ON SUPER EXOGENEITY: A COMPARISON OF TESTING PROCEDURES

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DEDICATION

This piece of work is dedicated

To my father ***Muhammad Muzaffar*** *(Late)*

To my mother ***Shehnaz Akhtar***

&

To all my family members

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DISCLAIMER

This document represents a part of the author's PhD thesis while at the Pakistan Institute of Development Economics, *PIDE*. The views stated therein are those of the author and not necessarily those of the Institute.

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In the name of Allah, the Lord of Mercy, the Bestower of Mercy.
All praise and glory belongs to Allah, Lord of the worlds. The Lord of Mercy, the Bestower of Mercy. Master of the Day of Judgment. You alone we worship; You alone we ask for help. Guide us to the straight path. The way of those on whom You have bestowed Your grace, Not of those who earned Your anger nor of those who went astray.

QUR'AN 1:1-7

ABSTRACT

There is hardly a concept in econometrics that is more mystifying and challenging than that of exogeneity in particular and its types in general. The study under considerations is an endeavor to argue that exogeneity is rather a simple concept, readily definable in terms of standard econometric models and deterministic relationship, and that the conceptual mystery primarily stems from improper usage of statistical vocabulary in a dynamic framework. Since, one purpose of econometric analysis is to device models for policy implications primarily relating to the concept of super exogeneity (SupExt, hereafter) sensing the underlying causal relation. There is an ample amount of tests that are available in literature for testing SupExt. The study compares the performance of SupExt tests on the basis of their size and power using Monte Carlo simulations combines with recently developed techniques of selecting data driven breaks or location shifts *i.e.* Indicator Saturation (like; *Impulse Indicator Saturation (IIS), Step Indicator Saturation (SIS) & Trend Indicator Saturation (TIS)*). To the best of our knowledge the performance of SupExt tests under stationary, non-stationary and dynamic settings using Monte Carlo Simulation has not been compared particularly combined with above mentioned break selections. Also, the study is not just limited to testing the performance of SupExt tests under IIS, SIS & TIS separately but it further extends its horizons to gauge the size and power of SupExt tests taking all these type of breaks into account jointly as well. While this is the main theoretical contribution of our study, from applied side; the study uses time series data to testify the hypothesis regarding SupExt of putative regressors using IIS, SIS & TIS in money demand function in case of Pakistan. The relevance of the empirical study to model money demand using tests of SupExt further strengthens the argument on a growing literature that empirically refutes the Lucas Critique for the determination of specific macroeconomic variables, resulting in to ensure the validity of policy simulations based on conditional model alone. After a detailed stability analysis in terms of SupExt testing of putative regressors in estimated money demand model, the empirical exercise concludes that the dynamic VECM is stable against the relevant class of interventions. Hence, estimated parsimonious model can be used for policy simulations. While taking power and size of these tests into account, one can use the optimal testing procedure to confirm the existence of SupExt of contemporaneous conditioning variables in their estimated model. Therefore, on the basis of detailed simulations, the study concludes that whatever is the type of break whether it is IIS, SIS & TIS or we used all these at a time (jointly), the test like *IB-Test* and *RB-Test* outperforms the performance of other SupExt tests. At the end, the power of the tests is increased significantly using all breaks at a time. Therefore, we recommend while testing SupExt using all breaks at time is more informative and useful than what is found to be in individual scenarios. The comparison of such SupExt testing procedures and selecting the best test out of these further blurs the demarcation of the critique raised by Lucas. Lastly, as far as the economic significance is concerned, the application of SupExt tests is not limited to test the Lucas critique but also help policy makers identify the existence of famous Ricardian equivalence indirectly as well.

Keywords: Exogeneity; Indicator Saturation; Super Exogeneity Tests; Simulation Analysis; Performance; Comparison; Money Demand

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LIST OF ABBERIVATIONS

ADF	Augmented Dickey-Fuller
AIC	Akaike Information Criterion
API	Asset Price Index
ARCH	Autoregressive Conditional Heteroskedasticity
ARDL	Autoregressive Distributed Lag
BIC	Bayesian information Criterion
CB-Test	Co-breaking Test
CK-Test	Charemza & Király Test
CVAR	Cointegrated Vector Autoregressive
DGP/s	Data Generating Process/es
DIB-Test	Double Index Based Test of Super Exogeneity
ECM	Error Correction Mechanism
e.g.	For Example
et al.	And others
etc.	And so forth
EG	Engle Granger
EHR	Engle, Hendry and Richards
FGLS	Feasible Generalized Least Square
FPE	Final Prediction Error
GC	Granger Causality
GETS	General-*to*-Specific
H-Test	Hendry Test
HQ	Hannan-Quinn
IB-Test	Index Based Test of Super Exogeneity
IFS	International Financial Statistics
i.i.d	Independent and Identically Distributes
IIS	Impulse Indicator Saturation
IRC	Interest Rate Corridor
IV	Instrumental Variables

JB	Jarque Bera
JJ	Johansen and Juselius
KPSS	Kwiatkowski Phillips Schmidt Shin
LDGP/s	Local Data Generating Process/es
LARD	Least Angle Regression
LASSO	Least Absolute Shrinkage and Selection Operator
LM	Lagrange Multiplier
LR	Likelihood Ratio
LSE	London School of Economics
MLE	Maximum Likelihood Estimate
OLS	Ordinary Least Square
PP	Phillips-Perron
RB-Test	Residual Based Test
SBP	State Bank of Pakistan
SC	Schwarz Criterion
SEM	Simultaneous Equation Model
SIC	Schwarz Information Criterion
SIS	Step Indicator Saturation
SL	Significance Level
StExt	Strong Exogeneity
STR	Smooth Transition Regression
SupExt	Super Exogeneity
s.t.	Such That
TIS	Trend Indicator Saturation
UK	United Kingdom
US	United States
VAR	Vector Autoregressive
VECM	Vector Error Correction Model
WDI	World Development Indicator
WeExt	Weak Exogeneity
w.r.t	With Respect to

CHAPTER 1

INTRODUCTION

This part of the study is designated to two unique and enormous school of thoughts exist in the field of econometrics enveloping exogeneity in general and how SupExt and its testing procedures came up to the canvas in particular. The one is Cowles Commission approach to exogeneity rendering endogeneity along with simultaneity as a part of Simultaneous Equations Model (SEM, hereafter) was mainly due to Koopmans and Haavelmo. The other is Engle, Hendry and Richard (EHR, hereafter) exogeneity and its types. Specifically, this section is an effort to incorporate how the concept of exogeneity and its types prevails over the time as researchers from these two schools of thought provide substantial evidences (research articles) on the issue from early 1940's to until EHR in 1983. One deals with causal description while other deals with statistical inference. The discussion here is a scuffle to reduce the density of the entanglements between these two thoughts and the journey of the term exogenous from (Tinbergen, 1940) to almost forty years later exogeneity by (Engle et al., 1983), its types *i.e.* Weak Exogeneity, Strong Exogeneity and especially *"Super Exogeneity"* (**SupExt**).

The importance of SupExt has been well documented in (Pearl, 2000, 2010) and (Hoover, 2001) in which they considered SupExt as an underlying causal structure if hold, and in many other as well like (Ahumada, 1992; Ericsson & Irons, 1994; Kurita, 2007). One purpose of econometric analysis is to device models for policy implications relating to the concept of SupExt (Hendry, 1988). In literature, numerous tests to identify the existence of SupExt have been developed (Charemza & Király, 1988; Engle & Hendry, 1993; Favero & Hendry, 1992; Hendry & Santos, 2006, 2010; Jansen & Teräsvirta, 1996; Krolzig & Toro, 2002; Psaradakis & Sola, 1996) which helps us in refuting the famous Lucas critique (Lucas, 1976). However, the performance of these testing procedures has never been equated especially under the shade of indicator saturation proposed in (Ericsson, 2012; Hendry et al., 2008) by considering stationary, non-stationary and dynamic data settings forcing a positive contribution to existing literature. Therefore, it is increasingly important to gauge the power as well as the size of SupExt testing procedures under indicator saturation.

Tests of SupExt used in this study for comparison were mainly due to (Charemza & Király, 1988; Engle & Hendry, 1993; Hendry, 1988) and some automatic tests of SupExt proposed in (Hendry & Santos, 2006, 2010). The aim is to check performance of these SupExt tests in presence of structural breaks (not theory but data driven). The types of breaks like; Impulse Indicator Saturation (IIS), Step Indicator Saturation (SIS) and Trend Indicator Saturation (TIS) that we used here being introduced first in (Hendry et al., 2008; Johansen & Nielsen, 2008) and its extensions were discussed in (Ericsson, 2012). The study is a juxtaposition of how these tests behave under different DGPs like stationary, non-stationary and dynamic settings.

Last but not the least, if SupExt is a model trait, it is enough to use the conditional models for analyzing the effects of policy changes (Castle et al., 2015, 2017; Favero & Hendry, 1992; Hendry & Santos, 2006; Nymoen, 2019). The testing procedures opted in this study has real world application in refuting famous Lucas critique and as well as the existence of famous Ricardian equivalence (*see;* Castle et al., 2015; Das & Mandal, 2000; Ericsson et al., 1998; Ericsson & Irons, 1994, 1995;

Favero & Hendry, 1992; Kónya & Abdullaev, 2015; Pretis, 2017; Qayyum, 2005b; Sachsida & Cardoso de Mendonça, 2006; Sachsida & Teixeira, 2000; Sachsida et al., 2010; Togay & Kose, 2013).

The discussion available in literature so far raises importance of testing SupExt by simply applying these tests. Nevertheless the literature didn't gauge which testing procedure should be adopted while testing SupExt under different data settings (stationary, non-stationary & dynamic) nor tells us about the selection of the structural breaks and its types. Therefore, this simulation study fill this gap in by addressing the questions raised just above. The application of this study in real world scenario is not limited to the dynamic macroeconometric modeling (*e.g.* the empirical modelling of money demand, wages, unemployment, prices, expenditure and federal reserves etc.) only but also allows us to move towards fully-coupled empirical climate-economic[1] models accounting for the necessary feedback to obtain empirical estimates (Pretis, 2021).

The following subsection will shed a light on how the term exogenous and exogeneity framed in Cowles and EHR sense of exogeneity. We will try to envelop these two schools of thought and how they inter linked with each other.

1.1 Cowles Exogeneity

Koopmans considered as a chief architect of the term *'Cowles Exogeneity'* and called exogenous variables as 'Determining Variables'. The researchers[2] at Cowles commission established a fact that the constancy in repeated samples was an important concept for new SEM analysis and figuring out Cowles Exogeneity as

[1] Climate Econometrics is a new research project launched at Nuffield College, University of Oxford in collaboration with the University of Victoria.

[2] Tjalling C. Koopmans, Jacob Marschak, Abraham Wald, Trygve Haavelmo, Leonid Hurwicz, Herman Rubin, Roy Leipnik and Lawrence Klein.

independence between disturbances and exogenous variables. The notion (constancy in repeated samples) had *apriori* existence and was not being considered as the part of SEM's formation till Haavelmo who joined the group in 1943; but an exception of (Koopmans, 1950) who pointed out that there is no loss of generality in treating the determining variables as fixed, considered as an ancestor of statistical completeness and weak exogeneity.

The theory of regression based on a series of work in (Fisher, 1925b, 1925d, 1925a, 1925c) was adapted by Koopmans to the model having errors in variables – but soon he accepted the point raised by (Frisch, 1934) that *"the economic variables being measured with errors has serious consequences for model estimation"*. He argued that this theory could not be applied to model those variables having measurement error. This argument need not to be labored as the research on estimating model having errors in variables was driven by the failure of least squares.

A dependent variable has two components, a systematic component and other one is an erratic component. The erratic part is the one, includes, measurement errors and the variables influencing the dependent variable but are not being added in the model (Frisch, 1934). Koopmans considered the systematic components as unknown parameters when estimating the errors in variables model in his search to estimate the parameters of causal relationship. However, the determination of exogeneity of a variable was a task of economic theory as discussed in (Koopmans, 1937). How Koopmans managed to combine both the causal and the inferential side has been discussed below.

The earlier work by (Haberler, 1937) described two theories; one which assume external disturbances and the other which rely solely on the movements that can be explained economically termed as Exogenous theories and Endogenous theories respectively. Following to what (Koopmans, 1950) pictured out two principles, *'The Departmental Principal'* and *'The Causal Principal'*. According to him, variables which rely solely or partially outside the canvas of economics are exogenous and their values remain fixed in repeated samples, and hence fall under the shade of Departmental Principal and those which influence the endogenous variables but are not influenced thereby fall under Causal Principal category. For Koopmans an equation is 'complete for statistical purpose' if its parameters are asymptotically unbiased. If so, then there is no need of specifying further equations; unlike to (Frisch, 1933b) and (Haavelmo, 1938) on account for a 'complete model'.

Taking into account the discussion on formalization of the causal principal, Koopmans argued that structural equations determining all the variables must be block recursive[3] and the errors associated to these blocks should be independent but leaving exogeneity of the variable unclear (Koopmans, 1945). A blurred form of exogeneity was introduced in (Haavelmo, 1947) therein he taken up the consistency by employing both Least Squares and Indirect Least Squares to Keynesian model selecting investment as exogenous variable in relation with consumption and income. Later, the stringency of exogeneity in block recursive models and the concept of predetermined variables[4] were introduced in (Koopmans, 1950). Koopmans & Hood (1953) applied the same block recursive methodology as proposed in (Koopmans, 1950) to a linear structure and define the concept of exogeneity and

[3] Suppose $\varphi(\alpha, X_t) = u_t$ is complete system where $X_t = (y_t, z_t)$ and $\varphi_1(\alpha_1, y_t, z_t) = u_{1t}$ and $\varphi_2(\alpha_2, z_t) = u_{2t}$ are two structural equations.

[4] A variable z_t is said to be predetermined at time t, if $E(z_t | u_t, u_{t+1}, u_{t+2}, \dots) = 0$

predeterminedness. They said that a variable is exogenous if it is independent of disturbances by means of all periods[5].

At this point, the Cowles Exogeneity via block recursiveness being considered as causal hierarchy where the first structural equation of the blocks is taken as fixed (*see*, footnote) for consistent estimation of the parameters of the model – using least square estimation technique with endogenous regressors is a big mistake. However, an essential point about the block recursiveness is that the disturbances associated with endogenous variables have no influence on the exogenous variables. Another mathematical observation was reported that the MLE[6] of subsystem's parameter α_1 was identical to that of the complete system. For more detailed review on Cowles Exogeneity (*see*, Chapter 2).

1.2 What Lacks in Cowles Exogeneity

Koopmans pledged the statistical necessities for the term exogeneity, but unfortunately was not sure about its validity in many applications with available data. However, in later practice, the problematic nature of the concept proposed by Koopmans in 1950, lead to some major critiques; (Orcutt, 1952c) discussed that the crucial assumption of orthogonality between exogenous and endogenous variables is "not operational". Also, (Orcutt, 1952a, 1952b) focused on the use of panel data to replace *apriori* classifications of exogeneity. He argued that econometrics could escape the identification trap by developing as an experimental science. By collecting data at the individual level one could hope to exploit better the process that was the source of the *apriori* knowledge in the first place. Koopmans (1952) argued that, in case where the null hypothesis of exogeneity for specific variable went false the

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[5] A variable x_t is said to be exogenous at time t, if $E(x_t|u_{t-1}, u_{t-2}, \dots u_t, u_{t+1}, u_{t+2}, \dots) = 0$
[6] Maximum Likelihood Estimate

parameter estimates went inconsistent. As a consequence it did not seem plausible to build their power functions that make a test more informative.

There were two main concerns related to exogeneity by that time; one was how actual world works and other was how a variable should be treated for estimation. The work by (Orcutt, 1952c) on correlation analysis and by (Simon, 1954) on causal ordering was mainly linked to the first concern. However, (Durbin, 1954) paper on the topic "Errors in Variables" was on the estimation concern. The study suggested the usefulness of Instrumental Variable (IV) to investigate the bias and remove it from estimation if found to be large. Later, (Sargan, 1958) provided a solution to the estimation procedures by introducing a theoretical and conceptual foundation to econometricians in terms of IV estimations along with general asymptotic distribution theory of IV estimators. The method of IV was originally explained in its modern form by a statistician (Reiersøl, 1945) on how to deal with the problem of errors in variables.

Later on (Sargan, 1964) proposed the idea to test the predeterminedness of IV and derived the asymptotic chi-square test. However, the leading econometricians did not concern about hypothesis testing at that time. Later, (Wu, 1973) and (Hausman, 1978) (a general approach) derived asymptotic chi-square tests of the hypothesis that there is no significant difference between the OLS and IV estimates.

1.3 The Role of Disturbance Term

The assumption the exogenous variables and the disturbances are independent or they have zero covariance seems to preclude the authors of EHR Exogeneity due to their critical approach towards the role disturbances in formulation of econometric models. Considering Koopmans and Haavelmo, their point was to develop structural equations with structural disturbances. From these structural equations[7], reduced form equations can be obtained which describes each endogenous variable as mean plus deviation[8]. Florens et al., (1976) has taken the reduced form into account via simultaneous equation model with all restrictions. From this point onwards, estimating parameters $\mathbf{B}, \mathbf{\Gamma}$ and $\mathbf{\Sigma}$ (*see*, footnote) there is no difference is selecting Cowles exogeneity or EHR exogeneity. In fact, disturbances from reduced form equations are more suitable because of their deviations from observable means. On this account, unlike to Cowles commission, Hendry describes in (Spanos, 1986) that the disturbances treated as derived but not autonomous.

Koopmans et al., (1950) emphasized that there exist only one structural representation but the evidence based on (Engle et al., 1983) in a bivariate case, where, they claimed that if one of the recursive representation is structural then other one is not and there is not testable difference between these two recursive blocks (*see*, footnote 3, above). Further, the usage of structural disturbances, lead a serious confusion. Richard (1980) argued that the errors being unobservable with zero covariance assumption has no implication to causality and exogeneity. Such criticism on exogeneity conditions further blurred the demarcation of faith in SEM's.

1.3.1 Exogeneity Paradox

In simple way, understanding the paradox of exogeneity, suppose we have the following model for instance:

$$y_t = \beta_0 + \beta_1 x_t + \beta_2 x_{t-1} + \varepsilon_t, \qquad \text{where } t = 1,2,3, \dots, T \tag{1.1}$$

Where, the variable x_t, the bone of contention (on lighter note), can be exogenous *or* it can be endogenous in econometric terms:

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[7] $\mathbf{B}\boldsymbol{y}_t + \mathbf{\Gamma}\boldsymbol{z}_t = \boldsymbol{u}_t, \boldsymbol{u}_t \sim \boldsymbol{N}(\mathbf{0}, \mathbf{\Sigma})$

[8] $\boldsymbol{y}_t = \mathbf{\Pi}\boldsymbol{z}_t + \boldsymbol{v}_t, \boldsymbol{u}_t \sim \boldsymbol{N}(\mathbf{0}, \mathbf{\Omega})$ with $\mathbf{\Pi} = -\mathbf{B}^{-1}\mathbf{\Gamma}$ and $\mathbf{\Omega} = \mathbf{B\Sigma B}^{-1}$

i) If there is no correlation between x_t and ε_t (but may not be with ε_{t-1}) *i.e.* $Corr(x_t, \varepsilon_t) = 0$, then x_t is said to be exogenous (or at least pre-determined)

ii) If $Corr(x_t, \varepsilon_t) \neq 0$, then it is endogenous.

Therefore, a paradox can be observed that the same variable x_t can either exogenous *or* endogenous at the same time and in a same model. The solution suggested for the problem is to establish suitable exogeneity assumptions *w.r.t* parameter of interest in which we are interested to model leads to the importance of testing SupExt.

1.4 EHR Exogeneity

EHR notation of exogeneity belongs to the canvas of statistical inference (Engle, 1980; Engle et al., 1983). However, importance of the issue of testing SupExt up to some extent is being discussed here.

SupExt is essentially an invariance concept (Engle et al., 1983). On one hand the parameters capturing the effects of variables such as tastes and technologies considered to be stable and remain invariant to policy fluctuations or breaks in other conditioning variables in the model under considerations. On the other hand, regression models that are specified as causal structure are prone to breaks when policy rules changes as argued by (Lucas, 1976). However, (Favero & Hendry, 1992) found evidence against the Lucas Critique in presence of SupExt of conditional model considering structural breaks in marginal models of putative regressors – this is what exactly the same that (Hoover, 2001) cites as serving to identify causal direction and which (Pearl, 2000, 2010) considered in terms of a causal structure. Although avoiding the term '*causality*', (Favero & Hendry, 1992) analysis of the Lucas critique in testing SupExt is also a variant on causal ordering posed by Simon in 1954

(Ericsson & Irons, 1994; Hoover, 2001). The following table is a classification of causal approaches in economics and can be helpful in identifying under which category Favero and Hendy SupExt testing procedure and others as well, falls.

Table 1.1: Classification of Approaches to Causality in Economics

	Structural	***Process***
Apriori	Cowles Commission: (Koopmans, 1950), (Koopmans & Hood, 1953a)	(Zellner, 1979)
Inferential	(Simon, 1953), (Hoover, 1990, 2001), (Favero & Hendry, 1992), **(Engle & Hendry, 1993),** **(Hendry & Santos, 2006, 2010)** Natural experiments: (Angrist & Krueger, 1999, 2001)	(Granger, 1969) Vector Auto Regressions: (Sims, 1980b)

Note: This table is extracted from (Hoover, 2006) with some changes made by the author as well.

In an article by (Koopmans, 1947) entitled *'Measurement Without Theory'*, he emphasized on a point that the current value of an economic variable is an accumulation of exogenous variables, a sequence of random shocks from recent past and the impulses exerted by exogenous variables. Further, he said, *"different impulses exerted successively by the same exogenous variables may produce different cycles of quite diverse appearance"*. So, our primary focus will remain on SupExt with constant conditional model parameters and structural invariance.

Several model selection algorithms are available in literature like; PcGets in (Hendry & Krolzig, 1999; Krolzig & Hendry, 2001), and more recently the introduction of *Autometrics* by (Doornik, 2007, 2009a). On shrinkage methods, the Least Absolute Shrinkage and Selection Operator (LASSO), introduced by (Tibshirani, 1996), and the adaptive LASSO proposed by (Zou, 2006), have received particular attention so far. However, on the selection of structural breaks experiments left a significant impact on the usefulness of *Autometrics* as compared with LASSO and Least Angle Regression (LARS) by (Efron et al., 2004), because LASSO and LARS work well for a single step shift, but once multiple breaks occur, because of the forward selection approach they adopt, selection over multiple step functions can fail to detect shifts, as no single step function is highly correlated with any of the multiple breaks (Castle et al., 2015). On the same fronts a recent key development in model selection *via* indicator saturation using R-Package *'**gets**'* by (Sucarrat et al., 2020) is freely available for all and has been successfully implemented in this study while modeling money demand in Pakistan (*see;* Chapter 6). However, the simulation analysis for this study is carried out in MATLAB. For details, on how the simulation designs is being opted in context of testing SupExt (*see;* Chapter 3-5) respectively.

The aim of the study is to highlight the significance of testing SupExt assumptions sensibly through the lens of (Charemza & Király, 1988, 1990; Engle & Hendry, 1993; Hendry, 1988; Hendry & Santos, 2006, 2010) when stipulating statistical models, in particular, when these models are to be used for the assessment of policies or intrusions. So, our primary emphasis will remain on SupExt with related concepts of parameter stability and invariance[9]. We will explain these testing procedures and compare their performance with amalgamation of Indicator Saturation and its types and further exemplify them under non- stationary and dynamic settings. Lastly, under policy changes, if SupExt is satisfied for the currently dated regressors, then the estimated conditional model can be taken as a feedback model (Hendry, 1988; Favero and Hendry, 1992). This type of the model encompasses a whole class of rational expectations models once SupExt happened that will invalidate the famous Lucas' critique (Hendry, 1988).

[9] Constancy is a property that parameters are independent of time while invariance is stability across interventions *i.e.* the process driving a variable does not change in the face of shocks.

1.4.1 The Problem of Exogeneity

The definitions of exogenous and endogenous variables encountered are often confusing. In this subsection, we will carefully define the exogeneity problem, with the help of econometric theory. For interests reader a collection of seminal papers on exogeneity can be found in (Ericsson & Irons, 1994), along with how the problem was step in to the structural equation modeling literature can be reach out in (Kaplan, 2004). Usually, the term that a variable is spawned from "outside the system" is another way of stating that the covariance between both the regressor and the error term is zero. However, by looking this statement closely make this statement a problematic because it doesn't clearly define what "outside the system" actually stands for.

As an example consider the problem of estimating the relationship between reading proficiency in young children as a function of parental reading activities (e.g., how often each week parents read to their children). We may represent this relationship by the simple model

$$y_t = \beta x_t + u_t \tag{1.2}$$

Where y_t represents reading proficiency, x_t represents the parental reading activities, β is the regression coefficient and u_t is the disturbance term, which is assumed to be *IID*(0, σ^2). The subscript t denotes the particular time point of measurement making a distinction that might be needed with the analysis of panel data. Now β is considered to be consistent if x_t is exogenous but what exogeneity in true sense means is not clear.

Typically, parental reading activities are treated as fixed. That is, at time t, levels of parental involvement in reading are assumed to be set and remain the same from that point on. If this assumption were true, then conditional estimation of reading proficiency given parental involvement in reading activities would be valid. However, it is probably not the case in practice that parental reading activities are fixed but rather are likely to be a function of past parental reading activities. That is, perhaps the mechanism that generates parental reading activities at time t is better represented by a first-order autoregressive model,

$$x_t = \gamma x_{t-1} + v_t \tag{1.3}$$

Where we will assume that $| \gamma | < 1$, ensuring a stable autoregressive process. Even if it were the case that the model in equation (1.3) generated parental reading activities *prior* to generating reading proficiency, that is still not a sufficient condition to render parental reading activities exogenous in this example. The reason is that such a condition does not preclude current disturbances in equation (1.2) to be related to past disturbances in equation (1.3) as

$$u_t = \varphi v_{t-1} + \varepsilon_t \tag{1.4}$$

Now if equation (1.4) holds for $\varphi \neq 0$, then

$$E(x_t, u_t) = E[(\gamma x_{t-1} + v_t)(\varphi v_{t-1} + \varepsilon_t)] = \gamma\, \varphi \sigma_v^2 \tag{1.5}$$

Therefore, x_t is correlated with u_t and hence is not exogenous. This simple counterexample serves to illustrate the subtleties of the problem of exogeneity. Despite treating parental reading activities as a fixed regressor and assuming that it is generated “from outside the system,” the fact is that the true mechanism that generates current values of the regressor yields a model in which the regressor is correlated with the disturbance term, suggesting that it is generated from inside the system as far as the model is concerned.

In short, a variable will be considered exogenous for a given purpose if a statistical analysis can be conducted conditionally on that variable without loss of relevant sample information. Whether or not a variable is exogenous depends on whether or not that variable can be taken as given without losing information for the purpose at hand. The literature differentiate various types of exogeneity in terms of statistical inference (estimation and testing), forecasting, and policy analysis are weak, strong (both for *efficient estimation*)[10], and SupExt respectively.

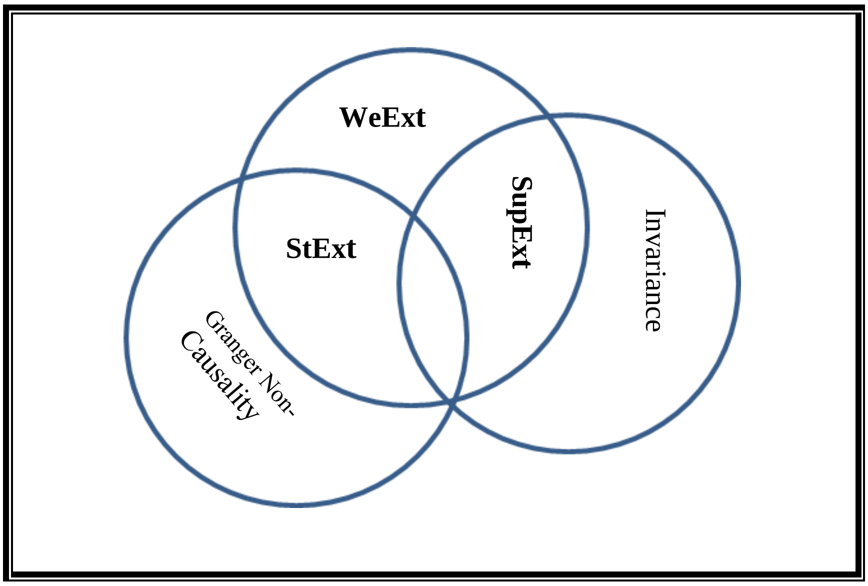
Valid exogeneity assumptions permit simpler modeling strategies, reduce computational cost, and help isolate invariants of the economic mechanism, with the last being particularly important in policy analysis. Invalid exogeneity assumptions may lead to inefficient or inconsistent inferences and result in misleading forecasts and policy simulations. Weak, strong, and super exogeneity are defined relative to parameters of interest, whereas pre-determinedness[11] and strict exogeneity[12] are not, making the latter two concepts of limited use for policy analysis (Engle et al., 1983). Nowadays it is a chic to differentiate between three types of exogeneity *i.e.* Weak, Strong and Super; the first two concerns with statistical inference and the latter is primarily related true structure (Pearl, 2000). Therefore, a rigorous testing procedure of SupExt is required that does not depend on the particular model under study but rather is based on the true structure of the system under investigation. The following

[10] Efficient estimation means inference without loss of relevant information not for efficiency of an estimator in small samples or any its properties.

[11] A variable is considered to be pre-determined in that equation if it is independent of contemporaneous and future errors in that equation.

[12] A variable is considered to be strictly exogenous if it is independent of contemporaneous and future and past errors in that equation.

diagram illustrates the different concepts of exogeneity corresponds to different intersection areas following (Ericsson, 1992):



Source: Extracted from the discussion in (Ericsson, 1992)

1.5 Objectives of the Study

At the time of writing this piece of work, one can't find a single work in which the comparison of these testing procedures has been encased leaving a loop behind to be fulfilled. Following are the key objectives of this thesis:

i) To compare the performance of SupExt testing procedures for stationary data settings using Monte Carlo Simulation design under the shade of Indicator Saturation (IIS, SIS, TIS & jointly).

ii) To analyze the significance and performance of these tests under non-stationary as well as dynamic settings under the shade of Indicator Saturation (IIS, SIS, TIS & jointly).

iii) To apply these tests on real data to establish a stable model (like, a model for money demand in Pakistan).

1.6 Literature Gap & Significance of the Study

Exogeneity lies at the heart of econometrics. However, the definitions and testing for SupExt procedures may differ when process switches from stationarity to non-stationary settings or to dynamic settings. There are numerous tests of SupExt available in literature but to the best of our knowledge the comparison among them does not exist. All the testing procedures for SupExt and their individual performance analysis are on the assumption of stationary data settings along with all derivations and Monte Carlo experiments that have been reported so far in literature are for static regression equations, the principles are general, and should apply to dynamic equations and to non-stationary settings (Castle et al., 2017; Hendry & Santos, 2010). Therefore, as a result on the basis of comparison one can use a test of SupExt whose performance seems to be more stable under Indicator Saturation as compared to others. A gap is identified which need to be fulfilled by considering the above data settings. So for theoretical side argument the study contributes to a step forward by testing the performance of these procedures under non-stationary and dynamic data settings with the help of simulation analysis. While for applied side, in case of Pakistan one can't single out a case study that used these tests for checking the stabilization of the model using types of dummies proposed by (Ericsson, 2012). Accordingly, the study provides a stable money demand model to fulfill our empirical (applied) side argument.

1.7 Motivation of the Study

There are three primary motivations behind the decision to analyze the above mentioned testing procedures. First, in the existing literature, there is scarce and incomplete evidence on how they behave in finite-sample and especially when considering non-stationary and dynamic data settings. This is due to the fact that some testing procedures are relatively recent and the underlying theory is still under development. The simulation analysis in this study filled this gap by providing evidence of their performance for a large class of models (like, money demand) usually adopted in applied economic research. Second, the Indicator Saturation under analysis constitutes alternative frameworks to the methodology of (Bai & Perron, 1998, 2003) and its extension to include non-stationary variables by (Kejriwal & Perron, 2008, 2010). These methods are theoretically well established though they rely on non-pivotal statistics to decide on the number of breaks which require extensive simulations to generate the appropriate critical values. On contrary, Indicator Saturation procedures do not rely on non-pivotal statistics to ascertain the number of structural breaks. Third, concerning about testing SupExt in particular studies like (Emory & Chang, 1996; Hess & Schweitzer, 2000; Mehra, 2000) have found that their estimated cointegrated relationship is stable over the long set of time or period but in certain sub-periods it breaks down.

This highlights a potential problem with splitting the sample to test for structural stability is that the resulting sub-periods may end up covering short time intervals. This makes testing for cointegration and even using the vector error correction model problematic as cointegration tests notoriously have a low power especially over short spans of time. This problem can be avoided by testing for SupExt, which is a test of invariance of parameter estimates to regime changes (Nourzad, 2012). Based on these arguments, we are able to establish the fact that there is a dire need to compare the performance of SupExt tests.

1.8 Thesis Outline

This thesis is organized in six chapters as discussed below:

Chapter 1 provides an overview the underpinnings of concept *"exogeneity"* both in Cowles sense of exogeneity and Engle, Hendry and Richard's sense of exogeneity. It discusses about the paradox of exogeneity, objectives of the study as well as its significance and motivation of this study.

Chapter 2 reviews available literature on exogeneity and how the idea stems from Cowles approach to Leamer's Bayesian methodology and explains some mathematical foundations of famous Lucas critique and reasons behind its negation. How the idea came into being in Hendry's LSE and it discusses some of the available empirical literature for late 19th and 20th century. Lastly, it covers different SupExt testing procedures, which is the prime focus of the study and a roadmap for exogeneity testing and its interpretation has been framed for the sake of brevity.

Chapter 3 envelops methodology and simulation strategy used in this study. The role of this chapter in this thesis is like a soul in a body. Further, it explains the usefulness of indicator saturation and its other types in SupExt perspective. It explains the DGPs and idea of SupExt in linear regression context and the conditions when it fails. At the end, all exogeneity testing procedures that have been used for comparison will be discussed in details to catch the mathematical aspects of these tests as well.

Chapter 4 contains the contribution of the study by assessing and comparing the size and power of several SupExt tests under autopsies. Note that the power and size in this chapter is for stationary DGPs, the first objective of the study.

Chapter 5 extend the horizons of SupExt tests in which power of these SupExt tests under different DGPs with non-stationary and dynamic data setting is be compared. This has never been equated in the past and leaving the impression to be added in a separate chapter uncovering the second objective of the study.

Chapter 6 illustrates an empirically stable money demand model in case of Pakistan implementing SupExt tests (invertibility test, index based test and double index based test) with an amalgamation of three different types of impulses (IIS, SIS & TIS) in detail. This kind of testing has never been instigated previously at the time writing this thesis and covers the applied side argument (objective) of the study. The estimated money demand model was found to be stable against relevant class of interventions and hence invalidating the famous Lucas critique.

The literature differentiates three major types of exogeneity as Weak, Strong and SupExt. It is the goal of this study to highlight the seriousness of testing exogeneity (SupExt) assumptions carefully when specifying statistical models, particularly if models are to be used for the evaluation of policies or interventions. So, our primary focus remains on SupExt with related concepts of parameter constancy and invariance. Methods for testing SupExt (Charemza & Király, 1988; Engle & Hendry, 1993; Hendry & Santos, 2006, 2010) will be explained. We compared the performance of these testing procedures of SupExt and further exemplify them under non- stationary and dynamic settings which do not exist in previously available literature.

CHAPTER 2

REVIEW OF LITERATURE

A plethora of literature on exogeneity in general and its types in particular is available. But one can hardly find such amount of literature exclusively dedicated to exogeneity and its types, make this section a worth reading for those who have interest to develop an in depth understanding of the topic. This section of the study provides a brief review of literature on exogeneity and its types primarily focusing on SupExt. It is an effort to encompass the historical perspective of exogeneity discussed in (Section, 2.1), the emergence of Bayesian methodology in (Section, 2.2), the VAR approach and the famous Lucas critique, how LSE approach figured out to the canvas of econometrics discussed in (Section 2.3). In (Section, 2.4) we briefly discuss about how the idea of exogeneity is being unfenced in LSE. A review of major studies after 1983 incorporating EHR exogeneity and its types is being discussed in (Section, 2.5). Lastly, SupExt and how its testing procedures operate will fall under (Section, 2.6).

2.1 Exogeneity: A Historical Perspective

In order to complete our portrayal of exogenous variables; we have to look at the expansions of econometrics in middle of the last century. As in the breakthroughs of the 1910s and 1920s (on identification problem) which revolved around the use of further information to reveal economic relations, here we focus on the importance of external information for gaining access to the unknown structures deemed to be the subject of econometric analyses. Exogeneity turns out to be important not only for estimating structural models and possibilities of policy intervention, thus continuing the preoccupations of Tinbergen's work into the modern era.

Specifically, while dealing with parameters constancy which is prime objective of SupExt, (Moore, 1914) tried to give a ‘concrete reality’ to economic relationships, using multiple regression analyses to accord their proper roles to dynamics and the multivariate structure of economic behavior. Such empirical relationships implicitly require at least within-sample constancy. Robbins (1932) strongly disagreed with Moore’s approach, although he actually directed his criticisms at (Schultz, 1928). In particular, Robbins claimed that the formal categories of economic theory could not be given numerical representations, since neither individual values nor technical causes were uniform over time or space. However, all forms of empirical evidence would be transient in Robbins’ formulation, which may well be true in the long run, but is an extreme view over short periods. Tinbergen (1940) making no direct replies to Robbins, but provide many empirical applications on his concerns about parameter constancy by testing parameter constancy of preferred models on several sub-periods; evaluate each equation’s performance through forecasting tests; and tested robustness of regression coefficients if other variables were added. Later, he introduced the concept of exogenous variable.

Tinbergen (1940) originally introduced the concept of exogenous variables in econometric models. Their major function was to increase the descriptive power of a complete system without adding to the number of equations to be estimated. Tinbergen was primarily interested in estimating the coefficients of the lagged endogenous variables that determined the oscillatory behavior of the system. The exogenous variables represented specific outside economic shocks that excited the equations. At that time estimation of their coefficients was not understood to involve any further statistical assumptions about the model (Epstein, 1987).

Marschak (1942) explicitly calls such variables 'extraneous'. Haavelmo (1943) calls them variables given from outside the model. Tinbergen refers to them as extraneous variables and says they are important because they behave as outside stimuli (Tinbergen, 1940). For Frisch, and Haavelmo, exogenous variables are ones which are unaffected by the internal workings of the economic system and thus have a power which the internal economic variables do not have. That power appears to be a causal power in the system, but also offers access to the system.

It was (Koopmans, 1950) who saw the statistical importance of such exogenous variables for the identification and estimation of simultaneous equations systems. He included exogenous variables in the model and this inclusion played a very significant role in two ways:

i) It offered solution to the identification problem.

ii) It gives ways to represent direct instruments of economic policy.

It follows from (Engle et al., 1983) that exogeneity is model dependent in the sense that variables are exogenous for a particular parameterization of a model. This is of interest as in the context of the long run the standard definitions of exogeneity can be directly tested (Ericsson & Irons, 1994).

The Cowles group started working during 1943 to develop the statistical tools that are required for a structural equation modelling. The group comprised young mathematicians and statisticians lead by Marschak until 1948 and after that by Koopmans, many other renowned personalities like; Haavelmo and Wald also join the group for letting them to achieve what they were aiming for. The major and striking contribution by the group to the literature was their two monographs (Monograph 10 & Monograph 14). Monograph 10 (a technical one and rare to find) was published in 1950 while Monograph 14 (relatively easy to understand) was published in 1953. These volumes set a new standard of rigor for econometric analyses, opened up many new paths to be explored and created a tradition of formalization which has been followed since then.

In the view of many practicing econometricians, the Cowles Commission founded modern econometrics (Arrow, 1991; Crist, 1994; Epstein, 1987). However, applied econometricians are aware that this claim in several important respects is unjustified. Conceptually (for example, structure, autonomy, identification and likelihood), and in terms of the probability approach, they adopted what had already been developed before. Nevertheless, it is hard to imagine the shape of modern econometrics without them. The famous paper by (Koopmans, 1950) from Monograph 10 was related to the term Exogeneity. Many other concepts in econometric literature, exogenous variables were treated as a notion before they were defined more precisely by Koopmans. Koopman's ideas about exogenous variables were associated with identification, with the nature of the system, with structure, and above all with certain statistical considerations.

Koopmans tries to define the notion of a statistically complete system, as well as to distinguish a closed from a complete system. A closed system has as many equations as variables, so all variables are modeled (a vector autoregressive representation is a member of this class). A complete system was initially construed as one with as many equations as endogenous variables, but that begs the very question Koopmans is addressing, namely under what conditions must any given variable be treated as endogenous? With a structure consisting of a closed and complete set of equations (such as the theoretical business cycle model in (Frisch, 1933a, 1933c) or the simultaneous equations systems actually considered by (Haavelmo, 1943).

Causal language and causal explanation are very much a part of Koopman's discussion. As a novel construct, exogenous variables are carefully defined in the context of a statistical model. In essence, a statistical system is complete for Koopmans if the inferences made within it about the parameters of interest are invariant to the treatment of the exogenous variables, whether the potentially exogenous variables are modeled or not should not affect inferences about the endogenous variables. Thus, completeness and exogeneity are tightly linked: when the non-modeled variables are indeed exogenous, the open system is complete for purposes of statistical inference. For example, valid inference in a conditional model requires that the conditioning variables are not jointly determined with the endogenous variables, or else least squares estimates will be subject to Haavelmo's simultaneity bias.

Enroute, Koopmans clarifies the notion of parameters of interest and links the concept to structure as the entity which captures such parameters, which fits in well with the context in which Frisch first used the term 'structural' in econometrics (Frisch & Waugh, 1933). One necessary condition for exogeneity is that the parameters in the two sets of relations (namely those determining endogenous and exogenous variables in the joint system) must be distinct.

2.2 Development of Bayesian Approach

This section shed a light on how Bayesian econometrics evolved over the time linking the apparent gap with exogeneity and its testing. The Bayesian econometrics relies on finding a parameter of an explicit specification of the prior distribution to those which are under considerations unlike classical econometrics. Before going into fancy details of this subsection, following aspect must be kept aside:

i) The development of Bayesian methodology linking with economics
ii) The development of Bayesian statistics and recent computing technological advancements, and
iii) Philosophical underpinnings

However, interested readers are referred for their detailed description which is available in different surveys and books (Koop, 1994; Koop et al., 2007; Poirier, 1988; Zellner, 1971, 1984, 2008). By reviewing at the literature, it can be inferred that the Bayesian econometrics emerged from the desire to compete Cowles Commission archetype. As a result, Edward E. Leamer set a foundation for Bayesian model specification approach. In a systematic way, Leamers's approach put emphasize on the empirical fragility in most of the available models, but was unable to give an alternative modeling strategy that enhance the robustness of the modeling strategy.

Although, the amalgamation of Bayesian to time series econometrics proved to have a significant impact and further blurred the ideological path between Bayesian and Classical econometrics. Therefore, the Bayesian methodology consistently showed its capability of generating new tools comparable to those generated by classical methodology. To answer a well-established question about the relationship of subjective probabilities and statistical methods (Marschak, 1954) used simple examples and showed that how ratio of the two Bayes' formulae can be used to match the degree of prior beliefs to its fusion with likelihood functions. The work by (Marschak, 1954) was about a decade before the establishment of Bayesian econometrics.

Fisher (1962) examined different effects of the model estimation using Bayes' theorem, induced by different purposes like prediction and for policy simulations. He derived two different set of coefficient estimates corresponding to two different aims (prediction and policy), which would minimize their respective loss functions. The procedure served the purpose by linking estimation to the desired welfare function with policy.

Drèze (1962) supposed to be the first case study to compete with Cowles commission within simultaneous equation model (SEM). The study focused on the issue of identification instead of estimation from Bayesian perspective. It further categorized the *apriori* information of SEM in two parts:

i) The bifurcation between endogenous and exogenous variables.
ii) All assumption on signs and magnitudes of structural parameters and on the covariance matric of error term.

After an impressive work by the last two, stood enough to inspire and pave the paths for (Rothenberg, 1963) in which he combined SEM to the loss function under the assumption of unknown and known error variance of the model in order to determine the effect of different priors on posterior parameter estimates which lead to conclude that in an exactly identified system Bayesian and classical solution are quite similar with 'weak' prior information. By the time Bayesian researchers put their focus to circumvent two main difficulties: the difficulty of handling the specifications of prior distributions and the distributions of posterior that joins the prior through likelihood functions (integral calculation). Enormous literature is available, showing the true picture of Bayesian estimation equivalents to AR model and COMFAC model (Shiller, 1973; Thornber, 1967; Zellner, 1971; Zellner & Geisel, 1970) that were mainly developed by classical researchers. The problem that mainly criticized on classical grounds was the wrong sign and magnitude of coefficient estimates not aligned with economic theory. However, Bayesian priors considered to have power of overcoming such indications. The difficulty of getting analytical solutions of the integral part, one need to put high dimensional priors in simultaneous equation models.

Therefore, Bayesian researchers focused on the development of getting solutions to the numerical integration only but on the development of computer software as well. In this race, (Kloek & van Dijk, 1978) considered as the first stepping stone to introduce the Monte Carlo integration procedures and able to solve analytically a wide range of priors. While on the other hand, the development of computer programs in Bayesian analysis was in progress. For a derailed review on developments of computer programs for Bayesian analysis, we refer you to go through (Press, 1980). But yet, selection of economic model based on statistical theory via regression remained in the picture.

Leamer (1983) is an encounter to the Bayesian route by introducing a statistical procedure for model selection. The study grouped all possible explanatory variables for a particular explained variable into two sets: one was the group of *'free variables'* which were *apriori* considered to be crucial for economic theory and the other is *'doubtful variables'*. Leamer derived the confidence intervals through the posteriors of the *free variables* and the sensitivity analysis carried out by imposing varying priors on the two coefficient sets. The largest interval among the selected ones was referred as *'extreme bounds'* and hence used as base model selection criterion. Models having wide and sensitive bounds were not been selected while those with narrow and insensitive extreme bounds *w.r.t* to a broader selection of priors were been selected and considered to be useful (Leamer, 1983b, 1985). Leamer was of the view that the *'anomalies'* in the data can't be enveloped in traditional econometric models resulting in the execution of *'Global Sensitivity Analysis'* (Leamer, 1983a, 1985) was essentially a ***'general-to-specific'*** modelling strategy.

However, the empirical efforts by employing the global sensitivity analysis left unfruitful. The money demand model proposed in (Cooley & LeRoy, 1981) was not the best alternative *via* global sensitivity analysis to the precise demarcation of available models. Although, they were left unpersuaded with traditional econometric models. One issue is that Leamer's systematic model selection strategy was criticized due to lack of *'Exogeneity'*. Later, Bayesian exogeneity tests were developed by (Lubrano et al., 1986; Zellner et al., 1988) in the context of SEMs. These test shifted the focus from probability distribution of structural coefficients to properties of residual terms and the parameters related to it. The conjecture of these two, enroute in developing new model selection tools, like *'encompassing'* (Hendry & Richard, 1989; Richard, 1995).

During 1980's, the idea of using VAR models (Sims, 1980b) came up in the picture with an immediate curse of dimensionality resulting to have an adverse effect on the forecast performance of the VARs. Nevertheless, the idea endorsed general-to-specific methodology propose by Leamer. Here Bayesian methodology came up as a helping candidate to overcome the problem (Sims, 1980a). Later, (Doan et al., 1984) played a crucial role in endorsing the usage of Bayesian Vector Autoregressive (BVAR) approach. The priors that could minimize the forecast error are being selected and their relationship with one-step-ahead forecast was observed.

The idea of applying Bayesian methods to VAR models was extended to dynamic stochastic general equilibrium (DSGE) models. A commonly recognized weakness of DSGE models was the arbitrary use of 'calibrated' parameters. A handy way to tackle this weakness was to assign Bayesian prior distributions to those parameters and simulate the model outcomes in probabilistic terms. The idea was initially explored by (Canova, 1994) and also (DeJong et al., 1996). Within the VAR model framework, the Bayesian methods were also extended to the production of confidence bands for impulse responses (Sims & Zha, 1998). These extensions helped to revitalize Bayesian econometrics, for more detailed surveys of subsequent developments one must read the work by (Geweke et al., 2011; Van Dijk, 2003). Noticeably in the revitalization, the subjectivist image of the Bayesian approach was weakened as econometricians' understanding of the versatility of Bayesian inference in model estimation broadened, and Bayesian econometrics reverted to an alternative technical division rather than a methodological one.

2.3 Unveiling VAR Methodology

This section is dedicated to assess the emergence of Sims' VAR approach came about in a historical perspective and how the concept evolves over the time leading us to a new era of econometrics. The present section is an amalgamation of what sort of issues can be tackled by the VAR approach and what methodological position it takes particularly with respect to Cowles Commission.

The first empirical encounter by means of VAR can be sketched in (Orcutt & Irwin, 1948). However, the theoretical side argument was missing in it and was later explored by (Sargan, 1959) where the dynamics of SEM was explored therein but it was (Wold, 1960, 1964) who discussed the dynamic representation of structural models under the belt of Causal Chain Models to argue Cowles Commission approach. In particular, the term VAR as an alternative methodology to the Cowles Commission was first coined in the joint venture by (Sargent & Sims, 1977). The following sub-sections *2.3.1-2.3.3* discuss the idea ignited by Lucas and its negation along with some reasons behind refuting this critique, at the same time when Sims' was working on the development of VAR methodology.

2.3.1 Foundations of Lucas Critique

The idea of SupExt considered to be incomplete, if someone, ignored the famous critique raised by (Lucas, 1976). Therefore, in this sub-section, the famous Lucas critique is framed in a general economic set up considering simple expectation model. Lucas considered an agent decision rule *F*(.) *via* optimized behavior and a policy response function *G*(.):

$$y_{t+1} = F(y_t, x_t, \boldsymbol{\theta}, \varepsilon_t) \tag{2.1}$$

$$x_t = G(y_t, x_{t-1}, \boldsymbol{\lambda}, \eta_t) \tag{2.2}$$

Where y_t is endogenous x_t is exogenous, $\boldsymbol{\theta}$, $\boldsymbol{\lambda}$ are the parameters and ε_t and η_t are corresponding *i.i.d* shocks to the functions *F*(.) and *G*(.). For example here, y_t and x_t be consumer expenditure and a government income supplement, *F*(.) is the empirically estimated consumption function (conditional) and *G*(.) is the rule for providing supplements (marginal). He argued that any agent *via* optimization, there is a chance that parameters in $\boldsymbol{\theta}$ may be dependent on $\boldsymbol{\lambda}$, and referred towards the case that changes in $\boldsymbol{\lambda}$ might cause serious disturbances in $\boldsymbol{\theta}$. Consequently, models that treat $\boldsymbol{\theta}$ as stable/fixed would fall apart when parameters in $\boldsymbol{\lambda}$ being changed through policy interventions.

Lucas had some models based on forward looking expectations in his imaginations which can be further elaborated by using following mathematical formation that describes how expectations generate problems in conditional models. Consider (2.1) and (2.2) are:

$$y_{t+1} = \gamma E(x_{t+1}|I_t) + \varepsilon_t \quad (2.3)$$

$$x_t = \boldsymbol{\lambda} x_{t-1} + \eta_t \quad (2.4)$$

Where, γ is structural parameter, $E(.)$ is expectations and I_t is taken as set of informations that were available to agent time *t*. In case if, $I_t = x_t$, then $E(x_{t+1}|I_t) = \boldsymbol{\lambda} x_t$ and therefore, (2.3) will become:

$$y_{t+1} = \boldsymbol{\theta} x_t + \varepsilon_t \quad (2.5)$$

Where, $\boldsymbol{\theta} = \boldsymbol{\theta}(\boldsymbol{\lambda}) = \gamma\boldsymbol{\lambda}$. Now avoiding the dependency of $\boldsymbol{\theta}$ on $\boldsymbol{\lambda}$, one estimate $\boldsymbol{\theta}$ using (2.5) only, leads to inefficient and misleading results. Furthermore, the value of $\boldsymbol{\theta}$ being estimated through (2.5) is not necessarily the value of agent optimization rule (2.3).

2.3.2 Negating Lucas Critique

Overviewing a bulk of literature available concerning to this sub-section we came up with concluding that the parameter $\boldsymbol{\theta}$ in (2.1) can be helpful in refuting Lucas critique under two properties:

Constancy of $\boldsymbol{\theta}$ as policy makers induce changes in $\boldsymbol{\lambda}$

It's invariance to $\boldsymbol{\lambda}$

On the basis of (Gordon, 1976) and later on (Neftçi & Sargent, 1978), the authors like (Engle & Hendry, 1993; Favero & Hendry, 1992; Hendry, 1988) have been able to develop testing procedures to confirm the validity/invalidity of Lucas critique.

Procedure 1:	Procedure 2:
First establish the constancy of $\boldsymbol{\theta}$ in (2.1) and then of $\boldsymbol{\lambda}$ in (2.2). Now, if $\boldsymbol{\theta}$ is stable but $\boldsymbol{\lambda}$ is not constant, then any changes in $\boldsymbol{\lambda}$ will not induce instability in $\boldsymbol{\theta}$, therefore, Lucas critique can't be applied.	First, construct (2.2) as a stable empirical model by simply adding dummies to it or other related factors like lags etc. and allow $\boldsymbol{\lambda}$ to vary over time, and then observe the significance of these added variables in (2.1). The insignificance of these additional variable in (2.1) showing the independence of $\boldsymbol{\theta}$ from $\boldsymbol{\lambda}$. Consequently, Lucas critique is invalid in such circumstances.

The *Procedure 2*, is taken as the test of SupExt of x_t, provided that invariance of $\boldsymbol{\theta}$ from $\boldsymbol{\lambda}$ holds. The existence of SupExt empirically, negates the presence of Lucas critique. The testing procedures used in this piece of work are mainly based on *Procedure 2*. The following sub-section provides a bird's eye view to some of reasons, refuting Lucas critique in econometric practice.

2.3.3 Reasons behind Refuting Lucas Critique

Econometricians round the globe dealing with Lucas critique found evidences by testing its validity (few) and invalidity (majority). A detailed bibliographical meta-analysis to this as a supporting argument can be found in (Ericsson & Irons, 1995). Researcher found several reasons behind the refutation of Lucas's argument which could be occurred due to incorrect functional form, dynamic mis-specification and due to omitted variable bias (Favero & Hendry, 1992). First, as these reasons can't be impede *apriori* so no information can be obtained about the critique through instability of conditional model and *via* VARs. Second, argued that testing *Procedure 1* (discussed above) do not require a full model for the policy variable as in (2.2) to negate Lucas critique. Third, under SupExt the unique parameters of the conditional model can be identified *via* invariance w.r.t $\boldsymbol{\lambda}$. Lastly, there exist problem of Type I and Type II errors in all those tests used to establish Lucas critique. In (Keith Cuthbertson & Taylor, 1990), they found these errors while testing Lucas critique using *Procedure 1*. Note that, this observation can also be applied to *Procedure 2* and to all those tests used to establish this critique.

2.4 Upsurge of LSE Approach

This section tries to project the development of London School of Economics (LSE, hereafter) approach and to bottle the river in a tumbler since we just explain a one narrow part (Dynamic Specifications) of what this approach has contributed to the field of econometrics. The approach discussed above is deriving probably from the work of (Hendry et al., 1984) and also Hendry's textbook *Dynamic Econometrics*. Before LSE, two major approaches that are available in the literature; one is the VAR approach proposed by (Sims, 1980b) and on the other hand the Bayesian specification search methodology introduced by (Leamer, 1983a, 1983b), the details of which has already been discussed above. The researchers at LSE have not explicitly criticized the Cowles Commission approach, unlike to Sims and Leamer. The LSE group has chosen a wide position, focusing not only on the compiled work of Cowles commission but also its historical root leading us to Frisch and others before him as well. The strategy has allowed the LSE group to put together a comprehensive framework for dynamic model choices and designs is considered as a major and well established development in the field of econometrics. A brief historical overview on LSE approach/philosophy can be found in (Mizon, 1995) and description about its origin in (Hendry & Krolzig, 2003).

The strategy has resolved much of the model selection issue which was left to one side by the Cowles commission group and, methodologically, it arguably goes further than the other two approaches (Sims & Leamer) in this respect. The fame of LSE approach symbolizes a collective and concentrated effort to improve the Cowles commission structural approach mainly with the help of time series statistical methods.

The origins of the LSE approach are described in (Gilbert, 1986, 1989). The essential features of this approach are being discussed in (Pagan, 1987, 1995) where they are compared with the Bayesian and the VAR approaches as well. Further historical material on this approach is available from interviews with leaders of the LSE approach *see;* (Ericsson, 2004; Phillips & Sargan, 1985) as well as the contributions by few potential key players, for example (Castle et al., 2015, 2017; Doornik, 2007, 2009a; Ericsson, 2012; Ericsson & Irons, 1994, 1995; Hendry, 2003; Hendry & Santos, 2010; Mizon, 1995; Nymoen, 2019; Pretis, 2017).

Leiva & Rubio-Varas (2020), emphasized that whether inference can be used for policy purpose or not, SupExt is a amazing property to identify causal relations. This approach contrasts from the traditional strategies to infer causality in literature *via* Granger Causality (GC, hereafter), like; (Kraft & Kraft, 1978) in case of bivariate, (Stern, 1993) considering multivariate scenario, which have been modified to account for integration (Wolde-Rufael, 2004), cointegration (Masih & Masih, 1996), and expansion to panel data settings (Lee, 2005) and incorporating regime shifts (Kocaaslan, 2013). However, these progressively developed and refined approaches suffer from the fact that in non-stationary data settings; GC doesn't imply any meaningful sense of causality (Hendry, 2004). This argument further strengthen the point in favour of SupExt as documented in (Pearl, 2000, 2010).

In literature, there is no ambiguity in that GC test considered as the most common approach for identification; though neither a necessary nor sufficient condition for causality (Hendry, 2004). The importance of testing SupExt further supported by arguments like; GC is a measure of forecast capability (Granger, 1980, 1988), and such capacity does not imply causality in non-stationary settings (Hendry & Mizon, 2000). Thus, Granger-causality should be used to study forecasting proficiency, and to establish causal links one should rest on theory or SupExt (Leiva & Rubio-Varas, 2020; Pearl, 2000).

2.5 Empirical Studies on Testing EHR Exogeneity

This subsection is dedicated to envelop major contributions on EHR exogeneity and its types from 1983, used for empirical investigation of different economic theories and hypothesis. On the basis of literature provided in this subsection, though not completely but upto some extent, one would be able to capture the empirical modelling strategies used to testify the hypothesis of exogeneity and its types in economic frameworks like; money demand, wages, unemployment, prices, expenditure and federal reserves etc. This subsection further divided into two sections one covering the empirical contributions by the researchers in 19th century and the other for studies in 20th century.

2.5.1 Empirical Studies in 19th Century (Selected)

Starting from (Engle et al., 1983) in which they provided the inescapable part of the idea of "exogeneity" in econometrics, it is fundamental to describe the ramifications of cases that specific factors are "exogenous" as indicated by definitions therein. Additionally, it is helpful to have definitions which require insignificant conditions but then are appropriate to as wide a class of important models as could be expected. Thus, general and unambiguous definitions are proposed for weak, strong and SupExt regarding the joint densities of recognizable factors and the boundaries of interest in given models, in the line and formalizing the methodology as proposed in (Koopmans, 1950).

Hendry (1988), shed light on two separate hypotheses; one is encompassing and other is SupExt. He developed techniques for differentiating between feedback and feedforward models. Adequate changes in the marginal processes lead to differentiate between feedback and feedforward models by testing the constancy of the proposed marginal models. The study highlighted the importance of SupExt testing in econometric models by incorporating two facets of the model *i.e.* weak exogeneity and parameter invariance. Hendry (1988) based on tests of constancy, proposed and used a test of SupExt while modeling UK money demand (M1) model. Hendry's test can be interpreted as an encompassing test of feed-back versus feed-forward models and is a test of the famous Lucas critique.

Cuthbertson (1991) emphasized on the issues of finite sample and model design proposed earlier in (Hendry, 1988). However, later on, (Favero & Hendry, 1992) discussed and deny criticisms raised by (Cuthbertson, 1991), focusing on the asymptotic and finite sample properties of the encompassing and SupExt test.

Johansen (1991) introduced the idea of cointegration in a statistical view point as it was ignored in (Granger, 1981). The outcome of the paper was two-fold; one concerned with order of integration of variables while other with testing weak exogeneity. The relevance of weak exogeneity is only in the case where the researchers want to use conditional model for long run parameter estimation. The long-run parameters are framed by implying parametric restriction on the adjustment coefficients and test the weak exogeneity hypothesis. The procedure is illustrated by using money demand data of UK.

Hunter (1992) gave the idea of cointegrating exogeneity (notion of long run exogeneity) and referred it as equivalent to strong exogeneity and give valid long run forecasts given the knowledge of cointegrating vector based on the idea that long run relations are basically block triangular. An implication of cointegrating exogeneity is that it ascertains a separation between the exogenous and endogenous variables. Unlike, SupExt, in cointegrating exogeneity, there is no need of parameters stability of conditional model regarding the associated changes in marginal model. The study focused on estimating PPP and uncovered interest rate parity in case of UK taking the data set used in (Johansen & Juselius, 1990).

Ahumada (1992) argued that a stable money demand model in case of Argentina can't be inverted following 'general-to-specific' methodology to obtain stable models for inflation and interest rate under the SupExt of these variables *via* (Chow, 1960) sequence of break point test. Also, (Nymoen, 1992) refuted the famous Lucas critique in favor of SupExt by establishing the real wage-unemployment model for Finish manufacturing data. The study observed the hysteresis effect in terms of wage rigidity rather than wage flexibility. Lastly, (Bårdsen, 1992) empirically found that the prices, real expenditure and interest rate were super exogenous in the estimated money demand model for Norway. The test used was proposed by (Hendry, 1988) and is an invariance based test.

Engle & Hendry (1993) examined the impact on a conditional model of changes in the moments of the conditioning variables, using a linear approximation: several SupExt tests were developed by replacing the unobservable changing moments by proxies based on processes that generate the conditional variables, also incorporating ARCH processes to capture changes in regimes for non-constant error variances. However, (Psaradakis & Sola, 1996) claim that such tests have relatively low power for rejecting the Lucas critique which later on refuted by (Ericsson et al., 1998).

Surprisingly, the empirical evidence in favor of Lucas critique is scarce. The statement is well supported by the work of (Ericsson & Irons, 1995). In which they gathered data about 590 research articles published during 1976-1990 citing Lucas critique based on Social Citation Index (SCI). If some studies found evidence in the favor of Lucas critique, they pointed out that the particular estimated models are empirically non-constant and flaws in their estimation processes. Also, the study highlighted a fact that there is a considerable increase in empirical literature testing SupExt with many cited articles on money demand.

Subsequent work re-assesses the exogeneity concepts for cointegrated systems. (Johansen, 1992b, 1992a) and (Urbain, 1992) derive sufficient conditions for weak exogeneity for short-run and long-run parameters in a conditional subsystem from a cointegrated VAR. Harbo et al., (1998) provide critical values for the likelihood ratio test for that form of cointegration; and (Johansen & Juselius, 1990) develop a general framework for testing restrictions on cointegrated systems, including exogeneity restrictions. (Ericsson, 1992) and (Ericsson et al., 1998) provide expository syntheses of cointegration and exogeneity.

Jansen & Teräsvirta (1996) extended the SupExt testing procedure offered in (Engle & Hendry, 1993) by considering non-linear specification in their marginal model through the lens of non-linear smooth transition regression (STR) model which originally discussed in (Granger & Teräsvirta, 1993). They first test the constancy of the parameters of their conditional model, excluding the coefficients of the variables of interest income and wealth. If stability is rejected they estimate an STR model and consider the constancy of its parameters. If at this point they no longer reject stability they proceed to testing SupExt. In order to do that, they estimate the marginal models and test their constancy. If it is rejected, they estimate STR models for these relationships and interpret the nonlinear part as potential evidence against SupExt. The residuals of these equations are needed in testing weak exogeneity. Finally, they test SupExt of the conditional equation with respect to the parameters of interest.

Ericsson (1999) validated the point that the inverted conditional model for variables which are super exogenous lead to invalid inferences. It was argues that parameter estimates of inverted regression differ drastically because regression inversion is not considered as inversion of a non-stochastic equation. Further, if prices or any other variable in an empirically stable conditional model is super exogenous then the corresponding inverted model will be non-constant (unstable) as previously reported in (Hendry, 1985) and (Hendry & Ericsson, 1991a, 1991b).

2.5.2 Empirical Studies in 20th Century (Selected)

Das & Mandal (2000) tested the hypothesis of weak, strong and SupExt while modeling money demand (M3) in India. The study provided the empirical evidence on whether M3 can be modeled while using a single partial equation conditional model or one have to consider the full model specifications like VARs. The weak exogeneity of prices leads to conclude that partial model is suitable. Further, the SupExt of prices and interest rate provided basis to argue that estimated M3 model can’t be inverted to get price and interest rate equations.

Sachsida & Cardoso de Mendonça (2006) estimated a relation between investment and domestic saving to verify capital mobility based on a famous (Feldstein & Horioka, 1980) puzzle. The paper performed exogeneity tests in order to determine the capacity of the Feldstein-Horioka equation for implementing economic policies in Brazil.

Kurita (2007) found the evidence against Lucas critique by establishing a dynamic system for yen-dollar rates for Japan. After implementing a rigorous cointegration analysis, a data-congruent SEM system for the real yen–dollar rate conditional on a set of weakly exogenous variables, was successfully estimated. Test of SupExt by (Engle & Hendry, 1993) is being opted. The study tests that both short and long term yield spread are super exogenous. The corresponding marginal model was obtained by estimating full model VAR first, and then reduced to the parsimonious model using encompassing tests at each step. The congruent marginal model is consistent with both theory and data. The out of sample forecasts were also being discussed.

Nourzad (2012) found unit labor cost to be weakly exogenous for both price indices (consumer price index and personal consumption expenditure deflator) while the two price indices are weakly exogenous for average hourly earnings per unit of output; unit labor cost is strongly exogenous for consumer price index but not for average hourly earnings also unit labor cost is super exogenous for consumer price index. Considering these findings together lead to conclude that unit labor cost is a reliable indicator of inflation but adjusted hourly earnings is not.

Togay & Kose (2013) implemented SupExt testing based on test of co-breaking as discussed in (Hendry & Santos, 2006) but used one-step ahead recursive residuals to validate his argument regarding SupExt. The foundation of this testing procedure was referred to be in (Charemza & Deadman, 2003 p.239). The study discussed that producer price index is not super exogenous *w.r.t* to graph of one-step ahead recursive residuals.

Choo & Kurita (2015) though did not focused on testing the SupExt but examined the weak and strong exogeneity of 10-years government bond yield, real

GDP and nominal effective US dollar exchange rate while modeling the nexus of US monetary policy rule and inflation over the past quarter century.

Unlike, traditional monetary side models like money demand which is mostly used in testing SupExt. Jawad (2014) using multivariate analysis, modeled the impact of policy environment on inflows of workers' remittances in case of Pakistan and found that the estimated model is invariant to changes in currently dated regressors like; policy variable, exchange rate and GDP. The study invalidated the existence of Lucas critique. But the selection of the breaks therein was not data driven.

Kónya & Abdullaev (2015) is a very different but unique study in the context of testing SupExt. As the study highlighted evidence on relationship between of Ricardian equivalence with Lucas critique with SupExt. The growth rates of per capita real GDP, per capita real public debt, the unemployment rate and the real interest rate are super exogenous in the conditional model for the growth rate of per capita real domestic savings. As a result, Ricardian equivalence held in case of Australia during the past half century, on the basis of idea given in (Sachsida & Teixeira, 2000) which says, existence of Lucas critique will lead to refute Ricardian equivalence hypothesis and *vice versa*.

