.113	0.131	0.110
	Somers'd	0.011	0.006	0.012	0.014	0.017	0.04	0.045	0.058	0.049	0.046	0.092	0.099	0.098	0.129	0.111
	Novel $\emptyset_k$	0.009	0.005	0.012	0.016	0.020	0.04	0.045	0.058	0.049	0.046	0.062	0.077	0.082	0.127	0.112

	Tests Name \ Sample Size	α = 0.01					α = 0.05					α = 0.10				
		50	100	200	400	800	50	100	200	400	800	50	100	200	400	800
SCV 4 X 4 CT	**Spearman ρ**	0.011	0.012	0.012	0.011	0.011	0.113	0.098	0.085	0.063	0.054	0.245	0.210	0.173	0.143	0.103
	Kendal τ-a	0.018	0.016	0.015	0.012	0.010	0.126	0.111	0.095	0.084	0.064	0.212	0.187	0.163	0.132	0.113
	Kendalτ-b	0.014	0.020	0.014	0.010	0.014	0.093	0.083	0.073	0.063	0.053	0.185	0.166	0.142	0.139	0.105
	Kendalτ-c	0.016	0.024	0.019	0.010	0.016	-0.010	-0.011	0.032	0.053	0.074	0.153	0.143	0.129	0.133	0.109
	Gd - Kγ	0.018	0.029	0.025	0.010	0.018	0.014	0.034	0.054	0.074	0.094	0.123	0.121	0.113	0.131	0.110
	Somers'd	0.011	0.012	0.012	0.011	0.011	0.113	0.098	0.085	0.063	0.054	0.092	0.099	0.098	0.129	0.111
	Novel $\emptyset_k$	0.011	0.012	0.012	0.011	0.011	0.126	0.111	0.095	0.084	0.064	0.062	0.077	0.082	0.127	0.112

Table 5. 10: Simulated Critical Values of Measure of Correlation in 5 X 5 and 6 X 6 CTs for Ordinal Data

	Sample Size / Tests Name	α = 0.01					α = 0.05					α = 0.10				
		75	150	300	600	1200	75	150	300	600	1200	75	150	300	600	1200
SCV 5 X 5 CT	Spearman ρ	0.089	0.07	0.051	0.032	0.013	0.04	0.045	0.058	0.049	0.046	0.153	0.143	0.129	0.133	0.109
	Kendall τ-a	0.008	0.009	0.01	0.011	0.012	0.04	0.045	0.058	0.049	0.046	0.123	0.121	0.113	0.131	0.110
	Kendall τ-b	0.01	0.011	0.012	0.013	0.014	0.04	0.045	0.058	0.049	0.046	0.185	0.166	0.142	0.139	0.105
	Kendall τ-c	0.023	0.02	0.017	0.014	0.011	0.04	0.045	0.058	0.049	0.046	0.153	0.143	0.129	0.133	0.109
	Gd Kruskl γ	0.007	0.009	0.011	0.013	0.015	0.072	0.071	0.051	0.052	0.051	0.121	0.141	0.122	0.121	0.171
	Somers'd	0.089	0.07	0.051	0.032	0.013	0.043	0.052	0.054	0.049	0.059	0.114	0.104	0.162	0.132	0.162
	$Novel\ \emptyset_k$	0.008	0.009	0.01	0.011	0.012	0.062	0.079	0.055	0.051	0.056	0.153	0.151	0.123	0.122	0.112

	Sample Size / Tests Name	α = 0.01					α = 0.05					α = 0.10				
		100	200	400	800	1600	100	200	400	800	1600	100	200	400	800	1600
SCV 6 X 6 CT	Spearman ρ	0.011	0.019	0.012	0.008	0.01	0.062	0.079	0.055	0.051	0.056	0.134	0.124	0.112	0.127	0.101
	Kendall τ-a	0.011	0.019	0.013	0.009	0.02	0.05	0.072	0.055	0.052	0.052	0.104	0.144	0.132	0.129	0.114
	Kendall τ-b	0.023	0.02	0.017	0.014	0.011	0.04	0.045	0.058	0.049	0.046	0.153	0.143	0.129	0.133	0.109
	Kendall τ-c	0.007	0.009	0.011	0.013	0.015	0.072	0.071	0.051	0.052	0.051	0.121	0.141	0.122	0.121	0.171
	Gd - Kruskal γ	.021	0.018	0.014	0.015	0.011	0.04	0.045	0.058	0.049	0.046	0.212	0.187	0.163	0.132	0.113
	Somers'd	0.017	0.011	0.010	0.008	0.006	0.04	0.045	0.058	0.049	0.046	0.185	0.166	0.142	0.139	0.105
	$Novel\ \emptyset_k$	0.008	0.009	0.01	0.011	0.012	0.04	0.045	0.058	0.049	0.046	0.123	0.121	0.113	0.131	0.110

Table 5. 11: Simulated Critical Values of Measure of Correlation in 12 × 12 Contingency Table for Ordinal Data

Sample Size[16] / Tests Name	α = 0.01					α = 0.05					α = 0.10				
	400	800	1600	3200	6400	400	800	1600	3200	6400	400	800	1600	3200	6400
Spearman ρ	0.017	0.011	0.010	0.008	0.006	0.04	0.045	0.058	0.049	0.046	0.185	0.166	0.142	0.139	0.105
Kendall τ-a	0.015	0.009	0.011	0.011	0.010	0.04	0.045	0.058	0.049	0.046	0.153	0.143	0.129	0.133	0.109
Kendall τ-b	0.013	0.008	0.011	0.012	0.014	0.04	0.045	0.058	0.049	0.046	0.123	0.121	0.113	0.131	0.110
Kendall τ-c	0.011	0.006	0.012	0.014	0.017	0.04	0.045	0.058	0.049	0.046	0.092	0.099	0.098	0.129	0.111
Gd - Kruskal γ	0.008	0.009	0.01	0.011	0.012	0.04	0.045	0.058	0.049	0.046	0.123	0.121	0.113	0.131	0.110
Somers'd	0.01	0.011	0.012	0.013	0.014	0.04	0.045	0.058	0.049	0.046	0.185	0.166	0.142	0.139	0.105
$Novel\ \emptyset_k$	0.011	0.019	0.013	0.009	0.02	0.05	0.072	0.055	0.052	0.052	0.104	0.144	0.132	0.129	0.114

[16] As the size of the contingency table increases, accordingly the size of sample size increases. Thus for 12 x 12 CT the sample size (SS: 400,800,1600,3200,6400)

5.5 Conclusion

In this chapter empirical simulated critical values (SCV) have been computed for small, medium, and large sample size with all cases under different specifications for a variety of DGP in 2×2 and in w × k CTs of tests of independence for nominal and ordinal data.

Keeping in view analysis of the chapters 5; we are now able to draw some conclusions from our MCS results. The powers of econometric procedures are comparable if the size remain the same. While comparing the tests, the process starts by finding out the critical values with fixed size, say 5%. Therefore 5% critical values for the entire procedures cannot be calculated. Instead, we can measure size distortion which is the difference between nominal and actual size of entire testing procedure; and the test with minimum size distortion and highest power would be the optimal test for ordinal data. Table 5.12 presents summary of SD for Nominal data.

Table 5. 12: Present Summary of Empirical Sizes for Nominal Data

	2×2 Contingency table (SCV)			(W ×K) 3×3 Contingency table (SCV)		
	α = 0.01	α = 0.05	α = 0.10	α = 0.01	α = 0.05	α = 0.10
χ^2 Statistics	.018	0.054	0.011	0.024	0.054	0.17
G^2 *Statistics*	.017	0.062	0.014	0.018	0.052	0.14
D^2 Statistics	.018	0.064	0.011	0.013	0.053	0.14
MDS |χ|	.017	0.0062	0.012	0.016	0.057	0.14
FTS	.016	0.062	0.012	0.016	0.057	0.12
LMS	.014	0.060	0.012	0.019	0.050	0.12
CRS	.016	0.062	0.012	0.015	0.072	0.18
BPS	.019	0.058	0.012	0.014	0.058	0.15

FSCV have been drawn out for Fisher exact test statistics (FES), Neyman modified chi squared test (NMCS) and Kullback - Leibler test (KLS) shown in Table 5.13.

Table 5. 13: Present Summary of Simulated Critical Values for Nominal Data

	2×2 Contingency table (FSCV)			**W×K (3 ×3) Contingency table (FSCV)**		
	α = 0.01	α = 0.05	α = 0.10	α = 0.01	α = 0.05	α = 0.10
FES	-.018	-.057	-.011	-.010	-.042	-.119
$NMCS$	.017	.058	.012	.011	.057	.136
KLS	.018	.045	.012	.018	.068	.139

FES, NMCS and KLS do not follow any standard or known distribution and thus simulated critical values are computed (SCV). These values are used in the computation of power analysis which are presented in chapter 6. In Table 5.14, simulated critical values have been carried out for seven tests of independence for ordinal data analysis in w × k CTs.

Table 5. 14: Present Summary of Simulated Critical Values for Ordinal Data

3×3 Contingency table (SCV)				**6×6 Contingency table (SCV)**		
	α = 0.01	α = 0.05	α = 0.10	α = 0.01	α = 0.05	α = 0.10
Spearman ρ	.024	.054	.125	.019	.079	.134
Kendall τ-a	.021	.058	.122	.019	.072	.144
Kendall τ-b	.017	.053	185	.023	.058	.153
Kendall τ-c	.015	.052	.153	.015	.072	.141
Gd - Kruskal γ	.014	.058	.123	.021	.058	.212
Somers'd	.017	.051	.129	.017	.058	.185
$Novel\ \emptyset_k$	.020	.049	.122	.012	.058	.131

FSCV are computed which are used and works in computation of power which is presented in chapter 7.

CHAPTER 6

POWER COMPARISON OF TESTS FOR NOMINAL DATA

This chapter has been documented of results based on solid estimations of MCS into two sections. Section I explains power comparison for selected eleven tests of independence namely (Pearson's) χ^2 test, log likelihood ratio (G^2) test, Fisher Exact Test (FES), Freeman and Tuckey Test (FTS), Cressie and Read Test (CRS), Kulber and Liaber test (KLS), Neyman Modified Chi Square Test (NMCS), BPS, Logarithmic Minimum Square (LMS) Test, Modular Test (MDS) and D Square (D^2) Test Statistics (MDS). We used five scenarios discussed in chapter 4 for 2×2 and $W \times K$ CTs.

Thus, in this connection eleven tests are compared, and we evaluated the most stringent test of independence using stringency criteria (SC) based on power envelope for 2×2 CT in section 1 while same procedure is adopted in section II for $w \times k$ CTs. The power of all these tests is defined as the probability of rejecting null hypothesis when it is false i.e.

$$Power = P(Rejecting\ H_0/H_1\ is\ True)$$

The sample size (small, medium, and large) has been used with nominal level α = 5%. As for calculation of size, to calculate the power, we used DGP described in chapter 4. For eight tests of independence ACV were used while for three tests of independence SCV are used.

[Section I]

6.1 Power Analysis of Tests of Independence for Nominal data in 2×2[17] CTs

We computed power analysis of tests of independence for nominal data for different scenarios (I-V) shown in Table 4.2 for a variety of DGP. The results are stated below in Table 6.1:

Asymptotic critical values (ACV) are used for eight tests of independence namely, Pearson's χ^2 test, log likelihood ratio (G^2) Freeman and Tuckey Test (FTS), Cressie and Read Test (CRS), BPS Test, Logarithmic Minimum Square Test, Modular Test (MDS) and D Square (D^2) Test while simulated critical values (SCV) are computed in Table 5.3 are used for three tests of independence namely, Fisher Exact Test (FES), Kulber-Liabler Test (KLS) and Neyman Modified Chi Square Test (NMCS). The results of 2×2 for CTs, N=25 shows that Fisher exact test, Logarithmic Minimum Square test and BPS have maximum power compared to others tests of independence.

We calculated Neyman Pearson Lemma (NPLT) point optimal test and calculated shortcomings to evaluate the most stringent test of independence in Scenario – I. We found that FES test of independence have minimum shortcomings compare to others tests of independence i.e., Pearson's χ^2 test, log likelihood ratio (G^2) , Freeman and Tuckey Test (FTS), Cressie and Read Test (CRS), BPS, Logarithmic Minimum Square Test, Modular Test (MDS) and D Square (D^2), Kulber-Liabler Test (KLS) and Neyman Modified Chi Square Test (NMCS).

[17] Computational Formulas for Tests of independence for nominal data for 2×2 CTs are presented in Appendix A.

Table 6. 1: Power Analysis of Tests of independence for 2 × 2 CT Scenario - I

Nominal Level (α) =5%	**Measure of Untruthfulness [MoU]**									**N=25**	
Tests Name	0.000	0.020	0.040	0.060	0.080	0.100	0.120	0.140	0.160	0.180	0.200
χ^2 Test	0.050	0.052	0.058	0.069	0.080	0.138	0.225	0.385	0.426	0.549	0.686
G^2 Test	0.050	0.054	0.062	0.077	0.085	0.145	0.153	0.195	0.226	0.364	0.497
D^2 Test	0.05	0.053	0.058	0.072	0.093	0.118	0.138	0.174	0.183	0.288	0.399
|χ| MDS	0.05	0.051	0.056	0.068	0.094	0.126	0.273	0.32	0.406	0.517	0.596
FES	0.05	0.051	0.168	0.276	0.384	0.518	0.666	0.814	0.892	0.96	1
NMCS	.050	0.052	0.066	0.087	0.112	0.137	0.150	0.180	0.195	0.240	0.374
FTS	.050	0.051	0.062	0.077	0.089	0.127	0.175	0.186	0.199	0.260	0.379
CRS	.050	0.051	0.058	0.072	0.084	0.118	0.166	0.172	0.188	0.197	0.398
KLS	.050	0.051	0.059	0.075	0.088	0.121	0.156	0.183	0.192	0.199	0.307
BPS	.052	0.059	0.119	0.202	0.336	0.432	0.568	0.7817	0.858	0.958	0.99
LMS	.051	0.056	0.109	0.1812	0.316	0.382	0.538	0.73	0.848	0.951	0.979
NPLT	0.05	0.09	0.19	0.3196	0.439	0.58	0.74	0.866	0.962	1	1

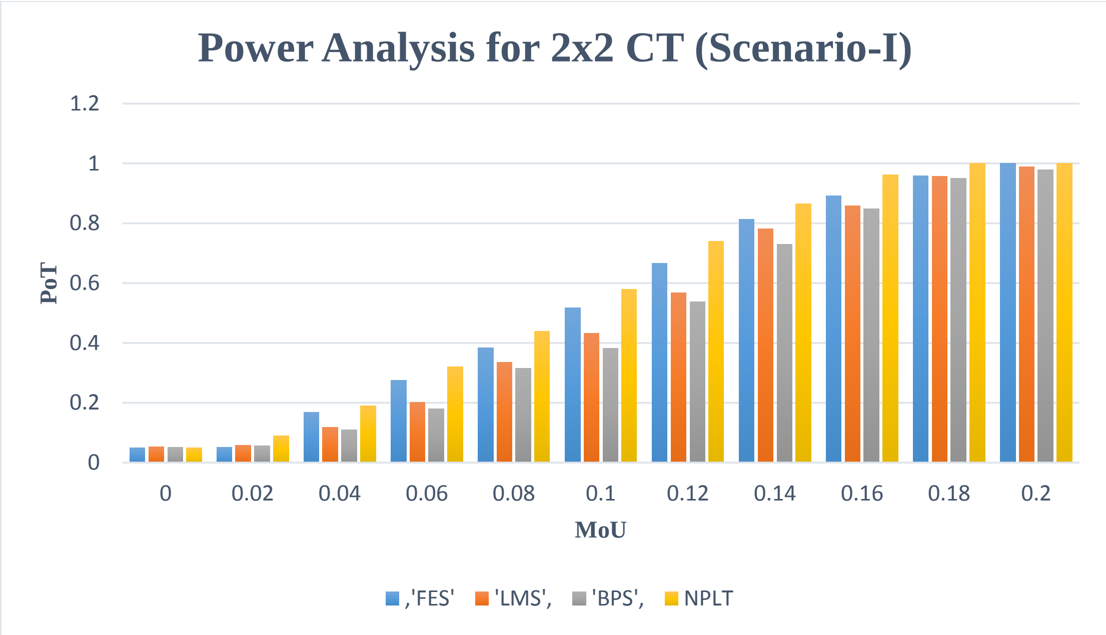


Figure 6.1: Shows Power of 2x2 CT.

Figure 6.1 shows estimated results of maximum power of four selected tests out of eleven tests of independence for nominal data. Considering scenario I, we observe that other tests have low power compared to FES. Therefore, we choose the top three tests of independence with maximum power presented in Figure 6.1. The result indicates that FES is the powerful tests of independence in scenario I. Furthermore, we calculated the power envelope and compared the power of all eleven tests of independence. We found that FES has minimum shortcomings. FES beats all others test of independence in scenario I.

Table 6. 2: Power Analysis of Tests of independence for 2 × 2 CT Scenario - II

Nominal Level (α) =5%	**Measure of Untruthfulness [MoU]**									**N=50**	
Tests Name	0.000	0.020	0.040	0.060	0.080	0.100	0.120	0.140	0.160	0.180	0.200
χ^2 Test	.051	0.054	0.065	0.067	0.075	0.145	0.248	0.352	0.43	0.553	0.69
G^2 Test	0.05	0.056	0.062	0.077	0.087	0.147	0.157	0.296	0.336	0.47	0.599
D^2 Test	.050	0.089	0.14	0.24	0.33	0.45	0.56	0.65	0.74	0.81	0.892
$\|\chi\|$ MDS	0.05	0.053	0.058	0.072	0.099	0.137	0.167	0.228	0.31	0.329	0.422
FES	0.05	0.14	0.245	0.407	0.494	0.6578	0.74	0.835	0.912	0.966	0.986
NMCS	0.05	0.052	0.082	0.086	0.131	0.168	0.172	0.184	0.21	0.271	0.318
FTS	0.05	0.052	0.061	0.07	0.092	0.147	0.184	0.198	0.221	0.277	0.380
CRS	0.05	0.052	0.06	0.077	0.087	0.138	0.171	0.181	0.192	0.21	0.312
KLS	0.05	0.051	0.062	0.071	0.086	0.132	0.173	0.188	0.192	0.223	0.382
BPS	0.05	0.11	0.205	0.25	0.38	0.51	0.67	0.75	0.83	0.9	0.970
LMS	0.05	0.09	0.15	0.23	0.34	0.46	0.61	0.71	0.79	0.86	0.990

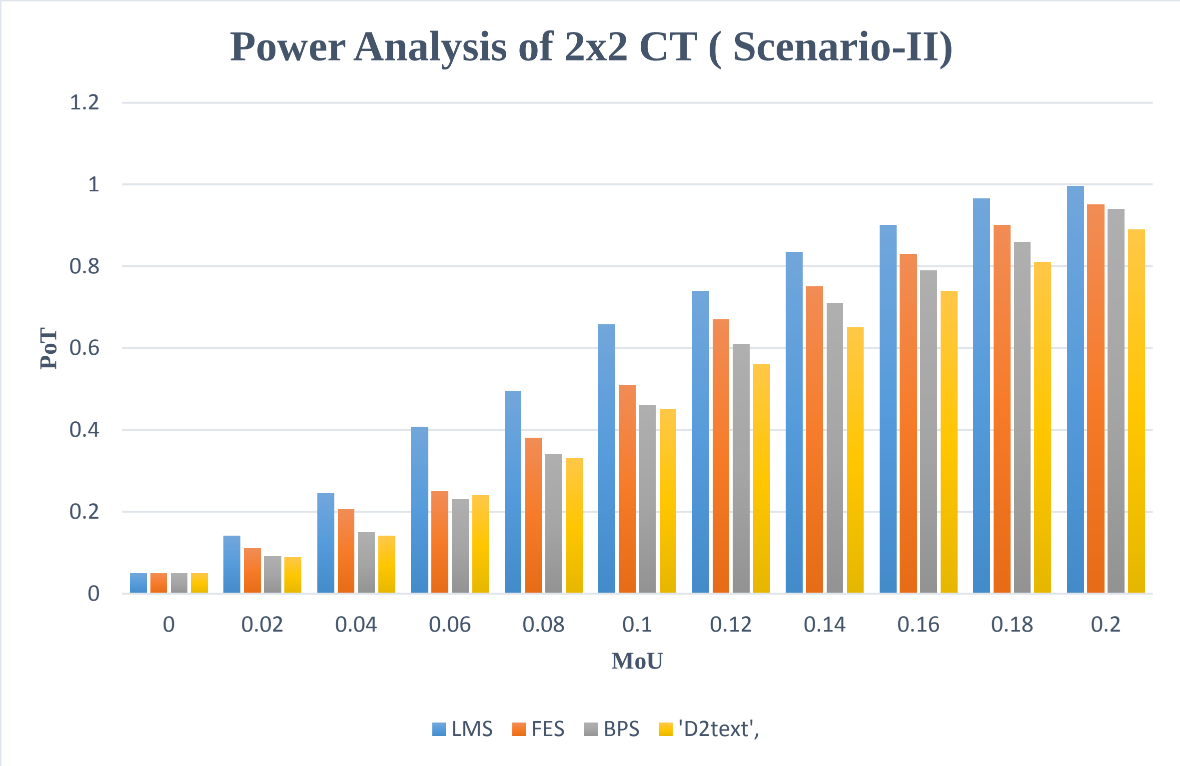


Figure 6.2: Shows Power Analysis of 2x2 CT.

Table 6.2 results indicates that LMS has the most powerful test as compared to others tests of independence in scenario II. The results contradict with scenario I due to different DGP. FES performs best at second while BPS and D square at third and fourth but performs betters as compared to others tests of independence.

Table 6. 3: Power Analysis of Tests of independence for 2 × 2 CT Scenario - III

Nominal Level (α) =5%	**Measure of Untruthfulness [MoU]**									**N=100**	
Tests Name	0.000	0.020	0.040	0.060	0.080	0.100	0.120	0.140	0.160	0.180	0.200
χ^2 Test	0.05	0.056	0.068	0.069	0.077	0.149	0.152	0.255	0.332	0.455	0.599
G^2 Test	0.05	0.058	0.061	0.079	0.089	0.148	0.158	0.299	0.346	0.482	0.512
D^2 Test	0.05	0.052	0.069	0.071	0.098	0.12	0.415	0.527	0.648	0.758	0.831
$\|\chi\|$ MDS	0.05	0.051	0.062	0.07	0.107	0.138	0.168	0.232	0.322	0.331	0.432
FES	0.05	0.051	0.063	0.078	0.197	0.22	0.482	0.674	0.723	0.932	1
NMCS	0.05	0.051	0.086	0.09	0.139	0.16	0.176	0.188	0.221	0.284	0.299
FTS	0.05	0.051	0.063	0.078	0.107	0.122	0.282	0.374	0.423	0.462	0.532
CRS	0.05	0.05	0.062	0.079	0.093	0.144	0.171	0.185	0.199	0.221	0.343
KLS	0.05	0.051	0.081	0.091	0.113	0.162	0.181	0.189	0.231	0.234	0.357
BPS	0.051	0.065	0.096	0.139	0.262	0.348	0.562	0.712	0.781	0.892	0.95
LMS	0.052	0.062	0.0097	0.121	0.23	0.311	0.528	0.641	0.757	0.83	0.957

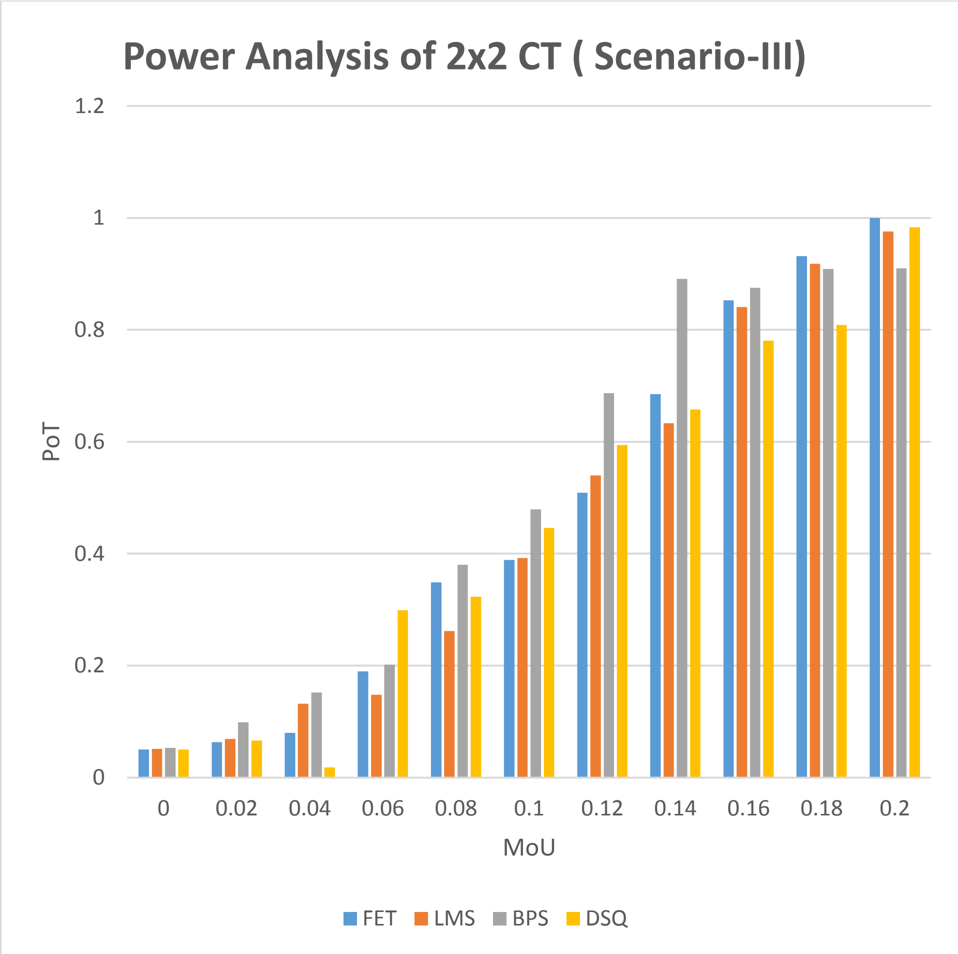


Figure 6.3: Power Analysis of 2x2 CT

Table 6.3 results of scenario –III indicates different result in contrast to scenario I-II. FES performs better as compared to LMS. BPS also shows better performance compared to others tests of independence.

Table 6. 4: Power Analysis of Tests of independence for 2 × 2 CT Scenario - IV

Nominal Level (α) =5%	Measure of Untruthfulness [MoU]									N=200	
Tests Name	0.000	0.020	0.040	0.060	0.080	0.100	0.120	0.140	0.160	0.180	0.200
χ^2 Test	0.05	0.053	0.063	0.088	0.097	0.148	0.172	0.201	0.335	0.485	0.574
G^2 Test	0.05	0.055	0.067	0.075	0.125	0.155	0.164	0.296	0.326	0.456	0.588
D^2 Test	0.05	0.051	0.062	0.078	0.093	0.221	0.437	0.757	0.806	0.847	0.856
|χ| MDT	0.052	0.063	0.068	0.089	0.194	0.218	0.276	0.389	0.401	0.456	0.511
FES	0.05	0.051	0.061	0.167	0.202	0.329	0.486	0.68	0.811	0.855	0.996
NMCS	0.05	0.051	0.052	0.057	0.091	0.117	0.145	0.156	0.163	0.181	0.232
FTS	0.05	0.052	0.057	0.062	0.094	0.126	0.156	0.169	0.176	0.189	0.278
CRS	0.05	0.059	0.078	0.094	0.243	0.264	0.387	0.495	0.553	0.577	0.665
KLS	0.052	0.094	0.1499	0.232	0.365	0.432	0.587	0.752	0.819	0.86	0.905
BPS	0.051	0.066	0.098	0.132	0.251	0.363	0.483	0.712	0.805	0.9011	0.929
LMS	0.052	0.094	0.1499	0.232	0.365	0.432	0.587	0.752	0.919	0.96	0.989

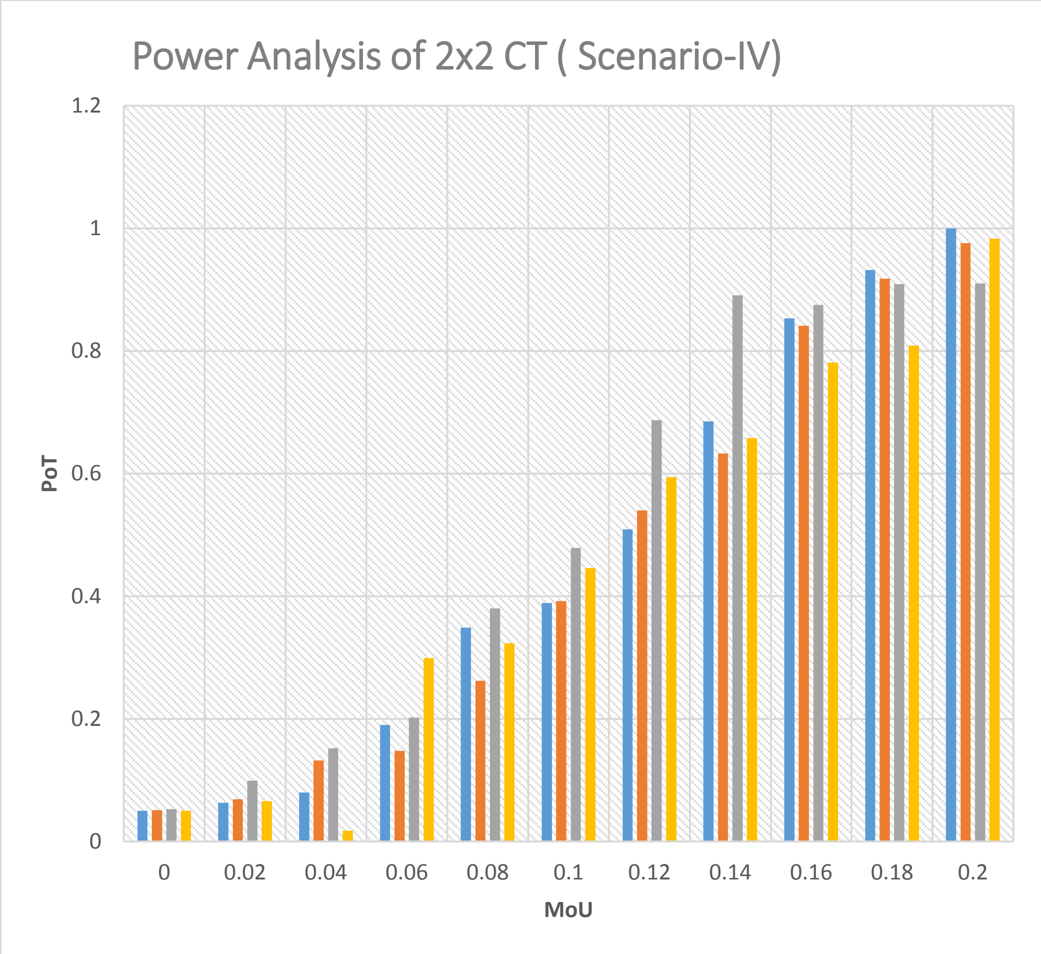


Figure 6.4: Shows Power Analysis of 2x2 CT.

Table 6.4 analysis explains that FES performs better in scenario IV as compared to LMS, BPS and D Square. Here KLS seems to be more power full as compared to D square test and others tests of independence.

Table 6. 5: Power Analysis of Tests of independence for 2 × 2 CT Scenario - V

Nominal Level (α) =5%	Measure of Untruthfulness [MoU]									N=400	
Tests Name	0.000	0.020	0.040	0.060	0.080	0.100	0.120	0.140	0.160	0.180	0.200
χ^2 Test	0.05	0.058	0.079	0.082	0.097	0.125	0.158	0.167	0.259	0.275	0.394
G^2 Test	0.05	0.066	0.069	0.091	0.094	0.131	0.163	0.216	0.271	0.294	0.419
D^2 Test	0.05	0.066	0.0178	0.299	0.323	0.446	0.594	0.658	0.781	0.809	0.983
|χ| MDT	0.05	0.055	0.067	0.081	0.112	0.14	0.179	0.229	0.346	0.387	0.482
FES	0.05	0.063	0.08	0.19	0.349	0.389	0.509	0.685	0.853	0.932	1
NMCS	0.05	0.058	0.071	0.098	0.142	0.168	0.195	0.198	0.251	0.298	0.382
FTS	0.052	0.056	0.072	0.079	0.107	0.138	0.141	0.149	0.153	0.167	0.188
CRS	0.051	0.057	0.08	0.083	0.093	0.099	0.108	0.14	0.205	0.281	0.3
KLS	0.05	0.057	0.074	0.099	0.153	0.171	0.188	0.197	0.265	0.323	0.397
LMS	0.051	0.069	0.132	0.148	0.262	0.392	0.54	0.633	0.841	0.918	0.976
BPS	0.053	0.099	0.152	0.202	0.38	0.479	0.687	0.891	0.875	0.909	0.910

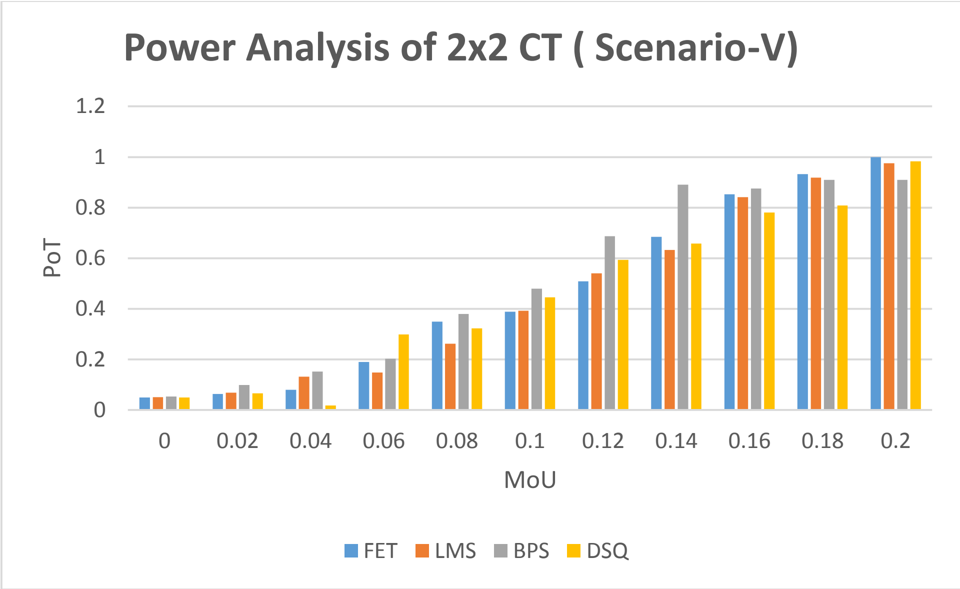


Figure 6.5: Shows Graph of PoT for 2x2 CT.

The results of scenario V in Table 6.5 indicates that FES have maximum power as compared to LMS, BPS and D Square tests of independence. This is the key point which makes confusion that which test is to be used for data in hand. Since different tests performs different under various DGP for 2x2 CT which leads us to evaluate the most stringent test using SC. The summary of different scenario is shown in Table 6.6.

Table 6. 6: Summary of Power for 2×2 Contingency Table

	2×2 Contingency table (Power)				
	α = 0.05				
	FES	LMS	BPS	DSQS	KLS
Scenario I	++++	+++	++	+	-
Scenario II	+++	++++	++	+	-
Scenario III	++++	+++	++	+	-
Scenario IV	+++	++	++++	-	+
Scenario V	++++	++	+	+++	-

(Note: “+” shows the power of tests as it increases shows the most powerful tests).

