

**EXTENSIONS OF GOLDFELD-QUANDT TEST FOR
HETEROSCEDASTICITY FOR UNKNOWN BREAK IN
VARIANCE**



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بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ

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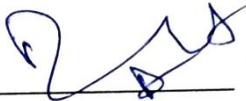
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To

My Parents

Mr. and Mrs. Muhammad Ramzan

Especially to my Mother (LATE)

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List of Abbreviations

GQ	Goldfeld-Quandt Test
MGQ	Modified Goldfeld-Quandt Test
Sup	Supremum
PGQ	Power of GQ test
PMGQ	Power of MGQ test
PMZ	Power of MZ test
PF	Power of F test
PSGQ	Power of supGQ test
PSMGQ	Power of supMGQ test
PSMZ	Power of supMZ test
PSF	Power of supF test

Abstract

The use of linear regression is central in applied research. However, when the base assumptions such as normality, homoscedasticity and independence of errors do not hold. Particularly, the assumption of homoscedasticity—the constancy of variance of errors often gets violated particularly in cross-sectional data. The OLS estimates in this situation though remain unbiased but they are no longer efficient, and the covariance matrix of OLS estimates becomes biased leading to wrong inferences related to hypothesis testing. Specifically, significant regressors might appear insignificant and vice versa if OLS is used under heteroskedasticity. Thus, it is especially important to evaluate this assumption via some existing tests for heteroskedasticity. One of the popular tests is the Goldfeld-Quandt (GQ, see, Goldfeld and Quandt, 1965) test which works well when there is a known break point, and one wants to assess if the variance before and after the break point is same. Rana et al. (2008) proposes a modified version of GQ—the MGQ test for the situations when there are outliers in the design matrix and shows that MGQ is a better choice when there are outliers in the data. It is emphasized that both tests, the GQ and the MGQ are designed to work for the case when the break point in variance is known. However, little is known on the performance of these tests when the break point in variance is unknown. This research takes a lead and proposes several extensions of GQ test for the case when break point in variance is unknown. Particularly, new tests have been proposed for the unknown break in variance by modifying a) the original GQ and b) the modified GQ for outliers (MGQ). The proposed tests are named supGQ and supMGQ and are designed for single but unknown split (break) in variance. The performance of these newly proposed tests is assessed via Monte-Carlo simulations and is compared with their conventional peers (GQ and the MGQ). Extensive Monte-Carlo simulations show the superiority of these newly proposed tests. A real-

World example is also provided to support the analysis. The results of comparison analysis indicate that the proposed tests demonstrate significantly better performance than the existing tests.

Key Words: Heteroskedasticity; Monte-Carlo simulations; outliers

CHAPTER 1

INTRODUCTION

1.1 Introduction to Chapter

Heteroskedasticity is more common in cross-sectional data this is because cross-sectional data usually includes observations from different groups, regions, or categories that may have different levels of variability in the dependent variable (Wooldridge, 2002). This can result in varying levels of variance of the error term in the regression model, leading to heteroskedasticity. Heteroskedasticity refers to the situation where the variance of the dependent variable (or the error term) in a regression model varies across different groups or observations within a dataset (Greene, 2003).

Measurement error can also contribute to heteroskedasticity in cross-sectional data. If there is more measurement errors in some observations compared to others, this can result in varying levels of variability in the dependent variable. It was not until the mid-20th century that heteroskedasticity became recognized as an issue in regression analysis. The term "heteroskedasticity" was first coined by the economist Ragnar Frisch to describe situations where the variance of the dependent variable varied across observations (Frisch & Waugh, 1933). Extreme values can also contribute to heteroskedasticity in cross-sectional data. If some observations have much larger or smaller values than others, this can result in varying levels of variability in the dependent variable. When the relationship between the independent and dependent variables is nonlinear, this can also result in heteroskedasticity in cross-sectional data (Wooldridge, 2002).

Heteroskedasticity can lead to incorrect statistical tests in regression analysis. It can lead to biased variance estimates in regression analysis. This is because the ordinary least squares

(OLS) method assumes that the variance of the dependent variable is constant across all observations. When this assumption is violated, OLS estimator may produce biased covariance estimates (Gujarati, 2022; Ramsey, 1969). This means that the standard errors of the coefficient estimates may be larger than they should be, making it harder to detect statistically significant relationships. This can reduce the power of statistical tests and increase the probability of type II errors. This is because the OLS estimator places more weight on observations with higher variance, leading to a larger standard error and a smaller coefficient estimate. Therefore, it is important to test for heteroskedasticity and, if present, to use appropriate methods to correct for it (Wooldridge, 2002).

The detection of heteroskedasticity in regression analysis has a history, and researchers have developed many different tests and methods for detecting this issue. The detection of heteroskedasticity in regression analysis dates to mid-20th century. In the 1950s and 1960s, researchers developed several tests for detecting heteroskedasticity, including the Park test (Park, 1966), the Glejser test (Glejser, 1969), the Breusch-Pagan test (Breusch-Pagan, 1979), the White and the Goldfeld-Quandt test (Goldfeld-Quandt, 1965).

Park (1966) proposed a test for heteroskedasticity that involves regressing the absolute residuals from the original regression on the independent variables. If heteroskedasticity is present, the absolute residuals will be correlated with the independent variables, leading to a significant coefficient in the Park test. The Glejser test, proposed by Herbert Glejser (Glejser, 1969), is another test for heteroskedasticity that involves regressing the absolute residuals from the original regression on one or more independent variables that are thought to be related to the variance of the dependent variable. If heteroskedasticity is present, the absolute residuals will be correlated with the independent variables, leading to a significant coefficient in the Glejser test.

The Breusch-Pagan test, proposed by Trevor Breusch and Adrian Pagan (1979), is one of the most used tests for detecting heteroskedasticity. It is based on regressing the squared residuals from the original regression on the independent variables. If heteroskedasticity is present, the squared residuals will be correlated with the independent variables, leading to a significant coefficient in the Breusch-Pagan test. The White test, proposed by Halbert White (White, 1980), is another commonly used test for heteroskedasticity. It involves regressing the squared residuals from the original regression on the independent variables and their cross-products. If heteroskedasticity is present, the squared residuals will be correlated with the independent variables and their cross-products, leading to a significant coefficient in the White test. The Goldfeld-Quandt (GQ) test, proposed by David Goldfeld and Richard Quandt (Goldfeld & Quandt, 1965), is a test for heteroskedasticity that involves dividing the sample into two subgroups based on a particular independent variable and comparing the variances of the residuals in the two subgroups. If heteroskedasticity is present, the variances of the residuals in the two subgroups will be significantly different. The Harvey-Collier test, proposed by David Harvey and Paul Collier (Harvey & Collier, 1976), is a test for heteroskedasticity that involves regressing the absolute residuals from the original regression on a set of predetermined independent variables. If heteroskedasticity is present, the absolute residuals will be correlated with the predetermined independent variables, leading to a significant coefficient in the Harvey-Collier test. These tests have varying degrees of power and efficiency in detecting heteroskedasticity, and the choice of test often depends on the specific characteristics of the data and the research question at hand (Harvey & Phillips, 1974).

Researchers have also developed graphical methods for detecting heteroskedasticity, such as residual plots and scale-location plots (Rosopa *et al*, 2013). These methods involve plotting

the residuals or the absolute residuals against the predicted values or the independent variables, and visually examining whether there is a pattern of increasing or decreasing variance (Evans & King, 1988).

The original GQ test is based on the idea that the break (split) point in variance is known and single and it tests the equality of variance across the two splits. The GQ is a good technique for quickly detecting whether two or more samples have statistically different distributions. Furthermore, because the test is based on non-parametric assumptions and does not require the data to follow an exact distribution, it is well-suited for analysing data sets of different complexity. The Goldfeld-Quandt test (Goldfeld-Quandt, 1972) is especially beneficial when there is a suspected shift in the variance of the residuals across distinct subsets of the data, such as with financial time series data or cross-sectional data with a known grouping structure (Zaman, 1994). A comparison of six regularly used techniques for heteroskedasticity identification reveals that the Harrison-McCabe test is the most powerful. While the White test has the least power of all the tests described (Uyanto, 2019). According to the literature, heteroskedasticity is an econometric problem that affects test technique and estimation; thus, identifying the problem is critical to resolving the problem (Abdul-Hameed & Matanmi, 2021). Because the existence of an outlier in a data set and subsequent identification of heteroskedasticity leads in biassed results, the study also proposed a modified version of the Breusch-pagan test for detecting heteroskedasticity in the presence of outliers.

Despite its simplicity and popularity, the original GQ test has many drawbacks highlighted in Tomak (1994) and some of these drawbacks are addressed by (Tomak, 1994) and a modified version of GQ, based on likelihood ration (LRGQ) test is proposed. Particularly, the LRGQ test considers the information on equality of regression coefficients across the split point

into account which is previously ignored by the statistic used in GQ test. Tomak (1994), shows that by incorporating this information into the test statistic, the performance of LRGQ improves substantially over the original GQ test.

In recent past, Rana et al. (2008) extends the GQ test to make it work when there are outliers present in the design matrix and propose a modified GQ (MGQ) test by replacing the OLS residuals with the least trimmed squares (LTS) residuals. This test is shown to have better power than the conventional (original) GQ, in the presence of outliers.

It is emphasized that the original GQ and the modified versions of GQ tests proposed by (Tomak, 1994) and (Rana et al., 2008) test the equality of variance when the split (break) point is known. However, no prior research is available on the performance of GQ and its variants when the break (split) point in variance is unknown. The present study takes the lead and addresses these issues by proposing various new tests.

1.2 Objectives of the Study

The Goldfeld-Quandt (GQ) test and its variants are used to test for assumption of homoscedasticity in linear model when split (break) point is known. The key objective of this study is to propose several extensions of conventional Goldfeld-Quandt (GQ) test with a focus on unknown (single) break (split) in variance. Specifically, the study proposes two new tests to test the null of homoscedasticity:

First: The heteroskedasticity test for a single unknown break (split) point when the design matrix contains usual observations.

Second: The heteroskedasticity test for a single unknown break (split) point when the design matrix contains outliers.

The performance of these tests is compared with the existing conventional GQ and modified-GQ tests designed respectively to test for the presence of heteroskedasticity when the design matrix contains no outlier and one or more outliers. It is emphasized that the proposed new tests are by structure better than their conventional counter parts as the conventional ones consider the issue of known break in variance while the newly proposed ones go one step ahead and consider the issue when the break point not known.

2.3 Significance of the Study

The major contribution of the present study is that it proposes two new tests for the single but unknown break in variance for the cases when there are no outliers and when there are outliers in the design matrix. The tests are designed in such a way that they are the process to make data in ascending order according to regressor as it is required in the original GQ test. Information loss is the basic idea to save the purity of original data without excluding the values as in the original GQ method. It is an auxiliary process in which no information loss occurs, and multi-process GQ statistics are obtained to get the Maximum value GQ at each point. The proposed tests are superior to existing ones as in proposed tests have no loss of information as no observations are omitted in proposed tests in contrast to conventional GQ and MGQ tests. The proposed tests are designed to capture and detect unknown break in variance whereas existing tests are designed for a given known break. There is no omission of data (data loss) as part in conventional test. This is never done earlier and thus the present study makes a significant contribution to the existing literature. It is highlighted that Monte-carlo simulation is not the ideal method to compare the power of such tests since it completely depends on the setting however the newly proposed tests are by structure better than the conventional (existing) tests for heteroskedasticity.

1.4 Limitations of the Study

One limitation of this research is that the proposed tests are designed to test for the presence of a single unknown break. The research is underway to develop tests that can be designed to test for multiple unknown breaks. These are left for future research.

1.5 Organization of the Study

The rest of the study is organized as:

Chapter two provides a review of existing literature and chapter three includes the discussion of regression model and heteroskedasticity tests. Chapter 4 provides the details and comparison of newly proposed test of heteroskedasticity for single but unknown break in variance when there are no outliers in design matrix, while Chapter 5 gives details of 2nd newly proposed test for heteroskedasticity for single but unknown break in variance when there are outliers in the design matrix. Chapter 6 compares the performance of supGQ with sup F and the sup MZ tests while Chapter 7 comprises of real-World example giving a comparison of proposed supGQ and supMGQ with existing tests and finally it discusses the overall conclusion of the study.

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction to the Chapter

This chapter summarizes the historical developments of heteroskedasticity detection tests for the linear regression models and also provides a brief comparison of these tests with the GQ test. Specifically, it is sub-divided into two sub-sections, the first sub-section entails the historical trajectory of tests for detection of heteroskedasticity while the second sub-section documents the literature of GQ test comparison with other related tests.

2.2 Historical Developments of Heteroskedasticity Tests

Heteroskedasticity tests are used to detect the presence of heteroskedasticity in a data set. It is an econometric term used to describe the presence of non-constant variance in the residuals of a regression. Heteroskedasticity can occur when the error terms of a regression are not independently and identically distributed (i.e., they are not homoskedastic). Heteroskedasticity is a form of non-constant variance in the residuals of a regression analysis. It is estimated that approximately 40% of all regression analysis suffer from heteroskedasticity (Greene, W. H., 2003). Graphical detection methods include plotting the residuals versus the independent variables and studying the pattern. If the points become more dispersed as the values of the independent variables increase, then heteroskedasticity may be present. Another graphical detection method is plotting the residuals versus the fitted values from the regression line. If the points become more dispersed as the fitted values increase, then heteroskedasticity may be present (Gujarati, D. N., 2022).

Statistical detection methods for heteroskedasticity include the Breusch-Pagan test (Breusch & Pagan, 1979), the White test (White, H., 1980), and the GQ test (Goldfeld & Quandt,

1965). The Breusch-Pagan test is a general test for heteroskedasticity and is used to test whether the variance of the residuals is related to the values of the independent variables. The White test is a more powerful version of the Breusch-Pagan test, which is used to test whether the variance of the residuals is related to the squared values of the independent variables. The Goldfeld-Quandt test is used to test whether the variance of the residuals is related to the fitted values from the regression line (Asteriou & Hall, 2015, 2017).

The Breusch-Pagan test, serves to scrutinize the presence of heteroskedasticity within a regression model. In the realm of statistical analysis, it is pivotal to discern whether the residuals exhibit uniform variance (homoskedasticity) or if they display varying degrees of dispersion across the range of predictor variables. The test operates under the framework of two hypotheses: the Null Hypothesis posits the existence of homoscedasticity, indicating that residuals are distributed uniformly with equal variance, while the Alternative Hypothesis suggests the presence of heteroskedasticity, implying that residuals do not share equal variance. Upon conducting the Breusch-Pagan test, if the derived p-value falls below a pre-defined significance level, commonly set at $\alpha = 0.05$, statisticians reject the null hypothesis. This outcome leads to the conclusion that heteroskedasticity is indeed present within the regression model, prompting further analysis and potential model adjustments. Begin by estimating the regression model using the relevant independent and dependent variables. After obtaining the regression estimates, compute the squared residuals by squaring the differences between observed and predicted values. Utilize the squared residuals obtained in the previous step as the response variable in a new regression model. This subsequent regression aims to capture any potential relationships between the squared residuals and the independent variables. The Chi-Square test statistic is determined by multiplying the total number of observations (n) by the R-squared value of the

updated constructed regression model. This statistic serves as a crucial indicator in assessing the significance of heteroskedasticity.

The Goldfeld-Quandt test is a test for heteroskedasticity in regression models (Goldfeld & Quandt, 1965, 1973). The test statistic is calculated by regressing the squared residuals on the independent variables and testing the significance of the resulting F-statistic. This test is used to detect heteroskedasticity in a linear regression model. It examines the correlation between the residuals and the independent variables. The GQ statistic is calculated by dividing the data into two groups based on the median value of the independent variables and then testing the difference in the variance of the residuals for the two groups for significance. If the variance in the two groups is significantly different, then the model suffers from heteroskedasticity.

The White test examine whether variance remains constant, an auxiliary regression analysis is conducted. This involves regressing the squared residuals from the initial regression model against a set of regressor's, including the original ones along with their squares and cross-products. The focus then shifts to evaluating the R-square value. The Lagrange multiplier (LM) test statistic is computed as the product of the R-square value and the sample size (nR^2). This statistic conforms to a chi-squared distribution, with degrees of freedom equivalent to $K-1$, where K represents the number of estimated parameters in the auxiliary regression.

The Park test is a test for heteroskedasticity in regression models (Park, 1966). The test statistic is calculated by regressing the squared residuals on the independent variables and testing the significance of the resulting F-statistic. This test is used to detect heteroskedasticity in a linear regression model. It examines the correlation between the residuals and the independent variables. The Park statistic is calculated by dividing the data into two groups based on the median value of the independent variables and then testing the difference in the variance of the

residuals for the two groups for significance. If the variance in the two groups is significantly different, then the model suffers from heteroskedasticity.

The Koenker-Bassett test is used to detect heteroskedasticity in a linear regression model. It examines the correlation between the absolute value of the residuals and the independent variables. The Koenker-Bassett statistic is calculated by regressing the absolute value of the residuals on the independent variables and then testing the resulting F-statistic for significance. If the F-statistic is significant, then the model suffers from heteroskedasticity.

The Breusch-Godfrey test is a test for heteroskedasticity in regression models (Breusch & Godfrey, 1978). The test statistic is calculated by regressing the squared residuals on the independent variables and testing the significance of the resulting F-statistic. The Bartlett test is a test for heteroskedasticity in regression models (Bartlett, 1937). The test statistic is calculated by regressing the squared residuals on the independent variables and testing the significance of the resulting F-statistic. The Harvey-Collier test is a test for heteroskedasticity in regression models (Harvey & Collier, 1976). The test statistic is calculated by regressing the squared residuals on the independent variables and testing the significance of the resulting F-statistic.

The Glejser test is a test for heteroskedasticity in regression models (Glejser, 1969). The test statistic is calculated by regressing the squared residuals on the independent variables and testing the significance of the resulting F-statistic. The Engle-Granger test is a test for heteroskedasticity in regression models (Engle & Granger, 1987). The test statistic is calculated by regressing the squared residuals on the independent variables and testing the significance of the resulting F-statistic (Andrew, 1993; Andrew & Ploberger, 1994).

Once heteroskedasticity is detected by any of the existing tests, the next task is to correct for the heteroskedasticity. There are two main routes for this, the first one is to use either Weighted Least Squares (WLS) or Generalized Least Squares (GLS) tests and the second one is to use heteroskedasticity consistent covariance matrix estimators (HCCMEs). Specifically, the Weighted Least Squares (WLS) is used to correct for heteroskedasticity in a linear regression model. It examines the correlation between the residuals and the independent variables. The WLS statistic is calculated by weighting the residuals according to the size of the independent variables and then testing the resulting F-statistic for significance. If the F-statistic is significant, then the model suffers from heteroskedasticity.

Generalized Least Squares (GLS) test is used to correct heteroskedasticity in a linear regression model. It examines the correlation between the residuals and the independent variables. The GLS statistic is calculated by weighing the residuals according to the size of the independent variables and then testing the resulting F-statistic for significance. If the F-statistic is significant, then the model suffers from heteroskedasticity (Kariya & Kurata, 2004).

The second route of using HCCMEs suggests using OLS method but with heteroskedasticity corrected standard errors. There are several heteroskedasticity consistent covariance matrix estimators, known as HC0 to HC7 and their variants as well. However, the focus of the present research is on heteroskedasticity testing and not heteroskedasticity corrections. So, we provide details of only heteroskedasticity tests.

2.3 Comparison of GQ Test with Other Related Tests

The literature highlights that heteroskedasticity is an econometric problem that impact test procedure and estimation hence the identification of problem is crucial to resolve the issue (Hameed, 2021). As the presence of outlier in a data set and subsequent identification of

heteroskedasticity lead to biased result, the study further suggested a modified version of Breusch-Pagan test for identification of heteroskedasticity in presence of outliers. The modified version derived by substituting non-robust components with the robust components and the Monte-Carlo simulation and real-time data was used to check the performance of modified version of test. Empirical results of modified Breusch-pagan test were relatively efficient in comparison with previous test and recommended for identification of heteroskedasticity in presence of outlier.

Similarly, Agboola *et al.*, (2020), explained that the heteroskedasticity is an econometric problem which arises due to deviation in variance of errors, omission of important variable, non-detail model, and presence of outlier in any given distribution. The study interpreted the results of five different heteroskedasticity tests: Park test, Glejser test, Breusch-pagan test, White test, and Goldfeld-test by applying simulated dataset with sample size ranging from 20 to 100 observations. The finding indicates that the Glejser test has strongest capacity to identify heteroskedasticity.

A comparison of six commonly used tests for detection of heteroskedasticity shows that the Harrison-McCabe test is the most powerful test. While the White test has the least power in mentioned tests (Uyanto, 2019). Alih and Ong (2015) applied robust procedure for identification of heteroskedasticity in presence of outlier in the data set. The application of tests on real time data set and simulated experiment suggested that proposed procedure for detection of heteroskedasticity overtake conventional GQ test and other tests.

The application of Monte-Carlo simulation on finite samples with Gaussian and Non-Gaussian distribution and inclusion of conventional as well as arch-type heteroskedasticity tests

provide dual desirable outcomes in terms of power and size control (Dufour *et al.*, 2004). The Bartlett's M Specification Error Test (BAMSET) is used to compare the null hypothesis, H_0 , that all population variances are equal, against the alternative that at least two are different. is the variance estimate as a group (Griffiths and Surekha, 1985).

Ali and Giaccotto (1984) explained that the different non-parametric tests of heteroskedasticity in presence of robust properties and results of all non-parametric tests are reasonably robust to OLS estimate. Further, the power of the test can be increased by increasing sample size and regressors variability while power of the test in presence of normally distributed observations can be reduced substantially. The unsuccessful generation of heteroskedasticity model led to inefficient and invalid coefficient and thus introduced a new model test for additive heteroskedasticity. The power functions of this new test compared with different tests of heteroskedasticity shows more powerful estimates in comparison with other types of tests (Evans and King, 1984).

For time series models, literature determines both single and multiple breaks. However, for cross-sectional data regression models, there are only a few methods to a single break. However, the solutions suggested may not be helpful in all instances. As a result, the tests created based on certain criteria have flaws or strengths. Under the same settings, a test's dominance over others is judged by having the most power (Hansen, 1992).

Harrison and McCabe (1979) proposed an exact test and bounds test for detection of heteroskedasticity by using F-distribution where the power of bound test equates with the power of Goldfeld-Quandt, Harvey and Phillipse, and Theil test in different circumstances. The simplicity in estimation of bound test as opposed to other tests make it lucrative test; in general, the power of exact test higher than other tests but its practical application was found to be

cumbersome. Harvey and Phillips (1974) proposed parametric test for identification of heteroskedasticity in a linear regression model and compared it with Goldfeld-Quandt test and BLUS test. It is found that under different circumstances above mentioned test has equitable power and on average BLUS test has higher power than recursive test.

In a variation of real-world scenarios, including macroeconomic data, regression analysis is vulnerable to the problem of heteroscedastic data. As a result, it is critical to test the data for potential heteroskedasticity. The Goldfeld-Quandt (GQ) test is a popular test for heteroskedasticity. However, its performance is called into question when the data contains one or more outliers. The adjusted edition of GQ, known as MGQ, incorporates outlier analysis when examining the likelihood of heteroskedasticity within the dataset (Rana et al., 2008). Though this is a useful addition to the existing set of heteroskedasticity tests, practitioners should use the MGQ test if there are outliers in the data to prevent reaching incorrect conclusions.

Overall, this chapter summarized the historical development of heteroskedasticity tests starting from Goldfeld & Quandt and pointed to presence of conditions for applicability of various tests for detection of heteroskedasticity. Furthermore, the last part of chapter documents the snapshot containing the comparison GQ test with other tests of heteroskedasticity. The results showed that the some test overtakes others owing to varying underlying conditions and properties of data sets. The focus of this research is on the GQ test and its variants (the modified GQ—MGQ). It is emphasized that the GQ as well as MGQ are designed to test for one single known break point. However, while working with real data, mostly the breakpoint is not known. Thus, there is a need to develop some suitable tests for unknown breakpoint. This is what is taken up in this research where two new tests are proposed which serve the purpose of testing for

heteroskedasticity in case of unknown break for the design matrices with and without outliers. Thus, the present gap in literature will be filled.

CHAPTER 3

REGRESSION MODEL AND HETEROSKEDASTICITY TESTS

3.1 Introduction to chapter

Several tests for heteroskedasticity exist in literature to test the regression errors for homoscedasticity when break point in variance is known. This chapter documents the details on the regression model used and summary of GQ (Goldfeld & Quandt, 1965) and Modified GQ (Rana et al., 2008) tests for heteroskedasticity for known break in variance.

3.2 The Regression Model

This section provides the main regression model used in this study followed by a discussion of the conventional and the modified versions of GQ tests.

Consider the multiple linear regression model:

$$Y = X\beta + \varepsilon \quad [3.1]$$

Where, Y is a $N \times 1$ vector containing observations on the dependent variable, β is $K \times 1$ vector of unknown parameters, X is a $N \times K$ matrix of the regressors, and ε is $N \times 1$ vector of unobserved errors which are assumed to be independent with mean zero vector (O of order $N \times 1$) and covariance matrix $\Sigma = \text{diag}(\sigma_n^2)$, $n = 1, 2, \dots, N$, i.e., $\varepsilon \sim N(O, \Sigma)$.

The usual OLS estimate of true parameter β is: $\hat{\beta}_{OLS} = (X'X)^{-1}X'Y$ with covariance matrix, $Cov(\hat{\beta}_{OLS}) = \hat{\sigma}^2(X'X)^{-1}$ (since OLS assumes homoscedasticity), where, $\hat{\sigma}^2 = \frac{RSS}{N-K}$, where RSS is residuals sum of squares obtained from estimating regression in equation [3.1] above.

When the assumption of homoscedasticity gets violated, the OLS estimates remain unbiased and consistent but no-longer efficient. In addition, the covariance matrix of OLS estimates, i.e., $Cov(\hat{\beta}_{OLS})$ becomes biased and this leads to wrong t & F statistics and related

confidence intervals and thus, significance of regressors also gets affected, specifically, a significant regressor may appear insignificant and vice versa. Thus, it is especially important to test the regression errors for homoscedasticity by using available tests for heteroskedasticity. If null of homoscedasticity gets rejected, then one can use heteroskedasticity consistent standard errors (HCSEs) while using OLS estimates to get valid inferences regarding significance of regressors. The true covariance matrix of OLS estimates under heteroskedasticity is: $Cov(\hat{\beta}_{OLS}) = (X'X)^{-1}X'\Sigma X(X'X)^{-1}$. Note that Σ contains unknown parameters, σ_n^2 , $n=1, 2, \dots, N$. So, usually it is replaced with its estimate, $\hat{\Sigma}$ leading to estimated covariance matrix as: $Cov(\hat{\beta}_{OLS}) = (X'X)^{-1}X'\hat{\Sigma}X(X'X)^{-1}$. Several variants of $\hat{\Sigma}$ are available in literature commonly known as heteroskedasticity consistent covariance matrix estimators (HCCMEs) (HC0 to HC5). The HCSEs are obtained by taking square root of diagonal entries of these HCCMEs. For details, see White, 1985; Hinkley, 1977; Horn et al., 1975; Mackinnon & White, 1985; Ahmed et al., 2017; Dutta & Zaman, 1989; Cook & Weisberg, 1983; among others.

An important challenge for the practitioner is to decide which test to use to test for the presence of heteroskedasticity. A large number of tests are available in literature, such as Goldfeld and Quandt (GQ) (1965), Glejser test (1969), Harrison–McCabe test (1979), Breusch–Pagan test (1969), White test (1985), White test (1980) among others.

Our focus is on the GQ test and its variant. The GQ test, a widely used tool in social sciences for detection of heteroskedasticity in simple regression model. After reviewing the literature, we identified a gap: while Maasoumi's MZ test and Andrew's F test have supremum versions for optimal solutions, there is no corresponding supremum version for the GQ test—the modified GQ which is designed for the situation when there are outliers in the data. Before proceeding

further, it is good to introduce first the conventional GQ test and then its variants. This is done in the following subsections:

3.2.1 The Goldfeld-Quandt Test

Goldfeld & Quandt (1965) propose a test (commonly known as GQ test) to detect Heteroskedasticity in linear regression model. This test is based on the idea of arranging the data in either ascending or descending order with respect to an identified variable with which residual variance is highly related. To set the stage for the GQ test and its variants, let us introduce a general framework which is used throughout in the discussion that follows from this point onward.

Consider a sample of T observations ranging from 1 to N ordered in such a way that variances are increasing. Divide the sample into two parts by choosing N_1 and N_2 in such a way that $1 < N_1 < N_2 < N$, and defining Y_1 and Y_2 to be $N_1 \times 1$ and $(N - N_2 + 1) \times 1$ vectors, with $N_1 \cong N/2$ and $N_2 \cong N_1 + 1$, as: $Y_1 = (y_1, y_2, \dots, y_{N_1})'$ and $Y_2 = (y_{N_2}, \dots, y_N)'$. Let X_1 and X_2 be $N_1 \times K$ and $(N - N_2 + 1) \times K$ matrices of corresponding values of the regressors and, ε_1 and ε_2 be the corresponding $N_1 \times 1$ and $(N - N_2 + 1) \times 1$ error vectors assumed to follow normal distribution with mean as zero vector and covariance matrices, $\sigma_1^2 I_{N_1}$ & $\sigma_2^2 I_{N-N_2+1}$ respectively.

Consider linear regression model separately for the two halves of the sample as:

$$Y_1 = X_1\beta_1 + \varepsilon_1 \quad [3.1A]$$

$$Y_2 = X_2\beta_2 + \varepsilon_2 \quad [3.1B]$$

Where, $\varepsilon_1 = N(0, \sigma_1^2 I_{N_1})$ and $\varepsilon_2 = N(0, \sigma_2^2 I_{N-N_2+1})$.

The focus of GQ is on testing the null hypothesis: $H_0: \gamma = 1$ against the alternative, $H_1: \gamma > 1$, where, $\gamma = \sigma_1^2 / \sigma_2^2$, under the assumption that regression coefficients across the two halves is same, i.e., $\beta_1 = \beta_2$.

Define $\hat{\beta}_1 = (X'_1 X_1)^{-1} X'_1 Y_1$ and $\hat{\beta}_2 = (X'_2 X_2)^{-1} X'_2 Y_2$ as OLS estimates of true parameters in each half of the sample and let $RSS_1^2 = \|Y_1 - X_1 \hat{\beta}_1\|^2$ and $RSS_2^2 = \|Y_2 - X_2 \hat{\beta}_2\|^2$ the corresponding sum of OLS squared residuals. The original GQ test proposes omitting a few observations from the middle to increase the contrast between the variances in the first half of the sample and that of the second half of the sample. The GQ-statistic is given by:

$$GQ = \hat{\sigma}_{2n}^2 / \hat{\sigma}_{1n}^2 = \frac{RSS_2^2 / df_2}{RSS_1^2 / df_1} \quad [3.2]$$

The GQ is an exact test and it follows an F -distribution under the null of homoscedasticity with $df_2 = N - (N_1 - K)$ and $df_1 = N_1 - K$ degrees of freedom and suggest to reject the null when calculated value of GQ-statistics (GQ_{cal}) is found to be greater than the critical value at 1%, 5% and 10% significance level (α). i.e.

$$GQ_{cal} > F(\alpha, df_2, df_1)$$

Basic assumption of GQ is that $\beta_1 = \beta_2$, however, this information is not considered in the GQ-statistic. Utilizing this information into the test statistic formula should lead to a better power of GQ test. This is done by Tomak (1994), and details are provided in the next subsection.

3.2.2 Modified Goldfeld Quandt Test

The modified GQ (MGQ) test proposed by Rana et al. (2008) is a modification of original Goldfeld-Quandt test for the case when there are outliers in the data. The key idea of MGQ test

is to identify the components of the GQ test which are affected by outliers and then these are replaced by their robust alternatives to get better inferences under Heteroskedasticity.

This test work in parallel to the original GQ test where one orders the observations with increasing variance and finding outliers via any robust Least Trimmed of Squares (LTS) method proposed by Rousseeuw and Leroy (1987) to estimate the regression models in [3.1A] and [3.1B] and then compute the deletion residuals (See Imon, 2003) for the entire data set based on a fit without the points identified as outliers by the LTS fit. The modified GQ (MGQ) test is obtained as a ratio of median of squared deletion residuals for the two halves of the entire sample, given as:

$$MGQ = \frac{MSDR_2^2/df_2}{MSDR_1^2/df_1} \quad [3.3]$$

Where, $MSDR_1^2 = med\|Y_1 - X_1\hat{\beta}_1\|^2$ and $MSDR_2^2 = med\|Y_2 - X_2\hat{\beta}_2\|^2$ are the median of the squared deletion residuals (MSDR) for first and second half of the sample. The MGQ test follows an F distribution with $df_2 \equiv N - (N_1 - K)$ and $df_1 \equiv N_1 - K$ degrees of freedom under the null of homoscedasticity while normality assumption holds true.

3.3 The Proposed Tests for Heteroskedasticity

The original GQ test and its variants presented above consider the situation when the split (break) point is known. However, in real world applications, usually this is not the case, so there is a need to develop the test when the split point is unknown. This study takes a lead and proposes three new tests by extending the tests documented in the previous section for the unknown split (break) point. The basic idea of these tests is borrowed from Andrews supF test and Ahmed et al. (2017) supMZ test. The following sub-sections provide details on our proposed tests for unknown break (in variance).

3.3.1 The supGQ Test

The first proposed test is the supGQ test which is the modification of the original GQ test for unknown split (break) in variance. The basic idea is to calculate usual GQ-statistic for all possible split (break) points. The supGQ test use the recursive estimation window through the entire sample period from 1 to N and split the data into two sub-samples. The tests are structured in such a way that they are the process of creating data in sequence according to regressor. In order to ensure that each sub-sample have reasonable number of observations, the cutoff point is 15% for N=100, each single data point is considered one observation at a time i.e., cutoff =0.15*N. Also, Andrews (1993) and Ahmed (2017) found that if structural change is too close to the endpoints creates difficulties in obtaining powerful tests. Accordingly, cutoff point is chosen so there is sufficient number of observations both before and after the breakpoint to allow for a reasonably powerful test. After calculating GQ-statistic given in equation [3.2] with a cutoff of 15% (0.15*N) denote this GQ-statistics as GQ₁. The cutoff point is shifted to one point ahead that is 16%, with this cut off first subsample contains (0.16*N) and the remain data are counted under second sample and denote GQ statistic at this point as: GQ₂, we continue this process till the first sample have 85% (0.85*N) data and second sample has 15% (0.15*N) data. GQ statistic is calculated for each sample through equation [3.2] and the maximum calculated value of GQ statistic is declared as supGQ. The procedure of supGQ test is illustrated as;

Definition 1: For all potential break points ‘*i*’, $c < a \leq i \leq b < N - c$,

$$supGQ = \max_{a \leq i \leq b} GQ_i, c < a \leq i \leq b < N - c$$

Where, GQ_{*i*} is the GQ-statistic calculated at each ‘*i*’, ‘*c*’ is the cutoff point (15% of sample, *c*=0.15N) and the constant ‘*a*’ and ‘*b*’ are selected in such a way that each sub-sample has reasonable number of observations.