Rodríguez-Caballero & Ventosa-Santaulària (2017) were found to be of the same view as many other researchers who pointed out that GC is not a causality test but predictability; tested the direction of causality between electric power consumption (EPC) and gross domestic product (GDP) in 19 countries including Canada and the US. The study opted the idea by (Hoover, 2001) sense of causality (SupExt) and assessing the four different hypotheses (growth, EPC $\rightarrow$ GDP; conservation, GDP $\rightarrow$ EPC; neutrality, EPC $\nLeftrightarrow$ GDP and feedback, EPC $\Leftrightarrow$ GDP).

Each hypothesis doesn’t seem to fit in every country but to a subset of different countries. The observed heterogeneity in hypotheses was due to the reason since different regions follow different economic, geographic, geological and institutional settings. The authors further argued that SupExt is a property that validates the usefulness of a model to plan economic strategies, is an appropriate and robust vehicle to disentangle the causal links between variables under autopsies.

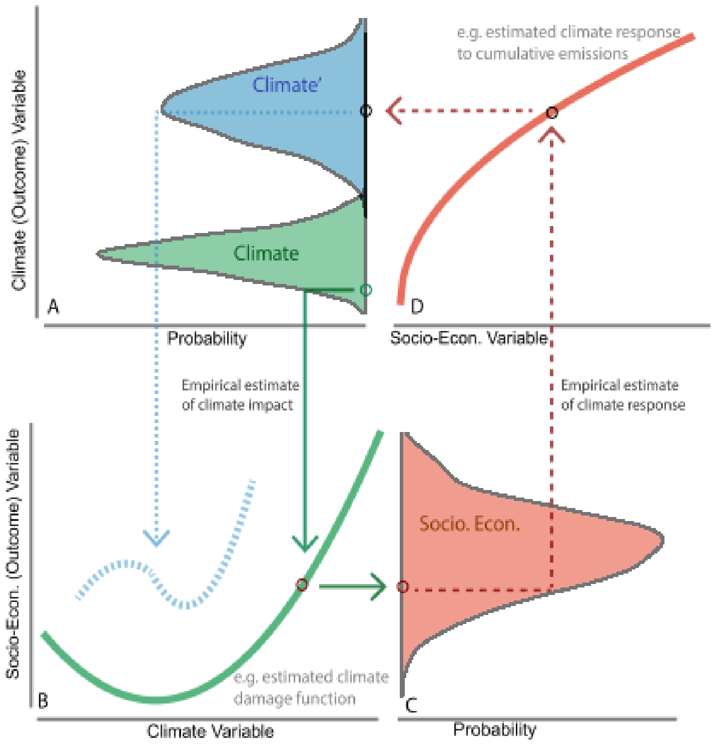
Jawad et al. (2022) tried to estimate Feldstein-Horioka equation taking into account the data driven structural breaks and testing exogeneity of savings in estimated FH-equation in case of Pakistan. The study found no evidence of cointegration between domestic savings and investment and super-exogeneity of savings holds which reflects that the conventional Lucas critique is not validate in Pakistan.

2.5.2.1 Exogeneity and Climate Econometrics

An initiative to launch a new project in the field of econometrics under the supervision of David F. Hendry and Felix Pretis named as Climate Econometrics is under discussion of the econometricians these days. Consequently, (Pretis, 2017) considered as a first ever attempt to introduce the concepts of exogeneity in environmental perspective. Later published in Energy Economics as (Pretis, 2021). The study is having rare but full of information for those knowledge seekers who have interest in modeling environment. The study shed light on weak, strong and SupExt along with indicator saturation in climate settings. Following his footprints, I’m currently working on modeling a stable climate model in case of Pakistan in which the types of exogeneity and their testing is also being incorporated.

The following figure depicts that for a given set of climate observations (A, green) empirical estimates of climate impacts derive the response of socio-economic variables (B, green). This response is then used to project socio-economic outcomes (C, red). Empirical climate model estimate the climate response given socio-economic variables (D, red). Weak exogeneity is the required condition to estimate empirical climate impacts, or empirical climate responses in conditional models alone (studying B & D in isolation respectively). A projected socio-economic outcome in the empirical impacts literature can lead to a different climate due to feedbacks which can be tested using test for strong exogeneity (A, expecting green while the outcome is blue). Shifts in the climate distribution (A, green to blue) can lead to different estimates of climate impacts (B, green vs. blue climate impacts curve) if super-exogeneity fails. How, these three concepts can be related with climate econometrics we refer to study (Pretis, 2017).

Figure 2.1: Explaining Exogeneity in Climate Econometrics



Note: This figure is taken from (Pretis, 2017)

2.6 Super Exogeneity and its Testing

Parameter invariance is essential in policy models, otherwise the fitted model will mis-predict under regime shifts or structural breaks. Thus, SupExt is a crucial requirement for economic policy as it combines parameter invariance with valid conditioning. The Lucas critique challenged the use of conditional econometric models for policy analysis with following words:

'Given that the structure of an econometric model consists of optimal decision rules for economic agents, and that optimal decision rules vary systematically with changes in the structure of series relevant to the decision maker, it follows that any change in policy will systematically alter the structure of econometric models.'

In other words, "*a model cannot be used for policy if implementing the policy would change the model on which that policy was based, since then the outcome of the policy would not be what the model had predicted*" (Hendry, 1995, p. 172).

The above statement by Lucas is essentially a theoretical claim denying SupExt under regime shifts. Thus, the Lucas critique is testable directly *via* tests of SupExt or indirectly *via* its encompassing implications. The fundamental weakness in Lucas's claim is the assertion that "optimal decision rules vary systematically with changes in the structure of series relevant to the decision maker", which need not apply to contingent decisions, and hence need not affect conditional relationships. To date, few investigators have found any evidence of induced instabilities following policy regime changes (Ericsson & Irons, 1994) who overview the literature on exogeneity, and (Hendry, 1995) who discussed the interpretation and testing of the Lucas critique in terms of SupExt. Indeed (Favero & Hendry, 1992) showed that location shifts were essential for detecting the Lucas critique. Consequently, we our primary focus is on tests of SupExt when location shifts occur in the marginal processes along with other SupExt tests like (Charemza & Király, 1988) in which we don't need any structural breaks.

Most SupExt tests are of cross-linkages between equations, which need not be conditional relations. Engle & Hendry (1993) proxied the effects of changes in the moments of conditioning variables on parameter estimate as one test of SupExt. Favero & Hendry (1992) derived a test of the impact of non-constant marginal on conditional models. Jansen & Teräsvirta (1996) introduced a smooth transition autoregressive (STAR) model based test, (Krolzig & Toro, 2002) proposed a deterministic shift co-breaking test of whether breaks cancel between processes, so some linear combinations are invariant to breaks.

However, the commonly used tests of SupExt need to be customized to specific settings. Few automatically computable tests, like those for autocorrelated errors, say, would be invaluable, and one can be based on indicator saturation. Such automatic tests of SupExt are the one which can be computed without additional user intervention and with no *ex-ante* knowledge of the timings, forms or magnitudes of breaks in the marginal processes for the conditioning variables, nor how the parameters of the conditional model will alter as a result. Moreover, the conditional model should not need to be over-identified.

This can be achieved when the breaks in the marginal models are determined by indicator saturation, using *Gets* to develop congruent, undominated models of their Local Data Generating Processes (LDGPs): the first stage is to have the desired null retention frequency. Then the significant indicators are added to the conditional model and tested for significance. This second stage also has the desired null rejection frequency (when there are no unit roots), and has power against failures of SupExt when location shifts occur, as we now explain. However, in this study, we compare performance of different tests of SupExt discussed using indicator saturation (either from Autometrics family or beyond) under stationary, non-stationary and dynamic settings as well.

In literature, we have two main types of tests that are used to examine the existence of SupExt. First one is the non-stability in the parameters of marginal density function (hereafter, M_D) and the stability in the parameters of conditional density function (hereafter, C_D). To validate the said process a M_D function can simply be obtained by flipping over the C_D function. So, in the presence of SupExt a stable C_D function of parameters of interest cannot be interpreted as a re-parameterization because the re-parameterization is a function of parameters depending upon time and some other the causal structural parameters of M_D process. Therefore, by inverting conditional model the steady marginal model cannot be obtained. Now, if the $C_D$function is not invertible into M_D model, then it can be used as a confirmation of super-exogeneity because the invertibility is illicit if the variables are super-exogenous for the parameters of the C_D model (Hendry & Ericsson, 1991a). Therefore, to find out that a M_D process is not stable while on the other hand C_D process is stable is sufficient enough to test SupExt (Perez, 2002). Additionally, the existence of SupExt confirms the weak exogeneity of currently dated regressors as well (Castle et al., 2017; Ericsson et al., 1998; Favero & Hendry, 1992; Hendry & Ericsson, 1991a; Jawad, 2014; Qayyum, 2005a).

Second one is to test SupExt of parameters of concern against the external shocks such as military regimes, oil price shocks or exchange rate shocks that creates instability in the parameters of M_D function. Now a M_D function can be developed by adding these dummies in the M_D process. Then add these significant dummies into the C_D function and check their significance by using a conventional *t-test* for individual significance and *F-test* for joint significance (Engle & Hendry, 1993) and later some recent developments of automatic SupExt tests in (Hendry & Santos, 2006, 2010). Hence, if these dummy variables are insignificant in the C_D suggesting the SupExt of C_D process.

In order to test SupExt of the parameter of the conditional model against the known identified external shocks, which can affect the constancy of the M_D process. The method of dummy saturation proposed by (Hendry et al., 2008; Johansen & Nielsen, 2008) initially introduced in (Hendry & Ericsson, 1991a) and later implemented in (Castle et al., 2015, 2017). The significance of dummy variable individually can be tested by *t-statistics,* while the joint significance of these dummies checked through *F-test*. The recent development of some automatic tests of SupExt with impulse saturation and co-breaking based test. The study discusses the power of these tests and conduct a simulation based analysis as well. Considering the argument that structural breaks in time series data are not always nuisance but these can work as a blessing while drawing small sample inference (Magnusson & Mavroeidis, 2014).

Literature differentiate three types of exogeneity *i.e. Weak*, *Strong* and *Super* and the purpose of their testing as discussed in (Engle et al., 1983). Here, the following Table 2.1 discusses all these types of exogeneity, their need and the main assumptions they follow. Lastly, several tests are available in literature on how one can test these exogeneity assumptions are reported below Table 2.1. Now as (Hoover, 2006; Pearl, 2000, 2010) highlighted the importance of testing SupExt and termed it as a causal one. Therefore, we remained our discussion around the term SupExt and its testing.

Table 2.1: Summary of types of Exogeneity

Type of Exogeneity	Needed for	Assumptions	Tests/Discussion
Weak	Statistical Inference	Joint distribution is normally distributed, no heteroskedasticity, linear in parameters, In ECM, setting adjustment coefficients of corresponding variable to zero	Mardia's measures; tests for linearity and heteroskedasticity, (Engle et al., 1983) (Hendry et al., 1990) (Boswijk, 1991) (Johansen, 1992b) (Urbain, 1992) (Kurita, 2010)
Strong	Forecasting	Weak exogeneity and Granger Non-Causality	Testing Weak Exogeneity *w.r.t* parameter of interest, test of significance of lagged endogenous variable (Granger, 1969) (Engle et al., 1983)
Super	Policy Purposes	Weak exogeneity, parameter stability, non-invertibility of conditional model, invariance	(Chow, 1960) test, (Hendry, 1988) (Charemza & Király, 1988, 1990) (Engle & Hendry, 1993) (Jansen & Teräsvirta, 1996) (Krolzig & Toro, 2002) (Hendry & Santos, 2006, 2010)

Note: This is based on authors understanding and subject to change for modifications

A key recent development in testing for parameter non-constancy is doing so by adding a complete set of impulse indicators to a marginal model (Hendry & Santos, 2005). This new technique is known as indicator saturation. Using *GETS* procedures, the authors establish the null distribution of the estimator of the mean in a location-scale model, after adding impulses equal to the sample size. A two-fold process is investigated, where half of the indicators are added and the significant ones recorded. Then, the other half is examined, and finally the two retained sets of indicators are combined. The average retention rate of indicators, under the null hypothesis that no indicator matters, is $T\alpha$, where α and T is the significance level and sample size respectively: hence there is no over-fitting. Moreover, (Hendry &

Santos, 2005) showed that other splits, namely $T/_3$ or so, do not affect the retention rate under the null.

The following lines enlist the testing procedures to be opted in this study. Based on the above discussion so far, we came up with some intrinsic view point that that SupExt can be tested in several ways as discussed in Table 2.1. However, the procedure discussed in (Hendry & Santos, 2006) can be applied in one of three ways:

i) As *m* indicator variables, matching the *m* dummies retained from the marginal model after impulse saturation; their joint significance is tested via a joint *F-test*. This type of test was first introduced in (Hendry, 1988) and later implemented in (Favero & Hendry, 1992; Hendry, 1992).

ii) As an index for the theory of indices or linear combinations of indicators, where each indicator carries a weight equal to its estimated coefficient in the marginal model; testing SupExt is now testing the individual significance of the index using *t-test* in the conditional model as discussed (Hendry & Santos, 2005, 2006).

iii) As two indices (the previous one and another where the weights are the previous ones multiplied by the values of the marginal variable at the dates for which dummies were retained); the SupExt test is now a test on the joint significance of the two indices using *F-test* in the conditional model.

In all three cases, rejection of the null is equivalent to rejecting the SupExt hypothesis. The tests outperform the criticism of *ad-hoc* selection of the dates for the dummy variables to be included in the conditional model highlighted in (Linde, 2001), as the procedure tests a dummy at each possible date, and can now be fully automated without any user intervention.

iv) This test is due to (Charemza & Király, 1988) and later implemented in (Charemza & Király, 1990; Nourzad, 2012) in which one regresses the forecast error of the conditional equation on the log-difference of the variable that is being tested for SupExt and its lagged values. The null of SupExt would not be rejected if the estimated coefficients on the regressors are not jointly statistically significant. The main advantage of this test is that it doesn't require any marginal model/equation.

v) This test of SupExt is suggested by (Engle & Hendry, 1993) where one includes squared residuals and their lagged values from the marginal density function in the conditional density function. As with the Charemza-Király test, the null of SupExt is not rejected if the estimated coefficients on the regressors are not jointly statistically significant.

vi) Test of SupExt with the help of co-breaking proposed in (Hendry & Santos, 2006) by checking whether the timing of the identified breaks in conditional matches with that in the marginal model or not. This type of test first discussed in (Krolzig & Toro, 2002) and implemented in (Hendry & Massmann, 2007; Togay & Kose, 2013).

It is hoped that reflecting on the importance of the testing SupExt assumption will lead to a critical assessment of the methods of statistical modeling for future developments. At the end, the following map describing the exogeneity testing and the corresponding interpretations of these procedures, if and if not hold:

Table 2.2: A Map for Exogeneity Testing and its Interpretation

Procedure Name	Validity	Explanation
Cointegration	Yes	This procedure can be applied if full model[13] has long term co-movements.
	No	If full model has no long term co-movements, then this procedure can't be opted.
Weak Exogeneity	Yes	Then decisions based on the conditional model[14] separately are valid. The policy variables determined outside the conditional model and authorities have full control over these exogenously determined variables. They can easily change it whenever they want.
	No	Then decisions based on the conditional model separately are not valid and accurate as the policy variables can't be determined outside the conditional model. The authorities are unable to perform its policies autonomously and can't articulate these policy variables.
Granger Causality	Yes	This means that the policy variables have effect on the target variable. Using full model these two, must be foredicted one period at a time. Policies are applicable. Further, the policy variable is valid instrument for changing target variable.
	No	This means that the policy variables do not have effect on the target variable. Policies are not applicable. Further, the policy variable is not a valid instrument for changing target variable.
Strong Exogeneity	Yes	It is an amalgamation weak exogeneity and Granger non causality and means that currently opted reforms don't have any impact of previously opted strategies. Therefore, we can disregard the information gathered from previous strategies.
	No	This means that previously opted reforms can't be disregarded and should be under autopsies for setting up recent reforms.
Invariance	Yes	Shocks in DGPs of marginal models don't effect the distribution of the conditional model.
	No	Shocks in DGPs of marginal models do effect the distribution of the conditional model.
Super Exogeneity	Yes	The preferred/conditional model can therefore be used alone for policy analysis and simulations as the reforms have their impact in the marginal model[15] but don't effect the parameter stability of the conditional model.
	No	The policy reforms have effect on the target variable *via* the marginal model and do effect the constancy of the conditional model. It is necessary to model both at a time for valid policy simulations.

Note: These interpretations are solemnly based on literature cited above and are subject to change, if found inappropriate.

2.7 Synthesis of Literature

To sum up the role of exogeneity that helps in identifying fundamental distinctions between theory, the DGP, and statistical models; exogeneity raises a

[13] $Y_t = \alpha + \beta X_t + \gamma Z_t + \varepsilon_t$
[14] $y_t | z_t = \alpha_0 + \alpha_1 Z_t + v_t$
[15] $z_t = \beta_0 + \beta_1 z_{t-1} + D_t + \vartheta_t$

number of important questions that are central to the practice of statistical modeling in the social and at the same time in behavioral sciences as well. One issue, for example, concerns the proper place of data mining as a pre-modeling strategy, when attention focuses on characterizing the joint distribution of the data, then data mining has a central role to play. Another issue arising from the context of exogeneity concerns the dynamic reality of the phenomenon under investigation. Granger non-causality and strong exogeneity force us to consider exogenous variables as possibly being responsive to their own dynamic structure and that this must be correctly modeled to obtain accurate estimates for prediction and forecasting. SupExt reminds that earlier models are sensitive to real-life changes in the process under investigation. Finally, serious consideration of the problem of testing exogeneity forces us to re-examine and clarify ambiguous concepts and historical developments.

Exogeneity is an adjective describing an assumed characteristic of a variable that is being chosen for theoretical reasons to be an exogenous variable. Exogeneity resides at the nexus of the actual data generating process (DGP) and the statistical model used to understand that process. In the simplest terms, the actual DGP is the real-life mechanism that generated the observed data. It is the reference point for both the theory and the statistical model. Some test used the concept of parameter constancy while other used invariance property to validate the existence of SupExt. The following Table 2.3 highlights the idea used to test the validity of SupExt testing procedures.

Table 2.3: SupExt Tests Proposed and their Description

Test Proposed by	Description/Idea
(Engle et al., 1983)	Tests for SupExt and invariance have been proposed.
(Hendry, 1988)	Considered the impact of non-constant marginal processes on conditional models, and concluded that location shifts were essential for detecting violations attributable to the (Lucas, 1976) critique.
(Engle & Hendry, 1993)	Examined the impact on a conditional model of changes in the moments of the conditioning variables, using a linear approximation.
(Charemza & Király, 1988, 1990)	This test has an advantage in relation to other SupExt tests, for it does not need a marginal equation. The idea is to estimate a regression where the forecast error of the conditional equation is the dependent variable.
(Psaradakis & Sola, 1996)	Claimed that tests allowing for non-constant error variances to capture changes in regimes have relatively low power for rejecting the Lucas critique based on structural invariance.
(Jansen & Teräsvirta, 1996)	Proposed self-exciting threshold models for testing constancy in the conditional model as well as SupExt by extending the idea to non-linear smooth transition models.
(Krolzig & Toro, 2002)	Developed SupExt tests based on a reduced-rank technique for co-breaking shown by the presence of common deterministic shifts, and demonstrated that their proposal dominated existing tests (on co-breaking, see (Clements & Hendry, 1999).
(Hendry & Santos, 2006, 2010)	Developed automatically computable tests for SupExt, using a variant of *general-to-specific* modelling. Based on the recent developments of impulse indicator saturation applied to marginal models.

Note: Based on Author's understanding and subject to change

CHAPTER 3

METHODOLGY AND SIMULATION STRATEGY

This chapter tries to contour the mathematical aspects that are being used in this study regarding Indicator Saturation and its types discussed in (Section, 3.1 through 3.4). The general-*to*-specific methodology through the lens of model selection strategies like; *PcGets* as well as *Autometrics* algorithms used in this study and their working principles will be discussed in (Section 3.5) and in (Section, 3.6) respectively. The mathematical foundations of SupExt in context of simple regression and the concept of different testing procedures (both from Autometrics family and out of this family) that are being compared here, the methodology along with how they can be implemented under the shade of indicator saturation will be discussed in (Section, 3.7). The data generating process, opted in this study will bring up to the canvas in (Section, 3.8). Lastly, the simulation strategy opted in this study will be explained and the diagrammatic view of this strategy *via* a flow chart will be discussed in (Section, 3.9).

3.1 Indicator Saturation and its Extensions

Indicator saturation is a powerful empirical tool for evaluating and improving existing empirical models. This section offers several possible extensions to this approach that may be more suitable than itself for detecting crises, jumps, and changes in regime. "Extended IIS" also provides a conceptual framework for interpreting existing tests of parameter constancy. Table 3.1 summarizes indicator saturation, some other existing tests, and some extensions of it, all in terms of the variables involved. Throughout in this section, T is the sample size, t is the index for time, i is the index for indicators, k is the index for economic variable x_{kt}, and K is the total number of potential regressors considered. A few remarks may be helpful for interpreting the entries in Table 3.1.

On empirical grounds several studies tried to use indicator saturation technique includes (Hendry et al., 2008) used impulse indicator saturation, (Castle et al., 2015) used step indicator saturation to detect location shifts or outliers while (Castle et al., 2019; Walker et al., 2019) used trend indicator saturation in their model to detect tend breaks in the data.

3.1.1 Impulse indicator saturation

This is the standard IIS procedure first proposed in (Hendry, 1999), with selection among the T zero-one impulse indicators $\{I_{it}\}$. A test of SupExt based on IIS was proposed in (Hendry & Santos, 2006) and building on this many researchers used IIS for their empirical modeling like (Castle et al., 2012; Ericsson & Reisman, 2012; Hendry & Mizon, 2011; Reade & Volz, 2011).

3.1.2 Super Saturation

In addition to searching across $\{I_{it}\}$, super saturation hunts through all possible one-off step functions $\{S_{it}\}$. Step functions are of economic interest because they may capture permanent or long-lasting changes in regime that are otherwise not incorporated into an empirical model. Statistically and numerically, a step function is a parsimonious representation of a sequential subset of impulse indicators that have equal coefficients. Hendry & Pretis (2012) investigate the statistical properties of a closely related saturation estimator–step indicator saturation (SIS), i.e., for only the variables $\{S_{it}\}$. A test of SupExt based on SIS can be found in (Castle et al., 2015) and its extension in (Castle et al., 2017).

3.1.3 Ultra Saturation

Partial sums of the impulse indicators may also be of economic interest, as those double-sums are broken linear trends $\{T_{it}\}$. Ultra saturation (earlier, sometimes called "super-duper" saturation) searches across $\{I_{it}, S_{it}, T_{it}\}$. Obvious extensions are broken quadratic trends, broken cubic trends, and so forth.

3.1.4 Sequential Pairwise Impulse Indicator Saturation

Extensions are based on partial sums of the impulse indicators and step functions over the remaining sample, i.e., for all $i \geq t$. Partial sums also can be constructed over fixed length windows of impulse indicators. The simplest case is sequential pairwise IIS, in which sequential pairs of impulse indicators are added together, i.e., $P_{it} = I_{it} + I_{i+1,t}$ Pairs (or triplets, or quadruplets, etc.) may parsimoniously capture effects that are persistent but not permanent. Non- sequential group wise IIS is also an option, with some non-sequential groups being of particular interest, such as groups at a seasonal frequency.

3.1.5 Zero-sum Pairwise IIS

Differences of impulse indicators may capture "zero-sum" effects, with $Z_{it} = \Delta I_{it}$, and leading to zero-sum pairwise IIS for empirical examples (Campos & Ericsson, 1999; Hendry, 1974).

3.2 Many-Many Variables

IIS provides a solution for dealing with more potential variables than observations, i.e., a set of K potential regressors $\{x_{kt}; k = 1.2.3 \ldots K\}$ for $K > T$. In the same spirit as IIS, block searches can be applied to a set of economic variables for which there are more variables than observations. Additionally, every economic series $\{x_{kt}\}$ is interpretable as the weighted sum of the impulse indicators $\{I_{it}\}$, where the weight on each impulse indicator I_{it} is the value of the economic series x_{kt} for the observation corresponding to the impulse indicator. Block searches across many-many variables are thus interpretable as searches across particular, economically interesting combinations of impulse indicators. Empirical models may involve (say) K data aggregation assumptions with $K > T$ in practice; those assumptions can now be tested. Ericsson & Reisman (2012) proposes this test of data aggregation and applies it to aggregation assumptions in a global vector auto-regression.

3.3 Factors

Factors and principal components are weighted sums of economic variables. The factors and principal components are weighted sums of the impulse indicators (Bernanke et al., 2005; Castle et al., 2011; Stock & Watson, 2002, 2005) for discussions.

3.4 Multiplicative Indicator Saturation

If a model's coefficient on x_{it} is suspected to have changed at a particular date i, a natural way to capture that change is by including $S_{it}x_{kt}$ in the model, in addition to x_{kt} itself, with the coefficient on $S_{it}x_{kt}$ picking up the incremental change in the original coefficient on x_{kt}. If the break-point i is itself unknown, block searches with more potential variables than observations permit considering $S_{it}x_{kt}$ for all break-point dates I and variables k. This approach precisely nests the (Andrews, 1993) unknown breakpoint test and the (Bai & Perron, 1998) multiple breakpoint test, aside from directly allowing the error variance to alter. (IIS does allow the error variance to alter, but IIS is not very parsimonious in the way that it does so).

Table 3.1: Indicator Saturation and Some Extensions

Type	Name	Description	Variables	Definitions
1.	**Impulse Indicator Saturation**	**Zero One Dummies**	$\{\boldsymbol{I_{it}}\}$	$\boldsymbol{I_{it}}$ **= 1 for** $\boldsymbol{t = i}$ **zero otherwise**
2.	**Step Indicator Saturation**	**Step Function**	$\{\boldsymbol{S_{it}}\}$	$\boldsymbol{S_{it}}$**= 1 for** $\boldsymbol{t \geq i}$ **zero otherwise**
3.	Super Saturation	Combining IIS with SIS	$\{I_{it}, S_{it}\}$	S_{it}= 1 for $t \geq i$ zero otherwise
4.	**Trend Indicator Saturation**	**Broken Linear Trend**	$\{\boldsymbol{T_{it}}\}$	$\boldsymbol{T_{it} = t - i + 1}$ **for** $\boldsymbol{t \geq i}$ **zero otherwise**
5.	Sequential Pairwise IIS	Zero One-One Dummies	$\{P_{it}\}$	P_{it} = 1 for $t = i, i + 1$; zero otherwise
6.	Zero-Sum Pairwise IIS	Plus-One & Minus-One Dummies	$\{Z_{it}\}$	Z_{it} = +1 for $t = i$ Z_{it} = −1 for $t = i + 1$; zero otherwise
7.	Many-Many Variables	More Variables then Observations	$\{x_{kt}\}$	$x_{kt} = \sum_{i-1}^{T} x_{kt} I_{it}$
8.	Factors	Factor Principal Component	$\{f_{jt}\}$	$f_{jt} = \sum_{\forall k} w_{jk} x_{kt}$
9.	Multiplicative Indicator Saturation	Partial Series	$\{x_{kt}^{(i)}, \forall i, k\}$	$x_{kt}^{(i)} = 0$ for $t < i$ $x_{kt}^{(i)} = x_{kt}$ for t ≥ i

Note: The table is extracted from (Ericsson, 2012)

An empirical example on modeling money demand in case of Pakistan has been given in Chapter 6, where the tests of SupExt under three types of indicator saturation like; (IIS, SIS and TIS) has been discussed in detail.

3.4.1 Detection of Multiple Structure Breaks

In this subsection, we will explain the underlying idea of indicator saturation procedures to detect breaks in linear regressions (breaks in deterministic part). Let's suppose,

$$y_t = \gamma_t^{\mathsf{T}} \boldsymbol{X}_t + \varepsilon_t = [\boldsymbol{\alpha}_t^{\mathsf{T}} \quad \boldsymbol{\beta}^{\mathsf{T}}] \begin{bmatrix} \boldsymbol{w}_t \\ \boldsymbol{z}_t \end{bmatrix} + \varepsilon_t, \quad t = 1, \dots, T \tag{3a}$$

Where $\boldsymbol{w}_t$ collects the deterministic parts, $\boldsymbol{z}_t$ is a k×1 vector of exogenous regressors as well as having lags of the endogenous variable and $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$ satisfies the condition $\mathrm{E}(\boldsymbol{z}_t, \varepsilon_t) = 0$. Given that a linear trend usually suffices for most economic applications, we can restrict $\boldsymbol{w}_t$ to consist of a constant without losing too much generality. Hereafter, we consider models that are nested in the following specification:

$$y_t = \alpha_t^{o} + \boldsymbol{\beta}^{\mathsf{T}}\boldsymbol{z}_t + \varepsilon_t, \quad t = 1, \ldots, T \tag{3b}$$

Conditionally on *m* unknown break dates $\{T_1, T_2, \ldots, T_m\}$. We can write a piece wise model of the form as below:

$$y_t = \alpha_t^{o} + \boldsymbol{\beta}^{\mathsf{T}}\boldsymbol{z}_t + \varepsilon_t \qquad T_0 = 0 \leq t \leq T_1$$

$$y_t = \alpha_t^{o} + \boldsymbol{\beta}^{\mathsf{T}}\boldsymbol{z}_t + \varepsilon_t \qquad T_1 \leq t \leq T_2$$

$$\vdots$$

$$y_t = \alpha_t^{o} + \boldsymbol{\beta}^{\mathsf{T}}\boldsymbol{z}_t + \varepsilon_t \qquad T_m \leq t \leq T_{m+1} = T$$

In the following, we want study the performance of indicator saturation and structural breaks in making inference about the vector of break dates $\{T_1, T_2, \ldots, T_m\}$.

3.4.2 Saturated Regressions

The idea of saturated regression approach originally introduced by (Hendry, 1999) and (Hendry et al., 2008) is a well-established method to test constancy of the estimated model by means of dummy saturation. The core idea is to saturate a linear model having *T* observations with *T* dummy variables (one dummy for each point) to capture outliers and structural breaks. The methods starts with an initial model where a break may happen at all times and then eliminates the statistically insignificant dummies following the "*general-to-specific*" approach,. The procedure is very general and dynamic and allowing us to test for the presence of multiple structural breaks.

In particular, the original approach as outlined in (Hendry, 1999; Hendry et al., 2008) encompasses a saturation with (0-1) impulses named as *impulse indicator saturation* (IIS). Later on, (Doornik et al., 2013) developed some theoretical properties of the *step indicator saturation* (SIS)[16]. Also (Ericsson, 2012), define *trend indicator saturation* (TIS) the version with also step dummies and the regression saturation involving trend dummies as well.

The indicator saturation principle tells us that inference about the vector of break dates is primarily based on the choice of the frugal representation from one of the following *saturated* regressions under considerations:

$$\text{Saturated Regressions} \begin{cases} IIS: & y_t = \alpha_t^{\text{o}} + \boldsymbol{\beta}^{\intercal}\boldsymbol{z}_t + \sum_{i=1}^{T} \gamma_i \, I_{i,t} + \varepsilon_t \\ SIS: & y_t = \alpha_t^{\text{o}} + \boldsymbol{\beta}^{\intercal}\boldsymbol{z}_t + \sum_{i=1}^{T} \varphi_i S_{i,t} + \varepsilon_t \\ TIS: & y_t = \alpha_t^{\text{o}} + \boldsymbol{\beta}^{\intercal}\boldsymbol{z}_t + \sum_{i=1}^{T} \omega_i T_{i,t} + \varepsilon_t \end{cases} \tag{3c}$$

Where $I_{i,t} = 1(t = i)$, $S_{i,t} = 1(t \geq i)$ and T_{it}= $(t - i + 1)1 \quad (t \geq i)$ for $i = 1, \dots, T$. Once the insignificant dummies are being dropped out, we are left with a set of dummies which can be labeled as outliers or breaks that affect the deterministic component of the process. Additionally, note that the coefficients of the dummies capture the magnitude of the changes in the coefficients of $\varphi_i = \alpha_i^{\text{o}} - \alpha_{i-1}^{\text{o}}$ and $\omega_i = \alpha_i^1 - \alpha_{i-1}^{\text{i}}$.

3.4.3 Solution to Multicollinearity and Dimensionality Problem

The methodology just introduced above face two major problems. Almost in all cases, particularly in SIS and TIS. First, the existence of multicollinearity among

[16] Where impulses are being replaced by partial sums of impulse dummies called it as Step Dummies (SIS).

some of the dummies clearly arises. Second, due to lack of degrees of freedom, the estimation of the saturated regressions is infeasible since number of regressor is more than observations. In principle, the multicollinearity problem within dummies can be solved quite easily by excluding some dummies (last step dummy or last trend dummy for instance). Typically, one sets $i = \ell y + 1, \ldots, T - 1$, where ℓy is the highest order of lagged dependent variables entering the process, and excludes the first step dummy, which is exactly collinear with the set of impulse dummies, and the last trend dummy.

On the other hand following the approach introduced in (Hendry et al., 2008), the possible solution to handle dimensionality (degree of freedom) problem is to divide the set of dummies in *J* blocks *s.t.* the number of lagged dependent variables and exogenous regressors (*k*) and the number of dummies in each block (N_j) plus the number of elements in the deterministic components, is less than the sample size ($i.e.\ N_j + 2 + k < T,$ for $j = 1, \ldots, J$). More explicitly, in the general case of trend indicator saturation, assume to form *J* blocks of about the same size[17] *s.t.* $\mathfrak{T}_1 = \{I_{i,t}, S_{i,t}, T_{i,t} : i = 1, \ldots, \lceil T/J \rceil\}$, $\mathfrak{T}_2 = \{I_{i,t}, S_{i,t}, T_{i,t} : i = \lceil T/J \rceil + 1, \ldots, \lceil 2T/J \rceil\}, \ldots, \mathfrak{T}_J = \{I_{i,t}, S_{i,t}, T_{i,t} : i = \lceil T(J-1)/J \rceil + 1, \ldots, T\}$. The technique is then can be implemented as:

- For $j = 1, \ldots, J$ include the $\mathfrak{T}_j$ subset of dummies in the equation of interest and estimate the partially saturated regression by noticing those dummies that are significant.
- Re-estimate the desired model by putting all relevant dummies obtained from previous iteration, assuming that the total number of the retained dummies from each subset $\mathfrak{T}_j$ is less than T (sample size). At the end, retain those

[17] In case of TIS, we set *J*> $\left\lceil \frac{3T}{T-2-k} \right\rceil$ in order to have enough *d.f.*

dummies that are significant. In this way one can get rid of these problems faced when using indicator saturation.

As explained in (Castle et al., 2012) under the null hypothesis of no breaks, the average retention rate of impulses is αT (α being the level of significance). For this reason, if we fix $\alpha \leq r/T$ we control the false null retention at r dummies. If we consider that we are evaluating the potential relevance of a large number of dummies that is a multiple of the sample size T, then this is fairly satisfactory. The approach outlined above is considered as the standard way to take structural breaks analysis into account.

The other more convenient way to implement the indicator saturation is through the algorithm for automated model selection *Autometrics*. This can be implemented in the software *OxMetrics*© which supports N > T and non-orthogonal candidate regressors. A regression saturated with dummies can be specified using *Autometrics* as a general unrestricted model (GUM), and statistically inconsequential (insignificant) regressors can be removed using a tree search algorithm. The method handles both the management of non-orthogonal regressors and the complete process of block formation. The entire procedure of block creation is carried out by the algorithm as well as the management of non-orthogonal regressors. An important aspect is that *Autometrics* provides different ways to create blocks in case of N > *T* (*i.e.* sequential random and cross blocks, etc.).

Finally, a theoretical investigation of the properties of the IIS framework only is developed by (Hendry et al., 2008) and generalized under less restrictive conditions in (Johansen & Nielsen, 2008). On one side, (Castle et al., 2012) used IIS approach to detect outliers and level shifts in several specifications including deterministic trends, unit roots, autoregressive processes as well as autoregressions with exogenous regressors with the help of simulation. On the other side (Castle et al., 2015) discussed about the performance of SIS. But so far the theoretical underpinnings of TIS are still under considerations. But neither test SupExt status under all types of indicator saturation. This gap is needed to be fulfilled. However, in this study we used IIS, SIS and TIS to test the performance of SupExt tests using these and all at a time (jointly) as well.

3.4.5 Enveloping Structural Breaks

Consider we have the following DGP in the form of a bivariate conditional model where both variables randomly drawn from normal distribution can be written as follows:

$$y_t = \alpha_t^{\mathrm{o}} + \beta^{\mathsf{T}} x_t + \varepsilon_t \tag{3d}$$

And the corresponding marginal model with IIS, SIS and TIS can be written as

$$\begin{aligned}
IIS: &\quad x_t = \alpha + \gamma x_{t-1} + \sum_{i=1}^{T} \gamma_i I_{i,t} + \varepsilon_{1t} \\
SIS: &\quad x_t = \alpha + \gamma x_{t-1} + \sum_{i=1}^{T} \varphi_i S_{i,t} + \varepsilon_{2t} \\
TIS: &\quad x_t = \alpha + \gamma x_{t-1} + \sum_{i=1}^{T} \omega_i T_{i,t} + \varepsilon_{3t}
\end{aligned} \tag{3e}$$

IIS, SIS & TIS:

$$x_t = \alpha + \gamma x_{t-1} + \sum_{i=1}^{T} \gamma_i I_{i,t} + \sum_{i=1}^{T} \varphi_i S_{i,t} + \sum_{i=1}^{T} \omega_i T_{i,t} + \varepsilon_{4t}$$

Now consider the above marginal models in each case and we want to capture all possible breaks at unknown time. The process is then starts by identifying those breaks that occurred at each point $t = 3, \dots, T-1$. So $\forall t$, we calculate the *F-statistic*, we call it F_t and may be defined as:

$$F_t = \frac{T-k-2(j+1)}{2} \times \frac{RSS_{T_{j-1}} - URSS_{T_{j=t}}}{URSS_{T_{j=t}}}$$

Where $RSS_{T_{j-1}}$ is residual sum of squares of restricted model and $URSS_{T_{j=t}}$ is the residual sum of squares of unrestricted model. However, the *d.f.* is $k+2(j+1)$, the number of parameters including those corresponding to the coefficients associated to the step and trend dummies $(2(j+1))$ in the unrestricted model, and 2 = $2(j+1)$ $-\ 2j$, the number of additional parameters resulting from adding one break date. Now the estimator $\hat{T}_j$ at break time T_j can be written as:

$$\hat{T}_j = argmax(F_t), \qquad \forall t$$

A case study by (Bai, 1994), he showed that in case of linear model using OLS the estimate of the break data T_j is asymptotically biased, however, the bias is small. Therefore, $\hat{T}_j = argmax(F_t) = argmin\left(URSS_{T_{j=t}}\right)$ and hence, the estimate $\hat{T}_j$ is then equivalent to OLS estimator.

3.4.6 Dummy Retention Rate

The validity of analysis of structural breaks depends on whether the agreed procedure is capable of detecting the correct number of breaks or not. The concept of dummy retention rate was introduced in (Castle et al., 2012). Therefore, following the precedence set by (Castle et al., 2012) we assume a GUM saturated with N dummies out of which $n <$ N have entered in our assumed DGP and M is the number of Monte Carlo simulations. The *rate of retention*, *r*, can be defined mathematically as:

$$\hat{r}_j = \frac{1}{M}\sum_{m=1}^{M} 1\left(\hat{\beta}_{j,m} \neq 0\right), \qquad j = 1, \dots, N \tag{3f}$$

Where $1(.)$ is indicator function and $\hat{\beta}_{j,m}$ is estimated coefficient of the break (dummy) at *j*th place found at the *m*th iteration. At a set level of significance, if the dummy is significant $(i.e.\ \hat{\beta}_{j,m} \neq 0)$[18], the indicator function becomes one. Therefore, the *gauge* and *potency* can mathematically be written as follows:

$$gauge = \frac{1}{N-m} \sum_{j=n+1}^{N} \hat{r}_j \tag{3g}$$

$$potency = \frac{1}{N} \sum_{j=1}^{n} \hat{r}_j \tag{3h}$$

where the *gauge* is the average retention rate of the inconsequential dummies and the average retention rate of those dummies that found to be significant is called *potency*.

a. Set j = 1 and i = 0:

1. Estimate the j^{th} break date as:

$$\hat{T}_j = argmax(F_t), \qquad t = 3, \dots, T-1$$

2. Test the significance of $\hat{T}_j$ $(H_0: j-1$ breaks$)$ figuring the *p*-value

3. If significant, re-estimate the previous $j-1$ breaks otherwise i = i + 1.

b. Set j = j + 1 and repeat steps 1-3. Stop when two consecutive break dates are not significant (i = 2)

c. Repeat for all the marginals and then impose the breaks in the conditional processes and check their significance.

3.5 General-*to*-Specific Modeling Strategy

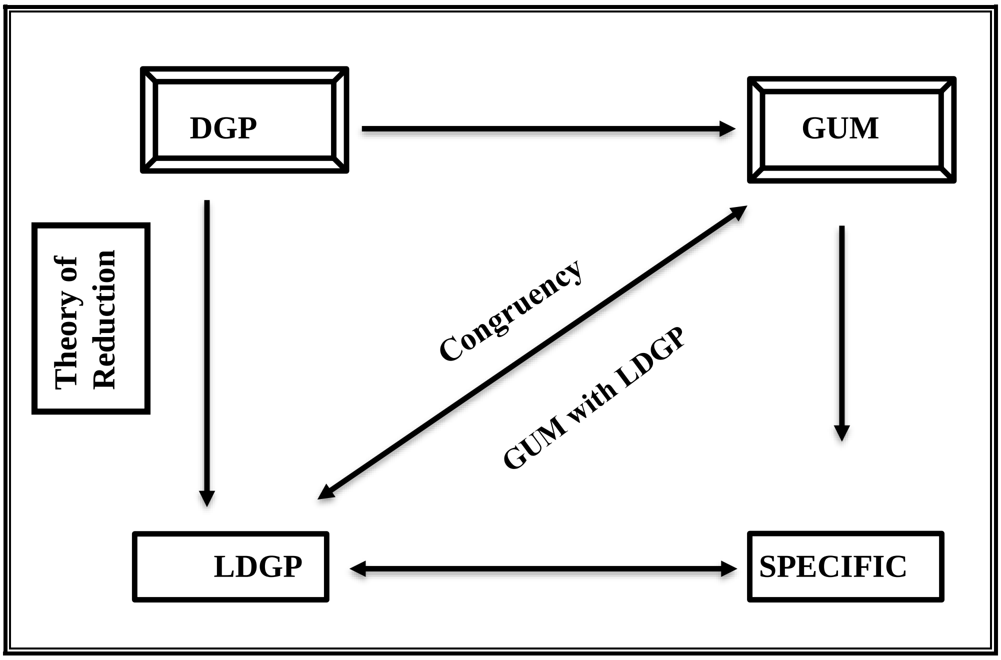
Before discussing the in depth details of *Autometrics* one should know how the developments evolved over the time from GETS to *PcGets* and then the advent of

[18] It is worth noting that is represents $\left|t_{\hat{\beta}_{j,m}}\right| \geq C_{\frac{\alpha}{2}}$, $t_{\hat{\beta}_{j,m}}$ represents *t-values* and $C_{\frac{\alpha}{2}}$ is the corresponding critical value at a desired significance level α.

latest *Autometrics* algorithm. In this section, we will first discuss about the conceptual framework of GETS and some about *PcGets* algorithm. The GETS modelling strategy which is also framed as 'Hendry Methodology' was initially introduced by David F. Hendry in 1980s.

The basic idea behind this strategy was to develop an econometric model as representation of probability distribution function or the sample data *i.e.* data generating process (Hendry, 1995). Based on theory of reduction, it started with a very general parameterization which is possible theoretical positions representing the DGP. After that DGP is reduced to obtain a 'local' DGP (LDGP) known as joint distribution enveloping subset of relevant variables by operating sequential cuts, valid conditioning and marginalization (Hendry, 1995). The intention is to develop a methodology which is robust and efficient in countering the fundamental problems raised while constructing econometric models by using specific-to-general approach.

Figure 3.1: Conceptual Framework of GETS Modeling



Note: It is adapted from (Hendry & Doornik, 2014)

Essentially on one side, the knowledge of economic theories *via* GUM vehicle is traceable. It has the possible features of the data along with previously available empirical results. On the other side, the congruency of the can be validate if it passes al the diagnostic tests like; autocorrelation, normality, heteroskedasticity and break test as well. Based on (Lovell, 1983) experiment, (Hoover & Perez, 1999) considered as the first case study to evaluate the performance of automatic GETS strategy using an algorithm. They came up with a conclusion that GETS algorithm performs better than the other model selection procedures like *'max-min-t'* selection, maximizing R^2 and stepwise regression. They pointed out that the size *(type-I error)* of the test is near as expected while the power *(type-II error)* is justifiable.

Later, (Hendry & Krolzig, 1999) developed a multi path search algorithm, capturing the drawbacks of initially feasible paths and then gets terminal models as a result of each search. In case, where huge number of terminals found, they pass through a filter of encompassing test for their union. As a result, a smallest model that nests all other relevant models will be obtained. *PcGets* algorithm achieved better power again based on (Lovell, 1983) experiment (Hendry & Krolzig, 2005, 1999, 2003; Krolzig & Hendry, 2001). The studies mentioned above, it is worth writing that *PcGets* is a more efficient approach in model selection procedures since it adds different specification tests.

Following GETS and *PcGets*, a new era of model selection has been introduced initially in (Doornik & Hendry, 2007) and later in (Doornik, 2009a) known as *Autometrics*. The *Autometrics* enriches the previously available model selection algorithms and give an opportunity to multi-step ahead by considering more paths, reducing time cost (hence, more efficient) and last but not the least, avoid those models who were selected repeatedly. Consequently, better results were obtained.

3.5.1 PcGets Algorithm

Based on the principal of general-*to*-specific approach, an automatic model selection algorithm for linear and dynamic regressions was proposed by (Krolzig & Hendry, 2001) and personified in *GiveWin 2.10*, named as *PcGets*. The problem face by using stepwise regression was successfully tackled in *PcGets* (Hendry & Krolzig, 2005). Each insignificant variable in general unrestricted model lead to determine a path and at a pre-specified chosen level of significance, these variables then converted into groups (Hendry & Krolzig, 1999). As it is a well-established fact that *PcGets* is a multi-path search algorithm. So, there is a high chance of getting more than the one potentially significant candidate variable that eventually left after reduction has been applied. Therefore, to wrestle with this issue a parsimonious encompassing test is applied. If, still terminals are there which are significant, then the algorithm takes the union of these terminals, the search will not be completed until a final model has been obtained based on *SIC*. At the end, in order to identify 'spuriously significant' predictor *PcGets* will use the sub-sample insignificance criteria. To support the argument of congruency, every selected model is then checked through different diagnostic tests (Hendry, 2000).

There are many tests available in literature for encompassing the nested and non-nested models. The following section is an effort to discuss briefly one of these encompassing tests and is named as *J*-test for non-nested hypothesis that is being used here in our simulation strategy.

3.5.2 Encompassing J-Test

The sub-section briefly discuss one of the tests used for comparing non-nested hypotheses named as the *J*-test which is proposed by (Davidson & MacKinnon, 1981). These tests arise in such situations where the alternate hypothesis (H_1) cannot be derived as a special case of the null hypothesis (H_0). The test appeared in several standard econometric textbooks like (Charemza & Deadman, 2003; Davidson & MacKinnon, 2004; Greene, 2003) and also included in standard econometrics programs like (*E-Views* and Shazam). There is huge empirical as well as theoretical literature available which discuss both pros and cons of *J*-test. An excellent expedition to this can be found in (McAleer, 1995).

The mathematical foundations of the test as discussed in (Davidson & MacKinnon, 1981) are being explained to some extent here for those who want to learn an in depth working of this test. Consider two alternate specifications of Y being considered as non-nested hypotheses.

$$H_0: Y = X\beta + \varepsilon_1 \tag{3.1a}$$

$$H_1: Y = Z\gamma + \varepsilon_2 \tag{3.1b}$$

Where, both *X* and *Z* has k_1 and k_2 independent regressors respectively. From above two models an artificially compound model can be written as follows:

$$Y = (1-\alpha)X\beta + \alpha Z\gamma + \eta \tag{3.1c}$$

If we estimate the model (3.1c), then we test the non-nested hypotheses by employing the parameter restrictions. Now, if $\boldsymbol{\alpha} = 0$ in this case, model in (3.1c) will reduce to (3.1a) and in case when $\boldsymbol{\alpha} = 1$ it will become (3.1b). It is suggested in (Davidson and MacKinnon, 1993) that replacing (3.1c) with the unknown parameter estimates of the models in (3.1a) and (3.1b) would be consistent provided that DGP actually belonged to the model they are defined. The test has two steps; first we estimate $\widehat{Y_Z}$ from regressing Y on Z to test (3.1a) and replace this estimate in (3.1c) to test hypothesis in (3.1a). Similarly for hypothesis in (3.1b), first we estimate $\widehat{Y_X}$ from regressing Y on X to test (3.1b) and replace this estimate in (3.1c). The *J*-test uses *t*-

stat. for the coefficient being estimated in (3.1c) $\widehat{Y_Z}$ and $\widehat{Y_X}$ one by one individually. A statistically significant *t-stat.* on $\widehat{Y_Z}$ will reject H_0 in considering (3.1a) as appropriate model while a statistically significant *t-stat.* on $\widehat{Y_X}$ will reject H_1 in considering (3.1b) as appropriate model.

3.5.2.1 Implementing J-Test

Now, how do we implement this encompassing test in this study can be seen in the following lines. Our aim is to determine between (M1 & M2) which model is better than the other. Here each model is testes against $\mathrm{M1} \cup \mathrm{M2}$. Suppose the predictors in M1 are $k_1 + k_2$ $(x_{1t},\ x_{2t})$, whereas in M2 are $k_2 + k_3$ $(x_{2t},\ x_{3t})$. Here it can be seen that x_{2t} is a common predictor and the union of these three in a compounded model will be $k = k_1 + k_2 + k_3$. Let RSS_{M1}, RSS_{M2} and RSS_{MC} denoted residual sum of squares from M1, M2 and compounded model respectively. Where, k_3 represents number of redundant variable in M1 and compounded model.

The following are the hypotheses to be tested:

H_0: $M1 \supset M2$ or $M2 \subset M1$ (M1 encompass M2)

H_1: $M1 \not\supseteq M2$

The test statistic for the test conducted at *5%* significance level is as below:

$$F = \frac{(RSS_{M1} - RSS_{MC})\big/_{k_3}}{RSS_{MC}\big/_{(T-k)}} \sim F(k_3, T-k)$$

3.6 Algorithm of *Autometrics*

Autometrics envelops all the properties of GETS modeling strategy and is considered as third generation of GETS model selection algorithm. It is rightly to say that *PcGets* developed by (Krolzig & Hendry, 2001) widely used for both linear and dynamic regression models. Unlike *PcGets*, the algorithm of *Autometrics* operates differently and considered as new generation of *PcGets* (Hendry & Krolzig, 2005). In general, *Autometrics* has three stages. First, the procedure to model with or without pre-search which were originally embedded in *PcGets* on ad-hoc basis, are now fully operational in *Autometrics.* Second, the usage of tree search procedure leads to select all possible candidate variables in GUM, unlike multi path search in *PcGets*. Using different information criteria, the implementation of pruning, bunching and chopping (discuss below) give more to be selected. Lastly, the repeated selection and estimation of a same model in *GETS* modeling has been enormously tackled in *Autometrics* resulting into improve the computational cost. The *Autometrics* works in five different stages. Stage-I is to estimate initial general unrestricted model. Stage-II is pre-search reduction at a loose significance level. Stage-III is about variable reduction over root branches. Stage-IV is search for nested terminals and Stage-V is selected a final model. A detailed overview on how *Autometrics* works at each stage can be seen in (Doornik, 2009a). However, for the sake of simplicity we took the liberty to just explain a little part of it from Stage-II at the time.

3.6.1 Criteria of Reduction

Since, GUM includes a set of all possible candidate variables; the tree search algorithm finds all possible models against these variables. The reduction principles help in reducing irrelevant models and enhance the computational efficacy. To obtain a unique model, the reduction principles search in a very systematic way to skip unnecessary path. The reduction principles that are implemented in *Autometrics* like pruning, bunching and chopping are as follows:

3.6.1.1 Pruning/Ignoring

In simple words it a single variable selection *i.e.* whenever, the removal of one or more variables individually is failed due to either greater *p*-value *w.r.t* desired significance level or any one of the diagnostic tests (normality, ARCH, Chow test for parameter constancy or autocorrelation etc.) is violated.

3.6.1.2 Bunching

The bunching principle allows removing pair, group or bunch of variables at a time in just one single step. In this study, for bunching a *5%* level of significance or p_α has been opted. While B_p reflects the amount of the bunching. Individually insignificant variables are then combined to make pairs and then groups as long as their smallest *p*-value is more than B_p^* which is defined as below:

$$B_p^* = B_p^{\frac{1}{2}} \left\{ 1 - \left(1 - B_p^{\frac{1}{2}} \right)^{k_b} \right\} \tag{3.1d}$$

Where, k_b is bunch size and B_p is as follows:

$$B_p = max \left\{ \frac{1}{2} B_p^{\frac{1}{2}} , B_p^{\frac{3}{4}} \right\} \tag{3.1e}$$

For example, at *5%* significance level, a two variable bunch will be removed if their *p*-value is more than 0.1862. On the other side the bunch will be removed from the path if *p*-value is less than 0.1862 for at least one of the variables in the group and hence the size of the bunch will be reduced to one variable.

3.6.1.3 Chopping

In this reduction principle, one or more variables will be removed permanently from the path search algorithm provided that it is highly insignificant from the branches of the model. When a variable is insignificant enough, the whole bunch could get the chop. No doubt, chopping saves computational time, but there is a chance that some relevant combinations of variables are missed.

3.6.1.4 Closed Lag

The removal of group lag variable is based of general to specific methodology *i.e.* starting from highest value to lowest. It starts with identification of each group *w.r.t* their lag number. Take for instance, the highest lag in each selected equation q, then, those variables which have lag q will form a group and k_p is denoted for number of variables involved. It is worth mentioning that the removal of each group in this reduction part is based on four conditions:

First, if *p*-values of all variables is more than the assumed level of significance, then this group is removed from the equation. Second, once the removal of the group has been taken place at first stage, after which the reduced model will again be estimated using FGLS and *F*-test will be used for their joint significance and is compared with the model before reduction took place. Third, unlike to the second condition, here we compare the reduced model with the initial GUM. Since, it is very important to have an eye on whether this removal is a valid reduction of the GUM. Fourth, the congruency of the model after each removal is being tested *via* several diagnostic tests. Now in case, if any of these conditions fails, the reduction process will be stopped and all the variables will be returned back to the equation. Otherwise, the process of reduction will continue until lag one.

3.6.1.5 Common Lag

This is another reduction criteria where the variables in their lag are grouped *w.r.t* their lag number. These lags then arranged into descending order after checking joint significance of these lags. The model reduced to the best fit after removing all insignificant lags starting from highest lag. The compact model is estimated using FGLS and test against the model before reduction to determine their joint *p*-values.

3.6.1.6 Common X-Lag

In this reduction procedure, the lag of dependent variable excluded from all the estimation procedure. The process works same like it works in common lag.

3.6.2 Working of Autometrics

The purpose of this subsection is to elaborate how Autometrics works in order to find a true model from a set of candidate variables. In broader sense, *Autometrics* helps the empirical modeller to apply the *"LSE Approach"* or *"GETS"* approach proposed by David F. Hendry. As a software it is being included in *OxMetrics*^{TM} and a latest upgraded version of *PcGets* developed by (Hendry & Krolzig, 2005, 1999) based on (Hoover & Perez, 1999).

We begin with a general unrestricted model (GUM) that contains all the predictors that the modeller believes may be important. Using a tree search algorithm and key variables chosen based on a battery of misspecification tests as well as individual significance tests *(t-tests)* on candidate regressors, *Autometrics* is able to choose a final model. Based on a criteria set by the user, the selection of the final model is performed using a reduction p-value (p_α). Additionally, *Autometrics* can handle non-orthogonal candidate regressors as well as more variables than observations $N > T$ (unidentified GUM). This is an interesting case when dealing with indicator saturation, so here we analyze a generic version of the algorithm involving block search. The case when $N < T$ is a special case of the block search algorithm.

Following (Doornik, 2009b), first we introduce the block-splitting component of the algorithm, then we present the overall algorithm. In particular, inside the block-search algorithm all the N variables entering the GUM, $\bar{\mathfrak{P}}$, are split in two sets: the *selected* variables at iteration i, denoted $\mathcal{S}_i$, and the *excluded* variables, denoted $\mathfrak{P}_0 = \overline{\mathfrak{P}} \setminus \mathcal{S}_i$ The excluded set is partitioned in blocks and two steps alternate in succession:

1. This step is called the *Expansion Step* that partition the excluded variables in blocks $(\mathfrak{P}_0^1, \mathfrak{P}_0^2, \mathfrak{P}_0^3, \dots, \mathfrak{P}_0^B)$ and run all over the blocks $\mathfrak{P}_0^1 \cup \mathcal{S}_i, \mathfrak{P}_0^2 \cup \mathcal{S}_i, \dots, \mathfrak{P}_0^B \cup \mathcal{S}_i$ to look for omitted variables $(\mathcal{O}_i)$. This is an iterative step and it stops when the number of regressors selected from the initial set of excluded variables is small enough.
2. After step 1, this step is called the reduction step that operates to find a new candidate set $\mathcal{S}_{i+1}$ from the model selection on $\mathcal{S}_i \cup \mathcal{O}_i$.

As explained above, the selection of the relevant variables is governed throughout by a *p-value* that can be modified by the user. However, a temporary increase of the *p-value* in the reduction step is used to improve the sensitivity of *Autometrics* at the cost of slightly increasing the risk of over fitting. For exposure of interested readers we define the following phases:

***Phase* 1:** Starts from an empty model, and is run only once.

***Phase* 2:** Is run until convergence, using p_α for the expansion step.

***Phase* 3:** Is run only once, using $2p_\alpha$ for the expansion step.

***Phase* 4:** Starts with $4p_\alpha$ for the expansion step, using p_α thereafter.

When *Phase 4* converges, the block search terminates.

After this detailed discussion the *Autometrics* algorithm runs as follows:

Set i = 0, $\mathcal{S}_{i-1}$= ∅, Phase = A.
1. *Expansion step* to $\mathcal{O}_i$ find ;
2. *Reduction step* to find $\mathcal{S}_i$;
3. Change the phase:
3a. **if** phase is A set Phase = B and go to *Continuation*;

3b. **if** phase is C set Phase = D and go to *Continuation*;

3c. **else** go to *Convergence* step;

4. *Convergence* **if** $\mathcal{S}_0 \cup \mathcal{S}_1 \cup \ldots \cup \mathcal{S}_i = \mathcal{S}_0 \cup \mathcal{S}_1 \cup \ldots \cup \mathcal{S}_{i-1}$ **then**

4a. **if** stage is B increment it to C,

4b. **else** terminate block search.

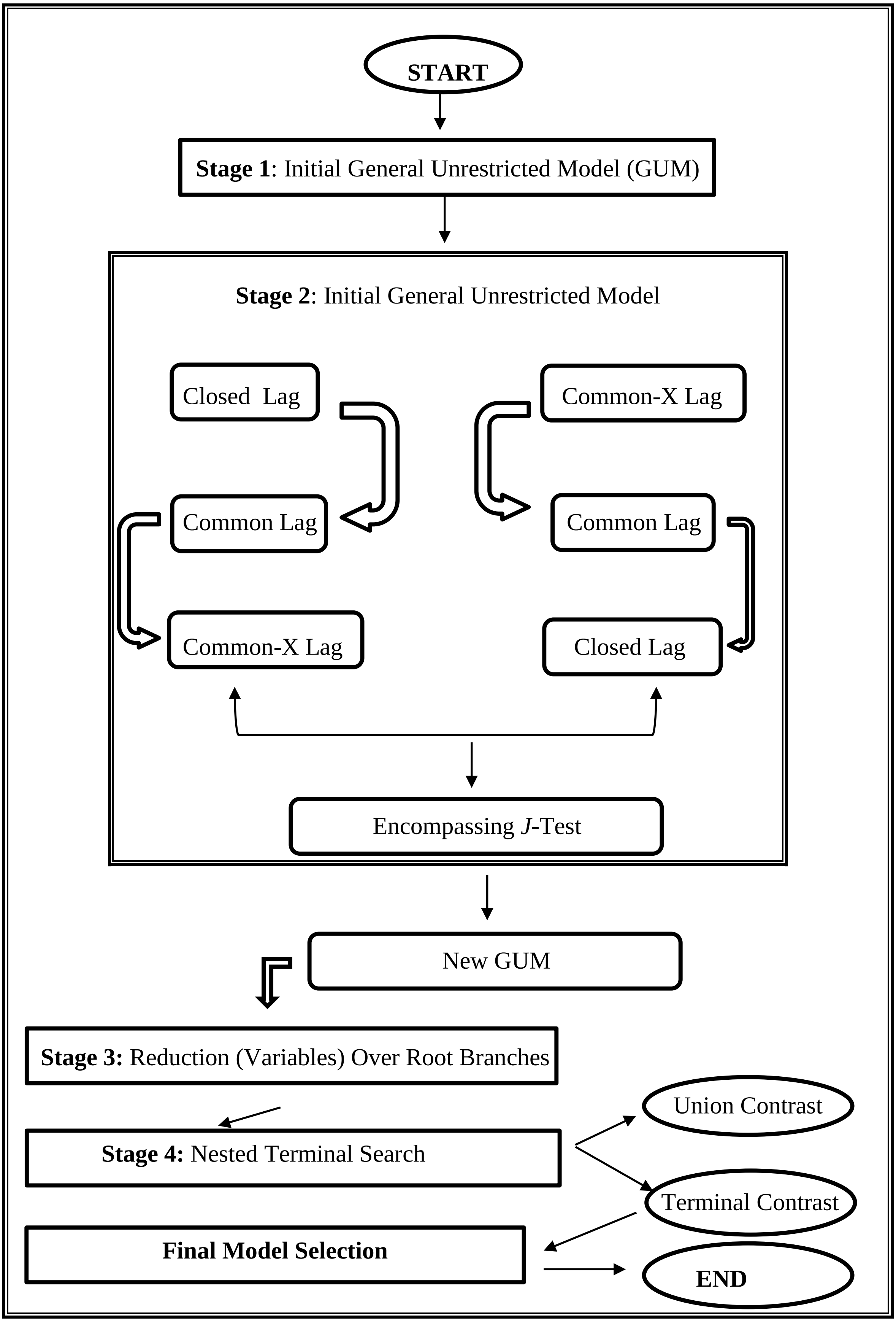
5. *Continuation* increment *i* and return to Step 1.

To improve on efficiency, the way in which the algorithm searches for significant variables is a tree-search type procedure where redundant branches are skipped. In the case there are multiple terminal models, *Autometrics* forms the final GUM as the union of these. Finally, the overall final model is chosen using the Schwarz criterion.

Additionally, there are different ways in which the blocks may be formed in the *Expansion* step. This option, together with the block size, can be set by the user. In particular, together with the *standard* block method where the blocks are formed sequentially, other options are the *random* blocks and two variants where the algorithms also search more extensively crossing the standard blocks. However, as also noted in (Doornik, 2009b), the overall algorithm is sensible to the ordering of the variables so a different way to constitute the blocks may lead to slightly different outputs as we observe in our simulation exercise.

For further details on the performances and the options of *Autometrics*, we invite the interested reader to refer to (Doornik, 2009a) and (Doornik, 2009b). However, the *Autometrics* framework can be seen in the following detailed framework in Figure 3.2.

Figure 3.2: Framework of Autometrics



Note: This figure is being developed on the basis of (Doornik, 2009a).

3.7 Super Exogeneity Testing Procedures

The type of exogeneity related with the structural invariance in the worlds of parameters uncertainty or change leads us towards the definition of SupExt (Engle et al., 1983). There are many testing procedures available in literature to test the hypothesis of SupExt. But all test performed under the assumption of stationary data settings. None of the testing procedures have checked the performance of these tests in the presence of unit root settings. However, this study will try to fill the gap and making a valuable contribution to the literature. Broadly, the test of SupExt can be divided in two types. First, to establish the parameter constancy using (Chow, 1960) break point test. However, the limitations of procedures based on this test has been well documented in (Spanos, 1986). Second, is to establishing the invariance property referred as parameters of model do not alter corresponding to changes in policy or policy interventions. Therefore, the tests used here are based on the assumptions of invariance. In this section, we our emphasize is on the methods of testing SupExt in details that we already have introduced above but first need to understand how SupExt can be explained in regression context.

3.7.1 Super Exogeneity and Simple Regression

A variable is said to be exogenous or not, depends primarily on parameter of interest in which investigator is interested and for what purpose investigator is going to model like; for inference, for forecasting or for policy analysis. The answer lies in three types of exogeneity described in (Engle et al., 1983). The explanation of exogeneity and its types therein in the line with (Richard, 1980) lead us to the term exogeneity based on the concept given in (Koopmans, 1950). Substantial exogeneity presumptions may allow more straightforward modeling techniques, diminish computational cost, and help separate invariants of economic mechanism. However, Invalid exogeneity suspicions may prompt conflicting inferences and results in deluding forecasts and policy analysis (Ericsson, 1991).

Since the prime focus of the study is on SupExt, therefore, without going into more fancy details, how SupExt can be dealt in simple regression perspective can be seen below:

Consider the joint DGP of an n-dimensional vector process $\{x_t\}$ can sequentially be partitioned as:

$$\prod_{t=1}^{T} D_X(\mathbf{x}_t|\mathrm{X}_{t-1},\Theta) = \prod_{t=1}^{T} D_{y|z}(\boldsymbol{y}_t|\boldsymbol{z}_t,\mathrm{X}_{t-1},\lambda_1) \,.\, \prod_{t=1}^{T} D_z(\boldsymbol{z}_t|\mathrm{X}_{t-1},\lambda_2) \tag{3.1}$$

Where, $\mathbf{x}_t' = (y_t' \colon z_t')$ and $\lambda = (\lambda_1' \colon \lambda_2') = f(\Theta) \epsilon \mathbb{R}^k$($k$-dimensional Euclidian space). This sequential cut may proceed and make the process $\boldsymbol{z}_t$ a weakly exogenous if the parameters of $\mathbf{y}$ and $\mathbf{z}$ are variation free for the parameters of interest of conditional model. However, this does not lead to conclude that $\lambda_1$will not change when $\lambda_2$changes. Therefore, in simple words SupExt is amalgamation of weak exogeneity and parameters invariance of conditional model. However, (Ericsson et al., 1998) pointed out that the weak exogeneity of putative conditioning regressors can be tested indirectly via testing them for super exogenous.

Now, in case if $D_X(.)$ is the multivariate Gaussian process, the above factorization can be expressed as unconditional model:

$$\begin{pmatrix} y_t \\ \boldsymbol{z}_t \end{pmatrix} \sim \mathrm{N}\left[\begin{pmatrix} \mu_{1,t} \\ \boldsymbol{\mu}_{2,t} \end{pmatrix}, \begin{pmatrix} \sigma_{11,t} & \boldsymbol{\sigma}'_{12,t} \\ \boldsymbol{\sigma}_{21,t} & \boldsymbol{\Omega}_{22,t} \end{pmatrix}\right] \tag{3.2}$$

Where $E\ [y_t] = \mu_{1,t}$ and $E\ [z_t] = \mu_{2,t}$ are functions of X_{t-1} in general.