Graphical analysis of 2x2 CT under scenarios (I-V)

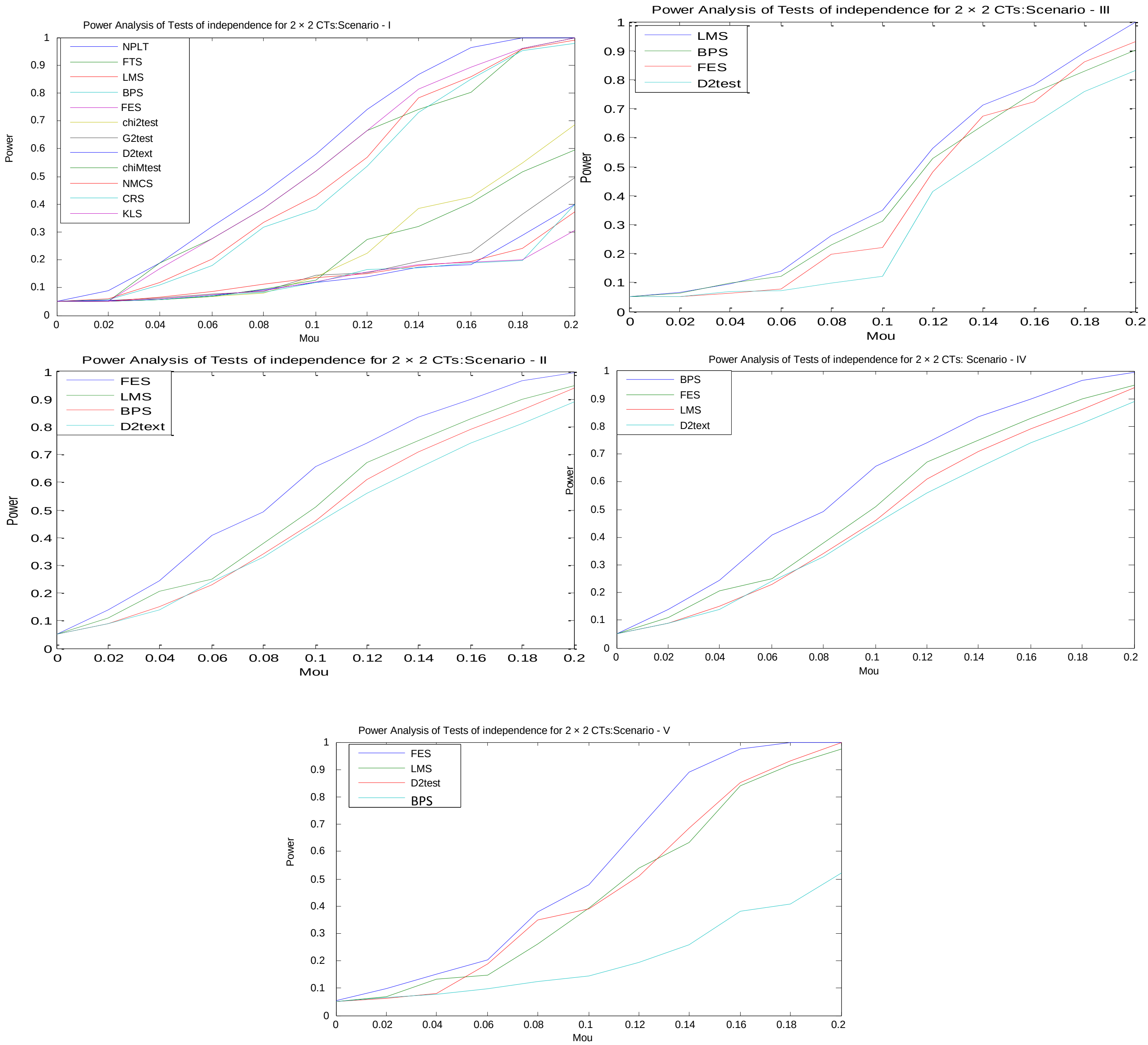


Figure 6.6: Shows Power Analysis Graphs for Nominal Data for 2 × 2 CT.

Different tests perform different output in various scenarios under consideration. Therefore, we use Stringency criteria (SC) to decide about the most stringent tests of independence in 2x2 Contingency table. We computed maximum likelihood, draw the power envelope calculated shortcomings of the numerous tests of independence that is the difference which is maximum between powers envelop and power curve of tests of Independence to evaluate most stringent tests of independence for nominal data in 2 × 2 CT.

$$\mathrm{S}\ (\mathrm{T}, \theta_{\mathrm{k}}) \quad = \quad \mathrm{P}\ (\mathrm{T}\theta_{\mathrm{k}}, \theta_{\mathrm{k}}) \ - \ \mathrm{P}\ (\mathrm{T}, \theta_{\mathrm{k}})$$

Shortcoming at specific alternative

$$\mathrm{S}\ (\mathrm{T}) = \mathrm{Max}\ [\mathrm{P}\ (\mathrm{T}\theta_{\mathrm{k}}, \theta_{\mathrm{k}}) - \mathrm{P}\ (\mathrm{T}, \theta_{\mathrm{k}})$$

Table 6. 7: Shortcoming of Tests of Independence for Nominal Data for 2 × 2 Contingency table

CT 2 × 2 α = 0.05	***Shortcomings***											***Most Stringent Test***
Sample Size	χ^2 Test	G^2 $Test$	D^2 Test	|χ| MD Test	FES	NMCS	FTS	CRS	KLS	LMS	BPS	
N=25	0.401	0.304	0.263	0.209	0.044	0.337	0.636	0.757	0.353	0.07	0.059	**FES**
N=50	0.427	0.331	0.273	0.265	0.045	0.246	0.43	0.69	0.32	0.08	0.093	**FES**
N=100	0.428	0.342	0.287	0.286	0.053	0.249	0.43	0.62	0.32	0.1	0.083	**FES**
N=200	0.432	0.363	0.288	0.294	0.052	0.243	0.48	0.69	0.15	0.09	0.073	**FES**
N=400	0.448	0.367	0.288	0.302	0.049	0.245	0.45	0.59	0.21	0.08	0.073	**FES**

Thus, from above table 6.7 and figure 6.6 results; this can be found and concluded that FES has minimum shortcoming and thus this is concluded that the most stringent test is Fisher Exact Test Statistics (FES) in 2 × 2 CTs.

[Section II]

6.2 Power Analysis of Tests of Independence for Nominal data in W × K CT

We investigated the power of tests of independence for nominal data in different scenarios presented in Table 4.2 for different CTs and found the following results stated in Tables.

Table 6. 8: Power Analysis of Tests of independence for 2× 3 CT

Scenario - I

Nominal Level (α) =5%	**Measure of Untruthfulness [MoU]**									**N=25**	
Tests Name	0.000	0.020	0.040	0.060	0.080	0.100	0.120	0.140	0.160	0.180	0.200
χ^2 Test	0.05	0.05	0.087	0.0798	0.145	0.252	0.289	0.398	0.421	0.577	0.689
G^2 Test	0.05	0.05	0.077	0.089	0.093	0.142	0.178	0.285	0.399	0.421	0.543
D^2 Test	0.05	0.058	0.098	0.102	0.177	0.249	0.352	0.455	0.532	0.678	0.788
$\lvert\chi\rvert$ MDT	0.05	0.06	0.068	0.089	0.099	0.188	0.258	0.399	0.446	0.582	0.699
FIT	0.05	0.062	0.078	0.098	0.109	0.298	0.351	0.471	0.58	0.698	0.723
NMCS	0.05	0.06	0.068	0.089	0.099	0.188	0.258	0.399	0.446	0.582	0.699
FTS	0.052	0.05	0.062	0.078	0.098	0.109	0.298	0.351	0.471	0.58	0.698
CRS	0.051	0.05	0.05	0.051	0.089	0.091	0.153	0.162	0.281	0.389	0.431
KLS	0.05	0.052	0.059	0.064	0.199	0.29	0.31	0.412	0.512	0.524	0.623
BPS	0.052	0.058	0.059	0.167	0.267	0.398	0.487	0.587	0.724	0.876	0.965
LMS	0.056	0.102	0.143	0.205	0.295	0.431	0.562	0.711	0.879	0.96	1.00
NPLT	0.05	0.233	0.365	0.488	0.595	0.622	0.762	0.811	0.979	0.999	1.00

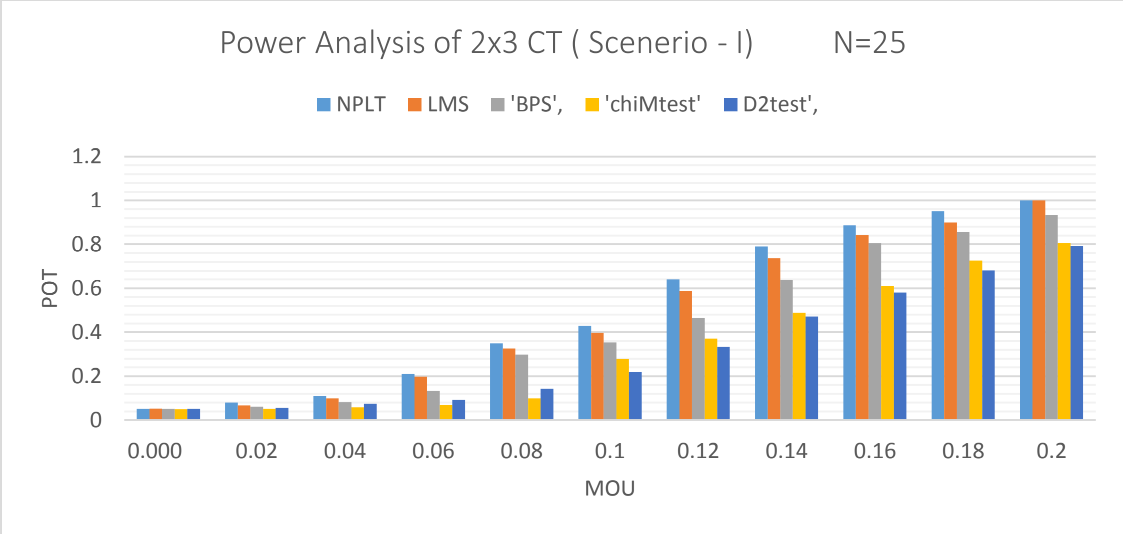


Figure 6.7: Shows Power Analysis of 2x3 CT (S-I)

Table 6.8 shows power of selected tests of independence for nominal data considering scenario I. We see from the results that other tests have low power therefore; we took only the top four tests of independence which has the maximum power in scenario I and compare the results shown in figure 6.8. The results indicates that LMS tests has maximum power. We have also compared the power envelope shown by NPLT with LMS, BPS and ChiMtest (MDS) test having maximum power and are used in evaluation of most stringent tests for nominal data using SC based on power envelop.

Table 6. 9: Power Analysis of Tests of independence for 3 × 3 Contingency table

Scenario - I

Nominal Level (α) =5%	**Measure of Untruthfulness [MoU]**									**N=50**	
Tests Name	0.000	0.020	0.040	0.060	0.080	0.100	0.120	0.140	0.160	0.180	0.200
χ^2 Test	0.051	0.056	0.067	0.075	0.087	0.188	0.242	0.252	0.33	0.41	0.491
G^2 $Test$	0.05	0.059	0.068	0.097	0.187	0.241	0.351	0.491	0.431	0.571	0.699
D^2 Test	0.05	0.055	0.064	0.091	0.196	0.223	0.241	0.289	0.386	0.492	0.51
|χ| MDT	0.05	0.057	0.059	0.073	0.199	0.188	0.199	0.328	0.412	0.521	0.629
FES	0.05	0.056	0.068	0.175	0.287	0.327	0.471	0.499	0.527	0.611	0.65

NMCS	0.05	0.053	0.087	0.089	0.231	0.368	0.379	0.489	0.51	0.698	0.789
FTS	0.05	0.056	0.068	0.098	0.192	0.257	0.398	0.499	0.598	0.698	0.732
CRS	0.05	0.052	0.068	0.099	0.102	0.138	0.271	0.381	0.492	0.51	0.612
KLS	0.05	0.051	0.087	0.089	0.186	0.198	0.273	0.381	0.492	0.523	0.612
LMS	0.052	0.068	0.091	0.178	0.276	0.387	0.599	0.756	0.877	0.899	1
BPS	0.052	0.068	0.091	0.178	0.276	0.387	0.599	0.756	0.877	0.899	0.96

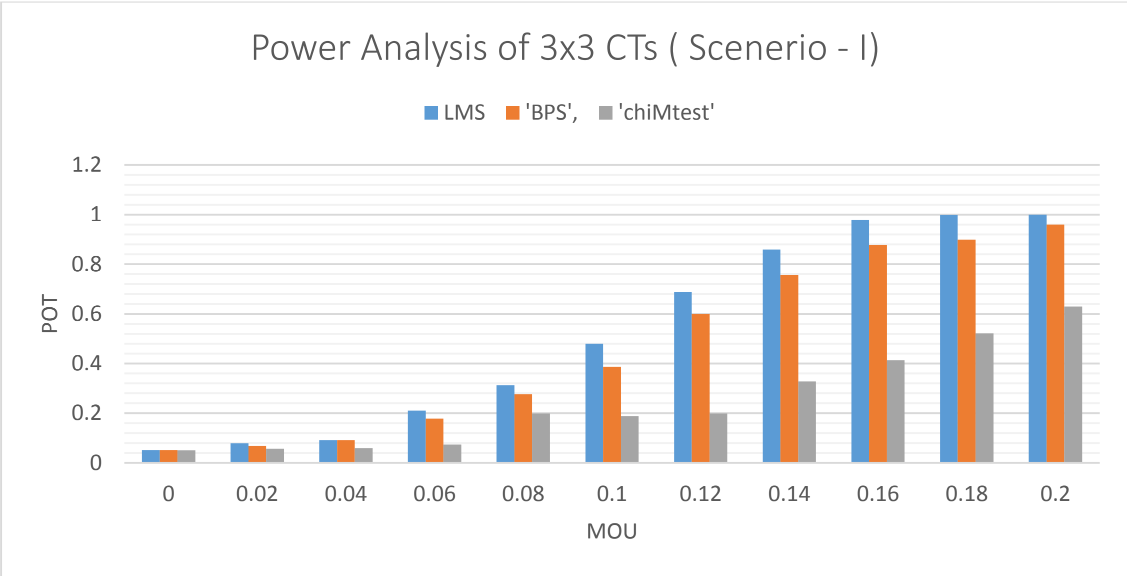


Figure 6.8: Shows Power Analysis of 3x3 CT (S-I)

Table 6.9 results indicates that LMS has the maximum power as compared to others tests of independence in scenario I for 3x3 CT. We also found the same result in scenarios – I for 2x3 CT that LMS, BPT and MDT tests performs betters as compared to other tests.

Table 6. 10: Power Analysis of Tests of independence for 4 × 4 Contingency table

Scenario - I

Nominal Level (α) =5%	Measure of Untruthfulness [MoU]									N=100	
Tests Name	0.000	0.020	0.040	0.060	0.080	0.100	0.120	0.140	0.160	0.180	0.200
χ^2 Test	0.05	0.058	0.098	0.102	0.177	0.249	0.252	0.355	0.432	0.578	0.688
G^2 Test	0.05	0.06	0.068	0.089	0.099	0.188	0.258	0.399	0.446	0.582	0.699
D^2 Test	0.05	0.055	0.064	0.091	0.196	0.223	0.241	0.289	0.386	0.492	0.51
|χ |MDT	0.05	0.057	0.059	0.073	0.199	0.188	0.199	0.328	0.412	0.521	0.629
FIS	0.05	0.055	0.063	0.078	0.1	0.12	0.282	0.374	0.422	0.582	0.632
NMCS	0.05	0.051	0.089	0.098	0.239	0.36	0.476	0.488	0.521	0.688	0.789
FTS	0.05	0.05	0.087	0.0798	0.145	0.252	0.289	0.398	0.421	0.577	0.689
CRS	0.05	0.05	0.077	0.089	0.093	0.142	0.178	0.285	0.399	0.421	0.543
KLS	0.05	0.051	0.089	0.091	0.153	0.162	0.281	0.389	0.431	0.534	0.557
LMS	0.051	0.078	0.092	0.21	0.312	0.4791	0.689	0.859	0.977	0.998	1
BPS	0.052	0.068	0.091	0.178	0.276	0.387	0.599	0.756	0.877	0.899	0.96

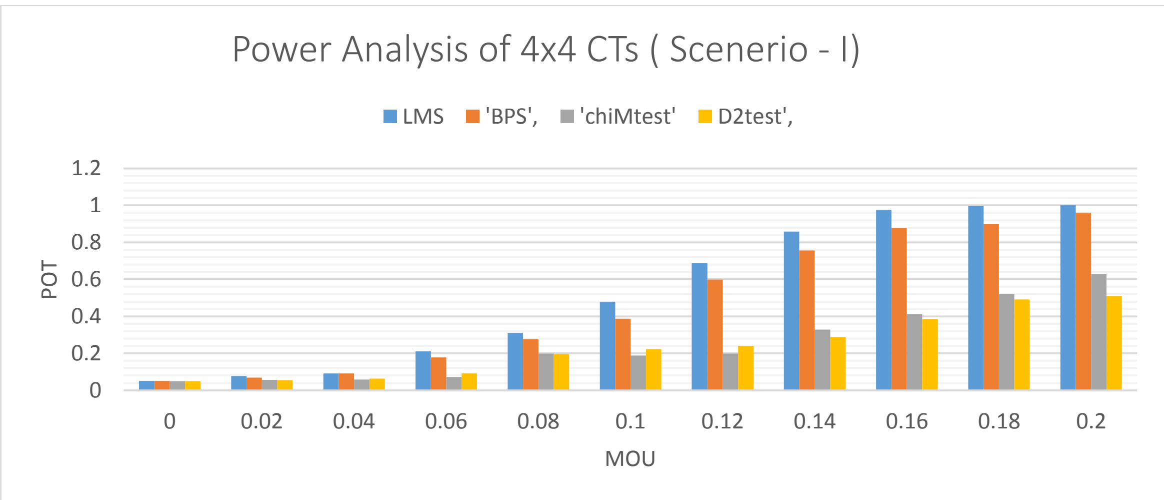


Figure 6.9: Shows power Analysis of 4x4 CT (S-I)

Table 6. 11: Power Analysis of Tests of independence for 5 × 5 Contingency table

Scenario - I

Nominal Level (α) =5%	Measure of Untruthfulness [MoU]									N=200	
Tests Name	0.000	0.020	0.040	0.060	0.080	0.100	0.120	0.140	0.160	0.180	0.200
χ^2 Test	0.05	0.051	0.089	0.098	0.239	0.36	0.476	0.488	0.521	0.688	0.789
G^2 $Test$	0.05	0.05	0.087	0.0798	0.145	0.252	0.289	0.398	0.421	0.577	0.689
D^2 Test	0.05	0.05	0.077	0.089	0.093	0.142	0.178	0.285	0.399	0.421	0.543
$\|\chi\|$ MDT	0.052	0.058	0.098	0.102	0.177	0.249	0.252	0.355	0.432	0.578	0.688
FES	0.05	0.06	0.068	0.089	0.099	0.188	0.258	0.399	0.646	0.882	0.999
NMCS	0.05	0.062	0.078	0.098	0.109	0.298	0.351	0.471	0.58	0.698	0.723
FTS	0.05	0.055	0.068	0.09	0.198	0.238	0.368	0.432	0.522	0.631	0.732
CRS	0.05	0.062	0.078	0.098	0.109	0.298	0.351	0.471	0.58	0.698	0.723
KLS	0.05	0.05	0.051	0.089	0.091	0.153	0.162	0.281	0.389	0.431	0.534
BPS	0.051	0.055	0.069	0.1199	0.298	0.311	0.499	0.512	0.624	0.823	0.923
LMS	0.052	0.063	0.117	0.299	0.365	0.487	0.687	0.734	0.823	0.925	1

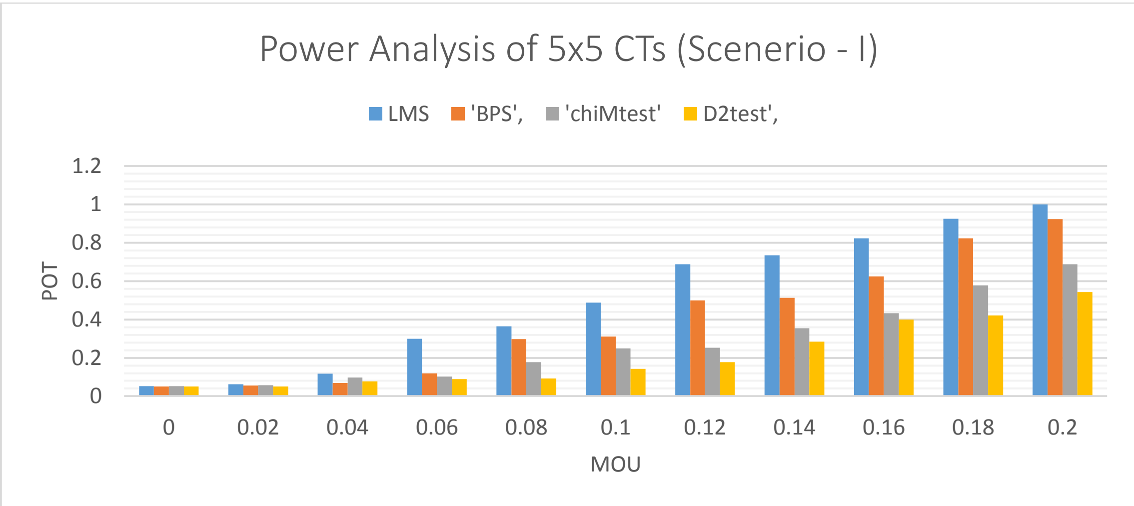


Figure 6.10: Shows Power Analysis of 5x5 CT (SI)

Table 6.11 analysis explains that LMS and BPS performs better in scenario I for 4x4 CT and 5x5 CT as compared to other statistics.

Table 6. 12: Power Analysis of Tests of independence for 6 × 6 Contingency Table

Scenario - I

Nominal Level (α) =5%	Measure of Untruthfulness [MoU]									N=400	
Tests Name	0.000	0.020	0.040	0.060	0.080	0.100	0.120	0.140	0.160	0.180	0.200
χ^2 Test	0.05	0.059	0.099	0.111	0.197	0.225	0.358	0.361	0.387	0.475	0.598
$G^2\ Test$	0.05	0.069	0.076	0.098	0.099	0.145	0.269	0.312	0.472	0.598	0.699
D^2 Test	0.05	0.068	0.099	0.193	0.223	0.346	0.494	0.552	0.681	0.688	0.698
|χ| MDT	0.05	0.057	0.087	0.089	0.167	0.175	0.279	0.329	0.446	0.587	0.682
FES	0.05	0.063	0.086	0.199	0.349	0.389	0.598	0.611	0.653	0.632	0.678
NMCS	0.05	0.058	0.077	0.099	0.187	0.198	0.295	0.398	0.451	0.598	0.682
FTS	0.052	0.056	0.078	0.099	0.198	0.154	0.187	0.234	0.353	0.467	0.5188
CRS	0.051	0.057	0.08	0.083	0.093	0.099	0.134	0.24	0.305	0.481	0.521
KLS	0.05	0.057	0.074	0.099	0.153	0.171	0.188	0.297	0.365	0.422	0.523
BPS	0.051	0.071	0.144	0.155	0.262	0.392	0.54	0.634	0.741	0.818	0.912
LMS	0.054	0.099	0.188	0.198	0.298	0.476	0.667	0.791	0.87	0.977	1

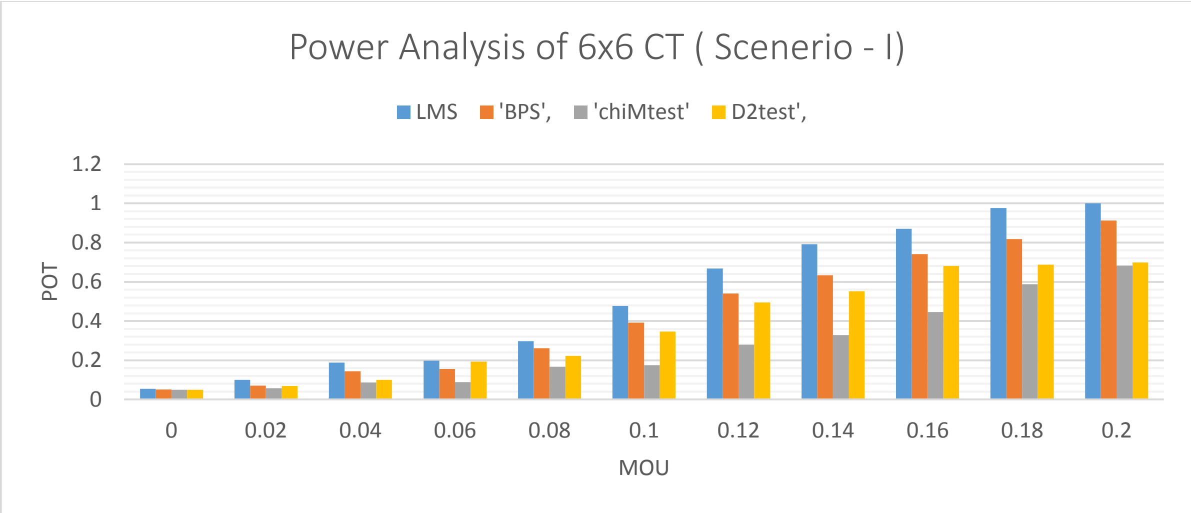


Figure 6.11: Shows Graph of PoT for 6x6 CT (SI)

Table 6. 13: Power Analysis of Tests of Independence for 12 × 12 CTs Scenario - I

Nominal Level (α) =5%	A measure of Untruthfulness [MoU]									N=400
Tests Name	0.000	0.020	0.040	0.060	0.080	0.100	0.120	0.140	0.160	0.180	0.200
χ^2 Test	0.05	0.05	0.087	0.0798	0.145	0.252	0.289	0.398	0.421	0.577	0.689
G^2 $Test$	0.05	0.05	0.077	0.089	0.093	0.142	0.178	0.285	0.399	0.421	0.543
D^2 Test	0.05	0.058	0.098	0.102	0.177	0.249	0.352	0.455	0.532	0.678	0.788
$|\chi|$	0.05	0.06	0.068	0.089	0.099	0.188	0.258	0.399	0.446	0.582	0.699
FES	0.05	0.062	0.078	0.098	0.109	0.298	0.351	0.471	0.58	0.698	0.723
NMCS	0.05	0.06	0.068	0.089	0.099	0.188	0.258	0.399	0.446	0.582	0.699
FTS	0.052	0.05	0.062	0.078	0.098	0.109	0.298	0.351	0.471	0.58	0.698
CRS	0.051	0.05	0.05	0.051	0.089	0.091	0.153	0.162	0.281	0.389	0.431
KLS	0.05	0.052	0.059	0.064	0.199	0.29	0.31	0.412	0.512	0.524	0.623
LMS	0.052	0.058	0.059	0.167	0.267	0.398	0.487	0.587	0.724	0.876	1
BPS	0.056	0.102	0.143	0.205	0.295	0.431	0.562	0.711	0.879	0.96	0.97

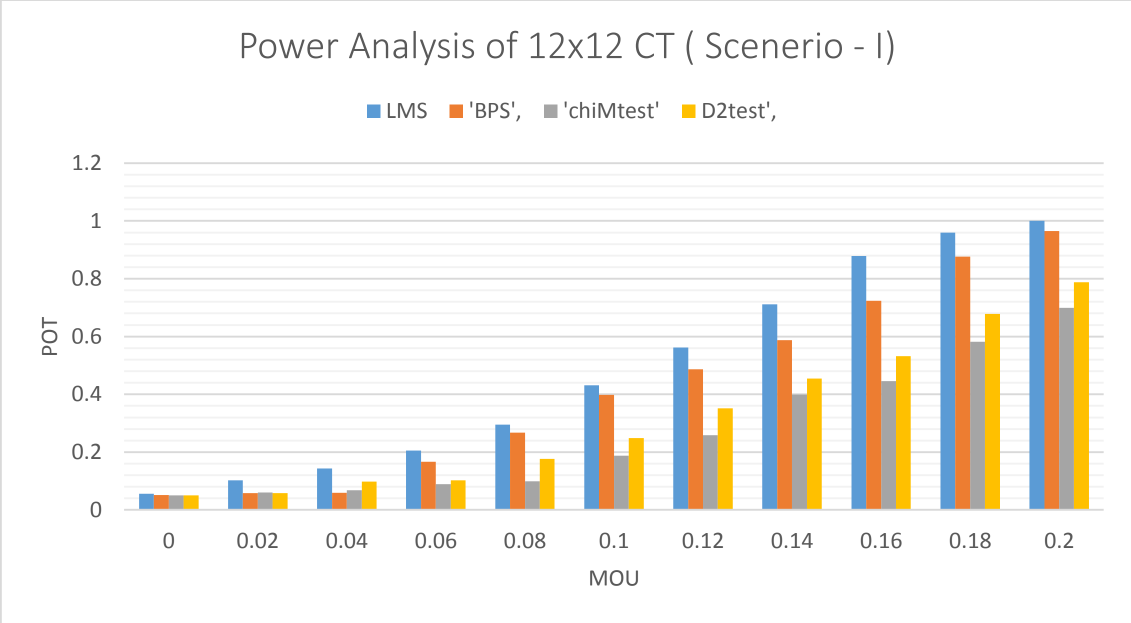


Figure 6.12: Shows Power Analysis of 12x12 CT (SI)

The results of scenario I for Table 6.13 indicates that LMS has maximum power as compared to BPS, D Square, and ChiMtests of independence in 6x6 and 12x12 CTs. The summary of scenario I for several types of CTs is shown in Table 6.14.

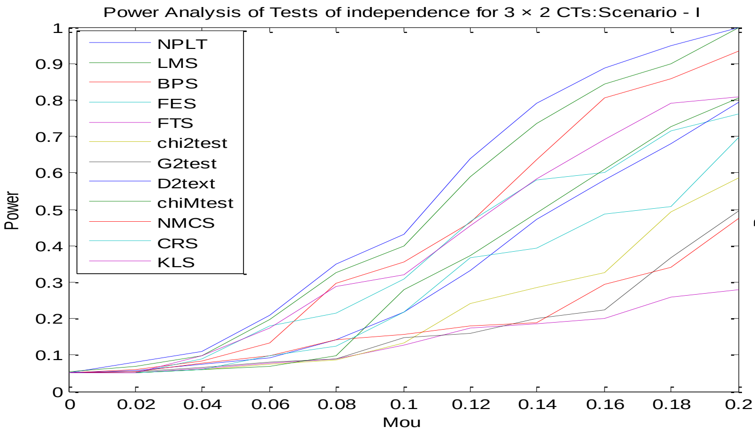
Table 6. 14: Summary of Power for w×k Contingency table Scenario - I

	W×K Contingency table (Power)			
		α = 0.05		
	BPS	LMS	ChiMtest	DSQ
2x3 CT	+++	++++	++	+
3x3 CT	+++	++++	++	+
4x4 CT	+++	++++	++	+
5x5 CT	+++	++++	++	+
6x6 CT	+++	++++	++	+
12x12 CT				

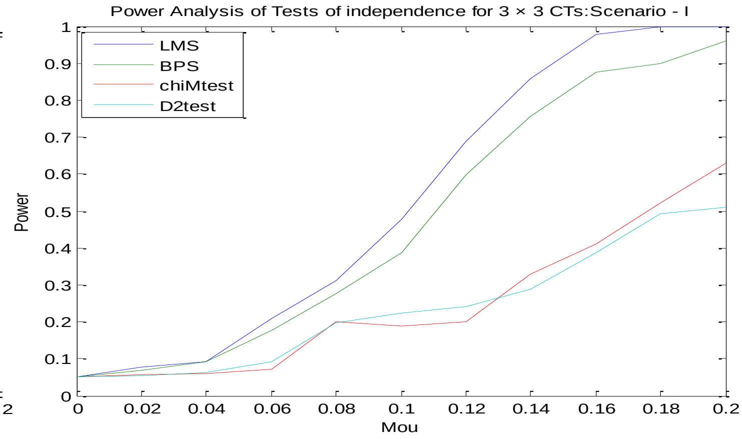
(Note: "+" shows the power of tests as it increases and shows the most powerful tests).

Graphical analysis of W x K CT under scenarios (I-V)

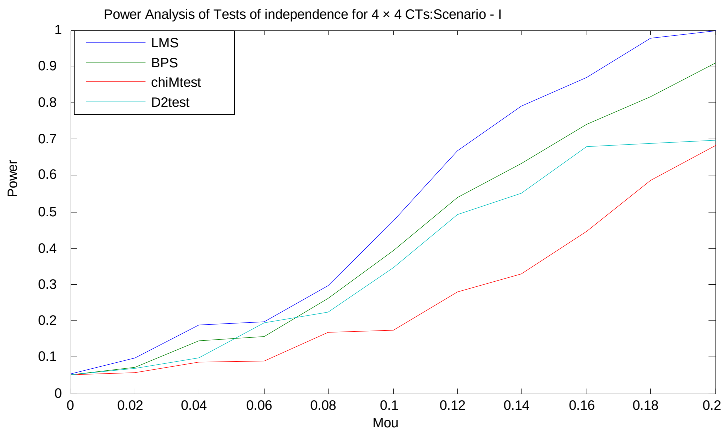
Scenario I: 2x3 CT



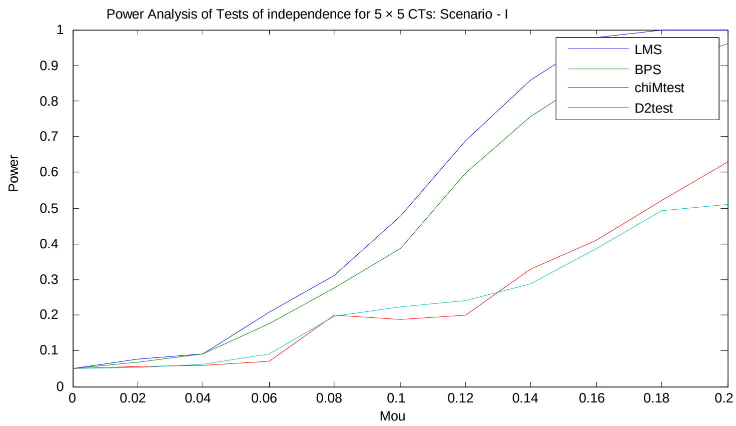
Scenario I: 3x3 CT



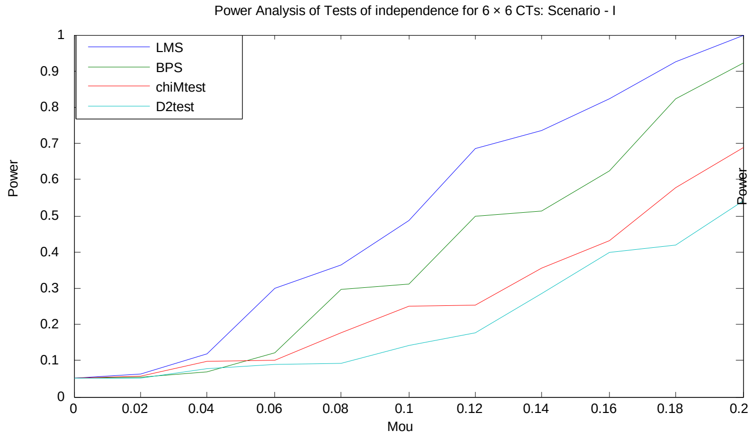
Scenario I: 4x4 CT



Scenario 5x5 CT



Scenario 6x6



Scenario 12x12

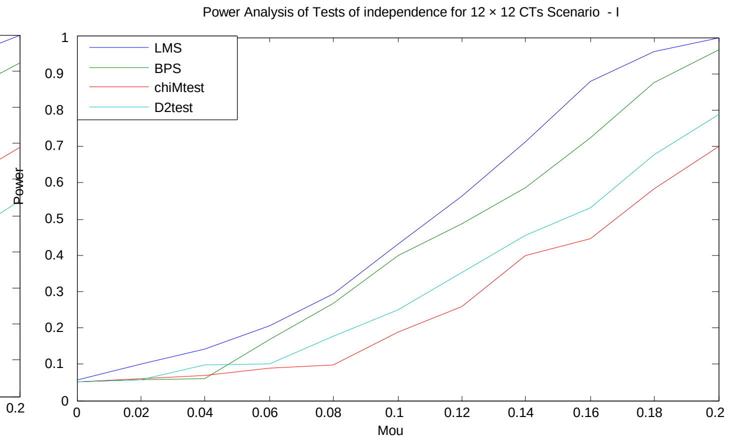


Figure 6.13: Shows Power Graph for W × K CTs

6.2.1 Summary of Power Analysis of CT – Scenario – I

The power is computed for W × K CTs i.e., for CTs 2x3, 3x3,4x4,5x5,6x6, and 12x12, and was found that LMS has the maximum power in all W × K CTs. BPS performs second and the Modular test performs on third number the maximum power among the eleven tests selected under the study.

6.3 Power Analysis of Tests of Independence for Nominal Data in W × K Contingency Table (Scenario II)

We investigated power analysis of tests of independence for nominal data in different scenarios II presented in Table 4.2 for different CTs and found the following results stated in Tables.