The GQ-statistic is calculated for each split starting at c of the sample to $(N-c)$ of the sample, where, c is the cut-off point taken in percentage and this way GQ-statistic is calculated $N-2c$ times. The supGQ is obtained by taking the maximum value of all GQ-statistics calculated over the grid of c to $N-c$ (15% to 85%) of the data.

Saving the original data's purity without removing any values, as was done in the GQ technique, is the fundamental notion behind information loss. In order to determine the maximum value of GQ at each location, multi-process GQ statistics are acquired during this auxiliary procedure, which doesn't cause information loss.

Definition 2: Note that the supGQ is equivalent to GQ test when the cut-off point for break is set at 50% of sample ($0.5N$), and it shares all properties of GQ and follows an F distribution under null of homoscedasticity with $df_2 \equiv N - (N_1 - K)$ and $df_1 \equiv N_1 - K$ degrees of freedom, where, N_1 are the value of the split (break) point where the GQ-statistic is maximum.

At 50% cut-off point set to data generating process, the supGQ and GQ will be equivalent as split-off point for GQ is taken as 50% and thus maximum of value of all GQ statistics calculated over the grid of 15% to 85% of the data will be at 50% of the data. Thus, both GQ and supGQ will be same and thus all properties will be shared. For a similar discussion, Andrews (1993).

R-studio is used to carry-out all simulation exercises.

3.3.2 The sup MGQ Test

The Second proposed test is the sup MGQ test which is the modification of the modified GQ (MGQ) test proposed in Rana et al. (2008). Note that the MGQ test is designed to provide robust inferences when there are outliers in the data by using LTS regression followed by median of squared deletion residuals for each sample. The sup MGQ is designed to extend the MGQ for the case of unknown split (break).

The working steps of sup MGQ are same as that of supGQ with the only difference in the test-statistic, where instead of calculating squared OLS residuals, now one needs to calculate median of squared deletion residuals and of course the regression is fitted via LTS (Rousseeuw et al. (1987) rather than OLS approach. The supMGQ test, which is a modification of the original MGQ test for unknown split (break), is the first proposed test. The tests are constructed so that they are the process of making data in order according to the regressor. The core concept is to compute the standard MGQ-statistic for all feasible split (break) points, with 15% of the data ($N=100$) in the first sample and 85% of the data in the second sample. Andrews (1993) and Ahmed et al. (2017) recommended a cut-off of 15%-85%. Following Andrews (1993) and Ahmed et al. (2017), a 15% cut-off is chosen and an LTS regression is run on two parts of data on Y on X, calculating LTS estimates of true parameters and then the LTS square residuals for each of the two parts, and finally calculating the MGQ-statistic given in equation [3.3]. This MGQ-statistic is denoted as MGQ1. Increase the number of observations in the first sample by including one additional observation from the second sample, lowering the second sample by one observation, such that we now have 16% of the data in the first sample and 84% of the data in the second sample. Calculate an LTS regression on each sample independently, then median of squared deletion residuals for each sample, and finally MGQ-statistics using equation [3.3] and indicate it as: MGQ2. Repeat this approach until we obtain 85% of the data in the first sample and 15% in the second sample. Using equation [3.3], calculate median of squared deletion residuals for each sample and, lastly, the MGQ-statistic. This test is shown by the following theorem.

The fundamental concept of information loss is the preservation of the original data's purity without removing any values, unlike the GQ approach of the past. There is no information loss

during this auxiliary operation, and multi-process GQ statistics are gathered to determine the maximum value of GQ.

Definition 3: For all potential break points ' i ', $c < a \leq i \leq b < N - c$,

$$supMGQ = \max_{1 \leq i \leq n} MGQ_i, c < a \leq i \leq b < N - c$$

Where, MGQ_i is the MGQ-statistic calculated at each 'i' and the constant 'a' and 'b' are selected in such a way that each sub-sample has reasonable number of observations.

The MGQ-statistic is calculated for each split starting at c of the sample to $(N-c)$ of the sample, where, c is the cut-off point taken in percentage and this way MGQ-statistic is calculated $N-2c$ times. The supMGQ is obtained by taking the maximum value of all MGQ-statistics calculated over the grid of c to $N-c$ (15% to 85%) of the data.

Definition 4: Note that the supMGQ is equivalent to MGQ test for a single break and it shares all properties of MGQ and follows an F distribution under null of homoscedasticity with $df_2 \equiv N - (N_1 - K)$ and $df_1 \equiv N_1 - K$ degrees of freedom, where, N_1 are the value of the split (break) point where the MGQ-statistic is maximum.

R-studio is used to carry-out all simulation exercises.

CHAPTER 4

HETEROSKEDASTICITY TESTING FOR SINGLE (UNKNOWN) BREAK IN VARIANCE: NON-OUTLIER CASE

4.1 Introduction to Chapter

This chapter provides a comparison of GQ test with the supGQ via extensive Monte-Carlo simulations. The details of Monte-Carlo simulations are provided in next subsection while the comparison results are provided in a separate subsection that follows the next subsection.

4.2 Monte-Carlo Setup for Comparison of GQ and supGQ Tests

The performance of our proposed tests and the existing GQ test is compared via extensive Monte-Carlo simulations. Linear regression model is considered where regressors are generated from different distributions to cover up all types of design matrices, the usual ones—without outliers.

Specifically, the following regression model $Y = X\beta + \varepsilon$ is used, where Y denotes a vector of size $N \times 1$ containing observations on the dependent variable, β represents a vector of size $K \times 1$ representing unknown parameters, X is a matrix of size $N \times K$ containing regressors, and ε is $N \times 1$ vector representing unobserved errors which are assumed to be independent, with a mean of zero (a Null vector of size $N \times 1$) and covariance matrix Σ that is a diagonal matrix with σ_n^2 as its diagonal elements for $n = 1, 2, \dots, N$. In other words, ε follows a multivariate normal distribution $N(O, \Sigma)$. In this analysis it is assumed that the other necessary conditions of linear regression model are met like there is no problem of multicollinearity, and serial correlation.

The distributional choice of regressors is done keeping in view all types of regressors into account (symmetric, asymmetric, equally spaced and the ones containing one or more outliers). All other distributions fall in one of these categories. Specifically, regressors are generated from a) additive sequence starting at $1(v)N$, where ‘v’ is the step size and N is the total number of sample points, b) standard normal, c) chi-square with 2 df, and Cauchy distribution. The treatment of outliers differs dramatically between the conventional normal distribution, the chi-square distribution, the Cauchy distribution, and the additive sequence. The standard normal distribution is extremely sensitive to outliers, as even a few extreme values can significantly affect the mean and standard deviation. This sensitivity is owing to its narrow tails, which means that the probability density reduces fast as data points move away from the mean. In practice, this makes the ordinary normal distribution less resilient in circumstances where outlier data points may present.

The chi-square distribution, on the other hand, is sensitive to outliers, particularly when working with lower degrees of freedom. Outliers can greatly exaggerate variance estimates. While the chi-square distribution's tail behaviour varies with degrees of freedom, it is often not as resilient as intended in the case of extreme values. The Cauchy distribution, on the other hand, is less sensitive to outliers due to its long tails. Extreme values have a reduced impact on summary statistics in this distribution. The heavy-tailed behaviour of the Cauchy distribution makes it more robust in dealing with outlier data points than the typical normal and chi-square distributions. Depending on the constituent distributions, additive sequences, which are formed of numerous probability distributions coupled through addition, exhibit a variety of behaviours. As a result, the treatment of outliers in additive sequences can vary greatly, and their robustness is dependent on the sequence's specific composition. As a result, while dealing with outliers in

additive sequences, it is critical to understand the constituent distributions and their interactions. The performance of tests is assessed over different sizes of sample (small, medium and large), N=30, 60, 120, 240, 480. The rationale regarding the selection of sample sizes in line of "Hameed (2021) and E Alih · (2015). This is to assess how power of the tests varies when sample size increases. Generally, power of a test increases with an increase in sample size. The regressors generated for N=30 is repeated to generate samples of higher sizes, this is to keep degrees of Heteroskedasticity fixed throughout samples. In addition, all regressors are kept fixed throughout simulations. The power of tests is also analyzed when the degree of heteroskedasticity varies from 0 (homoskedasticity) to any value higher than zero, showing an increased level of heteroskedasticity.

The errors are generated from various distributions, a) standard normal for the two sub-samples—this is to calculate and compare size of all tests under null, b) non-standard by increasing variance of second sub-sample (and thus, degree of Heteroskedasticity) systematically to assess its impact on the power of all tests, while keeping variance of first sub-sample fixed. Several forms of heteroskedasticity (variance structures) are considered, such as additive, multiplicative etc., and the degree of heteroskedasticity(*Het*) are measured via guidelines provided in Maasoumi et al. (2010) and Ahmed et al. (2017). Specifically, the degree of heteroskedasticity(*Het*) is measured as:

$$Het = \log(W_1\sigma_1^2 + W_2\sigma_2^2) - (W_1 \log \sigma_1^2 + W_2 \log \sigma_2^2)$$

Where, $W_1=N_1/N$, $W_2=N_2/N$, σ_1^2 is fixed at 1 while σ_2^2 is varied from 1(0.2)4, i.e., from 1 to 4 with a step of size 0.2. The degree of heteroskedasticity (HET) is varied by varying the value of

σ_2^2 , i.e., the variance of the 2nd regime. Table 4.1 provides value of HET for different values of σ_2^2 . The results of Monte-Carlo analysis are provided in the next section.

Table 4.5.1: Degree of Heteroskedasticity (HET)

σ_2^2	1	1.2	1.4	1.6	1.8	2	2.2	2.4	2.6	2.8	3	3.2	3.4	3.6	3.8	4
HET	0	.10	0.18	0.26	0.34	0.41	0.47	0.53	0.59	0.64	0.69	0.74	0.79	0.83	0.88	0.92

4.3 Results of supGQ and its Comparison with GQ

To demonstrate the efficacy and robustness of the supGQ technique, a number of cases utilizing a variety of data distribution sets have been taken into consideration. The standard GQ strategy for dealing with heteroskedasticity has an insufficiently long convergence time span to arrive at a solution. The Power of supGQ test (PSGQ) manages to get ahead of the traditional GQ power (PGQ) and arrive at early convergence. Table 4.2 presents the results of normal regressor case. It is important to note that, in terms of solution convergence, PSGQ performs significantly better than traditional GQ. In the case of a normal distribution with a breakpoint located at 30 percent of the data, both the proposed supGQ test and the GQ test maintain their size even when simulated crucial values are applied.

The power of the proposed test started to increase as the sample size increased, which was more than the base test GQ throughout the sample size increment. This occurred when the sample size increased. As the data's standard deviation rises, the suggested test's power to reliably predict outcomes surpasses that of the GQ test with each incremental rise above the crucial standard deviation.

Table 5.2: Power Comparison of GQ and supGQ Tests (Normal Distribution Case)

N	Test statistic	Degree of Heteroskedasticity (HET)															
		0	0.10	0.18	0.26	0.34	0.41	0.47	0.53	0.59	0.64	0.69	0.74	0.79	0.83	0.88	0.92
30	PGQ	0.047	0.055	0.078	0.128	0.151	0.206	0.244	0.235	0.266	0.299	0.317	0.312	0.317	0.355	0.327	0.341
	PSGQ	0.042	0.054	0.062	0.084	0.097	0.104	0.124	0.127	0.167	0.17	0.228	0.255	0.265	0.308	0.353	0.346
60	PGQ	0.039	0.068	0.135	0.203	0.275	0.352	0.388	0.409	0.452	0.489	0.487	0.503	0.528	0.534	0.506	0.57
	PSGQ	0.044	0.088	0.15	0.259	0.407	0.555	0.706	0.822	0.917	0.945	0.978	0.989	0.997	0.998	1	1
120	PGQ	0.055	0.101	0.308	0.41	0.54	0.625	0.7	0.718	0.739	0.771	0.806	0.803	0.828	0.836	0.852	0.855
	PSGQ	0.047	0.115	0.368	0.672	0.886	0.965	0.995	0.998	1	1	1	1	1	1	1	1
240	PGQ	0.056	0.183	0.459	0.684	0.8	0.875	0.888	0.952	0.96	0.973	0.972	0.973	0.978	0.981	0.979	0.98
	PSGQ	0.042	0.213	0.659	0.957	0.997	1	1	1	1	1	1	1	1	1	1	1
480	PGQ	0.039	0.302	0.767	0.939	0.977	0.989	0.996	0.998	0.998	0.999	0.999	1	1	1	1	0.999
	PSGQ	0.034	0.417	0.977	1	1	1	1	1	1	1	1	1	1	1	1	1

Note: PGQ= power of GQ test; PSGQ= Power of supGQ test.

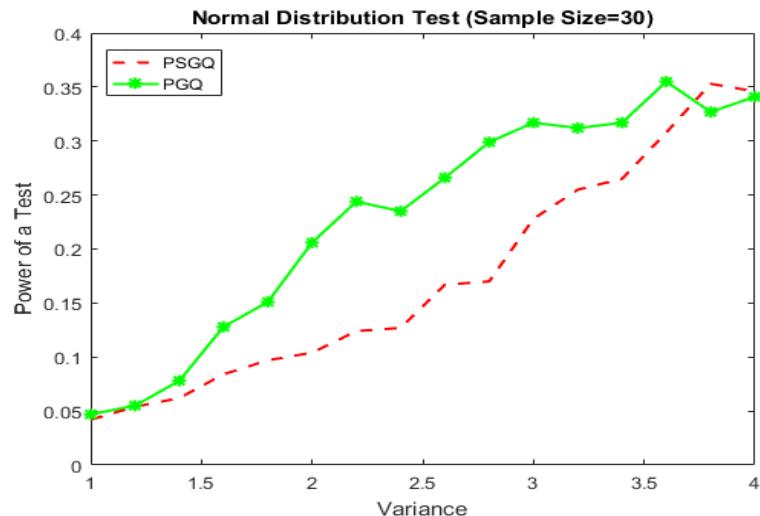
In addition, the effectiveness of the proposed approach is illustrated in Figures 4.1 (panel a—e).

This demonstrates the PSGQ approach's early convergence in comparison to the GQ method.

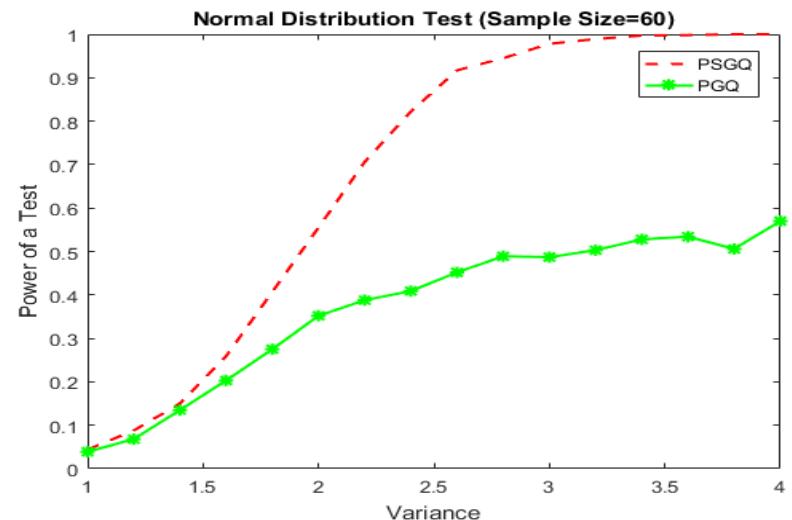
As shown in Figure 4.1 (panel a—e), even though the sample size is small ($n=30$), the PSGQ strategy overtakes the standard GQ approach when degree of heteroskedasticity high, however, at some points where degree of Heteroskedasticity is low, GQ test has a little edge on supGQ, this little edge is performance gets deteriorated when sample size and degree of Heteroskedasticity increases and supGQ overpasses GQ test and achieves early convergence. As the sample size increases from 60 to 240, the observability appears to become more distinct, and early convergence of the PSGQ method can be detected. Moreover, it is obvious from the preceding figures that the PSGQ method handles normal distribution data more efficiently by requiring a shorter time interval to reach a solution.

A normal distribution portrays a symmetrical bell-shaped curve having most data points clustering around the mean and deviations from the mean occurring less frequently as one moves farther out in either direction. However, real-world data is usually complex and may contain outliers, which are data points that differ significantly from the rest of the data. Outliers in a normal distribution indicate that a few data points depart greatly from the rest of the data, skewing the distribution and making it less representative of the total dataset. For sample size 30, when degree of heteroskedasticity is low power of GQ test is high and power of supGQ starts to increase when degree of heteroskedasticity tends to increase as shown in table 4.2. but as sample size increases to degree of heteroskedasticity is low at start and our proposed test overtakes GQ in term of power.

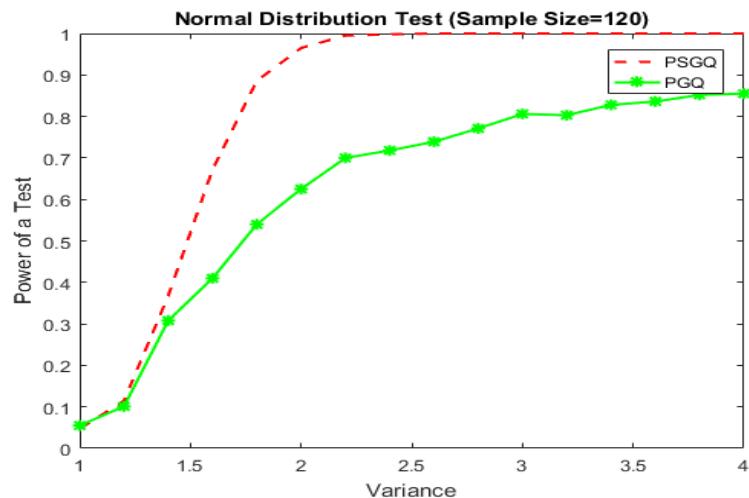
Figure 5.1 Comparison of GQ and supGQ (Standard Normal Distribution Case)



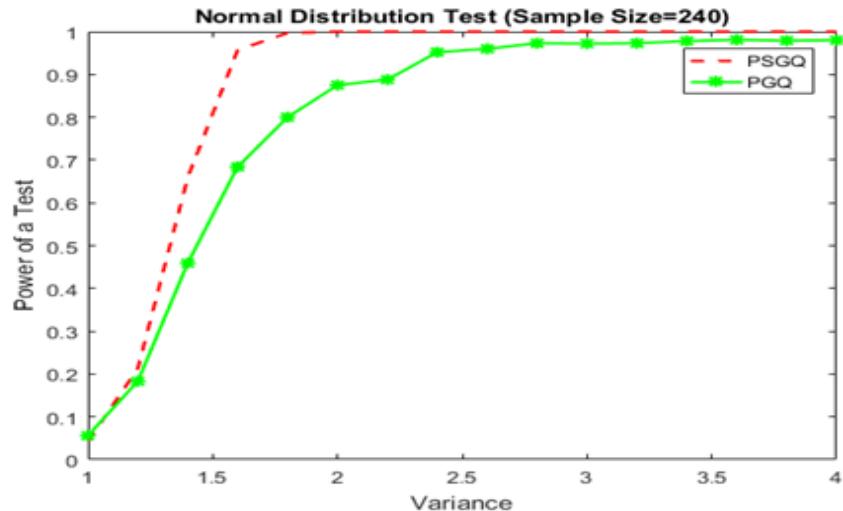
Panel (a)



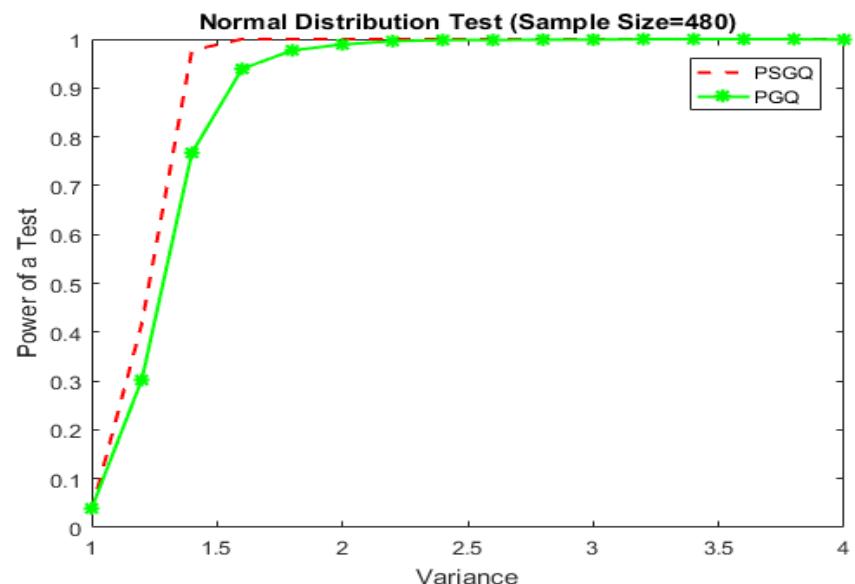
Panel (b)



Panel (c)



Panel (d)



Panel (e)

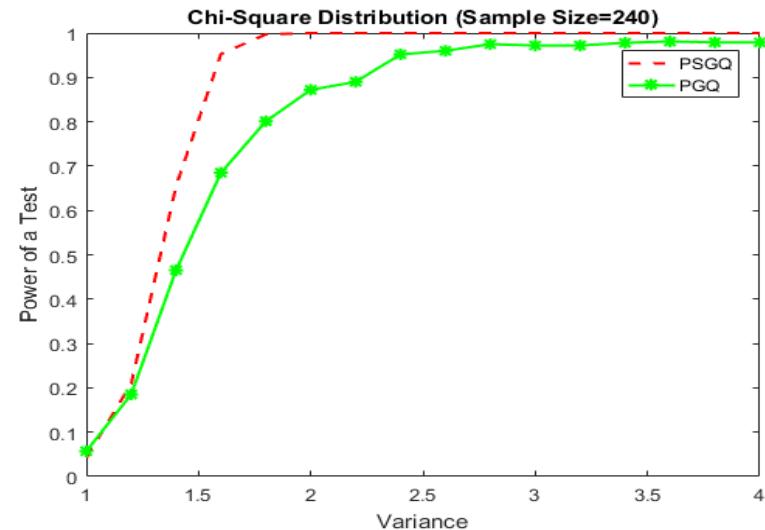
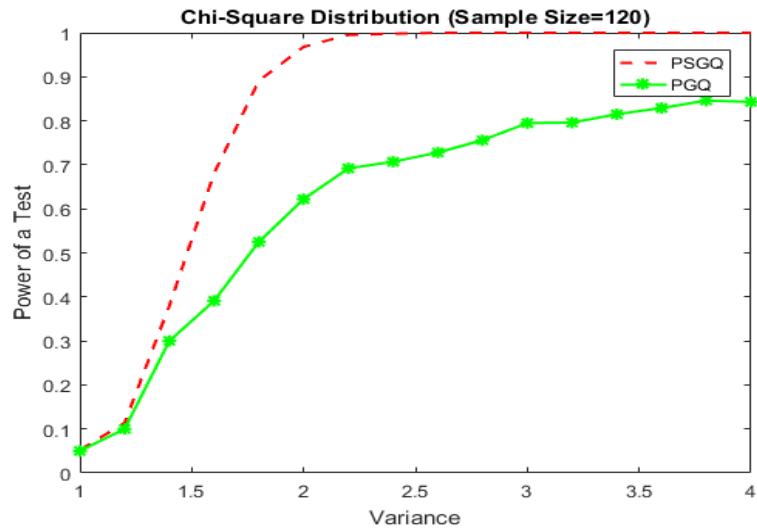
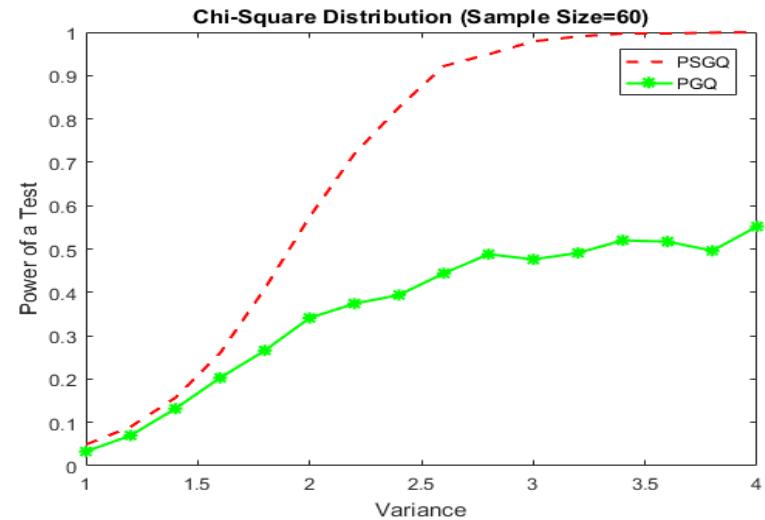
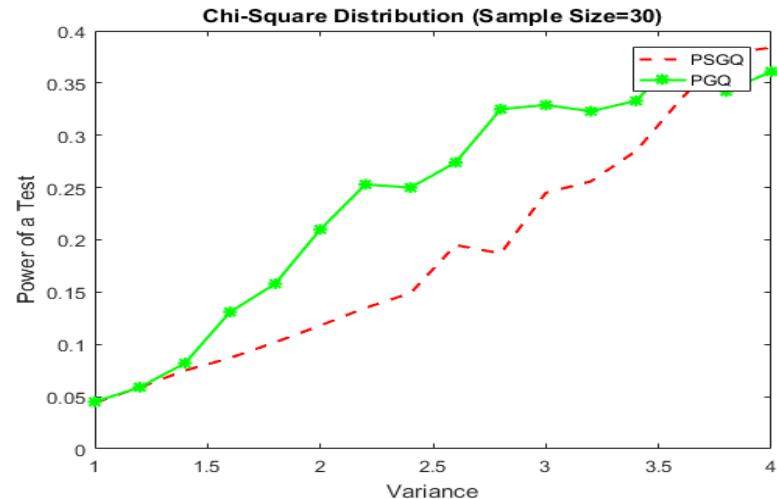
Table 4.3 shows comparison where regressor follows a Chi-Square distribution. It is worth noting that PGQ overtakes classic GQ in terms of solution convergence. Both the suggested supGQ test and the GQ test remain reasonably sized when applied to data from a Chi distribution with a breakpoint at 30%, even when using simulated critical values. As the sample size grew, so did the suggested test's power; in fact, during each iteration of the sample size rise, the proposed test's power was more than the basic test's GQ. With each increase in the data's standard deviation above the critical standard deviation, the recommended test's power to reliably predict outcomes surpasses that of the GQ test. Figure 4.2 (panel a—e) further demonstrates the efficacy of the proposed method. This illustrates that the PGQ methodology converges faster than the GQ. Figure 4.2 shows that PSGQ overtakes the traditional GQ technique and reaches early convergence despite the limited sample size ($n=30$). Early convergence of the PSGQ approach can be identified as the sample size grows from 60 to 240. The observability also looks to become clearer. And as can be seen in the figures above, the PSGQ approach makes better use of time when working with data from Chi-Square distributions.

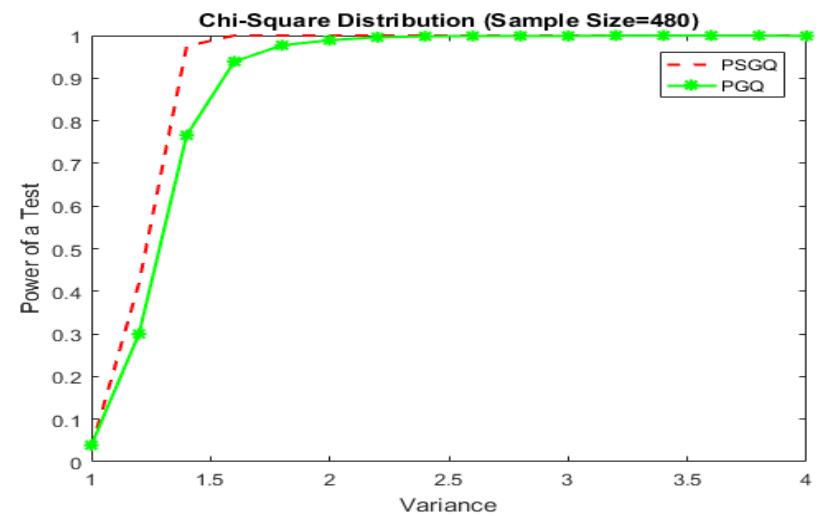
The Chi-Square distribution does not produce outliers by definition because it is a mathematical model for describing the distribution of a sum of squared normal random variables. The chi-square distribution is widely employed in hypothesis testing, particularly in the context of categorical data analysis. When degree of heteroskedasticity is low for sample size 30 as in table 4.3 based test performs in edge to proposed test and as degree of heteroskedasticity increases the proposed test power tends to increase. As sample size increases the degree of heteroskedasticity is low but the supGQ test detect heteroskedasticity more efficiently and with increase in sample size and degree of heteroskedasticity the proposed test significantly performs better than original GQ test.

Table 5.3 Data set demonstrating Chi-Square distribution.

Degree of Heteroskedasticity (HET)																	
N	Tests	0	0.10	0.18	0.26	0.34	0.41	0.47	0.53	0.59	0.64	0.69	0.74	0.79	0.83	0.88	0.92
30	PGQ	0.045	0.059	0.082	0.131	0.158	0.21	0.253	0.25	0.274	0.325	0.329	0.323	0.333	0.377	0.342	0.361
	PSGQ	0.044	0.059	0.075	0.087	0.102	0.118	0.135	0.149	0.195	0.187	0.245	0.256	0.285	0.335	0.378	0.384
60	PGQ	0.033	0.07	0.132	0.203	0.266	0.341	0.374	0.394	0.444	0.488	0.476	0.491	0.52	0.517	0.496	0.552
	PSGQ	0.049	0.09	0.157	0.26	0.409	0.574	0.719	0.827	0.922	0.949	0.979	0.99	0.997	0.997	0.999	1
120	PGQ	0.05	0.1	0.3	0.391	0.525	0.622	0.692	0.707	0.728	0.756	0.795	0.796	0.815	0.829	0.846	0.843
	PSGQ	0.052	0.114	0.38	0.681	0.891	0.968	0.995	0.998	1	1	1	1	1	1	1	1
240	PGQ	0.057	0.185	0.465	0.685	0.801	0.872	0.89	0.952	0.96	0.975	0.972	0.972	0.978	0.981	0.979	0.979
	PSGQ	0.042	0.209	0.655	0.953	0.997	1	1	1	1	1	1	1	1	1	1	1
480	PGQ	0.039	0.3	0.766	0.939	0.977	0.989	0.996	0.998	0.999	0.999	0.999	1	1	1	1	0.999
	PSGQ	0.032	0.419	0.975	1	1	1	1	1	1	1	1	1	1	1	1	1

Figure 5.2 Results of Chi Square Distribution for GQ and supGQ





Similar pattern can be seen in the distributions presented in Tables 4.4 and 4.5. In contrast to past GQ approaches, the suggested supGQ method quickly reaches unity power of test, as was discussed earlier. The curves shown in Figures 4.3 and 4.4 provide further evidence that the intended data driven PSGQ approach is far less complicated to put into practice by hand and deals with all types of data in a consistent manner. In addition to this, it has a lower requirement for computational overheads and is more practical to implement.