The parameters of interest can be then be defined by using formulation of economic theory as follows:

$$\mu_{1,t} = \mu + \gamma' \boldsymbol{\mu}_{2,t} + \delta' \mathbf{x}_{t-1} \tag{3.3}$$

Where, the parameter γ is of prime interest. Now from (3.2) and (3.3):

$$\begin{aligned} E\ [y_t | \mathbf{z}_t, \mathbf{x}_{t-1}] &= \mu_{1,t} + \boldsymbol{\sigma}'_{12,t} \boldsymbol{\Omega}^{-1}_{22,t} (\mathbf{z}_t - \boldsymbol{\mu}_{2,t}) + \delta' \mathbf{x}_{t-1} \\ &= \mu + \beta_{1,t} + \beta'_{2,t} \mathbf{z}_t + \delta' \mathbf{x}_{t-1} \end{aligned} \tag{3.4}$$

Where, $\beta_{1,t} = (\gamma - \beta'_{2,t}) \boldsymbol{\mu}_{2,t}$ and $\beta'_{2,t} = \sigma'_{12,t} \Omega^{-1}_{22,t}$. Also from above the conditional variance can be written as $\vartheta_t^2 = \sigma_{11,t} - \boldsymbol{\beta}'_{2,t} \boldsymbol{\sigma}_{21,t}$. Now on this ground the parameters of both densities *i.e.* conditional and marginal can therefore be observed respectively: $\lambda_{1,t} = \{\mu, \beta_{1,t}, \beta_{2,t}, \delta, \vartheta_t^2\}$ and $\lambda_{1,t} = \{\boldsymbol{\mu}_{2,t}, \boldsymbol{\Omega}_{22,t}\}$. In the case, if (3.4) modeled as a regression with constant parameters for $t = 1, \dots, T$:

$$y_t = \mu + \gamma' \mu_{2,t} + \boldsymbol{\delta}' \mathbf{x}_{t-1} + \varepsilon_t, \text{ Where } \varepsilon_t \sim \text{IN}[0, \vartheta^2] \tag{3.5}$$

Following (Engle & Hendry, 1993), $\mathbf{z}_t$ is said to be super exogenous for the parameters of interest, if the conditions given below are satisfied:

i) $\beta_{2,t} = \beta_2$ is constant $\forall t \epsilon T$

ii) $\boldsymbol{\delta}' = \beta_2$

iii) $\lambda_{1,t}$ is stable against shocks in $\lambda_{2,t}$ $\forall t \epsilon T$ (Invariance Property)

iv) $\vartheta_t^2 = \vartheta^2$ $\forall t \epsilon T$

In case, when conditions (i)-(iv) are satisfied, then $\mathbf{z}_t$ is considered to super exogenous for parameter of interest $\boldsymbol{\beta}$ and model can be written as:

$$E\ [y_t | \mathbf{z}_t] = \mu_0 + \boldsymbol{\beta}' \mathbf{z}_t \text{ with } \boldsymbol{\sigma}'_{12,t} = \boldsymbol{\beta}' \boldsymbol{\Omega}_{22,t}\ \forall t \tag{3.6}$$

Finally, under the scenario of SupExt the joint density function can be written as:

$$\begin{pmatrix} y_t \\ \boldsymbol{z_t} \end{pmatrix} = \sim \mathrm{N}\left[\begin{pmatrix} \mu_0 + \gamma'\boldsymbol{\mu}_{2,t} \\ \boldsymbol{\mu}_{2,t} \end{pmatrix}, \begin{pmatrix} \vartheta_t^2 + \boldsymbol{\beta}'_{2,t}\boldsymbol{\Omega}_{22,t}\boldsymbol{\beta} & \boldsymbol{\beta}'\boldsymbol{\Omega}_{22,t} \\ \boldsymbol{\Omega}_{22,t}\boldsymbol{\beta} & \boldsymbol{\Omega}_{22,t} \end{pmatrix}\right] \tag{3.7}$$

Therefore, the sequential cut or factorization of joint density into conditional and marginal is:

$$\begin{pmatrix} y_t|\boldsymbol{z_t} \\ \boldsymbol{z_t} \end{pmatrix} = \sim \mathrm{N}\left[\begin{pmatrix} \mu_0 + \gamma'\boldsymbol{z_t} \\ \boldsymbol{\mu}_{2,t} \end{pmatrix}, \begin{pmatrix} \vartheta_t^2 & \boldsymbol{0}' \\ \boldsymbol{0} & \boldsymbol{\Omega}_{22,t} \end{pmatrix}\right] \tag{3.8}$$

Accordingly, without changing the parameters of (3.5) the marginal model is:

$$\mathbf{z}_{\boldsymbol{t}} \sim \mathrm{N}[\boldsymbol{\mu}_{2,t}, \boldsymbol{\Omega}_{22,t}]\ \forall t \tag{3.9}$$

3.7.2 Cases when Super Exogeneity Fails

The literature so far has been able to identify main reasons of SupExt failure

i) The case when estimates of conditional model overlap with $\boldsymbol{\beta}$ *i.e. the* case where $\boldsymbol{z_t}$ fail to be weakly exogenous for parameter of interest $\boldsymbol{\beta}$ (condition for weak exogeneity)
ii) The estimates of conditional model are not constant (non-constancy condition)
iii) The case when parameters in marginal model induce changes in the set of parameters of interest $\boldsymbol{\beta}$ (condition of non-invariance)

Now after discussing the concept of SupExt in basic regression models, we will now move towards the testing procedures of SupExt.

3.7.2.1 *Test 1 procedure: Invertibility Test*

As discussed in (Engle et al., 1983), SupExt is a concept that the parameters of a preferred model are invariant to shifts in the DGPs of weakly exogenous conditioning variables. An eloquent discussion on several concepts of exogeneity can also been seen in (Hendry, 1995). The test we used first was mainly due to (Hendry, 1988) named as ***H-Test***, later implemented while modelling UK demand for narrow money (Hendry & Ericsson, 1991a) and is known as invertibility test of SupExt. By inverting the conditional model to become a marginal one. But now a key recent development is that of testing for non-constancy by adding a complete set of indicators to a marginal model (Hendry et al., 2004).

Using a *general-to-specific* procedure, those authors analytically establish the null distribution of the estimator of the mean in a location-scale IID-distribution after adding T impulse indicators when the sample size is T. Using *Autometrics* algorithm all significant dummies were retained and then added to the desired model. All these steps we used here are being implemented using *Autometrics*. The average retention rate of impulse indicators under the null is αT when the significance level of an individual test is set at α, so for $\alpha = 0.01$, for example, $0.01T$ indicators will be retained. Moreover, (Hendry et al., 2004) showed by simulation that other splits, such as reordering the impulses, or using three splits of size T/3, do not affect the retention rate under the null, or the simulation-based distribution of the estimated mean.

This procedure can be applied to the marginal models for the putative super-exogenous regressors. First, the associated significant dummies in the marginal processes are recorded. Secondly, those which are retained are tested as an added variable set in the conditional model. Specifically, after the first stage when m impulse indicators are retained in the inverted model, the marginal models then can be extended to following form:

$$IIS: \qquad x_t = \alpha + \gamma y_t + \delta x_{t-1} + \sum_{i=1}^{T} \gamma_{i,\alpha_1} I_{i,t} + \varepsilon_{1t}$$

$$SIS: \qquad x_t = \alpha + \gamma y_t + \delta x_{t-1} + \sum_{i=1}^{T} \varphi_{i,\alpha_1} S_{i,t} + \varepsilon_{2t}$$

$$TIS: \qquad x_t = \alpha + \gamma y_t + \delta x_{t-1} + \sum_{i=1}^{T} \omega_{i,\alpha_1} T_{i,t} + \varepsilon_{3t} \qquad (3.10)$$

IIS, SIS & TIS:

$$x_t = \alpha + \gamma y_t + \delta x_{t-1} + \sum_{i=1}^{T} \gamma_{i,\alpha_1} I_{i,t} + \sum_{i=1}^{T} \varphi_{i,\alpha_1} S_{i,t} + \sum_{i=1}^{T} \omega_{i,\alpha_1} T_{i,t} + \varepsilon_{4t}$$

Where, the coefficients of the significant impulses are denoted γ_{i,α_1} , φ_{i,α_1} and ω_{i,α_1} to emphasize their dependence on the significance level α_1 used in the marginal model. Note, this test has the appropriate null rejection frequency. The second stage of the testing procedure is to add these set of *m* retained dummies from the marginal model to the conditional model as below:

$$IIS: \qquad y_t = \alpha + \gamma x_t + \sum_{i=1}^{T} \gamma_{i,\alpha_2} I_{i,t} + \varepsilon_{1t}$$

$$SIS: \qquad y_t = \alpha + \gamma x_t + \sum_{i=1}^{T} \varphi_{i,\alpha_2} S_{i,t} + \varepsilon_{2t} \qquad (3.11)$$

$$TIS: \qquad y_t = \alpha + \gamma x_t + \sum_{i=1}^{T} \omega_{i,\alpha_2} T_{i,t} + \varepsilon_{3t}$$

IIS, SIS & TIS:

$$y_t = \alpha + \gamma x_t + \sum_{i=1}^{T} \gamma_{i,\alpha_2} I_{i,t} + \sum_{i=1}^{T} \varphi_{i,\alpha_2} S_{i,t} + \sum_{i=1}^{T} \omega_{i,\alpha_2} T_{i,t} + \varepsilon_{4t}$$

Then conduct an *F-test* for the significance of impulses at level α_2.Under the null of SupExt, check the joint significance of the *m* included impulse indicators in the conditional model with *F-test*.

3.7.2.2 Test 2 procedure: An Index based Test

Following (Hendry & Santos, 2006), a variant of the test discussed above, which could have different power characteristics, is to combine the *m* retained impulses detected in all the equations or from marginal model and then form an Index of these impulses with weights equal to the coefficients of the impulses retained in the marginal models by considering the following scenario. We call this as ***IB-Test***.

Suppose the third term for first three cases on R.H.S in (3.13) represents significant dummies entering into our marginal model and third, forth and fifth term in last case represents significant dummies entering into our marginal model for instance:

$$IIS: \quad x_t = \alpha + \gamma x_{t-1} + \sum_{i=1}^{T} \gamma_{i,\alpha_2} I_{i,t} + \varepsilon_{1t}$$

$$SIS: \quad x_t = \alpha + \gamma x_{t-1} + \sum_{i=1}^{T} \varphi_{i,\alpha_2} S_{i,t} + \varepsilon_{2t}$$

$$TIS: \quad x_t = \alpha + \gamma x_{t-1} + \sum_{i=1}^{T} \omega_{i,\alpha_2} T_{i,t} + \varepsilon_{3t} \qquad (3.13)$$

IIS, SIS & TIS:

$$x_t = \alpha + \gamma x_{t-1} + \sum_{i=1}^{T} \gamma_{i,\alpha_2} I_{i,t} + \sum_{i=1}^{T} \varphi_{i,\alpha_2} S_{i,t} + \sum_{i=1}^{T} \omega_{i,\alpha_2} T_{i,t} + \varepsilon_{4t}$$

Here (3.14) is representing the formation of the index which will be used for checking the stability of the conditional model.

$$I_{1i,t} = \sum_{i=1}^{m} \hat{\gamma}_{i,\alpha_1} 1_{\{t=t_i\}}\ ;\ Where\ \hat{\gamma}_{i,\alpha_1} = \sum_{i=1}^{n-1} \hat{\gamma}_{i,\alpha_1} 1_{\{t=t_i\}}$$

$$I_{2i,t} = \sum_{i=1}^{m} \widehat{\varphi}_{i,\alpha_1} 1_{\{t\geq t_i\}}\ ;\ Where\ \widehat{\varphi}_{i,\alpha_1} = \sum_{i=1}^{n-1} \widehat{\varphi}_{i,\alpha_1} 1_{\{t\geq t_i\}}$$

$$I_{3i,t} = \sum_{i=1}^{m} \widehat{\omega}_{i,\alpha_1} 1_{\{t-i+1\}}\ ;\ Where\ \widehat{\omega}_{i,\alpha_1} = \sum_{i=1}^{n-1} \widehat{\omega}_{i,\alpha_1} 1_{\{t-i+1\}} \tag{3.14}$$

$$I_{4i,t} = \sum_{i=1}^{m} \widehat{\omega}_{i,\alpha_1} 1_{\{t-i+1\}} + \sum_{i=1}^{m} \widehat{\varphi}_{i,\alpha_1} 1_{\{t\geq t_i\}} + \sum_{i=1}^{n-1} \widehat{\omega}_{i,\alpha_1} 1_{\{t-i+1\}}$$

After that we use these indices in our conditional model and test the null hypothesis that $\varphi = 0$ in each scenario

$$IIS: \quad y_t = \alpha + \gamma x_t + \varphi I_{1i,t} + \varepsilon_{1t}$$

$$SIS: \quad y_t = \alpha + \gamma x_t + \varphi I_{2i,t} + \varepsilon_{2t}$$

$$TIS: \quad y_t = \alpha + \gamma x_t + \varphi I_{3i,t} + \varepsilon_{3t} \tag{3.15}$$

$$\text{IIS, SIS \& TIS:} \quad y_t = \alpha + \gamma x_t + \varphi I_{4i,t} + \varepsilon_{4t}$$

An alternative test with *T-n-1 d.f.* and approximately distributed as *t-distribution* under the null of SupExt. Furthermore, (Hendry & Santos, 2006) suggested that the index based test for a known break point is considered to be equated with (Chow, 1960) test but in general this similarity does not hold where changes are periodic.

3.7.2.3 *Test 3 procedure: A Double Index based Test*

Now for testing the SupExt in relevant class of models following (Hendry & Santos, 2006, 2010) another index based test is formed in a way that the indices iterated with the conditioning variable or x_t as follows:

$$
\begin{aligned}
II_{1i,t} &= \sum_{i=1}^{m} \hat{\gamma}_{i,\alpha_1} x_{j,t} 1_{\{t=t_i\}}\,; \; Where\ \hat{\gamma}_{i,\alpha_1} = \sum_{i=1}^{n-1} \hat{\gamma}_{i,\alpha_1} x_{j,t} 1_{\{t=t_i\}} \\
II_{2i,t} &= \sum_{i=1}^{m} \widehat{\varphi}_{i,\alpha_1} x_{j,t} 1_{\{t\geq t_i\}}\,; \; Where\ \widehat{\varphi}_{i,\alpha_1} = \sum_{i=1}^{n-1} \widehat{\varphi}_{i,\alpha_1} x_{j,t} 1_{\{t\geq t_i\}} \\
II_{3i,t} &= \sum_{i=1}^{m} \widehat{\omega}_{i,\alpha_1} 1_{\{t-i+1\}}\,; Where\ \widehat{\omega}_{i,\alpha_1} = \sum_{i=1}^{n-1} \widehat{\omega}_{i,\alpha_1} x_{j,t} 1_{\{t-i+1\}} \\
II_{4i,t} &= \sum_{i=1}^{m} \widehat{\omega}_{i,\alpha_1} x_{j,t} 1_{\{t-i+1\}} + \sum_{i=1}^{m} \widehat{\varphi}_{i,\alpha_1} x_{j,t} 1_{\{t\geq t_i\}} + \sum_{i=1}^{n-1} \widehat{\omega}_{i,\alpha_1} x_{j,t} 1_{\{t-i+1\}}
\end{aligned}
\tag{3.16}
$$

We call this test as ***DIB*-Test**. Once the indices are being formed then, test the individual significance of these new iterated indices and the previous indices or joint significance *i.e.* $\varphi = \theta = 0;$ under the null of SupExt in the conditional model using *F-test* with 2 *d.f.*

$$
\begin{aligned}
IIS&: \quad y_t = \alpha + \gamma x_t + \varphi I_{1i,t} + \theta II_{1i,t} + \varepsilon_{1t} \\
SIS&: \quad y_t = \alpha + \gamma x_t + \varphi I_{2i,t} + \theta II_{2i,t} + \varepsilon_{2t} \\
TIS&: \quad y_t = \alpha + \gamma x_t + \varphi I_{3i,t} + \theta II_{3i,t} + \varepsilon_{3t} \\
\text{IIS, SIS \& TIS}&: \quad y_t = \alpha + \gamma x_t + \varphi I_{14,t} + \theta II_{4i,t} + \varepsilon_{4t}
\end{aligned}
\tag{3.17}
$$

3.7.2.4 *Test 4 procedure: Forecaste Error Based Test*

This test is due to (Charemza & Király, 1988) and later implemented in (Charemza & Király, 1990), we call it as ***CK-Test***. The advantage of this test is that it doesn't require any marginal model. Suppose in our analysis x_t is a variable that is being tested for SupExt.

$$IIS: \quad y_t = \alpha + \gamma x_t + \sum_{i=1}^{T} \gamma_{i,\alpha_2} I_{i,t} + \varepsilon_{1t}$$

$$SIS: \quad y_t = \alpha + \gamma x_t + \sum_{i=1}^{T} \varphi_{i,\alpha_2} S_{i,t} + \varepsilon_{2t}$$

$$TIS: \quad y_t = \alpha + \gamma x_t + \sum_{i=1}^{T} \omega_{i,\alpha_2} T_{i,t} + \varepsilon_{3t} \tag{3.18}$$

IIS, SIS & TIS:

$$y_t = \alpha + \gamma x_t + \sum_{i=1}^{T} \gamma_{i,\alpha_2} I_{i,t} + \sum_{i=1}^{T} \varphi_{i,\alpha_2} S_{i,t} + \sum_{i=1}^{T} \omega_{i,\alpha_2} T_{i,t} + \varepsilon_{4t}$$

Estimate the forecast errors of each of the above conditional models and denote by $(\hat{\varepsilon}_{fc_{1t}})$ for the case of IIS $(\hat{\varepsilon}_{fc_{2t}})$ for SIS $(\hat{\varepsilon}_{fc_{3t}})$ for TIS and lastly $(\hat{\varepsilon}_{fc_{4t}})$ for joint break detection case and then regress these forecast errors on *ln(*y_t) and its lagged values for stationary DGPs and on *ln(*Δy_t) and its lagged values for non-stationary DGPs and on *ln(*y_t) and its lagged values for dynamic DGPs and check the joint significance of the parameters. The null of SupExt would not be rejected if the estimated coefficients on the regressors are not jointly significant.

3.7.2.5 Test 5 procedure: Residual Based Test

This simple test is based on the ground of following mathematical expressions and can be implemented with linear regressions only. Engle and Hendry (1993), purposes a test of SupExt and invariance. In which under H_0 the parameters of conditional model remains stable while under H_1 breaks in the marginal model of independent variables causing shifts in conditional model. We call this test as ***RB-***

Test. However, this test is prone to changing variances and covariances. To show how this procedure works, let us start with the following:

$$\begin{pmatrix} y_t \\ z_t \end{pmatrix} = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} \begin{pmatrix} y_{t-1} \\ z_{t-1} \end{pmatrix} + \begin{pmatrix} \mu_1^y & \mu_2^y \\ \mu_1^z & \mu_2^z \end{pmatrix} \begin{pmatrix} d_{t1} \\ d_{t2} \end{pmatrix} + \begin{pmatrix} \varepsilon_{yt} \\ \varepsilon_{zt} \end{pmatrix} \tag{3.18a}$$

$$y_t = A_{11}y_{t-1} + A_{12}z_{t-1} + \mu_1^y d_{t1} + \mu_2^y d_{t2} + E(\varepsilon_{yt}|\varepsilon_{zt}) \tag{3.18b}$$

Where $= \sum_{yz}\sum_{zz}^{-1}$ and $\widetilde{\varepsilon_{yt}} = \varepsilon_{yt} - \omega\varepsilon_{zt}$, and can be written as

$$\begin{aligned} = {} & A_{11}y_{t-1} + A_{12}z_{t-1} + \mu_1^y d_{t1} + \mu_2^y d_{t2} \\ & + \omega(z_t - A_{21}y_{t-1} - A_{22}z_{t-1} - \mu_1^z d_{t1} - \mu_2^z d_{t2}) + \widetilde{\varepsilon_{yt}} \end{aligned} \tag{3.18c}$$

$$\begin{aligned} y_t = {} & \omega z_t + (A_{11} - \omega A_{21})y_{t-1} + (A_{12} - \omega A_{22})z_{t-1} \\ & + (\mu_1^y - \omega\mu_1^z)d_{t1} + (\mu_2^y - \omega\mu_2^z)d_{t2} + \widetilde{\varepsilon_{yt}} \end{aligned} \tag{3.18d}$$

While the marginal model can be written in the form as follows:

$$z_t = A_{21}y_{t-1} + A_{22}z_{t-1} - \mu_1^z d_{t1} - \mu_2^z d_{t2} + \varepsilon_{zt} \tag{3.18e}$$

Now for SupExt condition, we ought to implement the restrictions like $\mu_1^y - \omega\mu_1^z = 0$ and $\left(\mu_2^y - \omega\mu_2^z\right) = 0$, considered as reduced rank conditions. Mathematically, these conditions can equivalently be written as $\frac{\mu_1^y}{\mu_1^z} = \frac{\mu_2^y}{\mu_2^z} = \omega$. The implication of these conditions will lead to reduce our conditional model to be like:

$$y_t = \omega z_t + (A_{11} - \omega A_{21})y_{t-1} + (A_{12} - \omega A_{22})z_{t-1} + \widetilde{\varepsilon_{yt}} \tag{3.18f}$$

Therefore, the set of parameters of conditional model $\lambda_C = \{\omega, A_{11} - \omega A_{21}, A_{12} - \omega A_{22}, \sum_{yy} - \sum_{yz}\sum_{zz}^{-1}\sum_{zy}\}$ and marginal model parameters are $\lambda_M = \{A_{21},\ A_{21}, \mu_2^z,\ \mu_2^z, \sum_{zy}\}$.

The parameter stability and their testing used in the above case critically depends on the underlying DGPs. Certainly, the test of constancy or invariance corresponds to a choice of $\boldsymbol{x_t}$ variables consisting upon (0, 1) dummies. In routine practice, it may not be possible for researcher to identify the full set $\boldsymbol{x_t}$ yet it is not impossible to implement the test by partitioning $\boldsymbol{x_t}$ into a set including dummy variables for shifts in the data, which in turn under the assumption of SupExt need not to enter in our conditional model significantly. A test for any linear combination of these significant dummies even can serve the purpose of SupExt test by regressing $\boldsymbol{x_t}$ these selected set of dummies and test significance of $\widehat{x_t}$ or equivalently, on the errors obtained by this regression or its squared value and its lagged values (Engle & Hendry, 1993). Engle and Hendry SupExt test is valid in the case of homoscedastic error. Therefore, for SupExt test, we add lagged values of squared residuals as well.

Suppose in our analysis (3.19) are the marginal models and (3.20) are the conditional model. Now test explains that one can find the $\widehat{\varepsilon^*}_t^2$ and $\sum \widehat{\varepsilon^*}_{t-i}^2$ (lagged values) from (3.19) and test the joint significance of these values in conditional model *i.e.* φ=$\theta_i = 0$ under the null of SupExt. The null of SupExt can't be rejected if the coefficients of the regressors are not jointly significant.

$$\begin{aligned}
&IIS: \quad x_t = \alpha + \gamma x_{t-1} + \sum_{i=1}^{T} \gamma_{i,\alpha_2} I_{i,t} + \varepsilon_{1t} \\
&SIS: \quad x_t = \alpha + \gamma x_{t-1} + \sum_{i=1}^{T} \varphi_{i,\alpha_2} S_{i,t} + \varepsilon_{2t} \\
&TIS: \quad x_t = \alpha + \gamma x_{t-1} + \sum_{i=1}^{T} \omega_{i,\alpha_2} T_{i,t} + \varepsilon_{3t}
\end{aligned} \tag{3.19}$$

IIS, SIS & TIS:

$$x_t = \alpha + \gamma x_{t-1} + \sum_{i=1}^{T} \gamma_{i,\alpha_2} I_{i,t} + \sum_{i=1}^{T} \varphi_{i,\alpha_2} S_{i,t} + \sum_{i=1}^{T} \omega_{i,\alpha_2} T_{i,t} + \varepsilon_{4t}$$

$$IIS: \quad y_t = \alpha + \gamma x_t + \varphi \widehat{\varepsilon^*}^2_{It} + \theta_i \sum \widehat{\varepsilon^*}^2_{It-i} + \varepsilon_{1t}$$

$$SIS: \quad y_t = \alpha + \gamma x_t + \varphi \widehat{\varepsilon^*}^2_{St} + \theta_i \sum \widehat{\varepsilon^*}^2_{St-i} + \varepsilon_{2t}$$

$$TIS: \quad y_t = \alpha + \gamma x_t + \varphi \widehat{\varepsilon^*}^2_{Tt} + \theta_i \sum \widehat{\varepsilon^*}^2_{Tt-i} + \varepsilon_{3t} \tag{3.20}$$

IIS, SIS & TIS:

$$y_t = \alpha + \gamma x_t + \varphi \widehat{\varepsilon^*}^2_{ISTt} + \theta_i \sum \widehat{\varepsilon^*}^2_{ISTt-i} + \varepsilon_{4t}$$

Where $\widehat{\varepsilon^*}^2_{It}$, $\widehat{\varepsilon^*}^2_{St}$, $\widehat{\varepsilon^*}^2_{Tt}$ & $\widehat{\varepsilon^*}^2_{ISTt}$ are the squared residuals obtained from the marginal models using IIS, SIS, TIS & jointly break detection respectively.

None of the previously available literature have test the performance of these SupExt tests in the presence of impulse saturation using its different types as described earlier. Therefore, the performance is being checked *via* simulation analysis and selected the types of indicator saturation where the performance of the test is optimal. The last test which we opted here is based on the concept of co-breaking based test of SupExt.

3.7.2.6 Test 6 procedure: Co-breaking Based Test

The idea of co-breaking can be related with the concept of cointegration: the unit roots in variables can be removed from the linear combination of these variables, if cointegration holds, while co-breaking can help us in removing effects of regime shifts by considering linear combinations of variables. Like impulse response, co-breaking is equally important for policy analysis so as the test of SupExt based on co-breaking is as well (Hendry & Massmann, 2007). For the sake of brevity, we call this test as ***CB-Test***.

Following this, first we come to the point that whether a given vector of random variables is conditioned upon structural breaks and second, to check whether, these identified breaks disappears in the linear combination of these random variables. Consequently, these models referred as co-breaking regressions as suggested in (Engle & Granger, 1987) which also termed as *co-feature* regressions in (Engle & Kozicki, 1993). Suppose $\mathbf{X}_t$ is the full set of variables which can be partitioned into y_t and $\mathbf{z}_t$. The following two steps may be implementation of co-breaking procedure, in practice.

i) First, a total of *m* shifts are significant in every component of $\mathbf{X}_t$.

ii) Second, to test the significance of these *m* shifts in conditional model into $\mathrm{y}_t|\mathbf{z}_t$.

On empirical grounds, the implementation of the general procedure mentioned above under different modeling setting can be observed in (Chapman & Ogaki, 1993) considering a piece-wise polynomial model, (Engle & Kozicki, 1993) with markov-switching model while (Hendry & Mizon, 1998) used a multivariate normal model to testify the co-breaking hypothesis. Now taking (Hendry & Mizon, 1998) under considerations where they pointed out that set $\mathbf{z}_t$ containing putative regressor is super exogenous if fewer number of breaks appeared in conditional model $(y_t|\mathbf{z}_t)$ than the full model. Initially, SupExt test based on co-breaking idea having breaks in deterministic part is proposed in (Krolzig & Toro, 2002).

A key difficulty in the test proposed by (Krolzig & Toro, 2002) is that it only considered the conditional co-breaking relationships. The solution to this problem is proposed by (Hatanaka & Yamada, 2003) for suggesting unconditional model to determine co-breaking and cointegration rank simultaneously. However, it would also be more interesting if one can check the behavior of SupExt test proposed in (Krolzig

& Toro, 2002) based on reduced rank condition under indicator saturation as this test has better performance than the test mentioned above. The empirical example of this test can be seen in (Schreiber, 2004).

A crucial assumption underlying the tests discussed above is that the power of impulse saturation tests to detect breaks and outliers was not applied to the conditional. In many situations, investigators will have done precisely that, vitiating the power of the direct super-exogeneity tests to detect failures. Conversely, one can utilize such results for a deterministic co-breaking based test of SupExt (Hendry & Santos, 2010).

Consider the following case in which the *s* dummies of each type are being added to the conditional model as:

$$IIS: \qquad y_t = \alpha + \gamma x_t + \sum_{i=1}^{s} \gamma_{i,\alpha_1} I_{i,t} + \varepsilon_{1t}$$

$$SIS: \qquad y_t = \alpha + \gamma x_t + \sum_{i=1}^{s} \varphi_{i,\alpha_1} S_{i,t} + \varepsilon_{2t}$$

$$TIS: \qquad y_t = \alpha + \gamma x_t + \sum_{i=1}^{s} \omega_{i,\alpha_1} T_{i,t} + \varepsilon_{3t} \tag{3.21}$$

IIS, SIS & TIS:

$$y_t = \alpha + \gamma x_t + \sum_{i=1}^{s} \gamma_{i,\alpha_1} I_{i,t} + \sum_{i=1}^{s} \varphi_{i,\alpha_1} S_{i,t} + \sum_{i=1}^{s} \omega_{i,\alpha_1} T_{i,t} + \varepsilon_{4t}$$

On the other side, *m* dummies are significant dummies in marginal model at the same level of significance as the conditional model does have were added:

$$IIS: \quad x_t = \alpha + \gamma x_{t-1} + \sum_{i=1}^{m} \gamma_{i,\alpha_2} I_{i,t} + \varepsilon_{1t}$$

$$SIS: \quad x_t = \alpha + \gamma x_{t-1} + \sum_{i=1}^{m} \varphi_{i,\alpha_2} S_{i,t} + \varepsilon_{2t}$$

$$TIS: \quad x_t = \alpha + \gamma x_{t-1} + \sum_{i=1}^{m} \omega_{i,\alpha_2} T_{i,t} + \varepsilon_{3t} \qquad (3.21)^*$$

IIS, SIS & TIS:

$$x_t = \alpha + \gamma x_{t-1} + \sum_{i=1}^{m} \gamma_{i,\alpha_2} I_{i,t} + \sum_{i=1}^{m} \varphi_{i,\alpha_2} S_{i,t} + \sum_{i=1}^{m} \omega_{i,\alpha_2} T_{i,t} + \varepsilon_{4t}$$

The test explains that check whether the timing of these impulses overlaps or not. However, a perfect match leads to conclude the failure of SupExt *i.e.* the significance of the dummies in conditional model that are retained in marginal model rejects the SupExt. However, a less number of dummies to be significant in conditional model leads to conclude the SupExt of x_t.

According to (Hendry & Santos, 2006, 2010), it is worthwhile here to note that all the testing procedures and their performance analysis are on the assumption of stationary data settings along with all derivations and Monte Carlo experiments that have been reported so far in literature are for static regression equations, the principles are general, and should apply to dynamic equations (probably with more approximate null rejection frequencies) and to non-stationary settings. A gap is identified which need to be fulfilled by considering the above case settings. Therefore, the study contributes to a step forward by testing the performance of these procedures under dynamic and non-stationary data settings with the help of simulation analysis. The next sub-section highlights the key concepts of DGP opt for simulation analysis in terms of above mentioned settings.

3.8 Data Generating Process

In this subsection we discusses about the data generating process that have been used for our simulation analysis.

We consider the bivariate DGP:

$$\begin{pmatrix} y_t \\ z_t \end{pmatrix} |X_{t-1} \sim N\left[\begin{pmatrix} \pi_{10} \\ \pi_{20} \end{pmatrix} + \begin{pmatrix} \pi_{11} & \pi_{12} \\ \pi_{21} & \pi_{22} \end{pmatrix}\begin{pmatrix} y_{t-1} \\ z_{t-1} \end{pmatrix}, \begin{pmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{pmatrix}\right] \tag{3.22}$$

Where $x_t = (y_t : z_t)', t = 1, \dots \dots, T,$ and therefore the matrix X_{t-1} must contain the information about the past values of both y_t, z_t clearly as follows:

$$X_{t-1} = (Y_{t-1} : Z_{t-1})' \tag{3.23}$$

As we all know that the joint density can further be factorized in both conditional and the marginal densities as:

$$D(x_t, z_t | X_{t-1}; \theta) = D_{y|z}(y_t | z_t, X_{t-1}; \lambda_1).\ D_z(z_t |, X_{t-1}; \lambda_2) \qquad (3.23)^*$$

Note that here $\theta \epsilon \boldsymbol{\Theta}$, and $\lambda_1 \epsilon\, \boldsymbol{\lambda}_1$and $\lambda_2 \epsilon\, \boldsymbol{\lambda}_2$where $\boldsymbol{\Theta}$, $\boldsymbol{\lambda}_1$ and $\boldsymbol{\lambda}_2$ are the parameter spaces for the joint , conditional and the marginal densities respectively. Given the normality assumption taking into account in (3.22), leads to make both marginal and conditional densities a Gaussian one. Therefore,

$$\begin{aligned}(y_t | z_t, X_{t-1}) \sim N[\pi_{10} + \pi_{11} y_{t-1} + \pi_{12} z_{t-1} \\ + \sigma_{12}\sigma_{22}^{-1}(z_t - \pi_{20} - \pi_{21} y_{t-1} - \pi_{22} z_{t-1});\ \sigma_{11} - \sigma_{12}\sigma_{22}^{-1}\sigma_{12}]\end{aligned} \tag{3.24}$$

And also

$$z_t | X_{t-1} \sim N[\pi_{20} + \pi_{21} y_{t-1} + \pi_{22} z_{t-1}; \sigma_{22}] \tag{3.25}$$

Clearly (3.24) a conditional density $D_{y|z}$ and (3.25) is the marginal one that have been represented in (3.23). However one can get the conditional expectation of y_t:

$$E(y_t|z_t, X_{t-1}) = \pi_{10} + \pi_{11}y_{t-1} + \pi_{12}z_{t-1} + \sigma_{12}\sigma_{22}^{-1}(z_t - \pi_{20} - \pi_{21}y_{t-1} - \pi_{22}z_{t-1}) \quad (3.26)$$

Whereas (3.27) is the marginal model for $z_t|X_{t-1}$:

$$z_t|X_{t-1} = \pi_{20} + \pi_{21}y_{t-1} + \pi_{22}z_{t-1} + \varepsilon_{z,t}, \text{ where } \varepsilon_{z,t} \sim N(0, \sigma_{12}) \quad (3.27)$$

It is worth to note that both equation (3.26) and (3.27) are important in terms of exogeneity concepts (*see, inter alia*, Hendry, 1995; Krolzig & Toro, 2002) specially for the implementation of the notion weak and SupExt. The basic feature in this DGP in that:

$$\pi_{21} \neq 0 \quad (3.28)$$

Therefore the marginal model will have an AD (1,1) structure. Therefore, equation (3.26) can also be written as:

$$E(y_t|z_t, X_{t-1}) = \pi_{10} - \sigma_{12}\sigma_{22}^{-1}\pi_{20} + (\pi_{11} - \sigma_{12}\sigma_{22}^{-1}\pi_{21})y_{t-1} + (\pi_{12} - \sigma_{12}\sigma_{22}^{-1}\pi_{22})z_{t-1} + \sigma_{12}\sigma_{22}^{-1}z_t \quad (3.29)$$

And this we can rewrite it as the following in terms of conditional density:

$$y_t = \beta_0 + \beta_1 y_{t-1} + \beta_2 z_{t-1} + \beta_3 z_t + \varepsilon_t \quad (3.30)$$

with $\varepsilon_t \sim N(0;\sigma_\varepsilon^2)$, also $\sigma_\varepsilon^2 = \sigma_{11} - \sigma_{12}\sigma_{22}^{-1}\sigma_{12}$. The parametric space for conditional density is $\boldsymbol{\lambda}_1 = (\beta_0 : \beta_1 : \beta_2 : \beta_3 : \sigma_\varepsilon^2)'$ and on the other hand the parametric space for the marginal model is $\boldsymbol{\lambda}_1 = (\pi_{20} : \pi_{21} : \pi_{22} : \beta_3 : \sigma_{22})'$.

Now without any loss of generality let's consider, that:

$$\pi_{20} = \pi_{21} = 0 \tag{3.31}$$

So that the unconditional mean of z_t will be zero. Furthermore, we don't make the joint stationarity assumption. In this case as well the random variable $y_t|z_t, X_{t-1}$ has expectations as:

$$E(y_t|z_t, X_{t-1}) = \pi_{10} + \pi_{11}y_{t-1} + (\pi_{12} - \sigma_{12}\sigma_{22}^{-1}\pi_{12})z_{t-1} + \sigma_{12}\sigma_{22}^{-1}z_t \tag{3.32}$$

It is clear that the weak exogeneity can easily be testes as conditions are verified. Therefore the tests of SupExt can be applied. So after having this detailed discussion we have been able to classify our DGP. We start several univariate (marginal models) and bivariate (conditional models) capturing all types of breaks individually as follows:

For stationary DGP

$$y_t = \alpha_t^{\mathrm{o}} + \beta^{\mathsf{T}} x_t + \varepsilon_t$$

$$IIS: \qquad x_t = \alpha + \gamma x_{t-1} + \sum_{i=1}^{T} \gamma_i \, I_{i,t} + \varepsilon_{1t}$$

$$SIS: \qquad x_t = \alpha + \gamma x_{t-1} + \sum_{i=1}^{T} \varphi_i \, S_{i,t} + \varepsilon_{2t}$$

$$TIS: \qquad x_t = \alpha + \gamma x_{t-1} + \sum_{i=1}^{T} \omega_i T_{i,t} + \varepsilon_{3t}$$

IIS, SIS & TIS:

$$x_t = \alpha + \gamma x_{t-1} + \sum_{i=1}^{T} \gamma_i I_{i,t} + \sum_{i=1}^{T} \varphi_i S_{i,t} + \sum_{i=1}^{T} \omega_i T_{i,t} + \varepsilon_{4t}$$

Here γ take value from 0.1 to 0.99.

For Non-stationary DGP

$$y_t = \alpha_t^{o} + \beta^{\top} x_t + \varepsilon_t$$

$$IIS: \qquad x_t = \alpha + x_{t-1} + \sum_{i=1}^{T} \gamma_i I_{i,t} + \varepsilon_{1t}$$

$$SIS: \qquad x_t = \alpha + x_{t-1} + \sum_{i=1}^{T} \varphi_i S_{i,t} + \varepsilon_{2t}$$

$$TIS: \qquad x_t = \alpha + x_{t-1} + \sum_{i=1}^{T} \omega_i T_{i,t} + \varepsilon_{3t}$$

IIS, SIS & TIS:

$$x_t = \alpha + x_{t-1} + \sum_{i=1}^{T} \gamma_i I_{i,t} + \sum_{i=1}^{T} \varphi_i S_{i,t} + \sum_{i=1}^{T} \omega_i T_{i,t} + \varepsilon_{4t}$$

We put $\gamma = 1$ for each of our marginal model. This will create non-stationary data but with each type of break and all at a time (jointly).

For Dynamic DGP

$$y_t = \alpha_t^{o} + \beta^{\top} x_t + \vartheta_1 y_{t-1} + \vartheta_2 y_{t-2} + \varepsilon_t$$

$$IIS: \qquad x_t = \alpha + \gamma x_{t-1} + \sum_{i=1}^{T} \gamma_i I_{i,t} + \varepsilon_{1t}$$

$$SIS: \qquad x_t = \alpha + \gamma x_{t-1} + \sum_{i=1}^{T} \varphi_i S_{i,t} + \varepsilon_{2t}$$

$$TIS: \qquad x_t = \alpha + \gamma x_{t-1} + \sum_{i=1}^{T} \omega_i T_{i,t} + \varepsilon_{3t}$$

IIS, SIS & TIS:

$$x_t = \alpha + \gamma x_{t-1} + \sum_{i=1}^{T} \gamma_i I_{i,t} + \sum_{i=1}^{T} \varphi_i S_{i,t} + \sum_{i=1}^{T} \omega_i T_{i,t} + \varepsilon_{4t}$$

Here γ take value from 0.1 to 0.99.

3.8.1 Null rejection frequency of the Impulse-Based Test

Here again reconsider the factorized DGP as discussed in (3.2), therefore under the null of SupExt equation form (3.4):

$$y_t = \mu + \beta_{1,t} + \beta'_{2,t}\mathbf{z}_t + \delta'\mathbf{x}_{t-1} + \boldsymbol{\varepsilon}_t \tag{3.33}$$

Or even with two lags it can be written as:

$$y_t = \mu + \beta_{1,t} + \beta'_{2,t}x_t + \delta^1 x_{t-i} + \delta^2 y_{t-i} + \varepsilon_t \text{ for } i = 1,2 \tag{3.34}$$

Now although the process generating x_t is constant over time. Let S_{α_1} denotes the dates of those dummies whether impulse, step or trend that are significant and retained in the marginal model:

$$x_t = \boldsymbol{\pi}_0 + \sum_{j=1}^{s} \Pi_j X_{t-j} + \sum_{i=1}^{m} \rho_{i1,\alpha_1} 1_{\{t=t_i,\}} + \sum_{i=1}^{m} \rho_{i1,\alpha_1} 1_{\{t\geq t_i,\}} + \sum_{i=1}^{m} \rho_{i3,\alpha_1} 1_{\{T-i+1\}} + \varepsilon_t^* \tag{3.35}$$

Where;

$$\left|t_{\widehat{\tau_{\iota,t_\iota}}}\right| > C_{\alpha_1} \tag{3.36}$$

Note that when C_{α_1} is C.V for significance level α_1. In the model (3.34) for $y_t|z_t, X_{t-1}$, conditioning on z_t with $\boldsymbol{\varepsilon_t}$ constant or fixed, so adding the impulses of each type like IIS, SIS and TIS and jointly would lead to:

$$E(y_t|z_t, X_{t-1}) = \mu + \beta_{1,t} + \beta'_{2,t}x_t + \delta^1 x_{t-i} + \delta^2 y_{t-i} + \boldsymbol{\delta 1_t} \tag{3.37}$$

Where $\delta = 0$ under H_0. Given a significance level α_2 a subset of impulses will be retained in the conditional model under consideration provided that they were also retained in the marginal model when:

$$\left|t_{\widehat{\delta_J}}\right| > C_{\alpha_2} \tag{3.38}$$

Therefore, when (3.36) hold, the probability of retaining any indicator in the conditional model can be written as:

$$Pr\left(\left|t_{\widehat{\delta_J}}\right| > C_{\alpha_2} | \left|t_{\widehat{\tau_{\iota,t_\iota}}}\right| > C_{\alpha_1}\right) = Pr\left(\left|t_{\widehat{\delta_J}}\right| > C_{\alpha_2}\right) = \alpha_2 \tag{3.39}$$

Since (3.36) holds, which only depends upon significance level C_{α_2} not on C_{α_1}. Now if, (3.36) doesn't hold, therefore, no impulse of any type will retained, hence

$*Pr\left(\left|t_{\widehat{\delta_j}}\right| > C_{\alpha_2}\right) = 0$, consequently SupExt test will reject under null. In the next section we will discuss about the simulation strategy opted in this study.

3.9 Simulation Strategy

The Monte Carlo experiment used in the study for null rejection frequencies (NRF's) of indicators in saturated stationary models with an extension to a unit root process and dynamic settings as well. The size and power of several SupExt testing procedures will be analyzed and compared under the null of no indicators in data generating process (DGP). With Monte Carlo experiments hoping on that there are no size distortions in impulse saturating stationary, non-stationary and dynamic processes, and that the procedure has good power properties in this class of models to detect level shifts at unknown dates. The detailed analysis can be found through Chapter 4 & 5 below.

Note that the reason to use impulse dummies is that these dummies are perfectly orthogonal to each other, so we do not have to cope with the problems of collinearity faced in more general settings (Hendry & Krolzig, 2003). Hendry (2000) advises orthogonalization of the regressors prior to model selection as a way to reduce model uncertainty. Also, the set of dummies can be changed to test the performance of the procedure and their impact on NRF's while using *t-test and F-test* (for joint significance) in large samples.

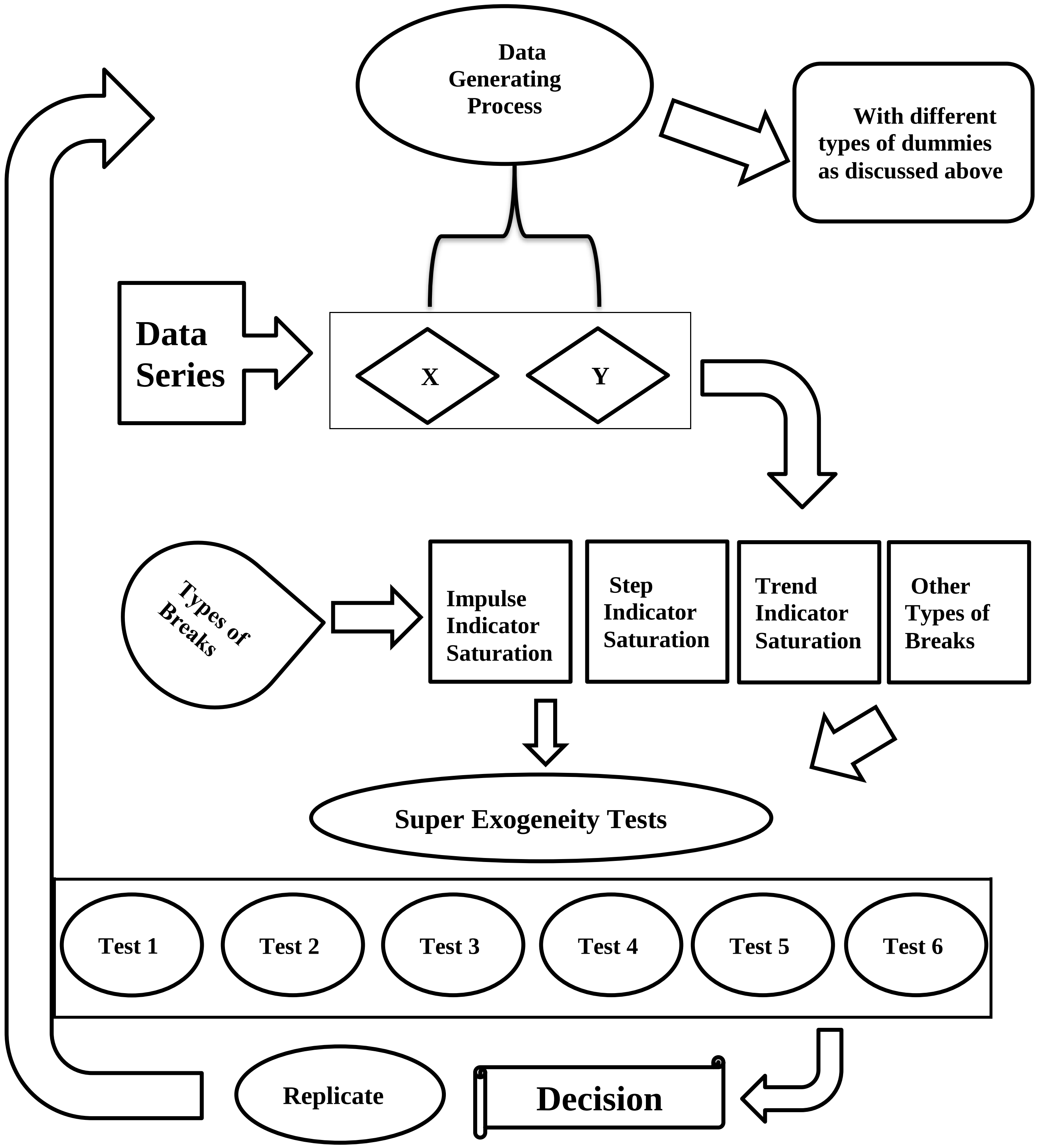
The sample sizes considered are T = 50, T = 100 and T = 200. Individual significance tests on the indicators saturation are conducted for a range of significance levels a, taking values from the set {0.01, 0.025, 0.05, … , 0.99}. While performing the experiments M = 100,000 times in MATLAB, we disregarded the first 100 observations in each case, in order to eliminate dependence on the initial values.

Nonetheless, the analysis of the unit root case would require considerable more evidence, whilst all we are doing here is to suggest that it would be possible to use dummy saturation in such models as well (Hendry & Santos, 2006). Following this statement we considered the unit root settings experiment with indicator saturation like IIS, SIS, and TIS to check whether there would be any significant NRFs problems in a random walk model. We checked discrepancy between real and nominal sizes, at all significance levels, it is our view that these are not sufficient to preclude the saturation of a unit root model.

The Monte Carlo Experiment comprises of the following components. First is the Data Generating Process (DGP), second is the simulation to get the critical values of the SupExt tests, third is the computation of size[19] of the test statistics under the null hypothesis and fourth is the computation of the power[20] of the test statistics under the alternative hypothesis leading to SupExt. Following is the tree diagram which explains the whole procedure.

[19] The size of a test is the probability of incorrectly rejecting the null hypothesis if it is true.
[20] The power of a test is the probability of correctly rejecting the null hypothesis if it is false.

3.9.1 Flow Chart for Simulation Strategy



3.10 Software Usage

The readily available packages or software modules to implement these simulations for SupExt tests are very limited and if some are available they are not in the form to use for indicator saturation. Since *OxMetrics*© has copy writes but recent development in an R-Package *GETS (Version 0.20)* gives some hints for implementing indicator saturation based on *Autometrics* algorithm and we used this Package for our analysis of money demand (*see;* Chapter 6). However, for simulation analysis we used *MATLAB* instead and some work has been performed in *OxMetrics* 7.

3.11 Summary of the Chapter

In above chapter we briefly discussed about the methodology and simulation strategy opted in this study. Several types of structural breaks are available in the literature. However, we used three of them like IIS, SIS and TIS while implanting SupExt tests. The detection and retention of multiple structural breaks in saturated regressions have been discussed. In past, researcher pointed out the problems faced when using multiple breaks in saturated regressions, for instance, multicollinearity and dimensionality problem. However, here we discussed how one can avoid these while using regressions saturated with IIS, SIS and TIS. The *'general-to-specific'* approach and the working of *Autometrics* and its algorithm along with detailed framework (like; punching, pruning, chopping & closed lags etc.) have been discussed. Also, for interested readers, the structural break detection and its procedure in the form of algorithm is being provided therein. Lastly, the DGPs and SupExt testing procedures using IIS, SIS, TIS and all at a time have been discussed in detail.

Chapter 4

Performance Under Stationary Settings

The chapter briefly explains and interprets the simulation results of SupExt tests under the shade of IIS, SIS and TIS both at 1% and 5% level of significance separately. Furthermore, we extend our analyses to compare their performance by considering IIS, SIS and TIS at a time under stationary data settings. Before we start any comparison of SupExt tests we need to find simulated critical values for each test.

The need for the simulated critical values is of high significance, as most of the tests rely on the basis of asymptotic critical values which loose its efficiency when we work with small samples, so that's the reason for the choice of simulated critical values. We used the asymptotic critical values of these SupExt tests for the calculation of size of the test. At 1% and 5% level of significance we estimate the simulated critical values of these SupExt tests. The simulated critical values of SupExt tests under stationary data settings using IIS, SIS, TIS and when using all these three at a time taking 1% and 5% level of significance for three different sample sizes of 50, 100 and 200 are being reported below in Table 4.1(a) - 4.1(d). However, to calculate the size of SupExt tests the data is generated using the DGP under the null hypothesis. We have used the asymptotic critical values to compare the nominal size with the empirical size, the empirical size of each SupExt tests is calculated by Monte Carlo sample size of 100,000 simulations using the asymptotic critical values for the various sample sizes *s.t.* 50, 100 and 200. The simulated critical values of SupExt tests are given at 5% and 1% level of significance for three different sample sizes of 50, 100 and 200 are given below in Table 4.1 (a) - 4.1 (d). Lastly, The Nominal size of

SupExt tests is calculated at 5% and 1% level of significance for three different sample sizes of 50, 100 and 200 are given below in Table 4.1 (a1) - 4.1 (d1).

4.1 Performance using IIS under Stationary Data

The following Table 4.1 envelops the simulated CV's of the SupExt tests when IIS is being taken into account. For the sample size 50 the observed values *CB-Test* and *IB-Test* are not very deviating *i.e.* both tests are showing very less size distortion. As on simulated critical values empirical size is approximately equal to nominal size which is obtained using simulated critical values. The difference between empirical size and nominal size is small which we obtained from Table 4.2. However, as sample size changes from 50 to 100 or even to 200 we found a small amount of size distortion in *CB-Test* and *H-Test* and more in *CK-Test* and *DIB-Test* as the nominal size is exceeding the empirical size in both cases. On the other hand, using IIS both *IB-Test* and *RB-Test* are no size distortion. As on simulated critical values empirical size is approximately equal to nominal size which is obtained using simulated critical values but for other tests the results of nominal size are exceeding the empirical size so here we face size distortion.

Table 4.1: Empirical Size of SupExt Tests using Asymptotic Critical Values under IIS

Under Stationary Data Settings												
Size of Test	***H-Test***		***CK-Test***		***RB-Test***		***IB-Tests***		***DIB-Test***		***CB-Test***	
	1%	5%	1%	5%	1%	5%	1%	5%	1%	5%	1%	5%
Sample Size: 50	0.012	0.053	0.017	0.059	0.013	0.052	0.010	0.051	0.015	0.053	0.010	0.050
Sample Size: 100	0.011	0.051	0.016	0.056	0.010	0.050	0.010	0.049	0.014	0.054	0.013	0.052
Sample Size: 200	0.011	0.051	0.017	0.057	0.010	0.050	0.009	0.050	0.014	0.055	0.011	0.052

Note: Author's own calculations

Table 4.2: Simulated Critical Values of SupExt Tests Under IIS

	Under Stationary Data Settings											
Size of Test	***H-Test***		***CK-Test***		***RB-Test***		***IB-Tests***		***DIB-Test***		***CB-Test***	
	5%	1%	5%	1%	5%	1%	5%	1%	5%	1%	5%	1%
Sample Size: 50	3.22	5.57	1.77	2.23	3.08	4.97	3.87	5.98	3.18	5.52	4.21	6.63
Sample Size: 100	4.17	5.79	1.81	2.33	4.11	5.51	4.45	6.69	4.40	6.61	3.13	3.78
Sample Size: 200	4.37	5.91	1.51	2.29	4.87	6.12	5.49	6.78	5.80	3.17	5.01	6.78

Note: Author's own calculations

4.1.1 Power of the Tests using IIS under Stationary Data

To calculate the power of each SupExt test the data is generated using the alternative hypothesis when IIS is considered. The power of all SupExt tests are calculated, the number of Monte Carlo simulation is 100,000 in each case of 1% and 5% of significance level. The power curves of SupExt tests for different alternative hypothesis at 1% level of significance for sample size of 50, 100 and 200 are given in the Table 4.3. Whereas at 5% level of significance power of SupExt tests for different alternative hypothesis for sample size of 50, 100 and 200 are given in the Table 4.4.

To obtain the power curves we will take different alternative hypothesis along the x-axis of the graph and the powers of each SupExt test (relative to the alternative hypothesis) on y-axis and we will draw the scatter plot graph at three different sample sizes of 50, 100 and 200. The power curves for each test under IIS are being portrayed below in Figure 4.1.

By looking at the graphs one can easily observe that when sample is 50 the power of *CB-Test* and *IB-Test* both at 1% as well as 5% level of significance is better than other tests. But as the sample size increase from 50 to 100 or 200 the power of *CB-Test* significantly reduced using IIS. However, the power of *IB-Test* and *RB-Test* showed relative improvement as compare to other tests considering IIS. But the power of *IB-Test* is much better than that of other SupExt tests. Lastly, the performance of *CK-Test* remains at the low at both significance levels under IIS.

Table 4.3: Powers under the Alternative Hypothesis at 1% Level of Significance Using Impulse Indicator Saturation (IIS)

Under Stationary Settings																		
1% SL[21]	Sample Size: 50						Sample Size: 100						Sample Size: 200					
H_1[22]	***H-test***	***CK-test***	***RB-test***	***IB-test***	***DIB-test***	***CB-test***	***H-test***	***CK-test***	***RB-test***	***IB-test***	***DIB-test***	***CB-test***	***H-test***	***CK-test***	***RB-test***	***IB-test***	***DIB-test***	***CB-test***
0.99	0.03	0.02	0.03	0.05	0.02	0.13	0.12	0.05	0.29	0.33	0.19	0.01	0.05	0.1	0.07	0.41	0.04	0.11
0.96	0.05	0.02	0.04	0.06	0.02	0.13	0.14	0.05	0.29	0.33	0.19	0.01	0.06	0.1	0.09	0.41	0.04	0.04
0.93	0.07	0.02	0.03	0.05	0.04	0.15	0.16	0.05	0.31	0.35	0.21	0.01	0.09	0.1	0.12	0.43	0.06	0.12
0.91	0.09	0.02	0.05	0.07	0.06	0.17	0.18	0.05	0.33	0.37	0.23	0.01	0.1	0.1	0.13	0.45	0.08	0.08
0.9	0.09	0.02	0.05	0.07	0.06	0.17	0.18	0.05	0.33	0.37	0.23	0.01	0.12	0.1	0.15	0.45	0.08	0.08
0.85	0.11	0.02	0.05	0.07	0.08	0.19	0.2	0.05	0.35	0.39	0.25	0.01	0.13	0.1	0.16	0.47	0.1	0.1
0.8	0.13	0.02	0.04	0.06	0.1	0.21	0.22	0.05	0.37	0.41	0.27	0.02	0.16	0.1	0.19	0.49	0.12	0.12
0.75	0.15	0.02	0.07	0.09	0.12	0.23	0.24	0.05	0.39	0.43	0.29	0.04	0.18	0.1	0.21	0.51	0.14	0.14
0.7	0.16	0.02	0.08	0.1	0.13	0.24	0.25	0.05	0.4	0.44	0.3	0.05	0.2	0.1	0.23	0.52	0.15	0.15
0.65	0.18	0.03	0.1	0.13	0.15	0.27	0.27	0.05	0.43	0.47	0.33	0.08	0.22	0.1	0.25	0.55	0.18	0.18
0.6	0.2	0.05	0.12	0.17	0.17	0.31	0.29	0.04	0.47	0.51	0.37	0.12	0.23	0.09	0.26	0.59	0.23	0.22
0.55	0.22	0.04	0.14	0.18	0.19	0.32	0.31	0.05	0.48	0.52	0.38	0.13	0.25	0.1	0.28	0.6	0.21	0.23
0.5	0.24	0.07	0.16	0.23	0.21	0.37	0.33	0.05	0.53	0.57	0.43	0.18	0.27	0.1	0.3	0.65	0.25	0.28
0.45	0.26	0.11	0.18	0.29	0.23	0.43	0.35	0.05	0.59	0.63	0.49	0.24	0.29	0.1	0.32	0.71	0.37	0.34
0.4	0.28	0.12	0.2	0.32	0.25	0.46	0.37	0.04	0.62	0.66	0.52	0.27	0.32	0.09	0.35	0.74	0.37	0.37
0.35	0.32	0.09	0.24	0.33	0.29	0.47	0.41	0.05	0.63	0.67	0.53	0.28	0.34	0.1	0.37	0.75	0.38	0.38
0.3	0.34	0.11	0.26	0.37	0.31	0.51	0.43	0.05	0.67	0.71	0.57	0.32	0.37	0.1	0.4	0.79	0.42	0.42
0.25	0.38	0.13	0.3	0.43	0.35	0.57	0.47	0.05	0.73	0.77	0.63	0.38	0.4	0.1	0.44	0.85	0.48	0.48
0.2	0.42	0.11	0.34	0.45	0.39	0.59	0.51	0.1	0.75	0.79	0.65	0.4	0.44	0.15	0.48	0.87	0.5	0.5
0.15	0.47	0.1	0.39	0.49	0.44	0.63	0.56	0.13	0.79	0.83	0.69	0.44	0.48	0.18	0.52	0.91	0.54	0.54
0.1	0.5	0.15	0.42	0.57	0.47	0.71	0.59	0.2	0.83	0.87	0.73	0.48	0.55	0.25	0.58	0.95	0.42	0.58
0.05	0.54	0.16	0.46	0.62	0.51	0.76	0.63	0.25	0.86	0.9	0.76	0.51	0.62	0.3	0.65	0.98	0.35	0.61
0.01	0.57	0.14	0.49	0.63	0.54	0.77	0.66	0.14	0.87	0.91	0.77	0.52	0.69	0.29	0.72	0.99	0.31	0.62

Note: Author's own calculations

[21] Significance Level
[22] Different Alternatives

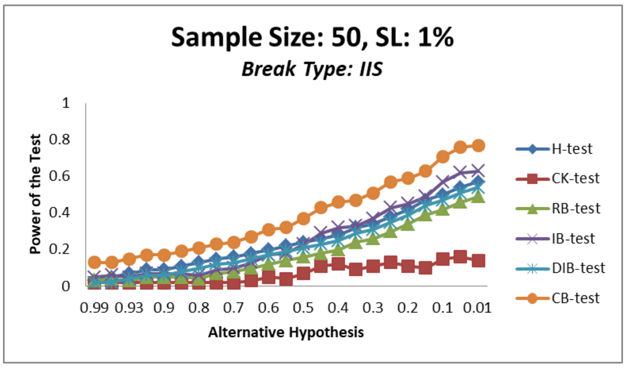
Table 4.4: Powers under the Alternative Hypothesis at 5% Level of Significance Using Impulse Indicator Saturation (IIS)

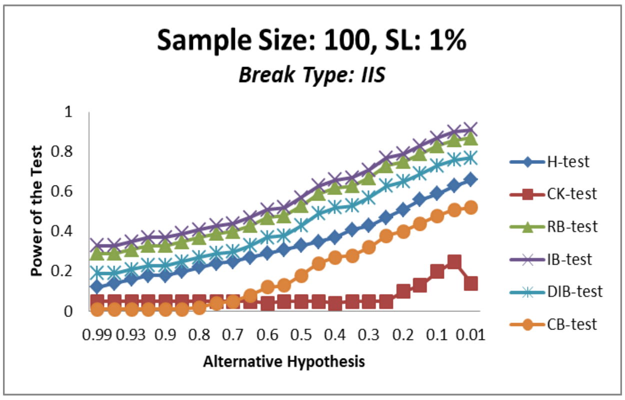
Under Stationary Data Settings																		
5% SL	Sample Size: 50						Sample Size: 100						Sample Size: 200					
H_1	***H-test***	***CK-test***	***RB-test***	***IB-test***	***DIB-test***	***CB-test***	***H-test***	***CK-test***	***RB-test***	***IB-test***	***DIB-test***	***CB-test***	***H-test***	***CK-test***	***RB-test***	***IB-test***	***DIB-test***	***CB-test***
0.99	0.03	0.03	0.04	0.07	0.02	0.14	0.12	0.05	0.19	0.336	0.09	0.1	0.24	0.1	0.39	0.506	0.15	0.2
0.96	0.05	0.03	0.05	0.08	0.02	0.14	0.14	0.05	0.2	0.336	0.09	0.11	0.26	0.1	0.4	0.506	0.16	0.21
0.93	0.07	0.03	0.08	0.11	0.04	0.16	0.16	0.05	0.23	0.356	0.11	0.1	0.28	0.1	0.43	0.526	0.15	0.2
0.91	0.09	0.03	0.09	0.12	0.06	0.18	0.18	0.05	0.24	0.376	0.13	0.12	0.3	0.1	0.44	0.546	0.17	0.22
0.9	0.09	0.03	0.09	0.12	0.06	0.18	0.18	0.05	0.24	0.376	0.13	0.12	0.3	0.1	0.44	0.546	0.17	0.22
0.85	0.11	0.03	0.11	0.14	0.08	0.20	0.2	0.05	0.26	0.396	0.15	0.12	0.32	0.1	0.46	0.566	0.17	0.22
0.8	0.13	0.03	0.13	0.16	0.1	0.22	0.22	0.05	0.28	0.416	0.17	0.11	0.34	0.1	0.48	0.586	0.16	0.21
0.75	0.15	0.03	0.15	0.18	0.12	0.24	0.24	0.05	0.3	0.436	0.19	0.14	0.36	0.1	0.5	0.606	0.19	0.24
0.7	0.16	0.03	0.17	0.20	0.13	0.25	0.25	0.05	0.32	0.446	0.2	0.15	0.37	0.1	0.52	0.616	0.2	0.25
0.65	0.18	0.04	0.18	0.22	0.15	0.28	0.27	0.05	0.33	0.476	0.23	0.17	0.39	0.1	0.53	0.646	0.22	0.27
0.6	0.2	0.06	0.21	0.27	0.17	0.32	0.29	0.04	0.36	0.516	0.28	0.19	0.41	0.09	0.56	0.686	0.28	0.29
0.55	0.22	0.05	0.23	0.28	0.19	0.33	0.31	0.05	0.38	0.526	0.26	0.21	0.43	0.1	0.58	0.696	0.26	0.31
0.5	0.24	0.08	0.25	0.33	0.21	0.38	0.33	0.05	0.4	0.576	0.3	0.23	0.45	0.1	0.6	0.746	0.3	0.33
0.45	0.26	0.12	0.27	0.39	0.23	0.44	0.35	0.05	0.42	0.636	0.31	0.25	0.47	0.1	0.62	0.806	0.31	0.35
0.4	0.28	0.13	0.29	0.42	0.25	0.47	0.37	0.04	0.44	0.666	0.42	0.27	0.49	0.09	0.64	0.836	0.32	0.37
0.35	0.32	0.10	0.32	0.42	0.29	0.48	0.41	0.05	0.47	0.676	0.43	0.31	0.53	0.1	0.67	0.846	0.36	0.41
0.3	0.34	0.12	0.35	0.47	0.31	0.52	0.43	0.05	0.5	0.716	0.47	0.33	0.55	0.1	0.7	0.886	0.38	0.43
0.25	0.38	0.14	0.39	0.53	0.35	0.58	0.47	0.05	0.54	0.776	0.53	0.37	0.58	0.1	0.74	0.946	0.42	0.47
0.2	0.42	0.12	0.44	0.56	0.39	0.60	0.48	0.1	0.59	0.796	0.55	0.41	0.6	0.15	0.79	0.966	0.46	0.51
0.15	0.47	0.11	0.48	0.59	0.44	0.64	0.51	0.13	0.63	0.836	0.59	0.46	0.65	0.18	0.83	0.95	0.51	0.56
0.1	0.5	0.16	0.51	0.67	0.47	0.72	0.53	0.2	0.66	0.916	0.51	0.49	0.66	0.25	0.86	0.96	0.51	0.59
0.05	0.54	0.17	0.55	0.72	0.51	0.77	0.52	0.25	0.7	0.966	0.49	0.53	0.64	0.3	0.9	0.95	0.49	0.6
0.01	0.57	0.15	0.58	0.73	0.54	0.78	0.55	0.14	0.73	0.976	0.49	0.56	0.67	0.29	0.93	0.94	0.49	0.61

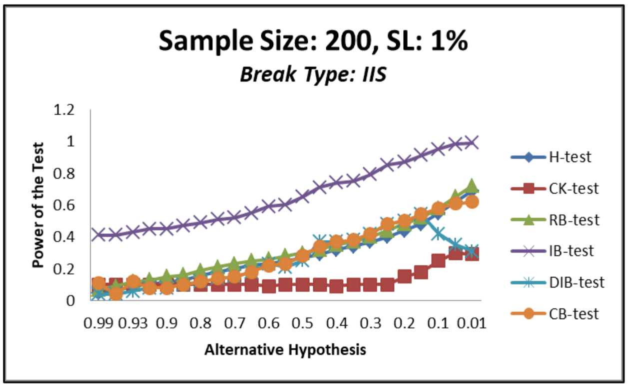
Note: Author's own calculation

Figure 4.1: Performance Under Impulse Indicator Saturation (IIS)

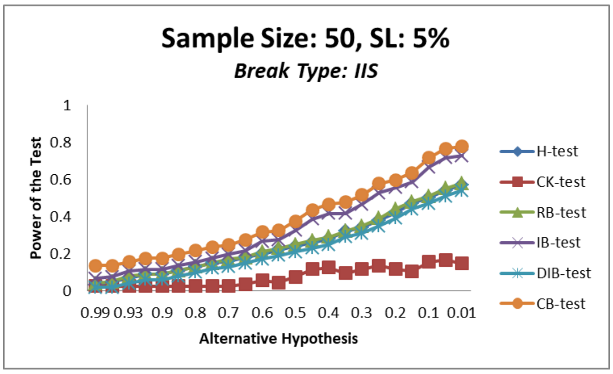
Performance Under 1% Level of Significance

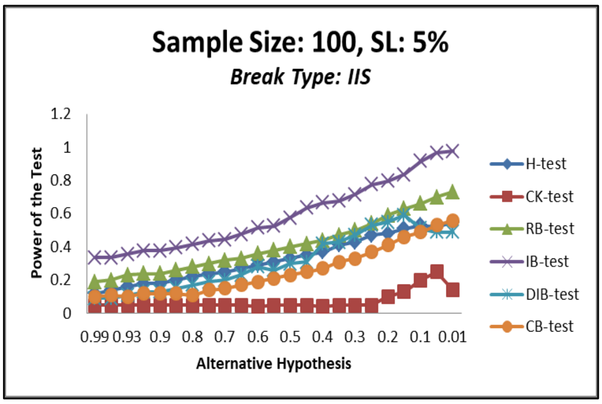


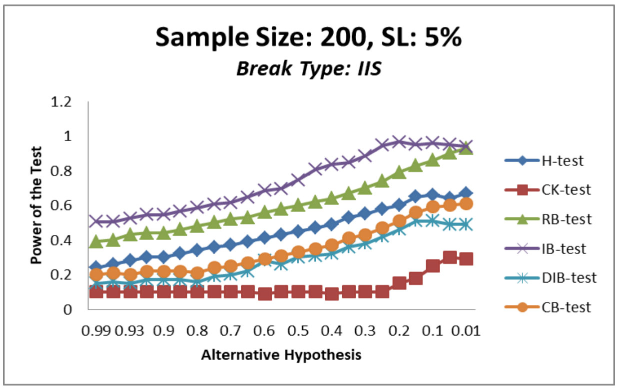




Performance Under 5% Level of Significance







Note: Author's Own Estimations

4.2 Performance using SIS under Stationary Data

The following Table 4.5 envelops the simulated CV's of the SupExt tests when SIS is being taken into account. For the sample size 50 the observed values *CB-Test* and *IB-Test* are not very deviating *i.e.* both tests are showing very less size distortion. As on simulated critical values empirical size is approximately equal to nominal size which is obtained using simulated critical values. The difference between empirical size and nominal size is small which we will obtain from Table 4.5. However, as sample size changes from 50 to 100 or even to 200 we found a size distortion in *CB-Test* and *DIB-Test* and more in *CK-Test* as the nominal size is exceeding the empirical size at both significance level. On the other hand, using SIS both *IB-Test* and *RB-Test* are no size distortion. As on simulated critical values empirical size is approximately equal to nominal size which is obtained using simulated critical values but for other tests the results of nominal size are exceeding the empirical size so here we will face size distortion.