Table 6. 15: Power Analysis of Tests of Independence for 2 × 3 CT

(Scenario II)

Nominal Level (α) =5%	A measure of Untruthfulness [MoU]									N=25	
Tests Name	0.00	0.02	0.04	0.06	0.08	0.1	0.12	0.14	0.16	0.18	0.2
χ^2 Test	0.051	0.059	0.045	0.087	0.099	0.197	0.256	0.287	0.345	0.455	0.498
$G^2\ Test$	0.050	0.064	0.098	0.102	0.197	0.241	0.397	0.498	0.431	0.586	0.699
D^2 Test	0.050	0.058	0.067	0.098	0.197	0.267	0.367	0.494	0.554	0.667	0.717
|χ| MDT	0.050	0.058	0.076	0.098	0.165	0.186	0.298	0.356	0.445	0.556	0.634
FES	0.050	0.057	0.072	0.185	0.296	0.345	0.485	0.445	0.578	0.647	0.676
NMCS	0.050	0.058	0.088	0.093	0.267	0.386	0.345	0.495	0.535	0.687	0.795
FTS	0.050	0.057	0.074	0.099	0.197	0.276	0.399	0.499	0.598	0.699	0.745
CRS	0.050	0.054	0.074	0.098	0.165	0.176	0.278	0.367	0.496	0.532	0.644
KLS	0.050	0.053	0.088	0.095	0.188	0.199	0.212	0.345	0.498	0.555	0.632
LMS	0.053	0.073	0.096	0.157	0.268	0.333	0.504	0.649	0.889	0.931	0.98
BPS	0.054	0.098	0.119	0.19	0.306	0.398	0.545	0.71	0.923	0.977	1

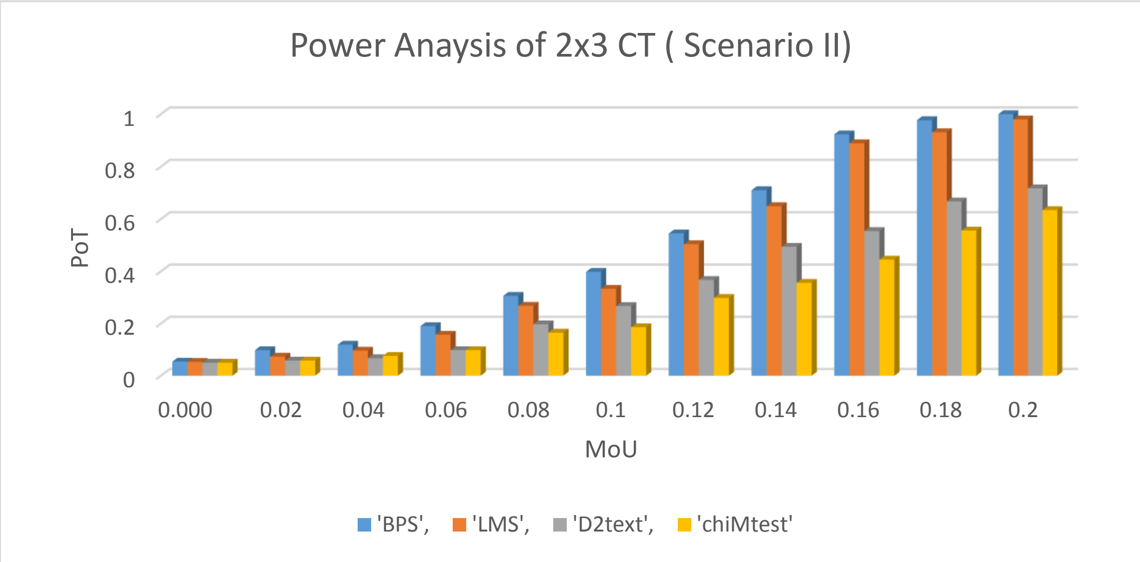


Figure 6.7: Shows the Power of 3x3 CT (SII)

Table 6. 16: Power Analysis of Tests of Independence for 3× 3 Contingency table (Scenario II)

Nominal Level (α) =5%	A measure of Untruthfulness [MoU]									N=50	
Tests Name	0.000	0.020	0.040	0.060	0.080	0.100	0.120	0.140	0.160	0.180	0.200
χ^2 Test	0.05	0.054	0.076	0.098	0.18	0.238	0.342	0.476	0.526	0.623	0.786
G^2 $Test$	0.05	0.054	0.062	0.077	0.085	0.145	0.153	0.195	0.226	0.264	0.297
D^2 Test	0.05	0.053	0.058	0.0972	0.1593	0.258	0.338	0.474	0.583	0.708	0.819
|χ| MDT	0.05	0.051	0.056	0.108	0.134	0.243	0.324	0.4451	0.493	0.571	0.642
FES	0.05	0.051	0.088	0.176	0.284	0.318	0.3738667	0.435	0.495	0.556	0.617
NMCS	0.05	0.052	0.066	0.087	0.112	0.137	0.15	0.173	0.194	0.215	0.236
FTS	0.05	0.051	0.062	0.077	0.089	0.127	0.1274	0.142	0.157	0.171	0.186
CRS	0.05	0.051	0.058	0.072	0.084	0.118	0.1174667	0.130	0.143	0.156	0.169
KLS	0.05	0.051	0.059	0.075	0.088	0.121	0.156	0.155	0.173	0.190	0.207
LMS	0.052	0.059	0.099	0.122	0.206	0.332	0.468	0.58	0.71	0.85	0.95
BPS	0.051	0.056	0.109	0.142	0.256	0.392	0.568	0.735	0.84	0.95	1

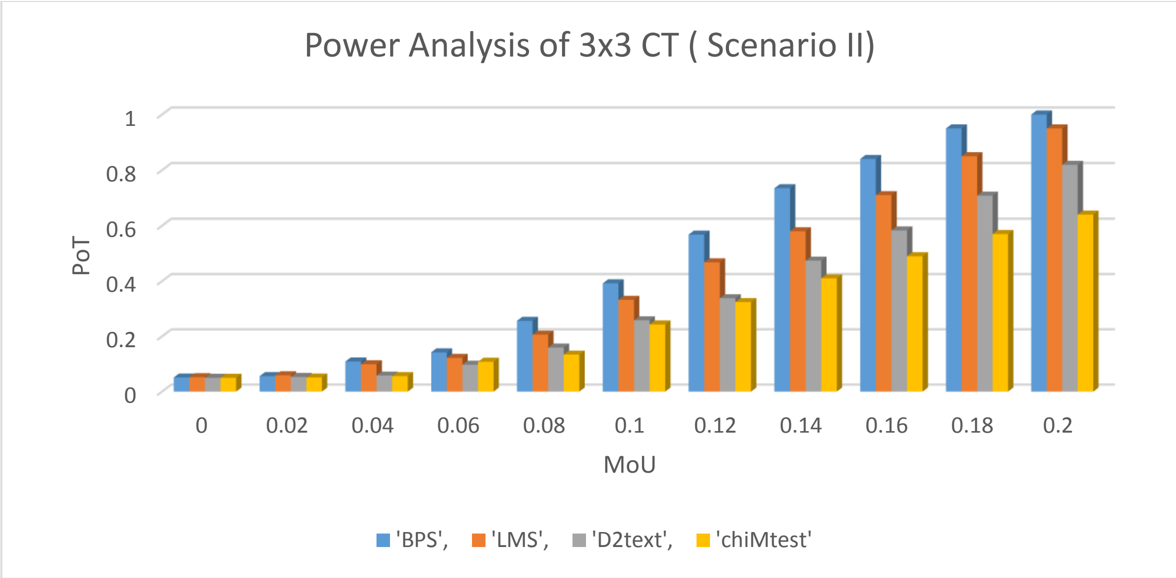


Figure 6.8: Shows Power of 3x3 CT (SII)

Table 6. 17: Power Analysis of Tests of independence for 4 × 4 CT

(Scenario II)

Nominal Level (α) =5%	Measure of Untruthfulness [MoU]									N=100	
Tests Name	0	0.02	0.04	0.06	0.08	0.1	0.12	0.14	0.16	0.18	0.2
χ^2 Test	0.05	0.053	0.061	0.087	0.095	0.167	0.267	0.298	0.387	0.492	0.585
G^2 *Test*	0.05	0.057	0.068	0.094	0.097	0.176	0.199	0.287	0.324	0.388	0.497
D^2 Test	0.051	0.054	0.078	0.095	0.187	0.235	0.401	0.533	0.641	0.7123	0.763
$\lvert\chi\rvert$ MDT	0.05	0.053	0.061	0.065	0.099	0.236	0.381	0.4521	0.59	0.65	0.721
FES	0.05	0.052	0.093	0.185	0.276	0.321	0.467	0.598	0.634	0.787	0.754
NMCS	0.05	0.056	0.079	0.099	0.165	0.187	0.195	0.197	0.298	0.345	0.498
FTS	0.05	0.053	0.065	0.079	0.094	0.107	0.121	0.134	0.148	0.162	0.176
CRS	0.05	0.054	0.066	0.091	0.165	0.234	0.238	0.274	0.311	0.347	0.384
KLS	0.05	0.053	0.067	0.188	0.297	0.334	0.392	0.457	0.522	0.587	0.652
BPS	0.054	0.067	0.144	0.176	0.235	0.387	0.551	0.75	0.89	0.98	1
LMS	0.053	0.0691	0.1604	0.2165	0.289	0.469	0.75	0.94	1	1	1

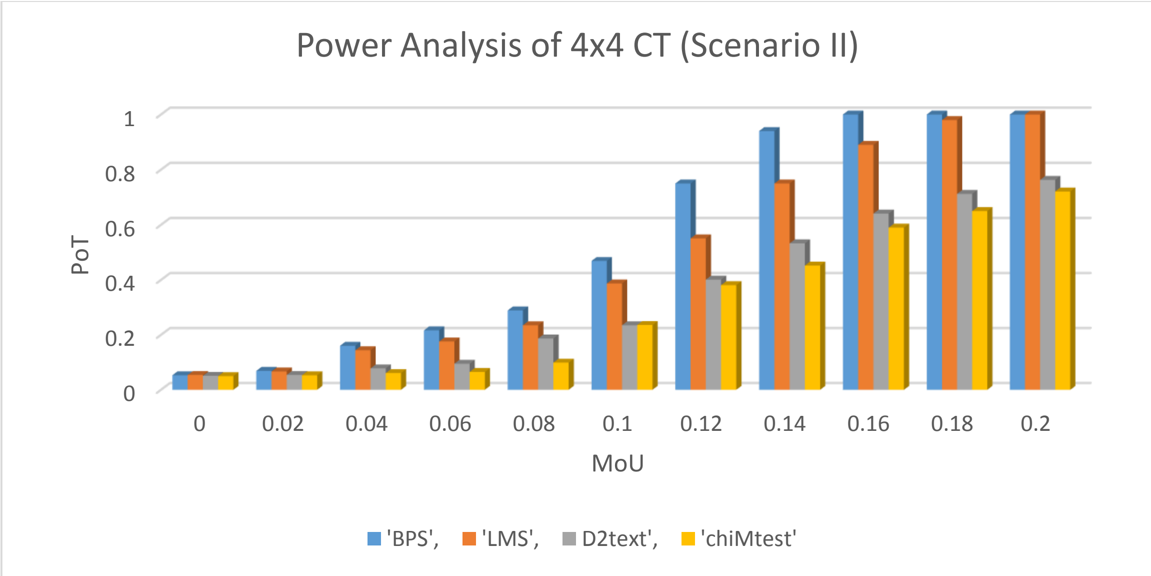


Figure 6.9: Shows Power of 4x4 CT (SII)

Table 6. 18: Power Analysis of Tests of independence for 5 × 5 Contingency Table

(Scenario II)

Nominal Level (α) =5%	Measure of Untruthfulness [MoU]									N=200	
Tests Name	0	0.02	0.04	0.06	0.08	0.1	0.12	0.14	0.16	0.18	0.2
χ^2 Test	0.050	0.066	0.069	0.091	0.094	0.131	0.163	0.216	0.271	0.294	0.419
G^2 *Test*	0.050	0.066	0.078	0.099	0.123	0.146	0.194	0.258	0.381	0.409	0.523
D^2 Test	0.050	0.055	0.067	0.148	0.312	0.414	0.517	0.559	0.646	0.717	0.762
|χ| MDT	0.050	0.063	0.08	0.12	0.249	0.338	0.459	0.511	0.612	0.699	0.741
FES	0.050	0.058	0.071	0.098	0.142	0.168	0.184	0.209	0.234	0.259	0.284
NMCS	0.052	0.056	0.072	0.079	0.089	0.098	0.108	0.118	0.127	0.137	0.147
FTS	0.051	0.057	0.08	0.083	0.093	0.105	0.116	0.128	0.138	0.149	0.160
CRS	0.050	0.057	0.074	0.099	0.153	0.171	0.202	0.233	0.264	0.295	0.325
KLS	0.051	0.069	0.132	0.148	0.262	0.168	0.256	0.290	0.323	0.357	0.391
LMS	0.053	0.099	0.152	0.172	0.338	0.449	0.531	0.631	0.775	0.891	0.971
BPS	0.050	0.098	0.179	0.282	0.392	0.547	0.691	0.821	0.94	0.97	1

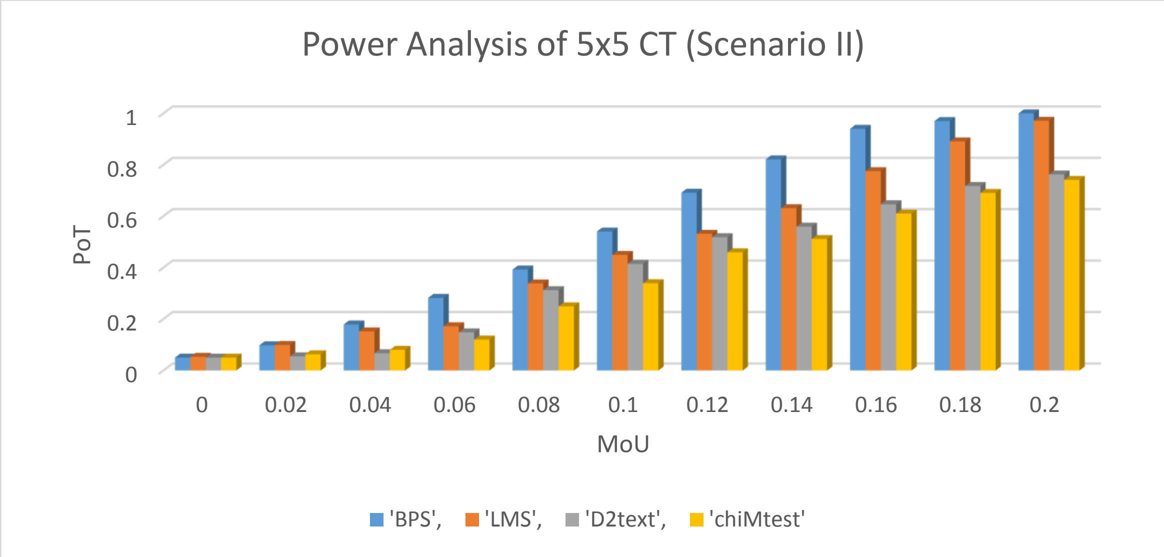


Figure 6.10: Shows Power of 5x5 CT (SII)

Table 6. 19: Power Analysis of Tests of independence for 6 × 6 CT

(Scenario II)

Nominal Level (α) =5%	Measure of Untruthfulness [MoU]									N=400	
Tests Name	0.000	0.020	0.040	0.060	0.080	0.100	0.120	0.140	0.160	0.180	0.200
χ^2 Test	0.051	0.089	0.098	0.239	0.36	0.476	0.488	0.521	0.688	0.789	0.051
G^2 $Test$	0.051	0.05	0.055	0.068	0.09	0.198	0.238	0.368	0.432	0.522	0.631
D^2 Test	0.050	0.05	0.122	0.178	0.298	0.389	0.498	0.551	0.671	0.78	0.898
|χ| MDT	0.052	0.05	0.1051	0.159	0.251	0.343	0.41	0.52	0.58	0.69	0.811
FES	0.053	0.058	0.098	0.102	0.177	0.249	0.252	0.355	0.432	0.578	0.688
NMCS	0.051	0.089	0.098	0.239	0.36	0.476	0.488	0.521	0.688	0.789	0.051
FTS	0.050	0.087	0.0798	0.145	0.252	0.289	0.368	0.432	0.522	0.631	0.668
CRS	0.055	0.05	0.077	0.089	0.093	0.159	0.258	0.33	0.38	0.48	0.57
KLS	0.052	0.058	0.098	0.102	0.177	0.198	0.238	0.368	0.432	0.522	0.631
BPS	0.050	0.07	0.187	0.2798	0.345	0.542	0.61	0.72	0.83	0.91	1
LMS	0.051	0.115	0.219	0.329	0.498	0.651	0.75	0.891	0.971	0.995	1

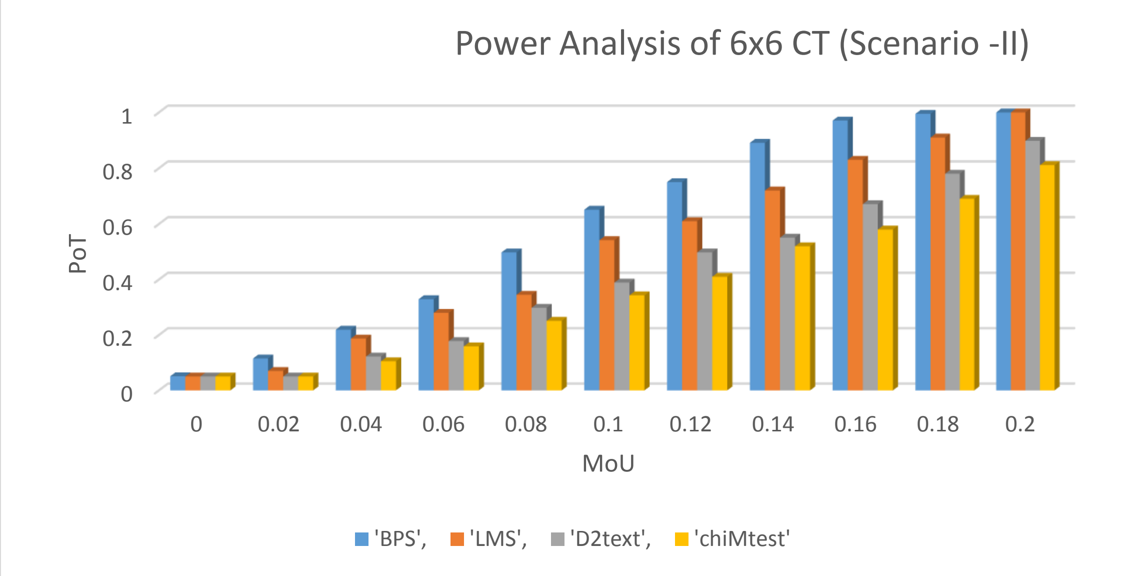


Figure 6.11: Shows Power of 6x6 CT (SII)

Table 6. 20: Power Analysis of Tests of independence for 12 × 12 CT (Scenario II)

Nominal Level (α) =5%	Measure of Untruthfulness [MoU]									N=400	
Tests Name	0.000	0.020	0.040	0.060	0.080	0.100	0.120	0.140	0.160	0.180	0.200
χ^2 Test	0.05	0.06	0.068	0.089	0.099	0.188	0.258	0.399	0.446	0.582	0.699
G^2 $Test$	0.052	0.05	0.062	0.078	0.098	0.109	0.298	0.351	0.471	0.58	0.698
D^2 Test	0.051	0.05	0.095	0.121	0.189	0.291	0.353	0.412	0.4981	0.589	0.731
|χ| MDT	0.05	0.06	0.078	0.109	0.159	0.258	0.33	0.38	0.48	0.57	0.69
FES	0.05	0.052	0.059	0.064	0.199	0.29	0.31	0.412	0.512	0.524	0.623
NMCS	0.051	0.089	0.098	0.239	0.36	0.476	0.488	0.521	0.688	0.789	0.811
FTS	0.05	0.087	0.0798	0.145	0.252	0.289	0.398	0.421	0.577	0.689	0.75
CRS	0.05	0.062	0.078	0.098	0.109	0.298	0.351	0.471	0.58	0.698	0.723
KLS	0.05	0.05	0.063	0.086	0.199	0.349	0.44	0.47	0.51	0.55	0.62
LMS	0.051	0.055	0.119	0.199	0.298	0.311	0.41	0.51	0.63	0.69	0.781
BPS	0.052	0.053	0.167	0.399	0.465	0.587	0.661	0.75	0.81	0.93	1
NPLT	0.052	0.134	0.286	0.417	0.556	0.718	0.851	0.96	0.998	1	1

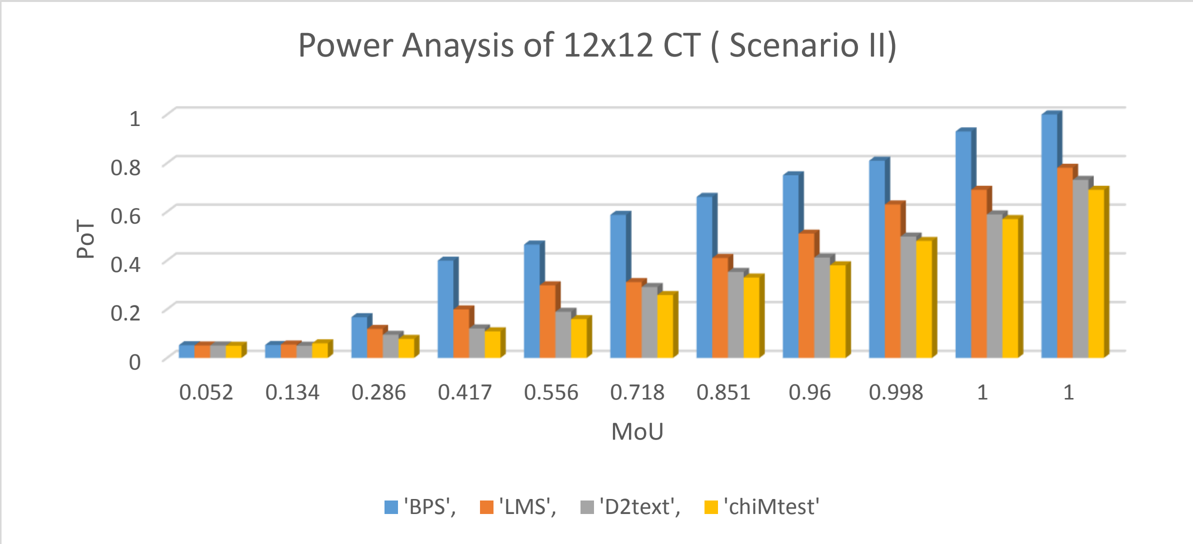


Figure 6.12: Shows Power of 3x3 CT (SII)

The results of scenario II for Table 6.15-20 indicates that BPS have maximum power as compared to LMS, D Square and MDT of independence in different specifications of w×k Contingency tables. The summary of scenario II for several types of CT are shown in Table 6.21.

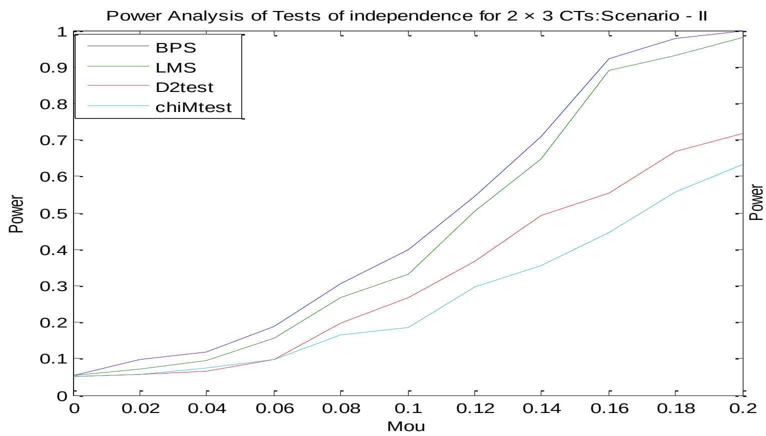
Table 6. 21: Summary of Power for w×k Contingency Table Scenario - II

	W×K Contingency table (Power)			
	LMS	α = 0.05 BPS	DSQ	MDT
2x3 CT	+++	++++	++	+
3x3 CT	+++	++++	++	+
4x4 CT	+++	++++	++	+
5x5 CT	+++	++++	++	+
6x6 CT	+++	++++	++	+
12x12 CT	+++	++++	++	+

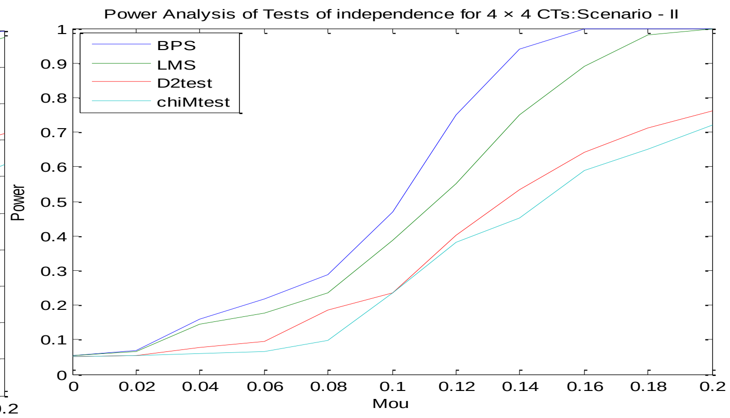
(Note: "+" shows the power of tests as it increases shows the most powerful test).

Graphical analysis of WxK CT under scenarios (II)

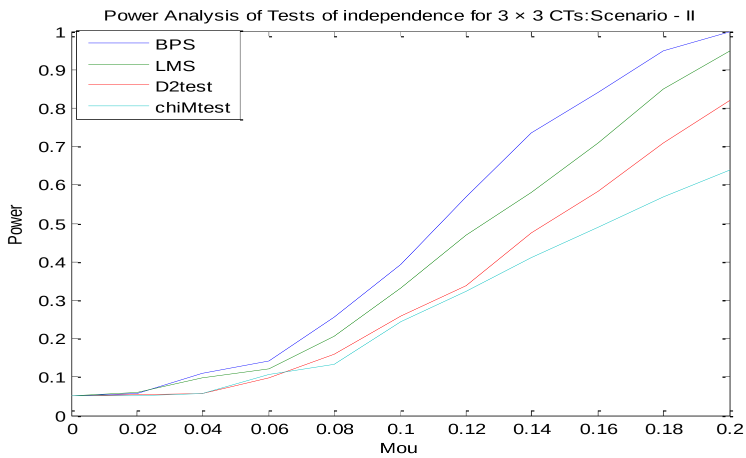
Scenario II: 2x3



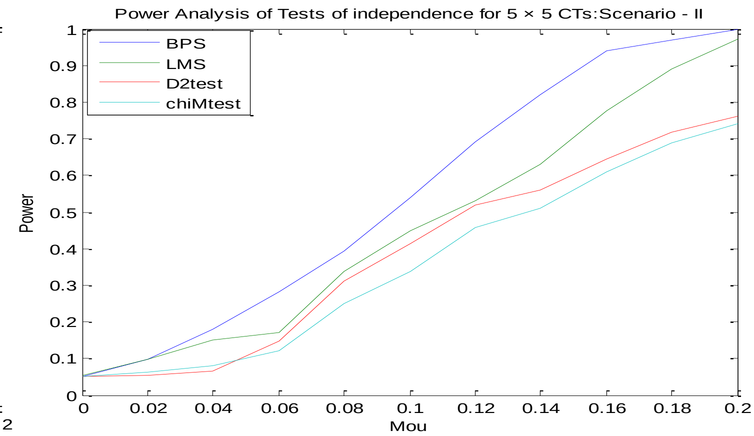
Scenario 4x4



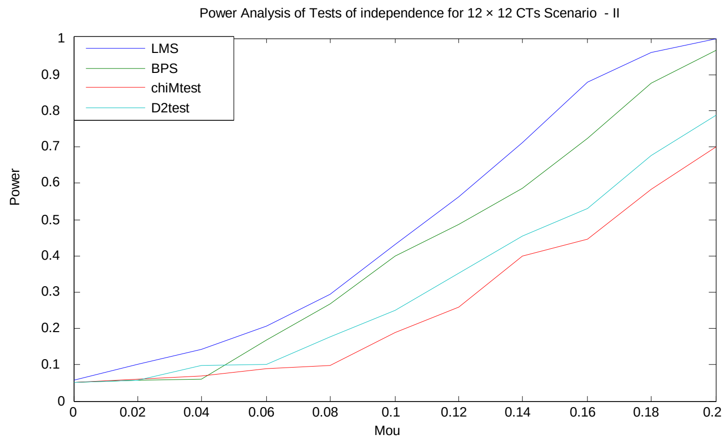
Scenario 3x3



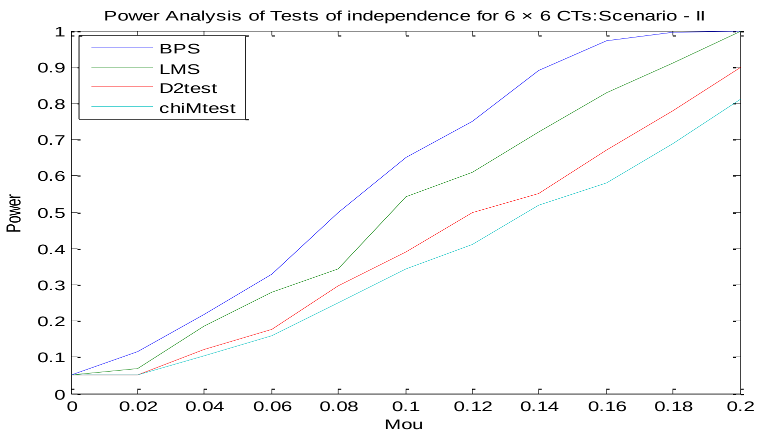
Scenario 5x5



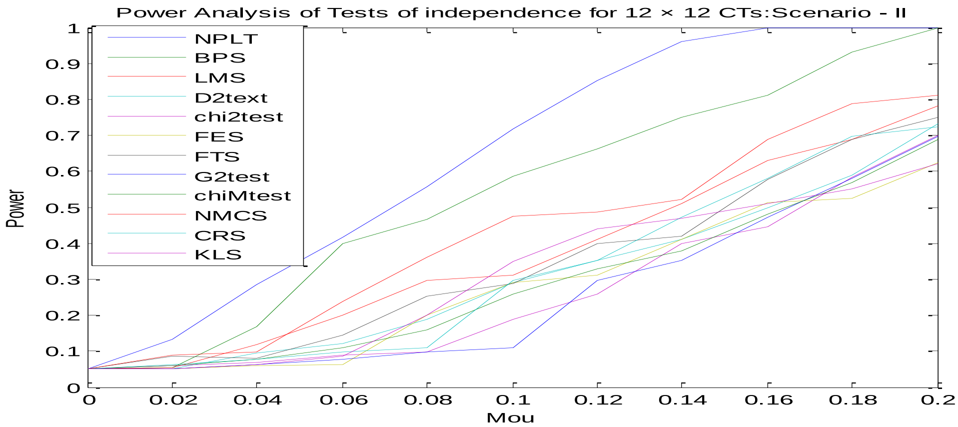
Scenario 12x12



Scenario 6x6



Scenario 12x12



6.3.1 Summary of Power Analysis of CT – Scenario – II

The power is computed for w×k contingency tables and was found that BPS has the maximum power in all higher order contingency tables under scenario II. LMS performs at second and MDT performs on third number the maximum power among the eleven tests selected under the study.

6.4 Power Analysis of Tests of Independence for Nominal data in W × K Contingency table (Scenario III)

We investigated power analysis of tests of independence for nominal data in different scenarios III for different Contingency table and found the following results stated in tables.

Table 6. 22: Power Analysis of Tests of independence for 2 × 3 CTs (Scenario – III)

Nominal Level (α) =5%	Measure of Untruthfulness [MoU]									N=25	
Tests Name	0	0.02	0.04	0.06	0.08	0.1	0.12	0.14	0.16	0.18	0.2
χ^2 Test	0.05	0.053	0.061	0.087	0.095	0.167	0.267	0.298	0.387	0.492	0.585
G^2 *Test*	0.051	0.057	0.068	0.094	0.097	0.176	0.199	0.287	0.324	0.388	0.497
D^2 Test	0.051	0.054	0.078	0.095	0.187	0.235	0.301	0.433	0.501	0.623	0.723
|χ| MDT	0.050	0.053	0.061	0.065	0.145	0.216	0.309	0.312	0.416	0.587	0.699
FES	0.052	0.052	0.093	0.185	0.276	0.321	0.367	0.498	0.534	0.579	0.604
NMCS	0.050	0.056	0.079	0.099	0.165	0.187	0.195	0.197	0.298	0.345	0.498
FTS	0.050	0.053	0.065	0.079	0.094	0.156	0.187	0.19	0.243	0.366	0.498
CRS	0.050	0.054	0.066	0.091	0.165	0.234	0.387	0.398	0.499	0.565	0.601
KLS	0.052	0.053	0.067	0.188	0.297	0.334	0.423	0.587	0.623	0.712	0.834
BPS	0.054	0.067	0.144	0.176	0.235	0.387	0.445	0.634	0.765	0.854	0.934
LMS	0.053	0.069	0.154	0.165	0.289	0.399	0.545	0.787	0.898	0.931	1

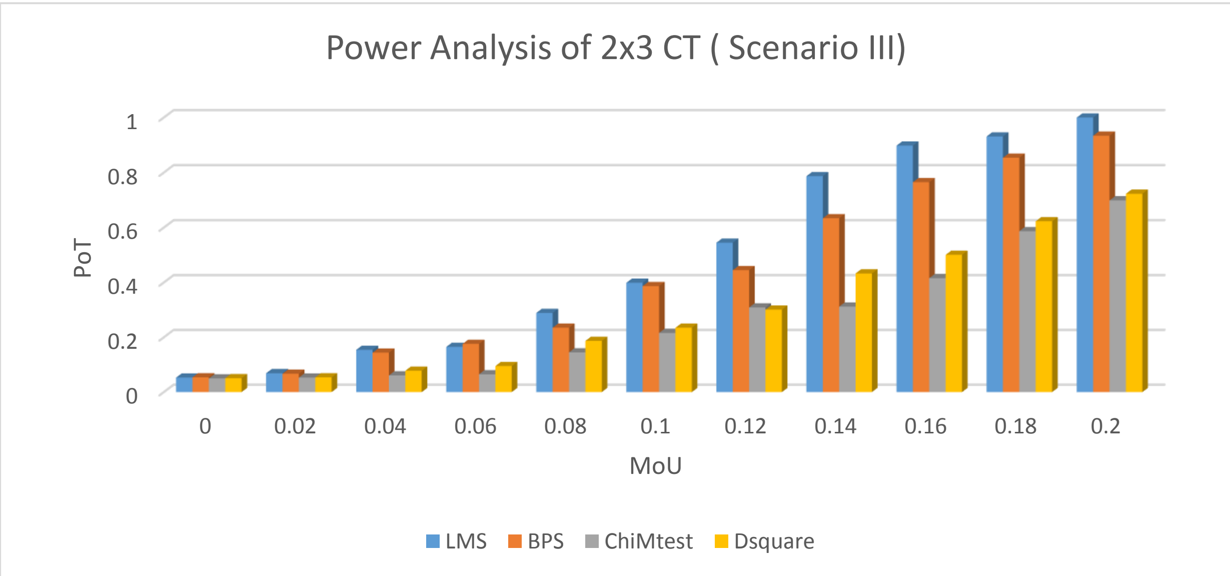


Figure 6.13: Power Analysis of Tests of independence for 3× 3 CTs (Scenario III)

Table 6. 23: Power Analysis of Tests of independence for 3× 3 Contingency table

(Scenario III)

Nominal Level (α) =5%	Measure of Untruthfulness [MoU]									N=50	
Tests Name	0.000	0.020	0.040	0.060	0.080	0.100	0.120	0.140	0.160	0.180	0.200
χ^2 Test	0.051	0.059	0.045	0.087	0.099	0.197	0.256	0.287	0.345	0.455	0.498
G^2 $Test$	0.051	0.064	0.098	0.102	0.197	0.241	0.397	0.498	0.431	0.586	0.699
D^2 Test	0.050	0.058	0.067	0.098	0.197	0.267	0.267	0.294	0.354	0.467	0.587
|χ| MDT	0.052	0.058	0.076	0.098	0.165	0.186	0.198	0.356	0.445	0.556	0.634
FES	0.052	0.057	0.072	0.185	0.296	0.345	0.485	0.445	0.578	0.687	0.776
NMCS	0.051	0.058	0.088	0.093	0.267	0.386	0.345	0.495	0.535	0.687	0.795
FTS	0.050	0.057	0.074	0.099	0.197	0.276	0.399	0.499	0.598	0.699	0.745
CRS	0.050	0.054	0.074	0.098	0.165	0.176	0.278	0.367	0.496	0.532	0.644
KLS	0.050	0.053	0.088	0.095	0.188	0.199	0.212	0.345	0.498	0.555	0.632
BPS	0.053	0.073	0.096	0.187	0.288	0.343	0.534	0.789	0.889	0.893	0.923
LMS	0.054	0.098	0.099	0.19	0.276	0.398	0.545	0.824	0.923	0.977	1