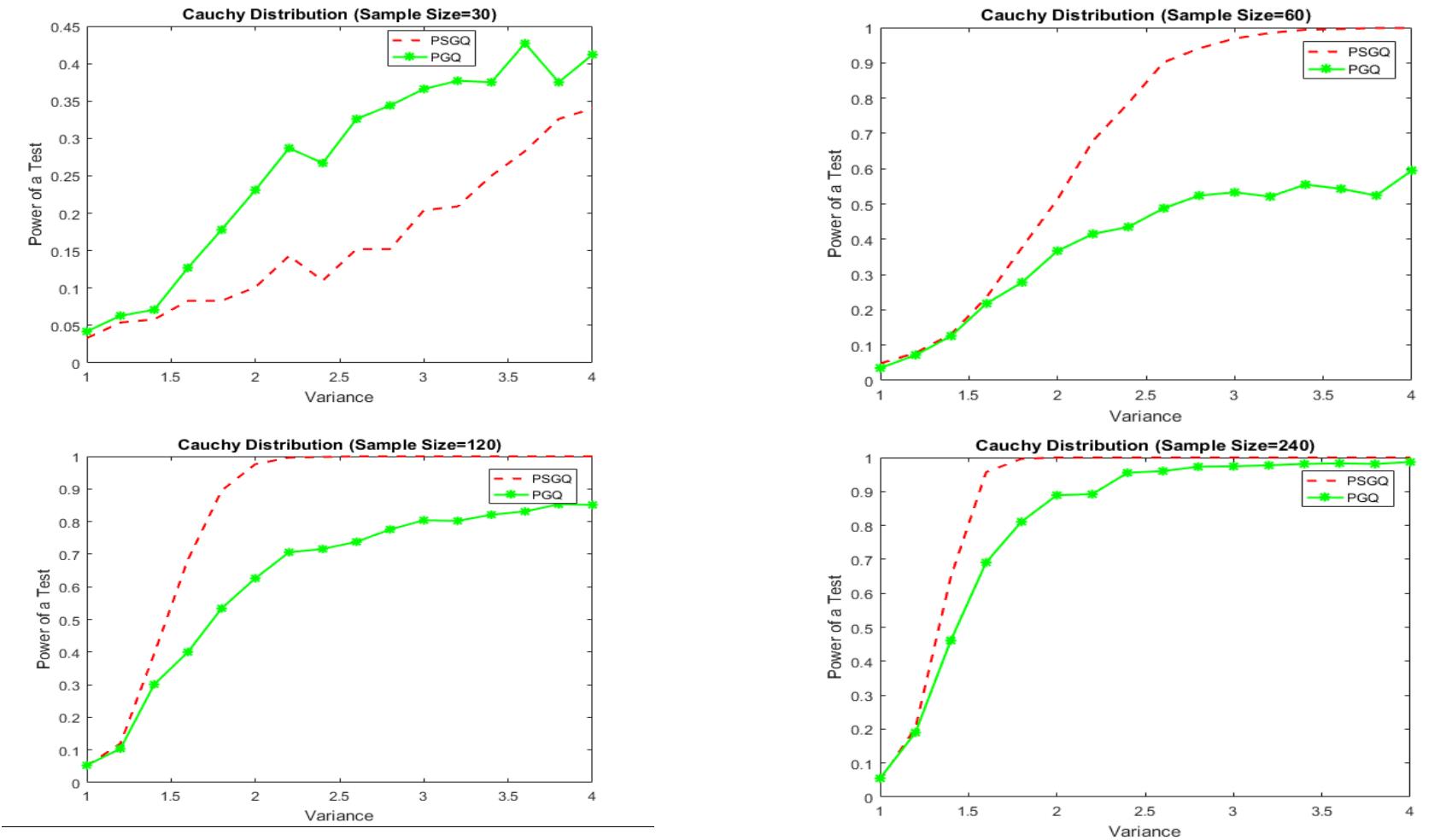
Table 5.4 Data set demonstrating Cauchy Distribution

Degree of Heteroskedasticity (HET)																	
N	Test	0	0.10	0.18	0.26	0.34	0.41	0.47	0.53	0.59	0.64	0.69	0.74	0.79	0.83	0.88	0.92
30	PGQ	0.042	0.063	0.071	0.127	0.178	0.231	0.287	0.267	0.326	0.344	0.366	0.377	0.375	0.427	0.375	0.412
	PSGQ	0.033	0.054	0.058	0.083	0.083	0.101	0.143	0.11	0.152	0.152	0.204	0.209	0.25	0.283	0.326	0.34
60	PGQ	0.035	0.072	0.126	0.218	0.278	0.367	0.415	0.435	0.488	0.524	0.533	0.521	0.555	0.543	0.524	0.595
	PSGQ	0.048	0.078	0.131	0.236	0.377	0.513	0.679	0.786	0.902	0.941	0.969	0.985	0.994	0.996	0.999	0.999
120	PGQ	0.053	0.104	0.301	0.4	0.534	0.626	0.706	0.716	0.738	0.776	0.804	0.802	0.821	0.831	0.853	0.851
	PSGQ	0.051	0.12	0.395	0.683	0.895	0.976	0.996	0.998	1	1	1	1	1	1	1	1
240	PGQ	0.055	0.19	0.461	0.691	0.811	0.889	0.892	0.955	0.96	0.973	0.974	0.977	0.981	0.983	0.981	0.987
	PSGQ	0.049	0.204	0.648	0.957	0.996	1	1	1	1	1	1	1	1	1	1	1
480	PGQ	0.045	0.322	0.786	0.944	0.983	0.991	0.996	0.998	0.999	1	0.999	1	1	1	1	1
	PSGQ	0.036	0.423	0.973	1	1	1	1	1	1	1	1	1	1	1	1	1

Table 5.5 Data set demonstrating Additive Sequence Distribution

Degree of Heteroskedasticity (HET)																	
N	Test	0	0.10	0.18	0.26	0.34	0.41	0.47	0.53	0.59	0.64	0.69	0.74	0.79	0.83	0.88	0.92
30	PGQ	0.045	0.059	0.082	0.131	0.158	0.21	0.253	0.25	0.274	0.325	0.329	0.323	0.333	0.377	0.342	0.361
	PSGQ	0.044	0.059	0.075	0.087	0.102	0.118	0.135	0.149	0.195	0.187	0.245	0.256	0.285	0.335	0.378	0.384
60	PGQ	0.034	0.066	0.125	0.211	0.272	0.347	0.381	0.408	0.454	0.487	0.489	0.502	0.535	0.536	0.509	0.567
	PSGQ	0.047	0.088	0.146	0.262	0.416	0.56	0.713	0.828	0.914	0.946	0.979	0.992	0.998	0.997	0.999	1
120	PGQ	0.047	0.097	0.283	0.379	0.518	0.607	0.68	0.702	0.721	0.76	0.792	0.787	0.814	0.822	0.835	0.842
	PSGQ	0.047	0.123	0.396	0.687	0.896	0.97	0.996	0.998	1	1	1	1	1	1	1	1
240	PGQ	0.045	0.168	0.438	0.674	0.797	0.871	0.881	0.948	0.962	0.969	0.968	0.973	0.979	0.98	0.977	0.981
	PSGQ	0.046	0.203	0.66	0.95	0.997	1	1	1	1	1	1	1	1	1	1	1
480	PGQ	0.046	0.321	0.781	0.949	0.978	0.99	0.996	0.998	0.999	1	1	1	1	1	1	1
	PSGQ	0.033	0.427	0.975	1	1	1	1	1	1	1	1	1	1	1	1	1

Figure 5.3 Results of Cauchy Distribution for GQ and supGQ.



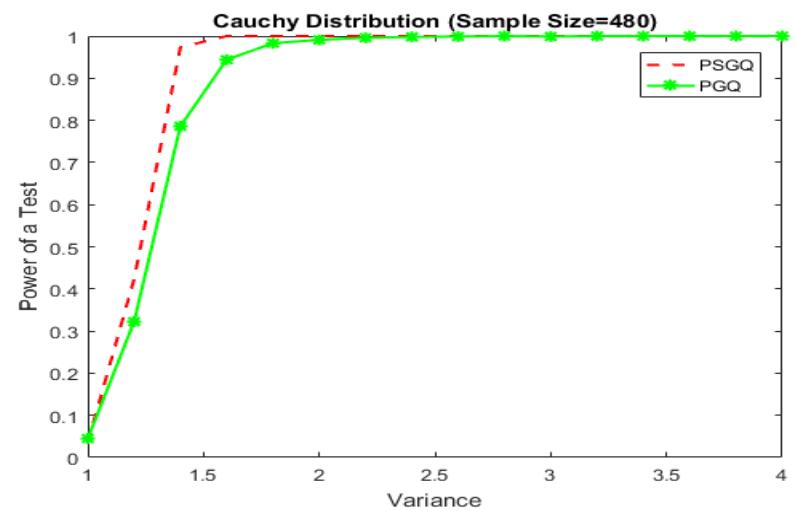
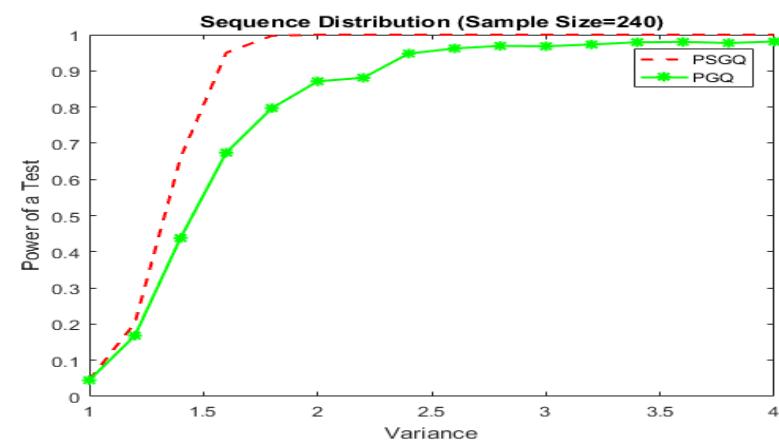
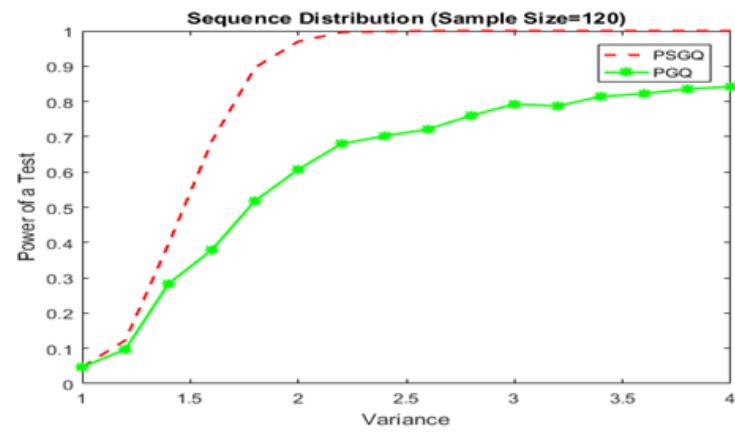
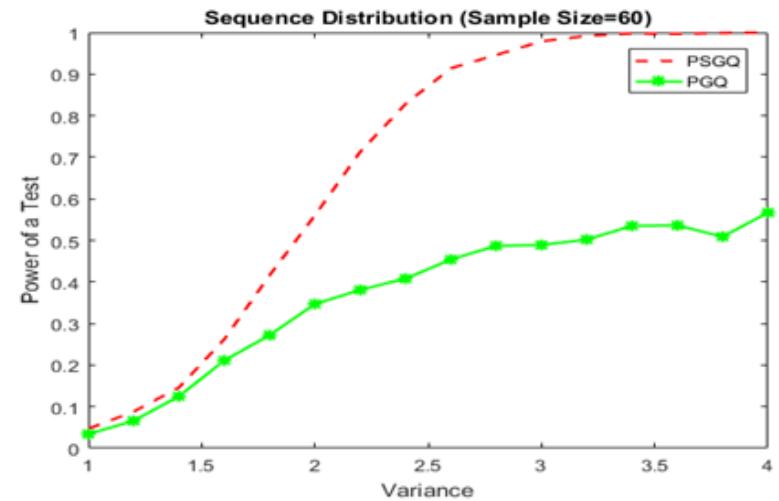
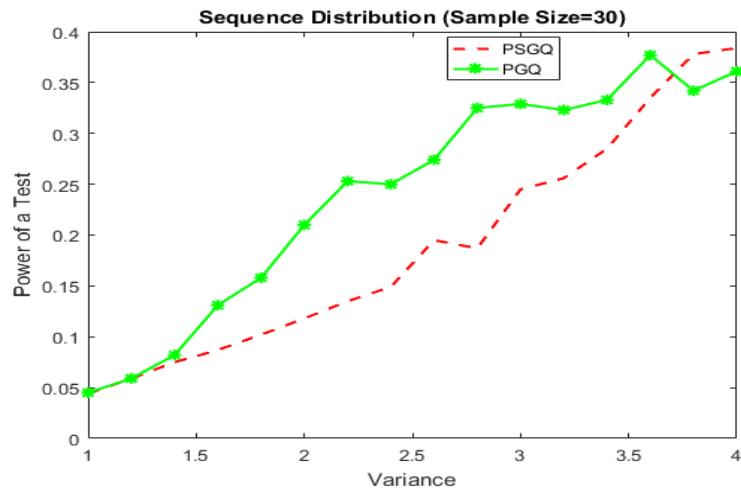
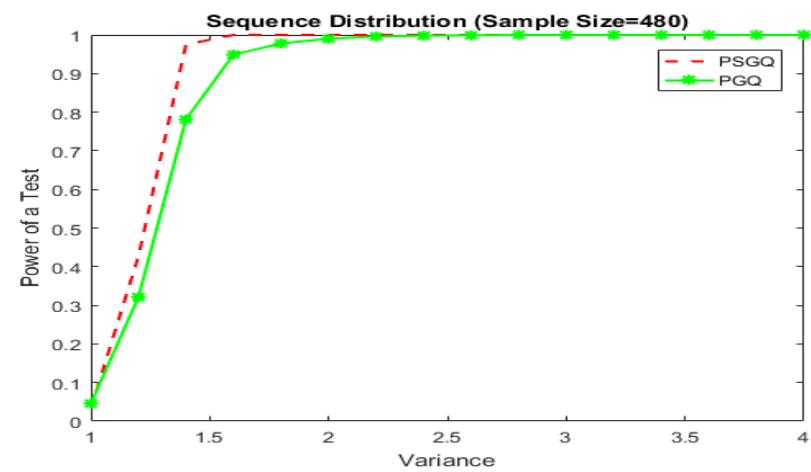


Figure 5.4 Results of Additive Sequence Distribution for GQ and supGQ





The Cauchy distribution is notable for having heavy tails, which implies it is more likely to have extreme values than other common distributions such as the normal distribution. Outliers are data points that differ greatly from the majority of the other data points in a dataset as a result of the Cauchy distribution's hefty tails. As sample size increases while keeping the degree of heteroskedasticity same the proposed test performs better. But as in table 4.4 shows that degree of heteroskedasticity is not much higher and sample size is 30 the original test also predicts heteroskedasticity issue efficiently but as the sample size increases the proposed test overtake it.

An additive sequence is a set of integers in which each number is the sum of the numbers before it and a constant value. Outliers can appear in an additive sequence if a data point deviates significantly from the pattern created by steady addition. The proposed test performs better as sample size increases while the degree of heteroskedasticity remains constant. However, as shown in table 4.4, the existing test predicts heteroskedasticity issues efficiently when the sample size is 30. As the sample size increases, the suggested test overtakes it.

Overall, the performance of proposed supGQ is shown to be far better than the conventional GQ test for different cases. The power increases as sample size increases and thus, we recommend using the supGQ test which has the ability to detect the unknown break in variance in contrast to the existing GQ test which works well only for known breaks.

CHAPTER 5

COMPARISON OF MGQ AND supMGQ

5.1 Introduction to Chapter

This chapter provides a comparison of MGQ test with the supMGQ via extensive Monte-Carlo simulations. The details of Monte-Carlo simulations are provided in next subsection while the comparison results are provided in a separate subsection that follow the next subsection.

5.2 Monte-Carlo Setup for Comparison of MGQ and supMGQ

The performance of our proposed tests and the existing GQ test is compared via extensive Monte-Carlo simulations. The equation $Y = X\beta + \varepsilon$ represents a linear model where Y is a vector of size $N \times 1$ containing observations on the dependent variable, β is a vector of size $K \times 1$ representing unknown parameters, X is a matrix of size $N \times K$ containing regressors, and ε is a vector of size $N \times 1$ representing unobserved errors. These errors, denoted by ε , are assumed to be independent with a mean of zero (a Null vector of size $N \times 1$) and a covariance matrix Σ that is a diagonal matrix with σ_n^2 as its diagonal elements for $n = 1, 2, \dots, N$. In simpler terms, ε follows a multivariate normal distribution $N(O, \Sigma)$. In this analysis, it is presupposed that all other requisite conditions of the linear regression model are fulfilled, such as the absence of multicollinearity and serial correlation issues.

Linear regression models are considered where regressors are generated from different distributions to cover up all types of design matrices, the usual ones—without outliers. For this, regressors are generated from a) additive sequence starting at $1(v)N$, where ‘ v ’ is the step size and N is the total number of sample points, b) standard normal. The performance of tests is assessed over different sizes of sample (small, medium and large), $N= 30, 60, 120, 240$. The rationale regarding the selection of sample sizes in line of "Hameed (2021) and E Alih · (2015).

The regressors generated for N=30 is repeated to generate samples of higher sizes, this is to keep degrees of Heteroskedasticity fixed throughout samples. In addition, all regressors are kept fixed throughout simulations.

The errors are generated from various distributions, a) standard normal for the two sub-samples—this is to calculate and compare size of all tests under null, b) non-standard by increasing variance of second sub-sample (and thus, degree of Heteroskedasticity) systematically to assess its impact on the power of all tests, while keeping variance of first sub-sample fixed. Forms of heteroskedasticity (variance structures) are considered, multiplicative etc., and the degree of heteroskedasticity (Het) are measured via guidelines provided in Ahmed et al. (2017).

5.3 Comparison of MGQ and supMGQ Test

Several cases utilizing various data distribution sets have been taken into consideration to show the effectiveness and robustness of the supMGQ over MGQ. The power of supMGQ (PMGQ) can surpass the power of MGQ (PMGQ). First the findings of standard Normal regressor case are presented in Table 5.1 over varying degree of heteroskedasticity and over different sample size from N=30 to N=240. The reason for using the same set of regressors repeatedly to obtain sample sizes of 30, 60, 120, or 240 is to maintain a consistent level of heteroskedasticity across varying sample sizes. The rationale regarding the selection of sample sizes in line of "Hameed (2021) and E Alih · (2015).

Table 6.1 Standard Normal Distribution

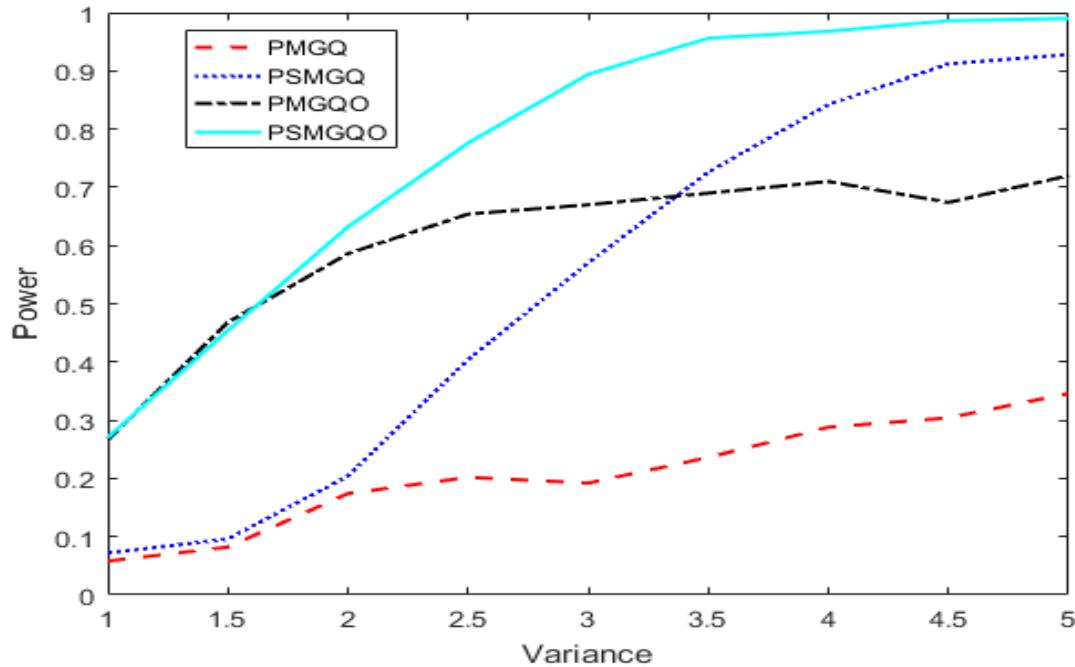
Sample size	Degree of Het.	Variance (σ^2)	Without Outliers		With Outliers	
			PMGQ	PSMGQ	PMGQ	PSMGQ
30	0.000	1	0.058	0.072	0.266	0.270
	0.373	2	0.174	0.204	0.586	0.632
	0.653	3	0.192	0.57	0.67	0.894
	1.131	4	0.288	0.842	0.71	0.968
	1.335	5	0.346	0.928	0.72	0.99
60	0.000	1	0.047	0.039	0.131	0.09
	0.373	2	0.357	0.196	0.612	0.374
	0.653	3	0.506	0.722	0.719	0.857
	1.131	4	0.537	0.966	0.747	0.989
	1.335	5	0.563	0.999	0.744	0.999
120	0.000	1	0.059	0.047	0.259	0.272
	0.040	1.1	0.075	0.054	0.355	0.326
	0.081	1.2	0.137	0.127	0.461	0.407
	0.191	1.3	0.195	0.168	0.534	0.486
	0.247	1.4	0.258	0.293	0.599	0.62
	0.300	1.5	0.322	0.423	0.677	0.708
	0.351	1.6	0.404	0.575	0.719	0.796
	0.399	1.7	0.455	0.685	0.738	0.858
	0.445	1.8	0.504	0.821	0.76	0.922
	0.489	1.9	0.588	0.881	0.82	0.963
240	0.000	1	0.044	0.06	0.387	0.509
	0.040	1.1	0.084	0.112	0.495	0.564
	0.081	1.2	0.186	0.243	0.64	0.689
	0.191	1.3	0.307	0.481	0.713	0.792
	0.247	1.4	0.453	0.746	0.799	0.895
	0.300	1.5	0.571	0.878	0.849	0.949
	0.351	1.6	0.675	0.958	0.867	0.994
	0.399	1.7	0.742	0.99	0.902	0.997
	0.445	1.8	0.784	0.997	0.921	0.999
	0.489	1.9	0.837	1	0.935	1

The proposed supMGQ test and the MGQ test maintain their size when simulated important values are used in the case of a normal distribution with a breakpoint situated at 30% of the data. When the sample size expanded, the power of the suggested test began to rise, remaining higher than the MGQ throughout the increment. When the sample size grew, this happened. For each incremental increase above the critical standard deviation, the recommended test's ability to accurately predict outcomes overtakes the MGQ test. It is also noted that when degree of heteroskedasticity is low, power of supMGQ is found to be less than MGQ. Also, Figures 5.1—5.4 show how effective the suggested strategy is for the standard normal regressor case. This indicates the early convergence of the PMGQ method in comparison to the MGQ method.

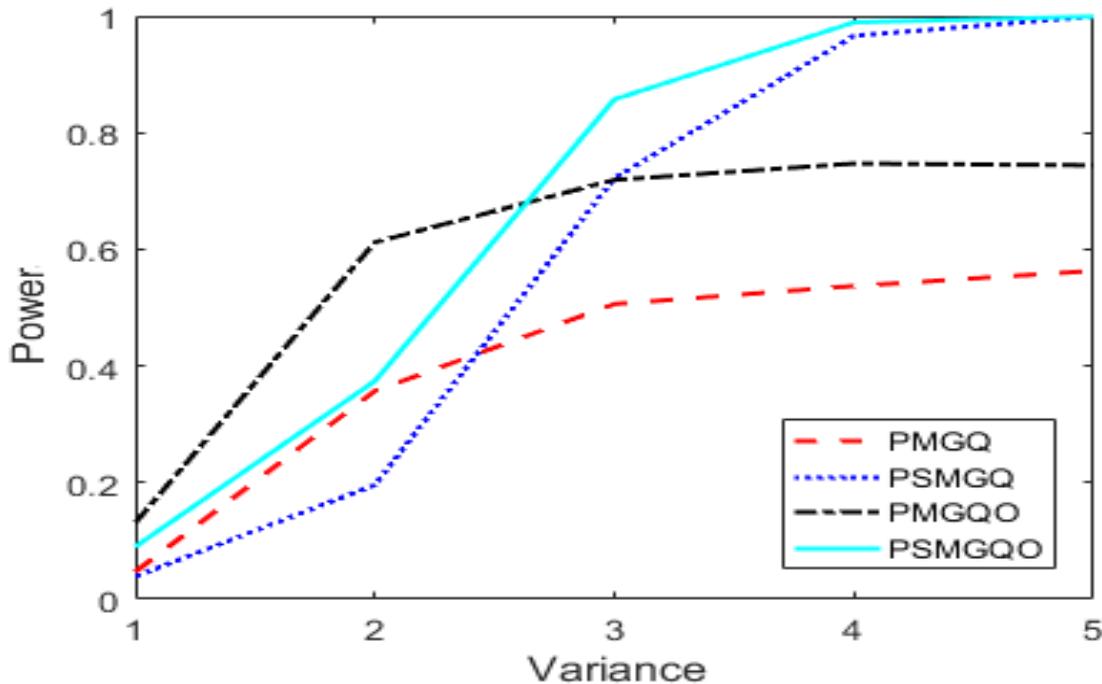
A normal distribution is a statistical notion that depicts a symmetrical bell-shaped curve with most data points clustering around the mean and deviations from the mean occurring less frequently as one moves farther out in either direction. Outliers in a normal distribution indicate that a few data points depart greatly from the rest of the data, skewing the distribution and making it less representative of the total dataset. For sample size 30 when outliers are not introduced. the power of MGQ is 0.058 and degree of heteroskedasticity is same when outlier is introduced the power increases to 0.266 which is much higher than without outlier which proves normal distribution is sensitive to outliers. as degree of heteroskedasticity increases the sensitivity of the outliers increases and power of original MGQ is higher with outliers than without outliers. as we can see in table 5.1 that throughout the power of proposed test remains high than the original test both in case of outlier and without outlier.

However, it is emphasized that since normal distribution is sensitive to outliers, so tests designed for outliers are likely to perform better when the regressors are chosen other than normal particularly, the ones containing one or more outliers.

Normal Distribution for N=30

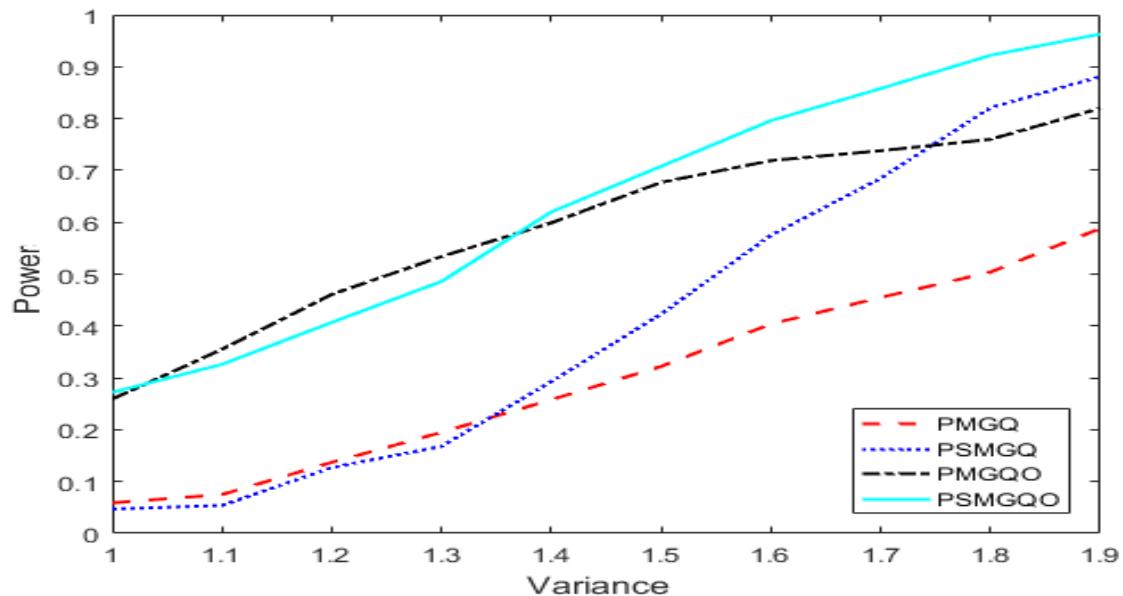


Normal Distribution for N=60



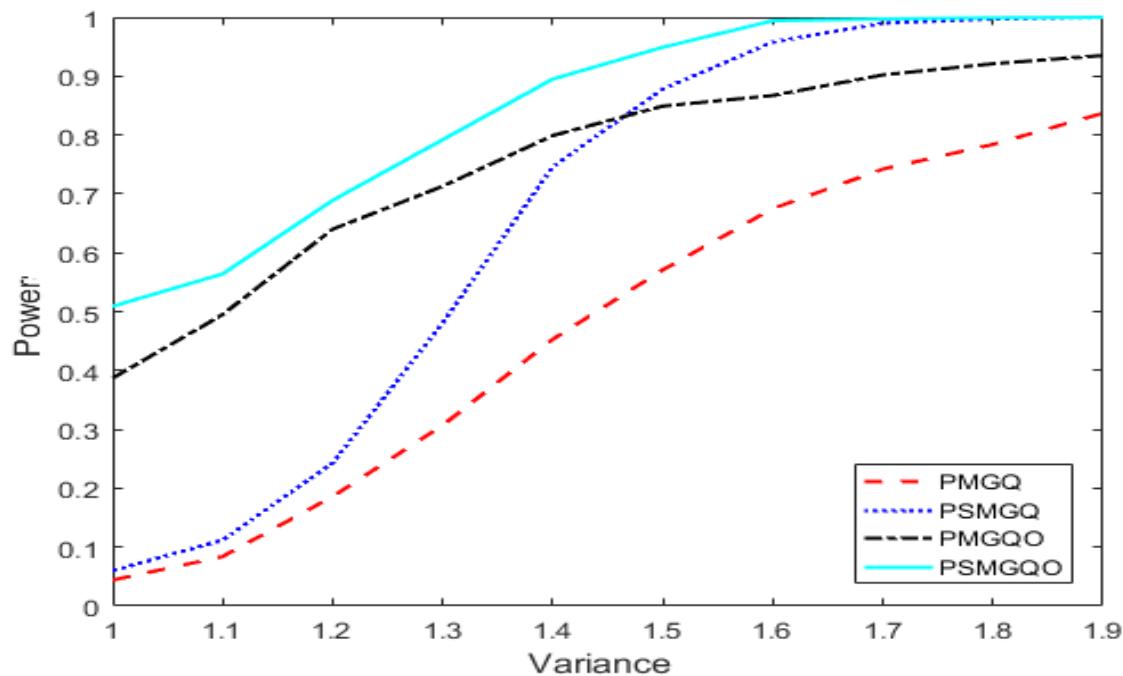
Note: PMGQO and MSMGQO are PMGQ and PSMGQ tests in case of outliers respectively.

Figure 6.1 Normal Distribution for N=120



Note: PMGQO and MSMGQO are PMGQ and PSMGQ tests in case of outliers respectively.

Figure 6.2 Normal Distribution for N=240



Note: PMGQO and MSMGQO are PMGQ and PSMGQ tests in case of outliers respectively.

As shown in Figure 5.1—5.4, even though the sample size is small ($n=30$), the PSMGQ test overtakes the standard MGQ test in case of outliers its ranges from 0.27 to 0.9 and in case of without outliers power of supMGQ ranges from 0.072 to 0.928 when degree of heteroskedasticity increases from 0 to 1.335. The sample size is small ($n=60$), the PSMGQ test overtakes the standard MGQ test. As the sample size increases from 120 and 240, the observability appears to become more distinct, and early convergence of the PSMGQ method can be detected. Moreover, it is obvious from the preceding figures that the PSMGQ method handles normal distribution data more efficiently by requiring a shorter time interval to reach a solution.

Table 5.2 shows the case of additive sequence regressor. It is worth noting that PMGQ overtakes classic MGQ in terms of solution convergence. Both the suggested sup-MGQ test and the MGQ test remain reasonably sized when applied to data from an additive sequence distribution with a breakpoint at 30%, even when using simulated critical values. As the sample size grew, so did the suggested test's power; in fact, during each iteration of the sample size rise, the proposed test's power was more than the basic test's MGQ. With each increase in the data's standard deviation above the critical standard deviation, the recommended test's power to reliably predict outcomes surpasses that of the MGQ test.

Table 6.2 Additive Sequence

Sample Size	Degree of Het.	Without Outlier		With Outlier	
		PMGQ	PSMGQ	PMGQ	PSMGQ
30	0.000	0.054	0.036	0.388	0.122
	0.373	0.198	0.15	0.706	0.456
	0.653	0.27	0.42	0.72	0.816
	1.131	0.334	0.734	0.804	0.96
	1.335	0.326	0.886	0.806	0.982
60	0.000	0.066	0.05	0.204	0.149
	0.373	0.4	0.471	0.59	0.65
	0.653	0.516	0.961	0.701	0.978
	1.131	0.568	1	0.744	1
	1.335	0.578	1	0.746	1
120	0.000	0.055	0.043	0.255	0.225
	0.040	0.076	0.049	0.347	0.302
	0.081	0.123	0.123	0.418	0.376
	0.191	0.196	0.18	0.509	0.472
	0.247	0.262	0.299	0.551	0.602
	0.300	0.325	0.425	0.635	0.695
	0.351	0.414	0.604	0.7	0.793
	0.399	0.437	0.717	0.706	0.875
	0.445	0.492	0.842	0.718	0.93
	0.489	0.564	0.906	0.766	0.965
240	0.000	0.04	0.049	0.358	0.382
	0.040	0.076	0.11	0.452	0.493
	0.081	0.178	0.241	0.573	0.626
	0.191	0.291	0.468	0.655	0.763
	0.247	0.448	0.734	0.762	0.896
	0.300	0.555	0.884	0.81	0.963
	0.351	0.664	0.958	0.86	0.992
	0.399	0.737	0.99	0.881	0.994
	0.445	0.769	0.997	0.924	0.999
	0.489	0.836	1	0.938	1

Figures 5.4—5.6 further demonstrate the efficacy of approach. The proposed method. This illustrates that the PMGQ methodology converges faster than the MGQ test.