Table 4.5: Simulated Critical Values of SupExt Tests Under SIS

Under Stationary Data Settings												
Size of Test	***H-Test***		***CK-Test***		***RB-Test***		***IB-Tests***		***DIB-Test***		***CB-Test***	
	5%	1%	5%	1%	5%	1%	5%	1%	5%	1%	5%	1%
Sample Size: 50	4.56	5.36	1.71	2.31	5.18	5.35	5.42	5.91	4.51	5.16	5.19	5.37
Sample Size: 100	5.73	6.61	1.93	2.88	5.43	6.54	6.48	6.87	5.55	6.14	5.42	6.01
Sample Size: 200	5.81	6.67	1.91	2.81	5.91	6.73	6.03	6.81	4.72	5.44	4.99	5.57

Note: Author's own calculations

Table 4.6: Empirical Size of SupExt Tests using Asymptotic Critical Values Under SIS

Size of Test	*H-Test*		*CK-Test*		*RB-Test*		*IB-Tests*		*DIB-Test*		*CB-Test*	
	Under Stationary Data Settings											
	1%	5%	1%	5%	1%	5%	1%	5%	1%	5%	1%	5%
Sample Size: 50	0.012	0.053	0.019	0.060	0.011	0.052	0.011	0.051	0.013	0.053	0.010	0.050
Sample Size: 100	0.011	0.051	0.019	0.059	0.010	0.048	0.010	0.050	0.012	0.053	0.013	0.054
Sample Size: 200	0.011	0.051	0.018	0.057	0.010	0.050	0.010	0.049	0.014	0.056	0.011	0.053

Note: Author’s own calculations

4.2.1 Power of the Tests using SIS under Stationary Data

The power curves for each test under SIS are given below in Figure 4.2. By looking at the graphs one can easily observe that when sample is 50 the power of *CB-Test* and *IB-Test* both at 1% as well as 5% level of significance is better than other tests though not as much as while considering IIS. But as the sample size increase from 50 to 100 or 200 the power of *CB-Test* significantly reduced using SIS. However, the power of *IB-Test* and *RB-Test* showed improvement as compare to other tests considering SIS. But the power of *IB-Test* is much better than that of other SupExt tests. At 5% level of significance and sample size 100 the power of *RB-Test* and *H-Test* is more or less equal. The performance of *DIB-Test* is not good as sample size increases under SIS. As a whole the *IB-Test* performs better than other SupExt tests while using SIS. Lastly, the performance of *CK-Test* remains at the low at both significance level under SIS.

Table 4.7: Powers under the Alternative Hypothesis at 1% Level of Significance Using Step Indicator Saturation (SIS)

Under Stationary Data Settings																		
1% SL	Sample Size: 50						Sample Size: 100						Sample Size: 200					
H_1	***H-test***	***CK-test***	***RB-test***	***IB-test***	***DIB-test***	***CB-test***	***H-test***	***CK-test***	***RB-test***	***IB-test***	***DIB-test***	***CB-test***	***H-test***	***CK-test***	***RB-test***	***IB-test***	***DIB-test***	***CB-test***
0.99	0.02	0.05	0.09	0.14	0.02	0.16	0.11	0.05	0.17	0.36	0.03	0.06	0.05	0.1	0.07	0.29	0.18	0.03
0.96	0.02	0.05	0.11	0.16	0.01	0.13	0.11	0.05	0.19	0.33	0.05	0.08	0.06	0.1	0.09	0.31	0.2	0.05
0.93	0.04	0.05	0.13	0.18	0.01	0.15	0.13	0.05	0.21	0.35	0.07	0.1	0.09	0.1	0.12	0.33	0.22	0.07
0.91	0.06	0.05	0.15	0.2	0.03	0.17	0.15	0.05	0.23	0.37	0.09	0.12	0.1	0.1	0.13	0.35	0.24	0.09
0.9	0.06	0.05	0.16	0.21	0.03	0.17	0.15	0.05	0.24	0.37	0.1	0.13	0.12	0.1	0.15	0.36	0.25	0.1
0.85	0.08	0.05	0.18	0.23	0.05	0.19	0.17	0.05	0.26	0.39	0.12	0.15	0.13	0.1	0.16	0.38	0.27	0.12
0.8	0.1	0.05	0.2	0.25	0.07	0.21	0.19	0.05	0.28	0.41	0.14	0.17	0.16	0.1	0.19	0.4	0.29	0.14
0.75	0.12	0.05	0.22	0.27	0.09	0.23	0.21	0.05	0.3	0.43	0.16	0.19	0.18	0.1	0.21	0.42	0.31	0.16
0.7	0.13	0.05	0.23	0.28	0.1	0.24	0.22	0.05	0.31	0.44	0.17	0.2	0.2	0.1	0.23	0.43	0.32	0.17
0.65	0.15	0.05	0.25	0.3	0.12	0.26	0.24	0.05	0.33	0.46	0.19	0.22	0.22	0.1	0.25	0.45	0.34	0.19
0.6	0.17	0.05	0.27	0.32	0.14	0.28	0.26	0.04	0.35	0.48	0.21	0.24	0.23	0.09	0.26	0.47	0.27	0.21
0.55	0.19	0.05	0.29	0.34	0.16	0.3	0.28	0.05	0.37	0.5	0.23	0.26	0.25	0.1	0.28	0.49	0.25	0.23
0.5	0.21	0.05	0.31	0.36	0.18	0.32	0.3	0.05	0.39	0.52	0.25	0.28	0.27	0.1	0.3	0.51	0.25	0.25
0.45	0.23	0.05	0.33	0.38	0.2	0.34	0.32	0.05	0.41	0.54	0.27	0.3	0.29	0.1	0.32	0.53	0.37	0.27
0.4	0.25	0.05	0.36	0.41	0.22	0.36	0.34	0.04	0.44	0.56	0.3	0.33	0.32	0.09	0.35	0.56	0.26	0.3
0.35	0.29	0.05	0.4	0.45	0.26	0.4	0.38	0.05	0.48	0.6	0.34	0.37	0.34	0.1	0.37	0.6	0.19	0.34
0.3	0.31	0.05	0.43	0.48	0.28	0.42	0.4	0.05	0.51	0.62	0.37	0.4	0.37	0.1	0.4	0.63	0.22	0.37
0.25	0.35	0.05	0.46	0.51	0.32	0.46	0.44	0.05	0.54	0.66	0.4	0.43	0.4	0.1	0.44	0.66	0.25	0.4
0.2	0.39	0.05	0.5	0.55	0.36	0.5	0.48	0.1	0.58	0.7	0.44	0.47	0.44	0.15	0.48	0.7	0.29	0.44
0.15	0.44	0.05	0.54	0.59	0.41	0.55	0.53	0.13	0.62	0.75	0.48	0.51	0.48	0.18	0.52	0.74	0.33	0.48
0.1	0.47	0.05	0.57	0.62	0.44	0.58	0.56	0.2	0.65	0.78	0.51	0.54	0.55	0.25	0.58	0.77	0.21	0.51
0.05	0.51	0.05	0.61	0.66	0.48	0.62	0.6	0.25	0.69	0.82	0.55	0.58	0.62	0.3	0.65	0.81	0.22	0.55
0.01	0.54	0.05	0.63	0.68	0.51	0.65	0.63	0.14	0.71	0.85	0.57	0.6	0.69	0.29	0.72	0.83	0.21	0.57

Note: Author's own calculations

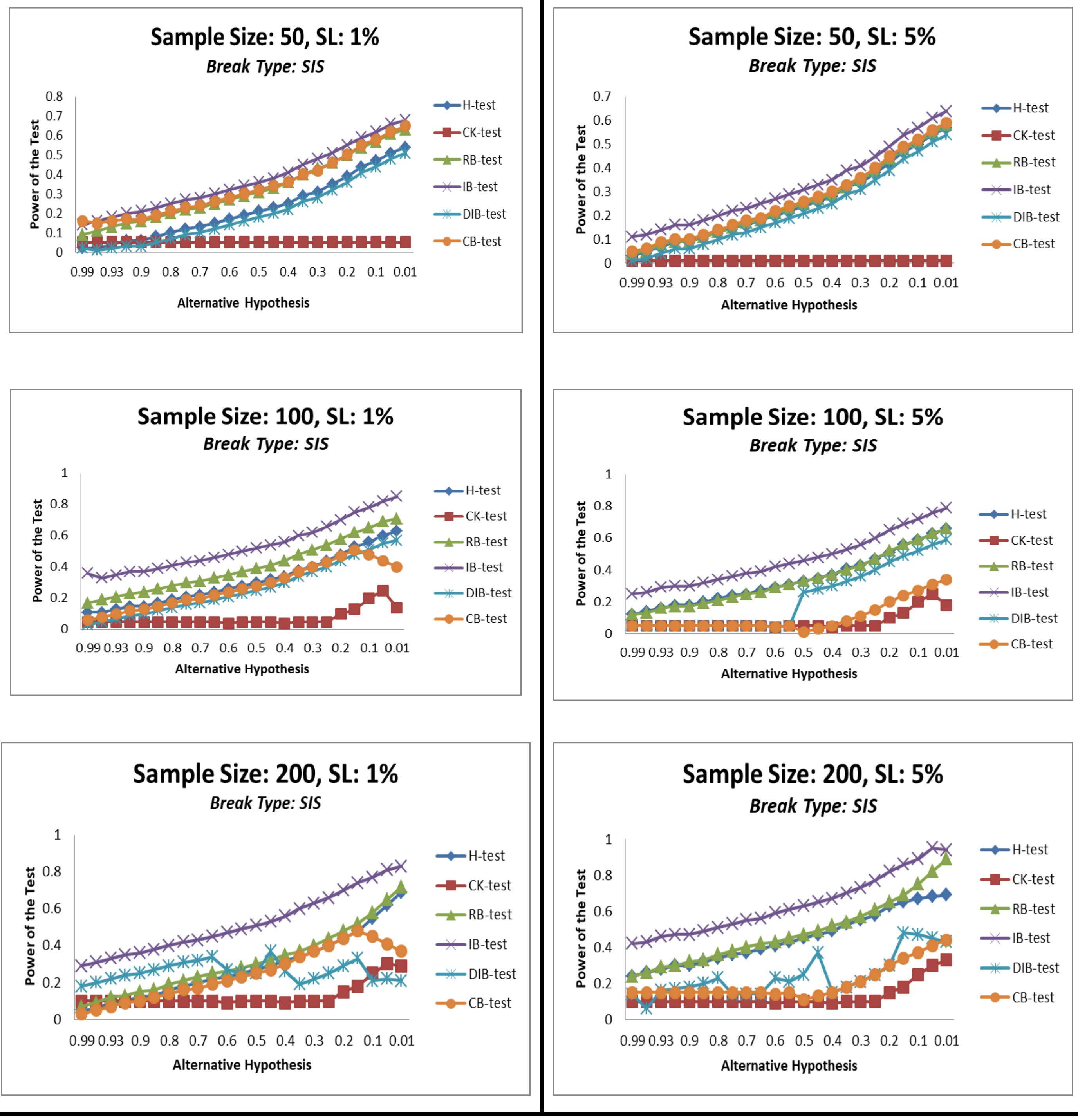
Table 4.8: Powers under the Alternative Hypothesis at 5% Level of Significance Using Step Indicator Saturation (SIS)

Under Stationary Data Settings																		
5% SL	Sample Size: 50						Sample Size: 100						Sample Size: 200					
H_1	***H-test***	***CK-test***	***RB-test***	***IB-test***	***DIB-test***	***CB-test***	***H-test***	***CK-test***	***RB-test***	***IB-test***	***DIB-test***	***CB-test***	***H-test***	***CK-test***	***RB-test***	***IB-test***	***DIB-test***	***CB-test***
0.99	0.03	0.01	0.04	0.05	0.01	0.11	0.12	0.05	0.12	0.31	0.26	0.01	0.24	0.1	0.24	0.48	0.11	0.11
0.96	0.05	0.01	0.05	0.06	0.02	0.12	0.14	0.05	0.13	0.32	0.27	0.02	0.26	0.1	0.26	0.49	0.12	0.12
0.93	0.07	0.01	0.08	0.09	0.04	0.14	0.16	0.05	0.16	0.34	0.3	0.05	0.28	0.1	0.29	0.51	0.15	0.15
0.91	0.09	0.01	0.09	0.1	0.06	0.16	0.18	0.05	0.17	0.36	0.31	0.06	0.3	0.1	0.3	0.53	0.16	0.16
0.9	0.09	0.01	0.09	0.1	0.06	0.16	0.18	0.05	0.17	0.36	0.31	0.06	0.3	0.1	0.32	0.53	0.16	0.16
0.85	0.11	0.01	0.11	0.12	0.08	0.18	0.2	0.05	0.19	0.38	0.33	0.08	0.32	0.1	0.33	0.55	0.18	0.18
0.8	0.13	0.01	0.13	0.14	0.1	0.2	0.22	0.05	0.21	0.4	0.35	0.1	0.34	0.1	0.36	0.57	0.2	0.2
0.75	0.15	0.01	0.15	0.16	0.12	0.22	0.24	0.05	0.23	0.42	0.37	0.12	0.36	0.1	0.38	0.59	0.22	0.22
0.7	0.16	0.01	0.17	0.18	0.13	0.23	0.25	0.05	0.25	0.43	0.39	0.14	0.37	0.1	0.4	0.6	0.24	0.24
0.65	0.18	0.01	0.18	0.19	0.15	0.25	0.27	0.05	0.26	0.45	0.4	0.15	0.39	0.1	0.42	0.62	0.25	0.25
0.6	0.2	0.01	0.21	0.22	0.17	0.27	0.29	0.04	0.29	0.47	0.43	0.18	0.41	0.09	0.43	0.64	0.23	0.28
0.55	0.22	0.01	0.23	0.24	0.19	0.29	0.31	0.05	0.31	0.49	0.45	0.2	0.43	0.1	0.45	0.66	0.21	0.3
0.5	0.24	0.01	0.25	0.26	0.21	0.31	0.33	0.05	0.33	0.51	0.47	0.22	0.45	0.1	0.47	0.68	0.25	0.32
0.45	0.26	0.01	0.27	0.28	0.23	0.33	0.35	0.05	0.35	0.53	0.49	0.24	0.47	0.1	0.49	0.7	0.37	0.34
0.4	0.28	0.01	0.29	0.3	0.25	0.35	0.37	0.04	0.37	0.55	0.51	0.26	0.49	0.09	0.52	0.72	0.36	0.36
0.35	0.32	0.01	0.32	0.33	0.29	0.39	0.41	0.05	0.4	0.59	0.54	0.29	0.53	0.1	0.54	0.76	0.39	0.39
0.3	0.34	0.01	0.35	0.36	0.31	0.41	0.43	0.05	0.43	0.61	0.57	0.32	0.55	0.1	0.57	0.78	0.42	0.42
0.25	0.38	0.01	0.39	0.4	0.35	0.45	0.47	0.05	0.47	0.65	0.61	0.36	0.58	0.1	0.61	0.82	0.46	0.46
0.2	0.42	0.01	0.44	0.45	0.39	0.49	0.51	0.1	0.52	0.69	0.66	0.41	0.63	0.15	0.65	0.86	0.51	0.51
0.15	0.47	0.01	0.48	0.49	0.44	0.54	0.56	0.13	0.56	0.74	0.7	0.45	0.65	0.18	0.69	0.91	0.48	0.55
0.1	0.5	0.01	0.51	0.52	0.47	0.57	0.59	0.2	0.59	0.77	0.73	0.48	0.67	0.25	0.75	0.94	0.47	0.58
0.05	0.54	0.01	0.55	0.56	0.51	0.61	0.63	0.25	0.63	0.81	0.77	0.52	0.68	0.3	0.82	0.95	0.45	0.62
0.01	0.57	0.01	0.58	0.59	0.54	0.64	0.66	0.14	0.66	0.84	0.8	0.55	0.69	0.29	0.89	0.94	0.43	0.65

Note: Author's own calculations

Figure 4.2: Performance Under Step Indicator Saturation (SIS)

Performance Under 1% Level of Significance	Performance Under 5% Level of Significance



Note: Author's Own Estimations

4.3 Performance using TIS under Stationary Data

The following Table 4.9 covers the simulated CV's of the SupExt tests when TIS is under consideration. For the sample size 50 the observed values *CB-Test* and *IB-Test* are not very deviating *i.e.* both tests are showing very less size distortion. As on simulated critical values empirical size is approximately equal to nominal size which is obtained using simulated critical values. The difference between empirical size and nominal size is small which we will obtain from Table 4.9. However, as sample size changes from 50 to 100 or even to 200 we found a size distortion in *CB-Test* and *DIB-Test* and more in *CK-Test* as the nominal size is exceeding the empirical size at both significance level. On the other hand, using SIS both *IB-Test* and *RB-Test* are no size distortion. As on simulated critical values empirical size is approximately equal to nominal size which is obtained using simulated critical values but for other tests the results of nominal size are exceeding the empirical size so here we will face size distortion.

Table 4.9: Simulated Critical Values of SupExt Tests Under TIS

Under Stationary Data Settings												
Size of Test	***H-Test***		***CK-Test***		***RB-Test***		***IB-Tests***		***DIB-Test***		***CB-Test***	
	5%	1%	5%	1%	5%	1%	5%	1%	5%	1%	5%	1%
Sample Size: 50	4.81	4.97	2.12	2.57	4.32	4.99	5.15	4.97	3.69	4.73	5.13	6.66
Sample Size: 100	4.78	5.97	1.91	2.1	4.89	5.99	5.51	5.76	5.66	5.74	4.47	4.63
Sample Size: 200	4.71	5.59	1.93	2.12	5.23	5.99	6.01	6.19	4.23	4.67	5.15	4.57

Note: Author's own calculations

Table 4.10: Empirical Size of SupExt Tests using Asymptotic Critical Values Under TIS

Under Stationary Data Settings												
Size of Test	***H-Test***		***CK-Test***		***RB-Test***		***IB-Tests***		***DIB-Test***		***CB-Test***	
	1%	5%	1%	5%	1%	5%	1%	5%	1%	5%	1%	5%
Sample Size: 50	0.010	0.051	0.016	0.054	0.011	0.051	0.011	0.051	0.013	0.053	0.010	0.050
Sample Size: 100	0.010	0.050	0.015	0.054	0.010	0.048	0.010	0.050	0.010	0.051	0.013	0.054
Sample Size: 200	0.010	0.050	0.013	0.055	0.010	0.050	0.010	0.048	0.012	0.054	0.011	0.053

Note: Author's own calculations

4.3.1 Power of the Tests using TIS under Stationary Data

The power curves for each test under SIS are given below in Figure 4.3. By looking at the graphs one can easily observe that when sample is 50 the power of *IB-Test* and *RB-Test* both at 1% as well as 5% level of significance is better than other tests though not as much as while considering TIS. When significance level is 5% the power of *CB-Test* improves than that of *IB-Test* and *RB-Test.* But as the sample size increase from 50 to 100 the power of *CB-Test* significantly reduced using TIS. However, the power of *IB-Test* and *DIB-Test* showed improvement as compare to other tests considering TIS when sample size is 100. But the power of *IB-Test* is much better than that of other SupExt tests when sample size is 200. At 5% level of significance and sample size 100 the power of *RB-Test* and *H-Test* is more or less equal. The performance of *DIB-Test* is not good as sample size increases under SIS. As a whole the *IB-Test* performs better than other SupExt tests while using SIS. Lastly, the performance of *CK-Test* improves when sample size is of 100 and 200 as compared to IIS and SIS but remains at the lowest at both significance levels under TIS.

Table 4.11: Powers under the Alternative Hypothesis at 1% Level of Significance Using Trend Indicator Saturation (TIS)

Under Stationary Data Settings																		
1% SL	Sample Size: 50						Sample Size: 100						Sample Size: 200					
H_1	***H-test***	***CK-test***	***RB-test***	***IB-test***	***DIB-test***	***CB-test***	***H-test***	***CK-test***	***RB-test***	***IB-test***	***DIB-test***	***CB-test***	***H-test***	***CK-test***	***RB-test***	***IB-test***	***DIB-test***	***CB-test***
0.99	0.03	0.03	0.09	0.14	0	0.14	0.12	0.05	0.17	0.34	0.31	0.06	0.05	0.1	0.07	0.29	0.16	0.16
0.96	0.05	0.05	0.11	0.16	0.02	0.16	0.14	0.05	0.19	0.36	0.33	0.08	0.06	0.1	0.09	0.31	0.18	0.18
0.93	0.07	0.07	0.13	0.18	0.04	0.18	0.16	0.05	0.21	0.38	0.35	0.1	0.09	0.1	0.12	0.33	0.2	0.2
0.91	0.09	0.09	0.15	0.2	0.06	0.2	0.18	0.05	0.23	0.4	0.37	0.12	0.1	0.1	0.13	0.35	0.22	0.22
0.9	0.09	0.09	0.16	0.21	0.06	0.2	0.18	0.05	0.24	0.4	0.38	0.13	0.12	0.1	0.15	0.36	0.23	0.23
0.85	0.11	0.11	0.18	0.23	0.08	0.22	0.2	0.05	0.26	0.42	0.4	0.15	0.13	0.1	0.16	0.38	0.25	0.25
0.8	0.13	0.13	0.2	0.25	0.1	0.24	0.22	0.05	0.28	0.44	0.42	0.17	0.16	0.1	0.19	0.4	0.27	0.27
0.75	0.15	0.15	0.22	0.27	0.12	0.26	0.24	0.05	0.3	0.46	0.44	0.19	0.18	0.1	0.21	0.42	0.29	0.29
0.7	0.16	0.16	0.23	0.28	0.13	0.27	0.25	0.05	0.31	0.47	0.45	0.2	0.2	0.1	0.23	0.43	0.3	0.3
0.65	0.18	0.18	0.25	0.3	0.15	0.29	0.27	0.05	0.33	0.49	0.47	0.22	0.22	0.1	0.25	0.45	0.32	0.32
0.6	0.2	0.2	0.27	0.32	0.17	0.31	0.29	0.04	0.35	0.51	0.49	0.24	0.23	0.09	0.26	0.47	0.23	0.34
0.55	0.22	0.22	0.29	0.34	0.19	0.33	0.31	0.05	0.37	0.53	0.51	0.26	0.25	0.1	0.28	0.49	0.21	0.36
0.5	0.24	0.21	0.31	0.36	0.14	0.28	0.26	0.05	0.39	0.53	0.53	0.28	0.27	0.1	0.3	0.51	0.25	0.38
0.45	0.26	0.18	0.33	0.38	0.15	0.29	0.27	0.05	0.41	0.55	0.55	0.3	0.29	0.1	0.32	0.53	0.37	0.4
0.4	0.28	0.15	0.36	0.41	0.17	0.31	0.29	0.04	0.44	0.58	0.58	0.33	0.32	0.09	0.35	0.56	0.43	0.43
0.35	0.32	0.12	0.4	0.45	0.2	0.34	0.32	0.05	0.48	0.62	0.62	0.37	0.34	0.1	0.37	0.6	0.47	0.47
0.3	0.34	0.09	0.43	0.48	0.215	0.355	0.335	0.05	0.51	0.65	0.65	0.4	0.37	0.1	0.4	0.63	0.5	0.5
0.25	0.38	0.09	0.46	0.51	0.235	0.375	0.355	0.05	0.54	0.68	0.68	0.43	0.4	0.1	0.44	0.66	0.53	0.53
0.2	0.42	0.08	0.5	0.55	0.255	0.395	0.375	0.1	0.58	0.72	0.72	0.47	0.44	0.15	0.48	0.7	0.54	0.57
0.15	0.47	0.05	0.54	0.59	0.275	0.415	0.395	0.13	0.62	0.76	0.76	0.51	0.48	0.18	0.52	0.74	0.54	0.61
0.1	0.50	0.05	0.57	0.62	0.295	0.435	0.415	0.2	0.65	0.79	0.79	0.54	0.55	0.25	0.58	0.77	0.52	0.64
0.05	0.54	0.05	0.61	0.66	0.315	0.455	0.435	0.25	0.69	0.83	0.83	0.58	0.62	0.3	0.65	0.81	0.5	0.68
0.01	0.57	0.05	0.63	0.68	0.335	0.475	0.455	0.14	0.71	0.85	0.85	0.6	0.69	0.29	0.72	0.83	0.5	0.7

Note: Author’s own calculations

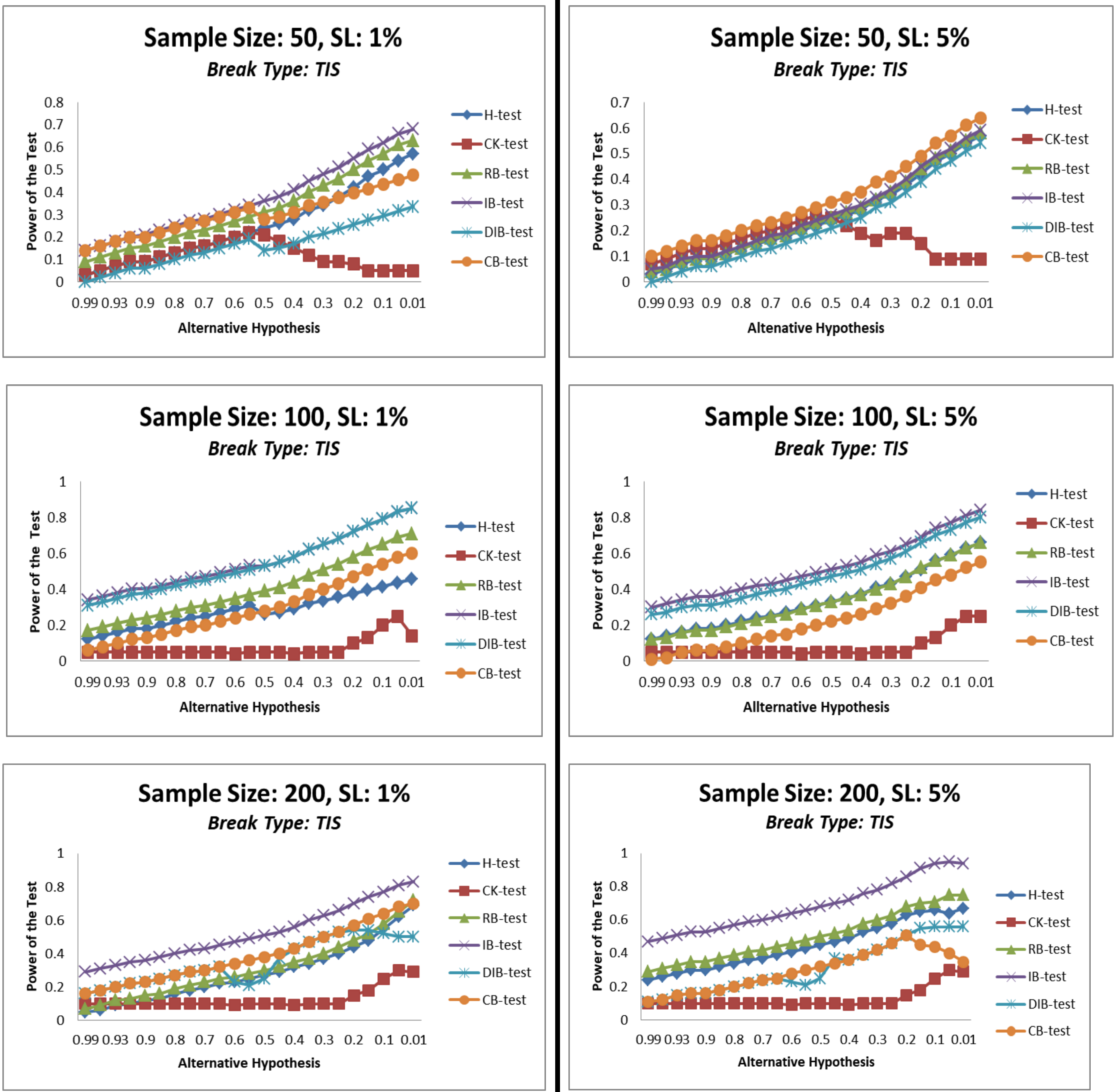
Table 4.12: Powers under the Alternative Hypothesis at 5% Level of Significance Using Trend Indicator Saturation (TIS)

Under Stationary Data Settings																		
5% SL	Sample Size: 50						Sample Size: 100						Sample Size: 200					
H_1	*H-test*	*CK-test*	*RB-test*	*IB-test*	*DIB-test*	*CB-test*	*H-test*	*CK-test*	*RB-test*	*IB-test*	*DIB-test*	*CB-test*	*H-test*	*CK-test*	*RB-test*	*IB-test*	*DIB-test*	*CB-test*
0.99	0.03	0.07	0.04	0.05	0	0.1	0.12	0.05	0.12	0.3	0.26	0.01	0.24	0.1	0.29	0.47	0.11	0.11
0.96	0.05	0.09	0.05	0.06	0.02	0.12	0.14	0.05	0.13	0.32	0.27	0.02	0.26	0.1	0.31	0.49	0.12	0.12
0.93	0.07	0.11	0.08	0.09	0.04	0.14	0.16	0.05	0.16	0.34	0.3	0.05	0.28	0.1	0.33	0.51	0.15	0.15
0.91	0.09	0.13	0.09	0.1	0.06	0.16	0.18	0.05	0.17	0.36	0.31	0.06	0.3	0.1	0.35	0.53	0.16	0.16
0.9	0.09	0.13	0.09	0.1	0.06	0.16	0.18	0.05	0.17	0.36	0.31	0.06	0.3	0.1	0.35	0.53	0.16	0.16
0.85	0.11	0.15	0.11	0.12	0.08	0.18	0.2	0.05	0.19	0.38	0.33	0.08	0.32	0.1	0.37	0.55	0.18	0.18
0.8	0.13	0.17	0.13	0.14	0.1	0.2	0.22	0.05	0.21	0.4	0.35	0.1	0.34	0.1	0.39	0.57	0.2	0.2
0.75	0.15	0.19	0.15	0.16	0.12	0.22	0.24	0.05	0.23	0.42	0.37	0.12	0.36	0.1	0.41	0.59	0.22	0.22
0.7	0.16	0.2	0.17	0.18	0.13	0.23	0.25	0.05	0.25	0.43	0.39	0.14	0.37	0.1	0.42	0.6	0.24	0.24
0.65	0.18	0.22	0.18	0.19	0.15	0.25	0.27	0.05	0.26	0.45	0.4	0.15	0.39	0.1	0.44	0.62	0.25	0.25
0.6	0.2	0.24	0.21	0.22	0.17	0.27	0.29	0.04	0.29	0.47	0.43	0.18	0.41	0.09	0.46	0.64	0.23	0.28
0.55	0.22	0.26	0.23	0.24	0.19	0.29	0.31	0.05	0.31	0.49	0.45	0.2	0.43	0.1	0.48	0.66	0.21	0.3
0.5	0.24	0.25	0.25	0.26	0.21	0.31	0.33	0.05	0.33	0.51	0.47	0.22	0.45	0.1	0.5	0.68	0.25	0.32
0.45	0.26	0.22	0.27	0.28	0.23	0.33	0.35	0.05	0.35	0.53	0.49	0.24	0.47	0.1	0.52	0.7	0.37	0.34
0.4	0.28	0.19	0.29	0.3	0.25	0.35	0.37	0.04	0.37	0.55	0.51	0.26	0.49	0.09	0.54	0.72	0.36	0.36
0.35	0.32	0.16	0.32	0.33	0.29	0.39	0.41	0.05	0.4	0.59	0.54	0.29	0.53	0.1	0.58	0.76	0.39	0.39
0.3	0.34	0.19	0.35	0.36	0.31	0.41	0.43	0.05	0.43	0.61	0.57	0.32	0.55	0.1	0.6	0.78	0.42	0.42
0.25	0.38	0.19	0.39	0.4	0.35	0.45	0.47	0.05	0.47	0.65	0.61	0.36	0.58	0.1	0.63	0.82	0.46	0.46
0.2	0.42	0.15	0.44	0.45	0.39	0.49	0.51	0.1	0.52	0.69	0.66	0.41	0.63	0.15	0.68	0.86	0.51	0.51
0.15	0.47	0.09	0.48	0.49	0.44	0.54	0.56	0.13	0.56	0.74	0.7	0.45	0.65	0.18	0.7	0.91	0.55	0.45
0.1	0.5	0.09	0.51	0.52	0.47	0.57	0.59	0.2	0.59	0.77	0.73	0.48	0.66	0.25	0.71	0.94	0.43	0.44
0.05	0.54	0.09	0.55	0.56	0.51	0.61	0.63	0.25	0.63	0.81	0.77	0.52	0.64	0.3	0.75	0.95	0.31	0.40
0.01	0.57	0.09	0.58	0.59	0.54	0.64	0.66	0.25	0.66	0.84	0.8	0.55	0.67	0.29	0.75	0.94	0.19	0.35

Note: Author's own calculations

Figure 4.3: Performance Under Trend Indicator Saturation (TIS)

Performance Under 1% Level of Significance	Performance Under 5% Level of Significance



Note: Author's Own Estimations

4.4 Performance using IIS, SIS & TIS under Stationary Data

The following Table 4.13 covers the simulated CV's of the SupExt tests when IIS, SIS and TIS are used jointly. For the sample size 50 the observed values *IB-Test* and *RB-Test* are approximately equal to the empirical size *i.e.* both tests are showing no size distortion. As on simulated critical values empirical size is approximately equal to nominal size which is obtained using simulated critical values. The difference between empirical size and nominal size is small which we will obtain from Table 4.13. At 5% lever of significance and sample size 200, we found size distortion in *CK-Test* and *DIB-Test* and more in *CB-Test* as the nominal size is exceeding the empirical size at both significance level. On the other hand, using all types of breaks jointly both *IB-Test* and *RB-Test* has observed no size distortion. As on simulated critical values empirical size is approximately equal to nominal size which is obtained using simulated critical values but for other tests the results of nominal size are exceeding the empirical size so here we will face size distortion. SO far we came up to that the *IB-Test* and *RB-Test* behaves better under all the types of data driven breaks, in particular, when we used them jointly.

Table 4.13: Simulated Critical Values of SupExt Tests Under IIS, SIS & TIS

Under Stationary Data Settings												
Size of Test	***H-Test***		***CK-Test***		***RB-Test***		***IB-Tests***		***DIB-Test***		***CB-Test***	
	5%	1%	5%	1%	5%	1%	5%	1%	5%	1%	5%	1%
Sample Size: 50	4.01	4.34	2.09	2.23	5.88	6.11	5.92	6.16	3.86	4.11	3.96	4.21
Sample Size: 100	4.61	5.34	1.98	2.12	5.93	6.12	6.01	6.23	4.73	5.16	3.99	4.29
Sample Size: 200	5.67	5.28	2.37	2.41	6.09	6.34	6.17	6.52	4.81	5.2	4.11	4.98

Note: Author's own calculations

Table 4.14: Empirical Size of SupExt Tests using Asymptotic Critical Values Under IIS, SIS & TIS

Under Stationary Data Settings												
Size of Test	***H-Test***		***CK-Test***		***RB-Test***		***IB-Tests***		***DIB-Test***		***CB-Test***	
	1%	5%	1%	5%	1%	5%	1%	5%	1%	5%	1%	5%
Sample Size: 50	0.012	0.053	0.014	0.053	0.010	0.050	0.010	0.050	0.013	0.052	0.013	0.052
Sample Size: 100	0.011	0.052	0.014	0.052	0.010	0.049	0.010	0.050	0.010	0.051	0.013	0.054
Sample Size: 200	0.012	0.055	0.013	0.053	0.010	0.050	0.010	0.050	0.012	0.052	0.011	0.056

Note: Author's own calculations

4.4.1 Power of the Tests using IIS, SIS & TIS under Stationary Data

The power curves for each test when IIS, SIS and TIS used jointly are given below in Figure 4.4. By looking at the graphs one can easily observe that when sample is 50 the power of *IB-Test* and *RB-Test* both at 1% as well as 5% level of significance is better than other tests though not as much as while considering all breaks jointly. However, the power of *IB-Test* and *RB-Test* showed improvement and remain above as compare to other tests considering all breaks at a time at all samples. But the power of *H-Test* and *RB-Test* is almost equal when sample size is 100 at 5% level of significance. The performance of *DIB-Test* is not good as sample size increases to 200. As a whole the performance of SupExt test improves a lot when we use all breaks at a time. However *IB-Test* and *RB-Test* remains at the top while using IIS, SIS & TIS jointly. Lastly, the performance of *CK-Test* improves when sample size is of 100 and 200 as compared to IIS and SIS but remains at the lowest at both significance levels.

Table 4.15: Powers under the Alternative Hypothesis at 1% Level of Significance Using IIS, SIS & TIS at a Time

Under Stationary Data Settings																		
1% SL	Sample Size: 50						Sample Size: 100						Sample Size: 200					
H_1	***H-test***	***CK-test***	***RB-test***	***IB-test***	***DIB-test***	***CB-test***	***H-test***	***CK-test***	***RB-test***	***IB-test***	***DIB-test***	***CB-test***	***H-test***	***CK-test***	***RB-test***	***IB-test***	***DIB-test***	***CB-test***
0.99	0.15	0.14	0.4	0.59	0.12	0.08	0.24	0.05	0.48	0.48	0.34	0.06	0.28	0.1	0.51	0.6	0.19	0.03
0.96	0.17	0.15	0.42	0.60	0.14	0.1	0.26	0.05	0.50	0.50	0.36	0.08	0.3	0.1	0.53	0.62	0.21	0.05
0.93	0.19	0.17	0.44	0.62	0.16	0.12	0.28	0.05	0.52	0.52	0.38	0.1	0.32	0.1	0.55	0.64	0.23	0.07
0.91	0.21	0.16	0.46	0.61	0.18	0.14	0.30	0.05	0.54	0.54	0.4	0.12	0.34	0.1	0.57	0.66	0.25	0.09
0.9	0.21	0.16	0.46	0.61	0.18	0.14	0.30	0.05	0.54	0.54	0.4	0.13	0.34	0.1	0.57	0.66	0.25	0.1
0.85	0.23	0.17	0.48	0.62	0.20	0.16	0.32	0.05	0.56	0.56	0.42	0.15	0.36	0.1	0.59	0.68	0.27	0.12
0.8	0.25	0.17	0.50	0.62	0.22	0.18	0.34	0.05	0.58	0.58	0.44	0.17	0.38	0.1	0.61	0.7	0.29	0.14
0.75	0.27	0.17	0.52	0.62	0.24	0.20	0.36	0.05	0.60	0.60	0.46	0.19	0.4	0.1	0.63	0.72	0.31	0.16
0.7	0.28	0.19	0.53	0.64	0.25	0.21	0.37	0.05	0.61	0.61	0.47	0.2	0.41	0.1	0.64	0.73	0.32	0.17
0.65	0.30	0.20	0.55	0.65	0.27	0.23	0.39	0.05	0.63	0.63	0.49	0.22	0.43	0.1	0.66	0.75	0.34	0.19
0.6	0.32	0.18	0.57	0.63	0.29	0.25	0.41	0.04	0.65	0.65	0.51	0.24	0.45	0.09	0.68	0.77	0.36	0.21
0.55	0.34	0.16	0.59	0.61	0.31	0.27	0.43	0.06	0.67	0.67	0.53	0.26	0.47	0.11	0.7	0.79	0.38	0.23
0.5	0.36	0.17	0.61	0.67	0.33	0.29	0.45	0.09	0.69	0.69	0.55	0.28	0.49	0.14	0.72	0.81	0.4	0.25
0.45	0.38	0.18	0.63	0.68	0.35	0.31	0.47	0.13	0.71	0.71	0.57	0.3	0.51	0.18	0.74	0.83	0.42	0.27
0.4	0.40	0.19	0.65	0.69	0.37	0.33	0.49	0.17	0.73	0.73	0.59	0.33	0.53	0.22	0.76	0.85	0.44	0.3
0.35	0.44	0.21	0.69	0.67	0.41	0.37	0.53	0.12	0.77	0.77	0.63	0.37	0.57	0.17	0.8	0.89	0.48	0.34
0.3	0.46	0.20	0.71	0.73	0.43	0.39	0.55	0.11	0.79	0.79	0.65	0.40	0.59	0.16	0.82	0.91	0.50	0.37
0.25	0.50	0.18	0.75	0.79	0.47	0.43	0.59	0.13	0.83	0.83	0.69	0.43	0.63	0.18	0.86	0.95	0.54	0.4
0.2	0.54	0.19	0.79	0.81	0.51	0.47	0.63	0.10	0.87	0.87	0.73	0.47	0.67	0.15	0.9	0.99	0.58	0.44
0.15	0.59	0.22	0.84	0.86	0.56	0.52	0.68	0.13	0.92	0.92	0.78	0.51	0.67	0.18	0.9	0.99	0.58	0.48
0.1	0.62	0.29	0.87	0.88	0.59	0.55	0.71	0.2	0.95	0.95	0.81	0.54	0.72	0.25	0.95	0.99	0.63	0.51
0.05	0.66	0.34	0.89	0.91	0.63	0.59	0.75	0.25	0.97	0.99	0.83	0.58	0.73	0.3	0.96	0.99	0.64	0.55
0.01	0.69	0.35	0.89	0.96	0.66	0.62	0.78	0.27	0.97	0.99	0.83	0.6	0.73	0.42	0.96	0.98	0.64	0.57

Note: Author's own calculations

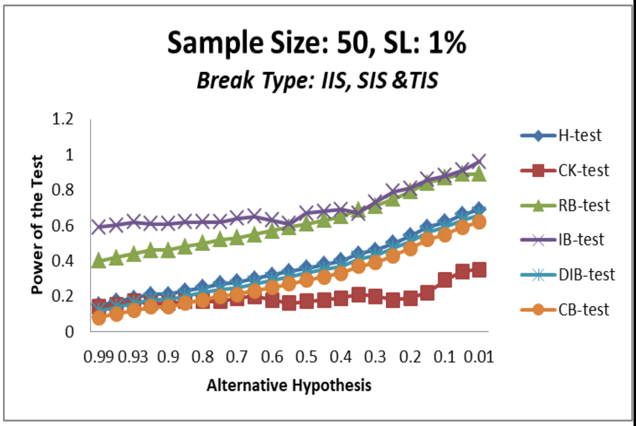
Table 4.16: Powers under the Alternative Hypothesis at 5% Level of Significance Using IIS, SIS & TIS at a Time

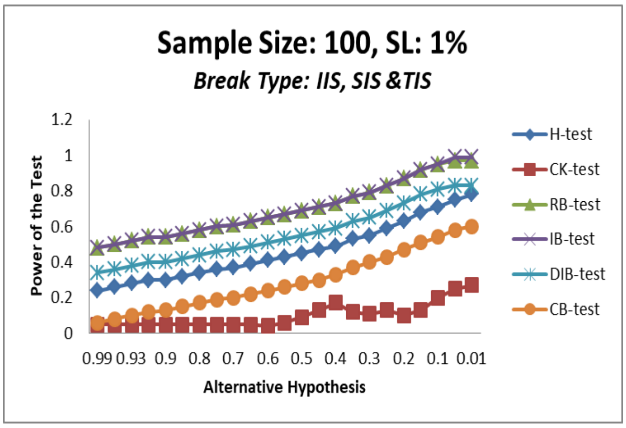
Under Stationary Data Settings																		
5% SL	Sample Size: 50						Sample Size: 100						Sample Size: 200					
H_1	***H-test***	***CK-test***	***RB-test***	***IB-test***	***DIB-test***	***CB-test***	***H-test***	***CK-test***	***RB-test***	***IB-test***	***DIB-test***	***CB-test***	***H-test***	***CK-test***	***RB-test***	***IB-test***	***DIB-test***	***CB-test***
0.99	0.38	0.14	0.57	0.6	0.34	0.06	0.47	0.15	0.49	0.51	0.35	0.22	0.59	0.203	0.68	0.68	0.2	0.32
0.96	0.4	0.14	0.59	0.62	0.36	0.08	0.49	0.15	0.51	0.53	0.37	0.24	0.61	0.203	0.7	0.7	0.22	0.34
0.93	0.42	0.16	0.61	0.64	0.38	0.1	0.51	0.17	0.53	0.55	0.39	0.26	0.63	0.223	0.72	0.72	0.24	0.36
0.91	0.44	0.18	0.63	0.66	0.4	0.12	0.53	0.19	0.55	0.57	0.41	0.28	0.65	0.243	0.74	0.74	0.26	0.38
0.9	0.44	0.2	0.63	0.66	0.4	0.13	0.53	0.21	0.55	0.58	0.41	0.28	0.65	0.263	0.74	0.75	0.26	0.38
0.85	0.46	0.27	0.65	0.68	0.42	0.15	0.55	0.28	0.57	0.6	0.43	0.3	0.67	0.333	0.76	0.77	0.28	0.4
0.8	0.48	0.29	0.67	0.7	0.44	0.17	0.57	0.30	0.59	0.62	0.45	0.32	0.69	0.353	0.78	0.79	0.3	0.42
0.75	0.5	0.3	0.69	0.72	0.46	0.19	0.59	0.31	0.61	0.64	0.47	0.34	0.71	0.363	0.8	0.81	0.32	0.44
0.7	0.51	0.23	0.7	0.73	0.47	0.2	0.6	0.24	0.62	0.65	0.48	0.35	0.72	0.293	0.81	0.82	0.33	0.45
0.65	0.53	0.14	0.72	0.75	0.49	0.22	0.62	0.15	0.64	0.67	0.5	0.37	0.74	0.203	0.83	0.84	0.35	0.47
0.6	0.55	0.13	0.74	0.77	0.51	0.24	0.64	0.14	0.66	0.69	0.52	0.39	0.76	0.193	0.85	0.86	0.23	0.49
0.55	0.57	0.15	0.76	0.79	0.53	0.26	0.66	0.16	0.68	0.71	0.54	0.41	0.78	0.213	0.87	0.88	0.21	0.51
0.5	0.59	0.18	0.78	0.81	0.55	0.28	0.68	0.19	0.7	0.73	0.56	0.43	0.8	0.243	0.89	0.9	0.25	0.53
0.45	0.61	0.22	0.8	0.83	0.57	0.3	0.7	0.23	0.72	0.75	0.58	0.45	0.82	0.283	0.91	0.92	0.37	0.55
0.4	0.63	0.26	0.82	0.85	0.59	0.33	0.72	0.27	0.74	0.78	0.6	0.47	0.84	0.323	0.93	0.95	0.45	0.57
0.35	0.67	0.21	0.86	0.89	0.63	0.37	0.76	0.22	0.78	0.82	0.64	0.51	0.88	0.273	0.93	0.99	0.49	0.61
0.3	0.69	0.2	0.88	0.91	0.65	0.4	0.78	0.21	0.8	0.85	0.66	0.53	0.9	0.263	0.93	0.99	0.51	0.63
0.25	0.73	0.22	0.92	0.95	0.69	0.43	0.82	0.23	0.84	0.88	0.7	0.57	0.8	0.283	0.93	0.99	0.55	0.67
0.2	0.77	0.19	0.96	0.99	0.73	0.47	0.86	0.20	0.88	0.92	0.74	0.61	0.77	0.253	0.93	0.99	0.59	0.71
0.15	0.82	0.22	0.96	0.99	0.78	0.51	0.91	0.23	0.88	0.96	0.74	0.61	0.65	0.283	0.93	0.99	0.48	0.71
0.1	0.85	0.29	0.96	0.99	0.81	0.54	0.94	0.20	0.88	0.99	0.74	0.61	0.67	0.25	0.94	0.99	0.47	0.71
0.05	0.89	0.34	0.95	0.99	0.83	0.58	0.94	0.16	0.87	0.99	0.73	0.6	0.68	0.21	0.94	0.99	0.45	0.7
0.01	0.92	0.36	0.96	0.99	0.83	0.6	0.94	0.15	0.88	0.99	0.74	0.61	0.69	0.3	0.94	0.99	0.43	0.71

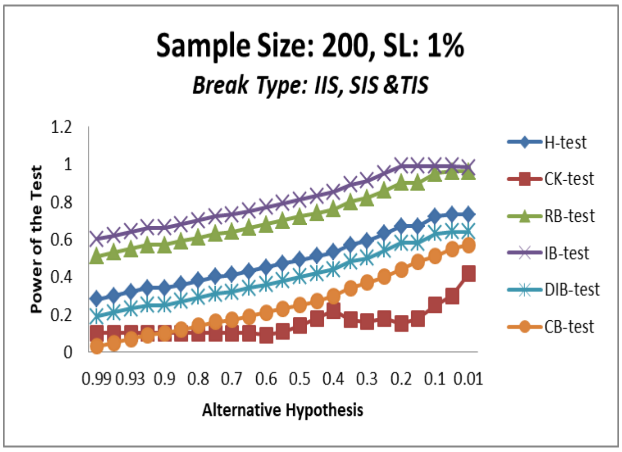
Note: Author's own calculations

Figure 4.4 : Performance Under IIS, SIS and TIS Jointly

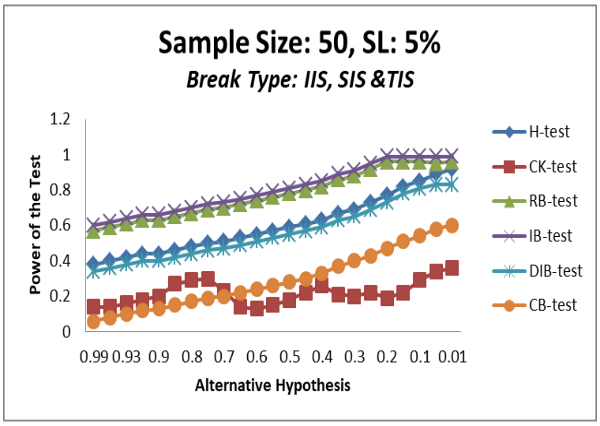
Performance Under 1% Level of Significance

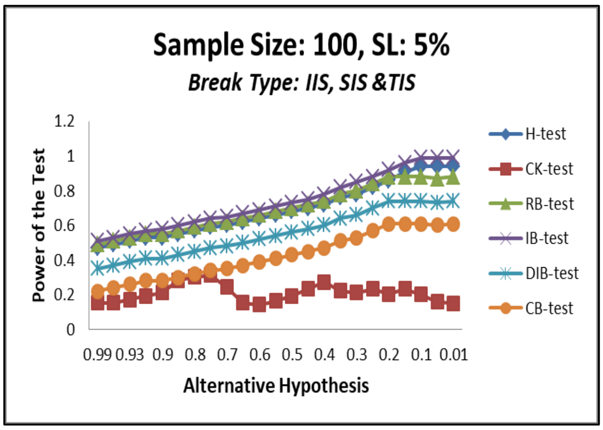


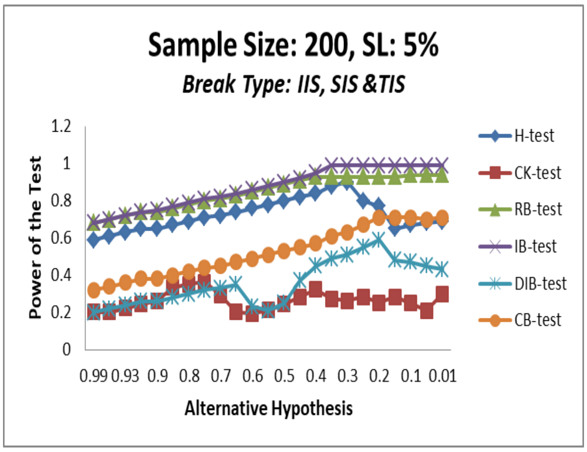




Performance Under 5% Level of Significance







Note: Author's Own Estimations

4.5 Conclusion

The chapter above covers the argument about the performance of SupExt tests under consideration. On the basis of above concrete and detailed analysis of SupExt tests and their performance under stationary data settings while considering structural break of the type IIS, SIS & TIS separately and all at a time jointly; we came up with a conclusion that for a small sample of 50 at both level of significance the *IB-Test* and *CB-Test* performs better using IIS. All experiments have been repeated for 100,000 times where simulations were done in MATLAB. But as sample changes from 50 to 100 and then 200 the power of *CB-Test* significantly dropped out. However, on the other hand the performance of *IB-Test* and *RB-Test* showed improvement. While the performance of *CK-Test* remains at the lowest stream.

Now under SIS, again for sample size of 50, the *IB-Test* and *CB-Test* performs good but as sample increases from 50 to 100 and then 200 the performance of *CB-Test* reduced significantly and also *DIB-Test* performs not very well using SIS as compared to IIS. The performance of *H-Test* improves as compared to SIS. However, on the other hand the performance of *IB-Test* and *RB-Test* showed improvement. While the performance of *CK-Test* remains at the lowest stream.

Considering TIS, it is worth noting that the power of *CK-Test* for small sample of 50 showed improvement at bot 1% and 5% level of significance initially but later dropped out. When sample size is 100 the performance of *DIB-Test* and *IB-Test* is more or less equal. But as sample size increases to 200 the performance of *DIB-Test* reduced but *IB-Test* and *RB-Test* showed improvement.

Lastly, while using all these breaks jointly, the overall performance of the tests significantly improves however the trend remains same. Both *IB-Test* and *RB-Test* performs better than that of other SupExt tests. However, *H-Test* also performs better for all samples but *DIB-Test* didn't perform well for sample size 200. Therefore, as a whole we can say that *IB-Test* and *RB-Test* perform well in all scenarios and the use of all breaks jointly at a time is recommended when the putative regressor in the conditional model is being tested for SupExt.

CHAPTER 5

PERFORMANCE UNDER NON-STATIONARY & DYNAMIC DATA SETTINGS

In last chapter, we covered the argument about performance of SupExt test under stationary data settings considering IIS, SIS & TIS and all at a time. Now this chapter will briefly explain and interpret the simulation results of SupExt tests while taking into account non-stationary as well as dynamic data settings using IIS, SIS and TIS at 1% and 5% level of significance separately. Furthermore, we extend our analyses to compare their performance by considering IIS, SIS and TIS at a time with non-stationary and dynamic data. Before we start any comparison of SupExt tests we need to find simulated critical values for each test.

The need for the simulated critical values is of high significance, as most of the tests rely on the basis of asymptotic critical values which loose its efficiency when we are working with small samples, so that's the reason for the choice of simulated critical values. We used the asymptotic critical values of the these SupExt tests for the calculation of size of the test, while computing size of the test we found that there was no size distortion as the taken sample size was of 50, 100 and 200. At 1% and 5% level of significance we will estimate the simulated critical values of these SupExt tests. The simulated critical values of SupExt tests under stationary data settings using IIS, SIS, TIS and when using all these three at a time taking 1% and 5% level of significance for three different sample sizes of 50, 100 and 200 are being reported below in Table 5.1 (a) - 5.1 (d).

5.1 Performance using IIS under Non-Stationary Data

The following Table 5.1 (a) envelops the simulated CV's of the SupExt tests when IIS is being taken into account when data non-stationary. It is worth noting that while taking non-stationary data setting the SupExt tests performs not as much well as were under the stationary data settings. In case of non-stationarity we face size distortions almost in each test. However, this size distortion is relatively less in *RB-Test* and in *IB-Test* as compared to other SupExt tests. For the sample size 50 the observed values SupExt tests are deviating. As on simulated critical values empirical size is not equal to nominal size which is obtained using simulated critical values. The difference between empirical size and nominal size can be seen in Table 5.1 (a.1). Also, as sample size changes from 50 to 100 or even to 200 we found size distortion in *CB-Test*, *H-Test*, *CK-Test* and *DIB-Test* as the nominal size is exceeding the empirical size in all cases. On the other hand, the amount of size distortion using IIS in both *IB-Test* and *RB-Test* is small. As on simulated critical values empirical size is approximately equal to nominal size which is obtained using simulated critical values as sample increases.

Table 5.1 : Empirical Size of SupExt Tests using Asymptotic Critical Values Under IIS

Under Non-Stationary Data Settings												
Size of Test	***H-Test***		***CK-Test***		***RB-Test***		***IB-Tests***		***DIB-Test***		***CB-Test***	
	5%	1%	5%	1%	5%	1%	5%	1%	5%	1%	5%	1%
Sample Size: 50	3.04	5.46	0.67	1.06	1.91	3.85	2.73	4.81	2.01	4.35	2.05	4.41
Sample Size: 100	1.97	2.61	0.64	1.16	2.94	4.34	3.28	5.52	3.23	5.44	3.00	4.62
Sample Size: 200	3.84	5.61	0.34	1.12	3.77	4.95	4.32	5.61	4.63	2.01	3.22	4.74

Note: Author's own calculations

Table 5.2: Empirical Size of SupExt Tests using Asymptotic Critical Values Under IIS

Under Non-Stationary Data Settings												
Size of Test	***H-Test***		***CK-Test***		***RB-Test***		***IB-Tests***		***DIB-Test***		***CB-Test***	
	1%	5%	1%	5%	1%	5%	1%	5%	1%	5%	1%	5%
Sample Size: 50	0.017	0.063	0.020	0.069	0.014	0.055	0.012	0.053	0.015	0.057	0.017	0.059
Sample Size: 100	0.017	0.061	0.019	0.065	0.013	0.054	0.012	0.053	0.014	0.056	0.017	0.057
Sample Size: 200	0.019	0.061	0.018	0.059	0.011	0.053	0.011	0.051	0.014	0.055	0.016	0.056

Note: Author's own calculations

5.1.1 Power of the Tests using IIS under Non-Stationary Data

By looking at the graphs in Figure (5.1) below one can easily observe that the power of each test reduced by a significant amount as compared with the one when data was stationary. When sample is 50 the power of *IB-Test* and *RB-Test* both at 1% as well as 5% level of significance is better than other tests under IIS. But as the sample size increase from 50 to 100 or 200 the power of *CB-Test* become more or less equivalent to *H-Test.* However, the power of *IB-Test* and *RB-Test* showed relative improvement as compare to other tests considering IIS. For the sample size of 100 the power of *IB-Test* and *RB-Test* for both 1% and 5% significance level remains almost same with some deviation. But the power of *IB-Test* is much better than that of other SupExt tests. Lastly, the performance of *CK-Test* remains at the low at both significance levels under IIS. Note that all the simulations experiments have been repeatedly done for 100,000 time.

Table 5.3: Powers under the Alternative Hypothesis at 1% Level of Significance Using Impulse Indicator Saturation (IIS)

Under Non-Stationary Settings																		
1% SL	Sample Size: 50						Sample Size: 100						Sample Size: 200					
H_1	***H-test***	***CK-test***	***RB-test***	***IB-test***	***DIB-test***	***CB-test***	***H-test***	***CK-test***	***RB-test***	***IB-test***	***DIB-test***	***CB-test***	***H-test***	***CK-test***	***RB-test***	***IB-test***	***DIB-test***	***CB-test***
0.99	0.11	0.02	0.19	0.25	0.02	0.04	0.2	0.1	0.34	0.34	0.03	0.01	0.08	0.12	0.07	0.34	0.05	0.08
0.96	0.11	0.05	0.19	0.25	0.08	0.13	0.2	0.1	0.34	0.34	0.09	0.13	0.08	0.12	0.09	0.34	0.11	0.1
0.93	0.11	0.08	0.19	0.25	0.08	0.13	0.2	0.1	0.34	0.34	0.09	0.02	0.08	0.12	0.12	0.34	0.11	0.1
0.91	0.11	0.11	0.19	0.26	0.08	0.13	0.2	0.1	0.34	0.35	0.09	0.14	0.08	0.12	0.13	0.34	0.11	0.14
0.9	0.11	0.08	0.19	0.32	0.08	0.13	0.2	0.1	0.34	0.38	0.09	0.14	0.08	0.12	0.15	0.34	0.11	0.14
0.85	0.11	0.07	0.19	0.32	0.08	0.15	0.2	0.1	0.34	0.41	0.09	0.14	0.08	0.12	0.16	0.34	0.11	0.14
0.8	0.13	0.07	0.21	0.34	0.1	0.17	0.22	0.1	0.36	0.43	0.11	0.13	0.1	0.12	0.19	0.36	0.13	0.13
0.75	0.15	0.07	0.23	0.36	0.12	0.19	0.24	0.1	0.38	0.45	0.13	0.16	0.12	0.12	0.21	0.38	0.15	0.16
0.7	0.16	0.11	0.24	0.37	0.13	0.19	0.25	0.1	0.39	0.46	0.14	0.17	0.13	0.12	0.23	0.39	0.16	0.17
0.65	0.18	0.11	0.26	0.39	0.15	0.19	0.27	0.1	0.41	0.48	0.16	0.19	0.15	0.12	0.25	0.41	0.18	0.19
0.6	0.2	0.11	0.28	0.39	0.17	0.2	0.29	0.1	0.43	0.48	0.18	0.21	0.17	0.12	0.26	0.43	0.2	0.21
0.55	0.22	0.11	0.3	0.39	0.19	0.21	0.31	0.1	0.45	0.48	0.2	0.23	0.19	0.12	0.28	0.45	0.22	0.23
0.5	0.22	0.11	0.3	0.39	0.19	0.22	0.31	0.1	0.45	0.48	0.2	0.25	0.19	0.12	0.3	0.45	0.22	0.25
0.45	0.22	0.11	0.3	0.39	0.19	0.23	0.31	0.1	0.45	0.48	0.2	0.27	0.19	0.12	0.32	0.45	0.22	0.27
0.4	0.22	0.12	0.3	0.39	0.19	0.24	0.31	0.12	0.45	0.48	0.2	0.29	0.19	0.14	0.35	0.45	0.22	0.29
0.35	0.22	0.12	0.3	0.39	0.19	0.23	0.31	0.13	0.45	0.48	0.2	0.3	0.19	0.15	0.37	0.45	0.22	0.3
0.3	0.22	0.12	0.3	0.39	0.19	0.21	0.31	0.15	0.45	0.48	0.2	0.31	0.19	0.17	0.4	0.45	0.22	0.31
0.25	0.23	0.12	0.31	0.4	0.19	0.19	0.32	0.18	0.46	0.49	0.2	0.3	0.2	0.2	0.4	0.46	0.22	0.3
0.2	0.24	0.12	0.32	0.4	0.19	0.17	0.33	0.19	0.47	0.49	0.2	0.3	0.21	0.21	0.4	0.47	0.22	0.3
0.15	0.25	0.12	0.33	0.4	0.19	0.15	0.34	0.2	0.48	0.49	0.2	0.3	0.22	0.22	0.43	0.48	0.22	0.3
0.1	0.26	0.12	0.34	0.4	0.19	0.15	0.35	0.2	0.49	0.49	0.2	0.3	0.23	0.22	0.45	0.49	0.22	0.3
0.05	0.26	0.12	0.34	0.41	0.2	0.15	0.35	0.2	0.49	0.5	0.21	0.3	0.23	0.22	0.45	0.49	0.23	0.3
0.01	0.26	0.12	0.34	0.41	0.21	0.15	0.35	0.2	0.49	0.5	0.22	0.3	0.23	0.22	0.45	0.49	0.24	0.3

Note: Author's own calculations

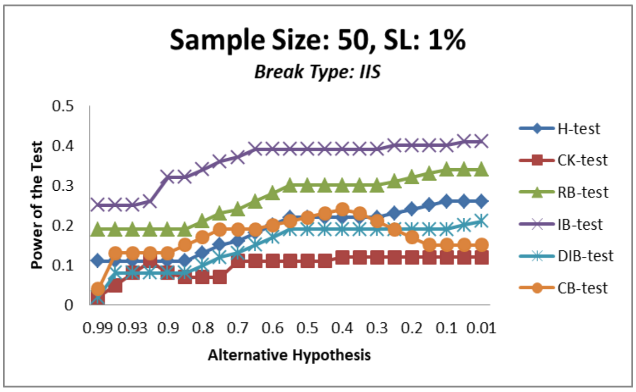
Table 5.4: Powers under the Alternative Hypothesis at 5% Level of Significance Using Impulse Indicator Saturation (IIS)

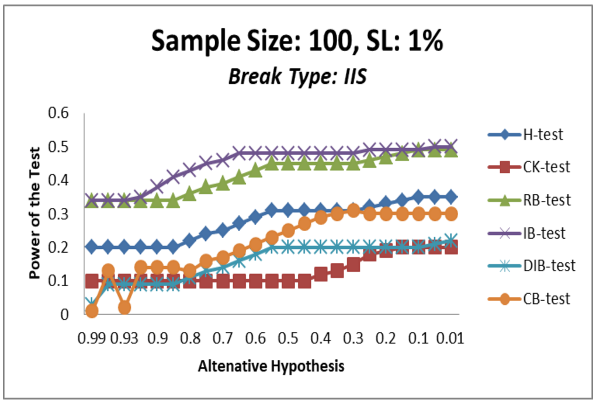
Under Non-Stationary Data Settings																		
5% SL	Sample Size: 50						Sample Size: 100						Sample Size: 200					
H_1	***H-test***	***CK-test***	***RB-test***	***IB-test***	***DIB-test***	***CB-test***	***H-test***	***CK-test***	***RB-test***	***IB-test***	***DIB-test***	***CB-test***	***H-test***	***CK-test***	***RB-test***	***IB-test***	***DIB-test***	***CB-test***
0.99	0.14	0.05	0.22	0.28	0.05	0.07	0.15	0.15	0.39	0.39	0.08	0.06	0.08	0.12	0.07	0.14	0.05	0.08
0.96	0.14	0.08	0.22	0.29	0.11	0.16	0.25	0.15	0.39	0.39	0.14	0.06	0.08	0.12	0.09	0.15	0.11	0.10
0.93	0.14	0.11	0.22	0.32	0.11	0.16	0.25	0.15	0.39	0.39	0.14	0.07	0.08	0.12	0.12	0.15	0.11	0.10
0.91	0.14	0.11	0.22	0.33	0.11	0.16	0.25	0.15	0.39	0.4	0.14	0.13	0.08	0.12	0.13	0.15	0.11	0.14
0.9	0.14	0.11	0.22	0.35	0.11	0.16	0.25	0.15	0.39	0.43	0.14	0.14	0.08	0.12	0.15	0.17	0.11	0.14
0.85	0.14	0.11	0.22	0.35	0.11	0.18	0.25	0.15	0.39	0.46	0.14	0.15	0.08	0.12	0.16	0.21	0.11	0.14
0.8	0.16	0.11	0.24	0.37	0.13	0.2	0.27	0.15	0.41	0.48	0.16	0.18	0.10	0.12	0.19	0.24	0.13	0.13
0.75	0.18	0.11	0.26	0.39	0.15	0.22	0.29	0.15	0.43	0.5	0.18	0.21	0.12	0.12	0.21	0.28	0.15	0.16
0.7	0.19	0.11	0.27	0.4	0.16	0.22	0.3	0.15	0.44	0.51	0.19	0.22	0.13	0.12	0.23	0.31	0.16	0.17
0.65	0.21	0.11	0.29	0.42	0.18	0.22	0.32	0.15	0.46	0.53	0.21	0.24	0.15	0.12	0.25	0.33	0.18	0.19
0.6	0.23	0.11	0.31	0.42	0.2	0.23	0.34	0.15	0.48	0.53	0.23	0.26	0.17	0.12	0.26	0.36	0.20	0.21
0.55	0.25	0.11	0.33	0.42	0.22	0.24	0.36	0.15	0.5	0.53	0.25	0.28	0.19	0.12	0.28	0.38	0.22	0.23
0.5	0.25	0.11	0.33	0.42	0.22	0.25	0.36	0.15	0.5	0.53	0.25	0.3	0.19	0.12	0.30	0.43	0.22	0.25
0.45	0.25	0.11	0.33	0.42	0.22	0.26	0.36	0.15	0.5	0.53	0.25	0.32	0.19	0.12	0.32	0.46	0.22	0.27
0.4	0.25	0.11	0.33	0.42	0.22	0.27	0.36	0.17	0.5	0.53	0.25	0.34	0.19	0.14	0.35	0.46	0.22	0.29
0.35	0.25	0.12	0.33	0.42	0.22	0.26	0.36	0.18	0.5	0.53	0.25	0.35	0.19	0.15	0.37	0.46	0.22	0.30
0.3	0.25	0.13	0.33	0.42	0.22	0.24	0.36	0.2	0.5	0.53	0.25	0.35	0.19	0.17	0.40	0.46	0.22	0.31
0.25	0.26	0.14	0.34	0.43	0.22	0.22	0.37	0.23	0.51	0.54	0.25	0.35	0.20	0.20	0.40	0.47	0.22	0.30
0.2	0.27	0.14	0.35	0.43	0.22	0.2	0.38	0.24	0.52	0.54	0.25	0.35	0.21	0.21	0.40	0.48	0.22	0.30
0.15	0.28	0.14	0.36	0.43	0.22	0.2	0.39	0.25	0.53	0.54	0.25	0.35	0.22	0.22	0.43	0.49	0.22	0.30
0.1	0.29	0.14	0.37	0.43	0.22	0.2	0.4	0.25	0.54	0.54	0.25	0.35	0.23	0.22	0.45	0.50	0.22	0.30
0.05	0.29	0.14	0.37	0.44	0.23	0.2	0.4	0.25	0.54	0.55	0.26	0.35	0.23	0.22	0.45	0.50	0.23	0.30
0.01	0.29	0.14	0.37	0.44	0.24	0.2	0.4	0.25	0.54	0.55	0.27	0.35	0.23	0.22	0.45	0.50	0.24	0.30

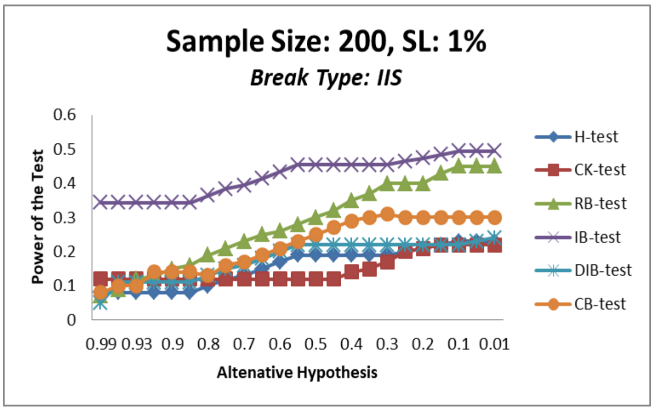
Note: Author's own calculations

Figure 5.1: Performance Under Impulse Indicator Saturation (IIS)

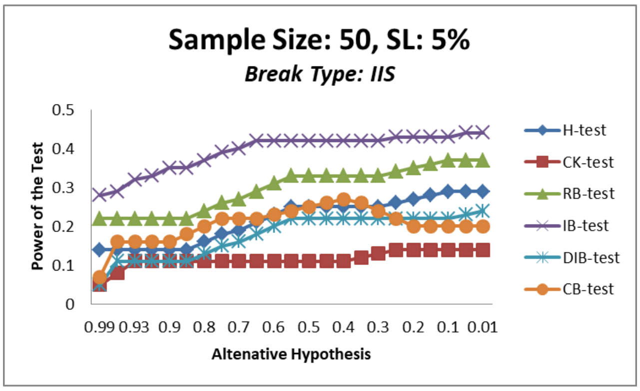
Performance Under 1% Level of Significance

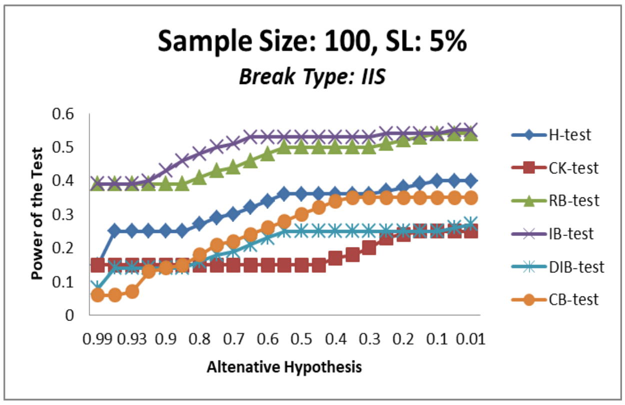


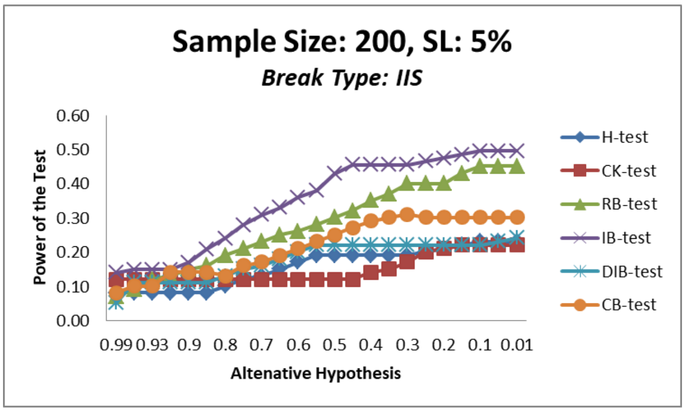




Performance Under 5% Level of Significance







Note: Author’s Own Estimations

5.2 Performance using SIS under Non-Stationary Data

The following Table 5.5 envelops the simulated CV's of the SupExt tests when SIS is being taken into account under non-stationary data. Again taking non-stationary data setting the SupExt tests performs not as much well as were under the stationary data settings. In case of non-stationarity we face size distortions almost in each test. However, this size distortion is relatively less in *RB-Test* and in *IB-Test* as compared to other SupExt tests. For the sample size 50 the observed values SupExt tests are deviating. As on simulated critical values empirical size is not equal to nominal size which is obtained using simulated critical values. The difference between empirical size and nominal size can be seen in Table 5.5. Also, as sample size changes from 50 to 100 or even to 200 we found size distortion in *CB-Test*, *H-Test*, *CK-Test* and *DIB-Test* as the nominal size is exceeding the empirical size in all cases. On the other hand, the amount of size distortion using SIS in both *IB-Test* and *RB-Test* is small. As on simulated critical values empirical size is approximately equal to nominal size which is obtained using simulated critical values as sample increases. On this ground we can say that both *IB-Test* and *RB-Test* performs better than that of other SupExt tests.