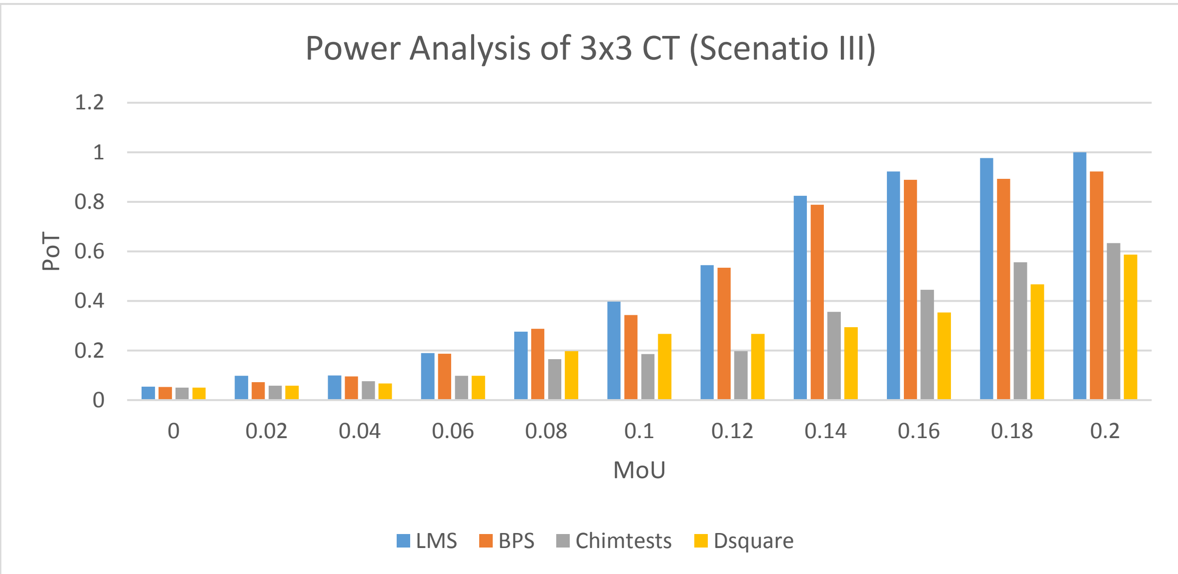


Figure 6.14: Power Analysis of Tests of independence for 3× 3 Contingency table

(Scenario III)

Table 6. 24: Power Analysis of Tests of independence for 4 × 4 CT (Scenario III)

Nominal Level (α) =5%	Measure of Untruthfulness [MoU]									N=100	
Tests Name	0.00	0.02	0.04	0.06	0.08	0.1	0.12	0.14	0.16	0.18	0.2
χ^2 Test	0.051	0.062	0.078	0.098	0.109	0.298	0.351	0.471	0.58	0.698	0.723
G^2 *Test*	0.051	0.055	0.068	0.09	0.198	0.238	0.368	0.432	0.522	0.631	0.732
D^2 Test	0.052	0.05	0.087	0.0798	0.145	0.252	0.289	0.398	0.421	0.577	0.689
|χ| MDT	0.050	0.05	0.077	0.089	0.093	0.142	0.178	0.285	0.399	0.421	0.543
FES	0.050	0.058	0.098	0.102	0.177	0.249	0.252	0.355	0.432	0.578	0.688
NMCS	0.051	0.06	0.068	0.089	0.099	0.188	0.258	0.399	0.446	0.582	0.699
FTS	0.050	0.051	0.089	0.091	0.153	0.162	0.281	0.389	0.431	0.534	0.557
CRS	0.051	0.059	0.045	0.087	0.099	0.197	0.256	0.287	0.345	0.455	0.498
KLS	0.050	0.064	0.098	0.102	0.197	0.241	0.397	0.498	0.431	0.586	0.699
BPS	0.052	0.069	0.102	0.156	0.287	0.367	0.438	0.567	0.778	0.898	0.923
LMS	0.052	0.076	0.109	0.187	0.234	0.388	0.556	0.798	0.835	0.987	1

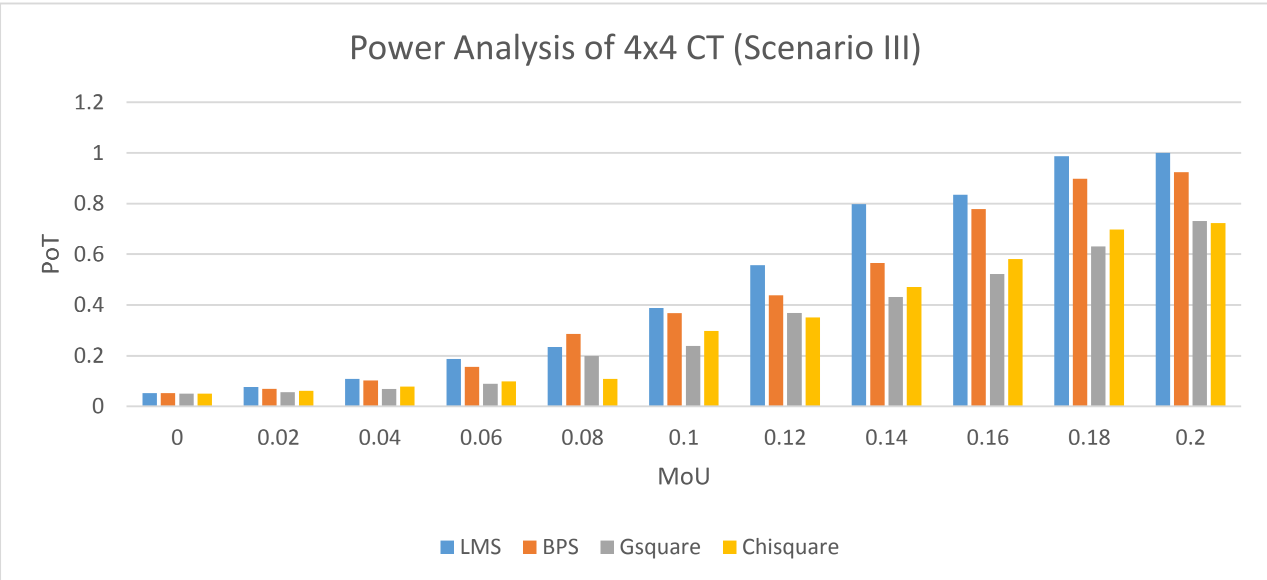


Figure 6.14: Shows Power of 4x4 CT (Scenario - III)

Table 6. 25: Power Analysis of Tests of independence for 5 × 5 CTs (Scenario III)

Nominal Level (α) =5%	Measure of Untruthfulness [MoU]									N=200	
Tests Name	0.00	0.02	0.04	0.06	0.08	0.1	0.12	0.14	0.16	0.18	0.2
χ^2 Test	0.05	0.05	0.055	0.068	0.09	0.198	0.238	0.368	0.432	0.522	0.631
G^2 $Test$	0.05	0.05	0.062	0.078	0.098	0.109	0.298	0.351	0.471	0.58	0.698
D^2 Test	0.05	0.05	0.051	0.089	0.091	0.153	0.162	0.281	0.389	0.431	0.534
|χ| MDT	0.05	0.058	0.098	0.102	0.177	0.249	0.252	0.355	0.432	0.578	0.688
FES	0.051	0.089	0.098	0.239	0.36	0.476	0.488	0.521	0.688	0.789	0.051
NMCS	0.05	0.087	0.0798	0.145	0.252	0.289	0.398	0.421	0.577	0.689	0.05
FTS	0.05	0.05	0.077	0.089	0.093	0.142	0.178	0.285	0.399	0.421	0.543
CRS	0.052	0.058	0.098	0.102	0.177	0.249	0.252	0.355	0.432	0.578	0.688
KLS	0.05	0.05	0.087	0.0798	0.145	0.252	0.289	0.398	0.421	0.577	0.689
BPS	0.051	0.055	0.069	0.099	0.298	0.311	0.499	0.512	0.624	0.823	0.923
LMS	0.052	0.053	0.067	0.299	0.365	0.487	0.687	0.734	0.823	0.925	1

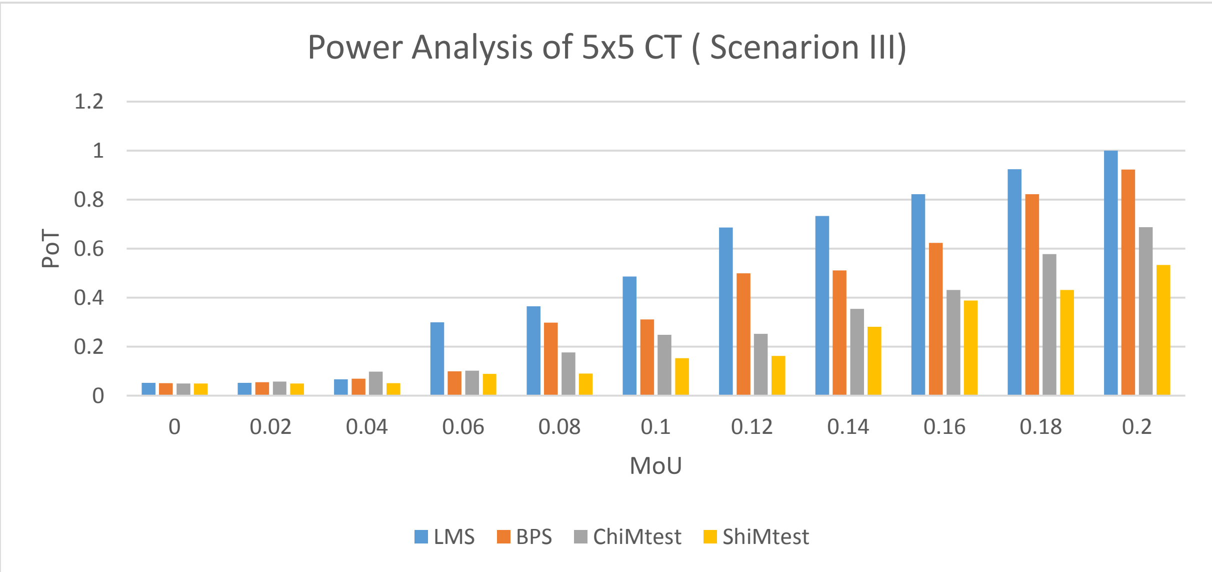


Figure 6.15: Shows Power of 5x5 CT (Scenario - III)

Table 6. 26: Power Analysis of Tests of independence for 6 × 6 CT (Scenario III)

Nominal Level (α) =5%	Measure of Untruthfulness [MoU]									N=400	
Tests Name	0.000	0.020	0.040	0.060	0.080	0.100	0.120	0.140	0.160	0.180	0.200
χ^2 Test	0.05	0.05	0.057	0.087	0.089	0.167	0.175	0.279	0.329	0.446	0.587
G^2 *Test*	0.05	0.05	0.063	0.086	0.199	0.349	0.389	0.598	0.611	0.653	0.632
D^2 Test	0.05	0.051	0.068	0.099	0.193	0.223	0.346	0.494	0.552	0.681	0.688
|χ| MDT	0.05	0.057	0.074	0.099	0.153	0.171	0.188	0.297	0.365	0.422	0.564
FES	0.058	0.098	0.102	0.177	0.249	0.252	0.355	0.432	0.578	0.688	0.586
NMCS	0.05	0.059	0.099	0.111	0.197	0.225	0.358	0.361	0.387	0.475	0.598
FTS	0.052	0.069	0.076	0.098	0.099	0.145	0.269	0.312	0.472	0.598	0.699
CRS	0.05	0.057	0.074	0.099	0.197	0.276	0.399	0.499	0.598	0.699	0.745
KLS	0.05	0.054	0.074	0.098	0.165	0.176	0.278	0.367	0.496	0.532	0.644
BPS	0.052	0.069	0.102	0.156	0.287	0.367	0.438	0.567	0.778	0.898	0.923
LMS	0.052	0.076	0.109	0.187	0.234	0.388	0.556	0.798	0.835	0.987	1

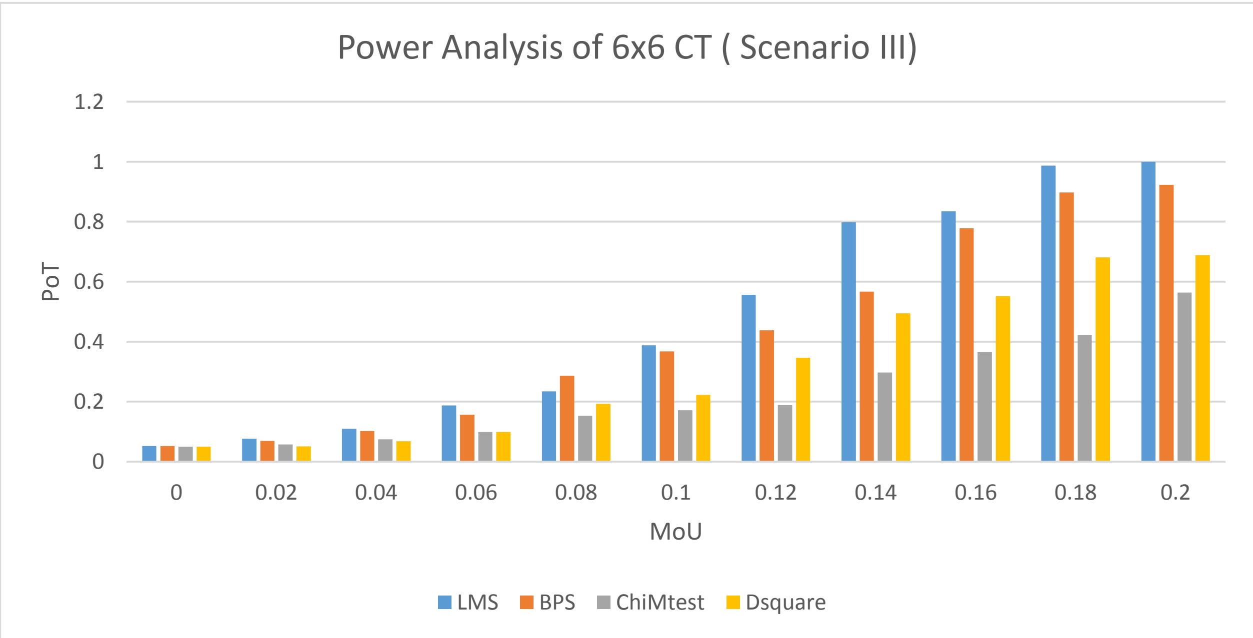


Figure 6.16: Shows Power of 6x6 CT (SIII)

Table 6. 27: Power Analysis of Tests of independence for 12 × 12 CT (Scenario III)

Nominal Level (α) =5%	Measure of Untruthfulness [MoU]									N=400	
Tests Name	0.000	0.020	0.040	0.060	0.080	0.100	0.120	0.140	0.160	0.180	0.200
χ^2 Test	0.05	0.06	0.068	0.089	0.099	0.188	0.258	0.399	0.446	0.582	0.699
G^2 $Test$	0.052	0.05	0.062	0.078	0.098	0.109	0.298	0.351	0.471	0.58	0.698
D^2 Test	0.051	0.05	0.05	0.051	0.089	0.091	0.153	0.162	0.281	0.389	0.573
|χ| MDT	0.05	0.06	0.068	0.089	0.099	0.188	0.258	0.399	0.446	0.582	0.699
FES	0.05	0.052	0.059	0.064	0.199	0.29	0.31	0.412	0.512	0.524	0.623
NMCS	0.051	0.089	0.098	0.239	0.36	0.476	0.488	0.521	0.688	0.789	0.051
FTS	0.05	0.087	0.0798	0.145	0.252	0.289	0.398	0.421	0.577	0.689	0.05
CRS	0.05	0.062	0.078	0.098	0.109	0.298	0.351	0.471	0.58	0.698	0.723
KLS	0.05	0.05	0.063	0.086	0.199	0.349	0.389	0.598	0.611	0.653	0.632
BPS	0.051	0.055	0.069	0.099	0.298	0.311	0.499	0.512	0.624	0.823	0.923
LMS	0.052	0.053	0.067	0.299	0.365	0.487	0.687	0.734	0.823	0.925	1
NPL	0.052	0.164	0.256	0.376	0.488	0.582	0.687	0.776	0.851	0.947	1

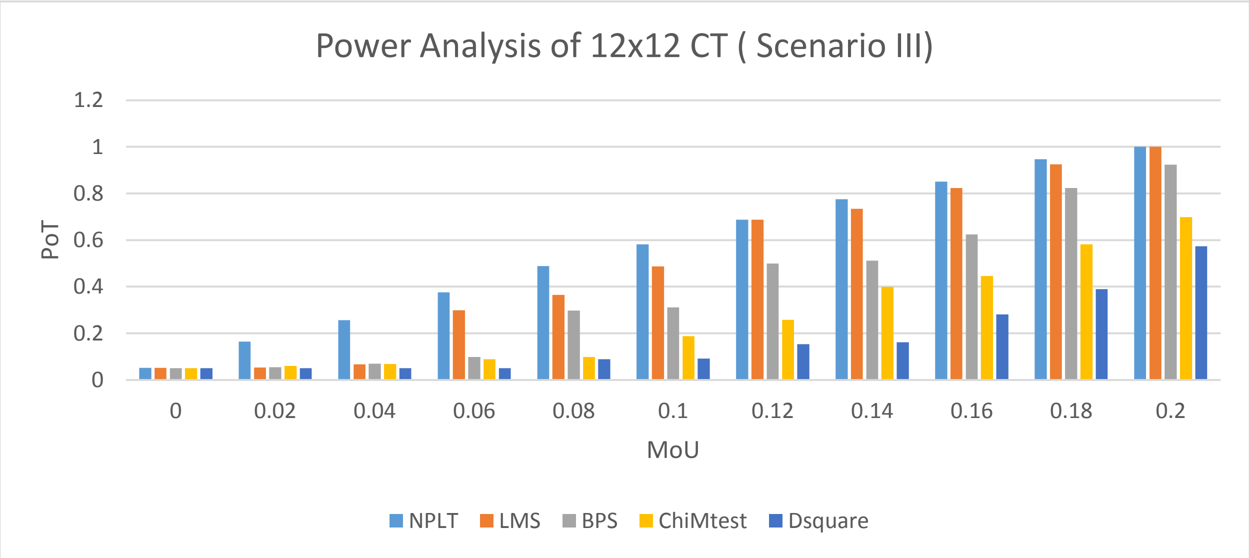


Figure 6.17: Shows Power of 12x12 CT (SIII)

The results of scenario IV for table 6.22-25 indicates that BPS has maximum power compared to LMS and D Square and MDT of independence in 6x6 and 12x12 contingency table. The summary of scenario I for several types of contingency table are shown in table 6.26.

Table 6. 28: Summary of Power for W×K Contingency table Scenario - III

	W×K Contingency table (Power)			
		α = 0.05		
	LMS	BPS	DSQ	MDT
2x3 CT	++++	+++	++	+
3x3 CT	++++	+++	++	+
4x4 CT	++++	+++	++	+
5x5 CT	++++	+++	++	+
6x6 CT	++++	+++	++	+
12x12 CT	++++	+++	++	+

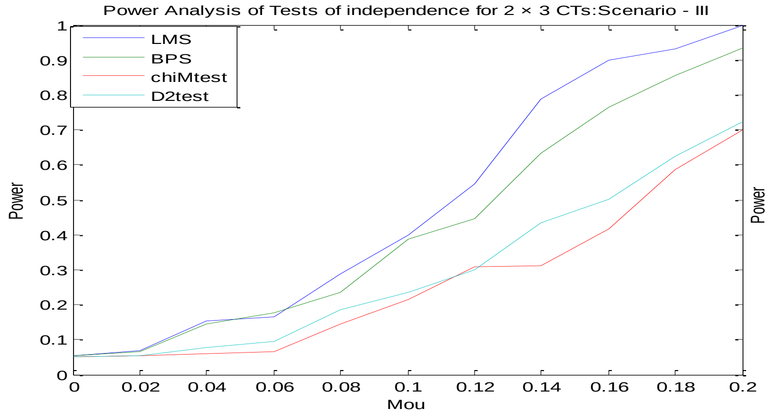
(Note: "+" shows the power of tests as it increases shows the most powerful tests).

From power analysis of of differents test of indepenence from scenerio 1-III, We observe that tests performs different under different DGP. There are some senerios where one test performs better while in other sceneraio other test performs better and thus there is confusion that which test ought to be used for better and reliable results. Therefore, we have used stringency cretion which are discussed in detail after analysis fo all sceneraios.

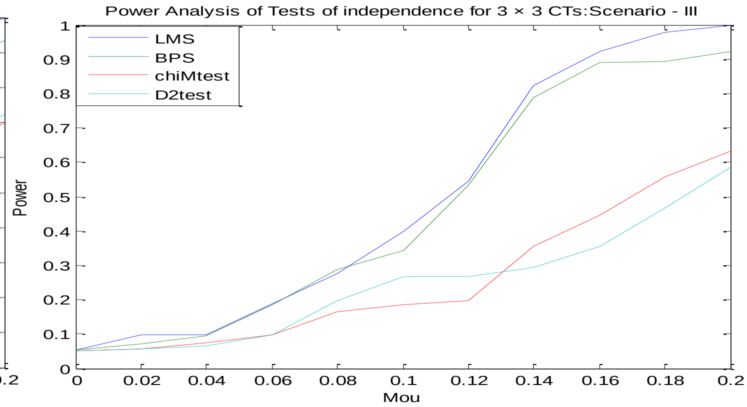
We also presented power comparisons for numerous tests of independence through visualizations i.e., graphical analysis presented below

Scenario III

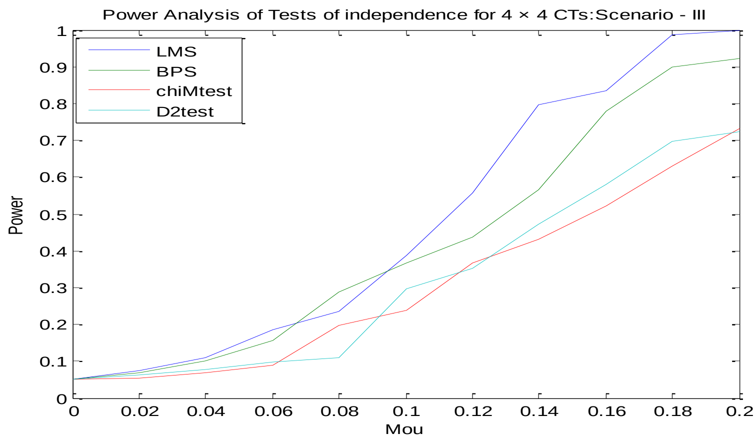
Scenario 2X3



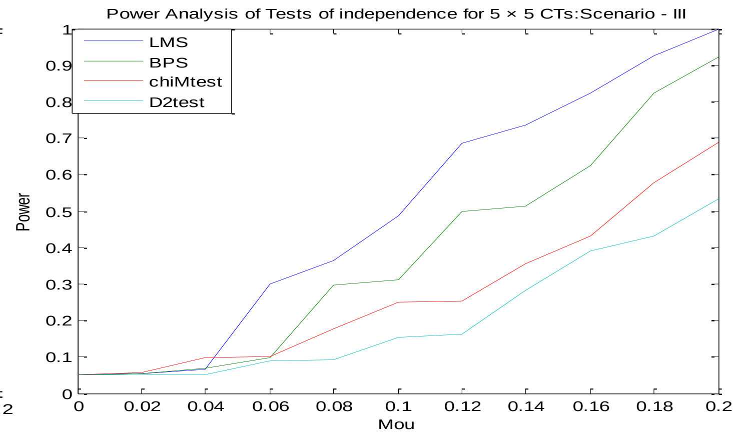
Scenario 3x3



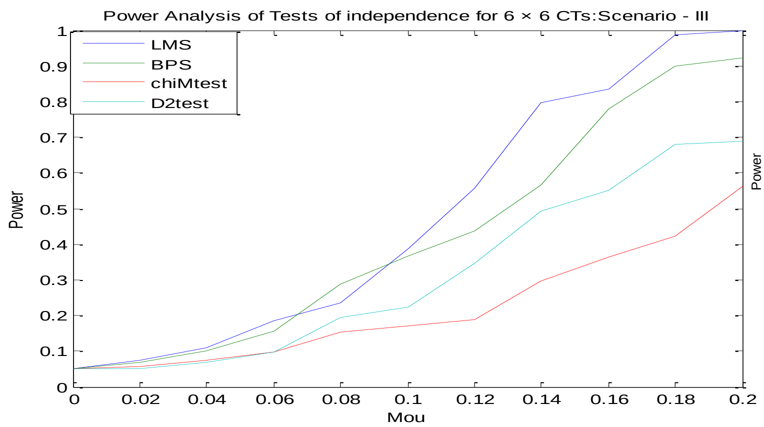
Scenario III 4x4



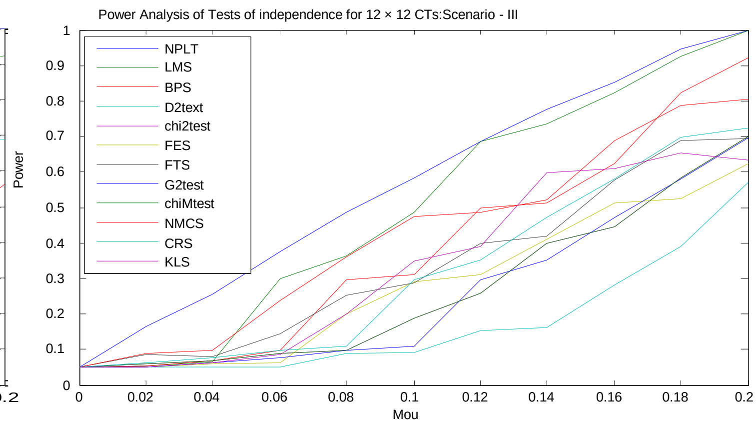
Scenario III 5x5



Scenario III 6x6



Scenario III 12x12



6.5 Power Analysis of Tests of Independence for Nominal data in W × K Contingency table (Scenario IV)

We investigated power analysis of tests of independence for nominal data in different scenarios IV for different Contingency tables and found the following results stated in tables.

Table 6. 29: Power Analysis of Tests of independence for 2 × 3 Contingency table (Scenario IV)

Nominal Level (α) =5%	Measure of Untruthfulness [MoU]									N=25	
Tests Name	0.00	0.02	0.04	0.06	0.08	0.1	0.12	0.14	0.16	0.18	0.2
χ^2 Test	0.051	0.059	0.045	0.087	0.099	0.197	0.256	0.287	0.345	0.455	0.498
$G^2\ Test$	0.050	0.064	0.098	0.102	0.197	0.241	0.397	0.498	0.431	0.586	0.699
D^2 Test	0.050	0.058	0.067	0.098	0.197	0.267	0.31	0.367	0.456	0.543	0.643
|χ| MDT	0.051	0.058	0.076	0.098	0.165	0.186	0.198	0.243	0.365	0.51	0.578
FES	0.052	0.057	0.072	0.185	0.296	0.345	0.485	0.445	0.478	0.511	0.576
NMCS	0.053	0.058	0.088	0.093	0.267	0.386	0.345	0.495	0.535	0.687	0.795
FTS	0.050	0.057	0.074	0.099	0.197	0.276	0.399	0.499	0.598	0.699	0.745
CRS	0.051	0.054	0.074	0.098	0.165	0.176	0.278	0.367	0.496	0.532	0.644
KLS	0.052	0.053	0.088	0.095	0.188	0.199	0.212	0.345	0.498	0.555	0.632
BPS	0.053	0.073	0.096	0.187	0.31	0.387	0.534	0.789	0.889	0.98	0.999
LMS	0.054	0.098	0.099	0.19	0.276	0.325	0.455	0.499	0.593	0.765	0.813

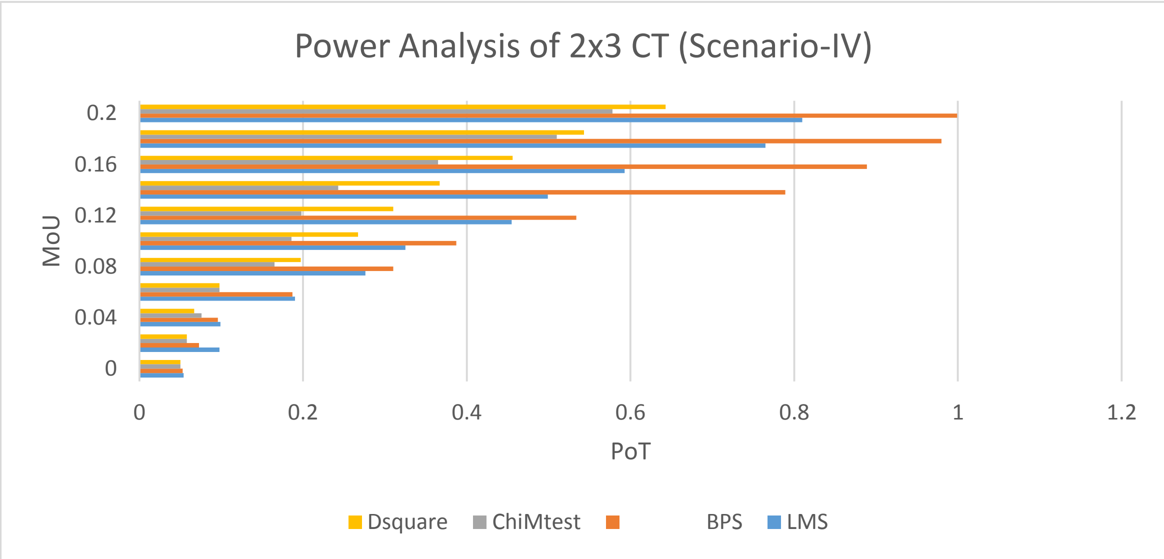


Figure 6.18: Shows Power of 2x3 CT (SIV)

Table 6. 30: Power Analysis of Tests of independence for 3× 3 Contingency table

(Scenario IV)

Nominal Level (α) =5%	Measure of Untruthfulness [MoU]									N=50	
Tests Name	0.000	0.020	0.040	0.060	0.080	0.100	0.120	0.140	0.160	0.180	0.200
χ^2 Test	0.05	0.054	0.076	0.098	0.18	0.238	0.342	0.476	0.526	0.623	0.786
G^2 *Test*	0.05	0.054	0.062	0.077	0.085	0.145	0.153	0.195	0.226	0.264	0.297
D^2 Test	0.05	0.053	0.056	0.072	0.13	0.18	0.21	0.28	0.345	0.42	0.53
$\lvert\chi\rvert$ MDT	0.05	0.054	0.079	0.068	0.14	0.198	0.256	0.398	0.456	0.587	0.713
FES	0.05	0.051	0.088	0.176	0.284	0.318	0.566	0.64	0.702	0.76	0.813
NMCS	0.05	0.052	0.066	0.087	0.112	0.137	0.15	0.18	0.195	0.24	0.274
FTS	0.05	0.051	0.062	0.077	0.089	0.127	0.175	0.186	0.199	0.26	0.279
CRS	0.05	0.051	0.058	0.072	0.084	0.118	0.166	0.172	0.188	0.197	0.298
KLS	0.05	0.051	0.059	0.075	0.088	0.121	0.156	0.183	0.192	0.199	0.307
BPS	0.052	0.059	0.122	0.189	0.298	0.467	0.768	0.937	0.998	0.998	1
LMS	0.051	0.056	0.089	0.112	0.214	0.411	0.598	0.701	0.81	0.823	0.818

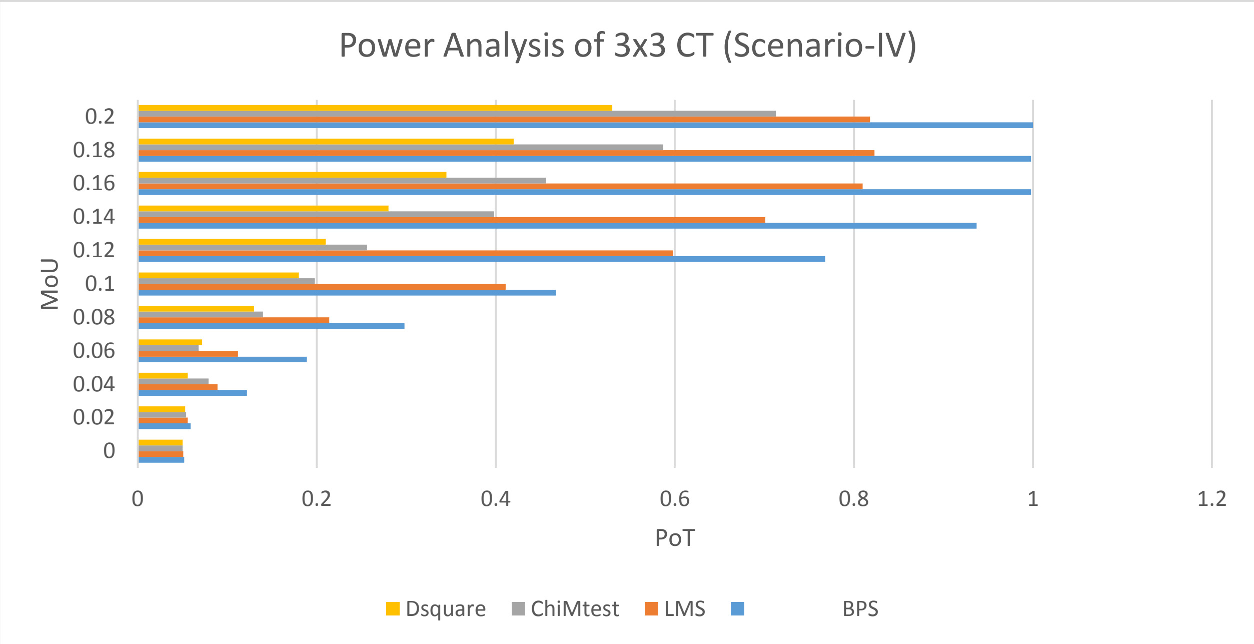


Figure 6.19: Shows Power of 3x3 CT (SIV)

Table 6. 31: Power Analysis of Tests of independence for 4 × 4 CT (Scenario IV)

Nominal Level (α) =5%	Measure of Untruthfulness [MoU]									N=100	
Tests Name	0.00	0.02	0.04	0.06	0.08	0.1	0.12	0.14	0.16	0.18	0.2
χ^2 Test	0.05	0.053	0.061	0.087	0.095	0.167	0.267	0.298	0.387	0.492	0.585
G^2 $Test$	0.05	0.057	0.068	0.094	0.097	0.176	0.199	0.287	0.324	0.388	0.497
D^2 Test	0.05	0.053	0.061	0.065	0.076	0.156	0.245	0.312	0.416	0.587	0.699
|χ| MDT	0.051	0.054	0.078	0.095	0.187	0.235	0.301	0.433	0.501	0.623	0.723
FES	0.05	0.052	0.093	0.185	0.276	0.321	0.187	0.193	0.243	0.366	0.498
NMCS	0.05	0.056	0.079	0.099	0.165	0.187	0.195	0.197	0.298	0.345	0.498
FTS	0.05	0.053	0.065	0.079	0.094	0.156	0.187	0.19	0.243	0.366	0.498
CRS	0.05	0.054	0.066	0.091	0.165	0.234	0.387	0.398	0.499	0.565	0.601
KLS	0.05	0.053	0.067	0.188	0.297	0.334	0.423	0.587	0.623	0.712	0.834
BPS	0.053	0.061	0.104	0.165	0.289	0.399	0.545	0.787	0.898	0.931	1
LMS	0.054	0.067	0.144	0.176	0.235	0.387	0.445	0.634	0.788	0.854	0.885

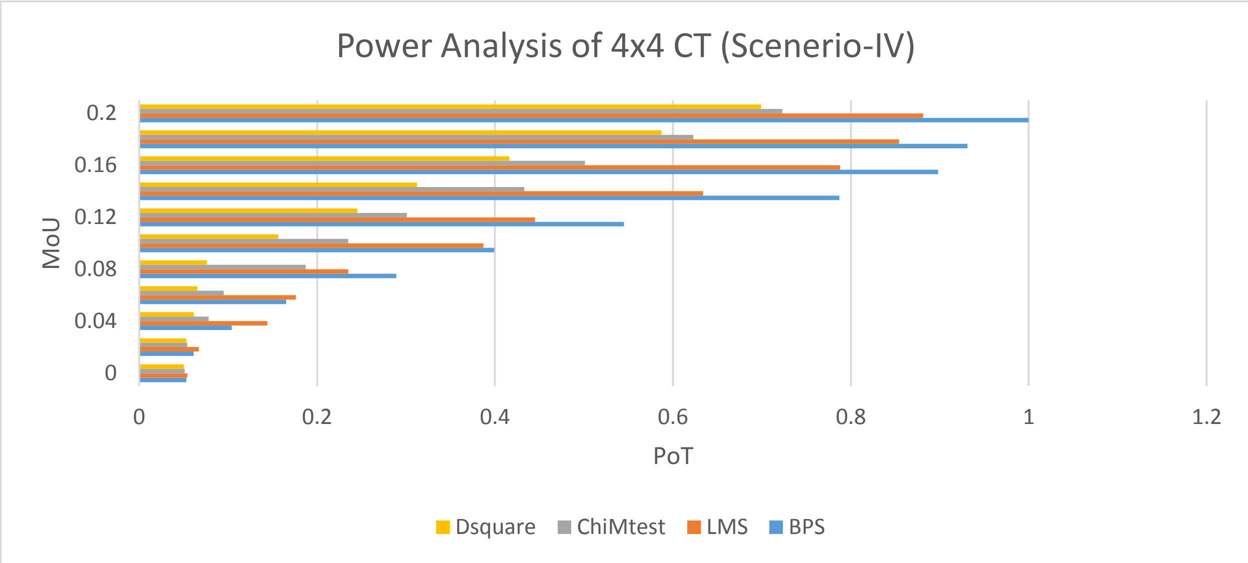


Figure 6.20: Shows Power of 4x4 CT (SIV)

Table 6. 32: Power Analysis of Tests of independence for 5 × 5 CTs (Scenario IV)