Figure 6.3 Additive Sequence for N=30

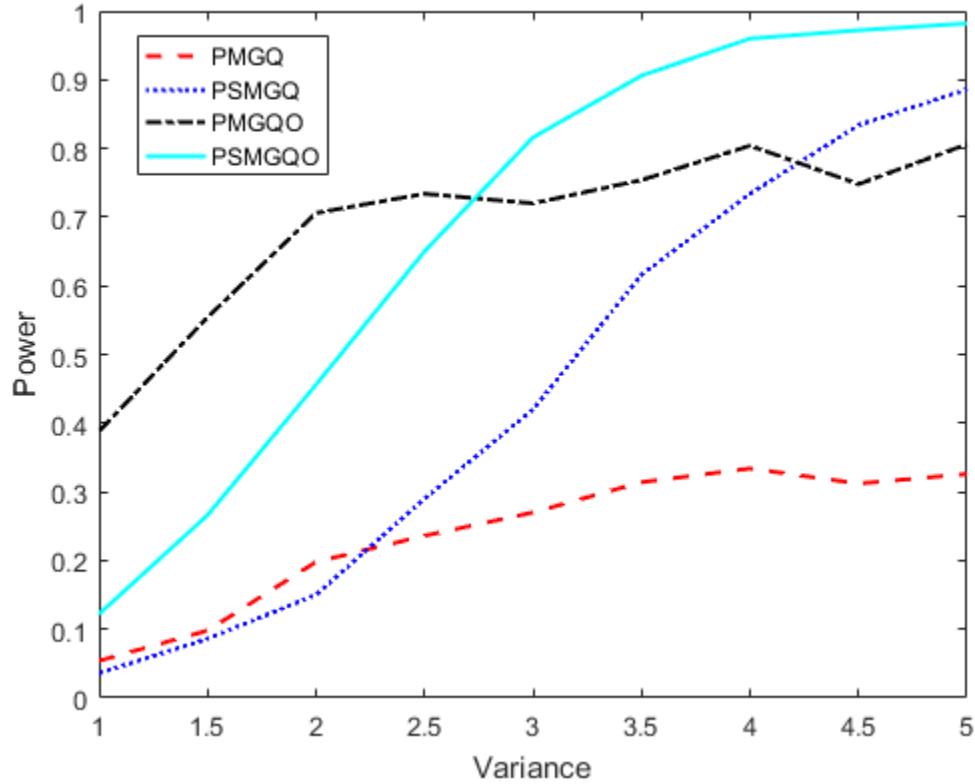
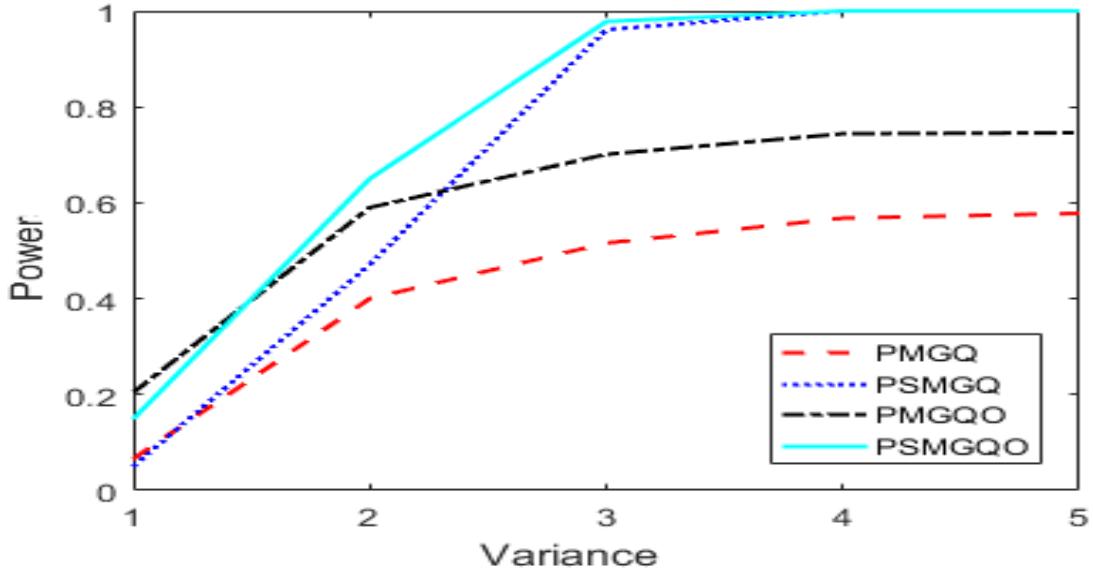
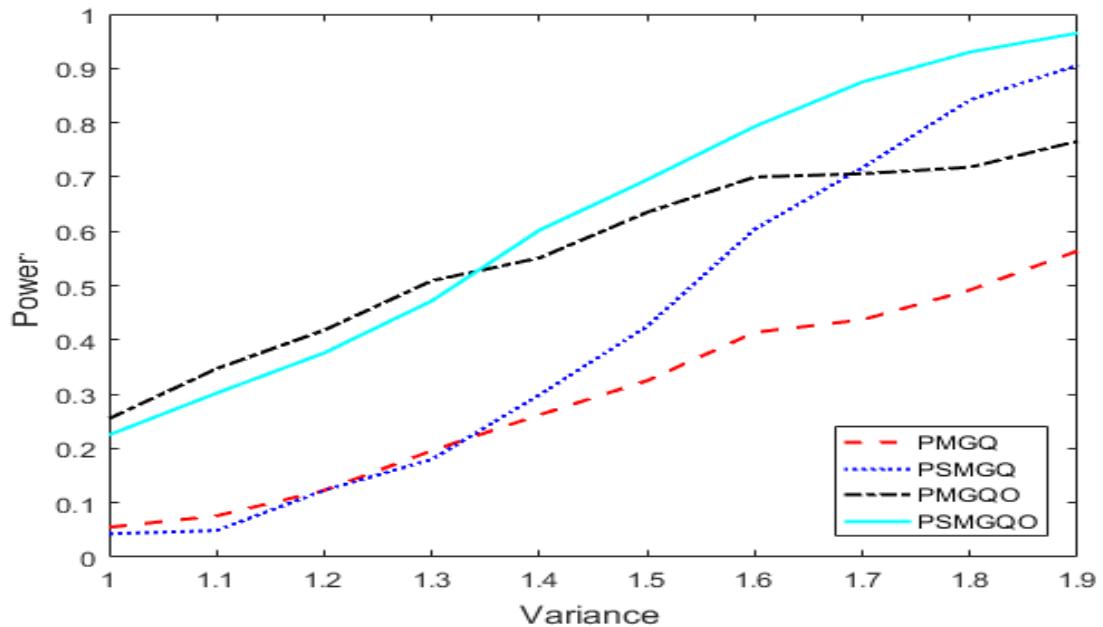


Figure 6.4 Additive Sequence for N=60



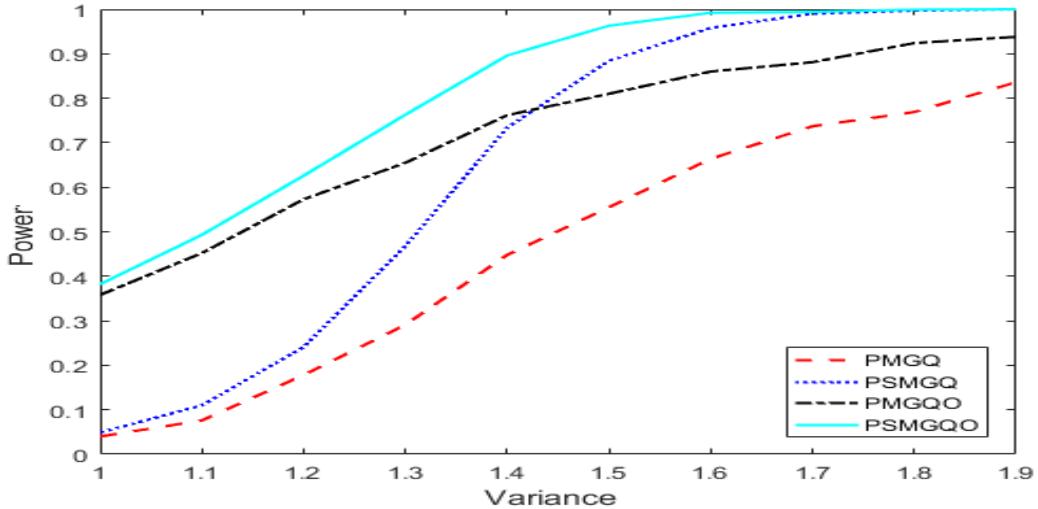
Note: PMGQO and MSMGQO are PMGQ and PSMGQ tests in case of outliers respectively.

Figure 6.5 Additive Sequence for N=120



Note: PMGQO and MSMGQO are PMGQ and PSMGQ tests in case of outliers respectively.

Figure 6.6 Additive Sequence for N=240



Note: PMGQO and MSMGQO are PMGQ and PSMGQ tests in case of outliers respectively.

Figure 5.5—5.8 for Additive sequence regressor, show that PSMGQ overtakes the traditional MGQ technique and reaches early convergence supMGS has high power than MGQ ranges from 0.036 to 0.886 in case without outlier and power ranges from 0.122 to 0.982 in case of outlier when sample size is 30, as degree of Heteroskedasticity increases. Despite the limited sample size ($n=60$). Early convergence of the PSGMQ approach can be identified as the sample size grows from 60 and 240. The observability also looks to become clearer. And as can be seen in the figures of Additive sequence distribution above, the PSMGQ approach makes better use of time when working with data from additive sequence distribution.

An additive sequence is a set of integers in which each number is the sum of the numbers before it and a constant value. Outliers can appear in an additive sequence if a data point deviates significantly from the pattern created by steady addition. When sample size is 30 as in table 5.2, the degree of heteroskedasticity is zero the power of MGQ test is higher 0.338 with outlier than the power without outliers 0.054. As degree of heteroskedasticity increases with the outliers the

power of the test MGQ increases significantly but the pattern remains same, the power in presence of outlier is higher. for sample original test may take lead but as sample size increases the proposed test overtakes in power and pattern of outliers and without outlier, the power with outlier is higher than power without outlier.

Overall, the supMGQ is better than modified GQ (MGQ) test proposed by (Rana et al., 2008) when the break (split) point in variance is unknown in the presence of outliers. The Standard normal and additive sequence distribution cases show that supMGQ test offers substantial improvements over the existing MGQ and performs superbly in the detection of heteroskedasticity in the presence of outliers.

CHAPTER 6

COMPARISON OF PROPOSED supGQ WITH supF and supMZ

6.1 Introduction to Chapter

Chapter 4 and 5 highlight how newly proposed tests—the supGQ and the supMGQ overtake the existing GQ and the MGQ tests. It is emphasized that both supGQ and supMGQ are by structure better than the conventional GQ and the MGQ tests respectively as the former are designed to cater the unknown break in variance as well, however, the later tests the null of homoskedasticity when the break point in variance is known.

Sup MZ tests simultaneously for break in regression coefficients as well as in variance. Since structural change can, and often does, involve changes in variances.

$$H_0; \beta_1 = \beta_2, \sigma_1^2 = \sigma_2^2$$

$$H_1; \beta_1 \neq \beta_2, \sigma_1^2 \neq \sigma_2^2$$

The Sup MZ statistics is defined as :

$$SupMZ = \max_{a \leq j \leq b} MZ_j, K < a \leq j \leq b < N - K$$

$$\text{Where: } MZ = (N - K) \log \hat{\sigma}_0^2 - \{(N_1 - K) \log \hat{\sigma}_1^2 + (N_2 - K) \log \hat{\sigma}_2^2\}$$

Sup MZ is for unknown break point and calculate MZ for all potential break points "j", $K < a \leq j \leq b < N - K$ and take the maximum value of these values.

A statistical test called the F test is used to assess whether there is a significant difference between the coefficients in two distinct regression models. It evaluates whether the relationship between the independent factors and the dependent variable exhibits a structural break.

H_0 ; There is no significant improvement in fit from running two regressions.

RSS_A is defined as the RSS using only subsample A, RSS_B is defined as the RSS using only subsample B and RSS_p is defined as the RSS using the entire (pooled) sample.

The F-test with the following F-statistic:

$$F = \frac{(RSS_p - RSS_A - RSS_B)/K}{(RSS_A - RSS_B)/(N - 2K)}$$

Under null hypothesis, the F-statistic follows an F-distribution with k and N- 2K degrees of freedom.

It is also emphasized that the supGQ and supMGQ cannot be directly comparable with any of the existing tests in literature; however, for the sake of completeness, in this chapter, we provide a comparison of power of supGQ with the existing supF and supMZ tests. However, since there is no test available in literature that can be compared with supMGQ, so we did not consider in this analysis and thus, this chapter provides a comparison of supGQ with supF and supMZ. Extensive Monte-Carlo simulations are done, and the results are provided. The next two sub-sections provide details of the simulations and the results obtained.

6.2 Monte-Carlo Setup of supGQ, supF and supMZ

Several Monte-Carlo simulations are used to compare how well our suggested tests and the current tests (F, MZ, and their sup variant) perform. Considered are linear regression models $Y = X\beta + \varepsilon$ represents a statistical model where Y is a vector of size $N \times 1$ containing observations on the dependent variable. The vector β has dimensions $K \times 1$ and represents unknown parameters. The matrix X is $N \times K$ in size and contains the regressors. The vector ε is $N \times 1$ and represents unobserved errors. These errors are assumed to be independent, with a

mean of zero (a Null vector of size $N \times 1$) and a covariance matrix Σ . The covariance matrix Σ is a diagonal matrix with σ_n^2 as its diagonal elements for $n = 1, 2, \dots, N$. In simpler terms, ε follows a multivariate normal distribution $N(O, \Sigma)$. This analysis assumes that the additional prerequisites of the linear regression model are satisfied, such as the absence of multicollinearity and serial correlation issues.

Regressors are constructed from various distributions to account for all common design matrices—without outliers. The distributional choice of regressors is done keeping in view all types of regressors into account (symmetric, asymmetric, equally spaced and the ones containing one or more outliers). All other distributions fall in one of these categories. For this, regressors are created using the following: a) an additive sequence beginning at $1(v)N$, where 'v' denotes the step size and 'N' denotes the total number of sample points; b) a standard normal distribution; c) a chi-square with two degrees of freedom and a Cauchy distribution. Several sample sizes (small, medium, and large) are used to evaluate test performance, with $N=30$, 60 , 120 , 240 , and 480 . To maintain degrees of freedom, the regressors created for $N=30$ is repeated to produce samples of larger sizes.

The errors are generated from different distributions: a1) a standard normal for the two sub-samples, which is used to calculate and compare the size of all tests under the null hypothesis; b) non-standard by systematically increasing the variance and coefficients of the second sub-sample (and thus, the degree of heteroskedasticity) while maintaining the variance of the first sub-sample.

6.3 Results of Comparison of supGQ, supF and supMZ

The distance (D) between coefficients (β_1 and β_2) of two subgroups is measured via a non-centrality parameter, following Massoumi et al. (2010), defined as:

$$Dist = (\beta_1 - \beta)^t (X'_1 X_1)^{-1} (\beta_1 - \beta) + (\beta_2 - \beta)^n (X'_2 X_2)^{-1} (\beta_2 - \beta)$$

The critical values for both supF and sup MZ tests³ are computed under the null hypothesis of no break in mean ($D = 0$) and variance ($H = 0$).

Table 6.3 Degree of Distance Normal

		Sample Size				
		30	60	120	240	480
Distance	1	0	0	0	0	0
	1.2	0.2586	0.0009	0.0004	0.0003	0.0001
	1.4	0.4823	0.0034	0.0016	0.0011	0.0005
	1.6	0.6796	0.0078	0.0036	0.0024	0.0011
	1.8	0.8561	0.0138	0.0065	0.0042	0.0020
	2	1.0158	0.0215	0.0101	0.0066	0.0031
	2.2	1.1616	0.0310	0.0145	0.0095	0.0044
	2.4	1.2959	0.0422	0.0198	0.0130	0.0060
	2.6	1.4202	0.0551	0.0258	0.0170	0.0078
	2.8	1.5360	0.0698	0.0327	0.0215	0.0099
	3	1.6444	0.0861	0.0403	0.0265	0.0122

From this point forward, we are going to discuss the variance coefficient surface differences between supF and supGQ. In addition, as the variance varies, the supGQ test gets more precise, and the supF test will be applicable for varying regression coefficients. The ordinary normal distribution, the chi-square distribution, the Cauchy distribution, and the additive sequence distribution will be utilized to portray the validity of the claim. In this regard, we will employ four distinct sample sizes (60,120,240, and 480) to demonstrate whether the proposed approach effectively manages the adaptive variance coefficient phenomena when compared to the test powers of all of the stated sample sizes.

6.3 NORMAL DISTRIBUTION CASE

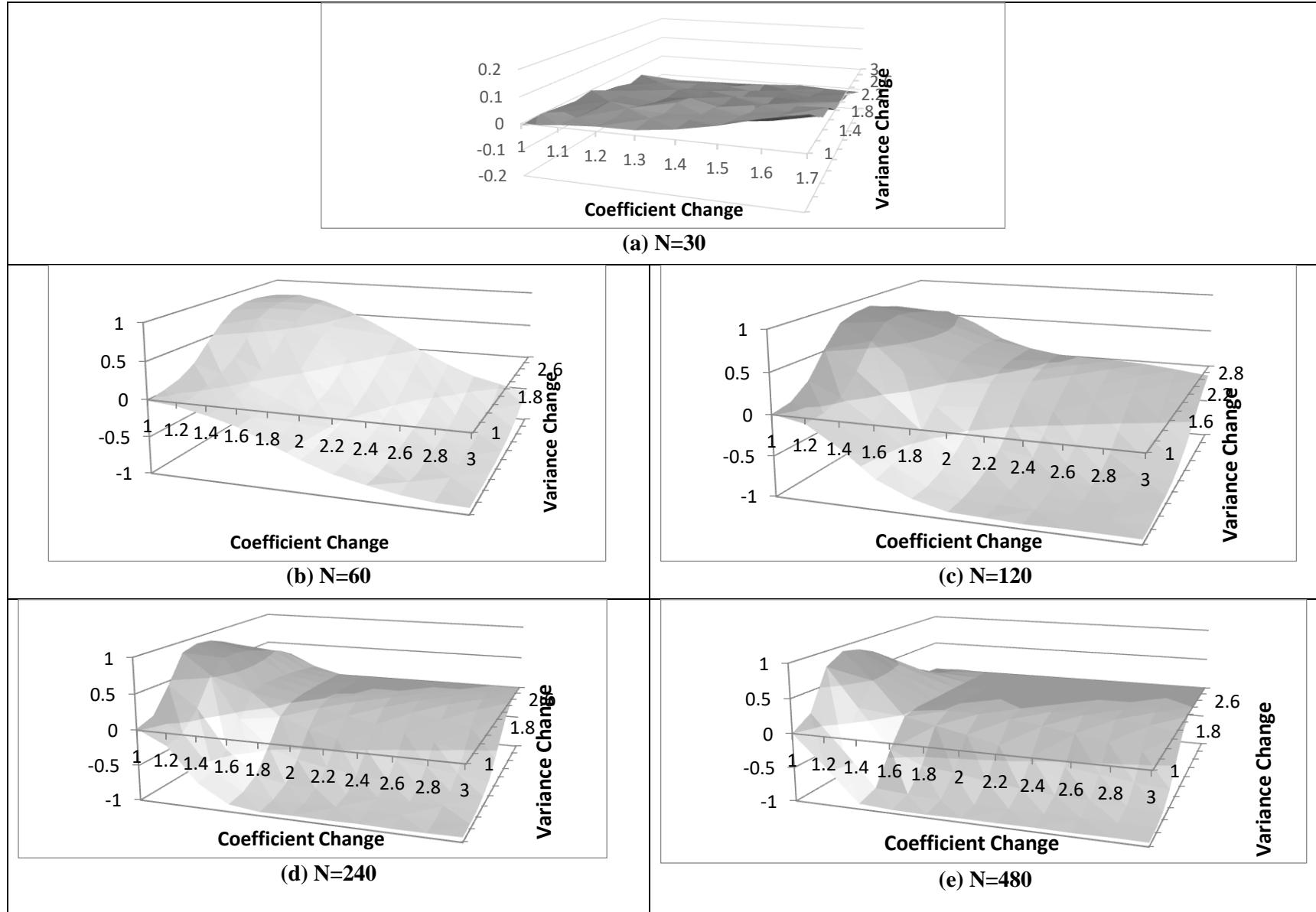
With a sample size of 30, the supGQ test overtakes when dealing with changes in variance in a steady standard normal distribution. The supF test, on the other hand, performs better when the coefficient varies. The supGQ test has greater power when the variance of the coefficient fluctuates, and the difference in powers between the supF and supGQ tests ranges from -0.006 to -0.1205 when the coefficient remains constant at 1 and the variance goes from 1

to 3. When the variance remains constant at one and the coefficient fluctuates from one to 1.7, the power difference between the supF and supGQ tests ranges from -0.006 to 0.1295. When both variance and coefficient are changed, the supGQ improves. For the sample size of 60, supF and supGQ tests performed better than the original GQ and F tests, and the difference between supF and supGQ tests follows the same pattern as the original GQ and F tests, but as the powers of supremum versions are greater than the original, their power differences are also greater. As depicted in the figure 6.1, the value of supF and supGQ in variance change space (1-3) is 0.91, whereas F-GQ has a value of 0.48, and coefficient change space ranges between 0.006 and 0.49. When the sample size increases from 60 to 120, the coefficient change space becomes elastic. When there is a small change in the coefficient change space, there is a substantial difference in power between the supF and supGQ tests. The difference increased from 0.006 to 0.80 due to a change in coefficients (1-2.2), whereas the difference between supF and supGQ increases with a change in coefficients from 1.8 to 2.2 and reaches 0.80. The supGQ test operates more robustly in variance-modified space ranging from 0.006 to 0.93. When the sample size hits 240, the supGQ test becomes more powerful and reaches its maximum strength, and the supF test follows suit. In coefficients variance altered space, the supGQ test behaves smoothly as the difference increases from 0.01 to 9.4 and the supF test behaves more strongly as the coefficients change from 1 to 1.8 to 0.01 to 0.90 but then declines to 0.19 when the coefficients change reaches 3. When the sample size increases to 480, both the supF and supGQ tests perform strongly and converge to 1, and their difference behaves accordingly. Conversely, when the sample size is less than 480, the supF test performs strongly in comparison to the supGQ test, but the difference between the two gradually decreases in coefficient change space. In spaces where the variance has altered, the supGQ test performs better, and the difference between the supF and supGQ tests

approaches 0.9 when the variance increases from 1 to 1.4. In coefficient variance space, we noticed that the supGQ difference increases smoothly while the supF power increases with rising return, but after reaching the maximum difference point, the supGQ difference begins to decrease with increasing return.

In coefficient variance space, we noticed that the supGQ difference increases smoothly while the supF power increases with rising return, but after reaching the maximum difference point, the supGQ difference begins to decrease with increasing return. Figure 6.1 to 6.4 express a functional relationship between a variance and coefficient change to show the power difference of the tests. Positive values shows that supF test is superior at that point and supGQ is superior where power difference is negative. Figures 6.1 to 6.4 display a mesh surface representing a set of three-dimensional data. These are the best power combinations between coefficient and variance data sets. By analogy with a topographic map, the colours and patterns represent places that have values in the same range. They are made up of three variables: coefficient, variance, and test power. Variance and coefficient are independent variables with horizontal axes. The other is Power of test, which is shown vertically. Figures 6.1 to 6.4 depict a functional relationship between a variance and coefficient change to show the power difference of the tests.

Figure 6.7: Difference in Powers of supF and supGQ (Normal Distribution Case)



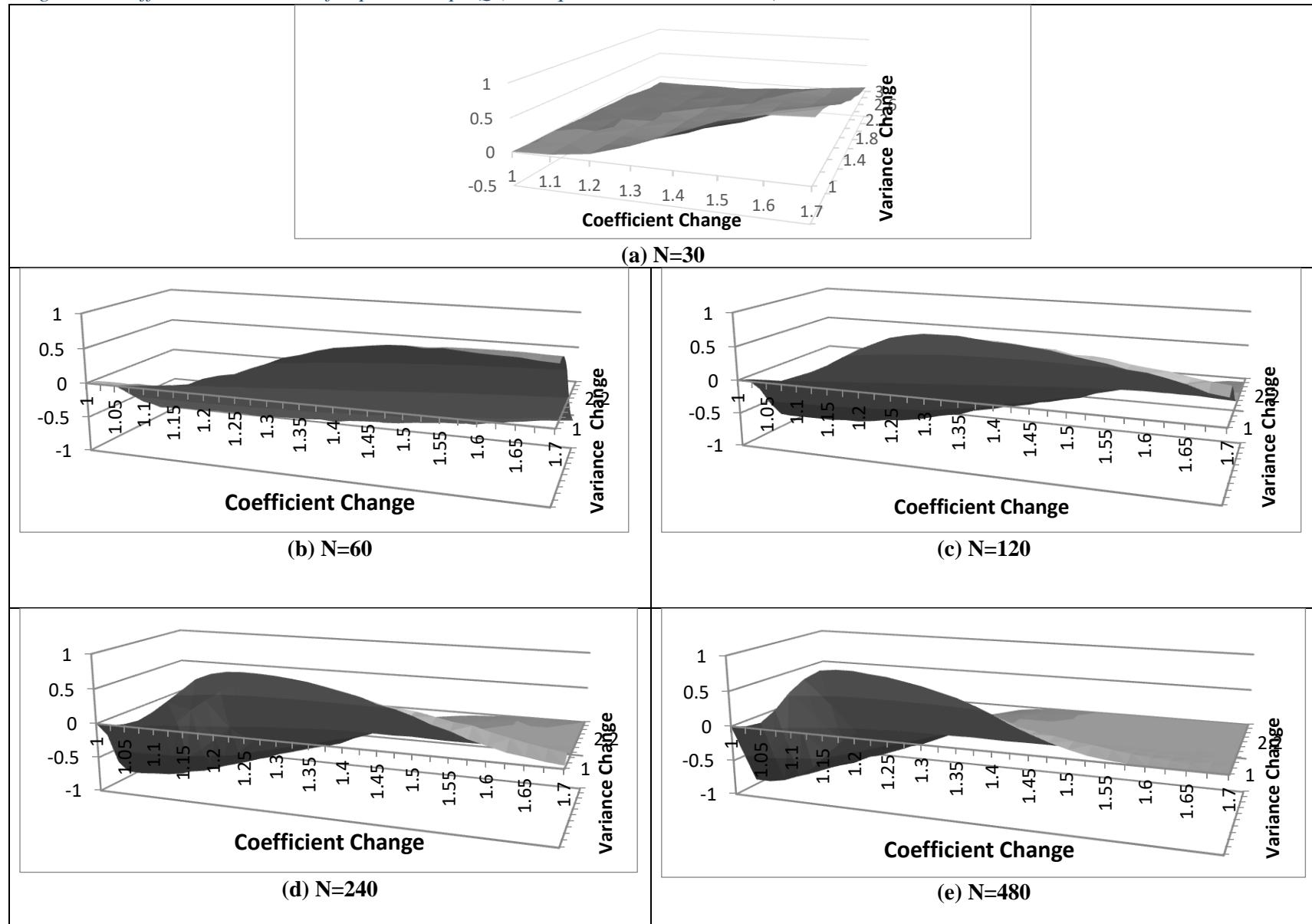
6.4 CHI-SQUARE DISTRIBUTION CASE

In a chi-square distribution with a sample size of 30, When the coefficient remains constant at one and the variance goes from one to three, the power difference between supF and supGQ ranges from 0.0085 to -0.1225. When the variance remains constant and the coefficient changes, the power difference of supF and supGQ varies from 0.0085 to 0.8795 in coefficient variance space. We can see that the power of supF for the chisquare distribution is greater than the power of the conventional normal distribution. When the variance remains constant and the coefficient changes, the variance coefficient path reveals that supF is stronger than supGQ, as the difference between supF and supGQ produces more positive values.

For a sample size of 60 in the chi-square distribution, the power of the supF test became elastic due to a negligible change in coefficients. It began at 0.04 and reaches 0.85 when the coefficient changes from 1 to 1.70, and the GQ test performs as a unified Normal distribution. Also, the power improves from 0.05 to 0.97 when the variance varies from 1 to 3. In coefficient variance space, as the coefficient changes, the difference between the supF and supGQ tests grows from 0.01 to 0.86 when the variance changes from 1 to 1.5, and then progressively diminishes. When sample size grows from 60 to 120, the variance coefficient changes for the supF and supGQ tests performs better. The supF test reaches its highest power when coefficients change reaches 1.37, while the supGQ test reaches its maximum power when variance change hits 1.4. In variance coefficients difference space, the supGQ test has a maximum difference power of 0.89 at a coefficient change of 1.32, and the difference declines to 0.37 as the coefficients change to 1.70. At a variance variation of 2.2%, GQ attains its maximum difference power of 0.93. When variance varies from to 3, the difference power of supGQ rises steadily from 0.01 to 0.93. The supF power achieves its highest when coefficient change is 1 to 1.17 and

supGQ test reaches its maximum power when variance change is 1 to 1.4. The supGQ test reaches 0.9 when the variance approaches 1.4. The differential power of supGQ has an ascending trend, whereas supF demonstrates a periodic one.

Figure 6.8 Difference in Powers of supF and supGQ (Chi square Distribution Case)



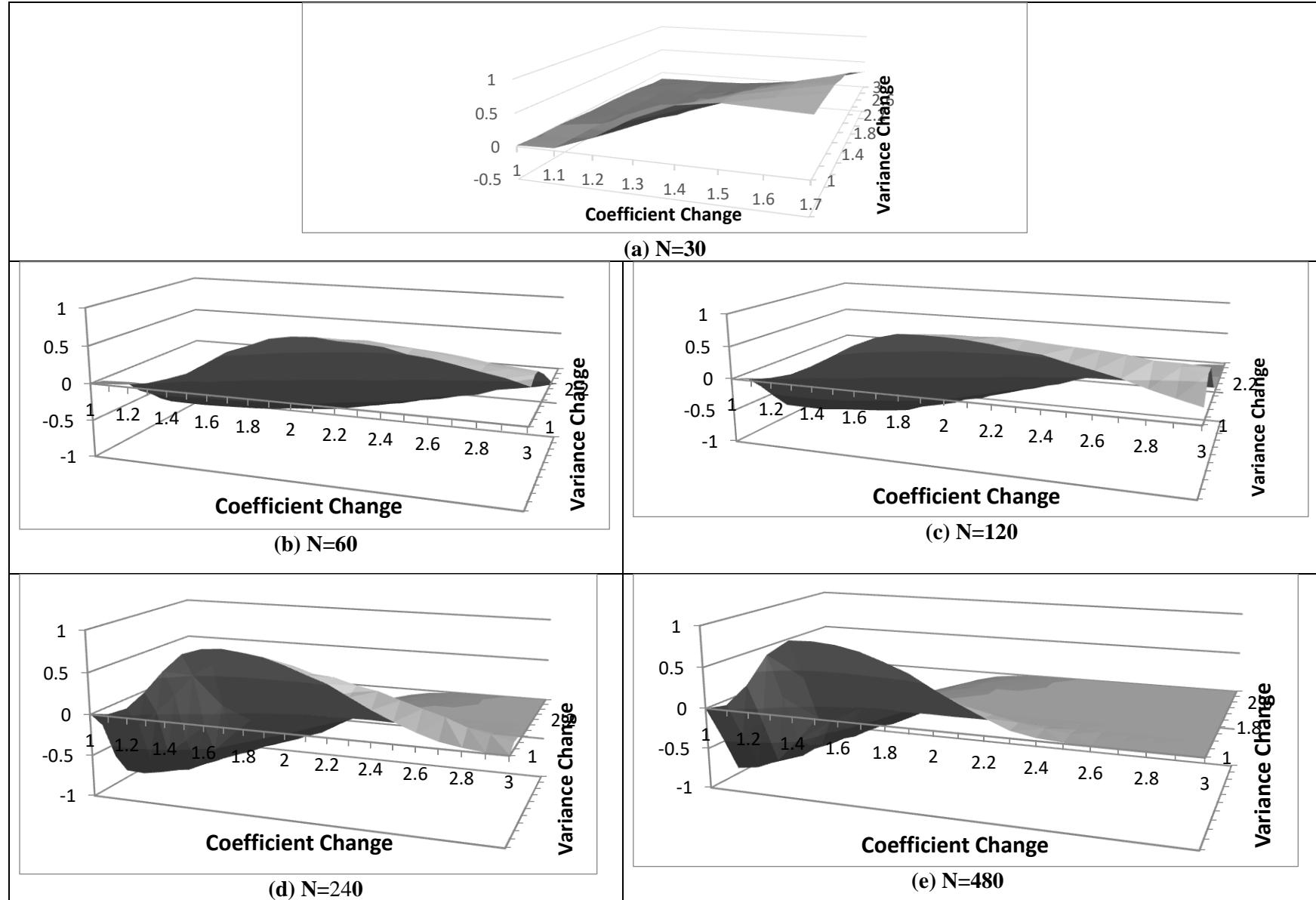
6.5 CAUCHY DISTRIBUTION CASE

In Cauchy distribution, supF overtakes supGQ for sample size 30 because supF's power is very responsive to coefficient change. We can see that when the coefficients vary and the variance remains constant at 1, the difference power ranges from 0.0195 to 0.8565. When the coefficient remains constant, the power difference ranges from 0.095 to -0.137. In a chauchy distribution with a sample size of 30, the supF test overtakes the supGQ test.

When the sample size is 60, the supF test overtakes the normal and chi-square distributions and reaches its maximum power when the coefficient changes from 1 to 2.2 are greater than one. The power of the supGQ test reaches 0.9 when the variance increases from 1 to 3. In coefficient variance space, supF has a periodic trend that begins at 0.00, peaks at 0.83 when the coefficient value is 2.10, and then declines to 0.45 when the coefficient value is 3. When sample size increases from 1 to 3, the supGQ shows a rising trend in the coefficient variance space, which reaches 0.90 from 0.00. When sample size is increased to 120, the supF test reaches its highest value when the coefficient value is 2 and the supGQ test reaches its maximum value when the variance value is 2.2. In comparison to smaller samples, the discrepancy between the two tests grows as sample size increases. supF difference exhibits periodic behavior and reaches a high of 0.8 when coefficient is equal to and declines to 0.2 when coefficient is equal to 3. The supGQ shows an increasing trend in difference as variance increases; the difference reaches 0.92 from 0.00 with an increasing trend. In a sample size of 240, the maximal power of supF and supGQ is 1 when the percentage change is 1.7% and the variance change is 1.8%, respectively. In difference of both supF and supGQ tests, supF responds periodically to the change in coefficient and reaches a maximum difference of 0.89 when the coefficient value is 1.70 and then decreases until it reaches a minimum difference of 0.00 when the coefficient value is 3. The

maximum performance of the supGQ test in the coefficient variance space is 0.95 when the variance change is 2 and growing. When the sample size is 480, both tests reach their maximum power elastic to coefficient and variance change; the supF test reaches its maximum power 1 when the coefficient changes from 1 to 1.5, and the supGQ test reaches its maximum power 1 when the variance changes from 1.6 to 1.6. Coefficient variance changes space supF reaches a maximum difference of 0.9 when coefficient is 1.5, whereas the supGQ test reaches a maximum difference of 0.94 when variance is 1.6. In the difference between the two tests, supF follows a periodic trend while supGQ follows an increasing trend.