Table 5.5: Simulated Critical Values of SupExt Tests Under SIS

Under Non-Stationary Data Settings												
Size of Test	***H-Test***		***CK-Test***		***RB-Test***		***IB-Tests***		***DIB-Test***		***CB-Test***	
	5%	1%	5%	1%	5%	1%	5%	1%	5%	1%	5%	1%
Sample Size: 50	2.54	3.34	1.52	2.12	3.16	3.33	3.4	3.89	2.49	3.14	3.17	3.35
Sample Size: 100	3.71	4.59	1.74	2.69	3.41	4.52	4.46	4.85	3.53	4.12	3.4	3.99
Sample Size: 200	3.79	4.65	1.72	2.62	3.89	4.71	4.01	4.79	2.7	3.42	2.97	3.55

Note: Author's own calculations

Table 5.6: Empirical Size of SupExt Tests using Asymptotic Critical Values Under SIS

Under Non-Stationary Data Settings												
Size of Test	***H-Test***		***CK-Test***		***RB-Test***		***IB-Tests***		***DIB-Test***		***CB-Test***	
	1%	5%	1%	5%	1%	5%	1%	5%	1%	5%	1%	5%
Sample Size: 50	0.020	0.061	0.023	0.069	0.014	0.055	0.012	0.054	0.016	0.057	0.018	0.059
Sample Size: 100	0.019	0.061	0.023	0.065	0.013	0.054	0.012	0.053	0.015	0.056	0.017	0.055
Sample Size: 200	0.020	0.062	0.019	0.057	0.012	0.054	0.011	0.051	0.015	0.055	0.017	0.055

Note: Author's own calculations

5.2.1 Power of the Tests using SIS under Non-Stationary Data

By looking at the graphs in Figure (5.2) below one can easily observe that the power of each test reduced by a significant amount as compared with the one when data was stationary. When sample is 50 the power of *IB-Test* and *RB-Test* both at 1% as well as 5% level of significance is better than other tests under SIS. But as the sample size increase from 50 to 100 or 200 the power of *CB-Test* become more or less equivalent to *H-Test.* The trend is same as was when we consider IIS. However, the power of *IB-Test* and *RB-Test* showed relative improvement as compare to other tests considering SIS. For the sample size of 100 the power of *IB-Test* and *RB-Test* for both 1% and 5% significance level remains almost same with some deviation. But the power of *IB-Test* is much better than that of other SupExt tests. Lastly, the performance of *CK-Test* shows a significant improvement when data is non-stationary though remains at the end and below *DIB-test.*

Table 5.7: Powers under the Alternative Hypothesis at 1% Level of Significance Using Step Indicator Saturation (SIS)

Under Non-Stationary Data Settings																		
1% SL	Sample Size: 50						Sample Size: 100						Sample Size: 200					
H_1	***H-test***	***CK-test***	***RB-test***	***IB-test***	***DIB-test***	***CB-test***	***H-test***	***CK-test***	***RB-test***	***IB-test***	***DIB-test***	***CB-test***	***H-test***	***CK-test***	***RB-test***	***IB-test***	***DIB-test***	***CB-test***
0.99	0.06	0.03	0.14	0.2	0.02	0.05	0.11	0.08	0.19	0.25	0.07	0.1	0.17	0.14	0.25	0.31	0.13	0.16
0.96	0.06	0.03	0.14	0.2	0.03	0.08	0.11	0.08	0.19	0.25	0.08	0.13	0.17	0.14	0.25	0.31	0.14	0.19
0.93	0.06	0.03	0.14	0.2	0.03	0.08	0.11	0.08	0.19	0.25	0.08	0.13	0.17	0.14	0.25	0.31	0.14	0.19
0.91	0.06	0.03	0.14	0.21	0.03	0.08	0.11	0.08	0.19	0.26	0.08	0.13	0.17	0.14	0.25	0.32	0.14	0.19
0.9	0.06	0.03	0.14	0.27	0.03	0.08	0.11	0.08	0.19	0.32	0.08	0.13	0.17	0.14	0.25	0.38	0.14	0.19
0.85	0.06	0.05	0.14	0.27	0.03	0.1	0.11	0.1	0.19	0.32	0.08	0.15	0.17	0.16	0.25	0.38	0.14	0.21
0.8	0.08	0.05	0.16	0.29	0.05	0.12	0.13	0.1	0.21	0.34	0.1	0.17	0.19	0.16	0.27	0.4	0.16	0.23
0.75	0.1	0.05	0.18	0.31	0.07	0.14	0.15	0.1	0.23	0.36	0.12	0.19	0.21	0.16	0.29	0.42	0.18	0.25
0.7	0.11	0.06	0.19	0.32	0.08	0.14	0.16	0.11	0.24	0.37	0.13	0.19	0.22	0.17	0.3	0.43	0.19	0.25
0.65	0.13	0.06	0.21	0.34	0.1	0.14	0.18	0.11	0.26	0.39	0.15	0.19	0.24	0.17	0.32	0.45	0.21	0.25
0.6	0.15	0.06	0.23	0.34	0.12	0.15	0.2	0.11	0.28	0.39	0.17	0.2	0.26	0.17	0.34	0.45	0.23	0.26
0.55	0.17	0.06	0.25	0.34	0.14	0.16	0.22	0.11	0.3	0.39	0.19	0.21	0.28	0.18	0.36	0.45	0.25	0.27
0.5	0.17	0.06	0.25	0.34	0.14	0.17	0.22	0.11	0.3	0.39	0.19	0.22	0.28	0.18	0.36	0.45	0.25	0.28
0.45	0.17	0.06	0.25	0.34	0.14	0.18	0.22	0.11	0.3	0.39	0.19	0.23	0.28	0.2	0.36	0.45	0.25	0.29
0.4	0.17	0.07	0.25	0.34	0.14	0.19	0.22	0.12	0.3	0.39	0.19	0.24	0.28	0.23	0.36	0.45	0.25	0.3
0.35	0.17	0.07	0.25	0.34	0.14	0.18	0.22	0.12	0.3	0.39	0.19	0.24	0.28	0.22	0.36	0.45	0.25	0.3
0.3	0.17	0.07	0.25	0.34	0.14	0.16	0.22	0.12	0.3	0.39	0.19	0.24	0.28	0.22	0.36	0.45	0.25	0.3
0.25	0.18	0.07	0.26	0.35	0.14	0.14	0.23	0.12	0.31	0.4	0.19	0.25	0.29	0.22	0.37	0.46	0.25	0.31
0.2	0.19	0.07	0.27	0.35	0.14	0.12	0.24	0.12	0.32	0.4	0.19	0.25	0.3	0.21	0.38	0.46	0.25	0.31
0.15	0.2	0.07	0.28	0.35	0.14	0.1	0.25	0.12	0.33	0.4	0.19	0.26	0.31	0.21	0.39	0.46	0.25	0.32
0.1	0.21	0.07	0.29	0.35	0.14	0.1	0.26	0.12	0.34	0.4	0.19	0.26	0.32	0.21	0.4	0.46	0.25	0.32
0.05	0.21	0.07	0.29	0.36	0.15	0.1	0.26	0.12	0.34	0.41	0.2	0.27	0.32	0.21	0.4	0.47	0.26	0.33
0.01	0.24	0.1	0.32	0.38	0.16	0.12	0.29	0.15	0.37	0.43	0.21	0.27	0.35	0.21	0.43	0.49	0.27	0.33

Note: Author's own calculations

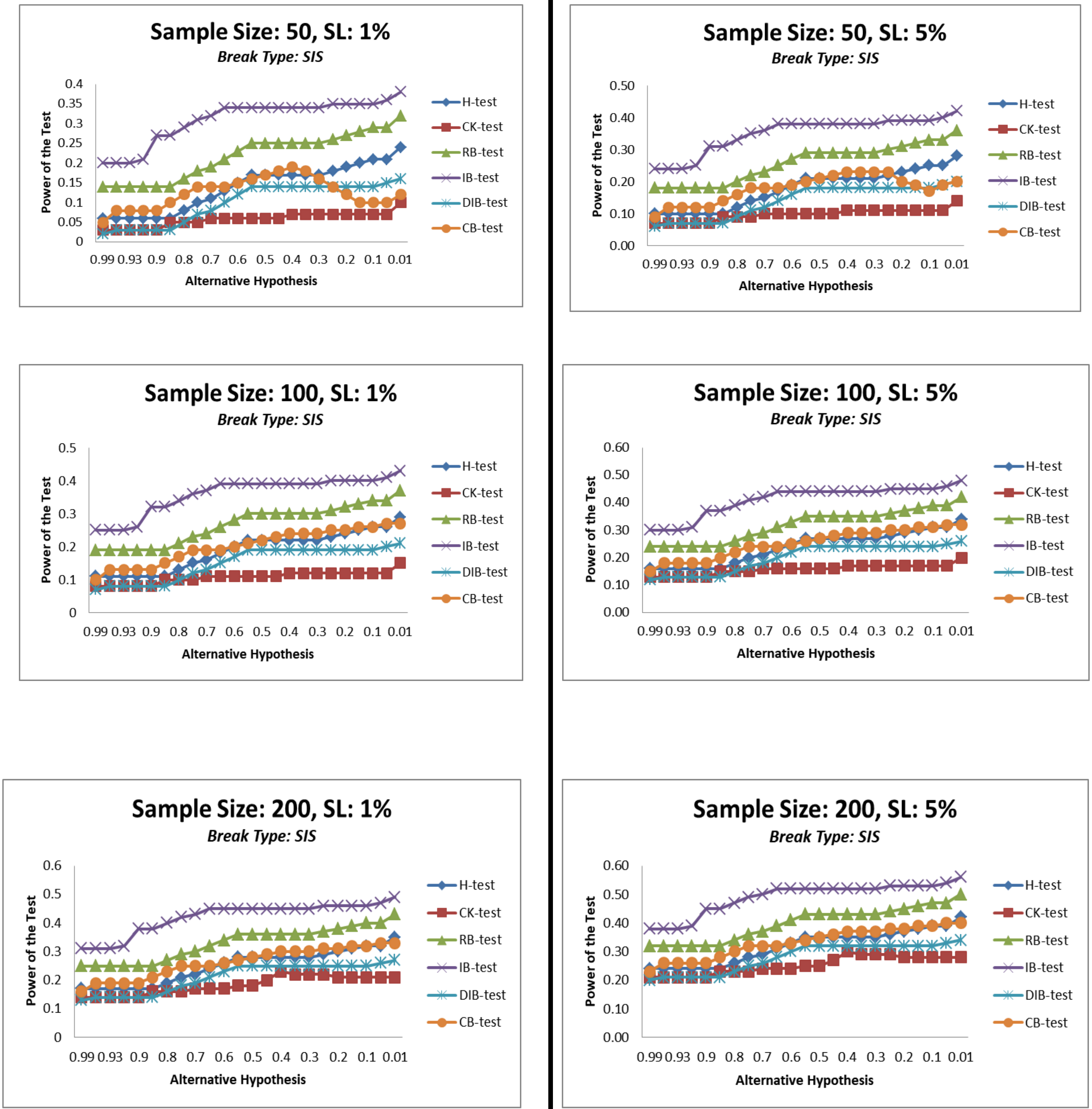
Table 5.8: Powers under the Alternative Hypothesis at 5% Level of Significance Using Step Indicator Saturation (SIS)

Under Non-Stationary Data Settings																		
5% SL	Sample Size: 50						Sample Size: 100						Sample Size: 200					
H_1	***H-test***	***CK-test***	***RB-test***	***IB-test***	***DIB-test***	***CB-test***	***H-test***	***CK-test***	***RB-test***	***IB-test***	***DIB-test***	***CB-test***	***H-test***	***CK-test***	***RB-test***	***IB-test***	***DIB-test***	***CB-test***
0.99	0.10	0.07	0.18	0.24	0.06	0.09	0.16	0.13	0.24	0.30	0.12	0.15	0.24	0.21	0.32	0.38	0.20	0.23
0.96	0.10	0.07	0.18	0.24	0.07	0.12	0.16	0.13	0.24	0.30	0.13	0.18	0.24	0.21	0.32	0.38	0.21	0.26
0.93	0.10	0.07	0.18	0.24	0.07	0.12	0.16	0.13	0.24	0.30	0.13	0.18	0.24	0.21	0.32	0.38	0.21	0.26
0.91	0.10	0.07	0.18	0.25	0.07	0.12	0.16	0.13	0.24	0.31	0.13	0.18	0.24	0.21	0.32	0.39	0.21	0.26
0.9	0.10	0.07	0.18	0.31	0.07	0.12	0.16	0.13	0.24	0.37	0.13	0.18	0.24	0.21	0.32	0.45	0.21	0.26
0.85	0.10	0.09	0.18	0.31	0.07	0.14	0.16	0.15	0.24	0.37	0.13	0.20	0.24	0.23	0.32	0.45	0.21	0.28
0.8	0.12	0.09	0.20	0.33	0.09	0.16	0.18	0.15	0.26	0.39	0.15	0.22	0.26	0.23	0.34	0.47	0.23	0.30
0.75	0.14	0.09	0.22	0.35	0.11	0.18	0.20	0.15	0.28	0.41	0.17	0.24	0.28	0.23	0.36	0.49	0.25	0.32
0.7	0.15	0.10	0.23	0.36	0.12	0.18	0.21	0.16	0.29	0.42	0.18	0.24	0.29	0.24	0.37	0.50	0.26	0.32
0.65	0.17	0.10	0.25	0.38	0.14	0.18	0.23	0.16	0.31	0.44	0.20	0.24	0.31	0.24	0.39	0.52	0.28	0.32
0.6	0.19	0.10	0.27	0.38	0.16	0.19	0.25	0.16	0.33	0.44	0.22	0.25	0.33	0.24	0.41	0.52	0.30	0.33
0.55	0.21	0.10	0.29	0.38	0.18	0.20	0.27	0.16	0.35	0.44	0.24	0.26	0.35	0.25	0.43	0.52	0.32	0.34
0.5	0.21	0.10	0.29	0.38	0.18	0.21	0.27	0.16	0.35	0.44	0.24	0.27	0.35	0.25	0.43	0.52	0.32	0.35
0.45	0.21	0.10	0.29	0.38	0.18	0.22	0.27	0.16	0.35	0.44	0.24	0.28	0.35	0.27	0.43	0.52	0.32	0.36
0.4	0.21	0.11	0.29	0.38	0.18	0.23	0.27	0.17	0.35	0.44	0.24	0.29	0.35	0.30	0.43	0.52	0.32	0.37
0.35	0.21	0.11	0.29	0.38	0.18	0.23	0.27	0.17	0.35	0.44	0.24	0.29	0.35	0.29	0.43	0.52	0.32	0.37
0.3	0.21	0.11	0.29	0.38	0.18	0.23	0.27	0.17	0.35	0.44	0.24	0.29	0.35	0.29	0.43	0.52	0.32	0.37
0.25	0.22	0.11	0.30	0.39	0.18	0.23	0.28	0.17	0.36	0.45	0.24	0.30	0.36	0.29	0.44	0.53	0.32	0.38
0.2	0.23	0.11	0.31	0.39	0.18	0.20	0.29	0.17	0.37	0.45	0.24	0.30	0.37	0.28	0.45	0.53	0.32	0.38
0.15	0.24	0.11	0.32	0.39	0.18	0.19	0.30	0.17	0.38	0.45	0.24	0.31	0.38	0.28	0.46	0.53	0.32	0.39
0.1	0.25	0.11	0.33	0.39	0.18	0.17	0.31	0.17	0.39	0.45	0.24	0.31	0.39	0.28	0.47	0.53	0.32	0.39
0.05	0.25	0.11	0.33	0.40	0.19	0.19	0.31	0.17	0.39	0.46	0.25	0.32	0.39	0.28	0.47	0.54	0.33	0.40
0.01	0.28	0.14	0.36	0.42	0.20	0.20	0.34	0.20	0.42	0.48	0.26	0.32	0.42	0.28	0.50	0.56	0.34	0.40

Note: Author's own calculations

Figure 5.2: Performance Under Step Indicator Saturation (SIS)

Performance Under 1% Level of Significance	Performance Under 5% Level of Significance



Note: Author's Own Estimations

5.3 Performance using TIS under Non-Stationary Data

The following Table 5.9 envelops the simulated CV's of the SupExt tests when TIS is being taken into account under non-stationary data. Again taking non-stationary data setting the SupExt tests performs not as much well as were under the stationary data settings. In case of non-stationarity we face size distortions almost in each test. However, this size distortion is relatively less in *RB-Test* and in *IB-Test* as compared to other SupExt tests. For the sample size 50 the observed values SupExt tests are deviating. As on simulated critical values empirical size is not equal to nominal size which is obtained using simulated critical values. The difference between empirical size and nominal size can be seen in Table 5.9. Also, as sample size changes from 50 to 100 or even to 200 we found size distortion in *CB-Test*, *H-Test* and *DIB-Test* as the nominal size is exceeding the empirical size in all cases. While *CK-Test* shows a less size distortion as compared with IIS and SIS and the power is almost equal to *DIB-Test.* On the other hand, the amount of size distortion using TIS in both *IB-Test* and *RB-Test* is small. As on simulated critical values empirical size is approximately equal to nominal size which is obtained using simulated critical values as sample increases. On this ground we can say that both *IB-Test* and *RB-Test* performs better than that of other SupExt tests.

Table 5.9: Simulated Critical Values of SupExt Tests Under TIS

Under Non-Stationary Data Settings												
Size of Test	***H-Test***		***CK-Test***		***RB-Test***		***IB-Tests***		***DIB-Test***		***CB-Test***	
	5%	1%	5%	1%	5%	1%	5%	1%	5%	1%	5%	1%
Sample Size: 50	3.24	3.56	1.93	2.38	2.75	3.42	3.58	3.4	2.12	3.16	3.56	5.09
Sample Size: 100	3.21	4.4	1.72	1.91	3.32	4.42	3.94	4.19	4.09	4.17	2.91	3.06
Sample Size: 200	3.14	4.02	1.74	1.93	3.66	4.42	4.44	4.62	2.66	3.1	3.58	3.00

Note: Author's own calculations

Table 5.10: Empirical Size of SupExt Tests using Asymptotic Critical Values Under TIS

Under Non-Stationary Data Settings												
Size of Test	***H-Test***		***CK-Test***		***RB-Test***		***IB-Tests***		***DIB-Test***		***CB-Test***	
	1%	5%	1%	5%	1%	5%	1%	5%	1%	5%	1%	5%
Sample Size: 50	0.019	0.059	0.018	0.055	0.014	0.052	0.014	0.053	0.015	0.057	0.018	0.059
Sample Size: 100	0.019	0.059	0.017	0.054	0.013	0.052	0.012	0.053	0.011	0.056	0.017	0.055
Sample Size: 200	0.020	0.060	0.018	0.054	0.012	0.053	0.011	0.051	0.014	0.055	0.017	0.055

Note: Author's own calculations

5.3.1 Power of the Tests using TIS under Non-Stationary Data

By looking at the graphs in Figure (5.3) below one can easily observe that the power of each test when data in non-stationary is reduced by a significant amount as compared with the one when data was stationary. When sample is 50 the power of *IB-Test* and *RB-Test* both at 1% as well as 5% level of significance is better than other tests under TIS. For the sample of 100 and 5% level of significance the power curves of *CB-Test* and *DIB-Test* becomes almost same. For the sample size of 200 the power of *CB-Test* becomes more or less equivalent to *H-Test*. However, the power of *IB-Test* and *RB-Test* showed relative improvement as compare to other tests considering SIS. For the sample size of 100 the power of *IB-Test* and *RB-Test* for both 1% and 5% significance level remains almost same with some deviation. But the power of *IB-Test* is much better than that of other SupExt tests. Lastly, the performance of *CK-Test* shows a significant improvement when data is non-stationary.

Table 5.11: Powers under the Alternative Hypothesis at 1% Level of Significance Using Trend Indicator Saturation (TIS)

Under Non-Stationary Data Settings																		
1% SL	Sample Size: 50						Sample Size: 100						Sample Size: 200					
H_1	***H-test***	***CK-test***	***RB-test***	***IB-test***	***DIB-test***	***CB-test***	***H-test***	***CK-test***	***RB-test***	***IB-test***	***DIB-test***	***CB-test***	***H-test***	***CK-test***	***RB-test***	***IB-test***	***DIB-test***	***CB-test***
0.99	0.03	0.02	0.09	0.15	0.04	0.03	0.06	0.08	0.14	0.25	0.07	0.04	0.17	0.14	0.25	0.31	0.13	0.16
0.96	0.03	0.02	0.09	0.16	0.04	0.03	0.06	0.08	0.14	0.27	0.08	0.04	0.17	0.14	0.25	0.31	0.14	0.19
0.93	0.03	0.02	0.09	0.18	0.04	0.03	0.06	0.08	0.14	0.28	0.08	0.04	0.17	0.14	0.25	0.31	0.14	0.19
0.91	0.03	0.02	0.10	0.20	0.04	0.03	0.06	0.08	0.15	0.30	0.08	0.04	0.17	0.14	0.25	0.32	0.14	0.19
0.9	0.03	0.02	0.10	0.22	0.04	0.03	0.06	0.08	0.18	0.32	0.08	0.04	0.17	0.14	0.25	0.38	0.14	0.19
0.85	0.03	0.03	0.10	0.22	0.07	0.05	0.06	0.10	0.19	0.32	0.08	0.06	0.17	0.16	0.25	0.38	0.14	0.21
0.8	0.03	0.03	0.15	0.24	0.07	0.07	0.06	0.10	0.20	0.34	0.10	0.08	0.19	0.16	0.27	0.40	0.16	0.23
0.75	0.05	0.05	0.16	0.26	0.07	0.09	0.08	0.10	0.21	0.36	0.12	0.10	0.21	0.16	0.29	0.42	0.18	0.25
0.7	0.06	0.05	0.18	0.27	0.07	0.09	0.09	0.11	0.23	0.37	0.13	0.10	0.22	0.17	0.30	0.43	0.19	0.25
0.65	0.08	0.06	0.18	0.29	0.07	0.09	0.11	0.11	0.23	0.39	0.15	0.10	0.24	0.17	0.32	0.45	0.21	0.25
0.6	0.10	0.08	0.18	0.29	0.07	0.10	0.13	0.11	0.23	0.39	0.17	0.11	0.26	0.17	0.34	0.45	0.23	0.26
0.55	0.12	0.08	0.20	0.29	0.09	0.11	0.15	0.11	0.25	0.39	0.19	0.12	0.28	0.18	0.36	0.45	0.25	0.27
0.5	0.12	0.08	0.20	0.29	0.09	0.12	0.15	0.11	0.25	0.39	0.19	0.13	0.28	0.18	0.37	0.45	0.25	0.28
0.45	0.12	0.09	0.20	0.29	0.09	0.13	0.15	0.11	0.25	0.39	0.19	0.14	0.28	0.20	0.39	0.45	0.25	0.29
0.4	0.12	0.09	0.20	0.29	0.09	0.14	0.15	0.12	0.25	0.39	0.19	0.15	0.28	0.23	0.39	0.45	0.25	0.30
0.35	0.12	0.09	0.20	0.29	0.09	0.13	0.15	0.12	0.25	0.39	0.19	0.14	0.28	0.22	0.42	0.45	0.25	0.30
0.3	0.12	0.09	0.20	0.29	0.09	0.11	0.15	0.12	0.25	0.39	0.19	0.12	0.28	0.22	0.45	0.45	0.25	0.30
0.25	0.13	0.09	0.21	0.30	0.09	0.11	0.16	0.12	0.26	0.40	0.19	0.12	0.29	0.22	0.47	0.46	0.25	0.31
0.2	0.14	0.09	0.22	0.30	0.09	0.11	0.17	0.12	0.27	0.40	0.19	0.12	0.30	0.21	0.47	0.46	0.25	0.31
0.15	0.15	0.10	0.23	0.30	0.09	0.12	0.18	0.12	0.28	0.40	0.19	0.13	0.31	0.21	0.49	0.48	0.25	0.32
0.1	0.16	0.10	0.24	0.30	0.09	0.12	0.19	0.12	0.29	0.40	0.19	0.13	0.32	0.21	0.52	0.51	0.25	0.32
0.05	0.16	0.10	0.24	0.31	0.10	0.13	0.19	0.12	0.29	0.41	0.20	0.14	0.32	0.21	0.52	0.52	0.26	0.33
0.01	0.19	0.10	0.27	0.33	0.11	0.14	0.22	0.15	0.32	0.43	0.21	0.15	0.35	0.21	0.52	0.53	0.27	0.33

Note: Author’s own calculations

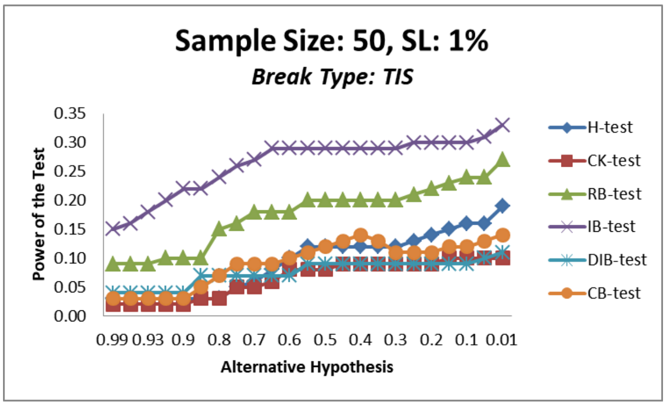
Table 5.12: Powers under the Alternative Hypothesis at 5% Level of Significance Using Trend Indicator Saturation (TIS)

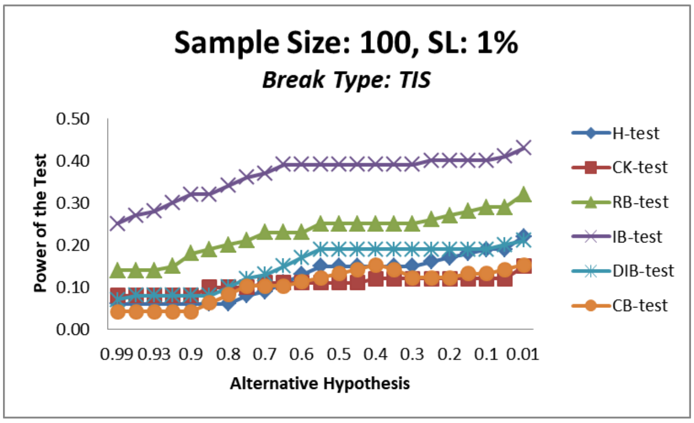
Under Non-Stationary Data Settings																		
5% SL	Sample Size: 50						Sample Size: 100						Sample Size: 200					
H_1	***H-test***	***CK-test***	***RB-test***	***IB-test***	***DIB-test***	***CB-test***	***H-test***	***CK-test***	***RB-test***	***IB-test***	***DIB-test***	***CB-test***	***H-test***	***CK-test***	***RB-test***	***IB-test***	***DIB-test***	***CB-test***
0.99	0.10	0.07	0.18	0.24	0.06	0.09	0.14	0.11	0.22	0.28	0.10	0.07	0.24	0.21	0.30	0.38	0.20	0.23
0.96	0.13	0.07	0.18	0.24	0.07	0.12	0.17	0.11	0.22	0.28	0.11	0.10	0.24	0.21	0.30	0.38	0.21	0.26
0.93	0.14	0.07	0.20	0.26	0.07	0.12	0.18	0.11	0.24	0.30	0.11	0.10	0.24	0.21	0.31	0.38	0.21	0.26
0.91	0.14	0.07	0.21	0.26	0.07	0.12	0.18	0.11	0.25	0.30	0.11	0.10	0.24	0.21	0.33	0.39	0.21	0.26
0.9	0.15	0.07	0.22	0.29	0.07	0.12	0.19	0.11	0.26	0.33	0.11	0.10	0.24	0.21	0.34	0.41	0.21	0.26
0.85	0.15	0.09	0.22	0.31	0.07	0.14	0.19	0.13	0.26	0.35	0.11	0.12	0.24	0.23	0.34	0.45	0.21	0.28
0.8	0.15	0.09	0.22	0.33	0.09	0.16	0.19	0.13	0.26	0.37	0.13	0.14	0.26	0.23	0.35	0.47	0.23	0.30
0.75	0.17	0.09	0.22	0.35	0.11	0.18	0.21	0.13	0.26	0.39	0.15	0.16	0.28	0.23	0.36	0.49	0.25	0.32
0.7	0.17	0.10	0.23	0.36	0.12	0.18	0.21	0.14	0.27	0.40	0.16	0.16	0.29	0.24	0.38	0.50	0.26	0.32
0.65	0.18	0.10	0.25	0.38	0.14	0.18	0.22	0.14	0.29	0.42	0.18	0.16	0.31	0.24	0.39	0.52	0.28	0.32
0.6	0.23	0.10	0.27	0.38	0.16	0.19	0.27	0.14	0.31	0.42	0.20	0.17	0.33	0.24	0.43	0.52	0.30	0.33
0.55	0.23	0.10	0.29	0.38	0.18	0.20	0.27	0.14	0.33	0.42	0.22	0.18	0.35	0.25	0.44	0.52	0.32	0.34
0.5	0.24	0.10	0.29	0.38	0.18	0.21	0.28	0.14	0.36	0.42	0.22	0.19	0.35	0.25	0.47	0.52	0.32	0.35
0.45	0.24	0.10	0.29	0.38	0.18	0.22	0.28	0.14	0.36	0.42	0.22	0.20	0.35	0.27	0.48	0.52	0.32	0.36
0.4	0.24	0.11	0.29	0.38	0.18	0.23	0.28	0.15	0.38	0.42	0.22	0.21	0.35	0.30	0.48	0.52	0.32	0.37
0.35	0.24	0.11	0.29	0.38	0.18	0.23	0.28	0.15	0.39	0.42	0.22	0.21	0.35	0.29	0.48	0.52	0.32	0.37
0.3	0.24	0.11	0.29	0.38	0.18	0.23	0.28	0.15	0.39	0.42	0.22	0.21	0.35	0.29	0.48	0.52	0.32	0.37
0.25	0.24	0.11	0.30	0.39	0.18	0.23	0.28	0.15	0.39	0.43	0.22	0.21	0.36	0.29	0.49	0.53	0.32	0.38
0.2	0.25	0.11	0.31	0.39	0.18	0.24	0.29	0.17	0.43	0.43	0.22	0.22	0.37	0.28	0.51	0.53	0.32	0.38
0.15	0.27	0.11	0.32	0.39	0.18	0.24	0.31	0.17	0.44	0.43	0.22	0.22	0.38	0.28	0.52	0.53	0.32	0.39
0.1	0.28	0.11	0.33	0.39	0.18	0.25	0.32	0.18	0.45	0.45	0.22	0.23	0.39	0.28	0.52	0.53	0.32	0.39
0.05	0.28	0.11	0.33	0.40	0.19	0.25	0.32	0.18	0.46	0.46	0.23	0.23	0.39	0.28	0.53	0.54	0.33	0.40
0.01	0.29	0.14	0.36	0.42	0.20	0.25	0.33	0.18	0.46	0.48	0.24	0.23	0.42	0.28	0.55	0.56	0.34	0.40

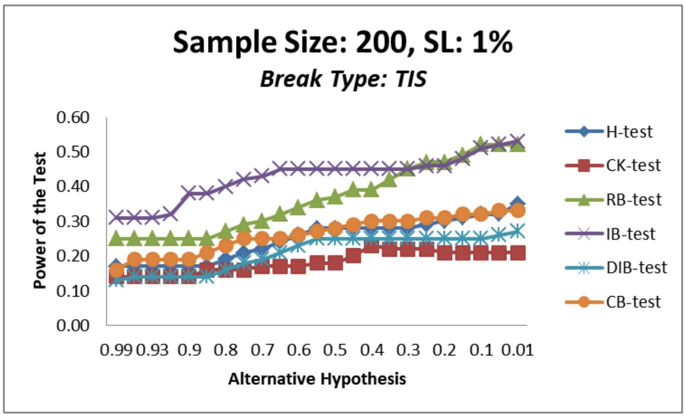
Note: Author's own calculations

Figure 5.3: Performance Under Trend Indicator Saturation (TIS)

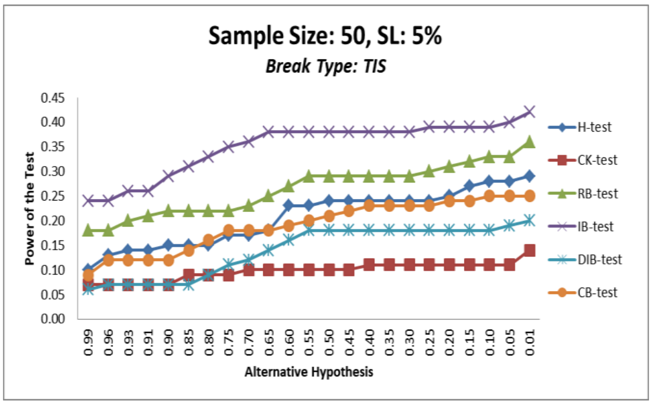
Performance Under 1% Level of Significance

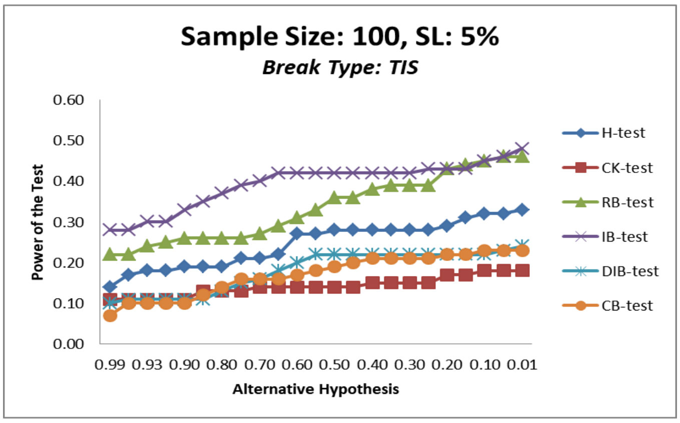


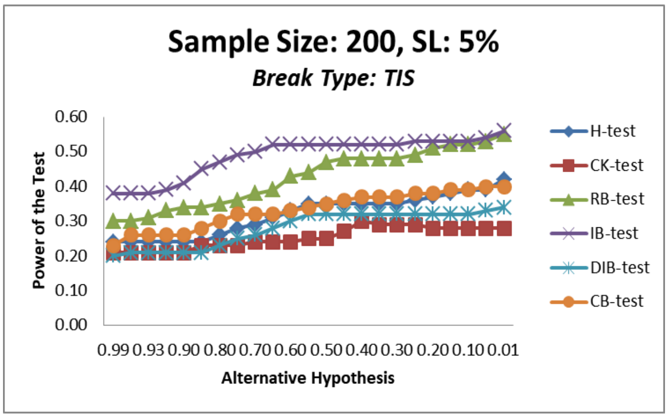




Performance Under 5% Level of Significance







Note: Author's Own Estimations

5.4 Performance using IIS, SIS & TIS under Non-Stationary Data

The following Table 5.13 envelops the simulated CV's of the SupExt tests when TIS is being taken into account under non-stationary data. Again taking non-stationary data setting the SupExt tests performs not as much well as were under the stationary data settings. In case of non-stationarity and using IIS, SIS and TIS at a time; we face size distortions. However, in this case there is no size distortion in *RB-Test* and in *IB-Test* as compared to other SupExt tests. Also for the sample size 50 the observed values of *CB-Test* are not deviating. As on simulated critical values empirical size is equal to nominal size which is obtained using simulated critical values. However, as sample increases we face size distortion in case of *CB-Test*. The difference between empirical size and nominal size can be seen in Table 5.13. Also, we found that there is some size distortion in *H-Test*, *CK-Test* and *DIB-Test* as the nominal size is exceeding the empirical size. But when sample is 200 the amount of size distortion is low in *H-Test* and *CB-Test.* On the other hand, the amount of size distortion using IIS, SIS & TIS in both *IB-Test* and *RB-Test* is small. As on simulated critical values empirical size is approximately equal to nominal size which is obtained using simulated critical values as sample increases. On this ground we can say that both *IB-Test* and *RB-Test* performs better than that of other SupExt tests.

Table 5.13: Simulated Critical Values of SupExt Tests Under IIS, SIS & TIS

Under Non-Stationary Data Settings												
Size of Test	***H-Test***		***CK-Test***		***RB-Test***		***IB-Tests***		***DIB-Test***		***CB-Test***	
	5%	1%	5%	1%	5%	1%	5%	1%	5%	1%	5%	1%
Sample Size: 50	2.87	3.2	1.94	2.08	4.74	4.97	4.78	5.02	2.72	2.97	2.82	3.07
Sample Size: 100	3.47	4.2	1.83	1.97	4.79	4.98	4.87	5.09	3.59	4.02	2.85	3.15
Sample Size: 200	4.53	4.14	2.22	2.26	4.95	5.2	5.03	5.38	3.67	4.06	2.97	3.84

Note: Author's own calculations

Table 5.14: Empirical Size of SupExt Tests using Asymptotic Critical Values Under IIS, SIS & TIS

Under Non-Stationary Data Settings												
Size of Test	***H-Test***		***CK-Test***		***RB-Test***		***IB-Tests***		***DIB-Test***		***CB-Test***	
	1%	5%	1%	5%	1%	5%	1%	5%	1%	5%	1%	5%
Sample Size: 50	0.012	0.053	0.017	0.059	0.013	0.051	0.010	0.051	0.015	0.053	0.010	0.050
Sample Size: 100	0.013	0.054	0.016	0.056	0.011	0.050	0.010	0.049	0.014	0.054	0.013	0.052
Sample Size: 200	0.011	0.051	0.017	0.057	0.011	0.050	0.009	0.050	0.014	0.055	0.011	0.052

Note: Author's own calculations

5.4.1 Power of the Tests using IIS, SIS & TIS under Non-Stationary Data

The graphs in Figure (5.4) below represent the power of each test when data is non-stationary and taking all IIS, SIS & TIS into account. When sample is 50 the power of *IB-Test* and *RB-Test* both at 1% as well as 5% level of significance is better than other SupExt tests under IIS, SIS & TIS. For the sample of 100 and at both 1% and 5% level of significance the power curves of *CK-Test* and *DIB-Test* becomes almost same though quite low. Also, *DIB-Test* is seems to be better than *CK-Test* for some points. The power of *H-Test* showed an overall improvement. For the sample size of 100 and 200 the power of *IB-Test* becomes more or less equivalent to *RB-Test*. However, the power of *IB-Test* and *RB-Test* showed relative improvement as compare to other tests considering IIS, SIS & TIS at a time. But the power of *IB-Test* is much better than that of other SupExt tests. Lastly, the performance of *CK-Test* shows a significant improvement for large sample when data is non-stationary.

Table 5.15: Powers under the Alternative Hypothesis at 1% Level of Significance Using IIS, SIS & TIS at a Time

Under Non-Stationary Data Settings																		
1% SL	Sample Size: 50						Sample Size: 100						Sample Size: 200					
H_1	***H-test***	***CK-test***	***RB-test***	***IB-test***	***DIB-test***	***CB-test***	***H-test***	***CK-test***	***RB-test***	***IB-test***	***DIB-test***	***CB-test***	***H-test***	***CK-test***	***RB-test***	***IB-test***	***DIB-test***	***CB-test***
0.99	0.18	0.09	0.26	0.32	0.09	0.11	0.27	0.17	0.41	0.41	0.1	0.15	0.29	0.2	0.45	0.45	0.12	0.18
0.96	0.21	0.12	0.27	0.32	0.15	0.2	0.27	0.17	0.41	0.41	0.16	0.2	0.29	0.2	0.45	0.45	0.18	0.23
0.93	0.22	0.15	0.29	0.34	0.15	0.2	0.27	0.17	0.41	0.41	0.16	0.2	0.29	0.2	0.45	0.45	0.18	0.23
0.91	0.21	0.18	0.32	0.36	0.15	0.2	0.27	0.17	0.41	0.42	0.16	0.21	0.29	0.2	0.45	0.46	0.18	0.24
0.9	0.23	0.15	0.32	0.38	0.15	0.2	0.27	0.17	0.41	0.45	0.16	0.21	0.29	0.2	0.45	0.49	0.18	0.24
0.85	0.24	0.14	0.33	0.39	0.15	0.22	0.27	0.17	0.41	0.48	0.16	0.21	0.29	0.2	0.45	0.52	0.18	0.24
0.8	0.26	0.14	0.35	0.41	0.17	0.24	0.29	0.17	0.43	0.5	0.18	0.2	0.31	0.2	0.47	0.54	0.2	0.23
0.75	0.28	0.14	0.35	0.43	0.19	0.26	0.31	0.17	0.45	0.52	0.2	0.23	0.33	0.2	0.49	0.56	0.22	0.26
0.7	0.28	0.18	0.35	0.44	0.2	0.26	0.32	0.17	0.46	0.53	0.21	0.24	0.34	0.2	0.5	0.57	0.23	0.27
0.65	0.31	0.18	0.38	0.46	0.22	0.26	0.34	0.17	0.48	0.55	0.23	0.26	0.36	0.2	0.52	0.59	0.25	0.29
0.6	0.32	0.18	0.38	0.46	0.24	0.27	0.36	0.17	0.5	0.55	0.25	0.28	0.38	0.2	0.54	0.59	0.27	0.31
0.55	0.34	0.18	0.38	0.46	0.26	0.28	0.38	0.17	0.52	0.55	0.27	0.3	0.4	0.2	0.56	0.59	0.29	0.33
0.5	0.36	0.18	0.39	0.46	0.26	0.29	0.38	0.17	0.52	0.55	0.27	0.32	0.4	0.2	0.56	0.59	0.29	0.35
0.45	0.34	0.18	0.39	0.46	0.26	0.3	0.38	0.17	0.52	0.55	0.27	0.34	0.4	0.2	0.56	0.59	0.29	0.37
0.4	0.35	0.19	0.39	0.46	0.26	0.31	0.38	0.19	0.52	0.55	0.27	0.36	0.4	0.22	0.56	0.59	0.29	0.39
0.35	0.35	0.19	0.41	0.46	0.26	0.31	0.38	0.2	0.52	0.55	0.27	0.37	0.4	0.23	0.56	0.59	0.29	0.4
0.3	0.35	0.19	0.41	0.46	0.26	0.32	0.38	0.22	0.52	0.55	0.27	0.38	0.4	0.25	0.56	0.59	0.29	0.41
0.25	0.36	0.19	0.41	0.47	0.26	0.32	0.39	0.25	0.53	0.56	0.27	0.37	0.41	0.28	0.57	0.60	0.29	0.4
0.2	0.38	0.19	0.42	0.47	0.26	0.32	0.4	0.26	0.54	0.56	0.27	0.37	0.42	0.29	0.58	0.60	0.29	0.4
0.15	0.38	0.19	0.42	0.47	0.26	0.33	0.41	0.27	0.55	0.56	0.27	0.37	0.43	0.3	0.59	0.60	0.29	0.4
0.1	0.39	0.19	0.42	0.47	0.26	0.33	0.42	0.27	0.56	0.56	0.27	0.37	0.44	0.3	0.6	0.60	0.29	0.4
0.05	0.4	0.19	0.42	0.48	0.27	0.33	0.42	0.27	0.56	0.57	0.28	0.37	0.44	0.3	0.6	0.61	0.3	0.4
0.01	0.4	0.19	0.43	0.48	0.28	0.33	0.42	0.27	0.56	0.57	0.29	0.37	0.44	0.3	0.6	0.61	0.31	0.4

Note: Author's own calculations

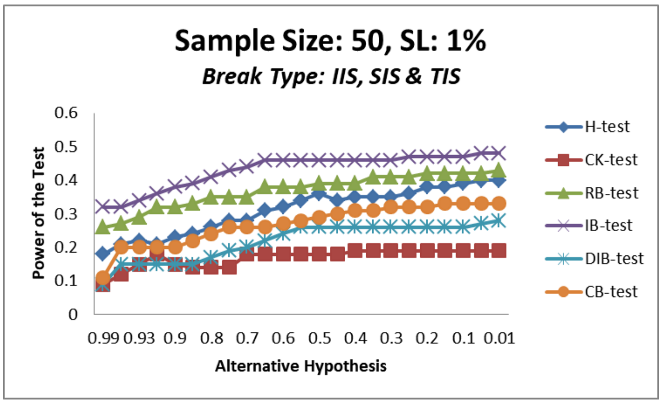
Table 5.16: Powers under the Alternative Hypothesis at 5% Level of Significance Using IIS, SIS & TIS at a Time

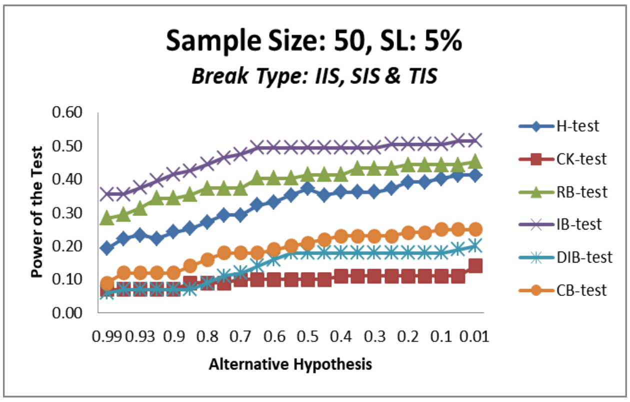
Under Non-Stationary Data Settings																		
5% SL	Sample Size: 50						Sample Size: 100						Sample Size: 200					
H_1	***H-test***	***CK-test***	***RB-test***	***IB-test***	***DIB-test***	***CB-test***	***H-test***	***CK-test***	***RB-test***	***IB-test***	***DIB-test***	***CB-test***	***H-test***	***CK-test***	***RB-test***	***IB-test***	***DIB-test***	***CB-test***
0.99	0.19	0.07	0.28	0.36	0.06	0.09	0.37	0.19	0.51	0.53	0.12	0.16	0.37	0.19	0.51	0.53	0.12	0.16
0.96	0.22	0.07	0.29	0.36	0.07	0.12	0.37	0.19	0.51	0.53	0.18	0.21	0.37	0.19	0.51	0.53	0.18	0.21
0.93	0.23	0.07	0.31	0.38	0.07	0.12	0.37	0.19	0.51	0.53	0.18	0.21	0.37	0.19	0.51	0.53	0.18	0.21
0.91	0.22	0.07	0.34	0.40	0.07	0.12	0.37	0.19	0.52	0.54	0.18	0.22	0.37	0.19	0.52	0.54	0.18	0.22
0.9	0.24	0.07	0.34	0.42	0.07	0.12	0.37	0.19	0.55	0.57	0.18	0.22	0.37	0.19	0.55	0.57	0.18	0.22
0.85	0.25	0.09	0.35	0.43	0.07	0.14	0.37	0.19	0.58	0.60	0.18	0.22	0.37	0.19	0.58	0.60	0.18	0.22
0.8	0.27	0.09	0.37	0.45	0.09	0.16	0.39	0.19	0.60	0.62	0.20	0.21	0.39	0.19	0.60	0.62	0.20	0.21
0.75	0.29	0.09	0.37	0.47	0.11	0.18	0.41	0.19	0.62	0.64	0.22	0.24	0.41	0.19	0.62	0.64	0.22	0.24
0.7	0.29	0.10	0.37	0.48	0.12	0.18	0.42	0.19	0.63	0.65	0.23	0.25	0.42	0.19	0.63	0.65	0.23	0.25
0.65	0.32	0.10	0.40	0.50	0.14	0.18	0.44	0.19	0.65	0.67	0.25	0.28	0.44	0.19	0.65	0.67	0.25	0.28
0.6	0.33	0.10	0.40	0.50	0.16	0.19	0.46	0.19	0.65	0.67	0.27	0.30	0.46	0.19	0.65	0.67	0.27	0.30
0.55	0.35	0.10	0.40	0.50	0.18	0.20	0.48	0.19	0.65	0.67	0.29	0.32	0.48	0.19	0.65	0.67	0.29	0.32
0.5	0.37	0.10	0.41	0.50	0.18	0.21	0.48	0.19	0.65	0.67	0.29	0.34	0.48	0.19	0.65	0.67	0.29	0.34
0.45	0.35	0.10	0.41	0.50	0.18	0.22	0.48	0.19	0.65	0.67	0.29	0.36	0.48	0.19	0.65	0.67	0.29	0.36
0.4	0.36	0.11	0.41	0.50	0.18	0.23	0.48	0.21	0.65	0.67	0.29	0.38	0.48	0.21	0.65	0.67	0.29	0.38
0.35	0.36	0.11	0.43	0.50	0.18	0.23	0.48	0.22	0.65	0.67	0.29	0.39	0.48	0.22	0.65	0.67	0.29	0.39
0.3	0.36	0.11	0.43	0.50	0.18	0.23	0.48	0.24	0.65	0.67	0.29	0.40	0.48	0.24	0.65	0.67	0.29	0.40
0.25	0.37	0.11	0.43	0.51	0.18	0.23	0.49	0.27	0.66	0.68	0.29	0.39	0.49	0.27	0.66	0.68	0.29	0.39
0.2	0.39	0.11	0.44	0.51	0.18	0.24	0.50	0.28	0.66	0.68	0.29	0.39	0.50	0.28	0.66	0.68	0.29	0.39
0.15	0.39	0.11	0.44	0.51	0.18	0.24	0.51	0.29	0.66	0.68	0.29	0.39	0.51	0.29	0.66	0.68	0.29	0.39
0.1	0.40	0.11	0.44	0.51	0.18	0.25	0.52	0.29	0.66	0.68	0.29	0.39	0.52	0.29	0.66	0.68	0.29	0.39
0.05	0.41	0.11	0.44	0.52	0.19	0.25	0.52	0.29	0.67	0.69	0.30	0.39	0.52	0.29	0.67	0.69	0.30	0.39
0.01	0.41	0.14	0.45	0.52	0.20	0.25	0.52	0.29	0.67	0.69	0.31	0.39	0.52	0.29	0.67	0.69	0.31	0.39

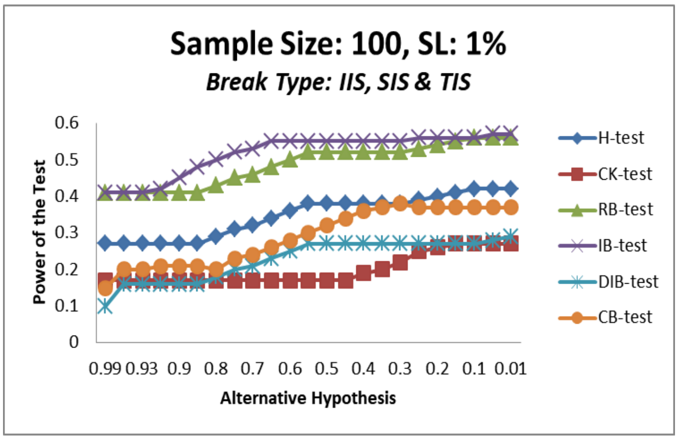
Note: Author's own calculations

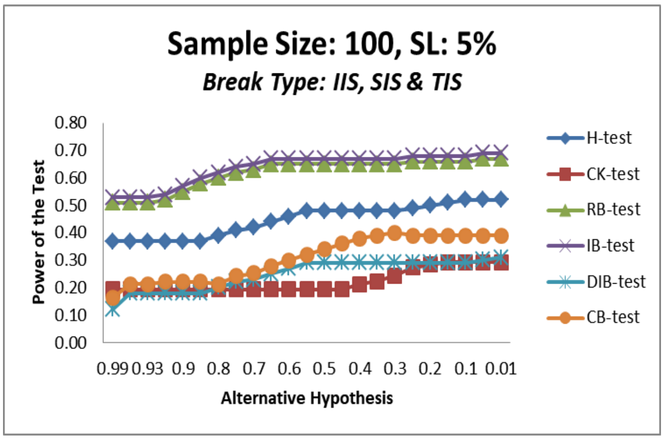
Figure 5.4: Performance Under IIS, SIS and TIS Jointly

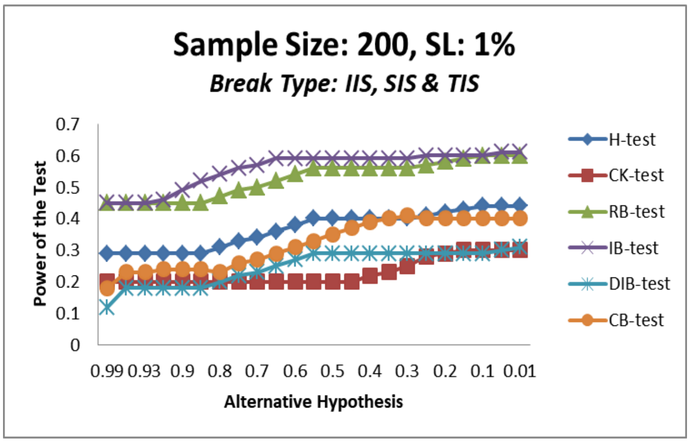
Performance Under 1% Level of Significance	Performance Under 5% Level of Significance

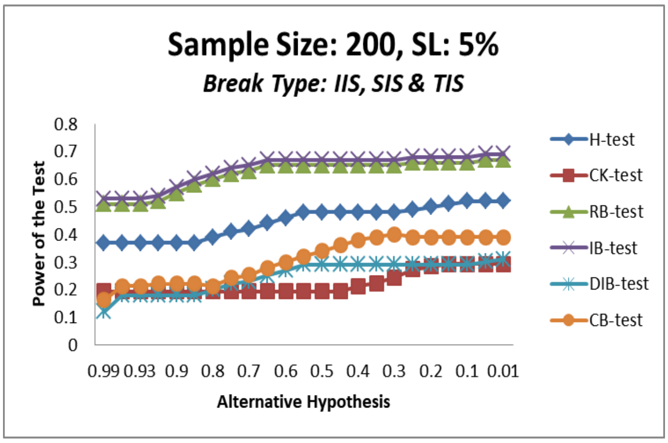












Note: Author's Own Estimations

5.5 Performance under Dynamic Settings

In Chapter 4, we have discussed the performance of SupExt tests considering stationary data. The DGP has been discussed in Chapter 3 *(Subsection 3.8)*. But no lags have been added at the time of simulations. In last section we discussed on how tests of SupExt behaves under non-stationary data settings taking IIS, SIS & TIS into account. Now in this subsection, our purpose is to compare the performance of SupExt tests while considering stationary but dynamic data settings by adding 2-lags in our conditional model. Though the choice of lags is independent but we restricted ourselves up to 2 lags in our conditional model.

5.5.1 Performance using IIS under Dynamic Data

The following Table 5.17 envelops the simulated CV's of the SupExt tests with stationary dynamic data settings when IIS is being taken into account. For the sample size 50 the observed values *CB-Test* and *IB-Test* are not very deviating *i.e.* both tests are showing very less size distortion. As on simulated critical values empirical size is approximately equal to nominal size which is obtained using simulated critical values. The difference between empirical size and nominal size is small which we will obtain from Table 5.1. However, as sample size changes from 50 to 100 or even to 200 we found a small amount of size distortion in *CB-Test* and more in *CK-Test* and *DIB-Test* as the nominal size is exceeding the empirical size in both cases but a small size distortion in *H*-Test. On the other hand, using IIS under dynamic data both *IB-Test* and *RB-Test* there is no size distortion. Since the empirical size is approximately equal to nominal size which is obtained using simulated critical values but for other tests the results of nominal size are exceeding the empirical size so here we will face size distortion.

Table 5.17: Simulated Critical Values of SupExt Tests Under IIS

Under Dynamic Data Settings (2-Lags)												
Size of Test	***H-Test***		***CK-Test***		***RB-Test***		***IB-Tests***		***DIB-Test***		***CB-Test***	
	5%	1%	5%	1%	5%	1%	5%	1%	5%	1%	5%	1%
Sample Size: 50	3.87	4.26	3.08	3.54	1.77	3.66	2.56	4.67	1.87	4.21	2.9	5.32
Sample Size: 100	2.86	4.48	3.12	3.64	2.8	4.2	3.14	5.38	3.09	5.3	1.82	2.47
Sample Size: 200	3.06	4.6	2.82	3.6	3.56	4.81	4.18	5.47	4.49	1.86	3.7	5.47

Note: Author's own calculations

Table 5.18: Empirical Size of SupExt Tests using Asymptotic Critical Values Under IIS

Under Dynamic Data Settings												
Size of Test	***H-Test***		***CK-Test***		***RB-Test***		***IB-Tests***		***DIB-Test***		***CB-Test***	
	1%	5%	1%	5%	1%	5%	1%	5%	1%	5%	1%	5%
Sample Size: 50	0.012	0.053	0.015	0.056	0.011	0.052	0.010	0.051	0.015	0.053	0.010	0.050
Sample Size: 100	0.011	0.052	0.016	0.056	0.010	0.050	0.010	0.049	0.014	0.055	0.013	0.052
Sample Size: 200	0.011	0.052	0.017	0.057	0.010	0.050	0.009	0.050	0.014	0.055	0.013	0.054

Note: Author's own calculations

5.5.2 Power of the Tests using IIS under Dynamic Data

The graphs in Figure 5.5 below represent the power curves of each SupExt test. One can easily observe that when sample is 50 the power of *CB-Test* both at 1% as well as 5% level of significance is better than other tests. But the power of *IB-Test, H-Test* and the power of *RB-Test, DIB-Test* is more or less same for the sample of 50. Now as the sample size increase from 50 to 100 or 200 the power of *CB-Test* significantly reduced using IIS though more or less same to *DIB-Test*. However, the power of *IB-Test* and *RB-Test* showed relative improvement as compare to other tests for both cases when sample is 100 and 200. But the power of *IB-Test* is comparatively better than that of other SupExt tests in this scenario. At the end, the performance of *CK-Test* showed a little bit improvement though stood at the last place.