Nominal Level (α) =5%	Measure of Untruthfulness [MoU]									N=200	
Tests Name	0.000	0.02	0.04	0.06	0.08	0.1	0.12	0.14	0.16	0.18	0.2
χ^2 Test	0.050	0.066	0.069	0.091	0.094	0.131	0.163	0.216	0.271	0.294	0.419
G^2 *Test*	0.050	0.066	0.078	0.099	0.123	0.146	0.194	0.258	0.381	0.409	0.523
D^2 Test	0.050	0.055	0.067	0.081	0.112	0.140	0.179	0.229	0.346	0.387	0.482
|χ| MDT	0.050	0.063	0.080	0.190	0.349	0.389	0.509	0.685	0.753	0.832	0.978
FES	0.050	0.058	0.071	0.098	0.142	0.168	0.195	0.198	0.251	0.298	0.382
NMCS	0.052	0.056	0.072	0.079	0.107	0.138	0.141	0.149	0.153	0.167	0.188
FTS	0.051	0.057	0.080	0.083	0.093	0.099	0.108	0.140	0.205	0.281	0.300
CRS	0.050	0.057	0.074	0.099	0.153	0.171	0.188	0.197	0.265	0.323	0.397
KLS	0.051	0.069	0.132	0.148	0.262	0.168	0.199	0.194	0.251	0.291	0.482
BPS	0.053	0.099	0.152	0.172	0.280	0.449	0.687	0.891	0.975	0.999	1.000
LMS	0.050	0.058	0.079	0.082	0.097	0.392	0.540	0.633	0.841	0.918	0.976

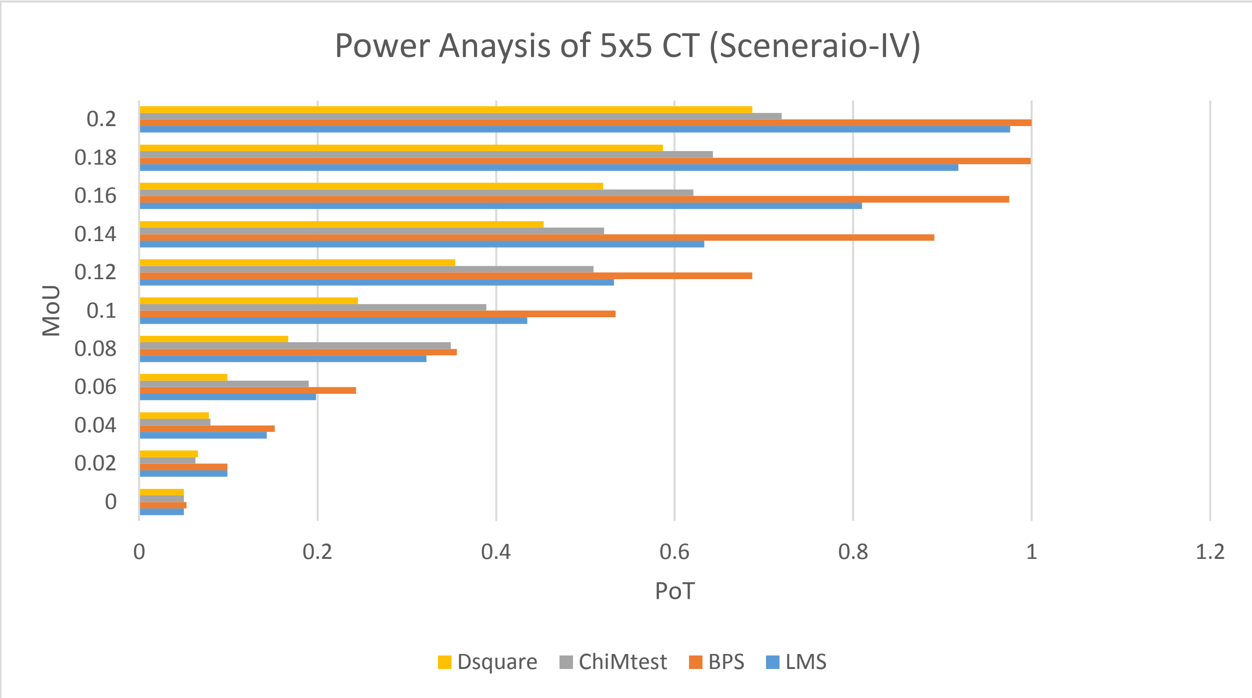


Figure 6.21: Shows Power of 5x5 CT (SIV)

Table 6. 33: Power Ana9ysis of Tests of independence for 6 × 6 CT (Scenario IV)

Nominal Level (α) =5%	Measure of Untruthfulness [MoU]									N=400	
Tests Name	0.000	0.020	0.040	0.060	0.080	0.100	0.120	0.140	0.160	0.180	0.200
χ^2 Test	0.05	0.066	0.088	0.113	0.246	0.273	0.332	0.497	0.559	0.657	0.789
G^2 *Test*	0.05	0.05	0.055	0.068	0.09	0.198	0.238	0.368	0.432	0.522	0.631
D^2 Test	0.05	0.05	0.062	0.1	0.2	0.26	0.31	0.41	0.523	0.598	0.678
|χ| MDT	0.05	0.064	0.06	0.11	0.24	0.27	0.332	0.494	0.556	0.618	0.78
FES	0.05	0.058	0.098	0.102	0.177	0.249	0.252	0.355	0.432	0.578	0.688
NMCS	0.051	0.089	0.098	0.239	0.36	0.476	0.488	0.521	0.688	0.789	0.051
FTS	0.05	0.087	0.0798	0.145	0.252	0.289	0.398	0.421	0.577	0.689	0.05
CRS	0.05	0.05	0.077	0.089	0.093	0.142	0.178	0.285	0.399	0.421	0.543
KLS	0.052	0.058	0.098	0.102	0.177	0.249	0.252	0.355	0.432	0.578	0.688
BPS	0.052	0.098	0.186	0.298	0.376	0.487	0.5443	0.654	0.854	0.93	1
LMS	0.051	0.088	0.069	0.12	0.28	0.387	0.499	0.543	0.72	0.823	0.923

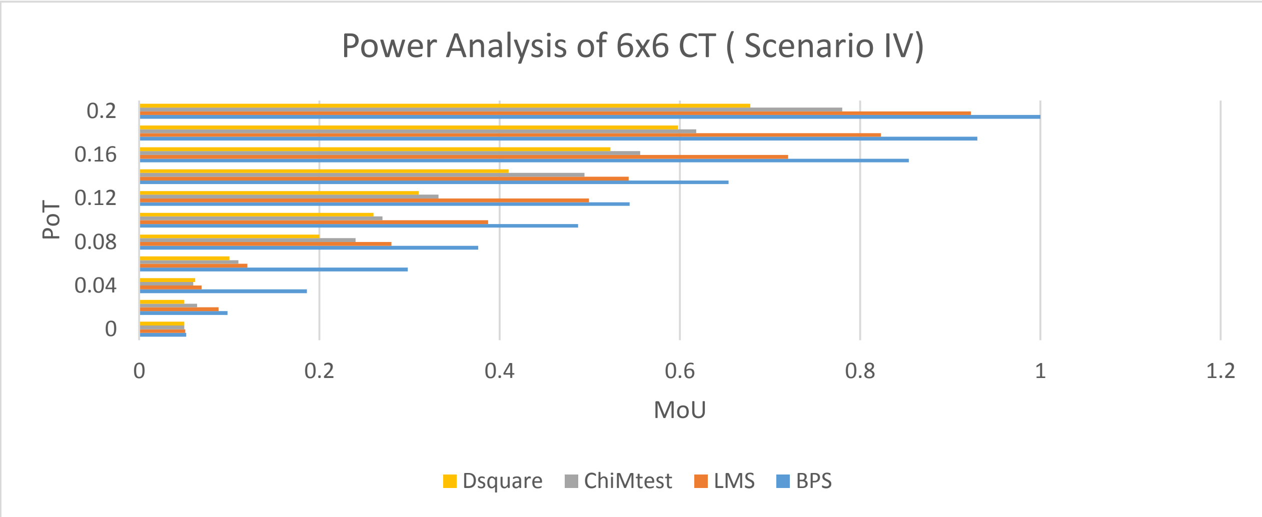


Figure 6.22: Shows Power of 6x6 CT (SIV)

Table 6. 34: Power Analysis of Tests of independence for 12 × 12 CT (Scenario IV)

Nominal Level (α) =5%	Measure of Untruthfulness [MoU]									N=400	
Tests Name	0.000	0.020	0.040	0.060	0.080	0.100	0.120	0.140	0.160	0.180	0.200
χ^2 Test	0.05	0.06	0.068	0.089	0.099	0.188	0.258	0.399	0.446	0.582	0.699
G^2 $Test$	0.052	0.05	0.062	0.078	0.098	0.109	0.298	0.351	0.471	0.58	0.698
D^2 Test	0.051	0.05	0.05	0.051	0.089	0.091	0.153	0.162	0.281	0.389	0.431
|χ| MDT	0.051	0.087	0.078	0.095	0.187	0.235	0.301	0.433	0.501	0.623	0.723
FES	0.05	0.052	0.059	0.064	0.199	0.29	0.31	0.412	0.512	0.524	0.623
NMCS	0.051	0.089	0.098	0.239	0.36	0.476	0.488	0.521	0.688	0.789	0.051
FTS	0.05	0.087	0.0798	0.145	0.252	0.289	0.398	0.421	0.577	0.689	0.05
CRS	0.05	0.062	0.078	0.098	0.109	0.298	0.351	0.471	0.58	0.698	0.723
KLS	0.05	0.05	0.063	0.086	0.199	0.349	0.389	0.598	0.611	0.653	0.632
BPS	0.051	0.088	0.069	0.12	0.28	0.387	0.499	0.543	0.72	0.976	1
LMS	0.052	0.053	0.067	0.299	0.365	0.487	0.687	0.734	0.823	0.925	1
NPL	0.052	0.098	0.256	0.387	0.486	0.598	0.787	0.876	0.956	0.999	1

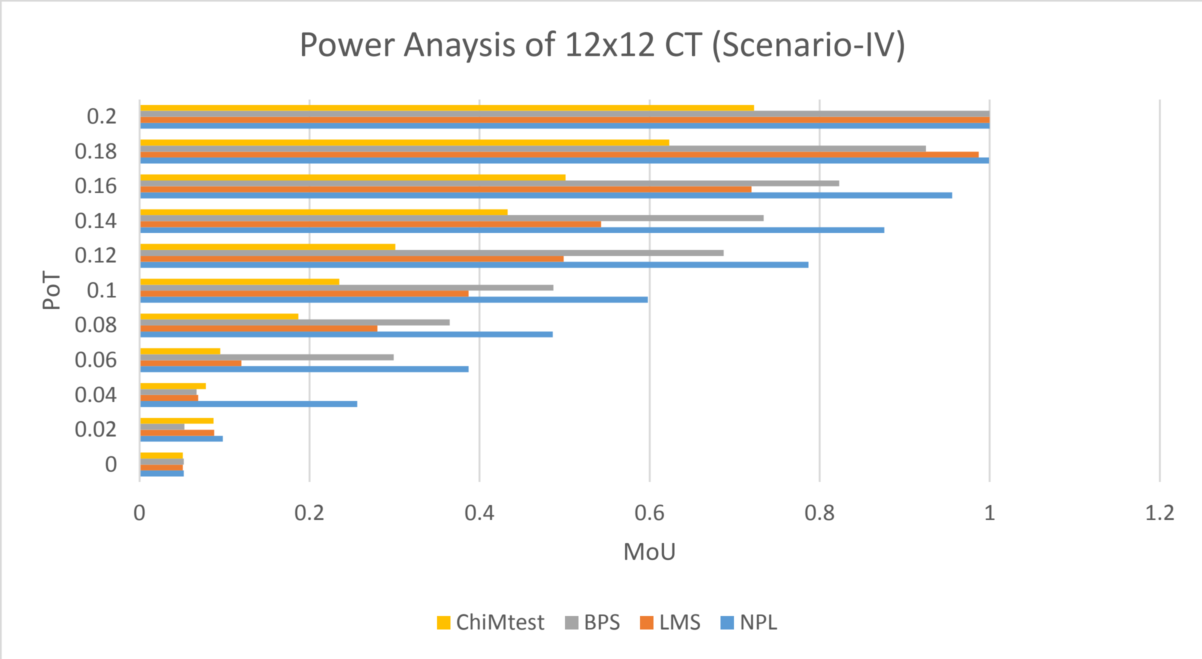


Figure 6.23: Shows Power of 12x12 CT (SIV)

The results of scenario I for table 6.4.5-6 indicates that BPS have maximum power as compared to LMS and D Square and ChiMtests of independence in 6x6 and 12x12 Contingency table. The summary of scenario I for several types of Contingency table are shown in Table 6.27.

Scenario IV 12x12

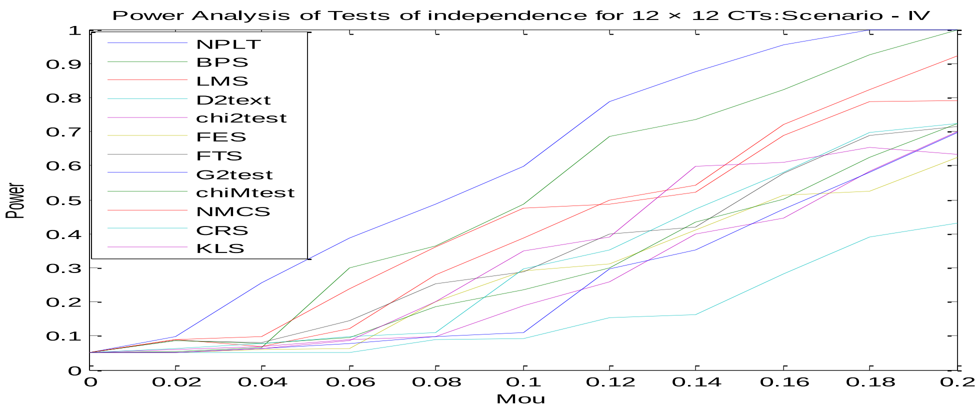


Table 6. 35: Summary of Power for W×K Contingency table Scenario - IV

	W×K Contingency table (Power)			
	α = 0.05			
	LMS	**BPS**	**ChiMtest**	**DSQ**
2x3 CT	+++	++++	++	+
3x3 CT	+++	++++	++	+
4x4 CT	+++	++++	++	+
5x5 CT	+++	++++	++	+
6x6 CT	+++	++++	++	+
12x12 CT	+++	++++	++	+

(Note: “+” shows the power of tests as it increases shows the most powerful tests).

6.5.1 Summary of Power Analysis of CT – Scenario – IV

The power is computed for higher order contingency table and was found that BPS has the maximum power in all the higher order contingency tables. LMS performs at second and MDT performs on third number the maximum power among the eleven tests selected under the study.

6.6 Power Analysis of Tests of Independence in W × K CTs (Scenario V)

We investigated power analysis of tests of independence for nominal data in different scenarios V for different Contingency table and found the following results stated in tables.

Table 6. 36: Power Analysis of Tests of independence for 2 × 3 CTs (Scenario V)

Nominal Level (α) =5%	Measure of Untruthfulness [MoU]									N=25	
Tests Name	0.000	0.020	0.040	0.060	0.080	0.100	0.120	0.140	0.160	0.180	0.200
χ^2 Test	0.050	0.054	0.076	0.098	0.18	0.238	0.342	0.476	0.526	0.623	0.786
G^2 $Test$	0.050	0.054	0.062	0.077	0.085	0.145	0.153	0.195	0.226	0.264	0.297
D^2 Test	0.051	0.066	0.078	0.124	0.253	0.31	0.429	0.59	0.632	0.699	0.71
|χ| MDT	0.050	0.097	0.1765	0.376	0.3987	0.473	0.576	0.672	0.722	0.865	0.965
FES	0.050	0.051	0.088	0.176	0.284	0.318	0.566	0.64	0.702	0.76	0.813
NMCS	0.052	0.052	0.066	0.087	0.112	0.137	0.15	0.18	0.195	0.24	0.274

FTS	0.051	0.051	0.062	0.077	0.089	0.127	0.175	0.186	0.199	0.26	0.279
CRS	0.050	0.051	0.058	0.072	0.084	0.118	0.166	0.172	0.188	0.197	0.298
KLS	0.052	0.051	0.059	0.075	0.088	0.121	0.156	0.183	0.192	0.199	0.307
BPS	0.053	0.099	0.124	0.346	0.4232	0.499	0.565	0.599	0.632	0.765	0.841
LMS	0.051	0.098	0.145	0.287	0.42	0.567	0.673	0.875	0.898	0.965	0.979

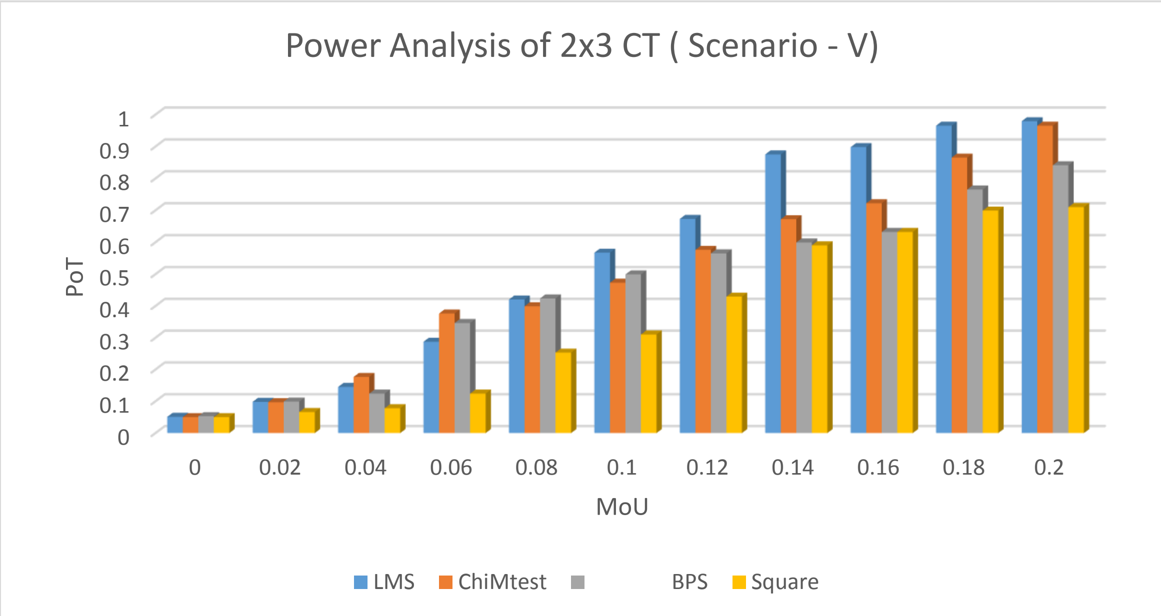


Figure 6.24: Power Analysis Graph 2x3 (SV)

Table 6. 37: Power Analysis of Tests of independence for 3× 3 CTs (Scenario – V)

Nominal Level (α) =5%	Measure of Untruthfulness [MoU]									N=50	
Tests Name	0.000	0.020	0.040	0.060	0.080	0.100	0.120	0.140	0.160	0.180	0.200
χ^2 Test	0.051	0.054	0.065	0.067	0.075	0.145	0.148	0.152	0.23	0.253	0.29
G^2 *Test*	0.051	0.056	0.062	0.077	0.087	0.147	0.157	0.196	0.236	0.27	0.299
D^2 Test	0.052	0.063	0.068	0.089	0.194	0.218	0.276	0.389	0.401	0.456	0.511
$\lvert\chi\rvert$ MDT	0.050	0.053	0.1	0.21	0.298	0.387	0.498	0.71	0.754	0.876	0.876
FES	0.052	0.052	0.06	0.075	0.087	0.227	0.278	0.364	0.42	0.472	0.479
NMCS	0.051	0.052	0.082	0.086	0.131	0.168	0.172	0.184	0.21	0.271	0.318
FTS	0.053	0.052	0.061	0.07	0.092	0.147	0.184	0.198	0.221	0.277	380
CRS	0.052	0.052	0.06	0.077	0.087	0.138	0.171	0.181	0.192	0.21	0.312

KLS	0.050	0.051	0.062	0.071	0.086	0.132	0.173	0.188	0.192	0.223	0.382
BPS	0.050	0.06	0.098	0.132	0.244	0.331	0.41	0.523	0.654	0.699	0.765
LMS	0.050	0.078	0.123	0.2876	0.309	0.435	0.567	0.731	0.887	0.99	1

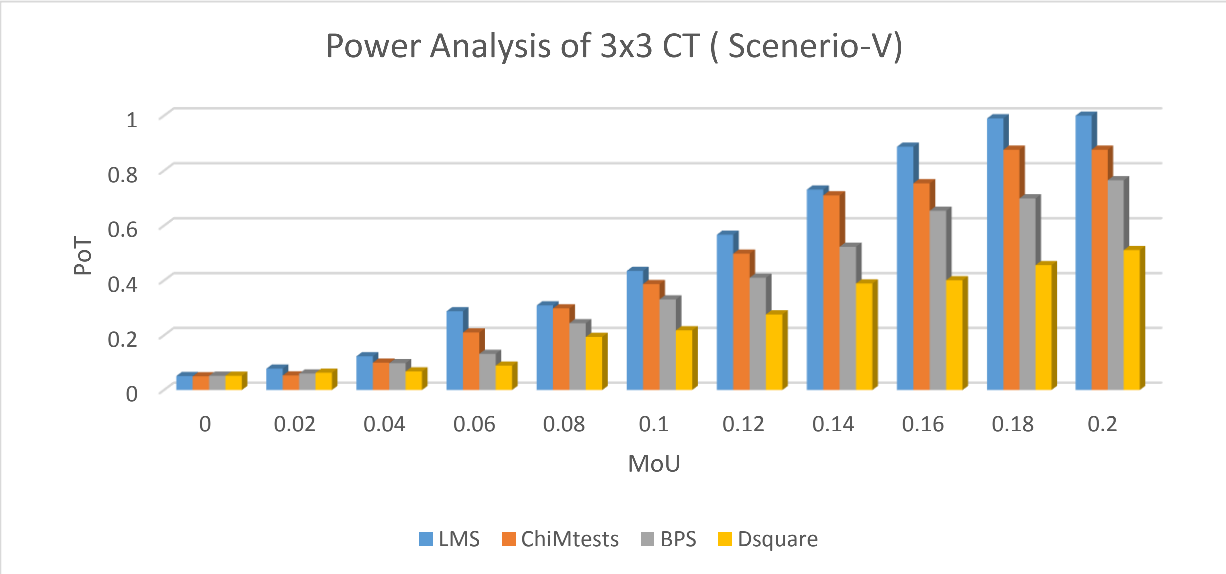


Figure 6.25: Power Analysis Graph 3x3 (SV)

Table 6. 38: Power Analysis of Tests of independence for 4 × 4 CTs Scenario - V

Nominal Level (α) =5%	Measure of Untruthfulness [MoU]									N=100	
Tests Name	0.000	0.020	0.040	0.060	0.080	0.100	0.120	0.140	0.160	0.180	0.200
χ^2 Test	0.05	0.056	0.068	0.069	0.077	0.149	0.152	0.155	0.232	0.255	0.299
G^2 $Test$	0.05	0.058	0.061	0.079	0.089	0.148	0.158	0.199	0.246	0.282	0.399
D^2 Test	0.05	0.052	0.069	0.071	0.098	0.12	0.151	0.171	0.18	0.198	0.222
|χ| MDT	0.05	0.051	0.062	0.07	0.107	0.138	0.168	0.232	0.322	0.331	0.432
FES	0.05	0.051	0.063	0.078	0.197	0.22	0.482	0.674	0.723	0.882	0.932
NMCS	0.05	0.051	0.086	0.09	0.139	0.16	0.176	0.188	0.221	0.284	0.299
FTS	0.05	0.05	0.066	0.072	0.11	0.156	0.184	0.198	0.221	0.277	0.38
CRS	0.05	0.05	0.062	0.079	0.093	0.144	0.171	0.185	0.199	0.221	0.343

KLS	0.05	0.051	0.081	0.091	0.113	0.162	0.181	0.189	0.231	0.234	0.357
BPS	0.051	0.06	0.095	0.123	0.242	0.348	0.462	0.572	0.781	0.8982	1
LMS	0.052	0.062	0.0097	0.136	0.26	0.341	0.578	0.741	0.877	0.93	1

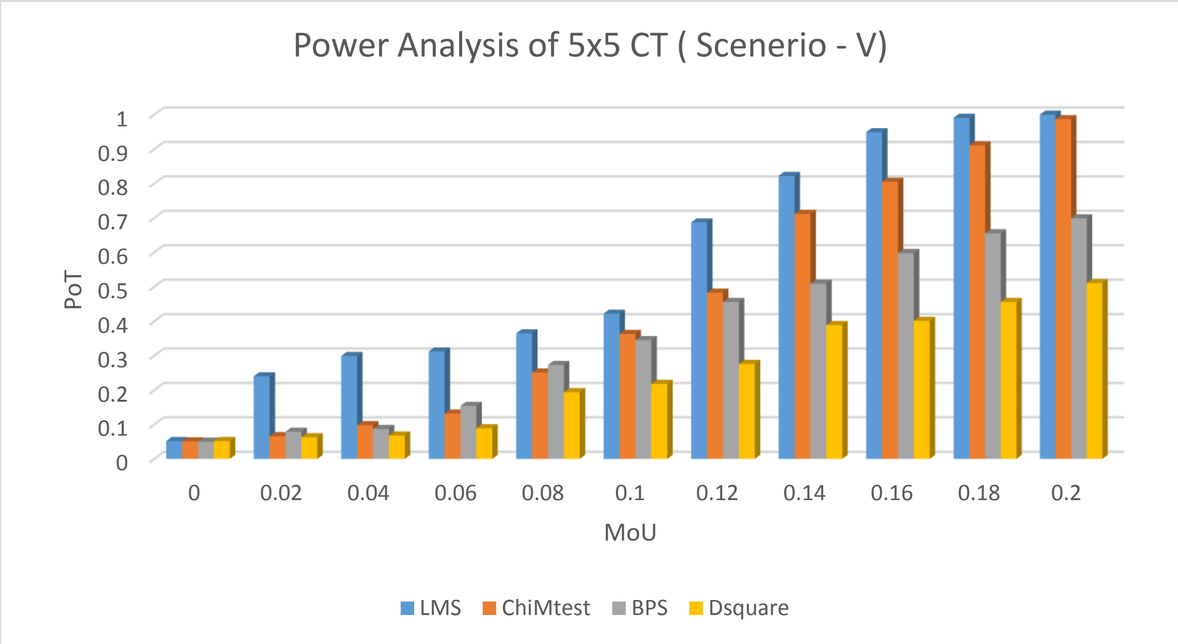


Figure 6.26: Power Analysis Graph 4x4 (SV)

Table 6. 39: Power Analysis of Tests of independence for 5 × 5 CTs Scenario - V

Nominal Level (α) =5%	Measure of Untruthfulness [MoU]									N=200	
Tests Name	0.000	0.020	0.040	0.060	0.080	0.100	0.120	0.140	0.160	0.180	0.200
χ^2 Test	0.05	0.053	0.063	0.088	0.097	0.148	0.172	0.201	0.235	0.285	0.274
G^2 $Test$	0.05	0.055	0.067	0.075	0.125	0.155	0.164	0.196	0.226	0.256	0.288
D^2 Test	0.052	0.063	0.068	0.089	0.194	0.218	0.276	0.389	0.401	0.456	0.511
$\lvert\chi\rvert$ MDT	0.051	0.066	0.098	0.132	0.251	0.363	0.483	0.712	0.805	0.911	0.987
FES	0.05	0.051	0.061	0.167	0.202	0.329	0.446	0.66	0.778	0.893	0.954
NMCS	0.05	0.051	0.052	0.057	0.091	0.117	0.145	0.156	0.163	0.181	0.232
FTS	0.05	0.052	0.057	0.062	0.094	0.126	0.156	0.169	0.176	0.189	0.278
CRS	0.05	0.059	0.078	0.094	0.243	0.264	0.387	0.495	0.553	0.577	0.665
KLS	0.05	0.062	0.081	0.097	0.251	0.272	0.397	0.499	0.571	0.584	0.687

BPS	0.05	0.079	0.087	0.154	0.273	0.345	0.456	0.509	0.598	0.6554	0.699
LMS	0.052	0.24	0.299	0.312	0.365	0.422	0.687	0.822	0.949	0.991	1

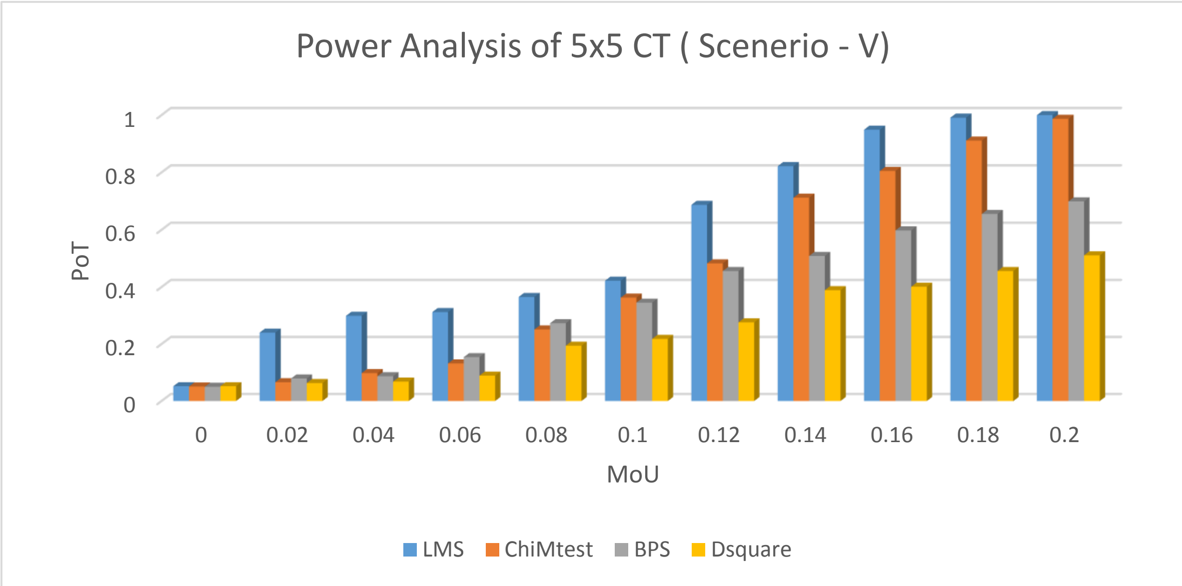


Figure 6.27: Power Analysis of 5x5 CT (SV)

Table 6. 40: Power Analysis of Tests of independence for 6 × 6 CTs Scenario - V

Nominal Level (α) =5%	Measure of Untruthfulness [MoU]									N=400	
Tests Name	0.000	0.020	0.040	0.060	0.080	0.100	0.120	0.140	0.160	0.180	0.200
χ^2 Test	0.05	0.058	0.079	0.082	0.097	0.125	0.158	0.167	0.259	0.275	0.394
G^2 *Test*	0.05	0.066	0.069	0.091	0.094	0.131	0.163	0.216	0.271	0.294	0.419
D^2 Test	0.05	0.055	0.067	0.081	0.112	0.14	0.179	0.229	0.346	0.387	0.482
|χ| MDT	0.051	0.069	0.132	0.148	0.262	0.392	0.54	0.633	0.841	0.918	0.976
FES	0.05	0.063	0.08	0.19	0.349	0.389	0.509	0.685	0.753	0.832	0.978
NMCS	0.05	0.058	0.071	0.098	0.142	0.168	0.195	0.298	0.351	0.498	0.582
FTS	0.052	0.056	0.072	0.079	0.107	0.138	0.141	0.149	0.253	0.367	0.488
CRS	0.051	0.057	0.08	0.083	0.093	0.099	0.108	0.14	0.205	0.381	0.4
KLS	0.05	0.057	0.074	0.099	0.153	0.171	0.188	0.197	0.265	0.323	0.474
BPS	0.05	0.066	0.078	0.099	0.123	0.146	0.194	0.258	0.381	0.409	0.523
LMS	0.053	0.099	0.152	0.172	0.28	0.449	0.687	0.891	0.975	0.999	1

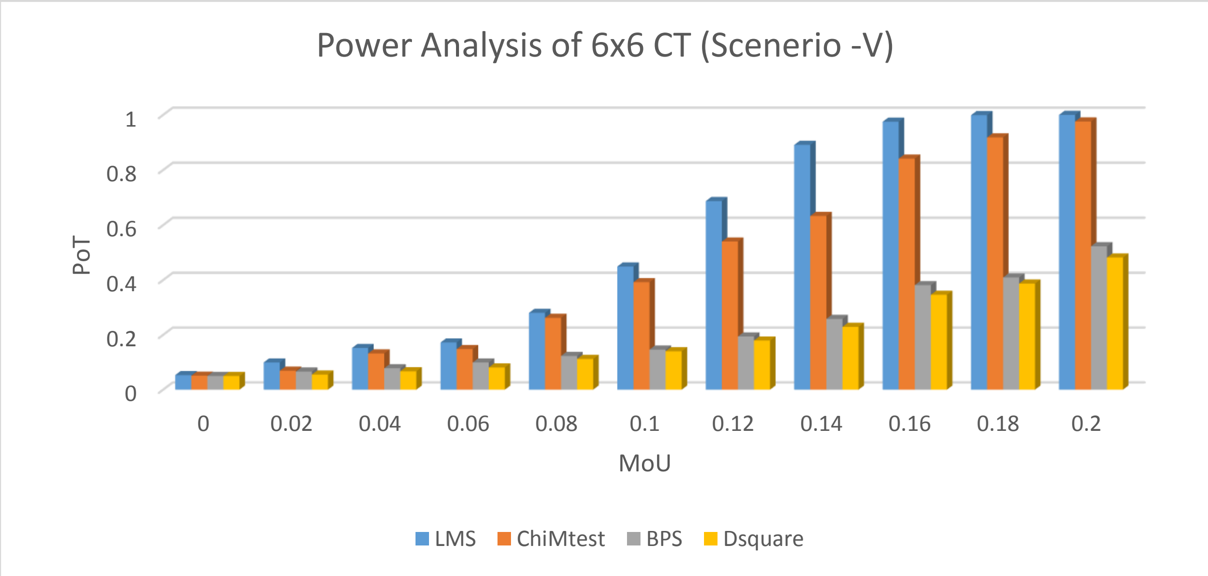


Figure 6.28: Power Analysis of 6x6 CT (SV)

Table 6.41: Power Analysis of Tests of independence for 12 × 12 CT Scenario - V

Nominal Level (α) =5%	Measure of Untruthfulness [MoU]									N=400	
Tests Name	0.000	0.020	0.040	0.060	0.080	0.100	0.120	0.140	0.160	0.180	0.200
χ^2 Test	0.05	0.058	0.079	0.082	0.097	0.125	0.158	0.167	0.259	0.275	0.394
G^2 $Test$	0.05	0.066	0.069	0.091	0.094	0.131	0.163	0.216	0.271	0.294	0.419
D^2 Test	0.05	0.066	0.078	0.099	0.123	0.146	0.194	0.258	0.381	0.409	0.523
|χ| MDT	0.051	0.069	0.132	0.148	0.262	0.392	0.54	0.633	0.741	0.831	0.91
FES	0.05	0.063	0.08	0.19	0.349	0.389	0.509	0.685	0.753	0.832	0.978
NMCS	0.05	0.058	0.071	0.098	0.142	0.168	0.195	0.198	0.251	0.298	0.382
FTS	0.052	0.056	0.072	0.079	0.107	0.138	0.141	0.149	0.153	0.167	0.188
CRS	0.051	0.057	0.08	0.083	0.093	0.099	0.108	0.14	0.205	0.281	0.3
KLS	0.05	0.057	0.074	0.099	0.153	0.171	0.188	0.197	0.265	0.323	0.397
BPS	0.05	0.066	0.078	0.099	0.123	0.146	0.194	0.258	0.381	0.409	0.523
LMS	0.058	0.088	0.156	0.278	0.384	0.449	0.587	0.691	0.775	0.899	1
NPLT	0.053	0.099	0.152	0.372	0.454	0.597	0.695	0.797	0.899	0.987	1

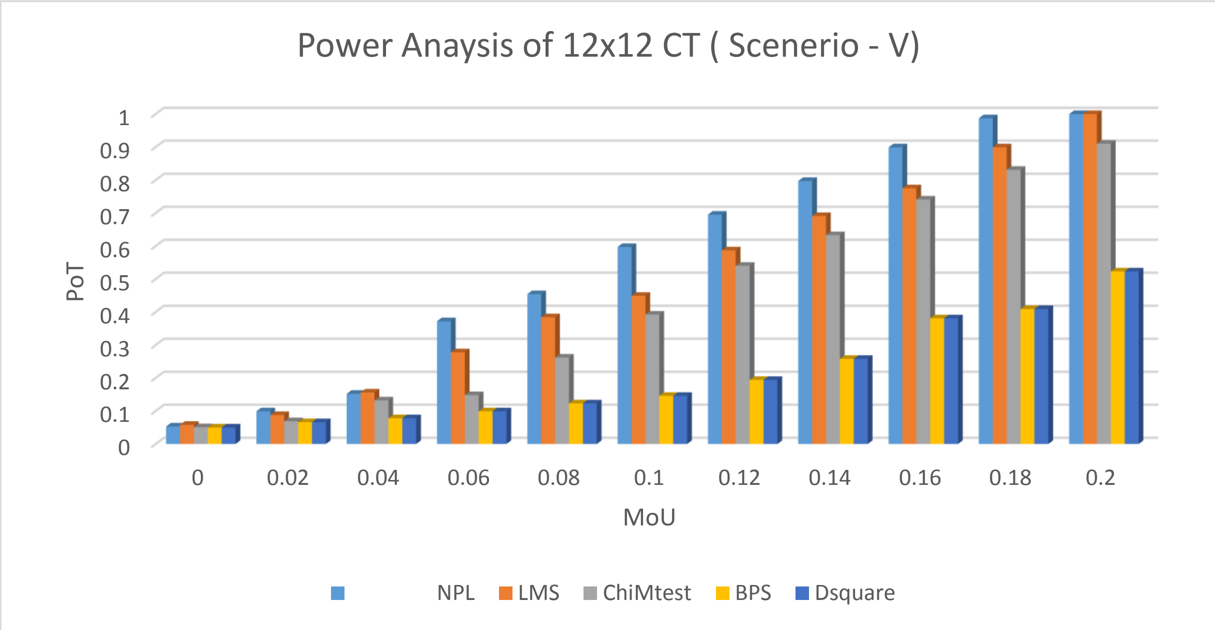


Figure 6.29: Power Analysis of 12x12 CT (SV)

The results of scenario V for tables above indicates that LMS have maximum power as compared to BPS and D Square and MDT of independence in 6x6 and 12x12 Contingency table. The summary of scenario V for distinct types of Contingency table are shown in Table 6.42.