Figure 6.9 Difference in Powers of supF and supGQ (Cauchy Distribution Case)



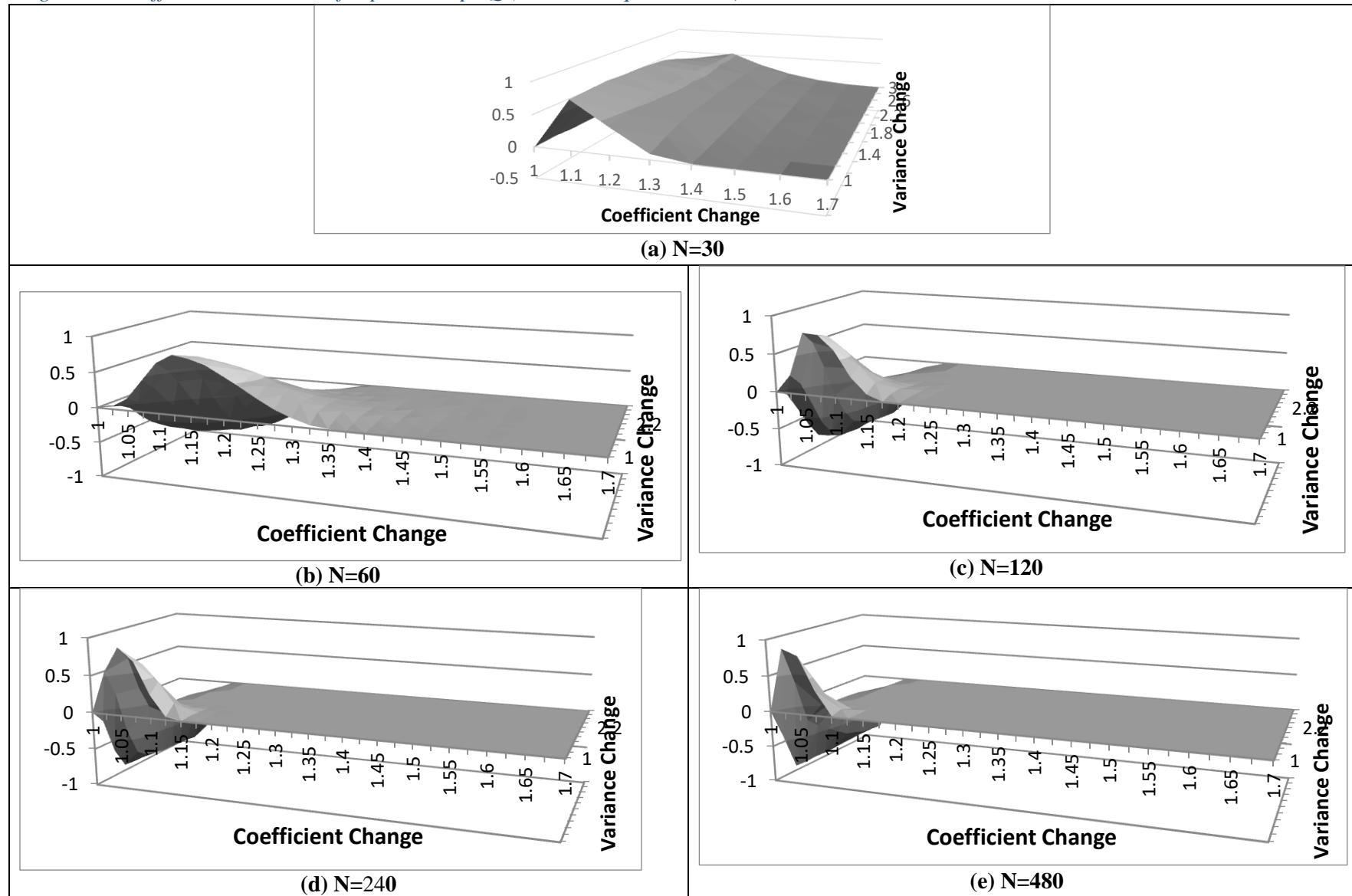
6.6 ADDITIVE SEQUENCE CASE

When the coefficient and variance remain constant, the power difference ranges from -0.011 to -0.15. When the coefficient and variance change, the power difference ranges from -0.112 to 0. This clearly shows that supF overtakes supGQ in additive sequence for sample size 30, but supGQ performs better when both the coefficient and variance change. More positive values are observed in the coefficient variance space, indicating that supF overtakes supGQ in the given variance coefficient change.

The supF test reaches its highest power with a coefficient change of 1.12, and the supGQ test reaches 0.98 power for the variance shift from 1 to 3. These results are based on a sample size of 60. The supF test is more sensitive to changes in the coefficient than any of the distributions that have been mentioned before. In the comparison between the two tests, the supF test reaches its maximum difference power of 0.82 at 1.12, whereas the supGQ test hits 0.88 at a variance change of 3. Both of the tests reached their maximum power in this distribution far more quickly than they did in any of the distributions that were discussed earlier for a sample size of sixty. When using a sample size of 120, the supF test reaches its maximum power at a variance change of 0.07, while the supGQ test reaches 1 when the variance change is 2.2. Both tests perform well in the additive sequence distribution in comparison to the other distributions that have been reviewed, and they reach their maximum power with only a little adjustment to the coefficient variance. In the coefficient variance space, the difference between the two tests reveals a periodic trend in the supF test and an increasing trend in the supGQ test. However, the periodic trend of the supF test is a relatively small span of 1.17, and it reaches its maximum difference of 0.79 at the coefficient change of 1.07. At a variance change of 2.6, the supGQ test shows a maximum power difference of 0.9. It is important to point out that the proposed method

follows the same trend for increases in sample size of 240 and 480, just like it did for sample sizes of 60 and 120, which were previously described.

Figure 6.10 Difference in Powers of supF and supGQ (Additive Sequence Case)



For sample size of 60 (N=60) standard normal distribution the power difference of F test and GQ test shows upper diagonal where coefficient change exists F test perform better, and GQ test detects heteroskedasticity better for variance change. And supremum version of F test and GQ test performs better in heteroskedasticity detection and get the maximum power more efficiently than original F test and GQ test. The proposed supGQ test detects heteroskedasticity more efficiently in case of coefficient variance change as shown in figure 6.1. More and strong power values are in result for supGQ when supF and supGQ difference is considered. For Chi-Square distribution F test power is higher than GQ test as shown in figure 6.2. The supGQ test perform better than GQ test and in variance coefficient change path supGQ test get some momentum of detecting heteroskedasticity but still supF test perform better, we may conclude that for Chi-square distribution supF test is better than supGQ test for the detection of heteroskedasticity. For Cauchy distribution F test perform better in coefficient variance path as shown in figure 6.3 but in supremum version comparison supGQ performs better we may conclude that for Cauchy distribution supGQ test is better than supF test for the detection of heteroskedasticity. In additive sequence F test and supF test perform better than GQ and supGQ test.

For standard normal distribution of sample size 120 (N=120) supGQ performs better than supF in coefficient variance path while F test and GQ test perform alike equal difference as shown in figure 6.1. For Chi-square distribution F test significantly performs better than GQ test. The supF test gains some power advantage over GQ test but still supF Test significantly better than supGQ test for the detection heteroskedasticity in variance coefficient change. For Cauchy distribution the same pattern is followed by supF test and supGQ test as in Chi-square distribution. As in additive sequence distribution both supF test and supGQ test detect heteroskedasticity than all

above discuss distribution. The supF test performs better than supGQ test in variance coefficient comparison path.

For standard normal distribution of sample size 240 (N=240) GQ test and F test performs in line as for small sample size supGQ test perform better in variance coefficient comparison path. For Chi-square distribution supF test takes a lead in variance coefficient comparison path over supGQ test. For Cauchy distribution supGQ test takes lead over supF test in occupation of variance coefficient comparison path and shows higher power values. While F test and GQ test difference exhibits the same pattern. In additive sequence supF test takes lead over supGQ in variance coefficient comparison path.

For standard normal distribution of sample size 480 (N=480) supGQ test and supF test gets higher power value for the detection of heteroskedasticity as sample size increases degree of heteroskedasticity increases but supGQ test gets higher power values over supF test. For Chi-square distribution supF test performs better than supGQ test and gains higher power values in variance coefficient combination path. The supF test reaches maximum power point superior to supGQ test as shown in figure 6.2. For Cauchy distribution supF test performs better than supGQ test as their difference figure 6.3 shows the clear scenario in variance coefficient comparison path. In additive sequence distribution supF test and supGQ test both perform fluoresently as degree of heteroskedasticity is higher for large sample size. While supF test takes a lead over supGQ test in difference as shown in figure 6.4 for variance coefficient combination path.

Table 0.1 to 0.60 in appendix show the power of supGQ, supF and supMZ for various regressors and for varying sample sizes for interested readers. Tables having a sample size 60 (N=60) present the results of the power of the supF test tends to grow, going from 0.0635 to 0.9545 as the coefficient is increased. However, at point variance is one, the power of the supGQ

is 0.984, which means that as the variance increases, the supGQ dramatically reaches the maximum power. When the variance and the coefficient are both altered, the performance of the supMZ tests is significantly superior to that of the supF and supGQ tests. A continuous distribution that has k degrees of freedom is referred to as a chi distribution. The distribution of a sum of squared random variables can be described with the help of this term. The coefficient is changing vertically ranges from 1 to 3 with the gap of 0.025 and the variance is changing horizontally with the gap of 0.2 ranges from 1 to 3. The proposed procedure's versatility can be observed in Chi as that power analysis of the supF test. When the variance and the coefficient are both altered, the performance of the supMZ tests is significantly superior to that of the supF and supGQ tests. The Cauchy distribution as well as the sequence distribution both exhibit the same phenomena.

The power of the supF test shown in the tables has sample size of 120($N=120$). Without a shred of doubt, these results show that the supF test overtakes. Both the coefficient and the variance vary between 1 and 3 with a 0.2-point jump in the vertical and the equivalent jump in the horizontal directions, respectively. supF test initial value is 0.0595 and reaches to its maximum when coefficient increase to 2.4 in standard normal distribution Yet, the supGQ's power is 0.0535 at this one sample variance level, indicating that the supGQ dramatically hits its maximum power as variance increases to 2.6. Performance of the supMZ tests is much better than that of the supF and supGQ tests when both the variance and the coefficient are varied. The chi-square distribution describes a continuous distribution with k degrees of freedom. This concept is useful in describing the distribution of a set of random variables that is the sum of squares. The coefficient varies from 1 to 3 with a 0.025-unit step, while the variance varies from 0.2-units to 1-unit steps on the horizontal axis. Power analyses of the supF test as the coefficient

is raised, the supF test's predictive power rises from 0.054 to 1 when coefficient is 1.525. With a variance of 1, the supGQ test has a power of 0.0485, whereas with a variance of 2.6, the power rises considerably to maximum for supGQ. Performance of supMZ tests is much better than that of the supF and supGQ tests when both the variance and the coefficient are varied. Similar patterns are seen in both the Cauchy and the sequence distributions.

When sample size increases to 240 (N=240) degree of heteroskedasticity increases than the 120-sample size as more variance and coefficient changes, supF test performs comparatively better in this condition and reaches to its maximum power point at 2 coefficient change. Our proposed supGQ test detects the heteroskedasticity more efficiently than small sample of 120 and gain maximum power point at point 2 variance change. The supMZ test performs better than supF and supGQ as by structure advantages and gain maximum point at 1.2 variance change. For Chi-distribution, the supF test perform effective than standard normal distribution and gain maximum power point 1.3 coefficient changes. The supGQ test performs equally and gain maximum power at point 2 variance change. The supMZ test considers both variance and coefficient changes and get maximum power point more efficiently than supGQ and supF tests. Proposed test of supGQ detects heteroskedasticity at point 2 variance change and supF test perform least than Chi-distribution in Cauchy distribution and gain maximum point at 1.8 coefficient change so we may conclude in this scenario supF test perform better in Chi-distribution and Standard normal distribution than in Cauchy distribution. In additive sequence distribution supF test high power than all discussed distribution and has maximum power point at point 1.075 coefficient change supGQ test incorporate variance change maximum power point at 2 but interestingly also effect by coefficient change and reaches to maximum point at 1.175.

For the sample size 480 ($N=480$) tables we compare the results of standard normal distribution for supGQ and supF performs significantly better for its detection. For Chi-square distribution degree of heterogeneity is higher than standard normal distribution by structure so supF and supGQ tests performs fluorescent then in standard normal distribution and reaches to its maximum power at coefficient change 1.225 and variance change 1.6. For Cauchy- distribution the supGQ gains its maximum value at variance change of 1.6 and it is equal to Chi-Square distribution power maximum point while supF test performs while slower than Chi-Square distribution and reaches to maximum point at coefficient change of 1.6. In additive sequence distribution supF test performs better than Standard normal, Chi-Square and Cauchy distribution and reaches to maximum power point at coefficient change of 1.05. The supGQ test reaches its maximum power point at 1.6 variance changes equal to maximum power point variance change of all above discussed distributions.

Overall, supGQ test overtakes the supF test as the variance in the data distribution shifts to the increasing side. In comparison to supGQ tests, the supF test is more efficient at capturing coefficient change. The supMZ test performs noticeably better than the supF and supGQ tests when both the variance and the coefficient are changed.

CHAPTER 7

REAL WORLD EXAMPLE AND CONCLUSION

This chapter provides real-world analysis to highlighting the superiority of the proposed test over the conventional GQ and MGQ approach to detect heteroskedasticity when there are outliers in the design matrix. The focus of the analysis is to examine the effect of human capital on monthly income and investigate whether proposed tests of heteroskedasticity.

7.1 A Real-World Example

Human capital includes health and education as well as learned, inherited, and accumulated talents that help people advance economically. Two routes of human capital accumulation that support economic growth have been identified in the literature. According to Lucas (1990), the accumulation of human capital can actively contribute to the production process by offering his services as a factor input. The level effect, which refers to the major factor influencing how quickly human capital accumulates in various countries, determines disparities in growth rates. However, according to Romer (1990), the rate effect, or the production of new knowledge and technology as a result of the accumulation of human capital, results in growth. These two effects are investigated empirically. Although not conclusive, rate effect is significant.

The importance of human capital accumulation is becoming more of an empirical question. In the growth literature, specific stand-ins for human capital have been employed. The primary and secondary enrollment rates are the proxies that are most frequently employed. Enrollment numbers do not account for dropout rates or academic quality, only a portion of the human capital investment. International student test scores, a stock variable adjusted for educational quality, have a considerable and favorable impact on the growth rate of the GDP per

capita. Literacy rate is defined as the percentage of people who are at least 15 years old and can read and write, which is insufficient as a proxy for human capital. Barro and Lee (1993) constructed a model using census and survey data particular to the country for the population aged 25 and over.

A faster rate of economic expansion is confirmed by a bigger stock of human capital (Romer, 1990). Human capital development boosts productivity and aids in technological adoption, which supports the pace of economic expansion (Nelson and Phelp, 1966; Barro and Lee, 2000). A country's ability to sustain its technological advantage depends on maintaining its educational level; otherwise, it can easily fall behind a country with a high level of education, regardless of its beginning level of technology (Benhabib and Spiegel, 1994). A higher human capital to physical capital ratio accelerates growth since it leads to the accumulation of both human and physical capital. Education evaluates human capital accumulation and makes people equally productive for all types of occupations. Two characteristics that set education apart from physical capital are as follows:

Knowledge is spread through education. However, study, development, and learning via experience all contribute to the transmission of knowledge. Firms internalize learning through experience, and research effort advances technology and fosters expansion. A change in the productivity parameter reallocates resources from experiential learning to research activities in the innovation process (Romer, 1990). In the research sector, where current research spillover influences the creation of future research, human capital is a crucial input. Accumulation of human capital is an asset, and the GDP pays for its creation. By allocating more resources, higher GDP helps to increase both the quality and quantity of human capital accumulation.

There is a wealth of information on the evaluation of students' earnings at various educational levels. Numerous researches used data from Pakistan to estimate the earnings functions. These studies used dummy variables to account for various levels of schooling and found low rates of return at each level compared to other emerging nations. Additionally, it has been noted that there is an inverse relationship between educational attainment and the degree of income inequality, but a positive relationship between earnings and educational levels (Haque, 1977; Hamdani, 1977; Guisinger et al., 1984; Khan and Irfan, 1985; Ahmad et al., 1991; Ashraf and Ashraf, 1993a, 1993b and 1996). Additionally, Nasir and Nazli (2000) examined how education, technical training, educational quality, and literacy affected wage earnings.

There have been numerous studies done in the past regarding secondary school and college graduates in the US. The theory of human capital, according to Becker (1964), regarded schooling participation as an investment in human capital due to the anticipated return in later life. Therefore, when the level of educational attainment rises, it can be claimed that more productive, more skilled, and people with greater knowledge are available in society. As a result, the degree of education had a significant impact on social parameters like fertility, mortality, and children's schooling as well as the distribution of income and life expectancy at birth. According to Schultz (1961), even the lesser returns on education were nearly equal to the higher returns on non-human capital.

Cross-sectional data were used by Tinbergen (1971) to explore the relationship between education and income distribution in the states of the United States, the provinces of Canada, and the Netherlands. He discovered that the degree of inequality in the United States, Canada, and the Netherlands would only be little reduced by an increase and smaller dispersion in educational years. Barro (2000) tried to ascertain how the growth rates were impacted by the physical capital

stock and the human capital stock. He examined data from 98 nations between 1965 and 1995. The study came to the conclusion that while human capital had a favorable impact on economic growth rate, the physical stock of capital had a negative relationship with growth rates.

The effects of gender imbalance in education on income, growth, and development were examined by Moheyuddin (2005). According to the study's findings, women's human capital should not receive insufficient funding. Second, it was bad for economic growth when there was gender inequality in schooling. Thirdly, a rise in gender equality was positively correlated with per capita income. Silles (2006) looked at the significance of social maladjustment for earning potential and educational attainment. In 2000, he collected data on 972 people. The study came to the conclusion that social competency in early life was associated with educational success and wage potential.

Using time series analysis, Kose and Guven (2007) investigated the effects of government spending on education, the secondary enrolment rate, and inflation on income disparity across 67 Turkish provinces. Between 1975 and 2001, they used an annual data set. As a dependent variable, they used the Gini index to measure income disparity. They came to the conclusion that Ministry of Education spending on education lowers income inequality in per capita GNP among Turkish provinces. Inequality grew as secondary enrollment rates rose. Additionally, inflation widened the income gap and made the income distribution worse.

The effect of higher education on women's incomes was demonstrated by Shah (2007). Female teachers employed in public sector educational institutions provided information on their years of experience, degree of education, and monthly pay. This analysis made use of the Mincerian model. The study found that education had a favorable effect on teachers' pay, and that their monthly salaries rose with each level of education attained.

In order to determine the effects of various educational levels on female labor force participation in Pakistan, Faridi et al. (2009) performed a study of the female labor force in the district of Bahawalpur. In their analysis, they took into account all levels of education except for basic level education up to middle level and found that all levels of education except for basic level education up to middle level had a positive impact on the female labor. They also included Matric level, Intermediate level education, Graduation level education, Masters, MPhil and PhD level education, Mother level education, Father level education, Spouse level education, Age in completed years, Marital Status, presence of assets, family setup, region of residence and household size. The employment of wives was positively impacted by the educated spouse (husband).

Soto (2009) investigated the causal relationship between education and overall income. The Macro Mincer return on education served as the model's foundation. He has employed panel regression for 83 nations to empirically analyze the relationship between educational attainment and GDP. He had come to the conclusion that one significant factor influencing wealth differences across nations was the caliber of education. The effect of education and job training on the labor income for migrant workers in China was studied by Messinis and Cheng (2009). They adopted a Mincerian strategy. In Hangzhou, Zhejiang province, they conducted a field study to gather information from 400 migrant workers. The study came to the conclusion that a significant factor in determining labor income is education and job experience. Second, incomes increase by 12.1% with lower middle school and higher education.

The current study uses data from the Pakistan Social and Living Standards Measurement (PSLM) survey for the final year 2019-20 (i.e., 2020), published by the Federal Bureau of Statistics, to evaluate the impact of education and experience on earnings. This poll included

176,790 households. Descriptive statistics of the date are given in table 7.2. This survey provides information at the micro level on crucial variables such as income, age, area, province, gender, education, nature of work, and type of school. Our sample includes 187395 wage earners with positive earnings who are 18 or older.

The proposed tests of heteroskedasticity are specific to the detection of heteroskedasticity in cross sectional data of simple regression model. As a way forward it can be extended to multiple regression analysis. Therefore, to match with the setting of our proposed tests we limit the regressor of Mincerian wage function to income only and do not take into account the demographic characteristics of the individuals.

Our empirical technique for estimating an earnings function is based on Becker's (1964) and Mincer's (1974) human capital model. Two variables are taken for the analysis, the logarithm of monthly earnings ($\ln E_i$) is a linear function of year of schooling (S_i) proxy of human capital, it is stated as;

$$\ln E_i = \alpha_0 + \alpha_1 S_i + u_i \quad \dots \dots 7.1$$

Table 7.1 Real world Example Variables Explanation

Dependent Variable			
$\ln E_i$	Logarithm of monthly earning	Only cash earnings from the first and second jobs are taken into account and the money earned from the second job is first converted into monthly earnings from annual earnings.	PKR
Independent Variables			
S_i	Year of schooling	primary education contains five to seven years of schooling, middle education contains eight to nine years of schooling, matriculation (10), Diploma (11), intermediate (12-13), bachelor (14-15) and Professional (16-24). Illiterate earners are considered as base category.	Continuous

Table 7.2 Descriptive Statistics

	Mean (S.D)	Proportion
Monthly Earning	24831.58 (10214.67)	
Year of schooling	Illiterate	21.79
	Primary	16.96
	Middle	14.61
	Matric	17.91
	Polytechnic Diploma	3.32
	Intermediate	9.70
	Bachelor	8.30
	Professionals	7.39

Table 7.3 Regression Result Comparison of Linear and Robust Regression

Linear Regression Model				
Coefficients	Estimate	Std. Error	t value	Pro.
Intercept	9.47658 ***	0.15238	62.192	0.000
Schooling	0.06877***	0.01502	4.579	0.000
Robust Regression Model				
Coefficients	Estimate	Std. Error	t value	Pro.
Intercept	-44.951***	6.388	-7.037	0.000
Schooling	5.310***	0.633	8.388	0.000

In table 7.3, the comparison of coefficients and standard errors between the linear regression and the robust linear regression, in linear regression intercept is 9.477 this is the estimated value of the dependent Income when the Schooling is zero. Coefficient of Schooling is 0.068 represents the estimated change in Income for a one-unit change in Schooling. For intercept in robust regression is -44.951, this is the estimated value of Income when Schooling is zero and coefficient of schooling is 5.310, this is the estimated change in Income for a one-unit change in Schooling, but it's obtained using a robust method that is less sensitive to outliers as compare to linear regression.

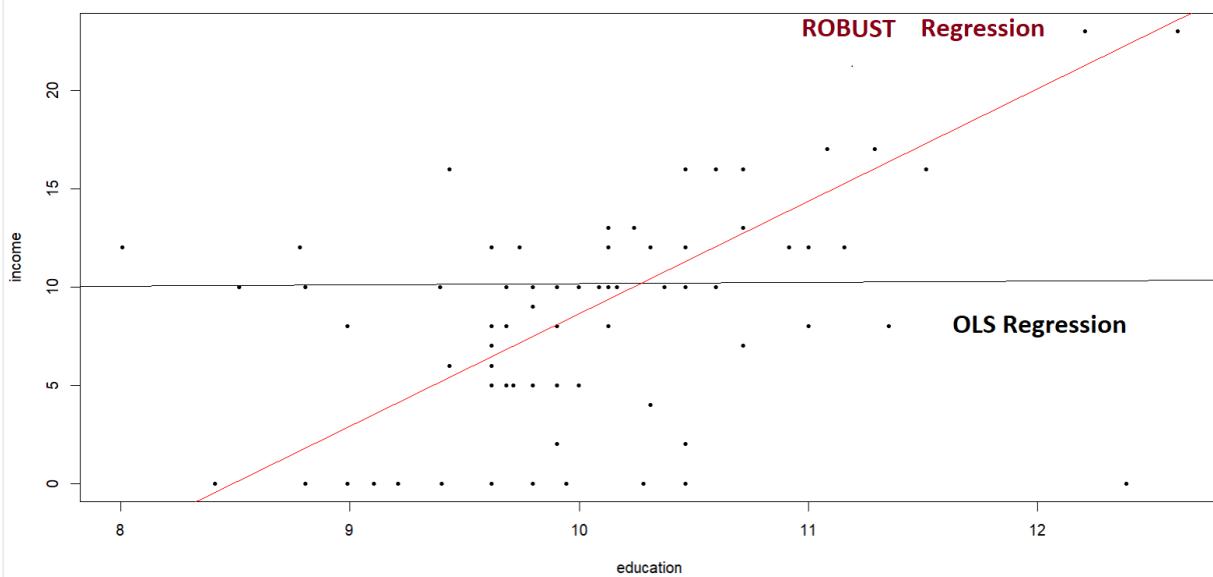
The linear model in table 7.3, standard errors are calculated based on the assumption of homoskedasticity (constant variance of errors). If there's heteroskedasticity, these standard errors

might be underestimated, leading to incorrect inference. Std. Error of Intercept is 0.152, represents the standard error (uncertainty) associated with the estimate of the Intercept. Intercept Std. Error is 6.388 and it represents the standard error associated with the estimate of the Intercept in the robust model. In the robust model, standard errors are calculated using a robust method. The larger standard error for the Intercept in robust regression is 6.388 compared to linear regression is 0.152 suggests that the robust model is less sensitive to potential outliers or heteroskedasticity in the data.

Schooling Std. Error is the 0.015 and represents the standard error associated with the estimate of the Schooling coefficient. Schooling Std. Error is 0.633 and it represents the standard error associated with the estimate of the Schooling coefficient in the robust model. The standard error for the Schooling coefficient in robust regression is 0.633 is also larger than in linear regression is 0.015, indicating that the robust model acknowledges and accounts for potential variations and outliers, providing more reliable standard error estimates.

If there are concerns about heteroskedasticity or the presence of outliers in the data, the robust regression model is more suitable. It provides coefficient estimates and standard errors that are less influenced by extreme values and are more robust in the face of violations of the homoskedasticity assumption. Larger standard errors in robust regression reflect the model's acknowledgment of potential data irregularities, making it a better choice when dealing with datasets exhibiting heteroskedasticity.

Figure 7.1 Linear and Robust Regression



A regression model with robust features can withstand the impact of outliers in a dataset as shown in figure 7.1. It is especially helpful when a regression model has a single outlier point that could otherwise bias the outcomes. This outlier can be recognized by the robust regression model, which can then discard while still capturing the general trend in the other data. They can thus deliver accurate answers that are unaffected by the outlier points, making them the perfect tool for studying datasets with outliers. In spite of the outlier point's influence, the robust regression model is successful in detecting the trend in the remaining data and resisting its influence. To assess the performance of proposed tests, apply 25000 Monte-carlo simulations are performed. The p-value of all tests considered is provided in Table 7.4.

Table 7.4 Comparison of Probability Values

Test Name	Test Statistic calculated value	p-value
GQ	1.545	0.181
supGQ	1.562	0.094
MGQ	2.541	0.182
supMGQ	2.792	0.074

Note: p-value is calculated via bootstrapping with 25,000 replications.

It can be seen from the results of Table 7.2. that the GQ test and MGQ test fails to detect heteroskedasticity even at 10% significance level. The newly proposed tests—the supGQ and the supMGQ detect heteroskedasticity as p-value is significant.

7.2 CONCLUSION

When the variance of the dependent variable is not constant across all observations, the ordinary least squares (OLS) method used in regression analysis can produce inaccurate coefficient estimates due to heteroskedasticity. To test for homoscedasticity in a linear model when the break (split) point is known, the Goldfeld-Quandt (GQ) and the modified GQ (MGQ) tests along with several other tests of heteroskedasticity are generally used. The usual Goldfeld-Quandt technique for dealing with heteroskedasticity does not have a long enough convergence time span to reach a conclusion.

The present study proposes two new tests (the supGQ and the supMGQ) which have the ability to test for an unknown break in variance. To assess the validity of proposed tests, regressors are chosen keeping in view all types of distributions into account (symmetric, asymmetric, equally spaced and the ones containing one or more outliers). Specifically, four distributions are considered: standard normal (symmetric), chi-square (skewed), Cauchy (contains one or more outliers), and Additive sequence distributions (equally spaced). The performance of test is compared via Monte-Carlo simulations for different sample sizes and over varying degree of heteroskedasticity. Our findings show support to the superiority of proposed tests over the existing tests. The research is conducted using limited sample sizes; however, the proposed tests are superior to conventional tests by structure regardless of sample size. Specifically, for large sample data, the proposed test outperforms conventional tests in detecting

the breakpoint. Therefore, we can conclude that the proposed tests are the best choice for breakpoint detection across various sample sizes, especially excelling with large datasets.

The power of the suggested tests increases as the sample size is increased, and it was greater than the power of the base tests (the rivals) throughout the sample size increment. When the break (split) point in variance is unknown, the supMGQ overtakes modified versions of GQ tests proposed by (Rana et al., 2008). The Standard normal and additive sequence distribution data sets, as well as Monte Carlo simulations, show that the supMGQ test overtakes the existing modified GoldfeldQuandt test in detecting heteroskedasticity in the presence of outliers.

The supF, supGQ, and sup MZ are compared using four distributions. The supGQ test becomes more precise as the variance increases, and the supF test becomes applicable for varying regression coefficients. When both the variance and the coefficient are varied, the supMZ tests perform much better than the supF and supGQ tests. The difference between the supF and supGQ tests is also taken into account to check the strength areas of both tests; as the variance varies, the supGQ test becomes more precise, while the supF test is applicable for varying regression coefficients.

Table 7 Performance Summary Table

	Cases	Distribution	Test			
			GQ	MGQ	supGQ	supMGQ
1.	No-Outlier	Normal	✓		✓	
	Outlier	Normal		✓		✓
2.	No-Outlier			GQ	supGQ	
		Normal			✓	
		Chi-Square			✓	
		Cauchy			✓	
		Additive Seq.			✓	
				supGQ	supF	supMZ
		Variance Change	Normal	✓		✓
		Coefficient Change	Normal		✓	✓

3.	Variance Change	Chi-Square	✓		✓
	Coefficient Change	Chi-Square		✓	✓
	Variance Change	Cauchy	✓		✓
	Coefficient Change	Cauchy		✓	✓
	Variance Change	Additive Seq.	✓		✓
	Coefficient Change	Additive Seq.		✓	✓

The strength of the proposed supGQ and supMGQ tests rises with increasing sample size, and during the entire sample size increment, it surpassed the power of the discussed base tests. When identifying heteroskedasticity, the supGQ test performs better than the Goldfeld-Quandt test and the supMGQ test surpasses the current modified Goldfeld-Quandt test in detecting heteroskedasticity in the presence of outliers.