Table 5.19: Powers under the Alternative Hypothesis at 1% Level of Significance Using Impulse Indicator Saturation (IIS)

Under Dynamic Settings (2-Lags)																		
1% SL	Sample Size: 50						Sample Size: 100						Sample Size: 200					
H_1[23]	***H-test***	***CK-test***	***RB-test***	***IB-test***	***DIB-test***	***CB-test***	***H-test***	***CK-test***	***RB-test***	***IB-test***	***DIB-test***	***CB-test***	***H-test***	***CK-test***	***RB-test***	***IB-test***	***DIB-test***	***CB-test***
0.99	0.06	0.07	0.07	0.06	0.03	0.18	0.12	0.05	0.28	0.33	0.25	0.13	0.16	0.09	0.32	0.37	0.29	0.17
0.96	0.08	0.07	0.08	0.07	0.03	0.18	0.14	0.05	0.28	0.33	0.26	0.13	0.18	0.09	0.32	0.37	0.30	0.17
0.93	0.1	0.07	0.07	0.06	0.05	0.2	0.16	0.05	0.3	0.35	0.25	0.15	0.20	0.09	0.34	0.39	0.29	0.19
0.91	0.12	0.07	0.09	0.08	0.07	0.22	0.18	0.1	0.32	0.37	0.27	0.17	0.22	0.14	0.36	0.41	0.31	0.21
0.9	0.12	0.07	0.09	0.08	0.07	0.22	0.18	0.1	0.32	0.37	0.27	0.17	0.22	0.14	0.36	0.41	0.31	0.21
0.85	0.14	0.07	0.09	0.08	0.09	0.24	0.2	0.1	0.34	0.39	0.27	0.19	0.24	0.14	0.38	0.43	0.31	0.23
0.8	0.16	0.07	0.08	0.07	0.11	0.26	0.22	0.15	0.36	0.41	0.26	0.21	0.26	0.19	0.40	0.45	0.30	0.25
0.75	0.18	0.07	0.11	0.10	0.13	0.28	0.24	0.16	0.38	0.43	0.29	0.23	0.28	0.20	0.42	0.47	0.33	0.27
0.7	0.19	0.07	0.12	0.11	0.14	0.29	0.25	0.16	0.39	0.44	0.3	0.24	0.29	0.20	0.43	0.48	0.34	0.28
0.65	0.21	0.08	0.14	0.14	0.16	0.32	0.27	0.16	0.42	0.47	0.32	0.27	0.31	0.20	0.46	0.51	0.36	0.31
0.6	0.23	0.1	0.16	0.18	0.18	0.36	0.29	0.16	0.46	0.51	0.34	0.31	0.33	0.20	0.50	0.55	0.38	0.35
0.55	0.25	0.09	0.18	0.19	0.20	0.37	0.31	0.18	0.47	0.52	0.36	0.32	0.35	0.22	0.51	0.56	0.40	0.36
0.5	0.27	0.12	0.2	0.24	0.22	0.42	0.33	0.18	0.52	0.57	0.38	0.37	0.37	0.22	0.56	0.61	0.42	0.41
0.45	0.29	0.16	0.22	0.30	0.24	0.48	0.35	0.18	0.58	0.63	0.4	0.43	0.39	0.22	0.62	0.67	0.44	0.47
0.4	0.31	0.17	0.24	0.33	0.26	0.51	0.37	0.2	0.61	0.66	0.42	0.46	0.41	0.24	0.65	0.70	0.46	0.50
0.35	0.35	0.14	0.28	0.34	0.30	0.52	0.41	0.2	0.62	0.67	0.46	0.47	0.45	0.24	0.66	0.71	0.50	0.51
0.3	0.37	0.16	0.3	0.38	0.32	0.56	0.43	0.23	0.66	0.71	0.48	0.51	0.47	0.27	0.70	0.75	0.52	0.55
0.25	0.41	0.18	0.34	0.44	0.36	0.62	0.47	0.24	0.72	0.77	0.52	0.57	0.51	0.28	0.76	0.81	0.56	0.61
0.2	0.45	0.16	0.38	0.46	0.40	0.64	0.51	0.24	0.74	0.79	0.56	0.59	0.55	0.28	0.78	0.83	0.60	0.63
0.15	0.5	0.15	0.43	0.50	0.45	0.68	0.56	0.25	0.78	0.83	0.61	0.63	0.60	0.29	0.82	0.87	0.65	0.67
0.1	0.53	0.2	0.46	0.58	0.48	0.76	0.59	0.27	0.82	0.87	0.64	0.71	0.63	0.31	0.86	0.91	0.68	0.75
0.05	0.57	0.21	0.5	0.63	0.52	0.81	0.63	0.28	0.85	0.9	0.68	0.76	0.67	0.32	0.89	0.94	0.72	0.80
0.01	0.6	0.19	0.53	0.64	0.55	0.82	0.66	0.3	0.86	0.91	0.71	0.77	0.70	0.34	0.90	0.95	0.75	0.81

Note: Author's own calculations

[23]

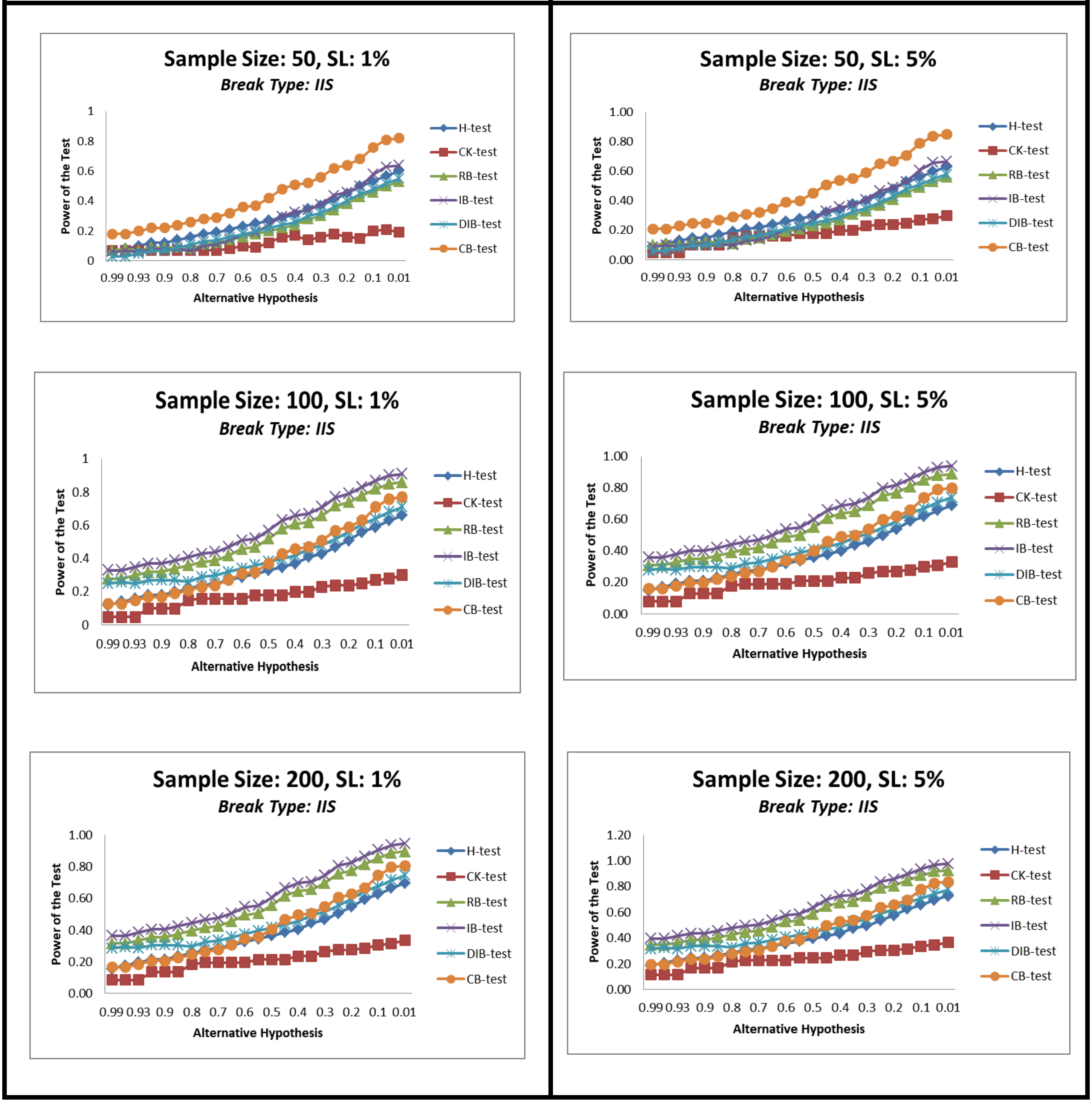
Table 5.20: Powers under the Alternative Hypothesis at 5% Level of Significance Using Impulse Indicator Saturation (IIS)

Under Dynamic Data Settings (2-Lags)																		
5% SL	Sample Size: 50						Sample Size: 100						Sample Size: 200					
H_1	***H-test***	***CK-test***	***RB-test***	***IB-test***	***DIB-test***	***CB-test***	***H-test***	***CK-test***	***RB-test***	***IB-test***	***DIB-test***	***CB-test***	***H-test***	***CK-test***	***RB-test***	***IB-test***	***DIB-test***	***CB-test***
0.99	0.09	0.10	0.10	0.09	0.06	0.21	0.15	0.08	0.31	0.36	0.28	0.16	0.19	0.12	0.35	0.40	0.32	0.20
0.96	0.11	0.10	0.11	0.10	0.06	0.21	0.17	0.08	0.31	0.36	0.29	0.16	0.21	0.12	0.35	0.40	0.33	0.20
0.93	0.13	0.10	0.10	0.09	0.08	0.23	0.19	0.08	0.33	0.38	0.28	0.18	0.23	0.12	0.37	0.42	0.32	0.22
0.91	0.15	0.10	0.12	0.11	0.10	0.25	0.21	0.13	0.35	0.40	0.30	0.20	0.25	0.17	0.39	0.44	0.34	0.24
0.9	0.15	0.10	0.12	0.11	0.10	0.25	0.21	0.13	0.35	0.40	0.30	0.20	0.25	0.17	0.39	0.44	0.34	0.24
0.85	0.17	0.10	0.12	0.11	0.12	0.27	0.23	0.13	0.37	0.42	0.30	0.22	0.27	0.17	0.41	0.46	0.34	0.26
0.8	0.19	0.10	0.11	0.10	0.14	0.29	0.25	0.18	0.39	0.44	0.29	0.24	0.29	0.22	0.43	0.48	0.33	0.28
0.75	0.21	0.10	0.14	0.13	0.16	0.31	0.27	0.19	0.41	0.46	0.32	0.26	0.31	0.23	0.45	0.50	0.36	0.30
0.7	0.22	0.10	0.15	0.14	0.17	0.32	0.28	0.19	0.42	0.47	0.33	0.27	0.32	0.23	0.46	0.51	0.37	0.31
0.65	0.24	0.11	0.17	0.17	0.19	0.35	0.30	0.19	0.45	0.50	0.35	0.30	0.34	0.23	0.49	0.54	0.39	0.34
0.6	0.26	0.13	0.19	0.21	0.21	0.39	0.32	0.19	0.49	0.54	0.37	0.34	0.36	0.23	0.53	0.58	0.41	0.38
0.55	0.28	0.12	0.21	0.22	0.23	0.40	0.34	0.21	0.50	0.55	0.39	0.35	0.38	0.25	0.54	0.59	0.43	0.39
0.5	0.30	0.15	0.23	0.27	0.25	0.45	0.36	0.21	0.55	0.60	0.41	0.40	0.40	0.25	0.59	0.64	0.45	0.44
0.45	0.32	0.19	0.25	0.33	0.27	0.51	0.38	0.21	0.61	0.66	0.43	0.46	0.42	0.25	0.65	0.70	0.47	0.50
0.4	0.34	0.20	0.27	0.36	0.29	0.54	0.40	0.23	0.64	0.69	0.45	0.49	0.44	0.27	0.68	0.73	0.49	0.53
0.35	0.38	0.17	0.31	0.37	0.33	0.55	0.44	0.23	0.65	0.70	0.49	0.50	0.48	0.27	0.69	0.74	0.53	0.54
0.3	0.40	0.19	0.33	0.41	0.35	0.59	0.46	0.26	0.69	0.74	0.51	0.54	0.50	0.30	0.73	0.78	0.55	0.58
0.25	0.44	0.21	0.37	0.47	0.39	0.65	0.50	0.27	0.75	0.80	0.55	0.60	0.54	0.31	0.79	0.84	0.59	0.64
0.2	0.48	0.19	0.41	0.49	0.43	0.67	0.54	0.27	0.77	0.82	0.59	0.62	0.58	0.31	0.81	0.86	0.63	0.66
0.15	0.53	0.18	0.46	0.53	0.48	0.71	0.59	0.28	0.81	0.86	0.64	0.66	0.63	0.32	0.85	0.90	0.68	0.70
0.1	0.56	0.23	0.49	0.61	0.51	0.79	0.62	0.30	0.85	0.90	0.67	0.74	0.66	0.34	0.89	0.94	0.71	0.78
0.05	0.60	0.24	0.53	0.66	0.55	0.84	0.66	0.31	0.88	0.93	0.71	0.79	0.70	0.35	0.92	0.97	0.75	0.83
0.01	0.63	0.22	0.56	0.67	0.58	0.85	0.69	0.33	0.89	0.94	0.74	0.80	0.73	0.37	0.93	0.98	0.78	0.84

Note: Author’s own calculations

Figure 5.5: Performance Under Impulse Indicator Saturation (IIS)

Performance Under 1% Level of Significance	Performance Under 5% Level of Significance



Note: Author's Own Estimations

5.5.3 Performance using SIS under Dynamic Data

The following Table 5.12 envelops the simulated CV's of the SupExt tests with dynamic data settings taking two lags when SIS is being taken into account. For the sample size 50 the observed values *CB-Test* and *IB-Test* are not very deviating *i.e.* both tests are showing no size distortion; unlike other SupExt tests. Since the empirical size is approximately equal to nominal size. The difference between empirical size and nominal size is small which we will obtain from Table 5.12 (a). However, as sample size changes from 50 to 100 or even to 200 we found size distortion in *CB-Test* and more in *CK-Test* and *DIB-Test* and even in *H-Test* as the nominal size is exceeding the empirical size in both cases *i.e.* 1% and 5% level of significance. On the other hand, using SIS under dynamic data both *IB-Test* and *RB-Test* there is no size distortion when sample is 100 and 200 and at both significance levels. Since the empirical size is approximately equal to nominal size which is obtained using simulated critical values but for other tests the results of nominal size are exceeding the empirical size so in case of SIS we will face size distortion in remaining SupExt tests.

Table 5.21: Simulated Critical Values of SupExt Tests Under SIS

Under Dynamic Data Settings (2-Lags)												
Size of Test	***H-Test***		***CK-Test***		***RB-Test***		***IB-Tests***		***DIB-Test***		***CB-Test***	
	5%	1%	5%	1%	5%	1%	5%	1%	5%	1%	5%	1%
Sample Size: 50	4.63	5.43	1.78	2.38	5.25	5.42	5.49	5.98	4.58	5.23	5.26	5.44
Sample Size: 100	5.8	6.68	2	2.95	5.5	6.61	6.55	6.94	5.62	6.21	5.49	6.08
Sample Size: 200	5.88	6.74	1.98	2.88	5.98	6.8	6.1	6.88	4.79	5.51	5.06	5.64

Note: Author's own calculations

Table 5.22: Empirical Size of SupExt Tests using Asymptotic Critical Values Under SIS

Under Dynamic Data Settings												
Size of Test	***H-Test***		***CK-Test***		***RB-Test***		***IB-Tests***		***DIB-Test***		***CB-Test***	
	1%	5%	1%	5%	1%	5%	1%	5%	1%	5%	1%	5%
Sample Size: 50	0.013	0.052	0.021	0.063	0.011	0.052	0.011	0.051	0.013	0.053	0.010	0.051
Sample Size: 100	0.011	0.051	0.015	0.057	0.010	0.050	0.010	0.050	0.012	0.053	0.013	0.054
Sample Size: 200	0.013	0.054	0.017	0.058	0.010	0.050	0.010	0.049	0.014	0.055	0.011	0.053

Note: Author’s own calculations

5.5.4 Power of the Tests using SIS under Dynamic Data

The graphs in Figure 5.6 below represent the power curves of each SupExt test for each case when sample is 50, 100 and 200. When sample is 50 the power of *IB-Test* both at 1% as well as 5% level of significance is better than *CB-Test*. But the power of *H-Test* and *DIB-Test* is more or less same for the sample of 50 and at both significance levels. Further, when sample is 50 the power of *H-Test* and *DIB-Test* is almost equal. Now as the sample size increase from 50 to 100 or 200 the power of *IB-Test* significantly improved for sample size 100 and 200. But the power of *CB-Test, RB-Test* and *H-Test* is almost same for sample size 100 and 200 at both levels. But still *IB-Test* remains at the top in terms of it performance whether the break is IIS and SIS. At the end, the performance of *CK-Test* showed a little bit improvement though stood at the last place.

Table 5.23: Powers under the Alternative Hypothesis at 1% Level of Significance Using Step Indicator Saturation (SIS)

Under Dynamic Data Settings (2-Lags)																		
1% SL	Sample Size: 50						Sample Size: 100						Sample Size: 200					
H_1	***H-test***	***CK-test***	***RB-test***	***IB-test***	***DIB-test***	***CB-test***	***H-test***	***CK-test***	***RB-test***	***IB-test***	***DIB-test***	***CB-test***	***H-test***	***CK-test***	***RB-test***	***IB-test***	***DIB-test***	***CB-test***
0.99	0.02	0.02	0.09	0.11	0.02	0.13	0.11	0.02	0.17	0.33	0.03	0.06	0.05	0.07	0.07	0.29	0.09	0.03
0.96	0.02	0.02	0.11	0.13	0.02	0.13	0.11	0.02	0.19	0.33	0.05	0.08	0.06	0.07	0.09	0.31	0.11	0.05
0.93	0.04	0.04	0.13	0.17	0.01	0.14	0.13	0.04	0.21	0.34	0.07	0.1	0.09	0.09	0.12	0.33	0.13	0.07
0.91	0.06	0.06	0.15	0.21	0.03	0.18	0.15	0.06	0.23	0.38	0.09	0.12	0.1	0.11	0.13	0.35	0.15	0.09
0.9	0.06	0.06	0.16	0.22	0.03	0.18	0.15	0.06	0.24	0.38	0.1	0.13	0.12	0.11	0.15	0.36	0.16	0.1
0.85	0.08	0.08	0.18	0.26	0.05	0.22	0.17	0.08	0.26	0.42	0.12	0.15	0.13	0.13	0.16	0.38	0.18	0.12
0.8	0.1	0.08	0.2	0.28	0.07	0.24	0.19	0.08	0.28	0.44	0.14	0.17	0.16	0.13	0.19	0.4	0.2	0.14
0.75	0.12	0.08	0.22	0.3	0.09	0.26	0.21	0.08	0.3	0.46	0.16	0.19	0.18	0.13	0.21	0.42	0.22	0.16
0.7	0.13	0.1	0.23	0.33	0.1	0.29	0.22	0.1	0.31	0.49	0.17	0.2	0.2	0.15	0.23	0.43	0.23	0.17
0.65	0.15	0.1	0.25	0.35	0.12	0.31	0.24	0.1	0.33	0.51	0.19	0.22	0.22	0.15	0.25	0.45	0.25	0.19
0.6	0.17	0.13	0.27	0.4	0.14	0.36	0.26	0.13	0.35	0.56	0.21	0.24	0.23	0.18	0.26	0.47	0.27	0.21
0.55	0.19	0.13	0.29	0.42	0.16	0.38	0.28	0.13	0.37	0.58	0.23	0.26	0.25	0.18	0.28	0.49	0.29	0.23
0.5	0.21	0.13	0.31	0.44	0.18	0.4	0.3	0.13	0.39	0.6	0.25	0.28	0.27	0.18	0.3	0.51	0.31	0.25
0.45	0.23	0.14	0.33	0.47	0.2	0.43	0.32	0.14	0.41	0.63	0.27	0.3	0.29	0.19	0.32	0.53	0.33	0.27
0.4	0.25	0.14	0.36	0.5	0.22	0.45	0.34	0.14	0.44	0.65	0.3	0.33	0.32	0.19	0.35	0.56	0.36	0.3
0.35	0.29	0.14	0.4	0.54	0.26	0.49	0.38	0.14	0.48	0.69	0.34	0.37	0.34	0.19	0.37	0.6	0.4	0.34
0.3	0.31	0.14	0.43	0.57	0.28	0.51	0.4	0.14	0.51	0.71	0.37	0.4	0.37	0.19	0.4	0.63	0.43	0.37
0.25	0.35	0.15	0.46	0.61	0.32	0.56	0.44	0.15	0.54	0.76	0.4	0.43	0.4	0.2	0.44	0.66	0.46	0.4
0.2	0.39	0.15	0.5	0.65	0.36	0.6	0.48	0.15	0.58	0.8	0.44	0.47	0.44	0.2	0.48	0.7	0.5	0.44
0.15	0.44	0.15	0.54	0.69	0.41	0.65	0.53	0.15	0.62	0.85	0.48	0.51	0.48	0.2	0.52	0.74	0.54	0.48
0.1	0.47	0.15	0.57	0.72	0.44	0.68	0.56	0.15	0.65	0.88	0.51	0.51	0.55	0.2	0.58	0.77	0.57	0.48
0.05	0.51	0.15	0.61	0.76	0.48	0.72	0.6	0.15	0.69	0.92	0.55	0.51	0.62	0.2	0.65	0.81	0.61	0.48
0.01	0.54	0.15	0.63	0.78	0.51	0.75	0.63	0.15	0.71	0.95	0.57	0.52	0.69	0.3	0.72	0.83	0.63	0.49

Note: Author's own calculations

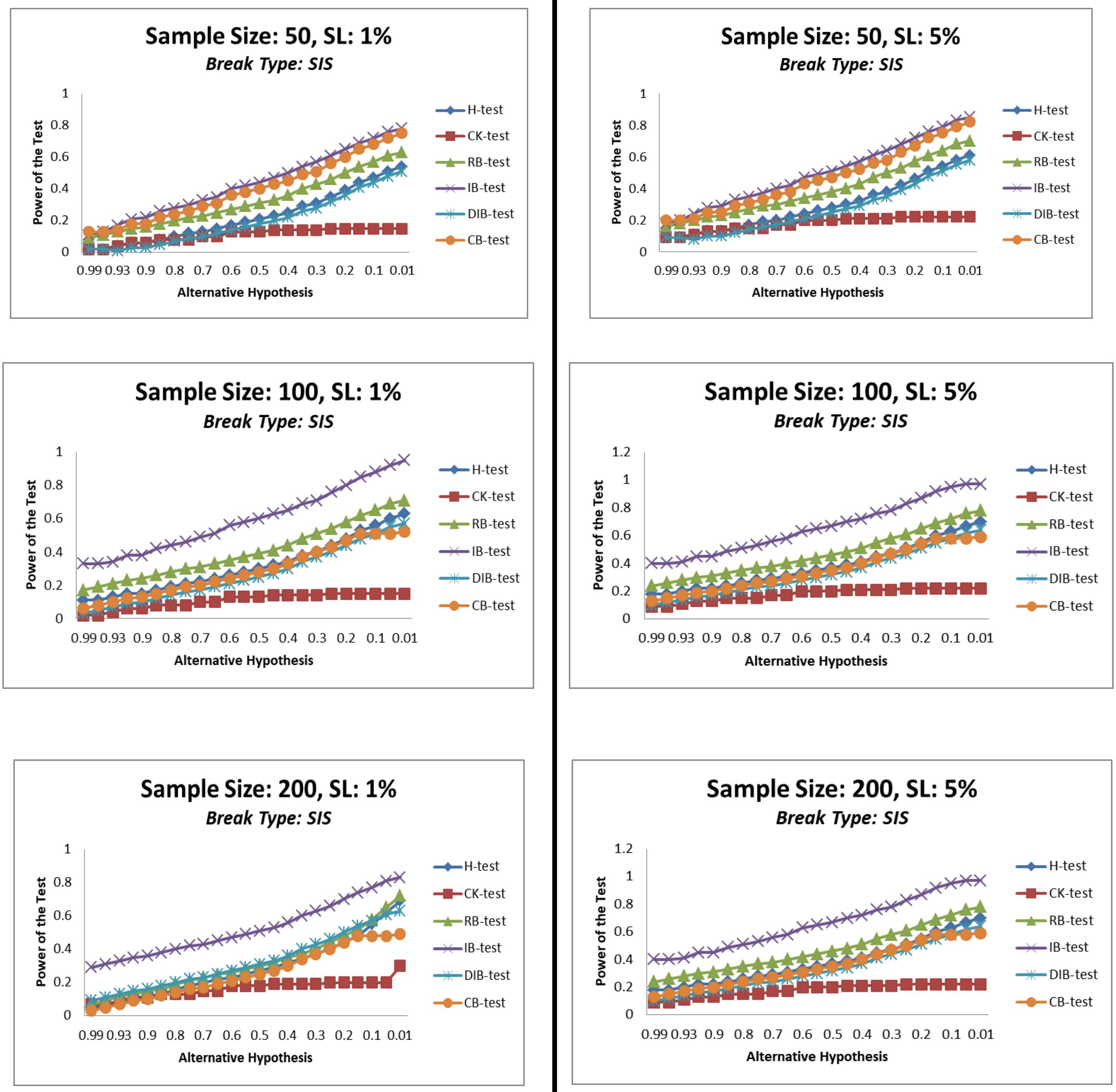
Table 5.24: Powers under the Alternative Hypothesis at 5% Level of Significance Using Step Indicator Saturation (SIS)

Under Dynamic Data Settings (2-Lags)																		
5% SL	Sample Size: 50						Sample Size: 100						Sample Size: 200					
H_1	***H-test***	***CK-test***	***RB-test***	***IB-test***	***DIB-test***	***CB-test***	***H-test***	***CK-test***	***RB-test***	***IB-test***	***DIB-test***	***CB-test***	***H-test***	***CK-test***	***RB-test***	***IB-test***	***DIB-test***	***CB-test***
0.99	0.09	0.09	0.16	0.18	0.09	0.2	0.18	0.09	0.24	0.4	0.1	0.13	0.18	0.09	0.24	0.4	0.1	0.13
0.96	0.09	0.09	0.18	0.2	0.09	0.2	0.18	0.09	0.26	0.4	0.12	0.15	0.18	0.09	0.26	0.4	0.12	0.15
0.93	0.11	0.11	0.2	0.24	0.08	0.21	0.2	0.11	0.28	0.41	0.14	0.17	0.2	0.11	0.28	0.41	0.14	0.17
0.91	0.13	0.13	0.22	0.28	0.1	0.25	0.22	0.13	0.3	0.45	0.16	0.19	0.22	0.13	0.3	0.45	0.16	0.19
0.9	0.13	0.13	0.23	0.29	0.1	0.25	0.22	0.13	0.31	0.45	0.17	0.2	0.22	0.13	0.31	0.45	0.17	0.2
0.85	0.15	0.15	0.25	0.33	0.12	0.29	0.24	0.15	0.33	0.49	0.19	0.22	0.24	0.15	0.33	0.49	0.19	0.22
0.8	0.17	0.15	0.27	0.35	0.14	0.31	0.26	0.15	0.35	0.51	0.21	0.24	0.26	0.15	0.35	0.51	0.21	0.24
0.75	0.19	0.15	0.29	0.37	0.16	0.33	0.28	0.15	0.37	0.53	0.23	0.26	0.28	0.15	0.37	0.53	0.23	0.26
0.7	0.2	0.17	0.3	0.4	0.17	0.36	0.29	0.17	0.38	0.56	0.24	0.27	0.29	0.17	0.38	0.56	0.24	0.27
0.65	0.22	0.17	0.32	0.42	0.19	0.38	0.31	0.17	0.4	0.58	0.26	0.29	0.31	0.17	0.4	0.58	0.26	0.29
0.6	0.24	0.2	0.34	0.47	0.21	0.43	0.33	0.2	0.42	0.63	0.28	0.31	0.33	0.2	0.42	0.63	0.28	0.31
0.55	0.26	0.2	0.36	0.49	0.23	0.45	0.35	0.2	0.44	0.65	0.3	0.33	0.35	0.2	0.44	0.65	0.3	0.33
0.5	0.28	0.2	0.38	0.51	0.25	0.47	0.37	0.2	0.46	0.67	0.32	0.35	0.37	0.2	0.46	0.67	0.32	0.35
0.45	0.3	0.21	0.4	0.54	0.27	0.5	0.39	0.21	0.48	0.7	0.34	0.37	0.39	0.21	0.48	0.7	0.34	0.37
0.4	0.32	0.21	0.43	0.57	0.29	0.52	0.41	0.21	0.51	0.72	0.37	0.4	0.41	0.21	0.51	0.72	0.37	0.4
0.35	0.36	0.21	0.47	0.61	0.33	0.56	0.45	0.21	0.55	0.76	0.41	0.44	0.45	0.21	0.55	0.76	0.41	0.44
0.3	0.38	0.21	0.5	0.64	0.35	0.58	0.47	0.21	0.58	0.78	0.44	0.47	0.47	0.21	0.58	0.78	0.44	0.47
0.25	0.42	0.22	0.53	0.68	0.39	0.63	0.51	0.22	0.61	0.83	0.47	0.5	0.51	0.22	0.61	0.83	0.47	0.5
0.2	0.46	0.22	0.57	0.72	0.43	0.67	0.55	0.22	0.65	0.87	0.51	0.54	0.55	0.22	0.65	0.87	0.51	0.54
0.15	0.51	0.22	0.61	0.76	0.48	0.72	0.6	0.22	0.69	0.92	0.55	0.58	0.6	0.22	0.69	0.92	0.55	0.58
0.1	0.54	0.22	0.64	0.79	0.51	0.75	0.63	0.22	0.72	0.95	0.58	0.58	0.63	0.22	0.72	0.95	0.58	0.58
0.05	0.58	0.22	0.68	0.83	0.55	0.79	0.67	0.22	0.76	0.97	0.62	0.58	0.67	0.22	0.76	0.97	0.62	0.58
0.01	0.61	0.22	0.7	0.85	0.58	0.82	0.7	0.22	0.78	0.97	0.64	0.59	0.7	0.22	0.78	0.97	0.64	0.59

Note: Author’s own calculations

Figure 5.6: Performance Under Step Indicator Saturation (SIS)

Performance Under 1% Level of Significance	Performance Under 5% Level of Significance



Note: Author's Own Estimations

5.5.5 *Performance using TIS under Dynamic Data*

The following Table 5.25 covers the simulated CV's of the SupExt tests when TIS is under consideration. For the sample size 50 the observed values *IB-Test* and *RB-Test* are not very deviating *i.e.* both tests are showing no size distortion. As empirical size is approximately equal to nominal size which is obtained using simulated critical values. The difference between empirical size and nominal size is small which we will obtain from Table 5.26. However, as sample size changes becomes 100 we found no size distortion in *DIB-Test* and *IB-Test*. On the other hand, using TIS and when sample size is 200 both *IB-Test* and *RB-Test* show no size distortion. As on simulated critical values empirical size is approximately equal to nominal size which is obtained using simulated critical values but for other tests the results of nominal size are exceeding the empirical size so here we will face size distortion. CK-Test showed improvement using TIS. As a whole the *IB-Test* performs relatively better than other SupExt tests under using TIS for all samples and *DIB-Test* for the sample of 100 but not better than *IB-Test*.

Table 5.25: Simulated Critical Values of SupExt Tests Under TIS

Under Dynamic Data Settings (2-Lags)												
Size of Test	***H-Test***		***CK-Test***		***RB-Test***		***IB-Tests***		***DIB-Test***		***CB-Test***	
	5%	1%	5%	1%	5%	1%	5%	1%	5%	1%	5%	1%
Sample Size: 50	4.65	4.81	1.96	2.41	4.16	4.83	4.99	4.81	3.53	4.57	4.97	6.5
Sample Size: 100	4.62	5.81	1.75	1.94	4.73	5.83	5.35	5.6	5.5	5.58	4.31	4.47
Sample Size: 200	4.55	5.43	1.77	1.96	5.07	5.83	5.85	6.03	4.07	4.51	4.99	4.41

Note: Author's own calculations

Table 5.26: Empirical Size of SupExt Tests using Asymptotic Critical Values Under TIS

Under Dynamic Data Settings												
Size of Test	***H-Test***		***CK-Test***		***RB-Test***		***IB-Tests***		***DIB-Test***		***CB-Test***	
	1%	5%	1%	5%	1%	5%	1%	5%	1%	5%	1%	5%
Sample Size: 50	0.015	0.057	0.019	0.063	0.011	0.051	0.011	0.050	0.015	0.057	0.017	0.059
Sample Size: 100	0.016	0.061	0.019	0.061	0.013	0.054	0.010	0.049	0.011	0.051	0.017	0.056
Sample Size: 200	0.019	0.061	0.016	0.059	0.011	0.051	0.010	0.051	0.014	0.055	0.016	0.054

Note: Author's own calculations

5.5.6 Power of the Tests using TIS under Dynamic Data

The power curves for each test using TIS for dynamic data settings are given below in Figure 5.7. One can observe that when sample is 50 the power of *IB-Test*, *RB-Test* and *CB-Test* initially move along but later the power of *CB-Test* become low as compared with *IB-Test*, *RB-Test* both at 1% as well as 5% level of significance. When sample is 100 the power of *IB-Test* and *DIB-Test* becomes alike. But as the sample size increase from 100 to 200 the power of *DIB-Test* significantly reduced using TIS. However, the power of *IB-Test* remains stable and above all other tests considering TIS when sample size is 200. The performance of *CK-Test* using TIS is not good for sample size 50 and 100 but slightly improved for sample size 200. As a whole the *IB-Test* performs better than other SupExt tests while using SIS. Lastly, the performance of *CK-Test* improves when sample size 200 as compared to IIS and SIS but remains at the lowest at both significance levels under TIS.

Table 5.27: Powers under the Alternative Hypothesis at 1% Level of Significance Using Trend Indicator Saturation (TIS)

Under Dynamic Data Settings (2-Lags)																		
1% SL	Sample Size: 50						Sample Size: 100						Sample Size: 200					
H_1	***H-test***	***CK-test***	***RB-test***	***IB-test***	***DIB-test***	***CB-test***	***H-test***	***CK-test***	***RB-test***	***IB-test***	***DIB-test***	***CB-test***	***H-test***	***CK-test***	***RB-test***	***IB-test***	***DIB-test***	***CB-test***
0.99	0.03	0.03	0.09	0.14	0.00	0.14	0.12	0.03	0.17	0.34	0.31	0.06	0.05	0.10	0.07	0.29	0.16	0.16
0.96	0.05	0.05	0.11	0.16	0.02	0.16	0.14	0.05	0.19	0.36	0.33	0.08	0.06	0.10	0.09	0.31	0.18	0.18
0.93	0.07	0.07	0.13	0.18	0.04	0.18	0.16	0.07	0.21	0.38	0.35	0.10	0.09	0.10	0.12	0.33	0.20	0.20
0.91	0.09	0.09	0.15	0.20	0.06	0.20	0.18	0.09	0.23	0.40	0.37	0.12	0.10	0.10	0.13	0.35	0.22	0.22
0.9	0.09	0.09	0.16	0.21	0.06	0.20	0.18	0.09	0.24	0.40	0.38	0.13	0.12	0.10	0.15	0.36	0.23	0.23
0.85	0.11	0.11	0.18	0.23	0.08	0.22	0.20	0.11	0.26	0.42	0.40	0.15	0.13	0.10	0.16	0.38	0.25	0.25
0.8	0.13	0.13	0.20	0.25	0.10	0.24	0.22	0.13	0.28	0.44	0.42	0.17	0.16	0.10	0.19	0.40	0.27	0.27
0.75	0.15	0.15	0.22	0.27	0.12	0.26	0.24	0.15	0.30	0.46	0.44	0.19	0.18	0.10	0.21	0.42	0.29	0.29
0.7	0.16	0.16	0.23	0.28	0.13	0.27	0.25	0.16	0.31	0.47	0.45	0.20	0.20	0.10	0.23	0.43	0.30	0.30
0.65	0.18	0.18	0.25	0.30	0.15	0.29	0.27	0.18	0.33	0.49	0.47	0.22	0.22	0.10	0.25	0.45	0.32	0.32
0.6	0.20	0.20	0.27	0.32	0.17	0.31	0.29	0.20	0.35	0.51	0.49	0.24	0.23	0.09	0.26	0.47	0.33	0.34
0.55	0.22	0.22	0.29	0.34	0.19	0.33	0.31	0.22	0.37	0.53	0.51	0.26	0.25	0.10	0.28	0.49	0.35	0.36
0.5	0.24	0.21	0.31	0.36	0.14	0.28	0.26	0.21	0.39	0.55	0.53	0.28	0.27	0.10	0.30	0.51	0.37	0.38
0.45	0.26	0.18	0.33	0.38	0.15	0.29	0.27	0.18	0.41	0.58	0.55	0.30	0.29	0.10	0.32	0.53	0.38	0.40
0.4	0.28	0.15	0.36	0.41	0.17	0.31	0.29	0.15	0.44	0.62	0.58	0.33	0.32	0.09	0.35	0.56	0.40	0.43
0.35	0.32	0.12	0.40	0.45	0.20	0.34	0.32	0.15	0.48	0.65	0.62	0.37	0.34	0.10	0.37	0.60	0.42	0.47
0.3	0.34	0.09	0.43	0.48	0.22	0.36	0.34	0.15	0.51	0.68	0.65	0.40	0.37	0.10	0.40	0.63	0.44	0.50
0.25	0.38	0.09	0.46	0.51	0.24	0.38	0.36	0.15	0.54	0.72	0.68	0.43	0.40	0.10	0.44	0.66	0.45	0.53
0.2	0.42	0.08	0.50	0.55	0.26	0.40	0.38	0.15	0.58	0.76	0.72	0.47	0.44	0.15	0.48	0.70	0.47	0.57
0.15	0.47	0.05	0.54	0.59	0.28	0.42	0.40	0.15	0.62	0.79	0.76	0.51	0.48	0.18	0.52	0.74	0.49	0.61
0.1	0.50	0.05	0.57	0.62	0.30	0.44	0.42	0.15	0.65	0.83	0.79	0.54	0.55	0.25	0.58	0.77	0.51	0.64
0.05	0.54	0.05	0.61	0.66	0.32	0.46	0.44	0.15	0.69	0.85	0.83	0.58	0.62	0.30	0.65	0.81	0.53	0.68
0.01	0.57	0.05	0.63	0.68	0.34	0.48	0.46	0.15	0.71	0.89	0.85	0.60	0.69	0.29	0.72	0.83	0.55	0.70

Note: Author's own calculations

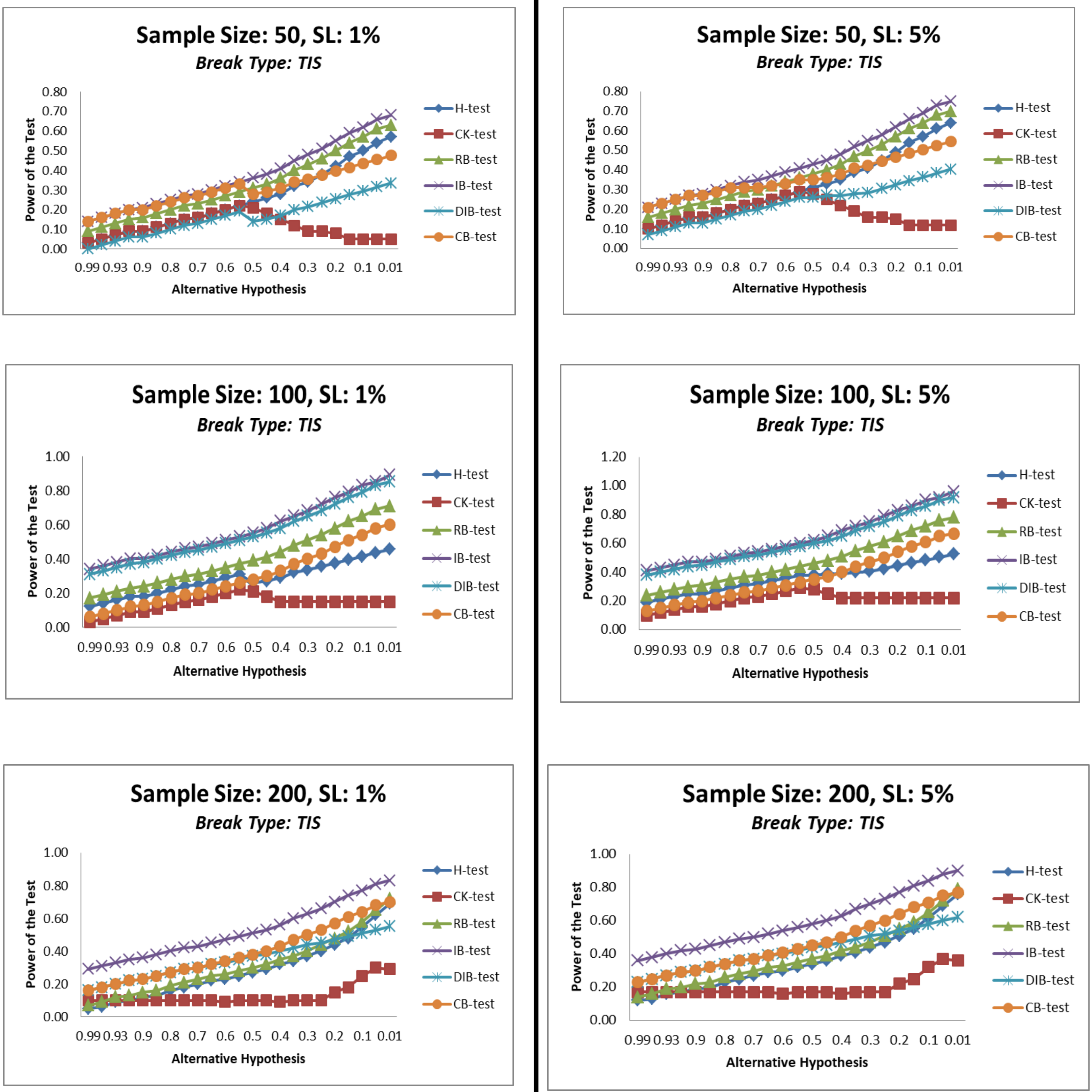
Table 5.28: Powers under the Alternative Hypothesis at 5% Level of Significance Using Trend Indicator Saturation (TIS)

Under Dynamic Data Settings (2-Lags)																		
5% SL	Sample Size: 50						Sample Size: 100						Sample Size: 200					
H_1	***H-test***	***CK-test***	***RB-test***	***IB-test***	***DIB-test***	***CB-test***	***H-test***	***CK-test***	***RB-test***	***IB-test***	***DIB-test***	***CB-test***	***H-test***	***CK-test***	***RB-test***	***IB-test***	***DIB-test***	***CB-test***
0.99	0.10	0.10	0.16	0.21	0.07	0.21	0.19	0.10	0.24	0.41	0.38	0.13	0.12	0.17	0.14	0.36	0.23	0.23
0.96	0.12	0.12	0.18	0.23	0.09	0.23	0.21	0.12	0.26	0.43	0.40	0.15	0.13	0.17	0.16	0.38	0.25	0.25
0.93	0.14	0.14	0.20	0.25	0.11	0.25	0.23	0.14	0.28	0.45	0.42	0.17	0.16	0.17	0.19	0.40	0.27	0.27
0.91	0.16	0.16	0.22	0.27	0.13	0.27	0.25	0.16	0.30	0.47	0.44	0.19	0.17	0.17	0.20	0.42	0.29	0.29
0.9	0.16	0.16	0.23	0.28	0.13	0.27	0.25	0.16	0.31	0.47	0.45	0.20	0.19	0.17	0.22	0.43	0.30	0.30
0.85	0.18	0.18	0.25	0.30	0.15	0.29	0.27	0.18	0.33	0.49	0.47	0.22	0.20	0.17	0.23	0.45	0.32	0.32
0.8	0.20	0.20	0.27	0.32	0.17	0.31	0.29	0.20	0.35	0.51	0.49	0.24	0.23	0.17	0.26	0.47	0.34	0.34
0.75	0.22	0.22	0.29	0.34	0.19	0.31	0.31	0.22	0.37	0.53	0.51	0.26	0.25	0.17	0.28	0.49	0.36	0.36
0.7	0.23	0.23	0.30	0.35	0.20	0.31	0.32	0.23	0.38	0.54	0.52	0.27	0.27	0.17	0.30	0.50	0.37	0.37
0.65	0.25	0.25	0.32	0.37	0.22	0.32	0.34	0.25	0.40	0.56	0.54	0.29	0.29	0.17	0.32	0.52	0.39	0.39
0.6	0.27	0.27	0.34	0.39	0.24	0.33	0.36	0.27	0.42	0.58	0.56	0.31	0.30	0.16	0.33	0.54	0.40	0.41
0.55	0.29	0.29	0.36	0.41	0.26	0.35	0.38	0.29	0.44	0.60	0.58	0.33	0.32	0.17	0.35	0.56	0.42	0.43
0.5	0.31	0.28	0.38	0.43	0.26	0.35	0.38	0.28	0.46	0.62	0.60	0.35	0.34	0.17	0.37	0.58	0.44	0.45
0.45	0.33	0.25	0.40	0.45	0.27	0.36	0.39	0.25	0.48	0.65	0.62	0.37	0.36	0.17	0.39	0.60	0.45	0.47
0.4	0.35	0.22	0.43	0.48	0.27	0.38	0.39	0.22	0.51	0.69	0.65	0.40	0.39	0.16	0.42	0.63	0.47	0.50
0.35	0.39	0.19	0.47	0.52	0.28	0.41	0.40	0.22	0.55	0.72	0.69	0.44	0.41	0.17	0.44	0.67	0.49	0.54
0.3	0.41	0.16	0.50	0.55	0.29	0.43	0.41	0.22	0.58	0.75	0.72	0.47	0.44	0.17	0.47	0.70	0.51	0.57
0.25	0.45	0.16	0.53	0.58	0.31	0.45	0.43	0.22	0.61	0.79	0.75	0.50	0.47	0.17	0.51	0.73	0.52	0.60
0.2	0.49	0.15	0.57	0.62	0.33	0.47	0.45	0.22	0.65	0.83	0.79	0.54	0.51	0.22	0.55	0.77	0.54	0.64
0.15	0.54	0.12	0.61	0.66	0.35	0.49	0.47	0.22	0.69	0.86	0.83	0.58	0.55	0.25	0.59	0.81	0.56	0.68
0.1	0.57	0.12	0.64	0.69	0.37	0.51	0.49	0.22	0.72	0.90	0.86	0.61	0.62	0.32	0.65	0.84	0.58	0.71
0.05	0.61	0.12	0.68	0.73	0.39	0.53	0.51	0.22	0.76	0.92	0.90	0.65	0.69	0.37	0.72	0.88	0.60	0.75
0.01	0.64	0.12	0.70	0.75	0.41	0.55	0.53	0.22	0.78	0.96	0.92	0.67	0.76	0.36	0.79	0.90	0.62	0.77

Note: Author's own calculations

Figure 5.7: Performance Under Step Indicator Saturation (TIS)

Performance Under 1% Level of Significance	Performance Under 5% Level of Significance



Note: Author's Own Estimations

5.5.7 *Performance using IIS, SIS & TIS under Dynamic Data*

The following Table 5.14 below covers the simulated CV's of the SupExt tests when IIS, SIS and TIS are used jointly. For the sample size 50 the observed values *IB-Test* and *RB-Test* are approximately equal to the empirical size *i.e.* both tests are showing no size distortion. Even when sample size changes from 50 to 100 or even to 200 we found no size distortion in both of these tests. The difference between empirical size and nominal size is small which we will obtain from Table 5.14 (a). When sample size is 100 we face a small amount of distortion in *DIB-Test* and *H-Test* but there is a significant amount of distortion in the remaining two. So far we came up to that the *IB-Test* and *RB-Test* behaves better under all the types of data driven breaks and for all sample sizes. Also the power of these test increased as well. However it is worth noting that the overall amount of distortion though found is less when we opt all breaks at a time as compared with the one when used individually. So we can say that one should use all these breaks at a time when testing for SupExt is being considered.

Table 5.29: Simulated Critical Values of SupExt Tests under IIS, SIS & TIS

Under Dynamic Data Settings (2-Lags)												
Size of Test	***H-Test***		***CK-Test***		***RB-Test***		***IB-Tests***		***DIB-Test***		***CB-Test***	
	5%	1%	5%	1%	5%	1%	5%	1%	5%	1%	5%	1%
Sample Size: 50	4.38	4.71	2.46	2.6	6.25	6.48	6.29	6.53	4.23	4.48	4.33	4.58
Sample Size: 100	4.98	5.71	2.35	2.49	6.3	6.49	6.38	6.6	5.1	5.53	4.36	4.66
Sample Size: 200	6.04	5.65	2.74	2.78	6.46	6.71	6.54	6.89	5.18	5.57	4.48	5.35

Note: Author's own calculations

Table 5.30: Empirical Size of SupExt Tests using Asymptotic Critical Values under IIS, SIS & TIS

Under Dynamic Data Settings												
Size of Test	***H-Test***		***CK-Test***		***RB-Test***		***IB-Tests***		***DIB-Test***		***CB-Test***	
	1%	5%	1%	5%	1%	5%	1%	5%	1%	5%	1%	5%
Sample Size: 50	0.012	0.053	0.014	0.053	0.09	0.050	0.010	0.050	0.013	0.053	0.013	0.052
Sample Size: 100	0.011	0.052	0.014	0.056	0.010	0.049	0.009	0.050	0.010	0.051	0.013	0.054
Sample Size: 200	0.011	0.052	0.013	0.053	0.010	0.050	0.010	0.048	0.012	0.051	0.011	0.056

Note: Author's own calculations

5.5.8 Power of the Tests using IIS, SIS & TIS under Dynamic Data

The power curves for each test when IIS, SIS and TIS used jointly are given below in Figure 5.8. By looking at the graphs one can easily observe that when sample is 50 the power of *IB-Test* and *RB-Test* both at 1% as well as 5% level of significance is better than other tests though not as much as while considering IIS, SIS and TIS jointly. However, the power of *H-Test* and *DIB-Test* is more or less same for the sample size of 50 and 200. The performance of *CB-Test* is not good as sample size increases to 200. As a whole the performance of SupExt test improves a lot when we use all breaks at a time. However *IB-Test* and *RB-Test* remains at the top while using IIS, SIS & TIS jointly. Lastly, the performance of *CK-Test* improves when sample size is of 100 and 200 but remains at the lowest at both significance levels.

Table 5.31: Powers under the Alternative Hypothesis at 1% Level of Significance Using IIS, SIS & TIS at a Time

Under Dynamic Data Settings (2-Lags)																		
1% SL	Sample Size: 50						Sample Size: 100						Sample Size: 200					
H_1	***H-test***	***CK-test***	***RB-test***	***IB-test***	***DIB-test***	***CB-test***	***H-test***	***CK-test***	***RB-test***	***IB-test***	***DIB-test***	***CB-test***	***H-test***	***CK-test***	***RB-test***	***IB-test***	***DIB-test***	***CB-test***
0.99	0.15	0.14	0.4	0.59	0.12	0.08	0.24	0.16	0.48	0.61	0.34	0.06	0.28	0.18	0.51	0.6	0.19	0.07
0.96	0.17	0.15	0.42	0.6	0.14	0.1	0.26	0.17	0.5	0.62	0.36	0.08	0.3	0.19	0.53	0.62	0.21	0.07
0.93	0.19	0.17	0.44	0.62	0.16	0.12	0.28	0.19	0.52	0.64	0.38	0.1	0.32	0.21	0.55	0.64	0.23	0.07
0.91	0.21	0.16	0.46	0.61	0.18	0.14	0.3	0.18	0.54	0.63	0.4	0.12	0.34	0.2	0.57	0.66	0.25	0.09
0.9	0.21	0.16	0.46	0.61	0.18	0.14	0.3	0.18	0.54	0.63	0.4	0.13	0.34	0.2	0.57	0.66	0.25	0.1
0.85	0.23	0.17	0.48	0.62	0.2	0.16	0.32	0.19	0.56	0.64	0.42	0.15	0.36	0.21	0.59	0.68	0.27	0.12
0.8	0.25	0.17	0.5	0.62	0.22	0.18	0.34	0.19	0.58	0.64	0.44	0.17	0.38	0.21	0.61	0.7	0.29	0.14
0.75	0.27	0.17	0.52	0.62	0.24	0.2	0.36	0.19	0.6	0.64	0.46	0.19	0.4	0.21	0.63	0.72	0.31	0.16
0.7	0.28	0.19	0.53	0.64	0.25	0.21	0.37	0.21	0.61	0.66	0.47	0.2	0.41	0.23	0.64	0.73	0.32	0.17
0.65	0.3	0.2	0.55	0.65	0.27	0.23	0.39	0.22	0.63	0.67	0.49	0.22	0.43	0.24	0.66	0.75	0.34	0.19
0.6	0.32	0.18	0.57	0.63	0.29	0.25	0.41	0.2	0.65	0.65	0.51	0.24	0.45	0.22	0.68	0.77	0.36	0.21
0.55	0.34	0.16	0.59	0.61	0.31	0.27	0.43	0.18	0.67	0.66	0.53	0.26	0.47	0.2	0.7	0.79	0.38	0.23
0.5	0.36	0.17	0.61	0.67	0.33	0.29	0.45	0.19	0.69	0.69	0.55	0.28	0.49	0.21	0.72	0.81	0.4	0.25
0.45	0.38	0.18	0.63	0.68	0.35	0.31	0.47	0.2	0.71	0.73	0.57	0.3	0.51	0.22	0.74	0.83	0.42	0.27
0.4	0.4	0.19	0.65	0.69	0.37	0.33	0.49	0.21	0.73	0.73	0.59	0.33	0.53	0.23	0.76	0.85	0.44	0.3
0.35	0.44	0.21	0.69	0.67	0.41	0.37	0.53	0.23	0.77	0.78	0.63	0.37	0.57	0.25	0.8	0.89	0.48	0.34
0.3	0.46	0.2	0.71	0.73	0.43	0.39	0.55	0.22	0.79	0.75	0.65	0.4	0.59	0.24	0.82	0.91	0.5	0.37
0.25	0.5	0.18	0.75	0.79	0.47	0.43	0.59	0.2	0.83	0.81	0.69	0.43	0.63	0.22	0.86	0.95	0.54	0.4
0.2	0.54	0.19	0.79	0.81	0.51	0.47	0.63	0.21	0.87	0.85	0.73	0.47	0.67	0.23	0.9	0.99	0.58	0.44
0.15	0.59	0.22	0.84	0.86	0.56	0.52	0.68	0.24	0.92	0.9	0.78	0.51	0.67	0.26	0.9	0.99	0.58	0.48
0.1	0.62	0.29	0.87	0.88	0.59	0.55	0.71	0.31	0.95	0.94	0.81	0.54	0.72	0.33	0.95	0.99	0.63	0.51
0.05	0.66	0.34	0.89	0.91	0.63	0.59	0.75	0.36	0.95	0.96	0.81	0.58	0.73	0.38	0.96	0.99	0.64	0.55
0.01	0.69	0.35	0.89	0.96	0.66	0.62	0.78	0.37	0.95	0.98	0.81	0.6	0.73	0.39	0.96	0.99	0.64	0.57

Note: Author's own calculations

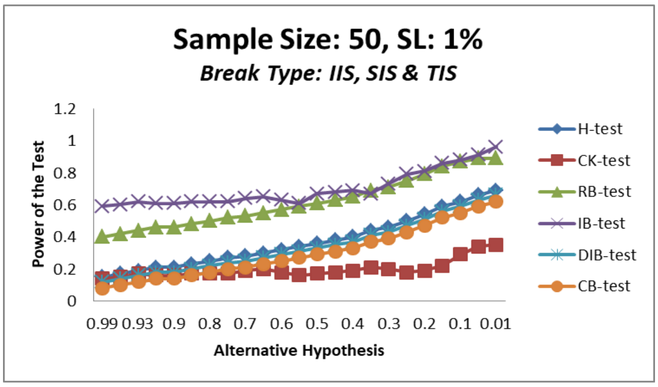
Table 5.32: Powers under the Alternative Hypothesis at 5% Level of Significance Using IIS, SIS & TIS at a Time

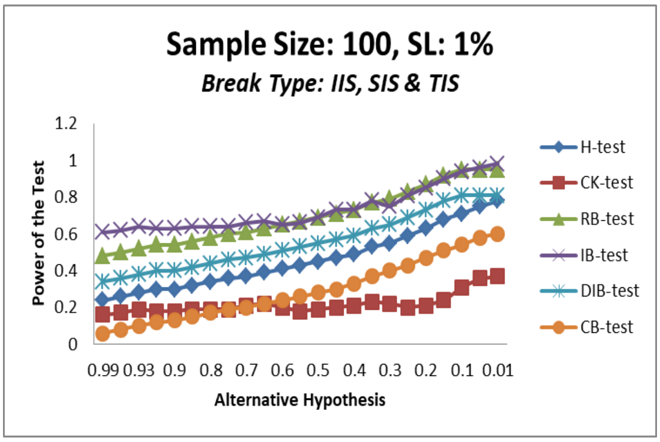
Under Dynamic Data Settings (2-Lags)																		
5% SL	Sample Size: 50						Sample Size: 100						Sample Size: 200					
H_1	***H-test***	***CK-test***	***RB-test***	***IB-test***	***DIB-test***	***CB-test***	***H-test***	***CK-test***	***RB-test***	***IB-test***	***DIB-test***	***CB-test***	***H-test***	***CK-test***	***RB-test***	***IB-test***	***DIB-test***	***CB-test***
0.99	0.38	0.17	0.57	0.6	0.34	0.06	0.38	0.17	0.57	0.6	0.34	0.06	0.5	0.22	0.68	0.77	0.38	0.16
0.96	0.4	0.18	0.59	0.62	0.36	0.08	0.4	0.18	0.59	0.62	0.36	0.08	0.52	0.23	0.7	0.79	0.4	0.18
0.93	0.42	0.2	0.61	0.64	0.38	0.1	0.42	0.20	0.61	0.64	0.38	0.1	0.54	0.25	0.72	0.81	0.42	0.2
0.91	0.44	0.19	0.63	0.66	0.4	0.12	0.44	0.19	0.63	0.66	0.4	0.12	0.56	0.24	0.74	0.83	0.44	0.22
0.9	0.44	0.19	0.63	0.66	0.4	0.13	0.44	0.19	0.63	0.66	0.4	0.13	0.56	0.24	0.74	0.83	0.44	0.23
0.85	0.46	0.2	0.65	0.68	0.42	0.15	0.46	0.20	0.65	0.68	0.42	0.15	0.58	0.25	0.76	0.85	0.46	0.25
0.8	0.48	0.2	0.67	0.7	0.44	0.17	0.48	0.20	0.67	0.7	0.44	0.17	0.6	0.25	0.78	0.87	0.48	0.27
0.75	0.5	0.2	0.69	0.72	0.46	0.19	0.5	0.20	0.69	0.72	0.46	0.19	0.62	0.25	0.8	0.89	0.5	0.29
0.7	0.51	0.22	0.7	0.73	0.47	0.2	0.51	0.22	0.7	0.73	0.47	0.2	0.63	0.27	0.81	0.9	0.51	0.3
0.65	0.53	0.23	0.72	0.75	0.49	0.22	0.53	0.23	0.72	0.75	0.49	0.22	0.65	0.28	0.83	0.92	0.53	0.32
0.6	0.55	0.21	0.74	0.77	0.51	0.24	0.55	0.21	0.74	0.77	0.51	0.24	0.67	0.26	0.85	0.94	0.55	0.34
0.55	0.57	0.19	0.76	0.79	0.53	0.26	0.57	0.19	0.76	0.79	0.53	0.26	0.69	0.24	0.87	0.96	0.57	0.36
0.5	0.59	0.2	0.78	0.81	0.55	0.28	0.59	0.20	0.78	0.81	0.55	0.28	0.71	0.25	0.89	0.98	0.59	0.38
0.45	0.61	0.21	0.8	0.83	0.57	0.3	0.61	0.21	0.8	0.83	0.57	0.3	0.73	0.26	0.91	0.99	0.61	0.4
0.4	0.63	0.22	0.82	0.85	0.59	0.33	0.63	0.22	0.82	0.85	0.59	0.33	0.75	0.27	0.93	0.99	0.63	0.43
0.35	0.67	0.24	0.86	0.89	0.63	0.37	0.67	0.24	0.86	0.89	0.63	0.37	0.79	0.29	0.93	0.99	0.67	0.47
0.3	0.69	0.23	0.88	0.91	0.65	0.4	0.69	0.23	0.88	0.91	0.65	0.4	0.81	0.28	0.93	0.99	0.69	0.5
0.25	0.73	0.21	0.92	0.95	0.69	0.43	0.73	0.21	0.92	0.95	0.69	0.43	0.81	0.26	0.93	0.99	0.73	0.53
0.2	0.77	0.22	0.96	0.99	0.73	0.47	0.77	0.22	0.96	0.99	0.73	0.47	0.81	0.27	0.93	0.99	0.77	0.57
0.15	0.82	0.25	0.96	0.99	0.78	0.51	0.82	0.25	0.96	0.99	0.78	0.51	0.82	0.3	0.93	0.99	0.82	0.61
0.1	0.85	0.32	0.96	0.99	0.81	0.54	0.85	0.32	0.96	0.99	0.81	0.54	0.82	0.37	0.94	0.99	0.85	0.64
0.05	0.89	0.37	0.95	0.99	0.83	0.58	0.89	0.37	0.96	0.99	0.83	0.58	0.82	0.42	0.94	0.99	0.87	0.68
0.01	0.92	0.38	0.96	0.99	0.83	0.6	0.92	0.38	0.96	0.99	0.83	0.6	0.82	0.53	0.94	0.99	0.87	0.7

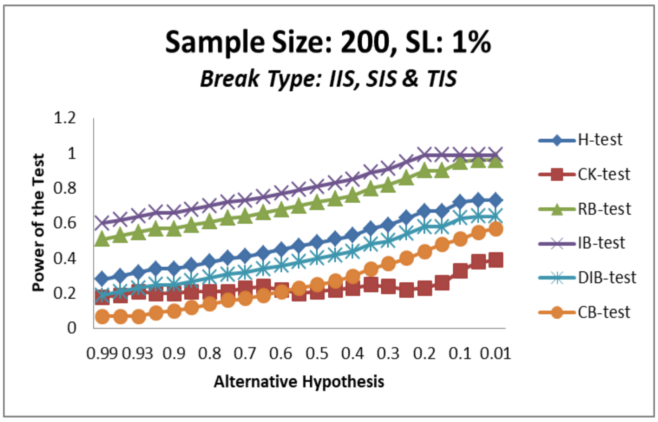
Note: Author’s own calculation

Figure 5.8: Performance Under IIS, SIS and TIS Jointly

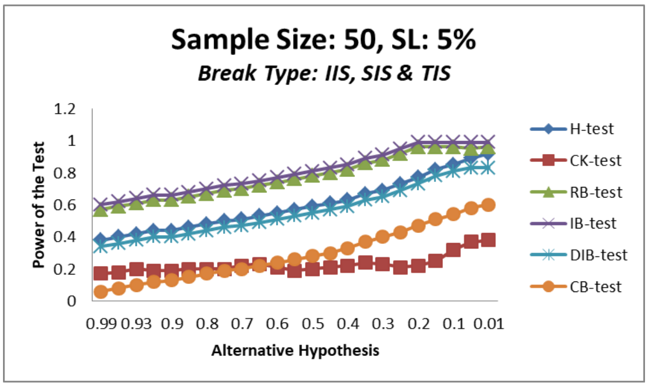
Performance Under 1% Level of Significance

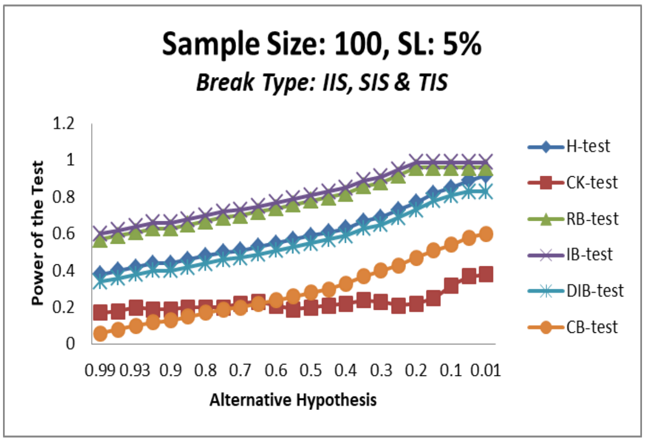


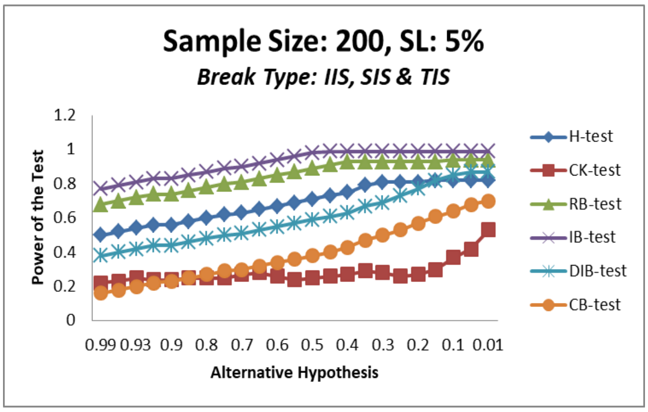




Performance Under 5% Level of Significance







Note: Author's Own Estimations

5.6 Some Future Extensions

So far our discussion evolved around single equation modelling and the power analysis discussed here is not in multivariate scenario. It is worth noting is someone could check and compare the performance and power of these SupExt tests cointegrated VAR models (hereafter CVAR) considering indicator saturation. The introduction of cointegration by (Granger, 1969) plays a critical role in time series econometrics, and a CVAR model explored by (Johansen, 1988, 1996) has become a major econometric tool for macroeconomists. The following debate will set a footstep for those who are interested in testing exogeneity in CVAR structure.

The problem of finding adjustment and cointegrating coefficients for the infinite order CVAR representation of a partially observed simple CVAR(1) model. The main tools are some classical results for the solution of the algebraic Riccati equation, and the results are exemplified by an analysis of CVAR(1) models for causal graphs in two cases where simple conditions for WeExt are derived in terms of the parameters of the CVAR(1) model and for the extensive empirical research using CVAR models in (Juselius, 2006). The CVAR model is well fitted in the *GETS* methodology, as its analysis usually commences with the investigation of general unrestricted VAR models. Hendry & Mizon (1993) discussed a model reduction procedure in the framework of the CVAR model, and present a parsimonious congruent model for UK money demand.

Multivariate cointegrated time series modeling usually embraces a number of model specification steps as, choosing the information set, picking the lag length and the determination of the cointegration properties. Finally, modeling the short run adjustment structure, i.e. the feedbacks to deviations from the long run relations, is an important step, because it can reveal information on the underlying economic structure. Modeling the feedback mechanisms in CVAR models is typically done by testing the significance of the feedback or loading coefficients. These significance tests are often called weak exogeneity tests, because certain sets of zero restrictions imply long run weak exogeneity with respect to the cointegrating parameters.

Without any loss of information, if the variables in multivariate setting are weakly exogenous for the set of parameters of interest, then single equation model conditional on weakly exogenous variables can be estimated in terms of statistical inference (Kurita, 2010).

Consider the unrestricted VAR (*k*) model for *p-dimensional* time series given by:

$$\Delta X_t = (\Pi, \Pi_l)\begin{pmatrix} X_{t-1} \\ t \end{pmatrix} + \sum_{i=1}^{k-1} \Gamma_i \Delta X_{t-i} \quad + \mu + \Phi D_t + \varepsilon_t\, , for\ t = 1,2,3, \ldots, T$$

Where D_t is a vector consisting deterministic term other than intercept and linear trend such as impulse and seasonal dummies and $\varepsilon_t \sim N(0, \Omega)$ also $\Pi, \Gamma_i, \Omega \in \mathcal{R}^{p\times p}, \Pi_l, \mu \in \mathcal{R}^p$ and $\Phi \in \mathcal{R}^{p\times s}$ vary freely and Ω is positive definite matrix, t is the deterministic trend with the parameter Π_l. Furthermore, it is a trend restricted model avoiding a quadratic trend in X_t. Note that, following regularity conditions must be fulfilled to perform the cointegration analysis for I(1):

i. The characteristic polynomial $A(z) = (1-z)I_p - \Pi z - \sum_{i=1}^{k-1} \Gamma_i (1-z) z^i$ obeys the equation $|A(z)| = 0$. The roots are either outside the unit circle or at one.

The above conditions ensures that the roots are neither explosive: $(|z| < 1)$ nor the seasonal $(|z| = 1 \text{ or } z = 1)$.

ii. $Rank(\Pi, \Pi_l) \leq r$, where $\alpha, \beta \in \mathcal{R}^{p\times r}$ for $r < p$.

The $\alpha - space$ is an adjustment space and the $\beta - space$ is called cointegrating space. The number of cointegrating vectors, $r(r < p)$ is given by the following reduced rank condition,

$$(\Pi, \Pi_l) = \alpha(\beta', \beta_l') = \alpha(\beta', \gamma'),$$

Where α and β are $p \times r$ matrices of full rank and β_l' is an r-vector. This equation shows that there are at least $p - r$ common stochastic trends and CI arises when $r \geq 1$.

iii. The third and final condition is that

$$Rank(\alpha_\perp' \Gamma \beta_\perp) = p - r$$

Where $\Gamma = I_p - \sum_{i=1}^{k-1} \Gamma_i$, $\alpha_\perp', \beta_\perp \in \mathcal{R}^{p\times(p-r)}$ are orthogonal complements s.t. $\alpha'\alpha_\perp = 0$ and $\beta'\beta_\perp = 0$ with $(\alpha, \alpha_\perp)$ and $(\beta, \beta_\perp)$ being of full rank. The third condition precludes the process from being I(2) or of higher order. If these conditions are satisfied, an I(1) cointegrated VAR model is defined as a sub-model of the original full model, for $\beta^*=(\beta', \gamma')'$ and $X_{t-1}^* = (X'_{t-1}, t)'$, as follows:

$$\Delta X_t = \alpha\beta^{*\prime} X_{t-1}^* + \sum_{i=1}^{k-1} \Gamma_i \Delta X_{t-i} + \mu + \Phi D_t + \varepsilon_t$$

This is to be considered as the basis for the subsequent cointegration analysis and model reduction. Since the cointegrating rank *r* is usually unknown to investigators, it needs to be determined using the data. A log-likelihood ratio (logLR) test statistic is given by the null hypothesis of *r* cointegration rank *H(r)* against the alternative hypothesis *H(p)*. The asymptotic quantiles for the logLR test statistic are provided in (Johansen, 1996). After determining the cointegrating rank, one is able to test various restrictions on *α, β* and γ in order to pursue the adjustment structure and cointegrating relationships subject to economic interpretation.