Scenario V for 12x12

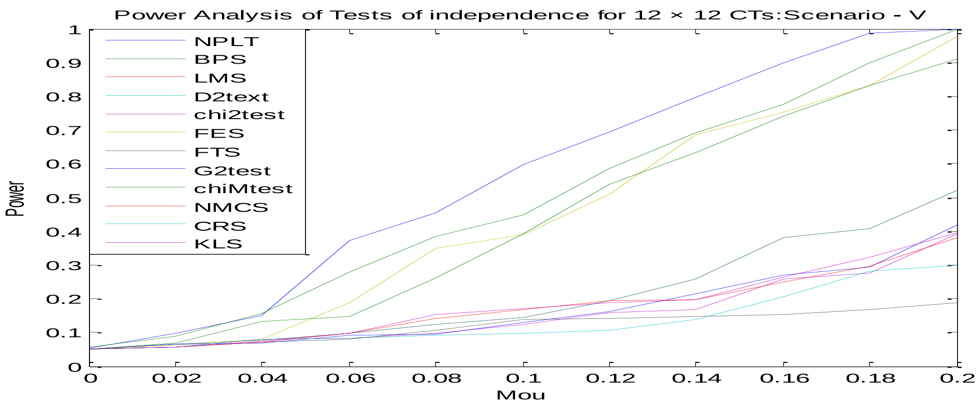


Table 6. 41: Summary of Power for W×K Contingency table Scenario - V

	W×K Contingency table (Power) α = 0.05			
	MDT	LMS	BPS	DSQ
2x3 CT	+++	++++	++	+
3x3 CT	+++	++++	++	+
4x4 CT	+++	++++	++	+
5x5 CT	+++	++++	++	+
6x6 CT	+++	++++	++	+
12x12 CT	+++	++++	++	+

(Note: "+" shows the power of tests as it increases shows the most powerful tests).

Now, let me present power analysis in comparisons of selected tests of independence in scenario V.

6.6.1 Summary of Power Analysis of CT – Scenario – V

The power was computed for higher order contingency table it was found that LMS has the maximum power in all the higher order contingency tables. MDT performs at second and BPS performs on third number the maximum power among the eleven tests selected under the study.

6.7 Conclusion

The power analysis for different tests of independence shows different results as stated from the summary tables of scenarios (I-V). The central problem of the study is to investigate and evaluate the most stringent test of independence for nominal data in w x k contingency tables. A special techniques of stringency criteria (SC) are used in this study to find out the most stringent test for w x k contingency table.

We computed maximum likelihood, draw the power envelope curve and calculated shortcomings of the numerous tests of independence. Shortcomings are the difference of power of the test and power envelope curve. The procedure of shortcoming is explained in chapter 4 as stated below.

$$S\,(T, \theta_{k)} \quad = \quad P\,(T\theta_k\,, \theta_k) \;-\; P\,(T\,, \theta_k)$$

Shortcoming at specific alternative

$$S\ (T) = \text{Max}\ [P\ (T\theta_k\ , \theta_k) - P\ (T, \theta_k)$$

Table 6.43 indicates a summary of shortcomings of tests of independence in different scenarios.

Table 6. 42: Summary of Power for W×K Contingency Tables

Scenarios (I-V)	W×K Contingency table (Shortcomings)		
		α = 0.05	
	MDT	LMS	BPS
Scenario I	0.069	0.050	0.068
Scenario II	0.782	0.054	0.061
Scenario III	0.074	0.052	0.062
Scenario IV	0.683	0.053	0.067
Scenario V	0.071	0.051	0.063

Thus, from the above W × K contingency table analysis it is found that the most stringent test is Logarithmic Minimum Square (LMS) test of independence which has the minimum shortcomings in maximum scenarios. Based on solid estimation results of MCS we concluded that LMS is the most stringent test of independence in W × K contingency tables.

CHAPTER 7

POWER COMPARISON OF TESTS OF INDEPENDENCE FOR ORDINAL DATA

One of the key proposed objectives of this study is to evaluate the most powerful test of independence/measure of correlation in w × k contingency table for ordinal data. Seven tests of independence have been chosen and are compared using power criteria (PC). The power of a test is defined as the probability of rejecting null hypothesis when it is false i.e.

$$Power = P(Rejecting\ H_0/H_1\ is\ True)$$

This study used small, medium and large sample size according to the size of CTs with nominal level (α level 5%). The study used simulated critical values (SCV) computed and presented in chapter five using numerous DGP explained in chapter 4.

7.1 Power Analysis of Tests of Independence for Ordinal Data in W × K CTs.

We investigated power analysis of tests of independence for ordinal data and found the following results stated in Tables 7.1.

Since we know from section 4.2.1, equation 4.15 states that,

$$Y = aX + bZ \quad ; a + b = 1$$

where, "a" determine strength of correlation in GDP. The result of table 7.1 indicates that as the strength of correlation increases, the power of the tests increases as well. The results further explains that Spearman ρ, Goodman Kruskal γ and Novel Φ_k has the maximum powers at nominal level (α = 5%) at sample size 25. The others test Kendall τ-a, Kendall τ-a, Kendall τ-a and Somers'd have lower power in this case.

Table 7. 1: Power Analysis of Tests of independence for Ordinal Data for 2 × 3 CT

Nominal Level (α) =5%	Strength of Correlation [SoC]									N=25	
Tests Name	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10
Spearman ρ	0.051	0.075	0.175	0.292	0.343	0.418	0.533	0.672	0.781	0.881	0.911
Kendall τ-a	0.050	0.078	0.112	0.175	0.287	0.334	0.422	0.485	0.526	0.592	0.598
Kendall τ-b	0.053	0.066	0.089	0.179	0.214	0.308	0.366	0.484	0.502	0.561	0.567
Kendall τ-c	0.054	0.051	0.068	0.175	0.287	0.327	0.472	0.499	0.526	0.671	0.683
Gd-Krskl γ	0.051	0.067	0.159	0.269	0.399	0.478	0.572	0.692	0.703	0.967	0.982
Somers'd	0.050	0.058	0.079	0.099	0.187	0.297	0.382	0.496	0.595	0.543	0.574
Novel Φ_k	0.055	0.099	0.187	0.295	0.398	0.493	0.567	0.687	0.754	0.947	1.000

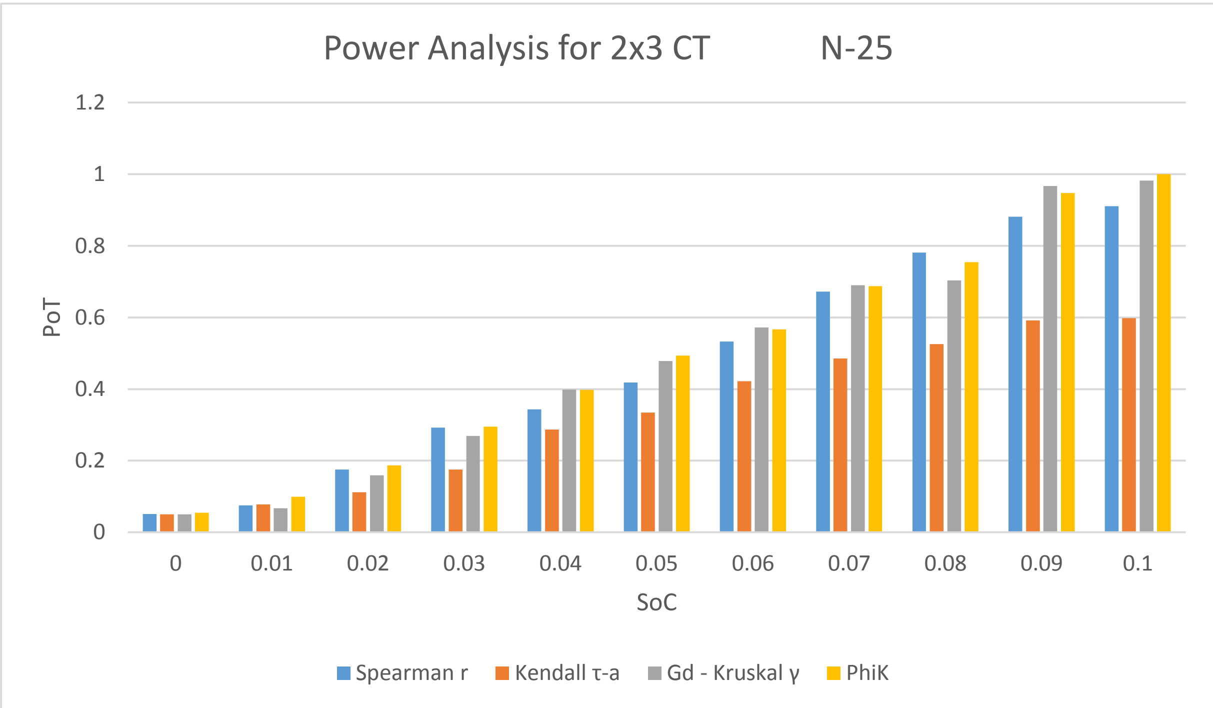


Figure 7.1: Shows Graph of Powerful Tests 2x3 CT for Ordinal Data

The Figure 7.1 shows graphical analysis of power of tests (POT) at various levels of strength of correlation. This results also indicates that Novel Φ_k has the maximum power while Goodman Kruskal γ performs at second and Spearman ρ at third.

We computed the power of tests for seven tests of independence for ordinal data in 3 × 3 contingency table shown in table 7.2.

Table 7. 2: Power Analysis of Tests of independence for Ordinal Data for 3 × 3 Contingency **table**

Nominal Level (α) =5%	Strength of Correlation [SoC]									N=50	
Tests Name	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10
Spearman ρ	0.050	0.055	0.163	0.278	0.332	0.423	0.582	0.674	0.722	0.882	0.921
Kendall τ-a	0.050	0.059	0.068	0.097	0.187	0.241	0.351	0.431	0.491	0.571	0.699
Kendall τ-b	0.051	0.055	0.064	0.091	0.196	0.223	0.241	0.286	0.389	0.492	0.516
Kendall τ-c	0.050	0.051	0.068	0.175	0.287	0.327	0.472	0.499	0.526	0.671	0.715
Gd Krskalγ	0.052	0.055	0.168	0.29	0.398	0.438	0.568	0.632	0.722	0.831	0.932
Somers'd	0.050	0.053	0.084	0.089	0.238	0.368	0.379	0.489	0.518	0.697	0.789
Novel Φ_k	0.050	0.098	0.196	0.298	0.411	0.457	0.598	0.699	0.898	0.998	1.000

The result of table 7.2 describes same situations as results of table 7.1 that as the strength of correlation increases, the power of the tests increases as well. The results further explains that Spearmanρ, Goodman Kruskal γ and Novel Φ_k has the maximum powers at nominal level (α) at sample size 50. The others test Kendall τ-a, Kendall τ-a, Kendall τ-a and Somers's have lower power in this case.

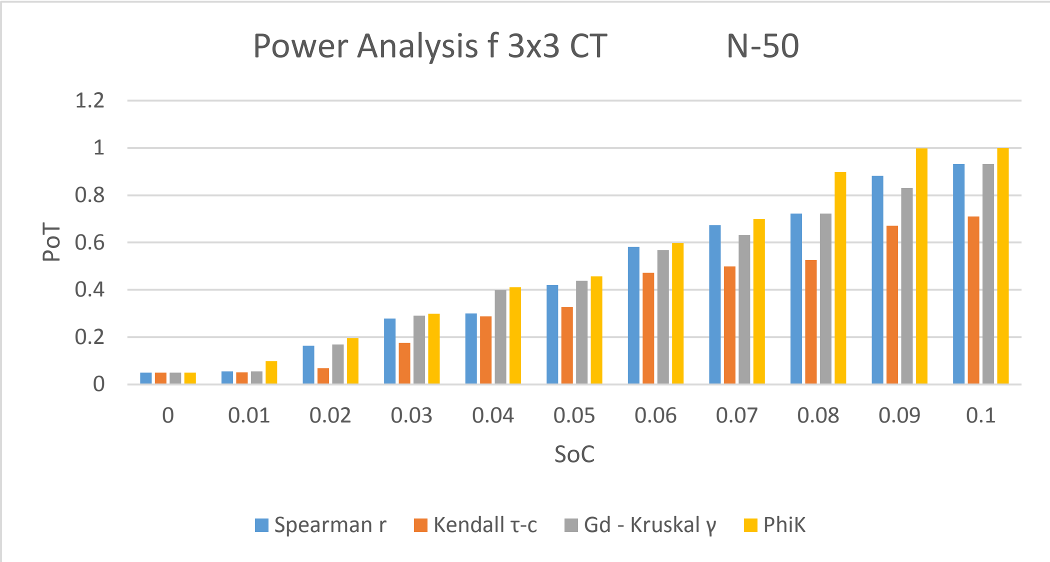


Figure 7.2: Shows Graph of Powerful Tests 3x3 CT for Ordinal Data (N-50)

The Figure 7.2 shows graphical analysis of power of tests (POT) at various levels of strength of correlation. This results also indicates that Novel Φ_k has the maximum power while Goodman Kruskal γ performs at second and Spearman ρ at third.

We computed power of tests for seven tests of independence for ordinal data in 4 × 4 contingency table shown in table 7.3.

Table 7. 3: Power Analysis of Tests of independence for Ordinal Data for 4 × 4 CTs

Level (α) =5%	Strength of Correlation [SoC]								N=100	
Tests Name	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10
Spearman ρ	0.050	0.058	0.144	0.202	0.334	0.398	0.532	0.655	0.732	0.791	0.823
Kendall τ-a	0.053	0.064	0.068	0.089	0.099	0.188	0.258	0.399	0.446	0.582	0.699
Kendall τ-b	0.052	0.062	0.078	0.098	0.109	0.298	0.351	0.471	0.582	0.698	0.723
Kendall τ-c	0.051	0.055	0.167	0.275	0.387	0.488	0.542	0.601	0.633	0.712	0.791
Gd Krskalγ	0.050	0.062	0.177	0.289	0.393	0.442	0.578	0.685	0.799	0.921	0.943
Somers'd	0.052	0.051	0.089	0.098	0.239	0.362	0.445	0.498	0.523	0.687	0.711
Novel Φ_k	0.050	0.076	0.187	0.349	0.365	0.452	0.589	0.698	0.721	0.977	1.000

The result of table 7.3 explains the same situations as results of table 7.2 that as the strength of correlation increases, the power of the tests increases as well. The results further explains that Spearman ρ, Goodman Kruskal γ and Novel Φ_k has the maximum powers at nominal level (α) at sample size 100. The others test Kendall τ-a, Kendall τ-a, Kendall τ-a and Somers's have lower power in this scenario.

The results further indicates that Novel Φ_k has the maximum power as compared to others tests of independence. In this scenario it is seen that Goodman Kruskal γ performs better as compared to Spearman ρ and other tests.

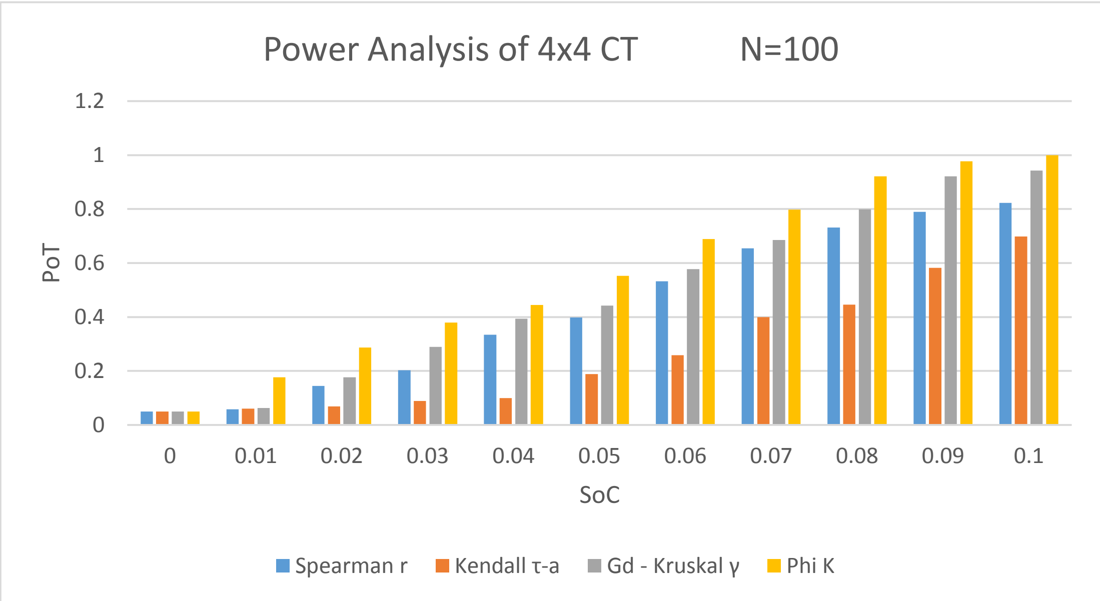


Figure 7.3: Shows Graph of Powerful Tests 4x4 CT for Ordinal Data (N-100)

The Figure 7.3 shows graphical analysis of power of tests (PoT) at various levels of strength of correlation. This results also indicates that Novel Φ_k has the maximum power while Goodman Kruskal γ performs at second and Spearman ρ at third. We computed power of tests for seven tests of independence for ordinal data in 4×4 contingency table shown in table 7.4.

Table 7. 4: Power Analysis of Tests of independence for Ordinal Data for 5 × 5 CTs

Nominal Level (α) =5%	Strength of Correlation [SoC]									N=200	
Tests Name	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10
Spearmanρ	0.053	0.051	0.089	0.098	0.239	0.364	0.476	0.488	0.521	0.688	0.789
Kendall τ-a	0.052	0.054	0.087	0.099	0.145	0.252	0.289	0.398	0.421	0.577	0.689
Kendall τ-b	0.053	0.062	0.168	0.289	0.399	0.488	0.551	0.571	0.683	0.698	0.723
Kendall τ-c	0.050	0.068	0.099	0.193	0.223	0.346	0.494	0.552	0.681	0.688	0.698
GdKrskalγ	0.052	0.058	0.198	0.202	0.377	0.449	0.552	0.655	0.732	0.878	0.988
Somers'd	0.050	0.062	0.078	0.098	0.109	0.298	0.351	0.471	0.583	0.698	0.723
Novel Φ_k	0.050	0.061	0.168	0.293	0.398	0.438	0.568	0.632	0.722	0.931	1.000

The result of table 7.4 explains the same situations as results of table 7.3 that as the strength of correlation increases, the power of the tests increases as well. The results further explains that Spearmanρ, Goodman Kruskal γ and Novel Φ_k has the maximum powers at nominal level (α) at sample size 200. The others test Kendall τ-a, Kendall τ-a, Kendall τ-a and Somers's have lower power in this scenario.

The results further indicates that Novel Φ_k has the maximum power as compared to others tests of independence. In this scenario it is seen that Goodman Kruskal γ performs better as compared to Spearman ρ and other tests.

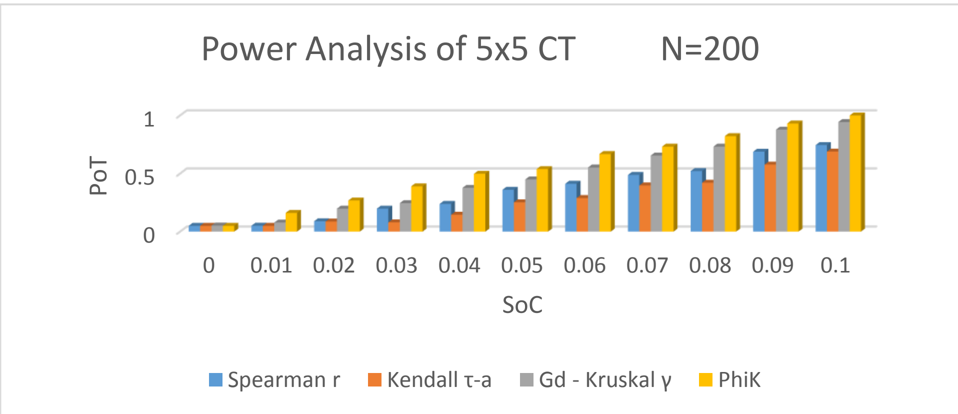


Figure 7.4: Shows Graph of Powerful Tests 5x5 CT for Ordinal Data (N-200)

The Figure 7.4 shows graphical analysis of power of tests (POT) at various level of strength of correlation at sample size 200. This results also indicates that Novel Φ_k has the maximum power while Goodman Kruskal γ performs at second and Spearman ρ at third.

We computed power of tests for seven tests of independence for ordinal data in 6 × 6 contingency table shown in table 7.5.

Table 7. 5: Power Analysis of Tests of independence for Ordinal Data for 6 × 6 CT

Nominal Level (α) =5%	Strength of Correlation [SoC]								N=400	
Tests Name	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10
Spearman ρ	0.050	0.059	0.199	0.211	0.397	0.425	0.558	0.661	0.787	0.875	0.923
Kendall τ-a	0.053	0.069	0.076	0.098	0.099	0.145	0.269	0.312	0.472	0.598	0.699
Kendall τ-b	0.054	0.068	0.099	0.193	0.223	0.346	0.494	0.552	0.681	0.688	0.698
Kendall τ-c	0.052	0.056	0.077	0.089	0.093	0.142	0.178	0.285	0.399	0.421	0.543
Gd Krskal γ	0.050	0.063	0.186	0.299	0.349	0.489	0.598	0.611	0.753	0.832	0.978
Somers'd	0.055	0.058	0.077	0.099	0.181	0.197	0.292	0.397	0.452	0.598	0.682
Novel Φ_k	0.050	0.169	0.278	0.399	0.498	0.554	0.687	0.734	0.853	0.967	1.000

The results indicates that Novel Φ_k has the maximum power as compared to others tests of independence. In this scenario it is seen that Goodman Kruskal γ performs better as compared to Spearman ρ and other tests.

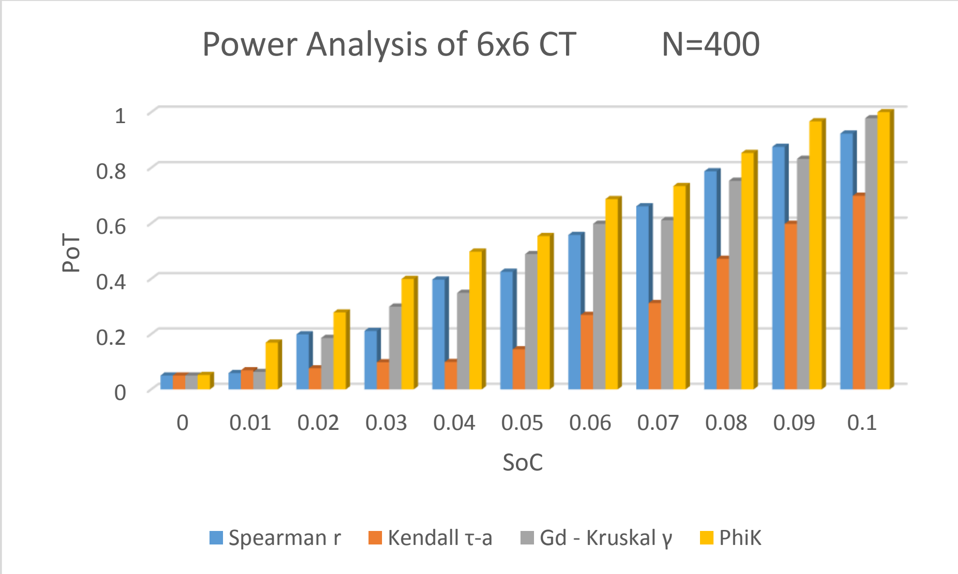


Figure 7.5: Shows Graph of Powerful Tests 6x6 CT for Ordinal Data (N-400)

The Figure 7.5 shows graphical analysis of power of tests (PoT) at various level of strength of correlation at sample size 400. This results also indicates that Novel Φ_k has the maximum power while Goodman Kruskal γ performs at second and Spearman ρ at third.

We computed power of tests for seven tests of independence for ordinal data in 12 × 12 contingency table shown in table 7.6.

Table 7. 6: Power Analysis of Tests of independence for Ordinal Data for 12 × 12 CTs

Nominal Level (α) =5%	Strength of Correlation [SoC]									N=800	
Tests Name	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10
Spearman ρ	0.050	0.065	0.087	0.198	0.245	0.352	0.489	0.598	0.621	0.771	0.889
Kendall τ-a	0.050	0.058	0.077	0.089	0.093	0.142	0.178	0.285	0.399	0.421	0.543
Kendall τ-b	0.051	0.053	0.098	0.102	0.177	0.249	0.252	0.355	0.432	0.578	0.688
Kendall τ-c	0.054	0.062	0.078	0.098	0.109	0.298	0.351	0.471	0.582	0.698	0.723
Gd - Kruskalγ	0.052	0.064	0.168	0.289	0.399	0.488	0.558	0.699	0.746	0.882	0.924
Somers'd	0.052	0.063	0.068	0.089	0.099	0.188	0.258	0.399	0.446	0.582	0.699
Novel Φ_k	0.050	0.198	0.262	0.378	0.498	0.556	0.698	0.751	0.871	0.938	1.000

The results further indicates that Novel Φ_k has the maximum power as compared to others tests of independence. In this scenario it is seen that Goodman Kruskal γ performs better as compared to Spearman ρ and other tests.

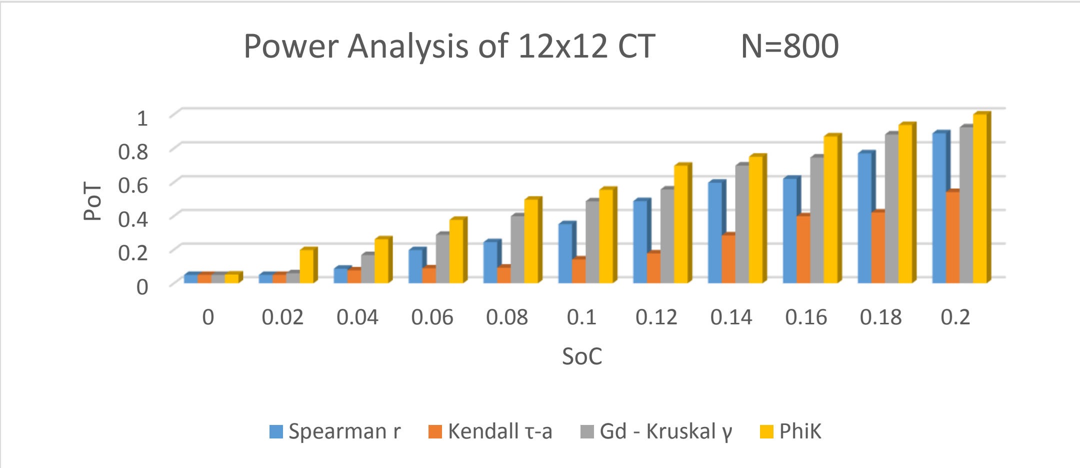


Figure 7.6: Shows Graph of Powerful Tests 12 x12 CT for Ordinal Data (N-800)

The figure 7.6 shows graphical analysis of power of tests (PoT) at various level of strength of correlation at sample size 800. This results also indicates that Novel $\emptyset_k$ has the maximum power while Goodman Kruskal γ performs at second and Spearman ρ at third. The power analysis of tests of independence indicates that the Novel Φ_k test of independence has maximum power as compared to others measure of correlation e.g. Spearman rank correlation, Somars'd, Kruskal Gamma. Goodman and Kruskal and Spearman rank correlation.

The summary of power analysis of seven tests of independence for ordinal data are described from below line charts. The figure 7.7 shows line charts of the power of tests of independence / measure of correlation for ordinal data. The results which are stated in above tables and bar charts shows exactly same results in below line charts.

The power analysis of tests of independence indicates in w x k at various sample size (small, medium and large) that the Novel Φ_k test of independence has maximum power as compared to others measure of correlation e.g Spearman rank correlation, Somars D, Kruskal Gamma. Goodman and Kruskal and Spearman rank correlation.

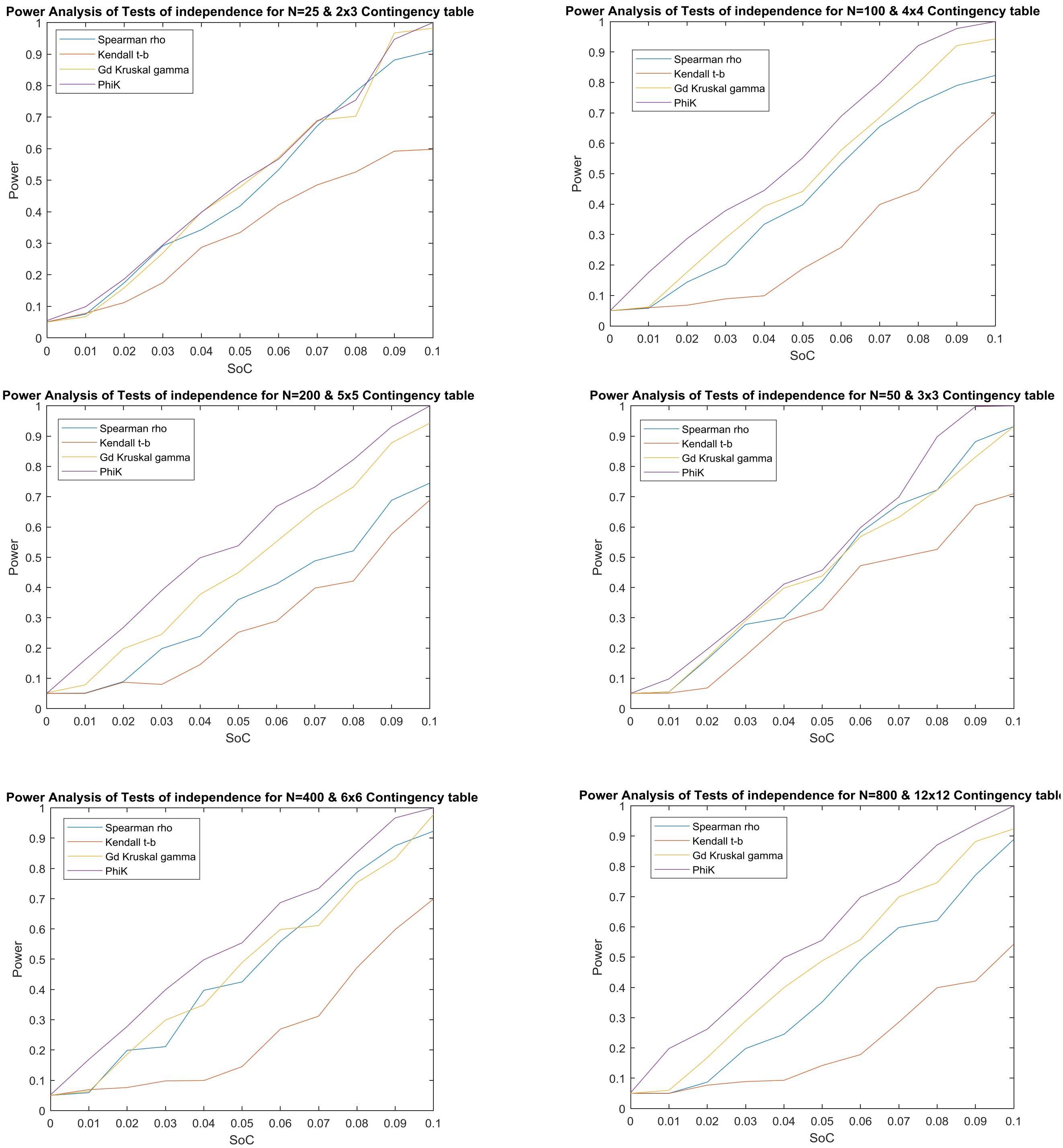


Figure 7.7: Power analysis of Test for Ordinal Data

7.2 Summary of Power of Tests of Independence in Ordinal Data in W x K CTs

All the above tests of independence are non-parametric, and we cannot find likelihood ratios for these measure of correlations. Therefore, we cannot use stringency criteria to evaluate most stringent test of independence. Thus, we used power analysis techniques (PC) through simulations and compared seven measure of correlation including a novel correlation known is Novel Φ_k . The results indicate that novel tests of independence i.e., Novel Φ_k shows better performance in W x K CTs. Goodman and Spearman rank correlation also perform reliable results compare to others measure of correlations. Table 7.7 summarizes results for the most powerful test of independence for ordinal data.

Table 7. 7: Present Summary of Power for Ordinal Data in W×K CT

	W×K Contingency table (Power)					
	N=50 3×3	N=100 4×4	N=200 5×5	N=400 6×6	N=800 12 × 12	Max Power of Test
Spearman ρ	.932	.823	.789	.923	.889	.932
Kendall τ-a	.699	.699	.689	.699	.543	.699
Kendall τ-b	.510	.723	.723	.698	.688	.723
Kendall τ-c	.710	.791	.698	.543	.723	.791
Gd - Kruskal γ	.932	.943	.988	.978	.924	.988
Somers'd	.789	.711	.723	.682	.699	.789
Novel Φ_k	.100	.100	.100	.100	.100	.100

In addition to above results, analysis based on Monte Carlo Simulation for ordinal data describes in table 7.8 that Novel Φ_k test is the powerful test of independence for ordinal data. The good characteristic of this test is that it not only capture the linear association among the variables but it also captures the nonlinear association among the variables. This Novel Φ_k correlation can further be used either for nominal, ordinal, ratio interval or especially for mixed variables as well as performs optimal in multivariate contingency table.

The calculation of the Novel Φ_k correlation is a bit tough as described in chapter 4. Therefore, Current study also found Kruskal Gamma and Spearman Rank correlations close to the Novel Φ_k correlation in terms of power comparison for higher order contingency table in analysis of ordinal data.

The study also concludes that in terms of powerful tests for ordinal data are ranked sequentially are Novel Φ_k, Gd - Kruskal γ, Spearman ρ, Somers'd, Kendall τ-c, Kendall τ-b and Kendall τ-a.

CHAPTER 8

APPLICATION OF THE MOST STRINGENT TEST ON NOMINAL REAL DATA SET

(NEXUS BETWEEN GIRLS ENROLLMENT IN EDUCATION) ACROSS PROVINCES OF PAKISTAN)

8.1 Introduction

Education is a prime requirement in this modern age of globalization. It does not only provide insights, but it has a significant role in building characters, grooming personality, giving skills, and inculcating moral values. The first stage for each human activity requires education in this phase of technological revolution. The welfare of the individuals and living standards is concerned with the vital role of education. The Education is a key factor that brings changes in human behavior. These changes insist that a human recognize his or her significant role in social,economic, and political life. To bring these changes, the equal opportunities of acquiring education to male and female are necessary. The education is a crucial tool to tackle the issues of income distribution and poverty along with facets of demographic, political and social developments. The human capital is important in developing countries as compared to developed countries because education is a core need for political, social, and economic transformation of institutions.