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Appendix

Table 0.1 Power of supF (Standard Normal Distribution N=30)

Coefficient Change	powersupF	Variance Change										
		1	1.2	1.4	1.6	1.8	2	2.2	2.4	2.6	2.8	3
	1	0.045	0.053	0.049	0.064	0.083	0.076	0.084	0.083	0.102	0.094	0.082
	1.025	0.053	0.046	0.059	0.067	0.067	0.083	0.076	0.095	0.088	0.080	0.087
	1.05	0.053	0.058	0.064	0.066	0.076	0.086	0.079	0.092	0.090	0.083	0.091
	1.075	0.064	0.051	0.064	0.055	0.066	0.080	0.085	0.081	0.079	0.081	0.088
	1.1	0.057	0.051	0.061	0.071	0.068	0.086	0.081	0.092	0.090	0.094	0.078
	1.125	0.056	0.050	0.065	0.068	0.069	0.073	0.068	0.087	0.089	0.098	0.094
	1.15	0.063	0.060	0.059	0.064	0.062	0.075	0.081	0.084	0.079	0.083	0.091
	1.175	0.052	0.055	0.061	0.067	0.073	0.078	0.081	0.078	0.081	0.091	0.082
	1.2	0.066	0.063	0.074	0.070	0.066	0.067	0.080	0.072	0.086	0.088	0.096
	1.225	0.064	0.067	0.060	0.070	0.070	0.076	0.092	0.074	0.075	0.075	0.101
	1.25	0.072	0.058	0.064	0.070	0.078	0.070	0.091	0.091	0.085	0.083	0.089
	1.275	0.068	0.071	0.071	0.075	0.071	0.076	0.084	0.084	0.086	0.095	0.082
	1.3	0.069	0.078	0.069	0.085	0.082	0.080	0.086	0.080	0.088	0.082	0.085
	1.325	0.081	0.076	0.066	0.068	0.083	0.085	0.076	0.081	0.086	0.074	0.085
	1.35	0.072	0.079	0.071	0.077	0.085	0.084	0.083	0.088	0.092	0.089	0.089
	1.375	0.089	0.094	0.080	0.076	0.079	0.081	0.081	0.085	0.087	0.081	0.093
	1.4	0.085	0.084	0.085	0.084	0.077	0.082	0.077	0.095	0.079	0.093	0.083
	1.425	0.102	0.085	0.074	0.086	0.077	0.074	0.087	0.087	0.090	0.087	0.093
	1.45	0.097	0.083	0.087	0.090	0.081	0.073	0.087	0.084	0.082	0.095	0.093
	1.475	0.125	0.094	0.085	0.082	0.085	0.095	0.094	0.083	0.093	0.085	0.105
	1.5	0.114	0.092	0.097	0.079	0.090	0.081	0.090	0.095	0.087	0.076	0.094
	1.525	0.121	0.104	0.077	0.084	0.076	0.093	0.078	0.099	0.088	0.089	0.090
	1.55	0.125	0.107	0.099	0.079	0.095	0.099	0.090	0.091	0.092	0.104	0.093
	1.575	0.141	0.109	0.099	0.084	0.102	0.098	0.088	0.089	0.101	0.092	0.082

	1.6	0.150	0.114	0.102	0.088	0.094	0.100	0.093	0.086	0.099	0.094	0.090
	1.625	0.159	0.134	0.105	0.114	0.096	0.096	0.100	0.100	0.095	0.097	0.081
	1.65	0.169	0.118	0.105	0.102	0.102	0.107	0.092	0.093	0.090	0.097	0.103
	1.675	0.165	0.135	0.119	0.110	0.105	0.095	0.091	0.097	0.103	0.091	0.104
	1.7	0.185	0.143	0.119	0.124	0.104	0.101	0.098	0.093	0.091	0.096	0.097

Table 0.2 Power of supGQ (Standard Normal Distribution N=30)

Coefficient Change	Test	Variance Change										
		1	1.2	1.4	1.6	1.8	2	2.2	2.4	2.6	2.8	3
	powersupGQ	0.051	0.052	0.056	0.079	0.085	0.109	0.127	0.145	0.147	0.188	0.203
	1	0.050	0.052	0.048	0.064	0.070	0.105	0.107	0.143	0.153	0.161	0.195
	1.025	0.049	0.055	0.062	0.077	0.087	0.099	0.121	0.131	0.150	0.179	0.202
	1.05	0.056	0.053	0.067	0.081	0.086	0.111	0.125	0.138	0.154	0.165	0.205
	1.1	0.048	0.051	0.063	0.072	0.081	0.108	0.119	0.135	0.159	0.186	0.213
	1.125	0.045	0.058	0.060	0.072	0.085	0.087	0.124	0.130	0.159	0.191	0.197
	1.15	0.053	0.049	0.067	0.080	0.087	0.091	0.129	0.148	0.154	0.174	0.194
	1.175	0.046	0.051	0.069	0.090	0.085	0.089	0.125	0.135	0.151	0.180	0.212
	1.2	0.049	0.056	0.059	0.077	0.096	0.108	0.116	0.135	0.154	0.181	0.216
	1.225	0.045	0.062	0.071	0.064	0.085	0.095	0.121	0.133	0.156	0.175	0.207
	1.25	0.052	0.053	0.057	0.079	0.084	0.109	0.123	0.148	0.164	0.169	0.190
	1.275	0.053	0.059	0.069	0.076	0.073	0.103	0.118	0.128	0.167	0.186	0.198
	1.3	0.050	0.054	0.063	0.069	0.081	0.108	0.116	0.137	0.158	0.183	0.202
	1.325	0.049	0.059	0.062	0.079	0.088	0.101	0.124	0.131	0.166	0.180	0.210
	1.35	0.057	0.049	0.061	0.082	0.082	0.095	0.127	0.134	0.157	0.183	0.209
	1.375	0.054	0.057	0.063	0.077	0.087	0.104	0.124	0.142	0.158	0.177	0.215
	1.4	0.049	0.052	0.069	0.076	0.090	0.094	0.127	0.145	0.173	0.173	0.214
	1.425	0.043	0.056	0.055	0.068	0.099	0.102	0.133	0.132	0.150	0.183	0.207
	1.45	0.052	0.051	0.055	0.082	0.087	0.084	0.129	0.149	0.165	0.197	0.214

	1.475	0.058	0.053	0.070	0.089	0.090	0.105	0.125	0.129	0.161	0.186	0.213
	1.5	0.051	0.051	0.072	0.078	0.091	0.113	0.126	0.133	0.151	0.162	0.197
	1.525	0.055	0.058	0.070	0.078	0.072	0.099	0.128	0.141	0.159	0.187	0.203
	1.55	0.050	0.056	0.063	0.078	0.087	0.096	0.131	0.129	0.157	0.202	0.207
	1.575	0.046	0.057	0.067	0.074	0.094	0.104	0.110	0.138	0.150	0.179	0.201
	1.6	0.049	0.057	0.060	0.069	0.087	0.105	0.117	0.127	0.166	0.170	0.209
	1.625	0.053	0.057	0.057	0.060	0.100	0.092	0.125	0.141	0.162	0.169	0.200
	1.65	0.055	0.056	0.063	0.065	0.090	0.107	0.112	0.135	0.157	0.186	0.220
	1.675	0.046	0.059	0.066	0.075	0.093	0.098	0.129	0.146	0.154	0.186	0.189
	1.7	0.055	0.057	0.072	0.076	0.101	0.099	0.115	0.119	0.149	0.180	0.211

Table 0.3 Power of supMZ (Standard Normal Distribution N=30)

Coefficient Change	Test	Variance Change										
		1	1.2	1.4	1.6	1.8	2	2.2	2.4	2.6	2.8	3
	powersupMZ	0.054	0.291	0.643	0.850	0.952	0.987	0.998	0.999	1.000	1.000	1.000
	1	0.047	0.313	0.644	0.862	0.949	0.985	0.997	0.999	1.000	1.000	1.000
	1.025	0.042	0.291	0.649	0.845	0.958	0.989	0.997	0.998	0.999	1.000	1.000
	1.05	0.050	0.303	0.632	0.846	0.952	0.986	0.998	1.000	0.999	1.000	1.000
	1.075	0.052	0.271	0.640	0.857	0.952	0.990	0.994	0.999	1.000	1.000	1.000
	1.1	0.047	0.286	0.639	0.865	0.951	0.991	0.995	0.997	1.000	1.000	1.000
	1.125	0.053	0.300	0.640	0.859	0.955	0.988	0.997	1.000	1.000	1.000	1.000
	1.15	0.055	0.294	0.621	0.868	0.959	0.982	0.996	0.997	1.000	1.000	1.000
	1.175	0.053	0.301	0.641	0.861	0.949	0.986	0.996	0.999	1.000	1.000	1.000
	1.2	0.059	0.296	0.634	0.861	0.959	0.986	0.995	0.999	1.000	1.000	1.000
	1.225	0.051	0.300	0.636	0.863	0.957	0.985	0.997	1.000	1.000	1.000	1.000
	1.25	0.063	0.312	0.650	0.858	0.957	0.989	0.996	0.999	1.000	1.000	1.000
	1.275	0.055	0.305	0.646	0.870	0.947	0.985	0.996	0.999	1.000	1.000	1.000
	1.3	0.063	0.305	0.646	0.870	0.947	0.985	0.996	0.999	1.000	1.000	1.000

	1.325	0.066	0.294	0.646	0.848	0.949	0.988	0.995	0.999	0.999	1.000	1.000
	1.35	0.061	0.314	0.665	0.865	0.965	0.984	0.997	1.000	1.000	1.000	1.000
	1.375	0.065	0.315	0.656	0.878	0.959	0.988	0.995	0.997	1.000	1.000	1.000
	1.4	0.066	0.331	0.647	0.874	0.964	0.986	0.997	0.998	1.000	1.000	1.000
	1.425	0.072	0.304	0.677	0.872	0.955	0.987	0.996	1.000	1.000	1.000	1.000
	1.45	0.077	0.323	0.670	0.873	0.962	0.983	0.996	0.998	1.000	1.000	1.000
	1.475	0.072	0.330	0.662	0.879	0.962	0.987	0.998	0.999	1.000	1.000	1.000
	1.5	0.077	0.332	0.667	0.867	0.965	0.993	0.996	1.000	1.000	1.000	1.000
	1.525	0.071	0.329	0.673	0.882	0.956	0.992	0.997	0.999	1.000	0.999	1.000
	1.55	0.081	0.358	0.677	0.880	0.959	0.982	0.997	0.998	1.000	0.999	1.000
	1.575	0.088	0.355	0.702	0.884	0.967	0.991	0.996	0.999	1.000	1.000	1.000
	1.6	0.081	0.371	0.699	0.888	0.965	0.988	0.995	1.000	1.000	1.000	1.000
	1.625	0.088	0.363	0.693	0.888	0.969	0.990	0.996	1.000	1.000	1.000	1.000
	1.65	0.098	0.378	0.721	0.882	0.963	0.990	0.998	1.000	1.000	1.000	1.000
	1.675	0.105	0.382	0.687	0.892	0.963	0.990	0.998	1.000	1.000	1.000	1.000
	1.7	0.106	0.384	0.704	0.886	0.963	0.992	0.999	0.999	1.000	1.000	1.000

Table 0.4 Power of supF (Chi-Square Distribution N=30)

Coefficient Change	Test	Variance Change										
		powersupF	1	1.2	1.4	1.6	1.8	2	2.2	2.4	2.6	2.8
	1	0.054	0.056	0.069	0.065	0.064	0.078	0.082	0.093	0.090	0.077	0.090
	1.025	0.049	0.056	0.066	0.049	0.067	0.081	0.087	0.083	0.084	0.085	0.090
	1.05	0.055	0.046	0.054	0.070	0.081	0.088	0.074	0.094	0.077	0.088	0.081
	1.075	0.054	0.054	0.070	0.066	0.070	0.083	0.082	0.082	0.091	0.090	0.089
	1.1	0.073	0.079	0.069	0.070	0.083	0.083	0.083	0.096	0.074	0.102	0.089
	1.125	0.088	0.070	0.079	0.093	0.079	0.087	0.089	0.103	0.079	0.091	0.098
	1.15	0.103	0.088	0.085	0.094	0.097	0.082	0.096	0.095	0.094	0.104	0.096

	1.175	0.119	0.106	0.105	0.098	0.091	0.098	0.104	0.107	0.093	0.105	0.116
	1.2	0.157	0.133	0.114	0.120	0.094	0.100	0.092	0.112	0.099	0.102	0.110
	1.225	0.206	0.161	0.132	0.126	0.116	0.122	0.111	0.115	0.109	0.108	0.108
	1.25	0.216	0.180	0.147	0.121	0.135	0.116	0.116	0.125	0.103	0.104	0.111
	1.275	0.276	0.204	0.188	0.149	0.128	0.137	0.119	0.113	0.109	0.114	0.114
	1.3	0.331	0.252	0.181	0.157	0.159	0.141	0.135	0.122	0.125	0.130	0.107
	1.325	0.384	0.304	0.237	0.181	0.174	0.156	0.141	0.137	0.128	0.135	0.120
	1.35	0.451	0.349	0.258	0.232	0.190	0.174	0.146	0.140	0.136	0.125	0.129
	1.375	0.521	0.396	0.320	0.241	0.214	0.195	0.162	0.162	0.162	0.137	0.130
	1.4	0.564	0.434	0.340	0.274	0.248	0.221	0.183	0.159	0.152	0.137	0.144
	1.425	0.640	0.492	0.401	0.327	0.248	0.225	0.203	0.202	0.164	0.162	0.159
	1.45	0.692	0.525	0.427	0.348	0.292	0.255	0.239	0.217	0.181	0.153	0.160
	1.475	0.753	0.606	0.472	0.391	0.308	0.290	0.240	0.217	0.212	0.170	0.168
	1.5	0.794	0.654	0.510	0.420	0.353	0.285	0.252	0.254	0.223	0.198	0.186
	1.525	0.828	0.708	0.578	0.480	0.391	0.337	0.289	0.242	0.219	0.199	0.191
	1.55	0.879	0.761	0.609	0.511	0.418	0.365	0.302	0.262	0.248	0.221	0.185
	1.575	0.910	0.809	0.679	0.561	0.453	0.402	0.345	0.299	0.266	0.224	0.219
	1.6	0.944	0.849	0.716	0.590	0.519	0.421	0.349	0.320	0.271	0.234	0.236
	1.625	0.950	0.862	0.754	0.638	0.514	0.467	0.386	0.330	0.298	0.274	0.238
	1.65	0.963	0.900	0.791	0.661	0.575	0.492	0.416	0.348	0.321	0.273	0.268
	1.675	0.977	0.919	0.826	0.710	0.628	0.508	0.474	0.387	0.334	0.300	0.268
	1.7	0.985	0.944	0.859	0.747	0.635	0.561	0.474	0.421	0.366	0.336	0.312

Table 0.5 Power of supGQ (Chi-Square Distribution N=30)

Coefficient Change	Test	Variance Change										
		1	1.2	1.4	1.6	1.8	2	2.2	2.4	2.6	2.8	3
	powersupGQ											
	1	0.046	0.059	0.068	0.077	0.095	0.109	0.131	0.141	0.185	0.209	0.213
	1.025	0.052	0.051	0.071	0.084	0.093	0.115	0.124	0.153	0.177	0.187	0.238
	1.05	0.049	0.054	0.068	0.082	0.095	0.110	0.139	0.137	0.167	0.183	0.215
	1.075	0.054	0.063	0.074	0.073	0.100	0.111	0.125	0.142	0.194	0.195	0.214
	1.1	0.046	0.055	0.057	0.084	0.090	0.114	0.125	0.136	0.160	0.206	0.220
	1.125	0.043	0.053	0.076	0.081	0.097	0.101	0.118	0.153	0.181	0.205	0.203
	1.15	0.058	0.066	0.069	0.075	0.092	0.107	0.133	0.146	0.176	0.199	0.214
	1.175	0.051	0.059	0.069	0.073	0.095	0.115	0.133	0.158	0.158	0.208	0.217
	1.2	0.058	0.058	0.065	0.083	0.095	0.099	0.127	0.150	0.195	0.191	0.229
	1.225	0.069	0.058	0.069	0.098	0.106	0.107	0.133	0.146	0.173	0.198	0.224
	1.25	0.068	0.071	0.069	0.088	0.100	0.112	0.135	0.137	0.177	0.191	0.238
	1.275	0.057	0.071	0.065	0.086	0.112	0.107	0.140	0.155	0.177	0.183	0.234
	1.3	0.059	0.062	0.074	0.084	0.095	0.127	0.132	0.164	0.172	0.203	0.237
	1.325	0.069	0.063	0.067	0.095	0.096	0.115	0.137	0.138	0.176	0.211	0.226
	1.35	0.068	0.062	0.083	0.084	0.110	0.108	0.139	0.152	0.163	0.191	0.207
	1.375	0.067	0.078	0.070	0.102	0.096	0.123	0.128	0.149	0.169	0.202	0.230
	1.4	0.068	0.077	0.072	0.088	0.104	0.113	0.137	0.162	0.175	0.192	0.224
	1.425	0.063	0.065	0.072	0.093	0.097	0.122	0.139	0.153	0.177	0.199	0.234
	1.45	0.065	0.072	0.078	0.095	0.105	0.116	0.140	0.153	0.176	0.179	0.239
	1.475	0.075	0.072	0.079	0.091	0.104	0.122	0.141	0.165	0.179	0.196	0.225
	1.5	0.070	0.068	0.091	0.092	0.097	0.129	0.137	0.162	0.190	0.191	0.231
	1.525	0.076	0.066	0.084	0.099	0.107	0.131	0.130	0.163	0.183	0.198	0.213
	1.55	0.068	0.076	0.084	0.093	0.113	0.113	0.132	0.160	0.179	0.209	0.230
	1.575	0.080	0.086	0.081	0.109	0.102	0.126	0.144	0.163	0.183	0.203	0.237
	1.6	0.087	0.076	0.088	0.084	0.113	0.124	0.137	0.156	0.186	0.202	0.215
	1.625	0.090	0.086	0.082	0.113	0.098	0.141	0.133	0.173	0.184	0.214	0.226

	1.65	0.093	0.091	0.084	0.107	0.108	0.128	0.142	0.164	0.177	0.202	0.230
	1.675	0.098	0.089	0.098	0.115	0.106	0.135	0.150	0.161	0.187	0.227	0.242
	1.7	0.106	0.095	0.092	0.099	0.113	0.125	0.149	0.160	0.193	0.211	0.236

Table 0.6 Power of supMZ (Chi-Square Distribution N=30)

Coefficient Change	Test	Variance Change										
	powersupMZ	1	1.2	1.4	1.6	1.8	2	2.2	2.4	2.6	2.8	3
	1	0.049	0.296	0.640	0.872	0.956	0.989	0.999	0.999	1.000	1.000	1.000
	1.025	0.050	0.297	0.661	0.861	0.957	0.987	0.997	1.000	1.000	1.000	1.000
	1.05	0.054	0.290	0.649	0.870	0.958	0.985	0.994	1.000	1.000	1.000	1.000
	1.075	0.054	0.293	0.643	0.871	0.963	0.987	0.995	0.999	1.000	1.000	1.000
	1.1	0.069	0.332	0.646	0.862	0.957	0.991	0.996	0.999	1.000	1.000	0.999
	1.125	0.061	0.322	0.655	0.868	0.963	0.989	1.000	0.999	1.000	1.000	1.000
	1.15	0.077	0.357	0.678	0.892	0.956	0.990	0.997	1.000	1.000	1.000	1.000
	1.175	0.079	0.346	0.669	0.877	0.955	0.990	0.999	0.999	1.000	1.000	1.000
	1.2	0.105	0.379	0.693	0.888	0.966	0.990	0.999	1.000	1.000	1.000	1.000
	1.225	0.113	0.407	0.722	0.894	0.974	0.994	0.997	0.998	1.000	1.000	1.000
	1.25	0.132	0.416	0.736	0.892	0.967	0.994	0.997	0.999	1.000	1.000	1.000
	1.275	0.142	0.441	0.752	0.903	0.971	0.993	0.997	0.999	1.000	1.000	1.000
	1.3	0.168	0.501	0.733	0.912	0.976	0.995	0.999	0.999	1.000	1.000	1.000
	1.325	0.194	0.504	0.804	0.931	0.967	0.993	0.996	1.000	1.000	1.000	1.000
	1.35	0.227	0.537	0.816	0.937	0.979	0.994	0.997	1.000	0.999	1.000	1.000
	1.375	0.287	0.591	0.849	0.934	0.985	0.997	0.998	0.999	1.000	1.000	1.000
	1.4	0.321	0.620	0.852	0.941	0.984	0.996	0.998	0.998	1.000	1.000	1.000
	1.425	0.364	0.685	0.864	0.952	0.986	0.994	1.000	1.000	1.000	1.000	1.000
	1.45	0.429	0.706	0.884	0.959	0.985	0.996	0.999	0.999	1.000	1.000	1.000
	1.475	0.474	0.730	0.888	0.971	0.988	0.997	0.998	0.999	1.000	1.000	1.000

	1.5	0.527	0.778	0.913	0.964	0.989	0.997	0.997	0.999	1.000	1.000	1.000
	1.525	0.596	0.804	0.925	0.970	0.990	0.997	0.998	1.000	1.000	1.000	1.000
	1.55	0.657	0.832	0.934	0.988	0.995	0.998	1.000	1.000	1.000	1.000	1.000
	1.575	0.697	0.884	0.957	0.986	0.994	0.998	1.000	1.000	1.000	1.000	1.000
	1.6	0.768	0.896	0.963	0.984	0.993	0.999	1.000	1.000	1.000	1.000	1.000
	1.625	0.814	0.912	0.966	0.990	0.998	1.000	1.000	1.000	1.000	1.000	1.000
	1.65	0.846	0.938	0.976	0.990	0.998	0.998	1.000	1.000	1.000	1.000	1.000
	1.675	0.871	0.949	0.983	0.992	0.998	0.999	1.000	1.000	1.000	1.000	1.000
	1.7	0.911	0.969	0.985	0.996	0.998	1.000	1.000	1.000	1.000	1.000	1.000

Table 0.7 Power of supF (Cauchy Distribution N=30)

Coefficient Change	Test	Variance Change										
		powersupF	1	1.2	1.4	1.6	1.8	2	2.2	2.4	2.6	2.8
	1	0.059	0.053	0.060	0.054	0.061	0.070	0.071	0.076	0.085	0.069	0.073
	1.025	0.054	0.058	0.053	0.059	0.069	0.073	0.083	0.064	0.067	0.083	0.071
	1.05	0.068	0.061	0.063	0.058	0.067	0.079	0.071	0.074	0.071	0.080	0.072
	1.075	0.082	0.063	0.069	0.068	0.066	0.079	0.084	0.071	0.078	0.068	0.078
	1.1	0.086	0.087	0.080	0.067	0.074	0.079	0.070	0.078	0.082	0.096	0.088
	1.125	0.129	0.103	0.088	0.093	0.079	0.081	0.079	0.091	0.080	0.083	0.091
	1.15	0.163	0.134	0.119	0.096	0.104	0.084	0.084	0.093	0.099	0.095	0.085
	1.175	0.229	0.168	0.143	0.123	0.112	0.103	0.101	0.099	0.101	0.089	0.100
	1.2	0.301	0.226	0.154	0.136	0.127	0.116	0.100	0.110	0.106	0.089	0.099
	1.225	0.358	0.282	0.203	0.158	0.129	0.126	0.123	0.114	0.100	0.112	0.113
	1.25	0.448	0.346	0.242	0.178	0.181	0.155	0.123	0.126	0.118	0.115	0.110
	1.275	0.578	0.409	0.329	0.239	0.169	0.149	0.153	0.125	0.126	0.108	0.117
	1.3	0.642	0.510	0.389	0.288	0.240	0.194	0.159	0.140	0.133	0.148	0.126
	1.325	0.729	0.581	0.447	0.335	0.269	0.227	0.205	0.163	0.138	0.143	0.127

	1.35	0.786	0.665	0.541	0.416	0.325	0.241	0.198	0.179	0.171	0.145	0.146
	1.375	0.859	0.740	0.599	0.482	0.375	0.289	0.262	0.204	0.202	0.161	0.150
	1.4	0.895	0.800	0.668	0.555	0.435	0.352	0.268	0.236	0.199	0.194	0.169
	1.425	0.938	0.846	0.740	0.630	0.494	0.408	0.336	0.266	0.245	0.207	0.201
	1.45	0.969	0.907	0.807	0.693	0.570	0.458	0.371	0.327	0.253	0.213	0.217
	1.475	0.981	0.933	0.869	0.741	0.626	0.526	0.425	0.348	0.279	0.260	0.234
	1.5	0.989	0.965	0.907	0.806	0.674	0.577	0.482	0.414	0.328	0.266	0.261
	1.525	0.994	0.973	0.942	0.859	0.742	0.649	0.543	0.439	0.371	0.322	0.277
	1.55	0.997	0.988	0.951	0.901	0.801	0.711	0.582	0.501	0.415	0.367	0.290
	1.575	0.996	0.992	0.972	0.936	0.873	0.748	0.665	0.555	0.461	0.391	0.343
	1.6	1.000	0.998	0.985	0.954	0.898	0.807	0.679	0.600	0.519	0.453	0.363
	1.625	1.000	0.998	0.989	0.967	0.919	0.846	0.756	0.657	0.564	0.473	0.422
	1.65	1.000	1.000	0.993	0.980	0.942	0.879	0.800	0.709	0.619	0.527	0.447
	1.675	1.000	1.000	0.998	0.987	0.960	0.917	0.839	0.754	0.648	0.556	0.496
	1.7	1.000	1.000	0.999	0.993	0.974	0.941	0.889	0.796	0.719	0.631	0.544

Table 0.8 Power of supGQ (Cauchy Distribution N=30)

Coefficient Change	Test	Variance Change										
		powersupGQ	1	1.2	1.4	1.6	1.8	2	2.2	2.4	2.6	2.8
	1	0.040	0.056	0.057	0.071	0.099	0.113	0.123	0.139	0.174	0.197	0.210
	1.025	0.053	0.044	0.065	0.081	0.086	0.113	0.117	0.153	0.173	0.182	0.217
	1.05	0.048	0.056	0.062	0.079	0.090	0.104	0.132	0.130	0.157	0.183	0.212
	1.075	0.053	0.055	0.079	0.066	0.082	0.105	0.122	0.131	0.185	0.178	0.208
	1.1	0.047	0.060	0.065	0.075	0.088	0.109	0.121	0.128	0.157	0.192	0.210
	1.125	0.049	0.052	0.074	0.075	0.103	0.095	0.109	0.155	0.168	0.199	0.205
	1.15	0.058	0.057	0.073	0.072	0.092	0.101	0.131	0.137	0.171	0.196	0.213
	1.175	0.053	0.051	0.061	0.077	0.094	0.109	0.122	0.154	0.158	0.209	0.204
	1.2	0.052	0.060	0.067	0.085	0.094	0.095	0.125	0.137	0.193	0.179	0.222

	1.225	0.067	0.057	0.077	0.095	0.106	0.113	0.133	0.150	0.168	0.184	0.215
	1.25	0.066	0.070	0.070	0.093	0.101	0.109	0.131	0.138	0.169	0.182	0.224
	1.275	0.063	0.068	0.064	0.080	0.107	0.112	0.136	0.144	0.167	0.172	0.236
	1.3	0.061	0.067	0.066	0.082	0.108	0.121	0.135	0.158	0.168	0.201	0.231
	1.325	0.073	0.065	0.072	0.085	0.099	0.109	0.136	0.134	0.171	0.206	0.220
	1.35	0.080	0.058	0.085	0.088	0.116	0.106	0.145	0.150	0.169	0.189	0.204
	1.375	0.072	0.079	0.078	0.096	0.099	0.125	0.131	0.143	0.168	0.195	0.214
	1.4	0.076	0.088	0.080	0.093	0.106	0.118	0.140	0.164	0.181	0.188	0.216
	1.425	0.080	0.088	0.084	0.095	0.103	0.135	0.144	0.155	0.181	0.197	0.228
	1.45	0.079	0.095	0.084	0.094	0.103	0.125	0.140	0.148	0.183	0.181	0.225
	1.475	0.079	0.077	0.091	0.093	0.117	0.128	0.136	0.164	0.181	0.193	0.224
	1.5	0.086	0.091	0.098	0.099	0.101	0.138	0.145	0.166	0.182	0.195	0.227
	1.525	0.097	0.078	0.102	0.100	0.115	0.134	0.148	0.170	0.188	0.204	0.217
	1.55	0.098	0.094	0.098	0.102	0.118	0.129	0.142	0.171	0.191	0.222	0.234
	1.575	0.101	0.095	0.099	0.120	0.116	0.134	0.153	0.174	0.185	0.209	0.243
	1.6	0.120	0.099	0.102	0.111	0.116	0.145	0.150	0.170	0.195	0.204	0.220
	1.625	0.123	0.111	0.104	0.113	0.111	0.154	0.154	0.189	0.193	0.217	0.243
	1.65	0.125	0.103	0.111	0.125	0.131	0.128	0.157	0.178	0.194	0.210	0.231
	1.675	0.131	0.125	0.116	0.133	0.125	0.152	0.154	0.172	0.197	0.231	0.258
	1.7	0.144	0.125	0.120	0.106	0.140	0.131	0.163	0.178	0.209	0.224	0.238

Table 0.9 Power of supMZ (Cauchy Distribution N=30)

ent Ch	Test	Variance Change									
		1	1.2	1.4	1.6	1.8	2	2.2	2.4	2.6	3
powersupMZ											

	1	0.049	0.282	0.628	0.854	0.949	0.984	0.999	0.997	1.000	1.000	1.000
	1.025	0.047	0.292	0.642	0.855	0.954	0.985	0.995	0.998	1.000	1.000	1.000
	1.05	0.057	0.294	0.637	0.855	0.951	0.984	0.996	1.000	1.000	1.000	1.000
	1.075	0.057	0.296	0.627	0.864	0.957	0.987	0.996	0.999	1.000	1.000	1.000
	1.1	0.076	0.343	0.650	0.858	0.956	0.991	0.996	0.998	1.000	1.000	1.000
	1.125	0.079	0.345	0.664	0.873	0.960	0.989	1.000	0.999	1.000	1.000	1.000
	1.15	0.105	0.388	0.706	0.882	0.961	0.988	0.997	1.000	1.000	1.000	1.000
	1.175	0.110	0.399	0.727	0.890	0.962	0.989	0.997	1.000	1.000	1.000	1.000
	1.2	0.151	0.457	0.755	0.902	0.974	0.993	0.998	1.000	1.000	1.000	1.000
	1.225	0.183	0.505	0.774	0.913	0.977	0.996	0.998	1.000	1.000	1.000	1.000
	1.25	0.245	0.549	0.809	0.931	0.977	0.994	0.998	0.999	1.000	1.000	1.000
	1.275	0.290	0.586	0.846	0.949	0.987	0.997	0.998	0.999	1.000	1.000	1.000
	1.3	0.328	0.660	0.850	0.951	0.990	0.998	1.000	0.999	1.000	1.000	1.000
	1.325	0.438	0.694	0.898	0.971	0.985	0.998	0.999	1.000	1.000	1.000	1.000
	1.35	0.497	0.757	0.925	0.973	0.992	0.998	1.000	1.000	1.000	1.000	1.000
	1.375	0.584	0.807	0.938	0.981	0.993	0.999	1.000	1.000	1.000	1.000	1.000
	1.4	0.650	0.853	0.950	0.989	0.995	1.000	1.000	1.000	1.000	1.000	1.000
	1.425	0.737	0.894	0.968	0.989	0.998	0.999	1.000	1.000	1.000	1.000	1.000
	1.45	0.799	0.924	0.982	0.996	0.999	1.000	1.000	1.000	1.000	1.000	1.000
	1.475	0.841	0.950	0.982	0.997	0.998	1.000	1.000	1.000	1.000	1.000	1.000
	1.5	0.901	0.969	0.993	0.998	0.999	1.000	1.000	1.000	1.000	1.000	1.000
	1.525	0.925	0.979	0.993	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	1.55	0.957	0.985	0.995	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	1.575	0.974	0.991	0.998	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	1.6	0.988	0.994	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	1.625	0.990	0.997	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	1.65	0.998	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	1.675	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	1.7	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Table 0.10Power of supF (Additive Sequence N=30)

Coefficient Change	Test	Variance Change										
	powersupF	1	1.2	1.4	1.6	1.8	2	2.2	2.4	2.6	2.8	3
	1	0.048	0.058	0.052	0.062	0.074	0.070	0.075	0.081	0.083	0.080	0.079
	1.025	0.299	0.243	0.185	0.177	0.151	0.128	0.113	0.107	0.107	0.111	0.110
	1.05	0.919	0.833	0.703	0.587	0.471	0.398	0.338	0.274	0.254	0.229	0.214
	1.075	1.000	0.999	0.982	0.959	0.881	0.783	0.721	0.621	0.541	0.480	0.424
	1.1	1.000	1.000	1.000	1.000	0.997	0.982	0.952	0.911	0.848	0.772	0.720
	1.125	1.000	1.000	1.000	1.000	1.000	0.998	0.996	0.992	0.974	0.950	0.918
	1.15	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.993	0.987
	1.175	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.999
	1.2	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	1.225	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	1.25	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	1.275	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	1.3	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	1.325	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	1.35	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	1.375	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	1.4	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	1.425	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	1.45	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	1.475	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	1.5	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	1.525	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	1.55	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	1.575	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	1.6	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	1.625	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

	1.65	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	1.675	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	1.7	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Table 0.11 Power of supGQ (Additive Sequence N=30)

Coefficient Change	Test	Variance Change										
		powersupGQ	1	1.2	1.4	1.6	1.8	2	2.2	2.4	2.6	2.8
	1	0.059	0.060	0.072	0.082	0.095	0.120	0.140	0.164	0.179	0.218	0.229
	1.025	0.064	0.063	0.075	0.106	0.116	0.123	0.162	0.163	0.188	0.202	0.232
	1.05	0.091	0.101	0.105	0.127	0.118	0.136	0.154	0.173	0.220	0.228	0.258
	1.075	0.160	0.125	0.163	0.147	0.147	0.166	0.185	0.204	0.229	0.253	0.260
	1.1	0.219	0.198	0.179	0.198	0.199	0.206	0.202	0.240	0.272	0.283	0.311
	1.125	0.305	0.257	0.248	0.241	0.251	0.245	0.262	0.280	0.296	0.319	0.338
	1.15	0.384	0.321	0.315	0.306	0.305	0.303	0.307	0.341	0.342	0.376	0.371
	1.175	0.469	0.421	0.401	0.378	0.376	0.359	0.369	0.384	0.390	0.388	0.447
	1.2	0.577	0.523	0.474	0.453	0.426	0.443	0.414	0.439	0.468	0.457	0.476
	1.225	0.687	0.587	0.550	0.516	0.511	0.511	0.477	0.514	0.505	0.511	0.528
	1.25	0.779	0.670	0.604	0.577	0.580	0.554	0.542	0.552	0.553	0.570	0.591
	1.275	0.846	0.770	0.700	0.648	0.637	0.624	0.624	0.606	0.636	0.612	0.620
	1.3	0.912	0.820	0.751	0.710	0.705	0.681	0.675	0.680	0.666	0.673	0.688
	1.325	0.934	0.872	0.810	0.776	0.758	0.736	0.704	0.735	0.719	0.724	0.730
	1.35	0.970	0.908	0.870	0.810	0.795	0.789	0.754	0.763	0.765	0.766	0.768
	1.375	0.985	0.941	0.888	0.866	0.844	0.823	0.817	0.803	0.807	0.790	0.816
	1.4	0.991	0.968	0.933	0.909	0.873	0.853	0.850	0.829	0.834	0.856	0.839
	1.425	0.996	0.980	0.953	0.922	0.918	0.891	0.887	0.874	0.877	0.884	0.876
	1.45	0.997	0.988	0.967	0.954	0.930	0.919	0.915	0.910	0.911	0.904	0.899
	1.475	1.000	0.994	0.988	0.970	0.950	0.940	0.932	0.916	0.930	0.926	0.905
	1.5	1.000	0.999	0.992	0.978	0.967	0.964	0.956	0.939	0.937	0.942	0.935

	1.525	1.000	0.999	0.995	0.988	0.980	0.973	0.962	0.970	0.965	0.956	0.948
	1.55	1.000	1.000	0.998	0.992	0.988	0.983	0.975	0.979	0.969	0.969	0.962
	1.575	1.000	1.000	1.000	0.995	0.992	0.985	0.983	0.980	0.977	0.976	0.975
	1.6	1.000	1.000	1.000	0.999	0.994	0.988	0.991	0.987	0.984	0.986	0.982
	1.625	1.000	1.000	1.000	0.999	0.995	0.996	0.992	0.988	0.989	0.988	0.986
	1.65	1.000	1.000	1.000	0.998	0.997	0.996	0.996	0.989	0.989	0.993	0.991
	1.675	1.000	1.000	1.000	1.000	1.000	0.998	0.998	0.999	0.997	0.996	0.994
	1.7	1.000	1.000	1.000	1.000	0.999	1.000	0.997	0.999	0.998	0.997	0.995

Table 0.12 Power of supMZ (Additive Sequence N=30)

Coefficient Change	Test	Variance Change											
		powersupMZ	1	1.2	1.4	1.6	1.8	2	2.2	2.4	2.6	2.8	3
	1	0.169	0.495	0.788	0.920	0.971	0.990	0.999	1.000	1.000	1.000	1.000	1.000
	1.025	0.736	0.890	0.962	0.987	0.997	0.999	1.000	1.000	1.000	1.000	1.000	1.000
	1.05	0.993	0.998	0.998	0.999	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	1.075	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	1.1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	1.125	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	1.15	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	1.175	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	1.2	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	1.225	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	1.25	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	1.275	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	1.3	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	1.325	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	1.35	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

	1.375	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	1.4	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	1.425	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	1.45	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	1.475	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	1.5	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	1.525	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	1.55	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	1.575	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	1.6	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	1.625	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	1.65	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	1.675	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	1.7	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Table 0.13 Power of supF (Standard Normal Distribution N=60)