5.6.1 Weak Exogeneity and Conditional Model

Considering the above I(1) CVAR model and let the process to decompose like; $X_t = (W_t', Z_t')'$ for $W_t \in \mathcal{R}^m$ and $Z_t \in \mathcal{R}^{p-m}$ and $m \geq r$. Also the set of parameters and the errors will decompose as follows:

$$\alpha = \begin{pmatrix} \alpha_w \\ \alpha_z \end{pmatrix}, \Gamma_i = \begin{pmatrix} \Gamma_{w,i} \\ \Gamma_{z,i} \end{pmatrix}, \mu = \begin{pmatrix} \mu_w \\ \mu_z \end{pmatrix}, \Phi = \begin{pmatrix} \Phi_w \\ \Phi_z \end{pmatrix} \varepsilon_t = \begin{pmatrix} \varepsilon_{w,t} \\ \varepsilon_{z,t} \end{pmatrix},$$

And error terms are mean-zero and having variance covariance matrix:

$$\Omega = \begin{pmatrix} \Omega_{ww} & \Omega_{wz} \\ \Omega_{zw} & \Omega_{zz} \end{pmatrix}$$

So decomposing CVAR into conditional model for W_t and marginal model for Z_t, *i.e.*

$$\Delta W_t = \boldsymbol{\omega} \Delta Z_t + (\alpha_w - \boldsymbol{\omega} \alpha_z) \beta^{*'} X_{t-1}^* + \sum_{i=1}^{k-1} \tilde{\Gamma}_{w,i} \Delta X_{t-i} \quad + \tilde{\mu}_w + \widetilde{\Phi}_w D_t + \tilde{\varepsilon}_{w,t} \ ,$$

$$\Delta Z_t = \alpha_z \beta^{*'} X_{t-1} + + \sum_{i=1}^{k-1} \Gamma_{z,i} \Delta X_{t-i} \quad + \mu_z + \Phi_z D_t + \varepsilon_{z,t} \ ,$$

Where $\boldsymbol{\omega} = \Omega_{wz} \Omega_{zz}^{-1}$, $\tilde{\Gamma}_{y,i} = \Gamma_{y,i} - \boldsymbol{\omega} \Gamma_{z,i}$, $\tilde{\mu}_w = \mu_w - \boldsymbol{\omega} \mu_z$, $\widetilde{\Phi}_w = \Phi_w - \boldsymbol{\omega} \Phi_z$ and

$\tilde{\varepsilon}_{w,t} = \varepsilon_{w,t} - \boldsymbol{\omega} \varepsilon_{z,t}$ and $\begin{pmatrix} \tilde{\varepsilon}_{w,t} \\ \varepsilon_{z,t} \end{pmatrix} = N \left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \Omega_{ww,z} & 0 \\ 0 & \Omega_{zz} \end{pmatrix} \right]$ for $\Omega_{ww,z} = \Omega_{ww} - \Omega_{wz} \Omega_{zz}^{-1} \Omega_{zw}$.

Now if the condition $\alpha_z = 0$ is satisfied, then both the conditional and marginal model can be written as follows:

$$\Delta W_t = \boldsymbol{\omega} \Delta Z_t + \alpha_w \beta^{*'} X_{t-1}^* + \sum_{i=1}^{k-1} \tilde{\Gamma}_{w,i} \Delta X_{t-i} \quad + \tilde{\mu}_w + \widetilde{\Phi}_w D_t + \tilde{\varepsilon}_{w,t} \ ,$$

$$\Delta Z_t = \sum_{i=1}^{k-1} \Gamma_{z,i} \Delta X_{t-i} \quad + \mu_z + \Phi_z D_t + \varepsilon_{z,t} \ ,$$

Under this condition Z_t is considered to be weakly exogenous for the set of parameter of interest $\boldsymbol{\omega}, \alpha_w, \beta^*, \tilde{\Gamma}_{w,i}, \tilde{\mu}_w, \widetilde{\Phi}_w$ and $\Omega_{ww,z}$. Note that cointegrating relations ${\beta^*}' X_{t-1}^*$ are not embedded in the marginal model. If the condition for weak exogeneity, $\alpha_z = 0$, is satisfied, the parameters can then be estimated from the conditional model without any loss of information, with no need for the estimation of the marginal model. Among the parameters in, α_w and β^* are of particular interest, as the parameters represent the adjustment mechanism and long-run economic relationships in the conditional model, respectively (Engle et al., 1983; Johansen, 1992b; Urbain, 1992) on weak exogeneity.

In case of VECM (Johansen, 1991) purposed that a joint test of a particular row of α matrix is zero is test of weak exogeneity of the corresponding variable. As these restrictions only correspond to coefficient nullities in the marginal model several conventional tests can be carried out (Likelihood Ratio test (LR), Lagrange Multiplier (LM) test, Wald test (W)). Such tests can easily be implemented in empirical applications using most statistical computer packages. Note that the LR test is generally preferable to the W and LM tests in this situation as the restrictions are nonlinear in Π, even if they are linear in α. The LR test is at least invariant to how those restrictions on Π are expressed.

5.6.2 Super Exogeneity in CVAR model

By considering the above details for testing SupExt in single equation models, now consider some of the ground footings for testing SupExt in CVAR models under the presence of structural breaks or dummies. However, we just introduce the process of testing SupExt under the presence of conditional co-breaking.

Following (Johansen, 1996), a trend restricted CVAR (k) model for a set of *p-variate* vector process X_t .

$$\Delta X_t = \alpha(\beta', \gamma)\binom{X_{t-1}}{t} + \sum_{i=1}^{k-1} \Gamma_i \Delta X_{t-i} \quad + \mu + \Phi D_t + \varepsilon_t \, , for \ t = 1,2,3, \dots, T \qquad (5.1)$$

Where $\varepsilon_t \sim N(0, \Omega)$ and D_t is vector representing outliers in the process. It is worth noting that the deterministic term D_t is called a drift term whereas the deterministic term in the expression for X_t is called a trend. Thus a constant drift term in the equation will generate a linear trend term in the process, and a linear drift term in the equation will generate a quadratic trend term in the process via Granger's representation theorem.

All the parameters involved in the above equation are considered to move freely Ω as positive definite matrix. Furthermore, if the linear trend is not involved in the above equation and the intercept μ is restricted to set as $\mu = \alpha\gamma$ leading as to get a constant restricted CVAR (*k*) model instead the trend restricted model. The following conditions must be hold for further considerations of the above equation.

i. $\left|(1-z)I_p - \Pi z - \sum_{i=1}^{k-1} \Gamma_i (1-z) z^i\right| = 0$

ii. $Rank(\alpha) = Rank(\beta) = r$

iii. $Rank(\alpha'_\perp \Gamma \beta_\perp) = p - r$ Where $\Gamma = I_p - \sum_{i=1}^{k-1} \Gamma_i$

Now in the presence of above assumptions (5.1) is called an I(1) CVAR (*k*) is called a joint system and with the help of Granger-Johansen representation, moving average of the equation (5.1) can be written as:

$$X_t = C \sum_{i=1}^{t} (\varepsilon_i + \Phi D_i) + C(L)(\varepsilon_t + \Phi D_t) + \tau_l t + \tau_c \qquad (5.2)$$

The concepts like weak, strong and SupExt can be considered under the belt of I(1) CAVR framework. Now weak exogeneity can be defined by splitting (5.1) into conditional and marginal model. By splitting the vector X_t into $(Y_t', Z_t')'$. In a similar way all the parmaeters apart from β and error can be expressed as follows:

$$\alpha = \begin{pmatrix} \alpha_y \\ \alpha_z \end{pmatrix}, \Gamma_i = \begin{pmatrix} \Gamma_{y,i} \\ \Gamma_{z,i} \end{pmatrix}, \mu = \begin{pmatrix} \mu_y \\ \mu_z \end{pmatrix}, \Phi = \begin{pmatrix} \Phi_y \\ \Phi_z \end{pmatrix} \varepsilon_t = \begin{pmatrix} \varepsilon_{y,t} \\ \varepsilon_{z,t} \end{pmatrix},$$

And error terms are mean-zero and having variance covariance matrix:

$$\Omega = \begin{pmatrix} \Omega_{yy}\ \Omega_{yz} \\ \Omega_{zy}\ \Omega_{zz} \end{pmatrix}$$

Keeping in mind that the variables Z_t is considered to be weakly exogenous w.r.t parameter of interest β if the condition $\alpha_z = 0$ holds. The conditional CVAR system for $Y_t|Z_t$ can be written as

$$\Delta Y_t = \omega \Delta Z_t + \alpha_y(\beta', \gamma)\binom{X_{t-1}}{t} + \sum_{i=1}^{k-1}(\Gamma_{y,i} - \omega \Gamma_{z,i})\Delta X_{t-i} + (\mu_y - \omega\mu_z) + (\Phi_y - \omega\Phi_z)D_t + (\varepsilon_{y,t} - \varepsilon_{z,t}) \tag{5.3}$$

Following (Kurita & Nielsen, 2018) one can allow different types of structural breaks in its deterministic component. Under $\alpha_z = 0$, weak exogeneity enables us to make conditional statistical inference about parameters of interest in β without any loss of information.

For strong exogeneity, we will define $\Gamma_{z,i} = (\Gamma_{zy,i}, \Gamma_{zz,i})$ and introduce a joint condition that both $\Gamma_{zy,i} = \alpha_z = 0$ holds for *i=1,2,3,...k-1*. If this joint condition holds, then Z_t is said to be strongly exogenous for parameters of interest β and the marginal model for Z_t is reduced to be in the following form:

$$\Delta Z_t = \sum_{i=1}^{k-1} \Gamma_{zz,i}\Delta Z_{t-i} + \mu_Z + \Phi_z D_t + \varepsilon_{z,t} \tag{5.4}$$

Therefore, Y_t does not granger cause Z_t. Under strong exogeneity, it is feasible to find multi-step forecasts Y_t using partial or conditional model (5.3) based on the series of forecasts of the marginal model Z_t in (5.4).

Lastly, to define the term SupExt, let us review the notion of invariance. The parameters of a partial model are said to be invariant to a class of interventions, such as changes in economic policy and regimes, if the parameters are constant over a certain period of time during which such interventions are actually observed. In the context of the above CVAR system, Z_t is judged to be super exogenous with respect to β if Z_t is weakly exogenous for β and the parameters of (5.3) are invariant to a class of interventions. There are various ways to model interventions. Clements & Hendry (1999) demonstrates that SupExt is closely associated with conditional co-breaking by linking interventions to deterministic breaks. For further details of co-breaking (*see;* Hendry & Massmann, 2007). In order to provide their argument here in a simplified manner using (5.3) and (5.4), we specify $D_t = 1_{(t=h)}$ for s = 1, where $1_{(.)}$ is an indicator function which is assigned 1 when a condition inside the brackets holds true and 0 otherwise, and h denotes a point of time over 1,…, T. That is, a deterministic break (corresponding to a mass of interventions) occurs at t = h. Conditional co-breaking occurs when

$$\Phi_z \neq 0 \text{ and } \left(I_q - \omega\right)\Phi = \Phi_y - \omega\Phi_z = 0 \tag{5.5}$$

under which the marginal model (5.4) is affected by the break but the partial model (5.3) is free from influences of the break, so that invariance holds for the partial model's parameters. See also (Kurita & Nielsen, 2018) for a different class of deterministic breaks in the partial CVAR framework. The combination of $\alpha_z = 0$ and the co-breaking conditions (5.5) allows us to regard Z_t as super exogenous w.r.t β. This combination is referred to as SupExt conditions. SupExt secures the partial model (5.3) against the Lucas critique, so that the model can be utilized for the purpose of conducting policy analysis. Justification of using the partial model as a policy-simulation tool is a great merit of SupExt, but there are some other useful implications of this concept for CVAR-based econometric studies. To the best of our knowledge, such implications associated with CVAR models have not been explicitly demonstrated yet in literature.

The above discussion here, gives an idea how one can adjust in CVAR models while testing exogeneity. However, it is still worth noting that and will be of a high value to the theoretical econometric literature if one could discuss and compare the performance of SupExt tests in case of CVAR models. This can be a new area for future research. In next chapter, we'll cover our applied side argument while modelling a stable money demand function in presence of structural break like IIS, SIS and TIS or others as discussed in Chapter 3 and testing SupExt of the putative regressors in the estimated conditional model.

5.7 Conclusion

The chapter above covers the argument about the performance of SupExt tests under consideration. On the basis of above concrete and detailed analysis of SupExt tests and their performance under non-stationary and dynamic data settings while considering structural breaks of the type IIS, SIS & TIS separately and all at a time jointly. All experiments have been repeated for 100,000 times where simulations were done in MATLAB. The results of the experiments highlight the fact that under non-stationary data settings the power of these test reduced by a significant amount. However, *IB-Test* and *RB-Test* perform better that other SupExt tests.

Now when we used dynamic data settings (2-lags) one can easily see that the amount of size distortion significantly decreased and the power of the SupExt test increased by a considerable amount. For small sample of 50 using IIS *CB-Test* seems good but as sample changes from 50 to 100 and then 200 the power of *CB-Test* reduced but the power of *IB-Test* and *RB-Test* show similar trend. Though, *IB-Test* seems better than other SupExt test by means of its power. As a whole we can conclude that whatever is the type of break the test like *IB-Test* and *RB-Test* are better while implementing SupExt of the putative regressors in conditional model. Lastly, as the power of the tests is increased using all breaks at a time. Therefore, we recommend while testing SupExt using all breaks at time is more informative and useful than individual scenario.

CHAPTER 6

EMPIRICAL MODELING OF MONEY DEMAND

"Essentially, all models are wrong, but some are useful"

Box, George E.P.

In this chapter we tried to analyse and to check the stability of money demand (M2) model in case of Pakistan under the shade of SupExt testing procedures following (Hendry & Ericsson, 1991b; Hendry & Santos, 2006) with an amalgamation of recently developed techniques of selecting breaks or location shifts (data driven) *i.e.* Indicator Saturation like; Impulse Indicator Saturation (IIS), Step Indicator Saturation (SIS) and Trend Indicator Saturation (TIS) proposed in (Ericsson, 2012). The estimated Vector Error Correction Model (VECM) of money demand (M2) with Real Income (GDP), Inflation Rate (CPI), short and long term interest rates as Call Money Rate and Government Bond Yield respectively, Financial Innovation and Financial Development respectively; reveals that the parsimonious model is structurally invariant and remain super exogenous to relevant class of interventions and for parameters of interest during the stipulated period (1972-2018) in Pakistan and hence can be used for policy purposes. Further, the application of several post estimation tests hinges that the estimated dynamic model is stable. This covers the empirical/applied side argument posted earlier in study objectives.

6.1 Introduction

Demand for money has received an enormous attention from researchers in Pakistan since early 1970's. Some of the studies have used classical econometric techniques like Classical Regression and OLS[24] in estimating the demand for money (Abe et al., 1975; Ahmad & Khan, 1990; Akhtar, 1974; Khan, 1980; Mangla, 1979; Nisar & Naheed, 1983), but results produced in these studies were mainly misleading due to the usage of small data sets and some found insignificant results or even did not pass the stability tests, if checked. Since the introduction of cointegration technique, many researchers in Pakistan have attempted to re-estimate the demand for money function. The use of cointegration technique has brought to light some controversies regarding the estimation of the money demand function. These controversies include the choice of satisfactory scale variable, opportunity cost measures of holding money, as well as, the appropriate functional form of the money demand equation. Although the importance of some measures of income in the money demand function has always been supported, there has been no consensus on the importance of the interest rate part. Some studies found interest rate to be a significant variable while other failed to find its significance in the demand for money. Furthermore, these studies have ignored the influence financial innovation, financial development and data driven structural breaks which brings up the need for further empirical modeling on the issue.

The State Bank of Pakistan (SBP) initially used narrow money (M0) to target broad money (M2) as an instrument till 2008 to achieve a dual macroeconomics objectives of price stability and output growth (Shafiq & Malik, 2018), but after that in August 2009, SBP established an Interest Rate Corridor (IRC) as a policy rate with

[24] Ordinary Least Square

SBP reverse repo rate and SBP repo rate named as ceiling and floor rate respectively. The goal of introducing IRC is to promote stability in money market and strengthening the transmission of monetary policy resulting in stable prices ultimately.

The standing literature so far has been able to classify several reasons of causing instability in money demand function includes – structural breaks in economy, degree of monetization, financial innovation and divergences between money supply and money demand (Khan & Hossain, 1994).

A plethora of studies have focused to single out the relevant determinants of money demand function in Pakistan from 1990's to onward like (Abbas & Husain, 2006; Ahad, 2017; Ahmad et al., 2007; Anwar & Asghar, 2012; Hossain, 1994; Iftekhar et al., 2016; Khan & Hossain, 1994; Khan & Hye, 2013; Qayyum & Azid, 2000; Qayyum & Khan, 2003; Qayyum, 1998; Sarwar et al., 2010) while on the stability of money demand function several studies are also available like (Asad et al., 2011; Faridi & Akhtar, 2013; Khan & Hye, 2013; Omer, 2010; Qayyum, 2005; Sarwar et al., 2013). For any monetary policy analysis, the stability of the demand for money is considered to be of prime interest. The success of monetary targeting based policy significantly relying on stability of money demand model. For a money demand function to be stable, it is considered that the quantity of money is in all likelihoods related to a small set of variables which is fact linked money to the real sector of the economy (Friedman, 1987; Judd & Scadding, 1982). The extensive overview on literature summaries and importance of the topic has been observed an increase in research and can be viewed in (Goldfeld & Sichel, 1990; Omer, 2010; Sriram, 1999).

It is argued that the stability of the money demand model is largely affected by location shifts in the economy and as a result leads to an ineffective monetary targeting. Researchers would may be in need of using data driven shifts (dummies) spanning over the sample against model constancy (Hendry & Ericsson, 1991b).

To the best of our knowledge so far, whilst modeling money demand round the globe in general and in Pakistan particularly; hardly one can find a single paper incorporating Indicator Saturation with its types proposed in (Ericsson, 2012) to capture the effect of shifts in the data. Further, the empirical modeling of money demand function along with application of SupExt testing procedures (Hendry & Ericsson, 1991b; Hendry & Santos, 2006) make this piece of study a worth producing in the field of applied econometrics.

The plan of the study is as follows: Section I will discuss the issues while modeling the demand for money and their remedial measures. A detailed review of literature on modeling money demand nationally and internationally will be discussed in Section II. Data, Model and Empirical Methodology opted in this study will be discussed in Section III. The key finding of this applied side argument and their interpretations will fall under Section IV and lastly, Section V will highlight the several conclusions and policy recommendations for deciding an optimal monetary strategy. Note that, key concepts like testing unit root, cointegration and estimating short run dynamics like ECM will precisely be discussed in Section IV not in Section III.

6.2 Review of Literature

This section will try to envelop the key literature on modeling the demand for money in Pakistan as well as a bit skirmish to cover it round the globe. This type of literature review is not being explicitly discussed and analysed in previously available studies in Pakistan, making this section a worth reading for those interested in modeling money demand.

(a) Literature available in Pakistan:

Despite an impressive and worth reading number of studies that tried to estimate the demand for money in Pakistan since early 1970's. Many researchers serve their valuable contributions to model demand for money in case of Pakistan. However, some of them have focused on in depth stability of demand for money and none of the available literature tried to model the effect of structural changes (data driven) while modeling money demand in Pakistan. On this ground one would say that all those models were not well specified and leaving a loop to be fulfilled in this field. The following are some mainly cited studies in case of Pakistan, mentioned in a chronological order.

Akhtar, (1974) considered as the first case study to be mentioned here and can't be ignored for estimating the demand for money using data on M1, M2, National Income, CPI, Aggregate Investment, Interest Rate (Include Govt. Bond Yield and Call Money Rate) over the sample period of 1951-1970. The methodology used in study was to compare the complementarity and substitution hypothesis by employing the classical regression techniques, came up with a conclusion that the substitution effect is dominant to complementarity effect and a more empirical analyses is required on the efficacy of complementarity hypothesis.

Abe et al. (1975) used M1, M2, Net National Product, CPI, Domestic Savings, Govt. Bond Yield and Call Money Rate data set as relevant determinants of money demand over the period of 1951-1970. The study pointed out the dominance of complementarity hypothesis over the substitution using classical regression technique & 2SLS.

Mangla (1979) concluded the fact that money demand function in Pakistan is stable over 1958-1971 with Real M1, Real GNP, Real Permanent Income, Call Money rate, Govt. Bond Yield for real side and same for nominal side model. The study suggested that a more dynamic method should be applied to check the parsimony and stability of the model.

Khan (1980) using OLS over the period of 1959-1978 with M1, M2, GNP, Permanent Income, GNP Deflator, Rate of Inflation, Call Money Rate, Interest Rate (time deposit), No. of Bank Branches proxy for degree of monetization as key determinants. The study single out that the real side model performed better as compared with nominal side. It further suggested that the rate of inflation is important determinant for post-1971 and emphasis on the usage of quarterly data and found is no evidence for usage of permanent income over measure income.

Nisar & Naheed (1983) focused on the usage of term structure interest rate while modeling demand for money along with M1, M2, GNP (Income), Call Money Rate and Rate of Inflation. The study used relatively large data set as compared to previously discussed literature over 1958-1979. The key findings of the study were stability of money demand under term structure specification using OLS. However, without it M2 equation for money demand did not pass the stability test. Further, it was suggested that the variable proxied for monetization renders to specification bias, if not included.

Ahmad & Khan (1990) used estimation technique of MLE with varying regression parameters capturing M1, M2, GNP (Income), Interest Rate (Call Money rate & Weighted Average of time Deposit Rate) as potential determinant resulting into an unstable money demand model. The study suggested that the stability can be achieved by inclusion of banking system based on Islamic law.

Khan (1992) reported that interest rate is insignificant while modeling money demand using M1 in case of Pakistan following classical linear regression methodology for the period of 1967-1987 with M1, M2, Income, CPI and Interest Rate as potential contributing factors to be used in the study.

Khan & Hossain (1994) introduced Engle Granger (EG) cointegration and Error Correction Mechanism to model money demand in Pakistan using quarterly data set for 1971:III-1993:II. The study concluded that incorporating financial liberalisation does not cause instability in the model with other relevant variables Real Income, Interest Rate (Short & Long Term) and Inflation Rate to model Real M1, Real M2.

Qayyum (1998) was the one to introduce Johansen and Juselius (JJ) cointegration approach and seasonal dummies with Error Correction Mechanism (ECM) to model money demand in Pakistan. The study used Real M2, Real Income, Govt. Bond Yield, Inflation Rate and seasonal dummies (but not data driven) as possible variables. It suggested that the Error Correction model is stable and an appropriate one to study money demand in Pakistan.

Khan et al. (2000) highlighted that the forecast performance of Cointegration equation is better than the Error Correction model for the period of 1971-1998 using M2, Real GDP and Inflation Rate. It was suggested that a disaggregated approach to model money demand is more useful in Pakistan.

Qayyum & Azid (2000) used quarterly data set for an extended period of 1960:I-1991:II enveloping variables Real M1, Real Sales, Vector of Interest Rate (Call Money rate, Govt. Bond Yield and Interest on Bank Advances) and Inflation Rate. The estimated dynamic model was stable using JJ and ECM methodology and identified that inflation rate is significant determinant of real money balance demand by business sector. Qayyum & Khan (2003) considered the degree of sterilisation that the Pakistan has used for controlling capital flows and the estimated frugal money demand model over 1982:III-2001:II is stable using JJ Cointegration and ECM. Furthermore, it is being suggested that to explore the degree of sterilisation, the development of credit policy reaction function is required.

Qayyum (2005) estimated a dynamic Error Correction model along with JJ cointegration. The only study found by applying post estimation stability test of SupExt in case Pakistan using annual sample space from 1960-1999. The author concluded that the estimated model is stable and super exogenous against the relevant class of interventions. However, the breaks introduced in the study were mainly a structural or theory driven (definitional) but not data driven, leaving a space to be fulfilled behind for not having all data driven breaks, that their estimated model remain unable to capture may cause an instability in the model.

Abbas & Husain (2006) showed that the feedback effect from price to money is weak and bidirectional causality between prices and money using Granger Causality and ECM. The annual data set over 1959-2003 and with M2, GNP and GDP deflator as key indicators is used to model the demand for money.

Ahmad et al. (2007) highlightened the importance of using Real Money Balances (M1, M2), Real Income and Interest Rate to model money demand by employing EG and JJ approach for the sample of 1953-2003. On concluding side, both real income and rate of interest were found to be weakly exogenous in long run money demand model and suggested that interest rate can't be used to regulate demand for money in Pakistan. Moinuddin (2009) tried to focus on log linear functional form using Real M2, Real GDP and Real Interest Rates, concluding that monetary aggregate targeting is not suitable as money demand function found to be unstable.

Azim et al. (2010) used ARDL methodology to model long run money demand function between broad money (M2) and exogenous variables like Real GDP, Inflation and Exchange rate for the sample period of 1973-2007. The stability of the model was confirmed using CUSUM and CUSUM square test. Asad et al. (2011) approached to model the demand for money over sample period 1980:I-2009:II. The study opted ARDL to model Real M2 against Real GDP, Domestic and Foreign Rate of Interest and Real Effective Exchange Rate. The key finding of the study is that amid high inflation periods people tend to invest in physical assets than monetary. While on the stability side the absence of structural break in the model was reported.

Anwar & Asghar (2012) used annual time series data from 1975-2009 following ARDL methodology and put emphasis on the long run stabilization of policy in Pakistan. The variables used in this study were M1, M2, Real GDP, GDP deflator and Exchange Rate and the model found to be stable without mentioning any structural break in data. Sarwar et al. (2013) used three aggregates of money like: M0, M1 and M2 with Real DGP, Interest Rate and Financial Innovation. The study argued that M2 is an appropriate measure or aggregate to provide stable money demand in Pakistan using JJ and Error Correction model for the period of 1972-2007.

Khan & Hye (2013) was of the pointed that the estimated model considered to be stable but finding no structural break during the selected period 1971-2009. While using JJ cointegration and ARDL approaches to model M2 with GDP, Exchange Rate, Interest Rate and Financial Liberalization as goal variables, found no structural break in the model amid the selected period. Faridi & Akhtar (2013) suggested that the higher degree of financial innovation needs to be hurled in order to promote business and economic activities. The study used ARDL approach to annual time series data varied over 1972-2011. The variable total population which was not being used in previous literature, considered among other relevant factors while modeling real money.

Iftekhar et al. (2016) has taken the annual sample data from 1972-2013 to model M2 with Exchange Rate, GDP per Capita, Fiscal Deficit (%GDP), Urban and Rural Population and Real Interest Rate. The methodology opted in study is ARDL and VECM, highlighting a fact that for stabilization, there is need to control unskilled workers in rural areas and high inflation and exchange rates.

Ghumro & Karim (2017) examined the dynamic relationship between the series of monetary aggregates M1 and M2 and real income, discount rate, inflation rate, real exchange rate, and remittances over the sample of 1972–2014 in case of Pakistan. Using ARDL bound testing approach money demand functions are stable and suggested that in Pakistan remittances are used for the consumption purposes. The speed of adjustment for M1 model is faster than that of M2 model with remittances.

Ahad (2017) used Bayer-Hanck combined CI, JJ and VECM taking Real Income, Industrial Production, Financial Development and Exchange Rate as key factors affecting M2. It is suggested that to stabilize money demand function, policy makers should focus on Financial Development in short and long run as well. Last but not the least, (Shafiq & Malik, 2018) used quarterly data for sample space of 1981:I-2017:II. The inclusion of Asset Price Index (API) is considered to be a cause of stabilizing the demand for money in Pakistan suggesting that the money demand model is not correctly specified, if API is not included in the model.

So far we are in the position to identify that what should be the key factors to be incorporated while modeling money demand. By looking at this detailed review of literature in case of Pakistan, different researchers tried different methodologies to model demand for money in Pakistan, but to the best of our knowledge, considering the effects of structural breaks (data driven) in the data was completely ignored. So, the study contributes a step ahead in literature by fulfilling this gap.

(b) Literature other than Pakistan:

In this subsection a meticulous review of literature from some selected countries over the past two decades, is being discussed. The studies here presented in a descending order for sake of concision to the readers, may help them out in identifying the specific year.

Barnett et al. (2022) investigated the long-term relationship between real money balances, real output, interest rate, and real effective exchange rate, using a modern version of the linear time-series macroeconometric model. Evidence of stable demand for money is found. Broad money, in general, captures a more stable demand for money than narrow money. The study used quarterly data for the European Monetary Union, India, Israel, Poland, the UK, and the US.

ISHII, (2022) discussed about the monetary policy framework has changed since the introduction of inflation targeting in Thailand. The study came up with four main outcomes. First, changes in the monetary policy framework did not change the model of the money demand function. Second, the adoption of inflation targeting policy leads to structural changes. Third, the effects of monetary policy changed with the adoption of inflation targeting policy. Interest rate elasticity is positive before the framework change but negative after the policy change. However, its value is weak. Fourth, the interest rate elasticities of M2 and r are stable and predictable.

Adil et al. (2022) checked the stability issues of real money balances considering financial development. The study found, real narrow (M1) and broad (M3) money demand in India during the post-financial reform, from 1996:Q2 to 2016:Q3. The study used the autoregressive distributed lag model of cointegration and other various time series techniques. After incorporating financial development into money demand, they determined short- and long-run relationships and a well-defined open-economy stable money demand specification (M1 and M3) in India.

Dritsaki & Dritsaki, (2022) aims to investigate the stability of money demand in the case of Korea. Since the economic reforms in Korea faced considerable structural changes, it was difficult to formulate a stable money demand function. The uses of unit root and cointegration tests with structural breaks suggest that economic and financial deregulations have influenced the stability of money demand function Korea.

Rathnasiri, (2021) empirically gauge the determinants of money demand function in Sri Lanka over the period 1977-2019. This study estimated both short run and long run money demand function using monetary aggregates M1 and M2 based on time series data. The stability test showed that the both M1 and M2 money demand functions are stable.

Adil et al. (2020) estimated demand for money in India during the post-reform period, from 1996:II-2016:III. The money demand function is estimated with the linear ARDL approach to cointegration with bounds testing approach. The study employs various proxies for financial innovation and weighs the relative importance of financial innovation variables in the money demand equation, and finds that financial innovation exhibits a very significant role in the money demand modelling and its stability.

Dritsaki & Dritsaki (2020) investigated the factors that influence money demand in Italy for the period 1960-2017. Real income, interest rate and inflation imitate with the expectations of monetary theory. ADF, PP and KPSS unit root tests were applied. ARDL and ECM were applied, while CUSUM and CUSUM of squares used to evaluate parameter's stability. The stability tests and unit circle confirmed the long run relationship among variables. At the end, the stability condition is satisfied when money demand is estimated using the demand for narrow money (M1).

Rasasi (2020) tried to investigate the stability of money demand function for Saudi Arabian economy over 2007:I-2018:II using various structural break tests. The estimated money demand function also shows the impact of real non-oil income on money demand is consistent with theory in addition to a positive impact of exchange rate and interest rate on the demand for money. JJ cointegration and ECM were applied and obtained results revealed the stability of money demand function.

Nel et al. (2020) expounded a key finding that the speed of adjustment to equilibrium for VECM is better than ARDL using data over 1995:I-2013:IV for the case of Hungary. The study further revealed that monetary policy should be based on M1 rather on M2. The results based on ARDL and VECM depicted that the estimated model is stable.

Ebadi (2019) elucidated the effect of government spending on money demand in the US using data over 1973:I–2013:IV. The related potential determinants were income, interest rate, exchange rate, and inflation. It proposed a new method of income decomposition to the public sector and private sector and applied ARDL. The results confirmed the long-run, significant effect of government spending on money demand, finding the elasticity of money demand with respect to government spending to be 0.62. In addition, worth noted that money demand tends to be unstable and shifts toward the edge of a structural break during recessions.

Adhikari (2018) tried to envelop the long run and short-run dynamics between broad money, consumption expenditure, capital stock and interest rate in Nepal over the period of 1975-2017 using ARDL bound testing approach. The empirical results show that the demand for money is affected by the interest rate and final consumption expenditure but not by the gross fixed capital formation. On contrast, interest rate is positively associated with Broad money demand, which is not consistent with theoretically. It suggested, correcting price fluctuation through the control of various expenditure components, particularly, real final consumption expenditure might be an important strategy for the long run.

Bahmani-Oskooee & Maki-Nayeri (2018) portrayed that when the linear ARDL model estimated, policy uncertainty had short-run effects only. However, estimates of the nonlinear ARDL model exposed that policy uncertainty have both short-run and long-run effects on money demand in the U.S. and were asymmetric.

They used data on M2, GDP, interest rate (3-months treasury bills), GDP deflator, index for nominal exchange rate and policy uncertainty over the sample for 1985:I-2017:IV.

Cho & Ramirez (2016) estimated the demand for real money in Korea over 1973:III-2014IV sample. Applying JJ cointegration methodology, the Pantula principle and granger causality, it is stated that a long-term relationship exists among the variables. The paper also estimated ECM as well as a VECM. It is found that, M2, served as a relatively better measure of the money aggregate than M1. The long-term interest (LR) rate also seems to provide better results than the short-term rate (SR), which is consistent with economic theory given that it refers to a long-run equilibrium relationship. Granger block causality tests and impulse response functions suggested that the traditional money demand function which places M as its 'dependent' variable, while including income and interest rates as its regressors displayed a robust and stable model for Korea.

Ogbonna (2015) conducted a study in Nigeria using monthly secondary data from 2005-2013 to examine the impact of the black market or official exchange rates on the demand for money function. Co-integration, CUSUM, CUSUMSQ test and VECM were used. The study revealed that in all variants of the demand for money model, coefficients of exchange rate variables went significant. Further, it was suggested that stability in foreign exchange market would foster stability in money demand in Nigeria.

Azeem & Mohammad (2015) conducted a study to examine the performance, money demand, interest rate and investment size in order to generate the prospective relationship of Turkey. Secondary data from electronic data distributed system of

Turkey's central bank for a periods of 1999:1 to 2014:4 were used to investigate the relationship between money and physical capital, methodology of ARDL test, ADF and PP test, unit root test, Dickey-Fuller test were estimated. The study estimated the demand for money demand equation and the investment rate in a statistically significant and positive interaction were detected. Turkey economy was based on limited complementary relationship between money and physical capital.

Ben-Salha & Jaidi (2014) aimed to find determinants of money demand in Tunisia using annual data series 1979-2011. The main object was to estimate the money demand function of Tunisia and stability of demand for money. ARDL bounds testing approach, Co-integration, ECM, Chow stability test, Hansen parameter instability test, CUSUM, CUSUMSQ test were used to testify the stability over a period. Saikkonen-Lutkepohl co-integration test with structural shift and Johnason-Mosconi structural break co-integration test were used to control for structural change. The study further argued that final consumption expenditure and interest rate were the main determinant for money demand function.

Nyong (2014) studied the demand for money with structural break and monetary policy in the Gambia over 1986-2012. Gregory-Hansen Co-integration technique, residual test, CUSUM test, CUSUMSQ test, Phillips-Perron unit root test, GLS , ECM were estimated for structural breaks to establish stability. A structural break was occurred in 1995 and affected the military coup and declined foreign aid. The existence of dynamic short run ECM to intercept best characterized the equilibrium relationship of the money demand function structural breaks.

Nduka (2014) examined a structural breaks and long run demand for real broad money function in Nigeria using annual data for the sample period of 1970-

2012, model used ADF and PP tests for unit root, Gregory and Hansen co-integration to seizure endogenous structural breaks in the model, CUSUM, CUSUMSQ test for structural stability were used. It revealed a long run relationship among real broad money, real domestic interest rate, real income, real exchange rate of inflation and foreign interest rate. CUSUMSQ test indicated that demand for money was temporary unstable.

Kumar (2014) determined a study on the stability of demand for money in India using monthly data from 2005:IV-2014:II. The objective of the study was to analysis short run and long run determinants and stability of money demand in India. For model specification, ECM, JJ cointegration, CUSUMSQ test, CUSUM test were used. CUSUM and CUSUMSQ test suggested that long run real broad money was stable and narrow real money was unstable in case of India.

Abdelnacer et al. (2013) examined the effect of black market exchange rate of demand for money in Algeria. Secondary quarterly data over the period from 1974:I-2003:III were taken from the International Finance Statistics. For estimation of black market exchange rate, ARDL, CUSUM test, CUSUMSQ test were used. The study indicated that inclusion of black market exchange rather than official rate issue. It examined a strong effect of demand for money in Algeria.

Suliman & Dafaalla (2011) tried to estimate a stable money demand function in Sudan during the period 1960-2010. The model enveloped real money balances, real GDP, the rate of inflation and exchange rate and applied cointegration and ECM. The estimated coefficients for long and short run are consistent with the economic theory and short run estimates were reported to be weaker in magnitude than those related to the long-run equilibrium. The estimated model is stable in case of Sudan.

Achsani (2010) conducted a study on the stability of money demand in an emerging market economy of Indonesia over the sample of 1990:I-2008:III and applied VECM and ARDL. The study implied that the real demand for money M3 was co-integrated with interest rate and real income and concluded that ARDL was more appropriate as compared to VECM.

Hossain (2006)investigated money demand behavior in Bangladesh by taking the annual data from 1973-2003 and estimated the dynamic behavior of demand for money. For econometric analysis, co-integration, ECM and Quandt Likelihood Ratio Test were used. The study implied that demand for money had structural break in the year 1987 in case of narrow money and in 1983 in case of broad money.

Ramachandran (2004) conducted a study on the stability relationship among m3 money, price and output in India. For the investigation of broad money, price and output stability, secondary data was used from the period from 1950-51 to 2000-01, and methodology of error correction model and co-integration were employed to test the structural break. The study revealed that the real m3 money demand and real income had stable relationship. During the period from 1978-1980, indicated the possibility of conventional stability.

Pradhan & Subramanian (2003) conducted a study on the stability of demand for money using monthly data set spanning over the period of 1970:IV-2000:III. The purpose of the study was to accentuate the financial innovation of the stability of demand for money in developing countries. The study used cointegration and VAR methodology and portrayed that the long run relationship of demand for money is stable in spite of financial regulation and innovation.

(c) Data, Model & Empirical Methodology

In this study we use annual time series data ranges over the period of 1972-2018. Data for Money Demand (Broad Money, M2), Narrow Money (M1), Price Level (GDP Implicit Price Deflator), Gross Domestic Product (Economic Activity, GDP), Call Money Rate (Short Term), Government Bond Yield (Long Term), and Rate of Inflation (CPI) is gathered from different data sources like; SBP, WDI and IFS[25]. For financial side components two proxies are used one is Financial Innovation and other is Financial Development.

According to (Friedman, 1987) theory has been able to identify key determinants of real money demand balances. Hence, we establish a money demand function relating the real money demand ($RM2_t = M2_t/P_t$) to real income ($RY_t = Y_t/P_t$) for scale variable, a vector interest rate variables representing opportunity cost of holding money and other key determinant found in literature. In functional form it may be written as:

$$M2_t/P_t = f(Y_t/P_t, CMR_t, BY_t, CPI_t, FI_t, FD_t, \mu_t) \qquad (6.1)$$

Where

$M1_t$ = Narrow Money in Billion Rs., comes from source (a)

$M2_t$ = Broad Money[26] in Billion Rs., comes from source (a & b)

P_t = Price Level (GDP Implicit Price Deflator), comes from source (b)

Y_t = Gross Domestic Product in Billion Rs., comes from source (b)

[25] **(a)** State Bank of Pakistan **(b)** World Development Indicator **(c)** International Financial Statistics

[26] Reason to take broad money definition of money because, SBP use M2 as the main target variable to demeanor Monetary Policy in Pakistan

CMR_t = Call money Rate, comes from source (a & c)

BY_t = Government Bond Yield, comes from source (a & c)

CPI_t = Consumer Price Index, comes from source (b)

FI_t = Financial Innovation $\langle {M2_t}/{M1_t} \rangle$, source already discussed

FD_t= Financial Development is Domestic Credit to Private Sector (% GDP), comes from source (b)

μ_t = White noise[27] error term

The following steps explain the structure of the empirical modeling strategy that used in this paper:

i) (Ehrlich & Gibbsons, 1977) and (Seaks & Layson, 1983) claim on theoretical and empirical grounds that the log linear form is considered to be more superior to simple linear form. Further, (Ehrlich, 1996) and (Schrooten & Stephan, 2005) suggested that a log-linear form is more likely to find evidence of a restraint effect than a linear form. So, all variables are used in in their natural logarithmic from to obtain more robust estimates of the parameter and their pliability can be seen via *Figure 1*. through *Figure 4 (see Appendix)*.

ii) Unit root tests like; Augmented Dickey-Fuller (ADF) Test by (Dickey & Fuller, 1979) and Phillips-Perron (PP) Test by (P. Phillips & Perron, 1988) use to find integration order of the series. The stationarity of these series is determined with the existence of a unit root.

[27] A stationary process with all of its autocorrelation functions equal to zero

iii) Variables found to be integrated of order one *i.e.* I (1), therefore (Johansen, 1988) maximum likelihood estimation approach was used to test the co-integration between variables, if exist.

iv) The lag length of the unrestricted VAR model proposed by (Sims, 1980b) is specified before the co-integration test. The VAR model treated every variable within the system and the equations of these variables were estimated with lagged values. Since the variables are I(1), the first cointegrating equation normalized on log of real money balances considered to be the long run equation.

v) The Dynamic Vector Error Correction Mechanism (VECM) by (Sargan, 1964) use to obtain the short run adjustment dynamics.

vi) Engle et al. (1983) explains different concepts on weak, strong and SupExt. Finally, we test the fragility of the estimated model *i.e.* whether it is used for forecasting or for policy analysis by analyzing its parsimony against SupExt testing procedures. Several tests on SupExt are available in literature like; (Hendry & Ericsson, 1991b) and (Hendry & Santos, 2006). The paper is a scuffle to apply these procedures on money demand model in case of Pakistan.

vii) Lastly, to capture breaks, crises or location shifts in the data, we use Impulse Indicator Saturation (IIS), Step Indicator Saturation (SIS) and Trend Indicator Saturation (TIS) proposed in (Ericsson, 2012), using a nicely written Package “gets” by (Sucarrat et al., 2020).

At the end, different diagnostics tests are applied to check the parsimony of the estimated model. For basic description of the methodology like unit root tests, VAR model, cointegration and VECM and their interpretation *see* section IV.

However, the concept of SupExt and its testing procedures have explicitly been discussed in this section for the reader.

Engle et al. (1983) explained the different concepts of weak, strong and SupExt. There are three main purposes of the model which are whether it can be used for the statistical analysis, for multi-step ahead prediction or it can be used for policy purposes. The answer lies in weak exogeneity, strong exogeneity and in SupExt respectively. A valid exogeneity assumption could encompass any or all of inference, forecasting, and policy. But if these assumptions are invalid, then estimation of the conditional model alone can lead to a wasteful or unreliable inferences, and thus the result obtained is a misleading one.

Now in statistical terms the dynamic joint density function can be written as $F_X(X_t, \Theta)$. If we bifurcate X_t into m_t[28] and z_t representing the determinants of the targeted variable *i.e.* (y_t, cr_t, by_t, i_t, fi_t, fd_t). Then the joint density function $F_X(X_t, \Theta)$ can be further factorized into the conditional density function of m_t given z_t i.e. $F_{m_t|z_t}(m_t|z_t, \lambda_1)$ times the marginal density function *i.e.* $F_{z_t}(z_t, \lambda_2)$. The relationship between the conditional and marginal model is

$$F_X(m_t, z_t|X_{t-1}, \Theta) = F_{m_t|z_t}(m_t|z_t, X_{t-1}, \lambda_1).F_{z_t}(z_t|X_{t-1}, \lambda_2) \qquad (6.2)$$

Now the dynamic conditional density function of money demand can be written as:

$$\Delta m_t = \omega_1 \Delta z_t + \omega_2 t + \sum_{i=1}^{k} \Gamma_i \Delta m_{t-i} + \Pi \sum_{i=1}^{k} \Psi_i X_{t-i} + \mu_t + D_{1t} + \varepsilon_{1t} \qquad (6.3)$$

And the marginal density function of z_t is written as:

[28] m_t or even others in small italic are variables in their logarithmic form.

$$\Delta z_t = \alpha_z \beta' X_{t-1} + \sum_{i=1}^{k} \Gamma_i \, \Delta X_{t-i} + \mu_t + D_{2t} + \varepsilon_{2t} \tag{6.4}$$

This sort of factorization allows us to test the system for the presence of weak exogeneity of the parameters in the (6.4). Weak exogeneity requires that the parameter of interest is the function of conditional model parameters only and parameters of conditional and marginal model are variation free[29]. If weak exogeneity holds, the model in (6.3) can be analyzed without specifying exactly how z_t, is determined. If weak exogeneity condition holds then testing and efficient estimation can be implemented by using conditional model and ignoring the information of the marginal model.

The conditional model in (6.3) considers the instant impact that change in z_t has on the change in m_t. The term ΠX_{t-1} (with the condition $\Pi = \alpha\beta' < 0$) indicates the impact on change in m_t of having m_{t-1} out of the equilibrium with βz_{t-1}. The long run error correction model requires that $m_t = \beta z_t$. The parameters of the conditional as well as the marginal model are interrelated if the cointegrating vector β enters into the (6.3) and as well as into the (6.4). So, to get inferences about the parameters efficiency, a full system is required. If the conditional model is not invertible into marginal model then it can be used as confirmation of super-exogeneity because the invertibility of conditional model into marginal model is prohibited if the variables are super exogenous for the parameters of the conditional model (Hendry & Ericsson, 1991b).

Considering the dummy saturation method, three of them are being used here in marginal model via their individual DGPs. IIS (Impulse indicator Saturation) is a

[29] Variation free means there is no cross restriction on the parameters of conditional and marginal models or parameters can take any values within their range.

set of zero-one dummies *i.e.* I_{it} = 1; for t = i and zero otherwise. SIS (Super Saturation) is set of step dummies *i.e.* S_{it}= 1; for $t \geq i$ and zero otherwise. TIS (Ultra Saturation) is a set of broken linear trend dummies *i.e.* $T_{it}= t - i + 1$; for $t \geq i$ and zero otherwise. Throughout in our analysis of capturing dummies for the average retention of each type, we deliberately opt significance level at $\alpha = 0.05$. This may cause of capturing more impulses as compare to much tight level of $\alpha = 0.025$ or even with $\alpha = 0.001$.

Initially a test of SupExt is proposed by (Hendry & Ericsson, 1991b). This procedure can be applied to the marginal models for the putative conditioning variables. First, the associated significant dummies of each type in the marginal processes are recorded. Secondly, those which are retained are then added as set of variables in the conditional model. Specifically, after the first stage when m impulse indicators are retained, a marginal model has been extended to following form:

$$\boldsymbol{z}_t = \boldsymbol{\pi}_0 + \sum_{j=1}^{s} \Pi_j X_{t-j} + \sum_{i=1}^{m} \rho_{i,\alpha_1} 1_{\{t=t_i\}} + \varepsilon_t^* \qquad (6.5)$$

Where, the coefficients of the significant impulses are denoted ρ_{i,α_1} to emphasize their dependence on the significance level α_1 used in the marginal model. Note, this test has the appropriate null rejection frequency. The second stage of the testing procedure is to add these set of m retained dummies from the marginal model to the conditional model as below:

$$y_t = \mu_0 + \boldsymbol{\beta}'\boldsymbol{z_t} + \sum_{i=1}^{m} \tau_{i,\alpha_2} 1_{\{t=t_i\}} + \varepsilon_t \qquad (6.6)$$

Then conduct an *F-test* for the significance of impulses at level α_2.Under the null of super-exogeneity, check the joint significance of the m included impulse indicators in the conditional model with *F-test*.

Following (Hendry & Santos, 2006), a variant of the test discussed above, which could have different power characteristics, is to combine the *m* retained impulses detected in all the equations or from marginal model to form an Index with weights equal to the coefficients of the retained dummies in the marginal models by considering the following scenario. Suppose the third term on R.H.S in (6.7) represents significant dummies entering into our marginal model for instance:

$$\boldsymbol{z}_t = \boldsymbol{\pi}_0 + \sum_{j=1}^{s} \Pi_j X_{t-j} + \sum_{i=1}^{m} \rho_{i,\alpha_1} 1_{\{t=t_i\}} + \varepsilon_t^* \tag{6.7}$$

Here (6.8) is representing the formation of the index which will be used for checking the stability of the conditional model.

$$I_{1,t} = \sum_{i=1}^{m} \hat{\rho}_{i,\alpha_1} 1_{\{t=t_i\}}\,; \qquad Where\ \hat{\rho}_{i,\alpha_1} = \sum_{i=1}^{n-1} \hat{\rho}_{j,i,\alpha_1} 1_{\{t=t_i\}} \tag{6.8}$$

After that we use this index in our conditional model and test the null hypothesis that $\varphi = 0$

$$y_t = \mu_0 + \boldsymbol{\beta}'\boldsymbol{z_t} + \varphi I_{1,t} + \varepsilon_t \tag{6.9}$$

An alternative test with $T - n - 1$ *d.f.* and approximately distributed as *t-dist.* under the null of SupExt. Now for testing the failure of invariance property another index is formed in a way that the indices iterated with the conditioning variable or $\boldsymbol{z_t}$ as follows:

$$I_{2,t} = \sum_{i=1}^{m} \sum_{j=1}^{n-1} \hat{\rho}_{j,i,\alpha_1} z_{j,t} 1_{\{t=t_i\}} \ \ Where\ \hat{\rho}_{j,i,\alpha_1} = \sum_{i=1}^{n-1} \hat{\rho}_{j,i,\alpha_1} z_{j,t} 1_{\{t=t_i\}}\,; \tag{6.10}$$

Once the index is being formed then, test the significance of this index and the previous index jointly in the conditional model using *F-test* with 2 *d.f.*

$$y_t = \mu_0 + \boldsymbol{\beta}'\boldsymbol{z_t} + \varphi I_{1,t} + \theta I_{2,t} + \varepsilon_t \tag{6.11}$$

(d) Empirical Results and Interpretation

In previous section we have discussed in detail about taking all variables in their natural logarithmic form to control the variability. Here, *Table 1.* below indicates some basic descriptive statistics of the variables under autopsies with 47 observations each. It climaxes the fact that maximum diaspora occurs in inflation rate and then in real money stock. An interesting fact via this table can be seen is that mean and median for all variables to 1st decimal place with a little deviance are approximately equal and may lead us to conclude the symmetriness of the data. While for three decimal places, the data appear to be little skewed to the left, which explains through those values where mean is smaller than the median and vice versa, which further can be reconfirmed via Skewness. For the spread, most of the data values lie within ($mean \pm 3 \times S.D$) range. The value of Kurtosis for cr_t is more than 3 having heavier tails than normal distribution and for the rest it is less than 3 means to have lighter tails than normal distribution.

Table 6.1: Descriptive Statistics of Variables

Descriptive Stat./Variable	m_t	y_t	cr_t	by_t	i_t	fi_t	fd_t
Mean	3.068	3.851	2.096	2.163	3.335	0.576	3.138
Median	3.248	3.945	2.176	2.225	3.412	0.574	3.185
Maximum	4.091	4.878	2.523	2.592	5.105	1.001	3.394
Minimum	1.802	2.665	0.761	1.227	1.159	0.184	2.734
Std. Dev.	0.673	0.649	0.348	0.340	1.111	0.197	0.175
Skewness	-0.411	-0.263	-1.803	-0.722	-0.047	0.042	-0.825
Kurtosis	1.949	1.915	7.468	2.985	1.942	2.693	2.880

Source: All Calculations are done by the Authors

(e) Explicating General Trends in Data

The graph of Real Money Balances (m_t) with Real Income (y_t) in Pakistan over stipulated period, is shown in *Figure 1.* The graph signifies as random walk with drift since a definite upward trend is there for both data series. Therefore, they considered to be non-stationary series. The graph of cr_t and by_t in Pakistan is shown in *Figure 2*. It displays that both variables have sluggish longer term movements and these movements are customarily thought as stochastic trend as well as there is a change in the level of the series that are not predictable from the past history. Therefore, both series are considered to be non-stationary in case of Pakistan. The graph incorporating the series of i_t and p_t in Pakistan is shown in *Figure 3* over the period of 1972-2018. The graph of CPI and GDP deflator has almost likely to be a definite upward trend and these trends are often considered as deterministic trend. Therefore, these series can be considered as non-stationary in case of Pakistan. The graph of fi_t and fd_t in Pakistan is shown in *Figure 4.* It shows that both variables have sluggish longer term movements and these movements are often thought as stochastic trend as well as there is a change in the level of the series that are not predictable from the past history. Therefore, both series are considered to be non-stationary. For all these graphs see *Appendix* at the end.

(f) Tests for Unit Root

Previously, the time series data were considered to be stationary but as time moved on, the opening out in time series econometrics exposed that most of the time series data were non-stationary and if the data is non-stationary then the use of OLS method to analyze such data isn't appropriate at all (Granger & Newbold, 1974). To determine the order of integration of the series several unit root test are available in literature. However, we use ADF and PP unit root tests to check the presence of unit root in the data with three different specifications. $H_o: \rho \geq 1 \quad H_a: \rho < 1$. The *ADF-test* statistic follows non-standard limiting distribution and not the asymptotic standard normal distribution. The critical values were obtained and are available in (MacKinnon, 1996). If the value of *ADF-test* statistic is less than critical value 5%

level of significance then null hypothesis will be rejected and we conclude that series is stationary.

However, for *PP-test* the null and the alternate hypothesis are $H_o: \rho = 1$ $H_a = \rho > 1$. For results *see Table 2.* and *Table 3*. From each table one can easily conclude that all the variables in their logarithmic form are non-stationary at levels but stationary at first difference.

Table 6.2: Augmented Dickey-Fuller (ADF) Unit Root Test

(*ADF Equation;* $\Delta y_t = \alpha + \beta t + \rho y_{t-1} + \sum_{k=1}^{n} \lambda_i \Delta y_{t-k} + \varepsilon_t$)

Variable/Test Stat.	Levels				First Difference				
	K	τ (No intercept no trend)	τ_μ (Intercept)	τ_t(Intercept and trend)	K	τ (No intercept no trend)	τ_μ (Intercept)	τ_t(Intercept and trend)	Specification
m_t	1	2.540	- 0.901	- 2.258	0	- 3.610*	- 4.858*	- 4.830*	C, No t
y_t	1	3.243	- 1.434	- 1.439	0	- 1.505	- 4.508*	- 4.692*	C, No t
cm_t	0	- 0.274	- 2.872	- 2.892	0	- 5.440*	- 5.378*	- 5.308*	None
by_t	0	- 0.179	- 3.166	- 3.117	1	- 6.406*	- 6.339*	- 6.288*	None
i_t	1	1.957	- 0.645	- 4.038**	0	-2.145**	- 3.390**	- 3.296	C, No t
fi_t	0	- 0.899	- 0.861	- 1.152	0	-6.200**	- 6.171**	- 6.330**	None
fd_t	1	- 0.495	- 2.220	- 2.533	0	-5.261**	- 5.214**	- 5.154**	None

Note: $H_0(\rho \geq 1)$: I(1) is being tested against $H_a(\rho < 1)$: I(0). The lag length K is based on author's own choice for precise results and reconfirmed through Akaike Information Criterion (AIC). Here, *and ** show significance level achieved at 1% and 5% respectively.

Table 6.3: Phillips-Perron (PP) Unit Root Test

(*Model Equation;* $\Delta y_t = \alpha + \rho y_{t-1} + \sum_{k=1}^{n} \lambda_i \Delta y_{t-k} + \varepsilon_t$)

Variable/Test Stat	Levels			First Difference			
	τ (No intercept no trend)	τ_μ (Intercept)	τ_t(Intercept and trend)	τ (No intercept no trend)	τ_μ (Intercept)	τ_t(Intercept and trend)	Specification
m_t	3.559	- 0.612	- 1.797	- 3.535*	- 4.787*	- 4.709*	C, No t
y_t	7.452	- 1.929	- 1.497	- 1.231	- 4.565*	- 4.766*	C, No t
cm_t	- 0.287	- 2.872	- 2.892	- 5.326*	- 5.245*	- 5.144*	None
by_t	0.401	- 2.574	- 2.471	- 7.676*	- 7.755*	- 7.325*	None
i_t	4.729	- 1.800	- 3.801**	- 2.146**	-3.353**	- 3.216	C, No t
fi_t	- 0.899	- 0.861	1.173	- 6.183*	- 6.151*	- 6.466*	None
fd_t	- 0.767	- 2.145	- 2.289	- 5. 230	- 5.184*	- 5.121*	None

Note: H_0: ρ=0 is being tested against H_a: ρ>1. (*) Significant at the 1%; (**) Significant at the 5% . Lag length K based on automatically computed Akaike's FPE test, hence not reported in table. The lag truncations for the Bartlett Kernel were chosen on the basis of (Newey & West, 1987). Probabilities were compared with (MacKinnon, 1996) one-sided p-values.

(g) VAR & Cointegration Analysis

Many researchers have focused on the application of ECM, like (Hendry & Ericsson, 1991b) and (Hendry, 1995) and they believed in that ECM has different formulations. However, (Johansen, 1988) reported that, one of the formulation of getting ECM is the application of VAR model. We apply conventional VAR/ECM to estimate interdependence of the variables. For this study we adopt a vector autoregressive (VAR) process, which generally can be written as:

$$X_t = A_o + A_1 \sum_{i=1}^{k} X_{t-i} + A_3 D_t + \varepsilon_t \tag{6.12}$$

Where X_t is the vector of variables used *i.e.* m_t, y_t, cr_t, by_t, i_t, fi_t, fd_t. D_t is an exogenous dummy named as DUM_{1989}[30] used in VAR model and ε_t is white noise error term. The adjusted Likelihood Ratio (LR) Test statistics, (Sims, 1980b) is used to determine the optimal lag length of the variables. The decision about the adjusted LR statistics is drawn from AIC, SBC and HQ criteria and chosen lag length is one *see Table 4*. The dynamics of VAR is difficult to interpret (Lutkepohl, 1993). However, some authors who made the interpretation of the coefficients of VAR as the long run elasticities *e.g.*(Hallam & Zanoli, 1993).

[30] The House Building Finance Corporation (HBFC) had shifted its rent sharing operations to interest based system.

Table 6.4: Lag Selection Criteria Results

Lags	LogL	LR	FPE	AIC	SC	HQ
0	-86.859	NA	1.40e-07	4.081	4.359	4.185
1	436.1989	864.183*	1.60e-16*	-16.530*	-14.304*	-15.697*

Note: (*) indicates appropriate lag length at 5% significance level selected by criterion LR: sequential modified LR test statistic, FPE: Final prediction error, AIC: Akaike information criterion, SC: Schwarz information criterion, HQ: Hannan-Quinn information criterion

According to (Engle & Granger, 1987), variables that are cointegrated must have an error correction representation, otherwise simple regression would lead to spurious correlations. Cointegration is a test for equilibrium between non-stationary variables having same order integration. It also has the lead of not imposing a priori assumptions of exogeneity of the variables. Ericsson (1991), highlights some facts about cointegration and said that "*cointegration ties with the long run relationship of economic variables and their pertinent statistical models which deliver an empirically operative dynamic ECM*".

The seminal work done by (Johansen, 1988) proposed two Likelihood Ratio (LR) test-statistics used to test the number of cointegrating relationships between real money and its determinants based on characteristic roots named as, Trace-Statistic and Maximum-Eigen Value Statistic. The Trace Test Statistic is as follows:

$$\lambda_{tra}(r) = -T \sum_{i=r+1}^{k} log_e (1 - \lambda_i) \tag{6.13}$$

Where λ_i are eigen values corresponding to eigen vectors v_i follow a descending order as $\lambda_1 > \lambda_2 > \lambda_3 > \cdots > \lambda_k$. Statistically two hypotheses can be stated as H_o: $Rank(\pi) \leq r;\ where\ \ 0 < r < k, H_a$: $Rank(\pi) > r$ **.**

While the Maximum Eigen value Test Statistic stated the null hypothesis of exactly r cointegrating vectors against the alternate of $r + 1$ cointegrating vectors. The test statistic can be seen as follows:

$$\lambda_{max}(r, r+1) = -T\, log_e(1-\lambda_i) \qquad (6.14)$$
$$= \lambda_{tra}(r) - \lambda_{tra}(r+1)$$

Also, statistical hypotheses can be tested as H_o: $Rank(\pi) = r$ and H_a: $Rank(\pi) = r+1$. Note that these test statistics are distributed as $\chi^2 \sim r(k-r)$ when $t \to \infty$.

The results of these cointegration tests have been reported below in *Table 5.* Trace test statistic indicates one cointegrating equation among the variables while maximum-eigen value test reports there exits two cointegrating vectors among them. As these two tests reports different number of cointegrating relations. In this case trace test is considered to be more powerful because it contains all $k-r$ values of the least eigen vector and in case of non-normality (Cheung & Lai, 1993) and (Hubrich et al., 2001) preferred trace test over maximum-eigen value test. Therefore, we use trace test to determine number of cointegrating relations.

Table 6.5: Johansen MLE Based Cointegration Test

(Variables = m_t, y_t, cr_t, by_t, i_t, fi_t, fd_t *and* Ord(VAR) is 1*)*

Null	*Unrestricted Cointegration Rank Test (Trace)*			
	Alternative	*Chi−square*	5% *C.V*	*Prob.***
r = 0	r ≥ 1	178.1550*	125.6154	0.0000
r ≤ 1	r ≥ 2	94.99470	95.75366	0.0601
r ≤ 2	r ≥ 3	60.36724	69.81889	0.2243
r ≤ 3	r ≥ 4	36.68925	47.85613	0.3624
r ≤ 4	r ≥ 5	1.26840	29.79707	0.3411
r ≤ 5	r ≥ 6	9.381156	15.49471	0.3313
r ≤ 6	r ≥ 7	0.272031	3.841466	0.6020

Note: (*) denotes rejection of the hypothesis at the 5% level and (**) are MacKinnon-Haug-Michelis (1999) p-values

Null	*Unrestricted Cointegration Rank Test (Maximum Eigenvalue)*			
	Alternative	*Chi−square*	5% *C.V*	*Prob**.*
r = 0	r = 1	62.16032*	46.23142	0.0005
r ≤ 1	r = 2	55.6744*	40.07757	0.0004
r ≤ 2	r = 3	23.67799	33.87687	0.4793
r ≤ 3	r = 4	5.42086	27.58434	0.7143
r ≤ 4	r = 5	11.8 724	21.13162	0.5588
r ≤ 5	r = 6	9.109125	14.26460	0.2772
r ≤ 6	r = 7	0.272031	3.841466	0.6020

Note: (*) denotes rejection of the hypothesis at the 5% level. (**) MacKinnon-Haug-Michelis (1999) p-values

Traditionally, the first normalized equation is considered as the long run equation and written in (6.15). Where *t-stat.* and χ^2-*values* are reported in (.) and [.] respectively. The income elasticity of 2.79 is positive and significant leads to diseconomies of scale. A 1% increase in real income in Pakistan increases the demand for money by 2.79 percent, implying that the SBP should increase the money supply by 2.79 percent for each 1 percent increase in real GDP. This may be due to inflexibilities in the economy. The results are in the line with (Bahmani-Oskooee & Barry, 2000; Bahmani, 2008). The coefficient of interest rate on bank deposits (own rate) is 1.10 positive significant (Adil et al., 2020) and 0.79 a negative significant for the government bond yield. The hypotheses of opportunity cost of holding money *i.e.* difference between call money rate and government bond rate, yield significant too. The results are aligned with some earlier studies by (Qayyum, 2001, 2005b). The coefficient of inflation rate is 0.80 negative and significant. It fulfills our theoretical expectations that when inflation rises the demand for real money decreases. This aspect of the result further can be seen in (Asad et al., 2011; Dou, 2019; Nel et al., 2020; Qayyum, 2001, 2005b). The coefficient of financial innovation is positive and significant. A 1 percent increase in financial innovation leads to 0.66 percent increase in real money demand. These key findings are in the line with (Adil et al., 2020; Columba, 2009; Hye et al., 2009; Odularu & Okunrinboye, 2009; Sarwar et al., 2013). However, the estimate for financial development in Pakistan found not to be in the line with (Ahad, 2017) as it is negative and significant.

$$m_t = 2.79\, y_t + 1.10\, cr_t - 0.79\, by_t - 0.80\, i_t + 0.66\, fi_t - 0.41\, fd_t \qquad (6.15)$$

(6.49)	(8.46)	(-7.18)	(-3.08)	(4.40)	(-1.95)	(t-stat.)
[42.12]	[71.57]	[51.55]	[9.49]	[19.36]	[3.80]	[χ^2-val.]

(h) Dynamic Error Correction Model (ECM)

The dynamic error correction model is estimated using General to Specific Methodology was first introduced in (Hendry & Ungern-Sternberg, 1981). We start with a general model including set of all variables and their optimal lags, dummy variables if significant and the lag of error correction term. The parsimonious model is obtained by leaving the insignificant variables behind by making sure that the sign of error correction term remains negative and significant throughout the estimation process. Using theorem 2 in (Johansen, 1995) the conditional distribution of m_t can then be represented by an error correction model that explains changes in m_t by its own lags, the error-correction term, and by simultaneous changes and their lags of the weak exogenous variables. The model may also contain deterministic terms like a constant and dummies which are represented here in the model as Dum_{2000}[31] variable. The coefficient $ect(-1)$ is expected to be negative and significant and shows the speed of adjustment in the model and remaining coefficients in the model are short rum dynamic coefficients which shows the adjustment of the long run equilibrium. The following equation (6.16) is the estimated short run equation for money demand in Pakistan.

$$\begin{array}{l} \Delta m_t = \underset{(3.83)^{32}}{1.05\, \Delta \boldsymbol{y_t}} + \underset{(2.03)}{0.05\, \Delta by_{t-2}} - \underset{(-3.09)}{0.51\, \Delta \boldsymbol{i_t}} + \underset{(3.86)}{0.29\, \Delta \boldsymbol{fi_t}} + \underset{(3.51)}{0.29\, \Delta \boldsymbol{fd_t}} \\ \qquad - \underset{(-2.00)}{0.10\, Dum_{2000}} - \underset{(-2.76)}{0.004\, ect_{t-1}} \end{array} \tag{6.16}$$

$R^2 = 0.64$, $\overline{R}^2 = 0.58$, Log lik. 220.78, Auto. LM χ^2 (1) = 0.81(0.36),
Norm. JB χ^2 (2) = 1.70 (0.42), BPG_{Hetro} χ^2 (7) = 6.30(0.51),
$ARCH_{Hetro}$ χ^2 (1) = 1.04(0.31), DW=1.72

Where $ect(-1)$ is the error correction term and can be defined as:

[31] General Musharraf took over the charge and leave a significant impact on financial side during his tenure

[32] (.) Values in there are *t-ratios*, also, Auto. LM is the Lagrange Multiplier test for autocorrelation, Norm. JB is Jarque-Bera normality test and White Hetro. ARCH Hetro. tests for heteroskedasticity and DW is Durbin Watson statistic.

$$ect = m_t - \alpha_0 - \beta_1 t - \beta_2 y_t - \beta_3 cm_t - \beta_4 by_t - \beta_5 i_t - \beta_6 fi_t - \beta_7 fd_t$$

As suggested by (Bahmani-Oskooee, 2001), some of the problems of instability could stem from scant modeling of the short-run dynamics characterizing departures from the long run relationship. Therefore, it is convenient to incorporate the short run dynamics for constancy of long run parameters. In view of this we apply the CUSUM (mean stability) and CUSUMSQ (variance stability) tests proposed by (Brown et al., 1975). The stability of the model can be seen in *Figure 5* and its coefficients with 2 S.E. bands in *Figure 6;* see *Appendix*.

(i) Testing Super Exogeneity

Since the aim of modeling money demand function is its usage for policy purposes or implication. So, SupExt comes into play to determine the constancy of the model. For that it is necessary to check whether the estimated conditional model remains stable against interventions (breaks) or not? To address this historically valid and significant question, we try to capture these intervention or breaks in the data set using recently developed technique of capturing impulses like IIS, SIS and TIS. The variables entering as a currently dated regressor in our frugal model is separately checked for their DGPs and all significant dummies are reported below for each case then these were added in their marginal models and then in conditional model and checked for their significance and insignificance respectively.

There are two main types of test that are used for testing SupExt. First test is to maintain the stability of the parameters of conditional model and the non-stability of the parameters of marginal model. To validate the said process a marginal model is obtained by simply inverting the conditional model. The test of SupExt requires constancy of the conditional model and non-constancy of the marginal process.