The Gender and Development approach identified that relations and roles of gender are the main factor to improve the lives of women, with a term 'gender' suggests that there is need to focuson both men and women. Recently, the desire to recognize how gender traverses with some characteristics like sexuality, ethnicity and age has been renowned. The approach of Gender and Development identifies that it will be insufficient to include girls and women into prevailing development processes, but it is also necessary to ask the question about why women remained excluded, supporting that the emphasis ought to be on demonstrating the imbalances of supremacy based on this exclusion. The Gender and Development method also defines the idea of‘development’ and its gentle nature, indicating that there is need to transform narrow understanding of development as economic growth into social development. The projects by Gender and Development approach are holistic and try to eliminate the discriminated forms of institutions against women’s interest, for instance, acquiring land rights, and living violence free lifestyle (Molyneux, 1985; Moser, 1989). The international agencies and developing countries diverted their focus towards human investment in 1980s.

The literacy rate of Pakistan was only 10%, when Pakistan came into existence. By then, Pakistan acquired only 10,000 elementary schools in inheritance. The number of the education institutions increased to 2, 65,538 (1, 14,302 women and 1, 51,236 for men) in 2019 as the outcomes of implementing different policies and reforms measures. The major task of the government was to enhance the education system in the elementary schools up to the economic, social, and ideological needs of the economy. An action plan contained on many reforms was developed during 1998-2010 to encourage higher education and literacy rate.

The objective of these reforms was to provide the facilities to those children who left schools due to unfavorable environment. In 2006, Pakistan has been ranked at 134th in Human development index and quoted as an example among those countries where female education is less (OCSD, 2007). The number of boys going to school is greater than the number of girls going to school with increasing age in Pakistan (Khan, 2008). The female higher education is the most effective education among primary, secondary, or higher secondary education because it is the level of education at which people pursue their pre - determined objectives. The higher education is defined as “all kinds of studies, training or trainingfor research at the post-secondary level, provided by universities or other educational establishments that are approved as institutions of higher education by the competent state authorities” (UNECSO, 1998).

In Pakistan, the bachelor, Master, M.Phil., and Ph.D. are considered as higher education that starts very after the higher secondary education or twelve years of schooling. The higher education is the mean for people to pursue their goals of life that people aim to achieve from their childhood (Yasmeen, 2005). The female education is a smart and most effective investment for the development and economic growth of any country in the world, but it remained ignored. The formulated policies are required to empower women in education for decision making, employment and career development (Salik & Zhiyong, 2014).

8.2 Literature Review

Many studies have made efforts to identify the socio-economic factors that influence the female enrollment and their level of education. The socio-economic factors are too many so that it is a difficult task to include all factors in a single empirical study. Therefore, few studies analyzed the macroeconomic variables causing lower school enrollment in a country while few studies analyzed microeconomic and social factors at household level. Different socio-economic factors and macroeconomic variables have been identified by different studies that significantly affect women enrollment.

There is lower participation of rural women in different types of employment in the India. The factor behind that is culture causing discouragement of female participation. The cultural aspect of joint family system insists female to opt agriculture employment but hinders the rural women to adopt non-farm employment. The joint family system decreases the working hours of rural women in non-farm employment. The social status of the women is not well defined due to lower education in the north India. The probability of rural women living in joint family system to work in non-farm sector is lower than the women living in nuclear family system. This gap has been lowered over the time but for those rural women who have tertiary education. The tertiary education of rural women overcomes the gender disparities prevailing in non-farm sector employment. The women have more drop out ratio than male in initial stage of education at 14 years' age. In the joint family system, the unemployment rate of rural women with a young child is higher than the rural women living in nuclear family system (Dhanraj and Mahambare, 2019). The female enrollment in higher education in Pakistan is lower due to the cultural and socio-economic problems. The travel freedom, sexual harassment, feudalism, religious misconception, lack of higher education institutes and gender discrimination are the dominant perceived factors affecting female education.

Most of the female have usually freedom to travel which means there is no much constraint of travel freedom. Sexual harassment is the mostly observed factor having negative influence on the female higher education. The parent permission for girls to acquire education has negligible outcome in lower female enrollment. The impact of feudalism is also an important factor behind lower female enrollment in higher education. Most of the people mis interpret the concept of religion regarding female education. So, religion misconception constraint adversely affects the female education. The security, lack of institutions and traditional customs are also important factors in determining the female education. The co-education causes a big challenge for female enrollment because parents oppose the co-education and do not allow girls to be enroll. The financial resources allow parents to bear the expenditures of female higher education but unfortunately, female higher education is restricted due to lower family income (Mehmood et al., 2018). Women's empowerment is complex and multifaceted, and its definition varies from community to community. Usually, female status refers to feelings of self-development among women, the ability to select from available choices as well as opportunities and the ability to manage your life outside and inside the house. The status of women is concerned with educational opportunities, labor improvement, birth control, decision-making rights, access to resources and decision-making on the reproductive process. Feminist economists indicate a masculine structure that increases gender inequality. To overcome these circumstances, women must challenge existing power relationships and exclude male dominant culture from society. The organizational development enhances the economic role of women. The communities need to raise awareness and to improve organizations to ensure equal opportunities and rights for both men and women. Recently, a large proportion of women occupy high posts in their workplaces, in trade unions, politics and the academic world. But gender inequality still prevails in most parts of the world. The discrimination between men and women affects economic development and partly true for human capital, but this view cannot explain the entire gender pay gap. The lives of women in Pakistan are governed by ancestral society. Such societies do not give women equal rights. It is widely evidenced that women face gender inequality in income, education, employment, healthcare, and control over assets.

Pakistan is one of those countries which has largest gender gap and discrimination between man and women in all aspects of life. According to World Economic Forum, women of 58 countries were able to achieve gender equality in five different sectors such as health and educational achievements, wealth, economic opportunities, economic and political participation, while Pakistan was at 56th out of these countries. In Pakistan, the growth rate of labor force participation for women was 15.9% in 2004 and increased to 18.9 percent for next two years (Ashraf & Ali, 2018). Access to higher education is probably going to turn out to be increasingly significant for developing nations but despite its pertinence this matter is understudied. In access to higher education, significant disparities exist between men and women emerging from background of parent, location, and household wealth. These Disparities are significant regarding distributional worries as well as on the grounds that they may have suggestions for the economic and social prospects of the nations. With regards to a huge writing on the formation human capital, inequalities in accessing higher education appear early and are apparent in the relationship between later enrollment and early methods of learning in higher education. However, children and parent aspirations for acquiring education hardly affect female education but household wealth significantly determines female education as liquidity improves (Sanchez & Singh, 2018).

Universities of Pakistan do not stand or secure position in the world ranking because their quality of education does not match international standard. The female education in rural areas of Pakistan is in very alarming situation. In developed countries, advanced infrastructure is provided to colleges and universities but in developing countries, even maintenance of schools is not possible. Government does not allocate desired and required budget for education in Pakistan. The government is focusing on the issues related to institutions and enrollment of the students since last decade whereas earlier state was unsatisfactory. Current state of country shows that government has taken few measures to improve education institutions for men and women at school and university levels. But these measures are not enough to get desired outcomes in the society. The findings in Pakistan show that female enrollment ratio, literacy rate and female participation of labor force have significant positive effects while fertility rate has negative and significant effect on economic growth in Pakistan. Hence, female literacy rate and female participation of labor force are necessary elements to achieve economic growth (Nosheen & Awan, 2018).

In most world communities, particularly in underdeveloped countries women are specified to household and men are specified to politics and public dealings. These dissimilarities between men and women are because of biological distinction. Women are born to give birth and house chores. Women give birth to children and keep themselves busy to feed up newly born babies/ children. They are deemed to be as domestic helper while men are physically strong and leave their children for extended periods. Therefore, men are more likely to be engaged in venture such as hunting and fighting and other socio-economic activity. There is greater gender discrimination in most developing countries. A girl-child has lower status and preferences, fewer rights, and benefits than a boy-child. Women at very young age are going through the inequality and facing difficulties. Women in Pakistan have been experiencing disadvantages since ages, their basic rights are being deprived. According to these social man-made norms, girls receive less food, less access to education, poor health care than a male child, and as a result, girls are more likely to die from childhood diseases. It has been reported that those girls who acquired training from vocational institutes have few chances to become teacher in vocational centers due to inefficiency of employment opportunities and lack of finances. According to Amnesty international, the girls' school's enrolment rate is very low, and according to the estimation of women organization groups, out of 28% of girls' school's enrolment at primary level, hardly 11% girls go to high school. The drop rate is very high and girls are kept home to do house chores or to take care of younger siblings/children, when requested by family or if the financial situation is very viable. The 24% females are literate as compared to males who are at 49%. To take estimation of women organization group, only 12 to 15 percent girls can read and write (Hirway & Mahadevia, 1996).

Keeping in view the empirical studies, it can be concluded that there are different socio-economic factors in different regions across the world that negatively influence the female enrollment in schools. In case of Pakistan, the findings of different studies illustrate that gender disparity, poverty, parent illiteracy, joint family system, lack of education institutes and facilities, poor health, family size, household income and assets, religion misconception and less travel freedom to women are the dominant constraints for female enrollment in Pakistan.

Table 8. 1: Description of Nominal Variables for W × K CTs

		Categorical Variable – Nominal Data
Variable Name	**Type**	**Description of the variable**
Female Enrollment	Categorical	0=If Female is not enrolled or left the school 1=If Female is currently enrolled
		Other Variable
Selected Provinces of Pakistan		
Province1	Categorical	The province female belongs to (1=Punjab 0=else)
Province2	Categorical	The province female belongs to (1=KPK 0=else)
Province3	Categorical	The province female belongs to (1=Sindh 0=else)
Province4	Categorical	The province female belongs to (1=Baluchistan 0=else)

This study includes targets four provinces of Pakistan named as Sindh, Punjab, Khyber Pakhtunkhwa and Baluchistan. The number of divisions, districts, Tehsils and union councils in Sindh are 7, 29, 119 and 1108, respectively. In Punjab, the number of divisions, districts, Tehsils and union councils are 9, 36, 146 and 7602 respectively whereas in Khyber Pakhtunkhwa, the number of divisions, districts, tehsils and union councils are 9, 35, 82 and 986 respectively. The divisions, districts, Tehsils and union councils in Balochistan are 7, 33, 141 and 86 respectively (PSLM, 2019-20). The following tables shows the demographic characteristics of provinces in Pakistan.

Table 8.2 : ***Demographic Characteristics of the Punjab***

	Urban	**Rural**
Households in Millions	6.39	10.71
Male Population in Millions	20.76	35.20
Female Population in Millions	19.62	34.43
Total Population in Millions	40.39	69.63
Transgender	4585	2124
Sex Ratio	105.81	102.25
Household Size	6.3	6.5

Table 8.3: ***Demographic Characteristics of the Sindh***

	Urban	**Rural**
Households in Millions	4.4	4.19
Male Population in Millions	13.01	11.92
Female Population in Millions	11.9	11.06
Total Population in Millions	24.91	22.98
Transgender	2226	301
Sex Ratio	109.31	107.8
Household Size	5.7	5.5

Table 8.4: Demographic Characteristics of the Khyber Pakhtunkhwa

	Urban	Rural
Households in Millions	0.74	3.10
Male Population in Millions	2.97	12.50
Female Population in Millions	2.76	12.30
Total Population in Millions	5.73	24.79
Transgender	690	223
Sex Ratio	107.83	101.6
Household Size	6.3	6.53

Table 8.5: Demographic Characteristics of the Balochistan

	Urban	Rural
Households in Millions	0.47	1.30
Male Population in Millions	1.79	4.69
Female Population in Millions	1.61	4.25
Total Population in Millions	3.40	8.94
Transgender	69	40
Sex Ratio	111.59	110.27
Household Size	7.2	6.93

This study utilized the secondary data from Pakistan Rural Household Panel Survey (PRHPS) conducted and provided by International Food Policy Research Institute (IFPRI) and Innovative Development Solution (IDS) in 2020.

The thesis has concluded based on a solid result of simulation that logarithmic minimum square test (LMS) is the most stringent test of Independence for nominal data sets. Therefore, we apply:

- → LMS tests and few others of independence on real nominal data set.
- → Computational details are given in chapter 3 while MATLAB Programing codes are presented in Appendix- B.
- → Hypothesis are presented as below.

$H_{0=}$ School enrolment and provincial domicile are statistically independent.

$H_{1=}$ School enrolment and provincial domicile are statistically dependent.

The objective is to assess evidence against the null hypothesis that the two variables Girl's school enrolment and provincial domicile are statistically independent.

8.3 Results and Discussion

We applied the most stringent tests of independence in read nominal data set arranged in w × k CT.

Table 8. 6: Results for W × K Contingency Table (Nominal Data)

Tests	Application of Logarithmic Minimum Square Test
α = 5%,	LMS
P Value	(0.0443)
	Decision: P value of Logarithmic Minimum Square test is less than 5% therefore, we reject null hypothesis and concludes that there is significant difference in girl's enrollment in education among different provinces in Pakistan.

8.4 Conclusion and Recommendations

The results of previous chapter i.e., Chapter 6 prove through a variety of Data Generating Process through Monte Carlo Simulations that the most stringent test for $\mathrm{w} \times k$ CTs for nominal data is logarithmic minimum square (LMS). Therefore, we are confident to apply LMS test on real nominal data set.

There is much discussion on gender differences in education but most of the discussion is without any statistical evidence. This study suggests that there is significant differences in gender enrollment in education.

CHAPTER 9

APPLICATION OF THE POWERFUL TEST ON ORDINAL REAL DATA

(NEXUS BETWEEN CORRUPTION PERCEPTION INDICES AND COUNTRIES BY PER CAPITA INCOME)

9.1 Introduction

Transparency International (TI) claim themselves to be a movement with a vision of corruption free world. Established in 1993, TI currently has chapters in 100 countries. The first ever Corruption Perception Index (CPI) was issued in 1995, since then every year Governments, Politicians, Civil Society, and Institutions anxiously started to wait for the new issue. Transparency International divides countries in six regions, AMERICAS (AME), ASIA PASIFIC (AP), EASTERN EUROPE & CENTRAL ASIA (ECA), WESTERN EUROPE & EUROPIAN UNION (WE-EU), MIDDLE EASTERN & NORTH AFRICA (MENA) and SUB-SAHARAN AFRICA (SSA).

In its current issue (2019) the CPI index ranks 180 countries divided into six regions. The index score varies from 0 to 100 from highly corrupt to very honest (dark RED to pale YELLOW in color scheme, see Fig# 9.1 below). About 67% of countries scored below 50; according to TI report no significant improvement is observed from previous years.

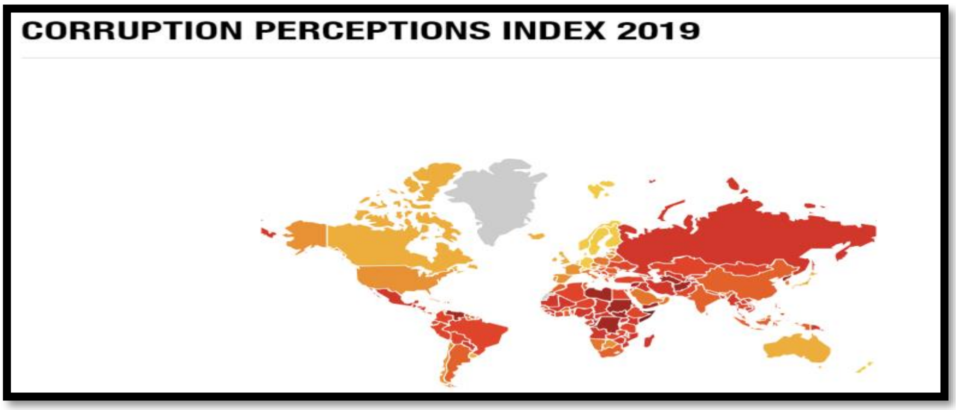


Figure 9.1: shows dark RED to pale YELLOW in color scheme.

Figure #9.1 above sign posts a strong link between ranks and Region. Another connotation can be observed between Rank and per capita income of the countries. To establish the fact, we sort the countries concerning scores in descending order (best performer to worst); we separate the group of countries with scores 50 and above and observe a pattern of regions. Figure # 9.2 below reveals the combination of better performing countries (50 & above score). About 44% of this group belong to WESTERN EUROPE & EUROPIAN UNION (WE-EU) region, proportion of other regions can be seen from figure # 9.2 below. Point to note that Region Americas (AME) includes Canada & USA, and the region Asia and Pacific contains Australia and New Zealand are among countries having score 50 and above.

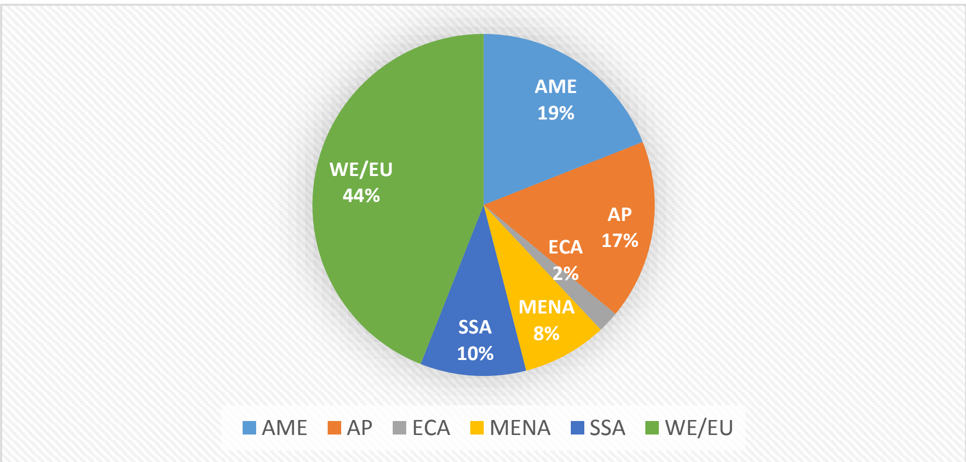


Figure 9.2: reveals the combination of better performing countries (50 & above score).

To establish the fact further, we simultaneously sort the countries below the median income per capita and above the median income per capita. We assign 1 point for countries performing well (50 and above score in CPI) and zero for bad performance (less than 50 score in CPI). In the similar fashion country having above median per capita income gets 1 point and zero for below median per capita income. Hence a country with 2 points is in the Best Performing Group (BPG) and the country with zero points belongs to Worst Performing Group (WPG).

The Best Performers:

In the best performing group 49% are from Western Europe and European Union (WE-EU) region; unfortunately no country from Eastern Europe and Central Asian group (ECA) qualify in this group. Proportion of other regions in this group can be seen in Figure#9.3 below. USA, Canada, Australia and New Zealand also belong to best performing group.

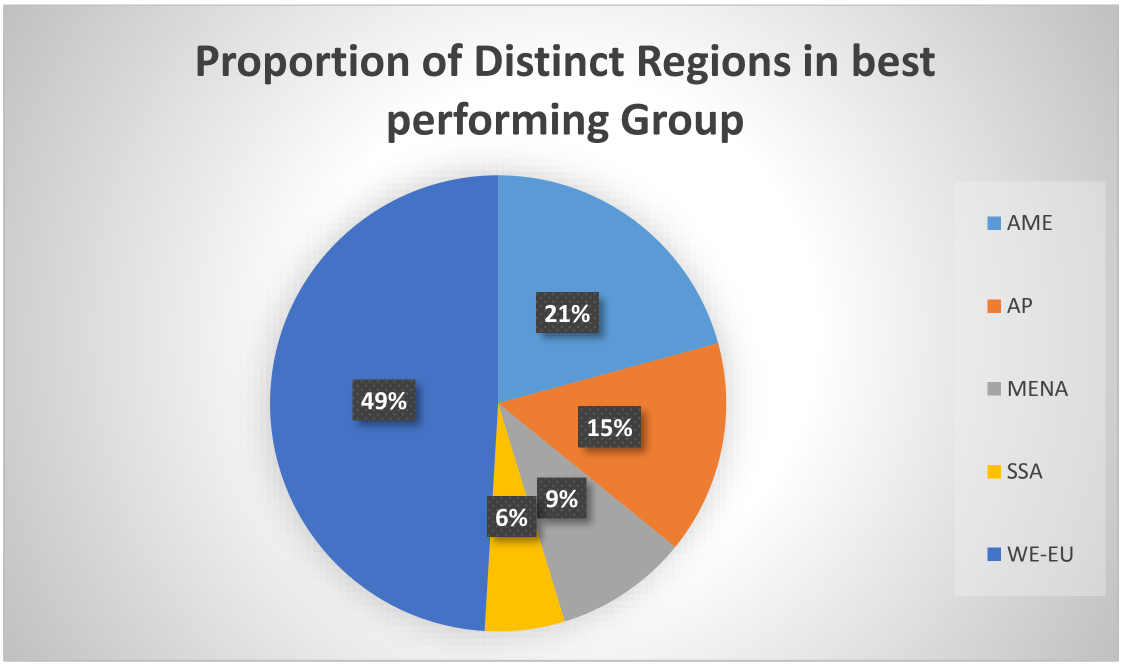


Figure 9.3: Reveals the proportion of distinct regions in best performing group.

To zoom in further it is observed that 84% countries, considered in 2019 CPI index, from WE-EU region fall in best performing group. No country from Eastern Europe and Central Asian Region (ECA) belongs to the best performing group. The proportions of other regions can be seen in Fig# 9.4 below

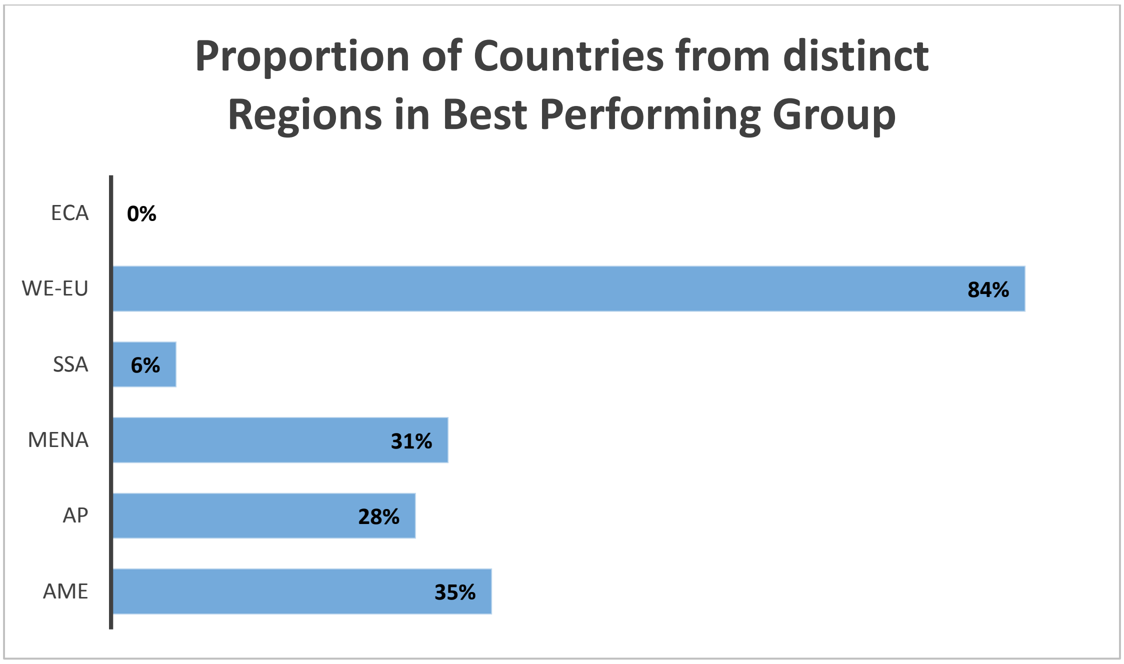


Figure 9.4: Proportion of Countries from distinct Regions in Best Performing Group

The Worst Performers:

In the worst performing group 46% are from Sub-Saharan Africa Region (SSA), as expected no country from Western Europe and European Union (WE-EU) Region in this group.

To zoom in deep it is observed that no country from WE-EU region fall in the worst performing group. About 79% countries, considered in 2019 CPI index, from Sub-

Saharan Africa (SSA) fall in the worst performing group. The proportions of other regions can be seen in Fig# 9.5 below.

Further it is observed that 62% countries with above median per capita income are also having good score (50 and above) in CPI.

Without using sophisticated statistical tools, a clear association can be established between CPI score and Region, per capita income. That is a country belongs to (WE/EU) Region and better income per capita might have good chance to score higher in CPI index.

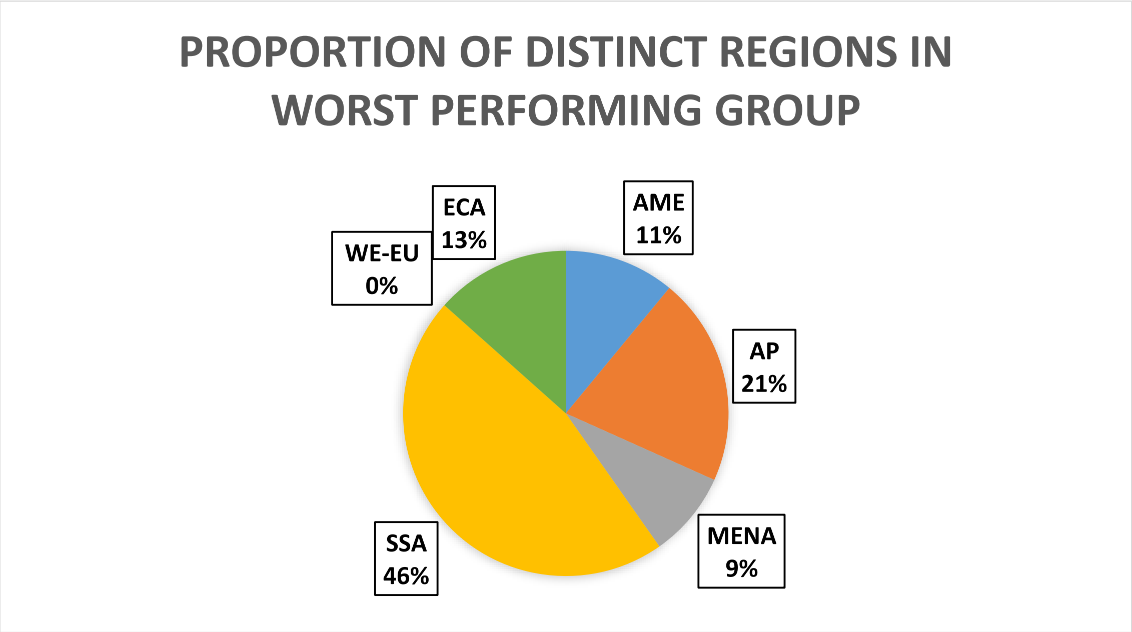


Figure 9.5: Proportion of Distinct Regions in Worst Performing Group

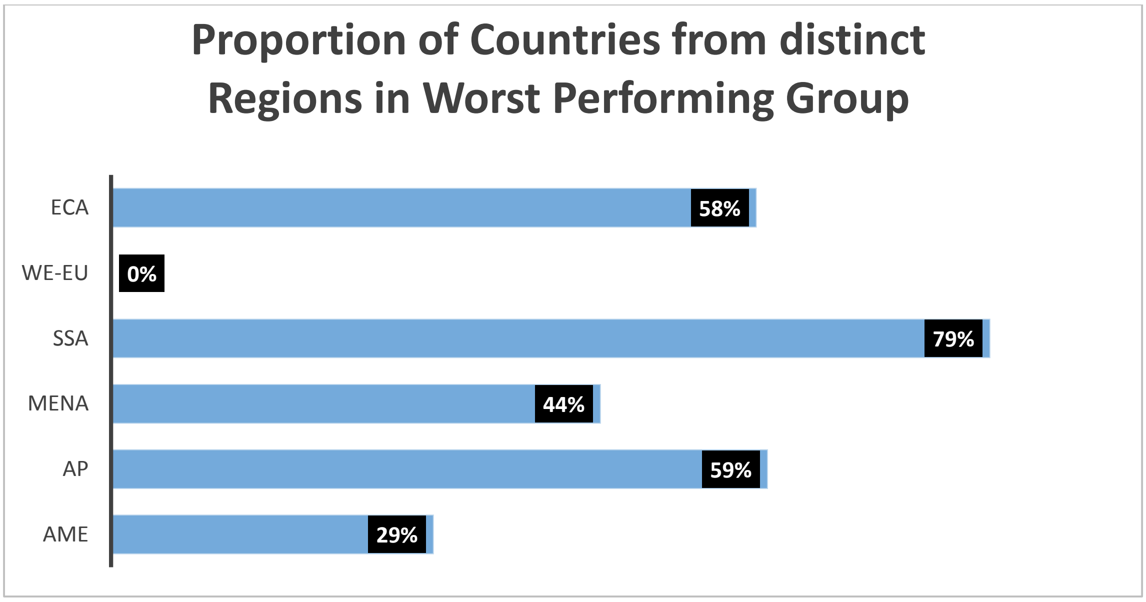


Figure 9.6: Proportions of other Regions in Worst Performing Group

9.2 Literature Review

There is an ongoing debate in the literature about the nature and direction of the link between corruption and income. There are two opposing views on the nature of the relationship, namely the efficiency-enhancing and the efficiency-diminishing view (Rehman and Naveed, 2007). The efficiency-enhancing view holds that corruption has positive effects on economic growth, which in turn increases per capita income (Leff, 1964; Huntington, 1968; Acemoglu and Verdier, 1998). According to the Efficiency Fette Hypothesis, corruption leads to greater efficiency (Mustapha, 2014). This is because it acts as a lubricant, motivating bureaucrats to be more productive and allowing investors to bypass time-consuming regulations or other transaction costs (Pak Hung Mo, 2001). Consistent with this, da Silva et al. (2001) the importance of the economic theory of bribery in studying corruption and income relations. Bureaucrats receive bribes, and firms accept the payment, both wanting to maximize their utility. However, the opposing view is that corruption has negative effects on the economy

(Kaufman and Wei, 1998; Aidt, 2009 and Mauro, 1995). Rehman and Naveed (2007) made it clear that corruption has a negative impact on efficiency due to what they consider to be an efficiency-reducing aspect. This is usually accompanied by a disincentive for investors to invest, leading to losses in productivity (Pak Hung Mo, 2001). In addition, corruption widens the gap between rich and poor and destroys any incentive to innovate. In addition, corruption increases the level of insecurity and political instability that hamper economic growth and development (da Silva et al., 2001). In summary, the impact of corruption on income can be viewed as a rent-seeking problem. According to Gyimah-Brempong (2002), corruption leads to misallocation of resources, loss of innovation, shift from productive activities to profit-oriented activities, and the creation of additional production costs, which in turn discourage investment. Da Silva et al. (2001) also emphasized that the level of corruption varies according to the type of institutional structure and the number of regulations.

Between these two opposing views, a third school of thought developed. This school deviates somewhat from the rather rigid ideology of the positive impact of corruption on income. It does this by tracking the impact of corruption on allocation efficiency. According to Rehman and Naveed (2007), allocation efficiency can be realized in the presence of corruption. Because although bureaucrats ignore the principle of bidding and award contracts to the highest bidder, it is usually the case that those who can afford to pay the highest bribes are those with the lowest costs. At the empirical level, the debate is still pronounced. For example, in their study of 65 countries, Li and Wu (2010) showed that trust offsets the negative impact of corruption on income. Furthermore, Blackburn and Forgues-Puccio (2009) examined the reason for the uneven impact of corruption in different countries in a dynamic general equilibrium model. Their results showed that countries with a well-organized corruption network will lead to lower bribery rates and higher growth rates. In addition, Rock et al. (2004) studied the relationship between corruption and economic growth in four different corruption data sets. Their findings showed that corruption slows growth in developing countries but boosts it in the newly industrialized large East Asian countries. In contrast, Mauro (1995) examined the relationship between investment and corruption in 58 countries. His findings revealed that corruption negatively impacts investment, which in turn negatively impacts the economy. Accordingly, Kaufman and Wei (1998) examined the impact of bribery payments on time and capital costs.

Their result contradicted the efficient grease hypothesis, as they found that those who pay bribes spend more time negotiating with bureaucrats, leading to higher capital costs. In addition, Aidt (2009) showed that growth and corruption exhibit a strong negative correlation. Igwike, Hussain and Noman (2012) came to the same conclusion in their study. They showed that there is an inverse relationship between corruption and economic development as measured by the annual growth rate of gross domestic product. Regarding the direction of the relationship between corruption and income, the debate is still ongoing, both at a theoretical and empirical level. On the one hand, as already mentioned, corruption affects income. Therefore, the direction of the relationship is from corruption to income. On the other hand, income can also affect corruption, leading to an ongoing debate as to whether corruption and income are unidirectionally or mutually related. According to Seldadyo and de Haan (2006), per capita income is one of the main determinants of corruption. The economic logic behind this is that corruption varies according to income level.

There are many studies that confirm this finding (Damania et al., 2004; Persson and Tabellini, 2003; and van Rijckeghem and Weder, 1997). However, there is further evidence that income has a negative impact on corruption, such as the case of Kunicova and Ackerman (2005), Lederman et al. (2005), Braun and Di Tella (2004), Chang and Golden (2004), Damania et al. (2004) and many others. Cole (2007) examined the relationship between income, corruption, and the environment. In his research, he acknowledged the existence of a reciprocal link between income and corruption. To do this, he used an IV estimation to avoid problems related to endogeneity. Therefore, the theoretical and empirical studies confirm that the debate on corruption income is not over yet.

9.3 Data and Methodology

The study include data on corruption perception index and countries categorized by per capita income for all counties listed in Transparency International reports in the year 2019 and Word Development Indicator (WDI).

Thus, hypothesis for w × k CT for this study is given below:

H_0: CPI and Countries Categorized by income per capita are statistically independent.

H_1: CPI and Countries Categorized by per capita income are statistically dependent.

9.4 Results and Discussion

Based on results of the current study implies that $Novel\ \emptyset_k$ is the most powerful test of independence for ordinal data. The test's findings are stated below.

Table 9. 1: Results for W × K Contingency Table (Ordinal Data)

Tests	Application of Powerful Test of Independence for Ordinal Data – Novel Phi_K (Φ_k)
α = 5%,	Novel Φ_k
P Value	(0.032)
Decision: P value of Novel $\emptyset_k$ test is less than 5% i.e. (0.032) therefore, we reject null hypothesis and concludes that there is significant difference in corruption perception index and countries categorized by income per capita which are dependent.	

9.5 Conclusion and Recommendations

We applied Novel Phi_K (Φ_k) measure of correlation on real data set of corruption perception index (CPI) and per capita income. From above analyses one can safely conclude that, there is fair chances to get high score in CPI if a Country belong to Western Europe and European Union (WE/EU) Region and have better income per capita. The other top four tests' results are also included which reject null hypothesis and same concludes as discussed above.

There is a lot of discussion about the relationship of corruption and development. We found that the relation is significant, however causal direction is not clear and the paper shows that high corruption is associated with lower level of income.

Based on the solid estimation of MCS presented in chapter # 07, we can recommend the Novel Phi_K (Φ_k) test of independence to be used for ordinal data

CHAPTER 10

CONCLUSION, RECOMMENDATION AND FUTURE DIRECTIONS

10.1 Conclusion

This dissertation describes the performance of tests of independence for nominal and ordinal data in $w \times k$ CTs. Keeping in view analysis of the chapters 5, 6 and 7; we are now able to draw some conclusions from our Monte Carlo simulations (MCS) results[18].

Tests of independence for nominal data have been examined and we found negligible distortion at various nominal level ($\alpha = 0.01$, $\alpha = 0.05$) at different sample size [small, medium and large]. Simulated critical values (SCV) have been computed for Fisher exact test, Neyman modified chi squared test and Kullback - Leibler test. Besides, simulated critical values are computed for seven tests of independence for ordinal data analysis in $w \times k$ CTs.

The analysis of chapter 5 concludes that there is no significant size distortion for selected eight tests of independence for nominal data at significance level ($\alpha = 0.05$) at different sample size [small, medium and large] for 2×2 and $w \times k$ CTs. Simulated critical values are computed for various tests of independence which does not follow

[18] The study is based on Monte Carlo simulations under numerous data generating process (DGP) described in chapter 4. It is possible that if the DGP of real data is not matching with DGP used, then results are not generalizable. However, we have created a large number of scenario to maximize the generalizability of results.

standard distribution for nominal and ordinal data which are further used in power computations.

Chapter 6 concludes power analysis that have been computed for different sample size [small, medium and large] at a specific MoU under several scenarios from I-V have been discussed in chapter 4. The results indicate for 2 ×2 order CTs that FES performs best among others tests of independence for nominal data in limitations IV and V. We also see that BPS tests performs best in scenario I, II and V. LMS performs in scenario III only. Similarly, the results for W × K CTs indicates that LMS performs better in scenario I, II and III. BPS performs better in scenario I and II. It is also concluded from chapter 6 results that tests performance are different in scenarios from I-V in W × K CTs such as Modular test (MDT), Likelihood ratio Test (G^2) and NMCS.

Thus, this study solves this complex problem by using the Stringency Criteria (SC) and it is finally concluded from shortcoming results that Fisher exact statistics (FES) has the lowest shortcomings and thus performs the best in small sample size of 2 x 2 order of CTs among the eleven sets of tests taken under current study. Similarly, LMS test has minimum shortcomings and performs best amongst others tests of independence for nominal data in W x K CTs.