Coefficient change	Test	Variance change										
		PsupF	1	1.2	1.4	1.6	1.8	2	2.2	2.4	2.6	2.8
	1	0.05	0.05	0.05	0.06	0.06	0.07	0.07	0.07	0.08	0.07	0.07
	1.2	0.08	0.07	0.06	0.09	0.06	0.07	0.07	0.07	0.08	0.08	0.08
	1.4	0.15	0.12	0.11	0.09	0.09	0.09	0.09	0.08	0.08	0.09	0.08
	1.6	0.28	0.20	0.16	0.13	0.12	0.11	0.09	0.09	0.09	0.10	0.09
	1.8	0.50	0.35	0.27	0.22	0.19	0.16	0.14	0.11	0.13	0.11	0.10
	2	0.70	0.56	0.45	0.35	0.27	0.20	0.19	0.17	0.15	0.13	0.12
	2.2	0.86	0.78	0.65	0.54	0.39	0.31	0.26	0.22	0.21	0.18	0.15
	2.4	0.96	0.89	0.79	0.70	0.55	0.46	0.37	0.30	0.27	0.24	0.21
	2.6	0.99	0.97	0.91	0.81	0.71	0.59	0.50	0.44	0.36	0.31	0.27

	2.8	1.00	1.00	0.97	0.90	0.83	0.72	0.64	0.55	0.48	0.39	0.36
	3	1.00	1.00	0.99	0.97	0.93	0.86	0.77	0.68	0.57	0.51	0.43

Table 0.14 Power of supGQ (Standard Normal Distribution N=60)

Coefficient change	Test	Variance change										
		PsupGQ	1.00	1.20	1.40	1.60	1.80	2.00	2.20	2.40	2.60	2.80
	1	0.06	0.10	0.18	0.30	0.47	0.64	0.78	0.87	0.94	0.97	0.98
	1.2	0.06	0.10	0.17	0.31	0.47	0.64	0.78	0.88	0.92	0.97	0.98
	1.4	0.06	0.09	0.18	0.30	0.46	0.64	0.77	0.88	0.94	0.96	0.99
	1.6	0.07	0.10	0.18	0.33	0.49	0.64	0.77	0.87	0.93	0.96	0.98
	1.8	0.07	0.11	0.18	0.32	0.47	0.65	0.78	0.87	0.94	0.96	0.99
	2	0.08	0.12	0.21	0.31	0.51	0.64	0.77	0.87	0.93	0.97	0.98
	2.2	0.10	0.12	0.22	0.33	0.50	0.65	0.77	0.88	0.95	0.97	0.98
	2.4	0.14	0.15	0.22	0.35	0.50	0.68	0.79	0.88	0.94	0.97	0.98
	2.6	0.17	0.19	0.24	0.36	0.51	0.67	0.79	0.88	0.94	0.96	0.98
	2.8	0.20	0.21	0.25	0.40	0.51	0.67	0.78	0.87	0.95	0.97	0.98
	3	0.24	0.23	0.30	0.42	0.54	0.68	0.80	0.89	0.94	0.97	0.99

Table 0.15 Power of supMZ (Standard Normal Distribution N=60)

Coefficient change	Test	Variance change										
		PsupMZ	1.00	1.20	1.40	1.60	1.80	2.00	2.20	2.40	2.60	2.80
1		0.05	0.43	0.85	0.98	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.2		0.04	0.43	0.86	0.98	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.4		0.07	0.48	0.88	0.98	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.6		0.10	0.55	0.89	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.8		0.15	0.63	0.93	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00
2		0.22	0.70	0.94	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
2.2		0.38	0.81	0.97	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
2.4		0.54	0.89	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
2.6		0.70	0.93	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
2.8		0.84	0.98	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
3		0.94	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Table 0.16 Power of supF (Chi Square Distribution N=60)

Coefficient change	Test	Variance change										
		1	1.2	1.4	1.6	1.8	2	2.2	2.4	2.6	2.8	3
	PsupF	0.04	0.05	0.04	0.05	0.05	0.05	0.06	0.06	0.05	0.06	0.06
	1	0.04	0.05	0.04	0.05	0.05	0.05	0.06	0.06	0.05	0.06	0.06
	1.025	0.04	0.05	0.05	0.05	0.05	0.05	0.05	0.06	0.06	0.07	0.05
	1.05	0.05	0.05	0.04	0.05	0.05	0.05	0.05	0.06	0.05	0.06	0.06
	1.075	0.07	0.06	0.06	0.05	0.05	0.06	0.06	0.06	0.06	0.06	0.06
	1.1	0.09	0.07	0.05	0.05	0.06	0.06	0.06	0.05	0.07	0.06	0.07
	1.125	0.12	0.09	0.07	0.07	0.06	0.07	0.08	0.07	0.07	0.05	0.07
	1.15	0.14	0.11	0.08	0.08	0.07	0.07	0.06	0.06	0.08	0.06	0.07
	1.175	0.19	0.12	0.12	0.09	0.08	0.07	0.08	0.07	0.07	0.06	0.07
	1.2	0.27	0.17	0.12	0.10	0.09	0.09	0.07	0.07	0.06	0.07	0.08
	1.225	0.33	0.23	0.17	0.13	0.10	0.09	0.07	0.09	0.08	0.08	0.08
	1.25	0.39	0.27	0.19	0.16	0.12	0.10	0.10	0.09	0.08	0.08	0.08
	1.275	0.48	0.35	0.24	0.19	0.15	0.11	0.11	0.09	0.09	0.08	0.08
	1.3	0.57	0.43	0.29	0.23	0.18	0.12	0.12	0.11	0.10	0.08	0.09
	1.325	0.67	0.48	0.35	0.28	0.22	0.16	0.14	0.11	0.10	0.10	0.10
	1.35	0.73	0.57	0.45	0.35	0.26	0.18	0.17	0.13	0.13	0.10	0.09
	1.375	0.80	0.67	0.50	0.39	0.27	0.21	0.18	0.14	0.13	0.13	0.11
	1.4	0.85	0.73	0.57	0.45	0.33	0.27	0.20	0.17	0.15	0.13	0.12
	1.425	0.89	0.77	0.64	0.52	0.40	0.29	0.25	0.20	0.16	0.15	0.12
	1.45	0.93	0.85	0.72	0.58	0.47	0.35	0.25	0.22	0.19	0.17	0.14
	1.475	0.96	0.88	0.78	0.64	0.51	0.42	0.32	0.26	0.22	0.19	0.16
	1.5	0.98	0.92	0.83	0.71	0.60	0.48	0.36	0.30	0.23	0.21	0.17
	1.525	0.99	0.95	0.87	0.77	0.65	0.55	0.41	0.33	0.27	0.23	0.18
	1.55	0.99	0.96	0.92	0.83	0.72	0.59	0.46	0.38	0.32	0.24	0.22
	1.575	1.00	0.98	0.96	0.88	0.77	0.64	0.52	0.44	0.34	0.28	0.24
	1.6	1.00	0.99	0.97	0.90	0.80	0.71	0.59	0.47	0.38	0.32	0.28
	1.625	1.00	1.00	0.98	0.93	0.85	0.75	0.65	0.54	0.44	0.36	0.31
	1.65	1.00	1.00	0.98	0.95	0.90	0.77	0.68	0.60	0.50	0.40	0.35
	1.675	1.00	1.00	0.99	0.96	0.92	0.85	0.73	0.65	0.54	0.43	0.37
	1.7	1.00	1.00	1.00	0.98	0.94	0.87	0.77	0.67	0.56	0.48	0.40

Table 0.17 Power of supGQ (Chi Square Distribution N=60)

Test	Variance change										
	PsupGQ	1	1.2	1.4	1.6	1.8	2	2.2	2.4	2.6	2.8
1	0.05	0.07	0.13	0.27	0.42	0.56	0.71	0.82	0.92	0.95	0.97
1.025	0.05	0.07	0.14	0.25	0.41	0.55	0.70	0.82	0.92	0.96	0.97
1.05	0.04	0.08	0.14	0.25	0.40	0.56	0.74	0.83	0.90	0.95	0.98
1.075	0.05	0.07	0.14	0.25	0.41	0.56	0.71	0.81	0.91	0.94	0.97
1.1	0.05	0.09	0.14	0.25	0.41	0.56	0.70	0.83	0.89	0.95	0.97
1.125	0.06	0.08	0.16	0.24	0.42	0.56	0.71	0.82	0.91	0.95	0.97
1.15	0.05	0.08	0.14	0.27	0.39	0.56	0.71	0.82	0.90	0.95	0.97
1.175	0.07	0.08	0.14	0.27	0.39	0.55	0.70	0.82	0.91	0.94	0.98
1.2	0.05	0.07	0.15	0.27	0.40	0.57	0.71	0.82	0.91	0.95	0.98
1.225	0.05	0.08	0.14	0.24	0.39	0.55	0.72	0.82	0.91	0.95	0.98
1.25	0.07	0.08	0.17	0.25	0.43	0.53	0.73	0.83	0.91	0.95	0.98
1.275	0.07	0.08	0.16	0.26	0.41	0.57	0.68	0.83	0.91	0.96	0.98
1.3	0.07	0.09	0.16	0.27	0.41	0.56	0.71	0.82	0.90	0.95	0.98
1.325	0.07	0.08	0.15	0.26	0.41	0.58	0.73	0.83	0.92	0.96	0.98
1.35	0.08	0.09	0.16	0.28	0.42	0.57	0.71	0.82	0.90	0.95	0.98
1.375	0.07	0.10	0.17	0.26	0.43	0.56	0.71	0.82	0.89	0.94	0.98
1.4	0.07	0.09	0.16	0.28	0.42	0.55	0.71	0.81	0.90	0.96	0.98
1.425	0.09	0.10	0.16	0.29	0.41	0.60	0.72	0.85	0.90	0.96	0.98
1.45	0.10	0.12	0.18	0.27	0.41	0.58	0.73	0.83	0.91	0.95	0.98
1.475	0.10	0.11	0.17	0.27	0.42	0.57	0.71	0.83	0.90	0.95	0.97
1.5	0.10	0.13	0.19	0.29	0.43	0.58	0.72	0.83	0.90	0.95	0.98
1.525	0.13	0.13	0.17	0.28	0.44	0.58	0.72	0.82	0.91	0.94	0.98
1.55	0.12	0.13	0.18	0.30	0.43	0.56	0.72	0.84	0.91	0.96	0.98
1.575	0.14	0.14	0.19	0.29	0.43	0.58	0.71	0.82	0.91	0.95	0.98
1.6	0.16	0.14	0.20	0.31	0.43	0.60	0.72	0.84	0.91	0.95	0.98
1.625	0.17	0.15	0.20	0.30	0.43	0.59	0.72	0.82	0.91	0.95	0.97
1.65	0.17	0.16	0.19	0.30	0.46	0.59	0.72	0.84	0.90	0.95	0.97
1.675	0.20	0.17	0.21	0.30	0.46	0.59	0.73	0.84	0.91	0.95	0.98
1.7	0.21	0.18	0.22	0.32	0.44	0.60	0.73	0.85	0.91	0.94	0.98

Table 0.18 Power of supMZ (Chi Square Distribution N=60)

Coefficient change	Test	Variance change										
		PsupMZ	1	1.2	1.4	1.6	1.8	2	2.2	2.4	2.6	2.8
	1	0.04	0.42	0.85	0.98	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.025	0.05	0.43	0.86	0.98	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.05	0.05	0.43	0.86	0.98	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.075	0.06	0.42	0.86	0.98	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.1	0.06	0.43	0.85	0.98	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.125	0.06	0.48	0.86	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.15	0.06	0.47	0.87	0.98	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.175	0.07	0.50	0.88	0.98	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.2	0.09	0.51	0.90	0.98	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.225	0.11	0.55	0.89	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.25	0.12	0.56	0.91	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.275	0.14	0.60	0.92	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.3	0.19	0.64	0.93	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.325	0.21	0.66	0.94	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.35	0.22	0.71	0.94	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.375	0.29	0.76	0.95	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.4	0.35	0.78	0.96	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.425	0.39	0.82	0.98	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.45	0.46	0.85	0.98	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.475	0.52	0.88	0.98	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.5	0.60	0.90	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.525	0.65	0.92	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.55	0.71	0.94	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.575	0.76	0.95	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.6	0.82	0.97	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.625	0.86	0.98	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.65	0.90	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.675	0.92	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.7	0.95	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Table 0.19 Power of supF (Cauchy Distribution N=60)

Coefficient change	Test	Variance change										
		PsupF	1	1.2	1.4	1.6	1.8	2	2.2	2.4	2.6	2.8
	1	0.05	0.05	0.05	0.05	0.05	0.06	0.06	0.06	0.06	0.07	0.06
	1.1	0.07	0.05	0.05	0.05	0.06	0.06	0.06	0.06	0.07	0.07	0.08
	1.2	0.09	0.07	0.05	0.07	0.06	0.07	0.07	0.07	0.08	0.06	0.08
	1.3	0.13	0.11	0.08	0.07	0.07	0.08	0.08	0.07	0.07	0.07	0.08
	1.4	0.22	0.15	0.13	0.10	0.09	0.09	0.07	0.08	0.08	0.08	0.08
	1.5	0.32	0.24	0.17	0.14	0.14	0.10	0.12	0.09	0.11	0.09	0.09
	1.6	0.48	0.36	0.28	0.22	0.17	0.15	0.13	0.12	0.11	0.10	0.10
	1.7	0.64	0.51	0.38	0.30	0.23	0.20	0.17	0.15	0.14	0.12	0.12
	1.8	0.77	0.62	0.51	0.39	0.31	0.25	0.20	0.19	0.16	0.16	0.14
	1.9	0.87	0.78	0.64	0.51	0.42	0.35	0.27	0.23	0.19	0.16	0.16
	2	0.95	0.87	0.74	0.64	0.52	0.43	0.36	0.30	0.25	0.22	0.19
	2.1	0.98	0.92	0.83	0.75	0.62	0.50	0.43	0.37	0.30	0.26	0.23
	2.2	0.99	0.97	0.91	0.84	0.72	0.58	0.52	0.43	0.38	0.32	0.26
	2.3	1.00	0.99	0.96	0.88	0.81	0.71	0.60	0.51	0.43	0.38	0.34
	2.4	1.00	1.00	0.98	0.93	0.88	0.79	0.68	0.58	0.52	0.48	0.38
	2.5	1.00	1.00	0.99	0.96	0.92	0.84	0.76	0.69	0.58	0.53	0.46
	2.6	1.00	1.00	0.99	0.99	0.96	0.91	0.82	0.75	0.68	0.58	0.50
	2.7	1.00	1.00	1.00	0.99	0.98	0.93	0.89	0.80	0.74	0.66	0.60
	2.8	1.00	1.00	1.00	1.00	0.98	0.96	0.93	0.86	0.80	0.72	0.65
	2.9	1.00	1.00	1.00	1.00	0.99	0.98	0.95	0.91	0.85	0.78	0.70
	3	1.00	1.00	1.00	1.00	1.00	0.99	0.98	0.93	0.88	0.83	0.77

Table 0.20 Power of supGQ (Cauchy Distribution N=60)

Coefficient change	Test	Variance change										
		PsupGQ	1	1.2	1.4	1.6	1.8	2	2.2	2.4	2.6	2.8
	1	0.05	0.08	0.16	0.27	0.44	0.56	0.72	0.84	0.92	0.95	0.98
	1.1	0.05	0.08	0.15	0.27	0.42	0.59	0.72	0.84	0.91	0.96	0.98
	1.2	0.05	0.09	0.16	0.27	0.42	0.59	0.73	0.85	0.90	0.96	0.98
	1.3	0.05	0.10	0.15	0.28	0.42	0.57	0.73	0.84	0.91	0.96	0.98
	1.4	0.06	0.08	0.15	0.28	0.42	0.57	0.73	0.84	0.91	0.96	0.98
	1.5	0.07	0.09	0.17	0.28	0.45	0.59	0.74	0.84	0.90	0.96	0.98
	1.6	0.07	0.11	0.18	0.29	0.44	0.60	0.73	0.85	0.91	0.96	0.98
	1.7	0.07	0.11	0.20	0.28	0.44	0.60	0.75	0.85	0.92	0.96	0.98
	1.8	0.10	0.12	0.19	0.31	0.46	0.62	0.74	0.85	0.92	0.95	0.97
	1.9	0.10	0.14	0.20	0.34	0.47	0.63	0.76	0.87	0.92	0.96	0.98
	2	0.13	0.16	0.23	0.33	0.48	0.64	0.75	0.85	0.93	0.95	0.98
	2.1	0.14	0.16	0.25	0.35	0.49	0.64	0.77	0.86	0.93	0.96	0.98
	2.2	0.19	0.21	0.29	0.37	0.51	0.66	0.77	0.86	0.93	0.96	0.98
	2.3	0.20	0.22	0.27	0.38	0.52	0.68	0.79	0.86	0.92	0.96	0.98
	2.4	0.22	0.25	0.32	0.43	0.54	0.68	0.78	0.88	0.93	0.96	0.98
	2.5	0.29	0.28	0.36	0.44	0.57	0.68	0.79	0.88	0.93	0.96	0.98
	2.6	0.32	0.32	0.37	0.46	0.61	0.69	0.80	0.88	0.94	0.97	0.98
	2.7	0.37	0.37	0.40	0.51	0.61	0.74	0.82	0.89	0.94	0.96	0.98
	2.8	0.42	0.41	0.46	0.52	0.63	0.73	0.84	0.91	0.95	0.97	0.99
	2.9	0.47	0.44	0.47	0.56	0.66	0.76	0.83	0.90	0.95	0.97	0.99
	3	0.55	0.48	0.50	0.59	0.67	0.77	0.85	0.91	0.95	0.97	0.99

Table 0.21 Power of supMZ (Cauchy Distribution N=60)

Coefficient change	Test	Variance change										
		PsupMZ	1	1.2	1.4	1.6	1.8	2	2.2	2.4	2.6	2.8
	1	0.05	0.44	0.86	0.98	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.1	0.05	0.44	0.87	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.2	0.05	0.45	0.86	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.3	0.06	0.49	0.88	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.4	0.09	0.53	0.89	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.5	0.11	0.59	0.91	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.6	0.17	0.62	0.94	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.7	0.23	0.70	0.95	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.8	0.31	0.77	0.96	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.9	0.41	0.83	0.97	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	2	0.53	0.87	0.98	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	2.1	0.64	0.91	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	2.2	0.76	0.95	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	2.3	0.85	0.98	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	2.4	0.92	0.98	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	2.5	0.95	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	2.6	0.98	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	2.7	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	2.8	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	2.9	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	3	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Table 0.22 Power of supF (Additive Sequence Distribution N=60)

Coefficient change	Test	Variance change										
		PsupF	1	1.2	1.4	1.6	1.8	2	2.2	2.4	2.6	2.8
	1	0.05	0.06	0.06	0.07	0.07	0.08	0.09	0.09	0.09	0.10	0.10
	1.025	0.09	0.08	0.07	0.07	0.09	0.08	0.10	0.09	0.09	0.09	0.11
	1.05	0.23	0.15	0.12	0.11	0.11	0.10	0.10	0.09	0.10	0.09	0.09
	1.075	0.51	0.40	0.27	0.20	0.16	0.15	0.13	0.12	0.12	0.11	0.13
	1.1	0.81	0.69	0.54	0.40	0.31	0.23	0.20	0.17	0.16	0.13	0.14
	1.125	0.97	0.90	0.79	0.67	0.51	0.40	0.33	0.27	0.23	0.20	0.16
	1.15	1.00	0.98	0.95	0.88	0.75	0.66	0.53	0.45	0.35	0.30	0.25
	1.175	1.00	1.00	1.00	0.98	0.93	0.85	0.75	0.61	0.53	0.44	0.37
	1.2	1.00	1.00	1.00	1.00	0.98	0.95	0.89	0.82	0.71	0.62	0.53
	1.225	1.00	1.00	1.00	1.00	1.00	0.99	0.97	0.94	0.87	0.80	0.70
	1.25	1.00	1.00	1.00	1.00	1.00	1.00	0.99	0.98	0.95	0.91	0.83
	1.275	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.99	0.96	0.93
	1.3	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.99	0.98
	1.325	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.99
	1.35	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.375	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.4	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.425	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.45	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.475	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.5	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.525	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.55	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.575	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.6	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.625	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.65	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.675	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.7	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Table 0.23 Power of supGQ (Additive Sequence Distribution N=60)

Coefficient change	Test	Variance change										
		PsupGQ	1	1.2	1.4	1.6	1.8	2	2.2	2.4	2.6	2.8
	1	0.05	0.09	0.15	0.26	0.44	0.60	0.72	0.83	0.91	0.96	0.98
	1.025	0.06	0.09	0.16	0.26	0.45	0.58	0.73	0.84	0.92	0.96	0.97
	1.05	0.06	0.09	0.17	0.30	0.45	0.60	0.74	0.85	0.90	0.95	0.98
	1.075	0.08	0.10	0.17	0.29	0.43	0.60	0.74	0.85	0.91	0.96	0.98
	1.1	0.11	0.14	0.20	0.30	0.45	0.61	0.76	0.84	0.91	0.96	0.98
	1.125	0.15	0.16	0.23	0.33	0.48	0.63	0.76	0.85	0.93	0.95	0.98
	1.15	0.21	0.21	0.29	0.39	0.52	0.65	0.78	0.86	0.93	0.96	0.98
	1.175	0.28	0.27	0.34	0.43	0.55	0.67	0.78	0.88	0.93	0.97	0.99
	1.2	0.41	0.36	0.39	0.49	0.59	0.71	0.81	0.89	0.93	0.97	0.99
	1.225	0.53	0.47	0.45	0.55	0.65	0.74	0.83	0.91	0.95	0.98	0.98
	1.25	0.67	0.56	0.56	0.63	0.69	0.80	0.87	0.92	0.95	0.98	0.99
	1.275	0.80	0.66	0.64	0.69	0.75	0.84	0.88	0.92	0.96	0.98	0.99
	1.3	0.89	0.75	0.74	0.76	0.81	0.86	0.93	0.94	0.97	0.98	0.99
	1.325	0.96	0.85	0.82	0.83	0.85	0.90	0.93	0.96	0.98	0.99	1.00
	1.35	0.98	0.92	0.87	0.87	0.90	0.92	0.95	0.96	0.99	0.99	0.99
	1.375	1.00	0.96	0.93	0.92	0.93	0.95	0.96	0.97	0.99	0.99	1.00
	1.4	1.00	0.99	0.97	0.94	0.95	0.97	0.98	0.99	0.99	0.99	1.00
	1.425	1.00	0.99	0.98	0.97	0.98	0.98	0.98	0.99	1.00	1.00	1.00
	1.45	1.00	1.00	0.99	0.98	0.98	0.98	0.99	1.00	1.00	1.00	1.00
	1.475	1.00	1.00	1.00	0.99	0.99	0.99	1.00	1.00	1.00	1.00	1.00
	1.5	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.525	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.55	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.575	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.6	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.625	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.65	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.675	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.7	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Table 0.24 Power of supMZ (Additive Sequence Distribution N=60)

Coefficient change	Test	Variance change									
		1	1.2	1.4	1.6	1.8	2	2.2	2.4	2.6	2.8
	PsupMZ										
	1	0.05	0.42	0.87	0.98	1.00	1.00	1.00	1.00	1.00	1.00
	1.025	0.05	0.44	0.86	0.98	1.00	1.00	1.00	1.00	1.00	1.00
	1.05	0.09	0.52	0.89	0.99	1.00	1.00	1.00	1.00	1.00	1.00
	1.075	0.19	0.65	0.94	0.99	1.00	1.00	1.00	1.00	1.00	1.00
	1.1	0.35	0.78	0.97	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.125	0.61	0.91	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.15	0.82	0.97	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.175	0.95	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.2	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.225	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.25	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.275	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.3	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.325	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.35	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.375	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.4	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.425	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.45	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.475	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.5	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.525	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.55	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.575	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.6	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.625	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.65	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.675	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.7	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Table 0.25 Power of supF (Standard Normal Distribution N=120)

Coefficient change	Test	Variance change										
		PsupF	1	1.2	1.4	PF	1	1.2	1.4	PF	1	1.2
1	1	0.06	0.05	0.05	0.05	0.06	0.06	0.07	0.06	0.07	0.06	0.06
1.2	1.2	0.10	0.08	0.06	0.07	0.06	0.06	0.06	0.07	0.07	0.07	0.06
1.4	1.4	0.27	0.19	0.14	0.12	0.12	0.10	0.10	0.09	0.09	0.09	0.09
1.6	1.6	0.58	0.42	0.31	0.26	0.21	0.18	0.14	0.14	0.12	0.11	0.10
1.8	1.8	0.86	0.74	0.60	0.47	0.38	0.28	0.25	0.22	0.17	0.17	0.14
2	2	0.97	0.93	0.83	0.70	0.60	0.50	0.41	0.34	0.27	0.24	0.21
2.2	2.2	1.00	0.99	0.96	0.89	0.82	0.72	0.62	0.52	0.43	0.38	0.32
2.4	2.4	1.00	1.00	0.99	0.97	0.95	0.88	0.77	0.70	0.60	0.53	0.45
2.6	2.6	1.00	1.00	1.00	1.00	0.99	0.96	0.92	0.83	0.76	0.68	0.61
2.8	2.8	1.00	1.00	1.00	1.00	1.00	0.99	0.98	0.95	0.89	0.82	0.75
3	3	1.00	1.00	1.00	1.00	1.00	1.00	0.99	0.99	0.96	0.92	0.87

Table 0.26 Power of supGQ (Standard Normal Distribution N=120)

Coefficient change	Test	Variance change										
		PsupGQ	1.00	1.20	1.40	1.60	1.80	2.00	2.20	2.40	2.60	2.80
1	1	0.05	0.12	0.37	0.69	0.87	0.97	0.99	1.00	1.00	1.00	1.00
1.2	1.2	0.05	0.13	0.37	0.66	0.88	0.97	0.99	1.00	1.00	1.00	1.00
1.4	1.4	0.06	0.15	0.38	0.66	0.88	0.97	1.00	1.00	1.00	1.00	1.00
1.6	1.6	0.06	0.15	0.38	0.68	0.88	0.98	0.99	1.00	1.00	1.00	1.00
1.8	1.8	0.07	0.17	0.37	0.68	0.90	0.97	1.00	1.00	1.00	1.00	1.00
2	2	0.12	0.17	0.40	0.69	0.88	0.97	1.00	1.00	1.00	1.00	1.00
2.2	2.2	0.12	0.21	0.44	0.70	0.89	0.97	1.00	1.00	1.00	1.00	1.00
2.4	2.4	0.20	0.26	0.46	0.73	0.91	0.98	1.00	1.00	1.00	1.00	1.00
2.6	2.6	0.26	0.31	0.50	0.74	0.91	0.98	1.00	1.00	1.00	1.00	1.00
2.8	2.8	0.38	0.37	0.55	0.75	0.93	0.98	1.00	1.00	1.00	1.00	1.00
3	3	0.51	0.45	0.60	0.81	0.92	0.99	1.00	1.00	1.00	1.00	1.00

Table 0.27 Power of supMZ (Standard Normal Distribution N=120)

Coefficient change	Test	Variance change										
		PsupMZ	1.00	1.20	1.40	1.60	1.80	2.00	2.20	2.40	2.60	2.80
	1	0.04	0.66	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.2	0.05	0.67	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.4	0.07	0.71	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.6	0.13	0.79	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.8	0.22	0.86	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	2	0.37	0.91	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	2.2	0.59	0.97	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	2.4	0.78	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	2.6	0.94	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	2.8	0.98	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	3	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Table 0.28 Power of supF (Chi Square Distribution N=120)

Test	Variance change										
	PsupF	1	1.2	1.4	1.6	1.8	2	2.2	2.4	2.6	2.8
1	0.06	0.05	0.06	0.06	0.06	0.06	0.06	0.07	0.07	0.07	0.07
1.025	0.07	0.06	0.05	0.06	0.06	0.07	0.06	0.06	0.07	0.07	0.07
1.05	0.09	0.08	0.07	0.07	0.08	0.06	0.08	0.07	0.07	0.07	0.07
1.075	0.10	0.11	0.07	0.08	0.06	0.07	0.07	0.06	0.08	0.09	0.08
1.1	0.18	0.12	0.10	0.09	0.10	0.09	0.09	0.08	0.09	0.08	0.08
1.125	0.28	0.19	0.15	0.13	0.12	0.10	0.11	0.09	0.09	0.08	0.08
1.15	0.39	0.28	0.21	0.17	0.14	0.12	0.11	0.09	0.10	0.09	0.10
1.175	0.52	0.40	0.29	0.22	0.20	0.16	0.15	0.10	0.10	0.10	0.11
1.2	0.65	0.52	0.39	0.30	0.25	0.19	0.18	0.15	0.14	0.13	0.12
1.225	0.76	0.62	0.51	0.39	0.31	0.23	0.20	0.17	0.15	0.14	0.13
1.25	0.87	0.76	0.61	0.48	0.38	0.31	0.26	0.21	0.19	0.17	0.16
1.275	0.92	0.83	0.72	0.58	0.44	0.38	0.31	0.25	0.23	0.20	0.15
1.3	0.96	0.90	0.83	0.69	0.57	0.47	0.39	0.33	0.27	0.24	0.21
1.325	0.98	0.96	0.88	0.78	0.65	0.56	0.46	0.39	0.32	0.28	0.22
1.35	1.00	0.97	0.93	0.86	0.75	0.63	0.54	0.48	0.39	0.32	0.29
1.375	1.00	0.99	0.98	0.90	0.83	0.74	0.64	0.52	0.45	0.37	0.33
1.4	1.00	1.00	0.98	0.94	0.88	0.80	0.70	0.63	0.52	0.45	0.37
1.425	1.00	1.00	0.99	0.98	0.93	0.87	0.78	0.69	0.59	0.52	0.43
1.45	1.00	1.00	1.00	0.99	0.96	0.90	0.84	0.76	0.67	0.57	0.48
1.475	1.00	1.00	1.00	1.00	0.98	0.94	0.88	0.81	0.71	0.63	0.57
1.5	1.00	1.00	1.00	1.00	0.99	0.96	0.92	0.86	0.79	0.72	0.63
1.525	1.00	1.00	1.00	1.00	0.99	0.98	0.96	0.90	0.85	0.77	0.69
1.55	1.00	1.00	1.00	1.00	1.00	0.99	0.97	0.94	0.89	0.80	0.75
1.575	1.00	1.00	1.00	1.00	1.00	1.00	0.99	0.96	0.91	0.85	0.79
1.6	1.00	1.00	1.00	1.00	1.00	1.00	0.99	0.97	0.94	0.89	0.84
1.625	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.98	0.97	0.93	0.89
1.65	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.99	0.97	0.95	0.91
1.675	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.99	0.99	0.97	0.94
1.7	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.99	0.98	0.95

Table 0.29 Power of supGQ (Chi Square Distribution N=120

Test PsupGQ	Variance change										
	1	1.2	1.4	1.6	1.8	2	2.2	2.4	2.6	2.8	3
1	0.05	0.12	0.32	0.63	0.86	0.95	0.99	1.00	1.00	1.00	1.00
1.025	0.03	0.11	0.34	0.62	0.84	0.96	0.99	1.00	1.00	1.00	1.00
1.05	0.04	0.11	0.33	0.61	0.86	0.96	0.99	1.00	1.00	1.00	1.00
1.075	0.05	0.12	0.32	0.61	0.87	0.97	0.99	1.00	1.00	1.00	1.00
1.1	0.04	0.10	0.31	0.63	0.85	0.96	0.99	1.00	1.00	1.00	1.00
1.125	0.05	0.10	0.32	0.62	0.87	0.97	0.99	1.00	1.00	1.00	1.00
1.15	0.05	0.13	0.32	0.61	0.86	0.96	0.99	1.00	1.00	1.00	1.00
1.175	0.05	0.12	0.35	0.64	0.86	0.96	0.99	1.00	1.00	1.00	1.00
1.2	0.05	0.12	0.35	0.65	0.85	0.96	0.99	1.00	1.00	1.00	1.00
1.225	0.06	0.13	0.35	0.63	0.85	0.97	0.99	1.00	1.00	1.00	1.00
1.25	0.07	0.14	0.34	0.66	0.88	0.97	0.99	1.00	1.00	1.00	1.00
1.275	0.07	0.15	0.37	0.65	0.88	0.96	0.99	1.00	1.00	1.00	1.00
1.3	0.08	0.15	0.35	0.65	0.86	0.97	0.99	1.00	1.00	1.00	1.00
1.325	0.09	0.17	0.36	0.68	0.87	0.96	0.99	1.00	1.00	1.00	1.00
1.35	0.10	0.17	0.39	0.67	0.87	0.97	1.00	1.00	1.00	1.00	1.00
1.375	0.13	0.20	0.40	0.67	0.87	0.97	0.99	1.00	1.00	1.00	1.00
1.4	0.15	0.20	0.42	0.68	0.88	0.96	0.99	1.00	1.00	1.00	1.00
1.425	0.16	0.22	0.41	0.68	0.88	0.97	0.99	1.00	1.00	1.00	1.00
1.45	0.18	0.25	0.44	0.70	0.88	0.97	0.99	1.00	1.00	1.00	1.00
1.475	0.21	0.25	0.46	0.69	0.88	0.97	0.99	1.00	1.00	1.00	1.00
1.5	0.25	0.30	0.49	0.73	0.89	0.98	0.99	1.00	1.00	1.00	1.00
1.525	0.29	0.30	0.48	0.73	0.89	0.98	0.99	1.00	1.00	1.00	1.00
1.55	0.33	0.32	0.49	0.74	0.90	0.97	0.99	1.00	1.00	1.00	1.00
1.575	0.38	0.37	0.55	0.75	0.91	0.98	0.99	1.00	1.00	1.00	1.00
1.6	0.40	0.40	0.55	0.77	0.90	0.98	0.99	1.00	1.00	1.00	1.00
1.625	0.45	0.43	0.59	0.77	0.91	0.97	0.99	1.00	1.00	1.00	1.00
1.65	0.51	0.47	0.62	0.79	0.91	0.98	1.00	1.00	1.00	1.00	1.00
1.675	0.59	0.51	0.64	0.82	0.93	0.98	0.99	1.00	1.00	1.00	1.00
1.7	0.62	0.54	0.66	0.83	0.94	0.99	1.00	1.00	1.00	1.00	1.00

Table 0.30 Power of supMZ (Chi Square Distribution N=120

Test	Variance change										
	PsupMZ	1	1.2	1.4	1.6	1.8	2	2.2	2.4	2.6	2.8
1	0.04	0.65	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.025	0.04	0.65	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.05	0.05	0.68	0.98	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.075	0.05	0.71	0.98	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.1	0.06	0.71	0.98	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.125	0.07	0.72	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.15	0.10	0.73	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.175	0.11	0.77	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.2	0.14	0.80	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.225	0.17	0.83	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.25	0.21	0.87	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.275	0.26	0.89	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.3	0.33	0.90	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.325	0.41	0.93	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.35	0.51	0.96	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.375	0.58	0.97	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.4	0.67	0.98	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.425	0.76	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.45	0.82	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.475	0.87	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.5	0.92	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.525	0.95	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.55	0.97	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.575	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.6	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.625	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.65	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.675	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.7	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Table 0.31 Power of supF (Cauchy Distribution N=120)