Chapter 7 concludes based on MCS for ordinal data that Novel Φ_k test is the powerful test of independence for ordinal data. The good characteristic of this test is that it not only capture the linear association among the variables but it also captures the nonlinear association among the variables. This Novel Φ_k correlation can further be used either for nominal, ordinal, ratio interval or especially for mixed variables as well as performs optimal in W x K CTs.

The computation of the Novel Φ_k correlation is a bit tough as discussed in chapter 4. Therefore, Current study also found Kruskal-Gamma and Spearman Rank correlations close to the Novel Φ_k correlation in terms of power comparison for W x K CTs in analysis of ordinal data.

Even though the study has some limitations, however we believe that our findings should prove beneficial for researchers and practitioner. In this regard following are some recommendations.

10.2 Recommendation

This dissertation gives very clear-cut recommendation to the practitioner regarding utilization of tests of independence for nominal and ordinal data. Results of Monte Carlo simulations indicates that:

- → There is no significant size distortion for eight specific tests of independence for categorical data at significance level ($\alpha = 0.05$) at different sample size [small, medium and large] for 2×2 and $w \times k$ contingency table.
- → The study recommends based on solid estimation of Monte Carlo simulation and algorithm for a variety of DGP in 2×2 CT. We came to conclusion and recommended clearly that Fisher exact statistics (FES) is the most stringent test and no other test can beat it for nominal data in 2×2 CTs.
- → Practitioners should know about the scenarios behind their data; however, we see from our results in maximum scenarios that LMS tests performs better than others test of independence for nominal data in $w \times k$ CTs. LMS

have the lowest shortcomings amongst others tests and therefore this test is recommended as the most stringent test for $w \times k$ CTs.

→ We are also able to rank test according to their shortcomings and we found minimum shortcomings of tests of independence sequentially; are LMS, BPS, MDS, D square, and G square for nominal data. The poorest test is KLS, CRS and NMCS.

→ Moreover, it may be noted that in analysis of measure of correlations/ tests of independence in ordinal data, the most powerful test of independence is Novel Φ_k, which is recommended to be used for $w \times k$ CTs for ordinal data.

→ We are also able to rank test according to their powers which concludes that in terms of powerful tests for ordinal data are ranked sequentially are Novel Φ_k, Goodman - Kruskal γ, Spearman ρ, Somers'd, Kendall τ-c, Kendall τ-b and Kendall τ-a.

10.3 Practical Implications

This research is specifically helpful to the statisticians/econometricians and other researchers who are connected and working directly or indirectly in national and international Research and Development (R&D) departments in educational sector, medical sector, agriculture sector, technological sector and any other sector in Pakistan or across the globe which are enabling them to apply the most appropriate test of independence for categorical data in 2×2 or $w \times k$ CTs.

This research is specifically helpful to the National Institute of Health, Islamabad (NIH), National Agriculture Research Center Islamabad (NARC), Higher Education Commission of Pakistan (HEC) and generally in education, medical and agriculture sector including all others Research and Development (R&D) departments of numerous industries in Pakistan and across the globe which are enabling them to apply the most appropriate test of independence for categorical data in 2×2 or $w \times k$ CTs.

The importance of the use of tests of independence for categorical data is common practice in many fields mostly due to its importance and application in statistics, education, biological and social science for example when a pharmacist of the field of medical science are interested to find Covid-19 vaccine effect for corona virus disease. He collects the data into two groups before and after the use of vaccine of corona disease. The two groups are called treatment and control group. The first one is to whom the vaccine is given and the other one those to whom vaccine have not been given. In both circumstances to find whether the vaccine has some effect or not. The researcher ought to use the most stringent test i.e., Fisher Exact Statistics (FES) in 2×2‘CTs and Logarithmic Minimum Square test (LMS) to find the correlation in nominal data in $w \times k$ CTs.

In education field when a researcher is interested to find the effect of some new developed techniques of teaching methodology, that whether the new methodology is effective or not. The researcher accumulates the student's grades before and after the implementation of new methodology and do assessments of the independence using any of the test statistics to find out the effectiveness of the new techniques in positive or negative sense

Similarly, in field of Biological Science when a biologist is interested to know the fertilizer's effect for a particular crop. The researcher takes the data and divide it into two groups called treatment group and control group. Thus here is again two groups, the first one is the class of plant which are fertilized and the other is that which is not. In both of the circumstances the researcher uses different tests statistics to find out the effect of the fertilizer that whether the fertilizer has a substantial effect or not for particular crop.

The above examples and discussion shows that the tests of independence are commonly used and their result and conclusion has a vital role in many fields and in social life. If the conclusion is wrong of a test of independence then it might have a very bad effect on a human life as well as on society.

10.4 Future Research

The study can be further improved to analyze tests of independence for Three - dimensions i.e., W x K x P CTs. There are many scenarios exists in analysis of the independence in three-fold CTs i.e., full independence, boundary independence, partial independence, total independence and conditional independence. In future research this can be carried out to investigate the most powerful test of independence for categorical data.

The study for tests of independence for ordinal data can be further modified and investigated through developing of a new test of independence that is free of sample size and table dimensions in three-fold CTs.

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Appendix A

This is an extension of this study by including and investigating the most stringent tests for 2×2 CTs for nominal data based on SC. Therefore, the computational formula for eleven tests of independence for nominal data for 2×2 CTs are given below.

Table A. 1: Computational Formulas for Test of Independence for 2×2 CTs

S. No	Test of Independence	Formula for 2×2 CTs	Standard Distribution	References
1	Chi Square Test (χ^2)	$\chi^2_{2\times2} = \frac{n(ad-bc)^2}{(a+b)(c+d)(a+c)(b+d)}$	Chi Square	Sulewski, P. (2017)
2	Likelihood Ratio (G^2) Test	$G^2_{2\times2} = \sum_{i=1}^{4} S_i$	Non - Central Chi Square	Sulewski, P. (2017)
3	Fisher Exact Test Statistics (FES)	$FES = \frac{\binom{a+b}{a}\binom{c+d}{c}}{\binom{n}{a+c}} = \frac{(a+b)!\,(c+d)!\,(a+c)!\,(b+d)!}{n!\,a!\,b!\,c!\,d!}$	Does not follow any Known /Standard Distribution	Sulewski, P. (2017)
4	Nyman Modified Chi Square Test Statistics (NMCS)	$NMCS_{2\times2} = \frac{(a-e_1)^2}{a} + \frac{(b-e_2)^2}{b} + \frac{(c-e_3)^2}{c} + \frac{(d-e_4)^2}{d}$	Does not follow any Known /Standard Distribution	Sulewski, P. (2017)
5	Kullback and Libeler Test Statistics (KLS)	$KLT_{2\times2} = 2\left[e_1 \ln\left(\frac{e_1}{a}\right) + e_2 \ln\left(\frac{e_2}{b}\right) + e_3 \ln\left(\frac{e_3}{c}\right) + e_4 \ln\left(\frac{e_4}{d}\right)\right]$	Does not follow any Known /Standard Distribution	Sulewski, P. (2017)
6	Freeman and Tuckey Test Statistics (FTS)	$FTT_{2\times2} = 4\left[\left(\sqrt{a}-\sqrt{e_1}\right)^2 + \left(\sqrt{b}-\sqrt{e_2}\right)^2 + \left(\sqrt{c}-\sqrt{e_3}\right)^2 + \left(\sqrt{d}-\sqrt{e_4}\right)^2\right]$	Non - Central Chi Square	Sulewski, P. (2017)
7	Cressie and Read Test Statistics (CRS)	$CRT_{2\times2} = \frac{9}{5}[aS_1 + bS_2 + aS_3 + dS_4]$	Non - Central Chi Square	Sulewski, P. (2017)
8	D Square ($\boldsymbol{D^2}$) Test Statistics (DSquare)	$D^2_{2\times2} = \sum_{i=1}^{2}\sum_{j=1}^{2} \frac{(n^*_{ij}-e^*_{ij})^2 - n^*_{ij}}{e^*_{ij}}$	Non - Central Chi Square	Sulewski, P. (2017)
9	Modular Test $\lvert\chi\rvert$ ($\lvert\chi\rvert$Mtest)	$MDT_{2\times2} = \lvert\chi\rvert = \sum_{i=1}^{2}\sum_{j=1}^{2} \frac{\lvert n^*_{ij}-e^*_{ij}\rvert}{e^*_{ij}}$	Non - Central Chi Square	Sulewski, P. (2017)
10	BP Tests Statistics (BPS)	$BPT_{2\times2} = n(p^* - p_0)^t A(p^* - p_0)$	Non - Central Chi Square	Sulewski, P. (2017)
11	Logarithmic Minimum Square Test (LMS)	$LMS_{2\times2} = -\sum_{i=1}^{2}\sum_{j=1}^{2} \ln\left[\frac{\min(n^*_{ij}, e^*_{ij})}{\max(n^*_{ij}, e^*_{ij})}\right]$	Non - Central Chi Square	Sulewski, P. (2017)

Appendix B

This section contains "MATLAB Programing" used in dissertation for Tests of Independence for nominal and ordinal data. The dissertation consists of eleven tests of independence for nominal data and seven tests of independence/ Measure of correlation for ordinal data. Additionally, the dissertation consists of a variety of Data Generating Process (DGP) for various order of contingency tables. Thus, a sample of MATLAB programing codes for tests of independence for higher order contingency table, computation of empirical size, computation of finite sample critical value (F.S.C.V) and computation of power are presented below.

Table B. 1: Codes for tests of independence / Measure of correlation for Nominal data

1. Chi Square Test

```
function Chisq=ConTbale(A)
A2=A/sum(sum(A));
[n, k]=size(A);
RT=sum(A,2);
CT=sum(A,1);
GT=sum(CT);
TT=zeros(n,k);
for i=1:n
   for j=1:k
      TT(i,j)=CT(1,j)*RT(i,1)/GT;
   end
end

Diff=((A-TT).*(A-TT));
Diff2=Diff./TT;
Chisq=sum(sum(Diff2));
```

2. Logliklihood Test (G^2)

```
function Gstat=Gtest(A)

[n, k]=size(A);
RT=sum(A,2);
CT=sum(A,1);
GT=sum(CT);
TT=zeros(n,k);
for i=1:n
   for j=1:k
      TT(i,j)=CT(1,j)*RT(i,1)/GT;
   end
end

   p1=A./(TT+eps)
   p2=log(p1+eps)
   p3=A.*p2
   p4=sum(sum(p3))

   Gstat=2*p4
```

3. Fisher Exact Test (FES)

```
function fet2=fetest(A)
A2=A/sum(sum(A));
CT=sum(A,1);

RT=sum(A,2);

GT=sum(RT);

[n,k]=size(A);

for i=1:n
   f(i,1)=log(factorial(RT(i,1)));
end

FACT1=sum(f);

for j=1:k
   f2(j,1)=log(factorial(CT(1,j)));
end

FACT2=sum(f2);
f3=log(factorial(A));
FACT4=sum(sum(f3));
fet2=FACT1+FACT2-log(factorial(GT))-log(FACT4
```

4. Kullber Liaber Test (KLS)

```
function KLTest=KLT(A)
A2=A/sum(sum(A));
[n, k]=size(A);
RT=sum(A2,2);
CT=sum(A2,1);
GT=sum(CT);
TT=zeros(n,k);
for i=1:n
   for j=1:k
      TT(i,j)=CT(1,j)*RT(i,1)/GT;
   end
end

Div=TT./(A2+eps);

KLTest=2*sum(sum(TT.*log(Div+eps)));
```

5. Freeman and Tuckey Test (FTS)

6. Crecent and Read Test (CRS)

```
function FTTEST=FTEST(A)
A2=A/sum(sum(A));
[n, k]=size(A);
RT=sum(A,2);
CT=sum(A,1);
GT=sum(CT);
TT=zeros(n,k);
for i=1:n
   for j=1:k
      TT(i,j)=CT(1,j)*RT(i,1)/GT;
   end
end
M= sqrt(A)- sqrt(TT);
M2=M.*M;

FTTEST= 4*sum(sum(M2));
```

```
function CRTEST=CRT(A)
[n, k]=size(A);
RT=sum(A,2);
CT=sum(A,1);
GT=sum(CT);
TT=zeros(n,k);
for i=1:n
   for j=1:k
      TT(i,j)=CT(1,j)*RT(i,1)/GT;
   end
end

k=A./TT;
k2=A.^(2/3)-1
K3=A.*k2
```

7. LMS Test

```
function LMSTest=LMST(A)
[n, k]=size(A);
RT=sum(A,2);
CT=sum(A,1);
GT=sum(CT);
TT=zeros(n,k);
for i=1:n
   for j=1:k
      TT(i,j)=CT(1,j)*RT(i,1)/GT;
   end
end
M= min(A,TT);
M2=max(A,TT);
M3= M./M2;
LMSTest= -(sum(sum(log(M3)));
```

8. BP Test (BPS)

```
function BPTest=BPT(A)
[n, k]=size(A);
RT=sum(A,2);
CT=sum(A,1);
GT=sum(CT);
TT=zeros(n,k);
for i=1:n
   for j=1:k
      TT(i,j)=CT(1,j)*RT(i,1)/GT;
   end
end
P1= A./n
P0 = A*TT
A=sum(po)^-1
SP= diag(p)-p^'p
BPTEST= n(P1-Po)'*A(P1-Po)
```

9. NMCS Test

```
function NMCTEST=NMC(A)
A2=A/sum(sum(A))
[n, k]=size(A)
RT=sum(A,2)
CT=sum(A,1)
GT=sum(CT)
TT=zeros(n,k)
for i=1:n

   for j=1:k
      TT(i,j)=CT(1,j)*RT(i,1)/GT;
   end
end

TT
U=(A2 - TT)
Dif=((A2-TT).*(A2-TT))
Dif2=Dif./(A2)
NMCTEST= sum(sum(Dif2))
[A2 TT]
```

10. D Square Test

```
function Dsquare=DSQT(A)
A2=A/sum(sum(A));
[n, k]=size(A);
RT=sum(A,2);
CT=sum(A,1);
GT=sum(CT);
TT=zeros(n,k);
for i=1:n
   for j=1:k
TT(i,j)=CT(1,j)*RT(i,1)/GT;
 end
end
Diff=((A-TT).*(A-TT));
Diff2=Diff-A;
Diff3=Diff2./TT;
Dsquare=sum(sum(Diff3));
```

11. Modular Test (MDS)

```
function MDTest=MDT(A)
A2=A/sum(sum(A));
[n, k]=size(A);
RT=sum(A,2);
CT=sum(A,1);
GT=sum(CT);
TT=zeros(n,k);
for i=1:n
   for j=1:k
      TT(i,j)=CT(1,j)*RT(i,1)/GT;
   end

   Z1= A - TT
   Z2= abs(Z1)./TT
   MDTest=sum(sum(Z2))
end
```

*****[19]NPLT/Point Optimal Test**

```
function NPS=nptest(X,TTh0,TTH1)

Lh1=LikelihoodCT(X,TTH1);
Lh0=LikelihoodCT(X,TTh0);

NPS=Lh1-Lh0;
```

Table A2.2: Codes for Data Generating Process for W × K CTs for Nominal Data

[19] Nyman Pearson lemma or point optimal test is used in power computations in comparison of tests of independence for nominal and ordinal data.

DGP 2 × 3 CT

```
function CT=CT23(n,TT23)
nn=sum(sum(TT23));
TT=TT23/nn;
OB=zeros(2,3);
   for i=1:n
      x=rand;
      if x<TT(1,1)
         OB(1,1)=OB(1,1)+1;
      elseif x<TT(1,1)+TT(1,2)
            OB(1,2)=OB(1,2)+1;
      elseif  x<TT(1,1)+TT(1,2)+TT(1,3)
            OB(1,3)=OB(1,3)+1;
      elseif  x<TT(1,1)+TT(1,2)+TT(1,3)+T
               OB(2,1)=OB(2,1)+1;
      elseif
x<TT(1,1)+TT(1,2)+TT(1,3)+TT(2,1)+TT(2
                  OB(2,2)=OB(2,2)+1;
      else
            OB(2,3)=OB(2,3)+1;
      end
   end
```

DGP 3 × 3 CT

```
CT=OB
function CT=CT33(n,TT33)
nn=sum(sum(TT33));
TT=TT33/nn;
OB=zeros(3,3);
   for i=1:n
      x=rand;
      if x<TT(1,1)
          OB(1,1)=OB(1,1)+1
       elseif x<TT(1,1)+TT(1,2)
         OB(1,2)=OB(1,2)+1;
       elseif  x<TT(1,1)+TT(1,2)+TT(1,3)
           OB(1,3)=OB(1,3)+1;
       elseif  x<TT(1,1)+TT(1,2)+TT(1,3)+TT(2,1)
               OB(2,1)=OB(2,1)+1;
       elseif x<TT(1,1)+TT(1,2)+TT(1,3)+TT(2,1)+TT(2,2)
                   OB(2,2)=OB(2,2)+1;
       elseif x<TT(1,1)+TT(1,2)+TT(1,3)+TT(2,1)+TT(2,2)+TT(2,3)
                     OB(2,3)=OB(2,3)+1;
       elseif
x<TT(1,1)+TT(1,2)+TT(1,3)+TT(2,1)+TT(2,2)+TT(2,3)+TT(3,1)
                   OB(3,1)=OB(3,1)+1;
       elseif
x<TT(1,1)+TT(1,2)+TT(1,3)+TT(2,1)+TT(2,2)+TT(2,3)+TT(3,1)+T
                              OB(3,2)=OB(3,2)+1;
       else
          OB(3,3)=OB(3,3)+1;
      end
   end
   CT=OB
```

Data Generating Process for 4 × 4 Contingency Table – [Sample Pattern]

```
function CT=CT44(n,TT44)
nn=sum(sum(TT44));
TT=TT44/nn;
OB=zeros(4,4);
   for i=1:n
      x=rand;
      if x<TT(1,1)
         OB(1,1)=OB(1,1)+1;
      else if x<TT(1,1)+TT(1,2)
            OB(1,2)=OB(1,2)+1;
      elseif  x<TT(1,1)+TT(1,2)+TT(1,3)
            OB(1,3)=OB(1,3)+1;
      elseif  x<TT(1,1)+TT(1,2)+TT(1,3)+TT(1,4)
               OB(1,4)=OB(1,4)+1;
      elseif x<TT(1,1)+TT(1,2)+TT(1,3)+TT(1,4)+TT(2,1)
                  OB(2,1)=OB(2,1)+1;
      elseif x<TT(1,1)+TT(1,2)+TT(1,3)+TT(1,4)+TT(2,1)+TT(2,2)+TT(2,2)
                        OB(2,2)=OB(2,2)+1;
      elseif x<TT(1,1)+TT(1,2)+TT(1,3)+TT(1,4)+TT(2,1)+TT(2,2)+TT(2,3)
                        OB(2,3)=OB(2,3)+1;
      elseif x<TT(1,1)+TT(1,2)+TT(1,3)+TT(1,4)+TT(2,1)+TT(2,2)+TT(2,3)+TT(2,4)
                        OB(2,4)=OB(2,4)+1;
      elseif x<TT(1,1)+TT(1,2)+TT(1,3)+TT(1,4)+TT(2,1)+TT(2,2)+TT(2,3)+TT(2,4)+TT(3,1)
                        OB(3,1)=OB(3,1)+1;
      elseif x<TT(1,1)+TT(1,2)+TT(1,3)+TT(1,4)+TT(2,1)+TT(2,2)+TT(2,3)+TT(2,4)+TT(3,1)
                        OB(3,2)=OB(3,2)+1;
      elseif x<TT(1,1)+TT(1,2)+TT(1,3)+TT(1,4)+TT(2,1)+TT(2,2)+TT(2,3)+TT(2,4)+TT(3,1)+TT(3,2)+TT(3,3)
                        OB(3,3)=OB(3,3)+1;
      elseif
x<TT(1,1)+TT(1,2)+TT(1,3)+TT(1,4)+TT(2,1)+TT(2,2)+TT(2,3)+TT(2,4)+TT(3,1)+TT(3,2)+TT(3,3)+TT(3,4)
```

```
                    OB(3,4)=OB(3,4)+1;
      elseif x<TT(1,1)+TT(1,2)+TT(1,3)+TT(2,1)+TT(2,2)+TT(2,3)+TT(3,1)+TT(3,2)+TT(3,3)+TT(3,4)+TT(4,1)
                    OB(4,1)=OB(4,1)+1;
      elseif
x<TT(1,1)+TT(1,2)+TT(1,3)+TT(1,4)+TT(2,1)+TT(2,2)+TT(2,3)+TT(2,4)+TT(3,1)+TT(3,2)+TT(3,3)+TT(3,4)+T
)+TT(4,2)
                    OB(4,2)=OB(4,2)+1;
      elseif
x<TT(1,1)+TT(1,2)+TT(1,3)+TT(1,4)+TT(2,1)+TT(2,2)+TT(2,3)+TT(2,4)+TT(3,1)+TT(3,2)+TT(3,3)+TT(3,4)+T
)+TT(4,2)+TT(4,3)
                    OB(4,3)=OB(4,3)+1;
      elseif
x<TT(1,1)+TT(1,2)+TT(1,3)+TT(1,4)+TT(2,1)+TT(2,2)+TT(2,3)+TT(2,4)+TT(3,1)+TT(3,2)+TT(3,3)+TT(3,4)+T
)+TT(4,2)+TT(4,3)+TT(4,4)
      else
                    OB(4,4)=OB(4,4)+1
         end
      end
   end
   CT=OB
```

Table A2.3: Codes for Computation of Empirical Size for Tests of Independence in W × K CTs for Nominal Data

```
TT2=[4 5 6
   8 10 12];
[r k]=size(TT2);
df=(r-1)*(k-1);
N=sum(sum(TT2));
TT=TT2/N;
Rejchisq=0;
RejGtest=0;
RejCRT=0;
RejFTEST=0;
RejKLT=0;
for j=1:20000
OB=zeros(2,3);
   for i=1:40
      x2=randn;
      x=normcdf(x2);
      if x<TT(1,1)
         OB(1,1)=OB(1,1)+1;
      else if x<TT(1,1)+TT(1,2)
            OB(1,2)=OB(1,2)+1;
         else if  x<TT(1,1)+TT(1,2)+TT(1,3)
            OB(1,3)=OB(1,3)+1;
            else if  x<TT(1,1)+TT(1,2)+TT(1,3)+TT(2,1)
               OB(2,1)=OB(2,1)+1;
               else if x<TT(1,1)+TT(1,2)+TT(1,3)+TT(2,1)+TT(2,2)
                  OB(2,2)=OB(2,2)+1;
                  else
                     OB(2,3)=OB(2,3)+1;
                  end
               end
            end
         end
      end
   end
   a=ConTbale(OB);
   b= Gtest(OB);
   c= CRT(OB);
   d= FTEST(OB);
   e=KLT(OB);
```

```
    CV= chi2inv(.95,df);
    if a>CV
       Rejchisq=Rejchisq+1;
    end
    if b>CV
       RejGtest=RejGtest+1;
    if c>CV
       RejCRT=RejCRT+1;
    if d>CV
       RejFTEST=RejFTEST+1;
    if e>CV
       RejKLT=RejKLT+1;
    end
end
    end
    end
end
size_chisq= Rejchisq/20000

size_Gtest= RejGtest/20000

size_CRT= RejCRT/20000

size_FTEST= RejFTEST/20000

size_KLT= RejKLT/20000
```

Table A2.4: Codes for Power Curve for Tests of Independence in W × K CTs for Nominal Data

```
% Program for calculating power curve of PO test
% Null ; MoU=0
% alternative; MoU=.01

TTH0_0 = [3 5  6
          6 10 12 ];
TTH0=TTH0_0/sum(sum(TTH0_0));
MCSS=1000;
SCT=50
TTH1=[0.0759146 0.1190476   0.1406141
0.1428571   0.2380952   0.2834713]
for i=1:MCSS
 CTBL=CT23(SCT,TTH0);

    b(i,1)=nptest(CTBL,TTH0,TTH1);
end
CV=prctile(b,95)
pTp01atp001=0

for j=1:MCSS
    CTBLE=CT23(SCT,TTH1);
    b2=nptest(CTBLE,TTH0,TTH1);
    if b2>CV
       pTp001atp001=pTp01atp001+1;
    end
end
PowerPTp001atp001=pTp001atp001/MCSS;

TTH1p002=[0.0804373 0.1190476   0.1383528
0.1428571   0.2380952   0.2812099]
pTp001atp002=0

for j=1:MCSS
    CTBLE=CT23(SCT,TTH1p002);
```

```
    b2=nptest(CTBLE,TTH0,TTH1);
    if b2>CV
        pTp001atp002=pTp001atp002+1;
    end
end
PowerPTp001atp002=pTp001atp002/MCSS;

TTH1p003=[0.0849974 0.1190476   0.1360728
0.1428571   0.2380952   0.2789299]
pTp001atp003=0

for j=1:MCSS
    CTBLE=CT23(SCT,TTH1p003);
    b2=nptest(CTBLE,TTH0,TTH1);
    if b2>CV
        pTp001atp003=pTp001atp003+1;
    end
end
PowerPTp001atp003=pTp001atp003/MCSS;
TTH1p004=[0.0895958 0.1190476   0.1337735
0.1428571   0.2380952   0.2766306]
pTp001atp004=0;

for j=1:MCSS
    CTBLE=CT23(SCT,TTH1p004);
    b2=nptest(CTBLE,TTH0,TTH1);
    if b2>CV
        pTp001atp004=pTp001atp004+1;
    end
end
PowerPTp001atp004=pTp001atp004/MCSS;
TTH1p005=[0.0942337 0.1190476   0.1314546
0.1428571   0.2380952   0.274311]

pTp001atp005=0;

for j=1:MCSS;
    CTBLE=CT23(SCT,TTH1p005);
    b2=nptest(CTBLE,TTH0,TTH1);
    if b2>CV
        pTp001atp005=pTp001atp005+1;
    end
end
PowerPTp001atp005=pTp001atp005/MCSS;
TTH1p006=[0.0989120 0.1190476   0.1291154
0.1428571   0.2380952   0.271972]
pTp001atp006=0;

for j=1:MCSS;
    CTBLE=CT23(SCT,TTH1p006);
    b2=nptest(CTBLE,TTH0,TTH1);
    if b2>CV
        pTp001atp006=pTp001atp006+1;
    end
end

PowerPTp001atp006=pTp001atp006/MCSS;

TTH1p007=[0.1036319 0.1190476   0.1267555
0.1428571   0.2380952   0.269612]

pTp001atp007=0;

for j=1:MCSS;
```

```
   CTBLE=CT23(SCT,TTH1p007);
   b2=nptest(CTBLE,TTH0,TTH1);
   if b2>CV
      pTp001atp007=pTp001atp007+1;
   end
end
PowerPTp001atp007=pTp001atp007/MCSS;

PowerPTp001atp007=pTp001atp007/MCSS;

TTH1p008=[0.1083943 0.1190476   0.1243743
0.1428571   0.2380952   0.2672314]

pTp001atp008=0;

for j=1:MCSS;
   CTBLE=CT23(SCT,TTH1p008);
   b2=nptest(CTBLE,TTH0,TTH1);
   if b2>CV
      pTp001atp008=pTp001atp008+1;
   end
end
PowerPTp001atp008=pTp001atp008/MCSS;

TTH1p009=[0.1132006 0.1190476   0.1219711
0.1428571   0.2380952   0.2648283]

pTp001atp009=0;

for j=1:MCSS;
   CTBLE=CT23(SCT,TTH1p009);
   b2=nptest(CTBLE,TTH0,TTH1);
   if b2>CV
      pTp001atp009=pTp001atp009+1;
   end
end
PowerPTp001atp009=pTp001atp009/MCSS;
TTH1p010=[0.1180522 0.1190476   0.1195453
0.1428571   0.2380952   0.2624025]

pTp001atp010=0;

for j=1:MCSS;
   CTBLE=CT23(SCT,TTH1p010);
   b2=nptest(CTBLE,TTH0,TTH1);
   if b2>CV
      pTp001atp010=pTp001atp010+1;
   end
end
PowerPTp001atp010=pTp001atp010/MCSS;


TTH1p011=[0.1229495 0.1190476   0.1170967
0.1428571   0.2380952   0.2599538]

pTp001atp011=0;

for j=1:MCSS;
   CTBLE=CT23(SCT,TTH1p011);
   b2=nptest(CTBLE,TTH0,TTH1);
   if b2>CV
      pTp001atp011=pTp001atp011+1;
   end
end
```

```
PowerPTp001atp011=pTp001atp011/MCSS;
TTH1p012=[0.1278948 0.1190476   0.1146240
0.1428571   0.2380952   0.2574812]
pTp001atp012=0;

for j=1:MCSS;
   CTBLE=CT23(SCT,TTH1p012);
   b2=nptest(CTBLE,TTH0,TTH1);
   if b2>CV
      pTp001atp012=pTp001atp012+1;
   end
end
PowerPTp001atp012=pTp001atp012/MCSS;

TTH1p013=[0.1328893 0.1190476   0.1121268
0.1428571   0.2380952   0.2549839]

pTp001atp013=0;

for j=1:MCSS;
   CTBLE=CT23(SCT,TTH1p013);
   b2=nptest(CTBLE,TTH0,TTH1);
   if b2>CV
      pTp001atp013=pTp001atp013+1;
   end
end
PowerPTp001atp013=pTp001atp013/MCSS;


TTH1p014=[0.1379343 0.1190476   0.1096043
0.1428571   0.2380952   0.2524614]

pTp001atp014=0;
for j=1:MCSS;
   CTBLE=CT23(SCT,TTH1p014);
   b2=nptest(CTBLE,TTH0,TTH1);
   if b2>CV
      pTp001atp014=pTp001atp014+1;
   end
end
PowerPTp001atp014=pTp001atp014/MCSS;

TTH1p015=[0.1430314 0.1190476   0.1070557
0.1428571   0.2380952   0.2499129]

pTp001atp015=0;

for j=1:MCSS;
   CTBLE=CT23(SCT,TTH1p015);
   b2=nptest(CTBLE,TTH0,TTH1);
   if b2>CV
      pTp001atp015=pTp001atp015+1;
   end
end
PowerPTp001atp015=pTp001atp015/MCSS;

PowerCurvePOp001=[
   .01  PowerPTp001atp001
   .02  PowerPTp001atp002
   .03  PowerPTp001atp003
   .04  PowerPTp001atp004
   .05  PowerPTp001atp005
   .06  PowerPTp001atp006
   .07  PowerPTp001atp007
```

.08 PowerPTp001atp008
.09 PowerPTp001atp009
.10 PowerPTp001atp010
.11 PowerPTp001atp011
.12 PowerPTp001atp012
.13 PowerPTp001atp013
.14 PowerPTp001atp014
.15 PowerPTp001atp015]

The Following table A2.2 presents complete set of Matlab Programming codes for tests of independence / Measure of correlations, Computation for finite sample critical values and Power analysis for ordinal data.

Table B. 2: Codes for tests of independence / Measure of correlation for ordinal data

1. Goodman Kruskal Test

```
function gk=goodmankruskal(X)

[Nc Nd]=NcNd(X)

gk=(Nc-Nd)/(Nc+Nd);
```

2. Kendal Tau (a)

```
function Kta=kandaltaua(X)

[a1 a2 a3 a4 a5 a6]=NcNd(X)
p1=2*(a1-a2);
p2=a6*(a6-1)

Kta= (p1/p2);
```

3. Kendal Tau b

```
function ktb=kendalltaub(x)

[Nc Nd Tx Ty]=NcNd(x)

k1= Nc-Nd
k2=(Nc+Nd+Tx)
k3=(Nc+Nd+Ty)

k4=sqrt(k2*k3)
ktb= k1/k4;
```

4. Kendal Tau C

```
function ktc=kendaltauc(x)

[Nc Nd]=NcNd(x)

m=min(n,k)

f1= Nc-Nd
f2=2*m*(f1)
f3=n*n*(m-1)

ktc=f2/f3;
```

5. Spearman Rho Correlation Test

```
n=100
  a=0.5
  b=0.5
for i=1:n
   x=randn

  if x  <-.8
     x1=1
  elseif x  <0.8
     x1=2
   else
    x1=3
  end
   y=a*x+b*randn
  if y <-.8
     y1=1
  elseif  y <0.8
     y1=2
  else
    y1=3
  end
    X(i)= x1
    Y(i)= y1
end
Rnk_x=tiedrank(X)
Rnk_y=tiedrank(Y)
```

6. Novel Φ_k

```
function rho=Phi_k(A)
A2=A/sum(sum(A));
[n, k]=size(A);
RT=sum(A,2);
CT=sum(A,1);
GT=sum(CT);
TT=zeros(n,k);
n_empty=0
% By default c=0
c=1
for i=1:n
   for j=1:k
      TT(i,j)=CT(1,j)*RT(i,1)/GT;
      if TT(i,j)==0
         n_empty = n_empty +1
      end

   end
end

Diff=((A-TT).*(A-TT));
Diff2=Diff./TT;
Chisq=sum(sum(Diff2));
nsdof = (n-1)*(k-1)-n_empty
Chise_ped = nsdof + c*sqrt(2*nsdof)
if Chisq < Chise_ped
```

```
cov_Rnk_x_Rnk_y=cov(Rnk_x,Rnk_y)

sperman_correlation=cov_Rnk_x_Rnk_y(1,2)/
(sqrt(cov_Rnk_x_Rnk_y(1,1))*sqrt(cov_Rnk_x_Rnk_y(2,2)))
```

```
    rho=0
else
    rho=1
Novel Φk = Invert(Chisq) % (N,r,K fixed )
End
```

7. Somer's D

```
function somd=somersd(x)

[Nc Nd Ty]=NcNd(x)

z1= Nc-Nd
z2=Nc+Nd+Ty

somd= z1/z2;
```

Program for Data Generating Process (DGP) for CT[20]	Data Generating Process (DGP) for Numerous Order CT [Sample / Pattern]

```
function T=OT23(n,a)
% a blongs to (-1,1)
b=1-a;
for i=1:n;
    x=randn;
    if x<-.8
        k=1;
    elseif x<.8
        k=2;
    else
        k=3;
    end
    w=randn;

    y=a*x+b*w;
    if y<-.8
        l=1;
    elseif y<.8
        l=2;
    else
        l=3;
    end
    CT(i,:)=[k l]
end
T=CT
function T=OT_CT55(n,a)

% a blongs to (-1,1)
b=1-a;
for i=1:n
    x=randn
    if x<-1.66
        k=1
    elseif x<-0.83
        k=2
    elseif x < 0.83
        k=3
    elseif x < 1.66
        k=4
```

```
function T=OT_CT44(n,a)
% a blongs to (-1,1)
b=1-a;
for i=1:n
    x=randn
    if x<-1.5
        k=1
    elseif x<0.5
        k=2
    elseif x < 1.5
        k=3
    else
        k=4
    end
    w=randn

    y=a*x+b*w
    if y<-.8
        l=1
    elseif y<.8
        l=2
    else
        l=3
    end
    CT(i,:)=[k l]
end
T=CT

for i=1:n
    x=randn
    if x< -1.7857
        k=1
    elseif x<-1.0714
        k=2
    elseif x <  -0.3571
        k=3
    elseif x < 0.3571
        k=4
    elseif x<1.0714
```

[20] We did programing for numerous order of CTs that is for 3×3, 4×4, 5×5, 6×6, 12×12 CTs, Since in ordinal data orders matters therefore, separate programing is coded for DGP in comparison of Tests of Independence in Ordinal data.

```
    else
        k=5
    end
    w=randn
    y=a*x+b*w
    if y<-.8
        l=1
    elseif y<.8
        l=2
    else
        l=3
    end
    CT(i,:)=[k l]
end
T=CT
```

```
        k=5
    else
        k=6

    end
    w=randn
    y=a*x+b*w
    if y<-.8
        l=1
    elseif y<.8
        l=2
    else
        l=3
    end
    CT(i,:)=[k l]
end
T=CT
```

Program for simulated Critical Values for Ordinal tests of independence	**Power Comparison for Tests of Independence/ Measu Correlation for Ordinal Data**

```
for i=1:20;
    CT=OT1(25,0);
    GK(i,1)=goodmankruskal(CT);
    Kta(i,1)=kendalltaub(CT);
end
CV5pgk=prctile(GK,95);
CV5pKta=prctile(Kta,95);
[CV5pgk CV5pKta]
```

```
function Power=Powerkgtest(NSimul,SS,a)
RegGK=0;
for i=1:NSimul;
    T=OT1(SS,a);
    ts=goodmankruskal(T);
    if ts>0.35
        RegGK=RegGK+1;
    end
end
Power=RegGK/NSimul;
```

The dissertation consists of multidimensional analysis in comparison of tests of independence for nominal and ordinal data. During Programing in MATLAB, We conducted separate programing for each test, DGP for several order of CTs namely 2×2, $S \times 2$, $2 \times S$, $W \times K$, i.e., 2×2, 2×3, 3×2, 3×3, 4×4, 5×5, 6×6, 12×12 CTs.

Programing are presented as a sample / pattern for size distortions, Computation of Simulated Critical Values, Power Computation for above mentioned CTs for selected tests of independence / Measure of correlation in nominal and ordinal data. For details and comprehensive complete codes folder sequentially, please contact me at shakeelshahzad_16@pide.edu.pk / shakeeleconometrics@gmail.com .