Coefficient change	Test	Variance change										
		PsupF	1	1.2	1.4	1.6	1.8	2	2.2	2.4	2.6	2.8
	1	0.05	0.05	0.05	0.06	0.06	0.06	0.07	0.07	0.06	0.08	0.07
	1.1	0.06	0.06	0.06	0.06	0.06	0.05	0.07	0.06	0.07	0.06	0.06
	1.2	0.12	0.08	0.08	0.07	0.07	0.07	0.08	0.08	0.08	0.07	0.08
	1.3	0.21	0.15	0.12	0.10	0.09	0.10	0.08	0.08	0.08	0.08	0.07
	1.4	0.35	0.26	0.17	0.14	0.12	0.10	0.11	0.09	0.10	0.09	0.08
	1.5	0.54	0.39	0.27	0.23	0.18	0.14	0.13	0.11	0.10	0.10	0.09
	1.6	0.73	0.59	0.47	0.35	0.26	0.18	0.16	0.14	0.11	0.11	0.10
	1.7	0.87	0.76	0.62	0.47	0.37	0.29	0.23	0.19	0.15	0.14	0.12
	1.8	0.95	0.87	0.77	0.64	0.51	0.40	0.30	0.27	0.21	0.18	0.15
	1.9	0.98	0.96	0.88	0.79	0.65	0.53	0.45	0.33	0.27	0.23	0.20
	2	1.00	0.98	0.95	0.88	0.79	0.68	0.55	0.48	0.36	0.31	0.24
	2.1	1.00	1.00	0.99	0.95	0.90	0.80	0.68	0.56	0.46	0.38	0.33
	2.2	1.00	1.00	1.00	0.98	0.95	0.88	0.78	0.70	0.59	0.49	0.40
	2.3	1.00	1.00	1.00	0.99	0.98	0.93	0.88	0.81	0.71	0.60	0.52
	2.4	1.00	1.00	1.00	1.00	0.99	0.97	0.94	0.88	0.81	0.72	0.61
	2.5	1.00	1.00	1.00	1.00	1.00	0.99	0.97	0.93	0.87	0.82	0.69
	2.6	1.00	1.00	1.00	1.00	1.00	1.00	0.98	0.97	0.93	0.87	0.80
	2.7	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.99	0.96	0.92	0.86
	2.8	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.99	0.99	0.96	0.92
	2.9	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.98	0.96
	3	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.99	0.98

Table 0.32 Power of supGQ (Cauchy Distribution N=120)

Coefficient change	Test	Variance change										
		PsupGQ	1	1.2	1.4	1.6	1.8	2	2.2	2.4	2.6	2.8
	1	0.05	0.12	0.34	0.61	0.87	0.97	1.00	1.00	1.00	1.00	1.00
	1.1	0.04	0.12	0.34	0.64	0.86	0.97	1.00	1.00	1.00	1.00	1.00
	1.2	0.04	0.13	0.34	0.65	0.88	0.96	0.99	1.00	1.00	1.00	1.00
	1.3	0.05	0.13	0.33	0.66	0.87	0.96	1.00	1.00	1.00	1.00	1.00
	1.4	0.04	0.12	0.34	0.65	0.87	0.96	0.99	1.00	1.00	1.00	1.00
	1.5	0.06	0.12	0.34	0.63	0.86	0.96	0.99	1.00	1.00	1.00	1.00
	1.6	0.06	0.11	0.32	0.64	0.87	0.97	0.99	1.00	1.00	1.00	1.00
	1.7	0.07	0.12	0.33	0.63	0.87	0.96	0.99	1.00	1.00	1.00	1.00
	1.8	0.06	0.13	0.34	0.63	0.86	0.96	0.99	1.00	1.00	1.00	1.00
	1.9	0.09	0.12	0.32	0.63	0.86	0.96	0.99	1.00	1.00	1.00	1.00
	2	0.11	0.13	0.35	0.63	0.87	0.97	0.99	1.00	1.00	1.00	1.00
	2.1	0.13	0.14	0.33	0.63	0.86	0.97	0.99	1.00	1.00	1.00	1.00
	2.2	0.16	0.13	0.32	0.62	0.87	0.96	0.99	1.00	1.00	1.00	1.00
	2.3	0.22	0.15	0.34	0.63	0.85	0.97	1.00	1.00	1.00	1.00	1.00
	2.4	0.25	0.18	0.36	0.63	0.87	0.96	0.99	1.00	1.00	1.00	1.00
	2.5	0.33	0.20	0.34	0.64	0.87	0.96	1.00	1.00	1.00	1.00	1.00
	2.6	0.41	0.22	0.36	0.64	0.87	0.97	0.99	1.00	1.00	1.00	1.00
	2.7	0.48	0.25	0.38	0.62	0.87	0.97	1.00	1.00	1.00	1.00	1.00
	2.8	0.59	0.30	0.39	0.65	0.87	0.97	0.99	1.00	1.00	1.00	1.00
	2.9	0.68	0.35	0.39	0.64	0.87	0.96	0.99	1.00	1.00	1.00	1.00
	3	0.77	0.42	0.41	0.66	0.87	0.96	0.99	1.00	1.00	1.00	1.00

Table 0.33 Power of supMZ (Cauchy Distribution N=120)

Coefficient change	Test	Variance change										
		PsupMZ	1	1.2	1.4	1.6	1.8	2	2.2	2.4	2.6	2.8
	1	0.04	0.66	0.98	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.1	0.04	0.66	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.2	0.06	0.70	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.3	0.07	0.71	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.4	0.09	0.76	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.5	0.11	0.78	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.6	0.17	0.81	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.7	0.22	0.88	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.8	0.31	0.90	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.9	0.41	0.93	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	2	0.54	0.96	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	2.1	0.65	0.98	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	2.2	0.78	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	2.3	0.85	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	2.4	0.93	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	2.5	0.96	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	2.6	0.98	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	2.7	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	2.8	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	2.9	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	3	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Table 0.34 Power of supF (Additive Sequence Distribution N=120)

Coefficient change	Test	Variance change										
		1	1.2	1.4	1.6	1.8	2	2.2	2.4	2.6	2.8	3
	PsupF											
	1	0.05	0.05	0.05	0.05	0.05	0.05	0.06	0.06	0.05	0.06	0.06
	1.025	0.31	0.23	0.14	0.11	0.10	0.09	0.09	0.07	0.09	0.08	0.08
	1.05	0.91	0.83	0.68	0.55	0.43	0.32	0.27	0.22	0.19	0.17	0.16
	1.075	1.00	1.00	0.99	0.96	0.90	0.81	0.71	0.63	0.51	0.45	0.35
	1.1	1.00	1.00	1.00	1.00	1.00	0.99	0.97	0.93	0.88	0.80	0.71
	1.125	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.98	0.97	0.94
	1.15	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.175	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.2	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.225	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.25	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.275	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.3	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.325	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.35	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.375	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.4	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.425	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.45	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.475	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.5	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.525	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.55	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.575	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.6	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.625	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.65	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.675	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.7	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Table 0.35 Power of supGQ (Additive Sequence Distribution N=120)

Coefficient change	Test	Variance change										
		PsupGQ	1	1.2	1.4	1.6	1.8	2	2.2	2.4	2.6	2.8
	1	0.05	0.14	0.40	0.68	0.89	0.97	1.00	1.00	1.00	1.00	1.00
	1.025	0.07	0.15	0.39	0.69	0.89	0.98	0.99	1.00	1.00	1.00	1.00
	1.05	0.10	0.19	0.41	0.71	0.90	0.98	0.99	1.00	1.00	1.00	1.00
	1.075	0.21	0.25	0.46	0.74	0.91	0.98	1.00	1.00	1.00	1.00	1.00
	1.1	0.39	0.40	0.56	0.79	0.91	0.98	1.00	1.00	1.00	1.00	1.00
	1.125	0.71	0.61	0.72	0.85	0.94	0.99	1.00	1.00	1.00	1.00	1.00
	1.15	0.92	0.82	0.84	0.91	0.97	1.00	1.00	1.00	1.00	1.00	1.00
	1.175	1.00	0.95	0.94	0.97	0.99	1.00	1.00	1.00	1.00	1.00	1.00
	1.2	1.00	0.99	0.99	0.99	0.99	1.00	1.00	1.00	1.00	1.00	1.00
	1.225	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.25	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.275	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.3	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.325	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.35	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.375	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.4	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.425	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.45	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.475	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.5	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.525	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.55	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.575	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.6	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.625	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.65	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.675	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.7	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Table 0.36 Power of supMZ (Additive Sequence Distribution N=120)

Test	Variance change										
	PsupMZ	1	1.2	1.4	1.6	1.8	2	2.2	2.4	2.6	2.8
1	0.05	0.65	0.98	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.025	0.08	0.74	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.05	0.27	0.89	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.075	0.72	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.1	0.96	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.125	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.15	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.175	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.2	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.225	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.25	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.275	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.3	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.325	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.35	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.375	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.4	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.425	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.45	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.475	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.5	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.525	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.55	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.575	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.6	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.625	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.65	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.675	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.7	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Table 0.37 Power of supF (Standard Normal Distribution N=240)

Coefficient change	Test	Variance change										
		PsupF	1.00	1.20	1.40	1.60	1.80	2.00	2.20	2.40	2.60	2.80
1.00	1.00	0.053	0.043	0.048	0.066	0.050	0.060	0.058	0.065	0.058	0.060	0.068
1.20	1.20	0.146	0.111	0.079	0.082	0.072	0.070	0.081	0.075	0.076	0.063	0.077
1.40	1.40	0.483	0.375	0.270	0.203	0.156	0.141	0.106	0.115	0.114	0.113	0.090
1.60	1.60	0.891	0.777	0.627	0.503	0.388	0.316	0.250	0.223	0.175	0.162	0.148
1.80	1.80	0.992	0.968	0.924	0.832	0.721	0.615	0.511	0.407	0.341	0.285	0.233
2.00	2.00	1.000	0.998	0.993	0.974	0.944	0.863	0.769	0.673	0.582	0.482	0.424
2.20	2.20	1.000	1.000	1.000	0.999	0.995	0.980	0.934	0.887	0.808	0.723	0.635
2.40	2.40	1.000	1.000	1.000	1.000	1.000	0.999	0.991	0.977	0.951	0.903	0.843
2.60	2.60	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.998	0.988	0.971	0.955
2.80	2.80	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.996	0.990
3.00	3.00	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.999

Table 0.38 Power of supGQ (Standard Normal Distribution N=240)

Coefficient change	Test	Variance change										
		PsupGQ	1.00	1.20	1.40	1.60	1.80	2.00	2.20	2.40	2.60	2.80
1.00	1.00	0.04	0.22	0.70	0.96	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.20	1.20	0.05	0.22	0.70	0.95	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.40	1.40	0.04	0.24	0.71	0.96	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.60	1.60	0.05	0.24	0.72	0.97	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.80	1.80	0.09	0.25	0.71	0.95	1.00	1.00	1.00	1.00	1.00	1.00	1.00
2.00	2.00	0.12	0.28	0.74	0.97	1.00	1.00	1.00	1.00	1.00	1.00	1.00
2.20	2.20	0.21	0.33	0.75	0.97	1.00	1.00	1.00	1.00	1.00	1.00	1.00
2.40	2.40	0.32	0.40	0.77	0.97	1.00	1.00	1.00	1.00	1.00	1.00	1.00
2.60	2.60	0.46	0.47	0.80	0.97	1.00	1.00	1.00	1.00	1.00	1.00	1.00
2.80	2.80	0.67	0.56	0.84	0.98	1.00	1.00	1.00	1.00	1.00	1.00	1.00
3.00	3.00	0.81	0.68	0.88	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Table 0.39 Power of supMZ (Standard Normal Distribution N=240)

Coefficient change	Test	Variance change										
		PsupMZ	1.00	1.20	1.40	1.60	1.80	2.00	2.20	2.40	2.60	2.80
	1.00	0.06	0.91	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.20	0.07	0.91	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.40	0.09	0.94	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.60	0.17	0.96	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.80	0.31	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	2.00	0.56	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	2.20	0.83	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	2.40	0.95	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	2.60	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	2.80	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	3.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Table 0.40 Power of F and supF (Chi Square Distribution N=240)

Coefficient change	Test	Variance change										
		PsupF	1	1.2	1.4	1.6	1.8	2	2.2	2.4	2.6	2.8
	1	0.05	0.04	0.04	0.04	0.05	0.05	0.04	0.05	0.06	0.06	0.05
	1.025	0.05	0.04	0.05	0.04	0.04	0.05	0.05	0.06	0.05	0.05	0.05
	1.05	0.09	0.07	0.06	0.06	0.06	0.05	0.06	0.06	0.06	0.05	0.06
	1.075	0.15	0.12	0.09	0.07	0.07	0.07	0.06	0.06	0.05	0.06	0.06
	1.1	0.30	0.19	0.14	0.11	0.10	0.09	0.08	0.10	0.07	0.07	0.08
	1.125	0.46	0.33	0.22	0.20	0.14	0.10	0.12	0.10	0.09	0.09	0.09
	1.15	0.64	0.51	0.36	0.28	0.21	0.17	0.14	0.12	0.12	0.11	0.11
	1.175	0.82	0.68	0.53	0.41	0.30	0.24	0.19	0.18	0.15	0.13	0.12
	1.2	0.92	0.82	0.70	0.53	0.44	0.34	0.26	0.21	0.19	0.15	0.15
	1.225	0.97	0.92	0.81	0.70	0.59	0.45	0.36	0.30	0.25	0.20	0.17
	1.25	0.99	0.97	0.92	0.83	0.71	0.58	0.48	0.39	0.31	0.27	0.22
	1.275	1.00	0.99	0.96	0.89	0.82	0.70	0.59	0.48	0.38	0.33	0.27
	1.3	1.00	1.00	0.99	0.96	0.90	0.83	0.71	0.60	0.50	0.44	0.34
	1.325	1.00	1.00	1.00	0.99	0.94	0.89	0.82	0.73	0.61	0.52	0.44
	1.35	1.00	1.00	1.00	0.99	0.98	0.94	0.89	0.83	0.71	0.63	0.52
	1.375	1.00	1.00	1.00	1.00	0.99	0.98	0.93	0.88	0.79	0.74	0.61
	1.4	1.00	1.00	1.00	1.00	1.00	0.99	0.97	0.93	0.87	0.79	0.72
	1.425	1.00	1.00	1.00	1.00	1.00	1.00	0.99	0.97	0.93	0.84	0.79
	1.45	1.00	1.00	1.00	1.00	1.00	1.00	0.99	0.98	0.96	0.90	0.87
	1.475	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.99	0.98	0.96	0.90
	1.5	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.99	0.98	0.94
	1.525	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.98	0.96
	1.55	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.99	0.98
	1.575	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.99
	1.6	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.625	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.65	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.675	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.7	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Table 0.41 Power of supGQ (Chi Square Distribution N=240)

Test	Variance change											
	PsupGQ	1	1.2	1.4	1.6	1.8	2	2.2	2.4	2.6	2.8	3
Coefficient change	1	0.04	0.26	0.75	0.96	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.025	0.05	0.24	0.72	0.97	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.05	0.05	0.23	0.74	0.97	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.075	0.05	0.26	0.71	0.96	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.1	0.05	0.25	0.74	0.97	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.125	0.05	0.26	0.73	0.97	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.15	0.07	0.25	0.74	0.96	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.175	0.06	0.26	0.73	0.96	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.2	0.08	0.27	0.72	0.97	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.225	0.08	0.26	0.74	0.96	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.25	0.10	0.26	0.75	0.97	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.275	0.11	0.29	0.75	0.97	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.3	0.13	0.32	0.75	0.97	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.325	0.15	0.33	0.74	0.97	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.35	0.18	0.35	0.79	0.98	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.375	0.22	0.36	0.78	0.97	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.4	0.27	0.38	0.79	0.98	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.425	0.32	0.42	0.80	0.98	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.45	0.39	0.46	0.80	0.98	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.475	0.44	0.48	0.82	0.97	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.5	0.51	0.51	0.84	0.98	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.525	0.60	0.56	0.84	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.55	0.66	0.60	0.87	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.575	0.74	0.63	0.88	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.6	0.80	0.69	0.89	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.625	0.86	0.74	0.91	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.65	0.90	0.79	0.91	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.675	0.93	0.83	0.92	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.7	0.96	0.87	0.95	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Table 0.42 Power of supMZ (Chi Square Distribution N=240)

Test	Variance change										
	PsupMZ	1	1.2	1.4	1.6	1.8	2	2.2	2.4	2.6	2.8
Coefficient change	1	0.05	0.91	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.025	0.05	0.91	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.05	0.06	0.91	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.075	0.07	0.92	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.1	0.07	0.94	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.125	0.09	0.94	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.15	0.13	0.94	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.175	0.15	0.96	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.2	0.19	0.96	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.225	0.26	0.98	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.25	0.32	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.275	0.43	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.3	0.55	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.325	0.64	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.35	0.72	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.375	0.82	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.4	0.89	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.425	0.94	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.45	0.97	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.475	0.98	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.5	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.525	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.55	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.575	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.6	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.625	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.65	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.675	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.7	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Table 0.43 Power of F and supF (Cauchy Distribution N=240)

Coefficient change	Test	Variance change										
		PsupF	1	1.2	1.4	1.6	1.8	2	2.2	2.4	2.6	2.8
	1	0.05	0.04	0.05	0.05	0.05	0.04	0.05	0.06	0.06	0.06	0.07
	1.1	0.08	0.06	0.06	0.05	0.06	0.06	0.06	0.07	0.07	0.07	0.07
	1.2	0.16	0.12	0.09	0.09	0.07	0.07	0.07	0.07	0.06	0.06	0.07
	1.3	0.37	0.27	0.18	0.14	0.10	0.10	0.10	0.08	0.08	0.08	0.07
	1.4	0.65	0.49	0.36	0.25	0.20	0.16	0.13	0.11	0.10	0.09	0.09
	1.5	0.86	0.73	0.58	0.44	0.34	0.27	0.22	0.16	0.15	0.13	0.13
	1.6	0.96	0.92	0.81	0.66	0.56	0.42	0.34	0.27	0.22	0.18	0.14
	1.7	0.99	0.98	0.94	0.86	0.74	0.62	0.49	0.39	0.33	0.26	0.22
	1.8	1.00	1.00	0.99	0.95	0.89	0.78	0.68	0.55	0.47	0.39	0.30
	1.9	1.00	1.00	1.00	0.99	0.96	0.91	0.85	0.72	0.62	0.54	0.44
	2	1.00	1.00	1.00	1.00	0.99	0.98	0.93	0.88	0.76	0.67	0.56
	2.1	1.00	1.00	1.00	1.00	1.00	0.99	0.98	0.94	0.89	0.82	0.72
	2.2	1.00	1.00	1.00	1.00	1.00	1.00	0.99	0.98	0.96	0.91	0.84
	2.3	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.98	0.96	0.93
	2.4	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.99	0.97
	2.5	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.99
	2.6	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	2.7	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	2.8	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	2.9	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	3	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Table 0.44 Power of supGQ (Cauchy Distribution N=240)

Coefficient change	Test	Variance change										
		PsupGQ	1	1.2	1.4	1.6	1.8	2	2.2	2.4	2.6	2.8
	1	0.04	0.23	0.70	0.96	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.1	0.05	0.24	0.72	0.97	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.2	0.05	0.25	0.72	0.97	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.3	0.05	0.24	0.72	0.97	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.4	0.06	0.23	0.72	0.97	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.5	0.06	0.22	0.73	0.96	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.6	0.09	0.27	0.70	0.97	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.7	0.10	0.26	0.73	0.97	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.8	0.13	0.28	0.71	0.96	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.9	0.17	0.29	0.73	0.98	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	2	0.24	0.29	0.73	0.97	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	2.1	0.32	0.37	0.76	0.98	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	2.2	0.41	0.39	0.76	0.97	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	2.3	0.53	0.46	0.77	0.97	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	2.4	0.65	0.50	0.80	0.98	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	2.5	0.76	0.58	0.82	0.97	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	2.6	0.85	0.65	0.85	0.98	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	2.7	0.92	0.72	0.86	0.98	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	2.8	0.96	0.82	0.90	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	2.9	0.99	0.86	0.92	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	3	1.00	0.92	0.94	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Table 0.45 Power of supMZ (Cauchy Distribution N=240)

Coefficient change	Test	Variance change										
		PsupMZ	1	1.2	1.4	1.6	1.8	2	2.2	2.4	2.6	2.8
	1	0.05	0.90	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.1	0.05	0.90	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.2	0.06	0.92	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.3	0.09	0.92	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.4	0.11	0.95	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.5	0.17	0.96	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.6	0.26	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.7	0.38	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.8	0.53	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.9	0.68	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	2	0.81	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	2.1	0.92	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	2.2	0.97	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	2.3	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	2.4	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	2.5	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	2.6	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	2.7	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	2.8	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	2.9	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	3	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Table 0.46 Power of supF (Additive Sequence Distribution N=240)

Coefficient change	Test	Variance change										
		PsupF	1	1.2	1.4	1.6	1.8	2	2.2	2.4	2.6	2.8
	1	0.04	0.04	0.05	0.04	0.06	0.06	0.06	0.05	0.06	0.05	0.07
	1.025	0.62	0.44	0.34	0.25	0.19	0.15	0.13	0.12	0.11	0.11	0.08
	1.05	1.00	0.99	0.97	0.93	0.86	0.75	0.65	0.53	0.43	0.37	0.28
	1.075	1.00	1.00	1.00	1.00	1.00	1.00	0.99	0.97	0.94	0.87	0.80
	1.1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.99
	1.125	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.15	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.175	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.2	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.225	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.25	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.275	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.3	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.325	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.35	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.375	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.4	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.425	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.45	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.475	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.5	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.525	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.55	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.575	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.6	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.625	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.65	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.675	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.7	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Table 0.47 Power of supGQ (Additive Sequence Distribution N=240)

Coefficient change	Test	Variance change									
		1	1.2	1.4	1.6	1.8	2	2.2	2.4	2.6	2.8
	PsupGQ										
	1	0.04	0.21	0.70	0.96	1.00	1.00	1.00	1.00	1.00	1.00
	1.025	0.05	0.21	0.69	0.96	1.00	1.00	1.00	1.00	1.00	1.00
	1.05	0.09	0.27	0.70	0.96	1.00	1.00	1.00	1.00	1.00	1.00
	1.075	0.27	0.35	0.75	0.96	1.00	1.00	1.00	1.00	1.00	1.00
	1.1	0.69	0.56	0.85	0.98	1.00	1.00	1.00	1.00	1.00	1.00
	1.125	0.96	0.83	0.93	0.99	1.00	1.00	1.00	1.00	1.00	1.00
	1.15	1.00	0.99	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.175	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.2	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.225	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.25	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.275	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.3	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.325	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.35	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.375	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.4	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.425	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.45	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.475	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.5	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.525	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.55	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.575	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.6	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.625	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.65	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.675	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.7	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Table 0.48 Power of supMZ (Additive Sequence Distribution N=240)

Test	Variance change										
	PsupMZ	1	1.2	1.4	1.6	1.8	2	2.2	2.4	2.6	2.8
1	0.05	0.91	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.025	0.11	0.94	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.05	0.45	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.075	0.93	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.125	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.15	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.175	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.2	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.225	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.25	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.275	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.3	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.325	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.35	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.375	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.4	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.425	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.45	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.475	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.5	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.525	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.55	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.575	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.6	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.625	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.65	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.675	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.7	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Table 0.49 Power of supF (Standard Normal Distribution N=480)

Coefficient change	Test	Variance change											
		PsupF	1	1.2	1.4	PF	1	1.2	1.4	PF	1	1.2	1.4
	1.00	0.05	0.04	0.05	0.05	0.05	0.06	0.05	0.05	0.06	0.05	0.06	0.06
	1.20	0.23	0.17	0.12	0.11	0.09	0.07	0.08	0.07	0.07	0.08	0.07	0.07
	1.40	0.83	0.69	0.58	0.40	0.34	0.27	0.22	0.17	0.16	0.13	0.11	
	1.60	1.00	0.99	0.95	0.88	0.78	0.67	0.55	0.45	0.39	0.32	0.27	
	1.80	1.00	1.00	1.00	1.00	0.98	0.95	0.88	0.81	0.73	0.64	0.55	
	2.00	1.00	1.00	1.00	1.00	1.00	1.00	0.99	0.98	0.95	0.90	0.82	
	2.20	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.99	0.97	
	2.40	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	
	2.60	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	
	2.80	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	
	3.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	

Table 0.50 Power of supGQ (Standard Normal Distribution N=480)

Coefficient change	Test	Variance change											
		PsupGQ	1.00	1.20	1.40	1.60	1.80	2.00	2.20	2.40	2.60	2.80	3.00
	1.00	0.05	0.48	0.97	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	
	1.20	0.05	0.50	0.97	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	
	1.40	0.06	0.50	0.98	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	
	1.60	0.08	0.49	0.98	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	
	1.80	0.13	0.54	0.98	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	
	2.00	0.25	0.60	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	
	2.20	0.43	0.66	0.98	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	
	2.40	0.66	0.75	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	
	2.60	0.87	0.86	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	
	2.80	0.97	0.93	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	
	3.00	1.00	0.98	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	

Table 0.51 Power of supMZ (Standard Normal Distribution N=480)

Coefficient change	Test	Variance change										
		PsupMZ	1.00	1.20	1.40	1.60	1.80	2.00	2.20	2.40	2.60	2.80
	1.00	0.04	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.20	0.07	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.40	0.11	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.60	0.25	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.80	0.50	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	2.00	0.83	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	2.20	0.97	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	2.40	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	2.60	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	2.80	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	3.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Table 0.52 Power of F and supF (Chi Square Distribution N=480)

Coefficient change	Test	Variance change										
		PsupF	1	1.2	1.4	1.6	1.8	2	2.2	2.4	2.6	2.8
	1	0.05	0.04	0.05	0.06	0.05	0.05	0.05	0.07	0.07	0.06	0.06
	1.025	0.07	0.07	0.05	0.06	0.06	0.06	0.06	0.07	0.06	0.07	0.06
	1.05	0.15	0.10	0.10	0.08	0.08	0.09	0.09	0.08	0.07	0.07	0.07
	1.075	0.34	0.26	0.18	0.16	0.12	0.12	0.10	0.11	0.09	0.09	0.08
	1.1	0.61	0.47	0.33	0.26	0.21	0.17	0.14	0.13	0.12	0.11	0.10
	1.125	0.81	0.70	0.57	0.44	0.33	0.24	0.22	0.19	0.14	0.13	0.13
	1.15	0.95	0.87	0.77	0.64	0.52	0.41	0.33	0.28	0.23	0.20	0.17
	1.175	0.99	0.97	0.91	0.80	0.72	0.58	0.48	0.39	0.31	0.29	0.23
	1.2	1.00	0.99	0.97	0.93	0.85	0.75	0.65	0.53	0.44	0.38	0.33
	1.225	1.00	1.00	0.99	0.98	0.94	0.88	0.80	0.69	0.58	0.49	0.43
	1.25	1.00	1.00	1.00	1.00	0.99	0.96	0.90	0.83	0.74	0.63	0.56
	1.275	1.00	1.00	1.00	1.00	1.00	0.99	0.97	0.91	0.84	0.77	0.69
	1.3	1.00	1.00	1.00	1.00	1.00	1.00	0.99	0.96	0.92	0.87	0.80
	1.325	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.99	0.98	0.93	0.89
	1.35	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.99	0.99	0.97	0.95
	1.375	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.99	0.98
	1.4	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.99
	1.425	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.99
	1.45	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.475	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.5	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.525	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.55	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.575	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.6	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.625	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.65	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.675	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.7	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Table 0.53 Power of supGQ (Chi Square Distribution N=480)

Coefficient change	Test	Variance change										
		PsupGQ	1	1.2	1.4	1.6	1.8	2	2.2	2.4	2.6	2.8
	1	0.05	0.52	0.98	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.025	0.05	0.50	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.05	0.06	0.52	0.98	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.075	0.06	0.50	0.98	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.1	0.06	0.49	0.98	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.125	0.06	0.51	0.98	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.15	0.07	0.51	0.98	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.175	0.08	0.51	0.98	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.2	0.09	0.53	0.98	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.225	0.12	0.53	0.98	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.25	0.14	0.53	0.98	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.275	0.18	0.57	0.98	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.3	0.22	0.58	0.98	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.325	0.28	0.61	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.35	0.34	0.63	0.98	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.375	0.43	0.65	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.4	0.52	0.70	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.425	0.64	0.71	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.45	0.73	0.76	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.475	0.81	0.79	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.5	0.88	0.84	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.525	0.94	0.86	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.55	0.97	0.90	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.575	0.99	0.93	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.6	0.99	0.96	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.625	1.00	0.97	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.65	1.00	0.98	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.675	1.00	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.7	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Table 0.54 Power of supMZ (Chi Square Distribution N=480)

Test	Variance change										
	PsupMZ	1	1.2	1.4	1.6	1.8	2	2.2	2.4	2.6	2.8
1	0.05	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.025	0.06	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.05	0.07	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.075	0.07	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.1	0.09	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.125	0.13	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.15	0.17	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.175	0.23	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.2	0.29	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.225	0.41	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.25	0.52	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.275	0.63	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.3	0.77	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.325	0.87	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.35	0.93	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.375	0.98	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.4	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.425	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.45	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.475	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.5	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.525	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.55	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.575	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.6	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.625	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.65	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.675	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.7	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Table 0.55 Power of supF (Cauchy Distribution N=480)

Coefficient change	Test	Variance change										
		PsupF	1	1.2	1.4	1.6	1.8	2	2.2	2.4	2.6	2.8
	1	0.04	0.05	0.05	0.05	0.05	0.06	0.06	0.06	0.06	0.06	0.06
	1.1	0.11	0.10	0.08	0.08	0.07	0.07	0.07	0.06	0.06	0.07	0.08
	1.2	0.38	0.29	0.21	0.17	0.12	0.12	0.10	0.10	0.09	0.09	0.09
	1.3	0.77	0.65	0.50	0.37	0.30	0.24	0.22	0.16	0.14	0.14	0.11
	1.4	0.97	0.92	0.82	0.71	0.56	0.45	0.37	0.31	0.27	0.21	0.18
	1.5	1.00	0.99	0.97	0.92	0.85	0.73	0.61	0.54	0.45	0.38	0.31
	1.6	1.00	1.00	1.00	0.99	0.97	0.93	0.84	0.77	0.65	0.56	0.51
	1.7	1.00	1.00	1.00	1.00	1.00	0.99	0.96	0.91	0.86	0.78	0.71
	1.8	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.98	0.96	0.92	0.88
	1.9	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.99	0.98	0.95
	2	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.99
	2.1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	2.2	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	2.3	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	2.4	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	2.5	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	2.6	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	2.7	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	2.8	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	2.9	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	3	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Table 0.56 Power of supGQ (Cauchy Distribution N=480)

Coefficient change	Test	Variance change										
		PsupGQ	1	1.2	1.4	1.6	1.8	2	2.2	2.4	2.6	2.8
	1	0.04	0.49	0.98	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.1	0.05	0.50	0.97	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.2	0.05	0.49	0.98	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.3	0.06	0.51	0.98	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.4	0.07	0.52	0.98	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.5	0.09	0.53	0.98	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.6	0.12	0.54	0.98	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.7	0.17	0.60	0.98	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.8	0.27	0.63	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.9	0.37	0.65	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	2	0.53	0.73	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	2.1	0.67	0.79	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	2.2	0.80	0.84	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	2.3	0.92	0.90	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	2.4	0.97	0.94	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	2.5	0.99	0.98	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	2.6	1.00	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	2.7	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	2.8	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	2.9	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	3	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Table 0.57 Power of supMZ (Cauchy Distribution N=480)

Coefficient change	Test	Variance change										
		PsupMZ	1	1.2	1.4	1.6	1.8	2	2.2	2.4	2.6	2.8
	1	0.05	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.1	0.06	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.2	0.08	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.3	0.11	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.4	0.17	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.5	0.32	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.6	0.50	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.7	0.70	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.8	0.86	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.9	0.95	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	2	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	2.1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	2.2	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	2.3	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	2.4	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	2.5	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	2.6	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	2.7	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	2.8	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	2.9	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	3	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Table 0.58 Power of supF (Additive Sequence Distribution N=480)

Coefficient change	Test	Variance change										
		1	1.2	1.4	1.6	1.8	2	2.2	2.4	2.6	2.8	3
	PsupF											
	1	0.05	0.05	0.05	0.06	0.06	0.06	0.06	0.07	0.07	0.07	0.08
	1.025	0.94	0.87	0.75	0.63	0.52	0.41	0.31	0.27	0.22	0.20	0.18
	1.05	1.00	1.00	1.00	1.00	1.00	0.99	0.98	0.96	0.91	0.86	0.78
	1.075	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.125	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.15	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.175	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.2	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.225	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.25	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.275	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.3	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.325	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.35	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.375	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.4	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.425	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.45	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.475	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.5	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.525	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.55	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.575	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.6	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.625	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.65	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.675	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.7	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Table 0.59 Power of supGQ (Additive Sequence Distribution N=480)

Coefficient change	Test	Variance change										
		PsupGQ	1	1.2	1.4	1.6	1.8	2	2.2	2.4	2.6	2.8
	1	0.05	0.49	0.98	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.025	0.05	0.49	0.98	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.05	0.19	0.57	0.98	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.075	0.66	0.75	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.1	0.98	0.94	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.125	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.15	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.175	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.2	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.225	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.25	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.275	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.3	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.325	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.35	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.375	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.4	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.425	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.45	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.475	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.5	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.525	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.55	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.575	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.6	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.625	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.65	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.675	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.7	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Table 0.60 Power of supMZ (Additive Sequence Distribution N=480)

Coefficient change	Test	Variance change										
		PsupMZ	1	1.2	1.4	1.6	1.8	2	2.2	2.4	2.6	2.8
	1	0.04	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.025	0.14	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.05	0.70	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.075	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.125	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.15	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.175	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.2	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.225	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.25	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.275	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.3	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.325	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.35	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.375	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.4	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.425	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.45	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.475	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.5	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.525	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.55	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.575	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.6	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.625	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.65	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.675	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.7	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00