Therefore, under SupExt the constant conditional money demand model is not interpretable as a re-parameterization because the re-parameterization is a function of the causal structural parameters and the time dependent parameters of the marginal process. Hence by inverting conditional model the steady marginal model cannot be obtained. The non-invertibility of the conditional model into marginal model can be used as evidence of super-exogeneity because the invertibility of conditional model into marginal model is prohibited if the variables are super exogenous for the parameters of the conditional model (Hendry & Ericsson, 1991b). Therefore, to estimate the instability of marginal process and stability of conditional process is sufficient to settle SupExt test (Perez, 2002).

Another test of SupExt of parameters of interest against the external shocks that may change the parameters of marginal density function is to develop the marginal model by adding dummies in the marginal process. Then add those dummy variables that are significant in the marginal model to the conditional model and test their significance by *F-statistics*. The *F-statistics* is being calculated by a conventional test of joint significance of interventional variables in the conditional models (Engle & Hendry, 1993). Therefore, the insignificance of dummy variables in the conditional model leads SupExt of conditional model. Moreover, the introduction of dummy variable into the marginal processes and then conditional process and their relevance in achieving the constancy of the marginal process and irrelevance in effecting the constancy the conditional process validates results of single equation. The preferred model (6.16) indicates that the income, inflation, financial innovation and financial development enter into the model at their current dates. These variables have been highlighted as a bold one in (6.16). Therefore, to test the SupExt we have to test firstly, the stability of marginal models of these four current value variables and show that the marginal models are instable and secondly, the stability of money demand model (6.16).

In order to test SupExt of the parameter of the money demand model against the unknown external shocks, which could have affected the stability of the marginal process, we used dummy saturation method proposed by (Ericsson, 2012). This method of dummy variables is suggested in (Hendry & Ericsson, 1991b) and used in (Hendry & Santos, 2006). The significance of dummy variable is tested by *t-statistics* for individual case, and *F-test* proposed by (Engle & Hendry, 1993) is used to test the joint significance of intervention dummies.

i.1) Testing for Δy_t:

For DGP of Δy_t (Keith Cuthbertson, 1988; Hendry & Ericsson, 1991b) have used autoregressive model in their studies. We start with 6th order AR process initially but left the outcome mentioned in *Table 5* to *Table 7*. Where *Table 5* is retaining all those dummies after implementing IIS. *Table 6* and *Table 7* are with those significant dummies while using SIS and TIS respectively. However, several post estimation test are reported at end of each DGPs and each estimated equation as well. Now the marginal model can be obtained by inverting our conditional model and letting Δy_t as dependent and Δm_t as regressor along with identified set of dummies in each table mentioned below.

$$\begin{aligned} \Delta y_t = {} & \underset{(1.13)}{0.09\,\Delta m_t} - \underset{(-1.29)}{0.02\,\Delta by_{t-2}} - 0.10\,\Delta i_t - \underset{(-0.74)}{0.08\,\Delta fi_t} - \underset{(-2.03)}{0.08\,\Delta fd_t} \quad \mathbf{(-0.22)} \\ & + \underset{\mathbf{(0.47)}}{0.01\,Dum_{2000}} + \underset{(2.58)}{0.05\,iis1978} + \underset{(3.19)}{0.05\,iis1980} + \underset{(1.65)}{0.03\,iis1985} + \\ & \underset{(1.70)}{0.03\,iis1992} - \underset{(-1.76)}{0.03\,iis1993} - \underset{(-1.16)}{0.03\,iis1997} - \underset{(-2.72)}{0.002\,ect_{t-1}} \end{aligned} \tag{6.17}$$

$R^2 = 0.51$ $\quad LM_{Auto.}\chi^2(1) = 9.73(0.02)$ $\quad JB_{Norm.}\chi^2(2) = 2.09(0.35)$

$ARCH_{Hetro.}\chi^2(1)=0.25(0.61)$ $\quad BPG_{Hetro.}\chi^2(13)=7.43(0.88)$

DW stat. = 1.21

It can be seen that most of the dummies significantly entered in marginal model of Δy_t but cause instability in marginal distribution of Δy_t. For the sake of brevity, *t-stats.* are being highlighted from here to own wards for insignificant regressors. Few diagnostic tests are reported below (6.17). Now for the case of SIS, we enter all those significant dummies from *Table 6* as independent regressor in our marginal model for Δy_t. Therefore, (6.18) capturing the effect of SIS on Δy_t. Some tests are also reported at the end of the each table like; (Ljung & Box, 1978) tests autocorrelation in residuals and squared residuals and (Jiao & Pretis, 2018) proportion and count outlier tests are for checking whether the proportion (or number) of outliers detected using IIS is different from the proportion (or number) of outliers expected under the null hypothesis of no outliers.

Data Generating Process for Δy_t

Table 6.6. ***Impulse Indicator Saturation***

Variable	Δy_t
Regression Type/DGP	IIS
Const.	0.022*
Δy_{t-1}	0.487*
iis1978	0.036*
iis1980	0.057*
iis1985	0.026**
iis1992	0.027**
iis1993	-0.052*
iis1997	-0.036*
Diagnostics and Fit	
Ljung-Box AR(1)	χ^2 (1) = 1.78 (0.18)
Ljung-Box ARCH(1)	χ^2 (1) = 0.45 (0.50)
Jiao-Pretis Prop.	Stat. 3.85 (0.01)
Jiao-Pretis Count	Stat. 6.00 (0.03)
S.E.= 0.012, R^2= 0.67, Log like.= 139.2	

Note: (*) and (**) represents 1% and 5 % significance level respectively. Values in (.) are respective p-value of the test statistic.

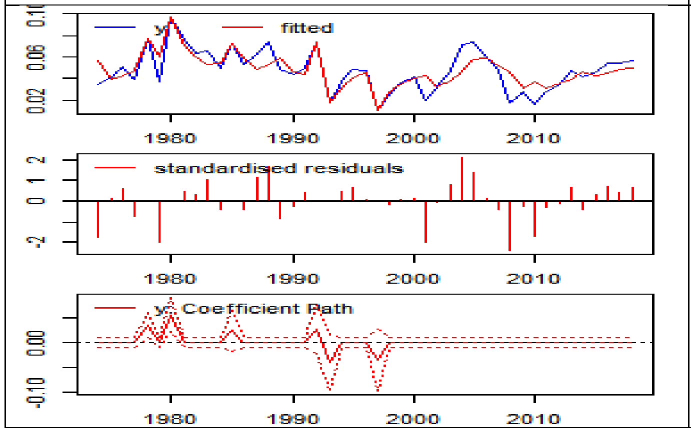


Table 6.7. Step Indicator Saturation

Variable	Δy_t
Regression Type/DGP	SIS
Const.	0.039*
Δy_{t-1}	0.048
sis1978	0.036*
sis1979	-0.042*
sis1981	-0.037*
sis1993	-0.044*
sis1994	0.029**
sis1997	-0.016**
sis2003	0.031*
sis2008	-0.034*
sis2013	0.024*
Diagnostics and Fit	
Ljung-Box AR(1)	χ^2 (1) = 0.19 (0.66)
Ljung-Box AR H(1)	χ^2 (1) = 0.00 (0.95)
S.E.= 0.010, R^2= 0.78, Log like.=148.38	

Note: (*), (**) represents 1% and 5 % significance level respectively. Values in (.) are respective p-value of the test statistic.

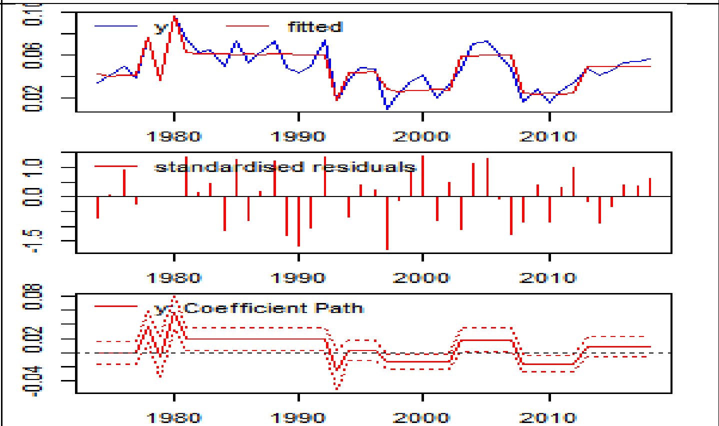
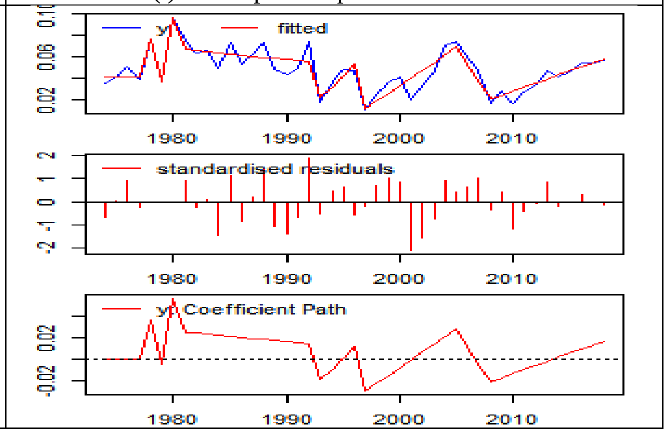


Table 6.8
Trend Indicator Saturation

Variable	Δy_t
Regression Type/DGP	TIS
Const.	0.040*
Δy_{t-1}	0.018
tis1978	0.036*
tis1979	-0.078*
tis1980	0.102*
tis1981	-0.092*
tis1982	0.030**
tis1993	-0.032*
tis1994	0.043*
tis1997	-0.051*
tis1998	0.048*
tis2006	-0.023*
tis2009	0.020
Diagnostics and Fit	
Ljung-Box AR(1	χ^2 (1) = 0.02 (0.90)
Ljung-Box ARCH(1)	χ^2 (1) = 0.98 (0.16
S.E.= 0.009, R^2= 0.81, Log like.= 150.58	

Note: (*), (**) represents 1%,and 5% significance level. Values in (.) are respective p-value of the test statistic.



Source: Authors own estimation

$$\Delta y_t = 0.09\,\Delta m_t - 0.01\,\Delta by_{t-2} - 0.10\,\Delta i_t - 0.08\,\Delta fi_t + 0.01\,\Delta fd_t$$
$$\quad \mathbf{(1.13)} \quad \mathbf{(-1.28)} \quad \mathbf{(-0.74)} \quad (-2.30) \quad \mathbf{(0.22)}$$
$$-0.01\,Dum_{2000} - 0.03\,sis1978 + 0.05\,sis1979 - 0.03\,sis1981 -$$
$$\quad \mathbf{(0.30)} \quad (-1.86) \quad (2.88) \quad (-3.17)$$
$$0.001\,sis1993 - 0.01\,sis1994 - 0.02\,sis1997 + 0.02\,sis2003 \qquad (6.18)$$
$$\quad (-0.45) \quad (-0.59) \quad (-1.65) \quad (1.29)$$
$$-0.02\,sis2008 - 0.002\,sis2013 - 0.006\,ect_{t-1}$$
$$\quad (-1.88) \quad (-0.11) \quad (-2.94)$$

$R^2 = 0.60$ $\quad LM_{Auto.}\chi^2\,(1) = 7.40(0.007)$ $\quad JB_{Norm.}\chi^2\,(2) = 8.69(0.01)$

$ARCH_{Hetro.}\chi^2(1)=0.07(0.79)$ $\quad BPG_{Hetro.}\chi^2(16)=18.04(0.32)$

DW stat. = 2.49

It can be seen from above equation that about half of the SIS impulses significantly entered in the marginal model but cause a severe instability in it. This may be due to the selection of $\alpha = 0.05$. Had it been settled at $\alpha = 0.001$, would lead to capture less number of impulses. However, we are still confident and hoping that at this level even then it will not affect the stability of our conditional model. Now for TIS in case of Δy_t, the significant trend impulses are given in *Table 7* below that are capture in DGP of Δy_t. The marginal model in their presence is:

$$\Delta y_t = -0.04\,\Delta m_t - 0.003\,\Delta by_{t-2} + 0.08\,\Delta i_t - 0.05\,\Delta fi_t +$$
$$\quad \mathbf{(-0.74)} \quad \mathbf{(-0.45)} \quad \mathbf{(1.12)} \quad (-2.33)$$
$$0.05\,\Delta fd_t + 0.009\,Dum_{2000} + 0.04\,tis1978 - 0.10\,tis1979 +$$
$$\quad (2.12) \quad \mathbf{(0.67)} \quad (3.83) \quad (-4.59)$$
$$0.12\,tis1980 - 0.10\,tis1981 + 0.03\,tis1982 - 0.04\,tis1993 + \qquad (6.19)$$
$$\quad (5.44) \quad (-4.56) \quad (2.74) \quad (-3.70)$$
$$0.05\,tis1994 - 0.05\,tis1997 + 0.04\,tis1998 - 0.02\,tis2006 +$$
$$\quad (3.75) \quad (-3.66) \quad (4.14) \quad (-4.48)$$
$$0.02\,tis2009 - 0.004\,ect_{t-1}$$
$$\quad (3.91) \quad (-3.44)$$

$R^2 = 0.88$ $\quad LM_{Auto.}\chi^2\,(1) = 7.01(0.008)$ $\quad JB_{Norm.}\chi^2\,(2) = 0.51(0.77)$

$ARCH_{Hetro.}\chi^2(1)=0.35(0.55)$ $\quad BPG_{Hetro.}\chi^2(18)=25.33(0.12)$

DW stat. = 2.58

Interestingly, all trend indicator saturation impulses highly significantly entered in our marginal process of currently dated Δy_t and causing instability in it. Now, after checking the significance of each type of impulse saturation in marginal model. We check the stability of conditional model in their presence one by one. Here (6.20) is the stability test of conditional model against TIS type of impulses. All the impulses went insignificant and don't cause any instability in the estimates of the remaining parameters of interest. Therefore, IIS type breaks in the marginal model of Δy_t don't alter the conditional distribution.

$$\Delta m_t = 1.16\,\Delta y_t + 0.05\,\Delta by_{t-2} - 0.48\,\Delta i_t + 0.29\,\Delta fi_t + 0.08\,\Delta fd_t$$

(2.85) (1.80) (-2.75) (3.64) (3.28)

$$-0.10\,Dum_{2000} + 0.02\,iis1978 - 0.03\,iis1980 - 0.04\,iis1985 - \qquad (6.20)$$

(1.90) **(0.38)** **(-0.51)** **(-0.91)**

$$0.04\,iis1992 + 0.03\,iis1993 - 0.03\,iis1997 - 0.002\,ect_{t-1}$$

(-0.88) **(0.74)** **(-0.72)** (-1.98)

$R^2 = 0.68$ $LM_{Auto.}\chi^2(1) = 0.21(0.64)$ $JB_{Norm.}\chi^2(2) = 1.90(0.64)$

$ARCH_{Hetro.}\chi^2(1)=2.62(0.11)$ $BPG_{Hetro.}\chi^2(13)=9.70(0.71)$

DW stat. = 1.83

For stability of the conditional model in presence SIS, we incorporate these dummies in (6.16) and check their significance. The results are reported in (6.20). All the step dummies are insignificant apart from the $sis2013$, even though it will not affect the stability of the model. However, Δy_t is significant at 10% level of significance. Therefore, estimated conditional ECM model remains stable in the presence of step impulses as well.

$$\Delta m_t = 0.48\,\Delta y_t + 0.03\,\Delta by_{t-2} - 0.93\,\Delta i_t + 0.26\,\Delta fi_t + 0.25\,\Delta fd_t$$

(1.73) (1.89) (-3.86) (3.37) (2.84)

$$-0.12\,Dum_{2000} - 0.01\,sis1978 - 0.05\,sis1979 - 0.02\,sis1981 -$$

(-2.39) **(-0.27)** **(-1.02)** **(-0.61)**

$$0.02\,sis1993 - 0.03\,sis1994 + 0.002\,sis1997 + 0.008\,sis2003 \qquad (6.21)$$

(-0.60) **(-0.63)** **(0.06)** **(0.31)**

$$-0.004\,sis2008 - 0.06\,sis2013 - 0.02\,ect_{t-1}$$

(-0.14) (-2.21) (-3.43)

$R^2 = 0.79$ $LM_{Auto.}\chi^2(1) = 1.94(0.16)$ $JB_{Norm.}\chi^2(2) = 1.44(0.48)$

$ARCH_{Hetro.}\chi^2(1)=0.20(0.65)$ $BPG_{Hetro.}\chi^2(16)=12.57(0.70)$

DW stat. = 2.37

At the end the stability of the conditional model in presence TIS, we incorporated these dummies in (6.16) and check their significance individually. The results are reported in (6.22). All the trend indicators are insignificant apart from the $tis2006$ and $tis2009$, even then it will not affect the stability of the model. However, Δy_t is significant at 10% level of significance. Therefore, estimated conditional ECM model remains stable in the presence of trend indicator saturation as well. Therefore, we can conclude that our estimated frugal model (6.16) is super exogenous against relevant class of interventions in the marginal process of one conditioning variable Δy_t .

$$\begin{array}{l} \Delta m_t = \underset{(-2.74)}{-0.57\,\Delta y_t} + \underset{(1.71)}{0.05\,\Delta by_{t-2}} - \underset{(-2.47)}{0.62\,\Delta i_t} + \underset{(2.62)}{0.22\,\Delta fi_t} + \\ \underset{(2.76)}{0.25\,\Delta fd_t} \underset{(-1.72)}{-0.08\,Dum_{2000}} + \underset{\mathbf{(0.47)}}{0.03\,tis1978} - \underset{\mathbf{(-0.57)}}{0.07\,tis1979} + \\ \underset{\mathbf{(0.48)}}{0.06tis1980} - \underset{\mathbf{(-0.76)}}{0.08\,tis1981} + \underset{\mathbf{(1.27)}}{0.06\,tis1982} - \underset{\mathbf{(-1.22)}}{0.06\,tis1993} + \\ \underset{\mathbf{(0.87)}}{0.05\,tis1994} - \underset{\mathbf{(-0.81)}}{0.05\,tis1997} + \underset{\mathbf{(1.35)}}{0.07\,tis1998} - \underset{(-2.24)}{0.05\,tis2006} + \\ \underset{\mathbf{(1.74)}}{0.04\,tis2009} - \underset{(-3.61)}{0.02\,ect_{t-1}} \end{array} \qquad (6.22)$$

$R^2 = 0.80$ $LM_{Auto.}\chi^2(1) = 0.83(0.36)$ $JB_{Norm.}\chi^2(2) = 1.49(0.78)$
$ARCH_{Hetro.}\chi^2(1)=0.42(0.52)$ $BPG_{Hetro.}\chi^2(18)=15.37(0.64)$
DW stat. = 2.20

i.2) Testing for Δi_t:

As difference of inflation variable contemporaneously enters in (6.16). Therefore, DGP for Δi_t using each type of impulse has been covered below in *Table 8* to *Table 10* retaining all the significant dummies in case of IIS, SIS and TIS at $\alpha = 0.05$ level of significance respectively. Taking Δi_t as dependent variable by inverting our conditional model leads to the estimates shown in (6.23) under the influence of IIS. The impulse dummies entered significantly in marginal model and lead to instability in the marginal process.

$$\begin{array}{l} \Delta i_t = \underset{(3.13)}{0.79\,\Delta y_t} + \underset{\mathbf{(1.13)}}{0.03\,\Delta by_{t-2}} - \underset{(-2.13)}{0.29\,\Delta m_t} + \underset{(3.40)}{0.22\,\Delta fi_t} - \underset{\mathbf{(-0.89)}}{0.08\,\Delta fd_t} \\ \underset{\mathbf{(-1.30)}}{-0.06\,Dum_{2000}} + \underset{(1.90)}{0.03\,iis1976} + \underset{(2.09)}{0.08\,iis2008} - \underset{(-4.38)}{0.004\,ect_{t-1}} \end{array} \qquad (6.23)$$

$R^2 = 0.35$ $LM_{Auto.}\chi^2(1) = 1.10(0.29)$ $JB_{Norm.}\chi^2(2) = 3.57(0.17)$
$ARCH_{Hetro.}\chi^2(1)=1.19(0.27)$ $BPG_{Hetro.}\chi^2(9)=11.57(0.24)$
DW stat. = 1.56

Now for the case of SIS, we enter all those significant dummies from *Table 9* as independent regressor in our marginal model for Δi_t. Therefore, (6.24) capturing the effect of SIS on Δi_t. The super saturated impulses entered in the marginal model to some extend significantly and causing instability in the marginal process of Δi_t.

$$\Delta i_t = 0.75\,\Delta y_t + 0.04\,\Delta by_{t-2} - 0.44\,\Delta m_t + 0.24\,\Delta fi_t - 0.06\,\Delta fd_t$$
(2.86) (1.72) (-3.60) (3.90) **(-0.77)**

$$-0.08\,Dum_{2000} + 0.05\,sis1976 - 0.10\,sis1977 - 0.04\,sis2008 + \qquad (6.24)$$
(-1.84) (1.80) (-2.90) **(-1.12)**

$$0.008\,sis2009 - 0.009\,ect_{t-1}$$
(0.23) (-4.27)

$R^2 = 0.47$ $LM_{Auto.}\chi^2\,(1) = 1.39(0.24)$ $JB_{Norm.}\chi^2\,(2) = 2.30(0.32)$

$ARCH_{Hetro.}\chi^2(1)=1.44(0.23)$ $BPG_{Hetro.}\chi^2(9)=14.69(0.20)$

DW stat. = 1.65

After implementing SIS for the marginal model of Δi_t, its time to check the significance of retained dummies in the marginal model for the type TIS. The estimated parameters for the said case are reported in (6.25). Most of the trend

$$\Delta i_t = 0.10\,\Delta y_t + 0.007\,\Delta by_{t-2} - 0.20\,\Delta m_t + 0.06\,\Delta fi_t + 0.02\,\Delta fd_t$$
(0.38) **(0.38)** (-1.99) (1.02) **(0.28)**

$$-0.03\,Dum_{2000} - 0.09\,tis1975 + 0.06\,tis1977 + 0.02\,tis1978 + \qquad (6.25)$$
(-0.90) (-2.26) (1.87) **(0.87)**

$$0.10\,tis2008 - 0.14\,tis2009 + 0.03 tis2010 - 0.009\,ect_{t-1}$$
(3.26) (-2.29) **(0.83)** (-4.09)

$R^2 = 0.69$ $LM_{Auto.}\chi^2\,(1) = 13.56(0.00)$ $JB_{Norm.}\chi^2\,(2) = 1.44(0.49)$

$ARCH_{Hetro.}\chi^2(1)=0.02(0.89)$ $BPG_{Hetro.}\chi^2(13)=10.28(0.67)$

DW stat. = 1.03

Impulses are significant and causing stark changes in the marginal process of Δi_t.

Data Generating Process for Δi_t

Table 6.9. *Impulse Indicator Saturation*

Variable	Δi_t
Regression Type/DGP	IIS
Const.	0.022*
Δi_{t-1}	0.677*
Iis1976	-0.082*
iis2008	0.113*
Diagnostics and Fit	
Ljung-Box AR(1)	χ^2 (1) = 0.15 (0.70)
Ljung-Box ARCH(1)	χ^2 (1) = 1.51 (0.22
Jiao-Pretis Prop.	Stat. 0.77 (0.44)
Jiao-Pretis Count	Stat. 2.00 (0.50)
S.E.= 0.02, R^2= 0.80, Log like.= 114.30	

Note: (*) represents 1% significance level. Values in (.) are respective p-value of the test statistic.

Table 6.10. *Step Indicator Saturation*

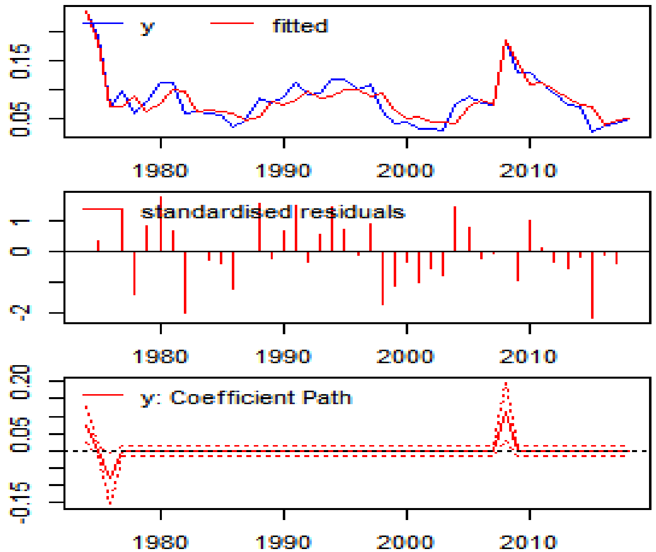
Variable	Δi_t
Regression Type/DGP	SIS
Const.	0.068*
Δi_{t-1}	0.652*
sis1976	-0.123*
sis1977	0.081*
sis2008	0.111*
sis2009	-0.119*
Diagnostics and Fit	
Ljung-Box AR(1)	χ^2 (1) = 0.01 (0.95)
Ljung-Box ARCH(1)	χ^2 (1) = 1.22 (0.27)
S.E.= 0.02, R^2= 0.79, Log like.=112.23	

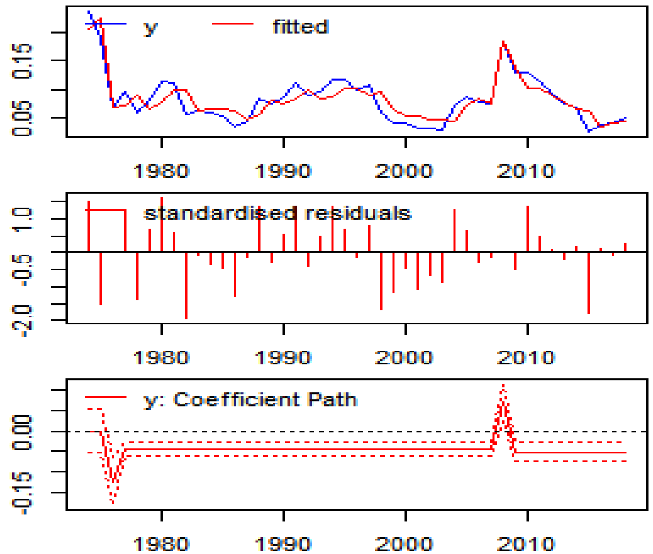
Note: represents 1% significance level. Values in (.) are respective p-value of the test statistic.

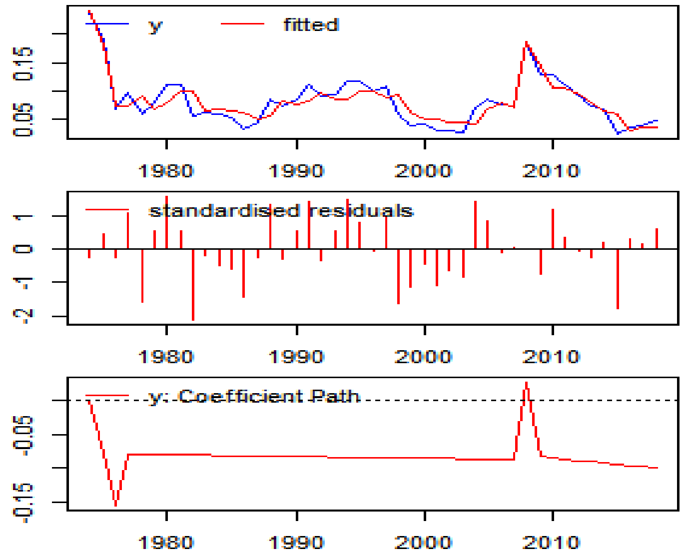
Table 16.11. *Trend Indicator Saturation*

Variable	Δi_t
Regression Type/DGP	TIS
Const.	0.112*
Δi_{t-1}	0.621*
tis1975	-0.078*
tis1977	0.154*
tis1978	-0.076*
tis2008	0.115*
tis2009	-0.226*
tis2010	0.109
Diagnostics and Fit	
Ljung-Box AR(1)	χ^2 (1) = 0.13 (0.72)
Ljung-Box ARCH(1)	χ^2 (1) = 1.82 (0.18)
S.E.= 0.02, R^2= 0.81, Log like.= 115.20	

Note: (*) represents 1% significance level. Values in (.) are respective p-value of the test statistic.







Source: Authors own estimation

So, we can conclude that the marginal distribution of contemporaneously happened regressors in (6.16), for instance Δi_t is largely affected by the shifts in its DGP. Now, we determine the stability of conditional distribution in the presence of these shifts. Below (5.15-5.16) is evidently depicting that IIS, SIS shifts do not alter the conditional distribution or preferred model and remain stable against these shocks. However, TIS alter the parameters of (6.16) as discussed in (6.28).

$$\begin{aligned}\Delta m_t = &\ \underset{(3.02)}{0.89}\ \Delta y_t + \underset{(2.19)}{0.06}\ \Delta by_{t-2} - \underset{(-2.13)}{0.39}\ \Delta i_t + \underset{(3.66)}{0.27}\ \Delta fi_t + \underset{(3.57)}{0.32}\ \Delta fd_t \\ &- \underset{(-1.79)}{0.09}\ Dum_{2000} + \underset{\mathbf{(0.27)}}{0.01}\ iis1976 - \underset{\mathbf{(-1.46)}}{0.08}\ iis2008 - \underset{(-2.75)}{1.46}\ ect_{t-1}\end{aligned} \tag{6.26}$$

R^2 = 0.65 LM$_{Auto.}\chi^2$ (1) = 0.48(0.49) JB$_{Norm.}\chi^2$ (2) = 1.75(0.42)
ARCH$_{Hetro.}\chi^2$(1)=1.19(0.27) BPG$_{Hetro.}\chi^2$(9)=7.16(0.62)
DW stat. = 1.78

$$\begin{aligned}\Delta m_t = &\ \underset{(3.54)}{1.04}\ \Delta y_t + \underset{(2.06)}{0.05}\ \Delta by_{t-2} - \underset{(-3.60)}{0.63}\ \Delta i_t + \underset{(4.29)}{0.30}\ \Delta fi_t + \underset{(2.04)}{0.20}\ \Delta fd_t \\ &- \underset{(-2.31)}{0.12}\ Dum_{2000} + \underset{\mathbf{(1.08)}}{0.09}\ sis1976 - \underset{\mathbf{(-1.47)}}{0.10}\ sis1977 - \underset{\mathbf{(-1.12)}}{0.06}\ sis2008 + \\ &\ \underset{\mathbf{(1.04)}}{0.05}\ sis2009 - \underset{(-1.70)}{0.005}\ ect_{t-1}\end{aligned} \tag{6.27}$$

R^2 = 0.71 LM$_{Auto.}\chi^2$ (1) = 1.14(0.71) JB$_{Norm.}\chi^2$ (2) = 1.21(0.55)
ARCH$_{Hetro.}\chi^2$(1)=1.44(0.23) BPG$_{Hetro.}\chi^2$(9)=13.12(0.29)
DW stat. = 1.87

$$\begin{aligned}\Delta m_t = &\ \underset{\mathbf{(1.26)}}{0.54}\ \Delta y_t + \underset{\mathbf{(1.53)}}{0.04}\ \Delta by_{t-2} - \underset{(-1.99)}{0.54}\ \Delta i_t + \underset{(2.85)}{0.24}\ \Delta fi_t + \underset{\mathbf{(2.78)}}{0.30}\ \Delta fd_t \\ &- \underset{(-1.93)}{0.10}\ Dum_{2000} + \underset{\mathbf{(-0.21)}}{0.01}\ tis1975 - \underset{\mathbf{(-0.20)}}{0.01}\ tis1977 + \underset{\mathbf{(0.05)}}{0.002}\ tis1978 - \\ &\ \underset{\mathbf{(-0.85)}}{0.04}\ tis2008 + \underset{\mathbf{(1.04)}}{0.11}\ tis2009 - \underset{\mathbf{(-1.05)}}{0.06}\ tis2010 - \underset{\mathbf{(-0.38)}}{0.005}\ ect_{t-1}\end{aligned} \tag{6.28}$$

R^2 = 0.69 LM$_{Auto.}\chi^2$ (1) = 1.41(0.24) JB$_{Norm.}\chi^2$ (2) = 1.59(0.45)
ARCH$_{Hetro.}\chi^2$(1)=0.32(0.57) BPG$_{Hetro.}\chi^2$(13)=10.28(0.67)
DW stat. = 1.68

Although, all impulses for TIS insignificantly enter in the model but model doesn't pass the stability test for this type of impulses. However, preferred model is stable against IIS and SIS and also pass the stability test of SupExt.

i.3) Testing for Δfi_t:

The DGP process using 6th order AR process and by dropping out the insignificant lags using general to specific modeling what we left with are being reported in the following *see Table 11-Table 13.* However, inverting our conditional model into marginal model for financial innovation, following three equations (5.18-5.20) signify the impact of IIS, SIS and TIS on Δfi_t .

$$\begin{aligned}\Delta fi_t = & -1.31\Delta y_t - 0.02\,\Delta by_{t-2} + 0.82\,\Delta i_t + 0.81\,\Delta m_t - 0.21\,\Delta fd_t \\ & \quad (-3.41) \quad \mathbf{(-0.58)} \quad (3.32) \quad (4.17) \quad (-1.74) \\ & +0.06\,Dum_{2000} + 0.07\,iis1975 - 0.12\,iis1997 - 0.17\,iis1999 - \\ & \quad \mathbf{(1.05)} \quad \mathbf{(1.28)} \quad (-2.17) \quad (-3.33) \\ & 0.28\,iis2006 + 0.003\,ect_{t-1} \\ & \quad (-5.30) \quad \mathbf{(1.71)}\end{aligned} \tag{6.29}$$

$R^2 = 0.76$ $LM_{Auto.}\chi^2(1) = 0.60(0.44)$ $JB_{Norm.}\chi^2(2) = 1.03(0.60)$
$ARCH_{Hetro.}\chi^2(1)=1.19(0.27)$ $BPG_{Hetro.}\chi^2(11)=14.38(0.21)$
DW stat. = 1.77

$$\begin{aligned}\Delta fi_t = & -1.84\,\Delta y_t - 0.08\,\Delta by_{t-2} + 0.98\,\Delta i_t + 1.01\,\Delta m_t - 0.24\,\Delta fd_t \\ & \quad (-3.08) \quad \mathbf{(-1.48)} \quad (3.06) \quad (3.73) \quad \mathbf{(-1.36)} \\ & +0.05\,Dum_{2000} - 0.06\,sis2000 + 0.06\,sis2006 - 0.03\,sis2007 + \\ & \quad \mathbf{(0.44)} \quad (-1.84) \quad (1.77) \quad \mathbf{(-0.43)} \\ & +0.005\,ect_{t-1} \\ & \quad \mathbf{(0.68)}\end{aligned} \tag{6.30}$$

$R^2 = 0.46$ $LM_{Auto.}\chi^2(1) = 0.10(0.75)$ $JB_{Norm.}\chi^2(2) = 15.7(0.00)$
$ARCH_{Hetro.}\chi^2(1)=1.44(0.23)$ $BPG_{Hetro.}\chi^2(10)=9.50(0.49)$
DW stat. = 2.01

$$\begin{aligned}\Delta fi_t = & -1.07\Delta y_t - 0.03\,\Delta by_{t-2} + 0.65\,\Delta i_t + 0.67\,\Delta m_t - 0.25\,\Delta fd_t \\ & \quad (1.91) \quad \mathbf{(1.53)} \quad (-1.99) \quad (2.85) \quad (2.78) \\ & +0.08\,Dum_{2000} + 0.12\,tis1997 - 0.26\,tis1998 + 0.25\,tis2000 - \\ & \quad (-1.93) \quad (1.87) \quad (-2.72) \quad (3.74) \\ & 0.15\,tis2002 - 0.16\,tis2006 + 0.32\,tis2007 - 0.14\,tis2009 \\ & \quad (-2.60) \quad (-1.70) \quad (3.01) \quad (-3.11) \\ & +0.05\,tis2016 + 0.002\,ect_{t-1} \\ & \quad (1.75) \quad \mathbf{(0.04)}\end{aligned} \tag{6.31}$$

$R^2 = 0.73$ $LM_{Auto.}\chi^2(1) = 2.26(0.13)$ $JB_{Norm.}\chi^2(2) = 0.61(0.74)$
$ARCH_{Hetro.}\chi^2(1)=0.32(0.57)$ $BPG_{Hetro.}\chi^2(15)=18.87(0.22)$
DW stat. = 2.27

As the inclusion of these identified external shocks into the marginal models caused instability in the parameters of the marginal model but the impact of these dummies is captured by each individual marginal model that is why these dummies are insignificant in preferred model staying behind the parameters of the conditional model stable against these identified external shocks *see (5.21-5.23).*

Data Generating Process for Δfi_t

Table 6.12 ***Impulse Indicator Saturation***

Variable	Δfi_t
Regression Type/DGP	I S
Const.	-0.003
Δfi_{t-1}	0.064*
iis1975	0.190*
iis1997	0.145*
iis1999	-0.198*
iis2006	-0.337*

Diagnostics and Fit

Ljung-Box AR(1)	χ^2 (1) = 0.02 (0.90
Ljung-Box ARCH(1)	χ^2 (1) = 0.01 (0.92)
Jiao-Pretis Prop.	Stat. 1.80 (0.07)
Jiao-Pretis Count	Stat. 4.00 (0.30)

S.E.= 0.06, R^2= 0.60, Log like.= 65.94

Note: (*) represents 1% significance level. Values in (.) are respective p-value of the test statistic.

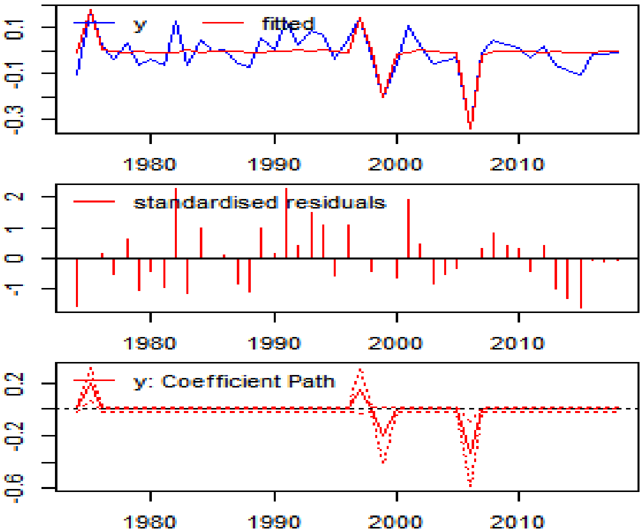


Table 6.13 ***Step Indicator Saturation***

ariable	Δfi_t
Regression Type/DGP	SIS
Const.	0.018
Δfi_{t-1}	-0.020
sis2000	0.196*
sis2006	-0.335*
sis2007	0.322*

Diagnostics and Fit

Ljung-Box AR(1)	χ^2 (1) = 0.42 (0.52)
Ljung-Box ARCH(1)	χ^2(1) = 0.52 (0.47)

S.E.= 0.07, R^2= 0.48, Log like.=59.87

Note: (*) represents 1% significance level. Values in (.) are respective p-value of the test statistic.

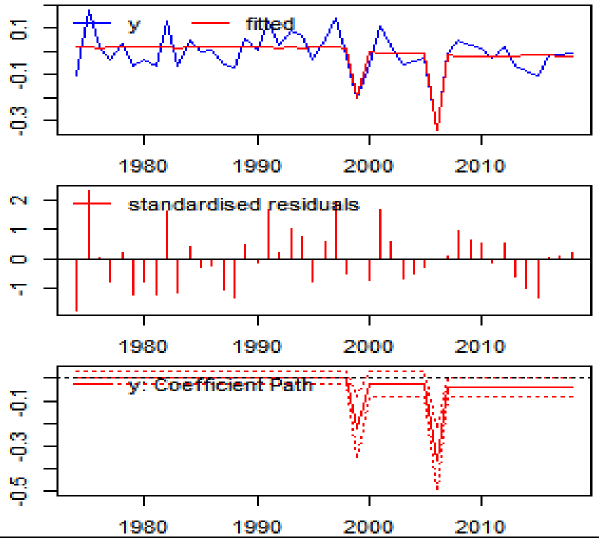


Table 6.14. ***Trend Indicator Saturation***

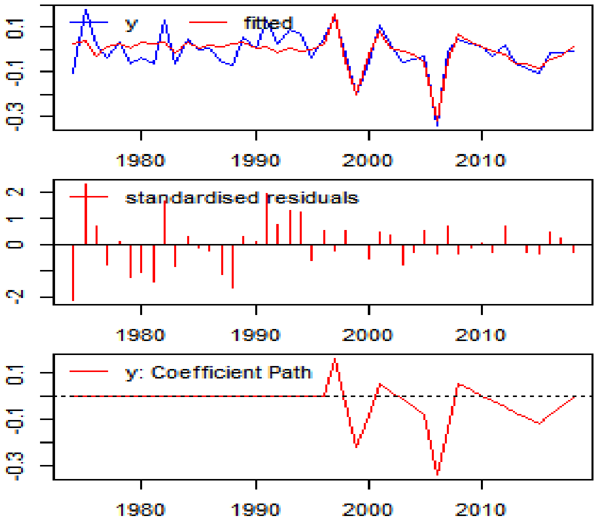
Variable	Δfi_t
Regression Type/DGP	TIS
Const.	0.016
Δfi_{t-1}	-0.238**
tis1997	0.159*
tis1998	-0.350*
tis2000	0.329*
tis2002	-0.171*
tis2006	-0.225*
tis2007	0.455*
tis2009	-0.221*
tis2016	0.063**

Diagnostics and Fit

Ljung-Box AR(1)	χ^2 (1) = 0.12 (0.73)
Ljung-Box ARCH(1)	χ^2 (1) = 6.26 (0.01)

S.E.= 0.02, R^2= 0.63, Log like.= 66.85

Note: (*) and (**) represents 1% and 5% significance level. Values in (.) are respective p-value of the test statistic.



Source: Authors own estimation

Clearly, it can be inferred that the conditional model remain stable during these external shocks that happened in marginal process of Δfi_t. far, our estimated model remain stable and pass almost all diagnostic tests under IIS, SIS and TIS.

$$\begin{aligned}
\Delta m_t &= \underset{(4.44)}{1.13}\,\Delta y_t + \underset{(1.93)}{0.05}\,\Delta by_{t-2} - \underset{(-4.14)}{0.66}\,\Delta i_t + \underset{(4.44)}{0.43}\,\Delta fi_t + \underset{(3.36)}{0.26}\,\Delta fd_t \\
&- \underset{(-2.27)}{0.10}\,Dum_{2000} + \underset{\mathbf{(1.14)}}{0.10}\,iis1997 + \underset{\mathbf{(1.31)}}{0.05}\,iis1999 + \underset{\mathbf{(1.57)}}{0.08}\,iis2006 \\
&- \underset{(-3.23)}{0.004}\,ect_{t-1}
\end{aligned} \tag{6.32}$$

$R^2 = 0.72$ LM$_{\text{Auto.}}\chi^2$ (1) = 0.62(0.43) JB$_{\text{Norm.}}\chi^2$ (2) = 2.18(0.37)
ARCH$_{\text{Hetro.}}\chi^2$(1)=0.49(0.48) BPG$_{\text{Hetro.}}\chi^2$(11)=10.87(0.36)
DW stat. = 1.75

$$\begin{aligned}
\Delta m_t &= \underset{(2.98)}{0.95}\,\Delta y_t + \underset{(1.92)}{0.05}\,\Delta by_{t-2} - \underset{(-2.80)}{0.49}\,\Delta i_t + \underset{(3.73)}{0.29}\,\Delta fi_t + \underset{(3.15)}{0.27}\,\Delta fd_t \\
&- \underset{(-1.94)}{0.10}\,Dum_{2000} + \underset{\mathbf{(0.84)}}{0.02}\,sis2000 - \underset{\mathbf{(0.79)}}{0.04}\,sis2006 + \underset{\mathbf{(0.11)}}{0.005}\,sis2007 + \\
&- \underset{(-1.94)}{0.004}\,ect_{t-1}
\end{aligned} \tag{6.33}$$

$R^2 = 0.66$ LM$_{\text{Auto.}}\chi^2$ (1) = 0.66(0.42) JB$_{\text{Norm.}}\chi^2$ (2) = 1.87(0.39)
ARCH$_{\text{Hetro.}}\chi^2$(1) = 0.42(0.52) BPG$_{\text{Hetro.}}\chi^2$(10) = 9.45(0.49)
DW stat. = 1.77

$$\begin{aligned}
\Delta m_t &= \underset{(1.73)}{0.51}\,\Delta y_t + \underset{(1.74)}{0.06}\,\Delta by_{t-2} - \underset{(-1.96)}{0.46}\,\Delta i_t + \underset{(2.94)}{0.34}\,\Delta fi_t + \underset{(2.81)}{0.26}\,\Delta fd_t \\
&- \underset{(-1.74)}{0.10}\,Dum_{2000} - \underset{(-2.04)}{0.09}\,tis1997 + \underset{(1.92)}{0.14}\,tis1998 - \underset{\mathbf{(-1.09)}}{0.06}\,tis2000 + \\
&\underset{\mathbf{(-1.09)}}{0.03}\,tis2002 - \underset{\mathbf{(-0.28)}}{0.02}\,tis2006 - \underset{\mathbf{(-0.43)}}{0.04}\,tis2007 + \underset{\mathbf{(1.37)}}{0.05}\,tis2009 \\
&- \underset{\mathbf{(-0.55)}}{0.01}\,tis2016 - \underset{(2.17)}{0.006}\,ect_{t-1}
\end{aligned} \tag{6.34}$$

$R^2 = 0.71$ LM$_{\text{Auto.}}\chi^2$ (1) = 1.46(0.23) JB$_{\text{Norm.}}\chi^2$ (2) = 1.23(0.52)
ARCH$_{\text{Hetro.}}\chi^2$(1) = 0.13(0.72) BPG$_{\text{Hetro.}}\chi^2$(15) = 13.67(0.55)
DW stat. = 1.67

i.4) Testing for Δfd_t:

The DGP using 6th order AR process and by dropping out the insignificant lags using GETS modeling what we left with are being testified in the following tables *see Table 14-Table 16.* However, inverting our conditional model into marginal model for financial development and then add significant dummies from DGP, following three equations (5.24-5.26) indicate the impact of IIS, SIS and TIS on Δfi_t

$$\begin{aligned}
\Delta fd_t &= -\underset{\mathbf{(-0.78)}}{0.41}\,\Delta y_t - \underset{\mathbf{(-1.18)}}{0.05}\,\Delta by_{t-2} - \underset{\mathbf{(-0.13)}}{0.04}\,\Delta i_t - \underset{\mathbf{(-1.19)}}{0.16}\,\Delta fi_t + \underset{(2.55)}{0.63}\,\Delta m_t \\
&- \underset{\mathbf{(-0.87)}}{0.07}\,Dum_{2000} + \underset{(1.89)}{0.10}\,iis1976 + \underset{(1.86)}{0.12}\,iis1985 - \underset{(-2.17)}{0.15}\,iis2009 \\
&- \underset{(-3.23)}{0.004}\,ect_{t-1}
\end{aligned} \tag{6.35}$$

$R^2 = 0.53$ LM$_{\text{Auto.}}\chi^2$ (1) = 1.73(0.19) JB$_{\text{Norm.}}\chi^2$ (2) = 0.91(0.63)
ARCH$_{\text{Hetro.}}\chi^2$(1)=0.81(0.37) BPG$_{\text{Hetro.}}\chi^2$(10)=9.27(0.51)
DW stat. = 1.60

Data Generating Process for Δfd_t

Table 6.15 *Impulse Indicator Saturation*

Variable	Δfd_t
Regression Type/DGP	IIS
Const.	-0.001
Δfd_{t-1}	0.257**
iis1976	0.155*
iis1985	0.160**
iis2009	-0.242*
Diagnostics and Fit	
Ljung-Box AR(1)	χ^2 (1) = 0.04 (0.55)
Ljung-Box ARCH(1)	χ^2 (1) = 0.00 (0.99)
Jiao-Pretis Prop.	Stat. 1.80 (0.07)
Jiao-Pretis Count	Stat. 3.00 (0.40)
S.E.= 0.07, R^2= 0.51, Log like.= 61.70	

Note: (*) and (**) represents 1% and 5% significance level. Values in (.) are respective p-value of the test statistic.

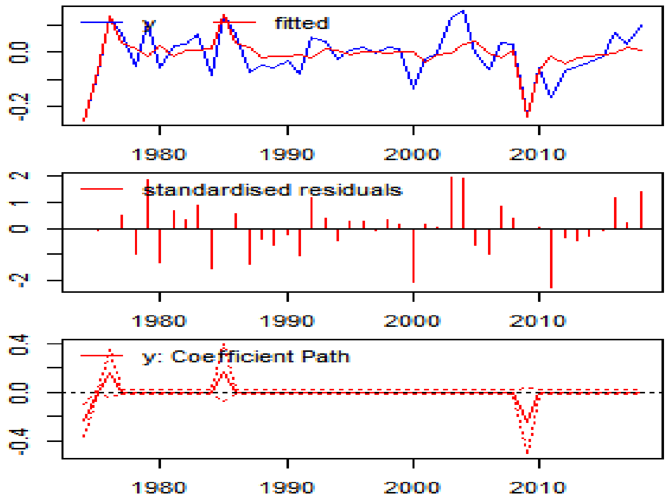


Table 6.16 *Step Indicator Saturation*

Variable	Δfd_t
Regression Type/DGP	SIS
Const.	-0.239*
Δfd_{t-1}	0.165
sis1975	0.249*
sis2009	-0.249*
sis2010	0.229*
Diagnostics and Fit	
Ljung-Box AR(1)	χ^2 (1) = 0.10 (0.76)
Ljung-Box ARCH(1)	χ^2 (1) = 0.15 (0.70)
S.E.= 0.07, R^2= 0.39, Log like.=56.64	

Note: (*) represents 1% significance level. Values in (.) are respective p-value of the test statistic.

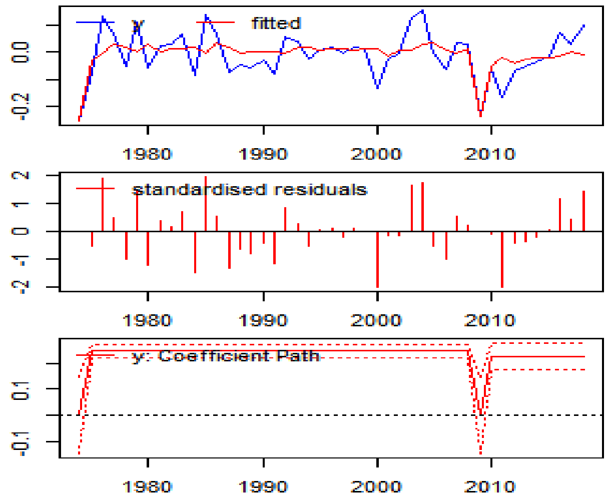
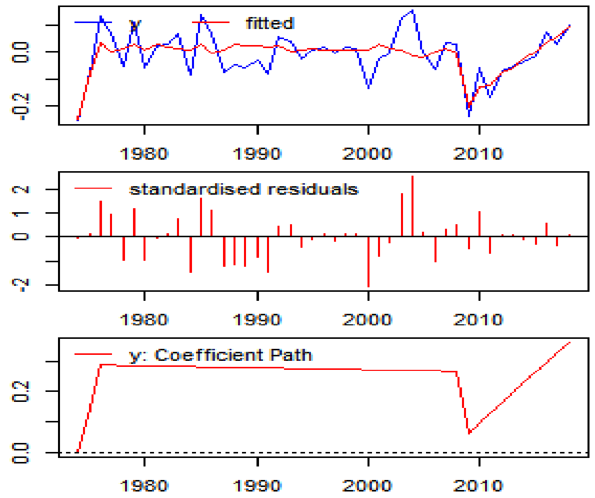


Table 6.19. *Trend Indicator Saturation*

Variable	Δfd_t
Regression Type/DGP	TIS
Const.	-0.263*
Δfd_{t-1}	-0.150
tis1975	0.144*
tis1977	-0.144*
tis2009	-0.200*
tis2010	0.234*
Diagnostics and Fit	
Ljung-Box AR(1)	χ^2 (1) = 0.49 (0.49)
Ljung-Box ARCH(1)	χ^2 (1) = 1.71 (0.19)
S.E.= 0.07, R^2= 0.50, Log like.= 61.12	

Note: (*) represents 1% significance level. Values in (.) are respective p-value of the test statistic.



Source: Authors own estimation

$$\Delta fd_t = -0.40\,\Delta y_t - 0.06\,\Delta by_{t-2} - 0.14\,\Delta i_t - 0.15\,\Delta fi_t + 0.65\,\Delta m_t$$
(-0.69) **(-1.31)** **(-0.43)** **(-1.06)** (2.45)

$$-0.10\,Dum_{2000} + 0.001 sis1975 - 0.16\,sis2009 + 0.11\,sis2010 + \qquad (6.36)$$
(-1.05) **(0.02)** (-2.16) (1.75)

$$-0.001\,ect_{t-1}$$
(-0.11)

$R^2 = 0.48$ $LM_{Auto.}\chi^2$ (1) = 0.16(0.68) $JB_{Norm.}\chi^2$ (2) = 1.25(0.54)
$ARCH_{Hetro.}\chi^2(1)$ = 0.31(0.57) $BPG_{Hetro.}\chi^2(10)$ = 6.65(0.75)
DW stat. = 1.82

$$\Delta fd_t = -0.51\,\Delta y_t - 0.05\,\Delta by_{t-2} + 0.41\,\Delta i_t - 0.15\,\Delta fi_t + 0.69\,\Delta m_t$$
(-0.78) **(-1.13)** **(1.14)** **(-1.08)** (2.90)

$$-0.08\,Dum_{2000} + 0.12\,tis1975 - 0.11\,tis1977 - 0.18\,tis2009 + \qquad (6.37)$$
(-0.94) **(1.39)** (-1.88) (-3.77)

$$0.20\,tis2010 + 0.03\,ect_{t-1}$$
(3.81) **(1.45)**

$R^2 = 0.58$ $LM_{Auto.}\chi^2$ (1) = 1.46(0.23) $JB_{Norm.}\chi^2$ (2) = 0.21(0.90)
$ARCH_{Hetro.}\chi^2(1)$ = 1.47(0.22) $BPG_{Hetro.}\chi^2(11)$ = 13.04(0.18)
DW stat. = 2.08

From above three equations we can easily see that each type of impulses in most of the cases entered significantly into the marginal process of Δfd_t and cause huge disturbances in the parameters of the model. Therefore, these dummies have their impact on the marginal distribution of Δfd_t. Lastly, we check their significance in the conditional model. Following three estimated equations (5.27-5.29) lead us to check the stability of the preferred model under these shocks. We incorporated these impulses in (6.16) for each subset of impulses and check their significance.

$$\Delta m_t = 1.06\,\Delta y_t + 0.05\,\Delta by_{t-2} - 0.51\,\Delta i_t + 0.29\,\Delta fi_t + 0.26\,\Delta fd_t$$
(3.69) (2.00) (-3.05) (3.81) (2.55)

$$-0.10\,Dum_{2000} + 0.02\,iis1976 - 0.03\,iis1985 - 0.04\,iis2009 \qquad (6.38)$$
(-2.04) **(0.42)** **(-0.69)** **(-0.90)**

$$-0.003\,ect_{t-1}$$
(-2.74)

$R^2 = 0.66$ $LM_{Auto.}\chi^2$ (1) = 0.27(0.61) $JB_{Norm.}\chi^2$ (2) = 1.05(0.59)
$ARCH_{Hetro.}\chi^2(1)$ = 1.38(0.24) $BPG_{Hetro.}\chi^2(10)$ = 9.89(0.45)
DW stat. = 1.82

$$\Delta m_t = 0.82\,\Delta y_t + 0.04\,\Delta by_{t-2} - 0.49\,\Delta i_t + 0.28\,\Delta fi_t + 0.23\,\Delta fd_t$$
(2.54) (1.77) (-2.84) (3.77) (2.45)

$$-0.12\,Dum_{2000} + 0.05\,sis1975 - 0.05\,sis2009 + 0.05\,sis2010 + \qquad (6.39)$$
(-2.23) **(1.19)** **(-0.99)** **(1.06)**

$$-0.001\,ect_{t-1}$$
(-2.19)

$R^2 = 0.66$ $LM_{Auto.}\chi^2$ (1) = 1.06(0.30) $JB_{Norm.}\chi^2$ (2) = 1.92(0.38)
$ARCH_{Hetro.}\chi^2(1)$ = 2.62(0.11) $BPG_{Hetro.}\chi^2(10)$ = 10.28(0.42)
DW stat. = 1.68

$$\Delta m_t = 0.56\,\Delta y_t + 0.04\,\Delta by_{t-2} - 0.68\,\Delta i_t + 0.24\,\Delta fi_t + 0.30\,\Delta fd_t$$
(1.86) (1.65) (-3.20) (2.87) (2.90)

$$-0.11\,Dum_{2000} + 0.12\,tis1975 + 0.003\,tis1977 + 0.03\,tis2009 - \qquad (6.40)$$
(-2.03) **(1.39)** **(0.01)** **(0.71)**

$0.03\ tis2010 - 0.01\ ect_{t-1}$
(-0.69) (-1.77)

$R^2 = 0.68$ $LM_{Auto.}\chi^2(1) = 1.82(0.18)$ $JB_{Norm.}\chi^2(2) = 1.56(0.46)$
$ARCH_{Hetro.}\chi^2(1) = 0.95(0.32)$ $BPG_{Hetro.}\chi^2(11) = 6.70(0.82)$
DW stat. = 1.64

From above three Equations one can inferred that the conditional model remain stable under the influence of all impulses when included in (6.16). On concluding remarks, the estimated frugal model is super exogenous against three types of impulses captured in the marginal models of all currently dated regressors, apart from one single time for a single type of impulse *i.e.* TIS *see* (6.28) above, where it doesn't pass the stability test, however remain stable for the other two types for the same marginal model. Therefore, the model can be used for policy purposes and forecasting as well.

6.3 An Index Based Test of Super Exogeneity

It may not be possible to add all those significant dummies at once in our conditional model to check its stability. So, it seems better to make and index for each type of impulses. This test is based on (Hendry & Santos, 2006). We generate three different indices using method discussed in (6.8) written above. We call these indices as I_{iis}, I_{sis} and I_{tis} stand for index based on IIS, SIS and TIS respectively. To form these indices we sum up all impulses of the same type from the marginal model of each putative regressor. The weight of each impulse assigned is its coefficient in that particular marginal model. For better understanding about how these indices are being generated *see Appendix.* After that we added these indices in conditional model (6.16) and check their individual significance via *t-stat*. However, one may test their joint significance as well. Equation (6.34) highlights an important aspect of the conditional model.

$$\Delta m_t = 1.18\ \Delta y_t + 0.05\ \Delta by_{t-2} - 0.57\ \Delta i_t + 0.40\ \Delta fi_t + 0.32\ \Delta fd_t \quad (6.41)$$
(3.11) (1.97) (-3.09) (3.61) (3.14)

$$-0.09\ Dum_{2000} - 0.22\ I_{iis} + 0.12\ I_{sis} - 0.02 I_{tis} - 0.004\ ect_{t-1}$$

(-1.76) **(-1.48)** **(0.66)** **(-0.28)** (-1.83)

$R^2 = 0.68$ LM$_{Auto.}\chi^2$ (1) = 2.57(0.11) JB$_{Norm.}\chi^2$ (2) = 2.31(0.32)

ARCH$_{Hetro.}\chi^2$(1) = 0.70(0.42) BPG$_{Hetro.}\chi^2$(10) = 5.74(0.84)

DW stat. = 1.54

As each index capture the effect of every single impulse of the same type, so we are nothing left with any significant dummy and jointly put them together even then the model remain stable. By applying joint test of significance of indices using Wald Test of linear restriction the $F_{(3,34)}$= 9.36(0.00) and χ^2(3) = 28.01(0.00) indicating the fact that these indices are jointly insignificant. Therefore, on this ground we conclude that our estimated model (6.16) is stable and super exogenous against relevant structural breaks like impulse shifts, step shifts and trend shifts in contemporaneous conditioning variables.

6.4 A Double Index Based Test

Hendry & Santos (2006) also proposed a double index based test of invariance lead to SupExt. The test suggested to incorporate the indices generated in the last test like I_{iis}, I_{sis} and I_{tis}. We also generate three more indices for each type of impulse in which each dummy is iterated with the corresponding variable's value at the break year following (6.10) and determine their joint significance using *F-test* as discussed in (6.11). These three new indices are named as II_{iis}, II_{sis} and II_{tis} stand for iterated index for IIS, SIS and TIS respectively. The results are being written in the following equation (6.42).

$$\Delta m_t = 0.58\,\Delta y_t + 0.04\,\Delta by_{t-2} - 0.78\,\Delta i_t + 0.46\,\Delta fi_t + 0.38\,\Delta fd_t$$

(1.64) **(1.40)** (-4.04) (4.01) (3.87)

$$-0.07\,Dum_{2000} - 0.26\,I_{iis} - 0.10\,I_{sis} - 0.16 I_{tis} + 0.003\,II_{iis} + \qquad (6.42)$$

(-1.41) **(-1.63)** **(-0.48)** (-1.72) **(0.60)**

$$0.005\,II_{sis} - 0.0001\,II_{tis} - 0.009\,ect_{t-1}$$

(1.74) **(-1.17)** (-2.61)

$R^2 = 0.75$ LM$_{Auto.}\chi^2$ (1) = 1.08(0.30) JB$_{Norm.}\chi^2$ (2) = 2.67(0.26)

ARCH$_{Hetro.}\chi^2$(1) = 0.91(0.34) BPG$_{Hetro.}\chi^2$(13) = 8.92(0.78)

DW stat. = 1.71

Although, none of the indices is significant at 5% level of significance. However, $I_{tis}$and II_{sis} are significant at 10% level of significance and causing a little deviance on the parameters of those variable which are not currently dated. This significance at 10% and insignificance of indices at 5% is possible due to the rejection of the null of invariance failure rather the rejection of weak exogeneity failure (Hendry and Santos, 2006a). Nevertheless, still one can see that currently dated regressors are yet to be destabilized. The joint significance of indices is determined by using Wald Test of linear restriction and *i.e.* $F_{(6,31)}$= 2.22 (0.06) and χ^2 (3) = 13.28 (0.04) indicating the fact that these indices are jointly insignificant. We conclude that with double index type of SupExt test our model is just fractionally deviant from the stability. Nevertheless, the parameters of interest remain stable.

6.5 Conclusion

On concluding remarks, the path of finding a stable and well-defined money demand function can be a nerve-racking task in the case of developing economies like Pakistan in which limited data available, high inflation and not much developed financial systems could prove to be significant constraints. However, using annual data series spanning over 1972-2018. There exists a long run relationship between the variables used in this study. The signs are well supported by the economic theory and previously available literature. Moreover, it was argued that the inclusion of structural break in the data may lead to generate instability in the parameter estimates of the model. This significantly valid question is being aptly carried out by incorporating location shifts of three different type of breaks like; IIS, SIS and TIS individually and also by using their indices as well, inter ilia though other types of breaks can also be tested as reported in (Ericsson, 2012). The retention of dummy saturation is set at $\alpha = 0.05$ significance level and can be fixed at a level of significance $\alpha = 0.025$ or even at $\alpha = 0.001$. The stability of the model is well tested via implying indicator saturation through SupExt tests. The stability of the model via such testing procedures strengthens the argument of previous studies that a stable money demand function could exist for Pakistan *see* Section-II (a). Since, estimated frugal model remain stable against the set of relevant class of interventions. Therefore, can use for policy purposes. However, these results may be different as reported here, if the weights of the indices set to be equal instead of their coefficients in the marginal model or if we divide the dummied with coefficients rather multiplying with them or even if someone use quarterly data set to estimate money demand model in case of Pakistan.

CHAPTER 7

CONCLUSIONS AND ROADMAP FOR FUTURE RESEARCH

Relationships that maintain parameter constancy *w.r.t* a wide range of different structural breaks and shocks that have had occurred elsewhere in the economy have a high degree of autonomy in comparison with those that break down more easily. Therefore, the autonomy of invariance property and SupExt are considered as relative concepts. A conditional model can have parameters that are super exogeneous *w.r.t* certain class of structural breaks, but not to all. So we use IIS, SIS, TIS & all at a time (jointly). However, one can use other types of breaks as well (for future research).

All estimated empirical models are fated to break apart, sooner or later! However, in the course of their life good models can be used for policy simulations and even for forecasting. A structural break may occur as result of some policy shift, sometimes referred as an intervention. For valid analysis of whether this policy change effect the endogenous variable or not, SupExt is the requisite property. Therefore, SupExt is linked with valid policy analysis from conditional models. In this study we estimate a money demand model in case of Pakistan. In our estimated conditional model all the putative regressors were found to be super exogeneous against relevant class of interventions. We found no evidence of Lucas critique. Therefore, the estimated model can be used for policy simulations. However, we didn't include the nonlinearities in our conditional model in presence of structural breaks which can be considered for an obvious case in future.

The advantage of this methodology in real world scenario is that, differently from the previous econometric procedures that have been used to test Ricardian equivalence, Lucas criticism can be tested in a more consistent and accepted way. In general, the use of SupExt tests is considered to be the most acknowledged methodology to test Lucas criticism. The SupExt test allows an econometric model to escape from the Lucas criticism. The variables that satisfied the conditions were labeled super exogenous. Whenever a variable is super exogenous, policy makers can use it to formulate economic policies. The application of SupExt tests is not limited to test the Lucas critique but also help policy makers to identify the existence of famous Ricardian Equivalence indirectly proposed in (Barro, 1974) and later implemented in (Barro, 1989). The evidence to the above statement can be found in (Kónya & Abdullaev, 2015; Sachsida & Teixeira, 2000; Sachsida et al., 2010).

Considering stationary data settings both *IB-Test* and *RB-Test* performs better than that of other SupExt tests. However, *H-Test* also performs better for all samples but *DIB-Test* didn't perform well for sample size 200. Therefore, as a whole we can say that *IB-Test* and *RB-Test* perform well in all scenarios and the use of all breaks jointly at a time is recommended when the putative regressor in the conditional model is being tested for SupExt. Now for non-stationary and dynamic settings, for small sample of 50 using IIS *CB-Test* seems good but as sample changes from 50 to 100 and then 200 the power of *CB-Test* reduced but the power of *IB-Test* and *RB-Test* show similar trend (improved). Though, *IB-Test* seems better than other SupExt test by means of its power. As a whole we can conclude that whatever is the type of break the test like *IB-Test* and *RB-Test* are better while implementing SupExt of the putative regressors in conditional model. Lastly, as the power of the tests is increased using all breaks at a time. Therefore, we recommend while testing SupExt using all breaks at time is more informative and useful than individual scenario.

On theoretical side, in this study we opted three types of breaks like IIS, SIS, TIS & all at a time in all SupExt tests while counting for their size and power. However, it would be an interesting case to check the relevance of other breaks that have been discussed in Section 3 *(subsection 3.1-3.4).* Also, we used a bivariate case in our conditional model. Therefore, a loop left behind to be fulfilled if one can extend the analysis to multivariate scenario in presence of dummy saturation along with capturing the non-linearities in their conditional as well as marginal models. For interested reader we refer to *see* Chapter 5 *(Subsection 5.6).*

REFERENCES

Abbas, K., & Husain, F. (2006). Money, Income and Prices in Pakistan: A Bi-variate and Tri-variate Causality. *South Asia Economic Journal*, *7*(1), 55–65. https://doi.org/10.1177/139156140500700103

Abdelnacer, B., Abdelhak, B., & Maliki, S. (2013). The Black Market Exchange Rate and Demand for Money in Algeria. *International Journal of Arts and Commerce*, *2*(10), 71–82. https://doi.org/10.1007/BF02295485

Abe, S., Fry, M. J., Min, B. K., Vongvipanond, P., & Yu, T. P. (1975). The Demand for Money in Pakistan: Some Alternative Estimates. *The Pakistan Development Review*, *14*, 249–257. https://ideas.repec.org/a/pid/journl/v14y1975i2p249-257.html

Achsani, N. A. (2010). Stability of money demand in an emerging market economy: An error correction and ARDL model for Indonesia. *Research Journal of International Studies*, *13*, 54–62.

Adhikari, P. (2018). Money Demand Function for Nepal : An Empirical View from Expenditure Component. *Economic Journal of Development Issues*, *25*&*26*(1–2), 12–25.

Adil, M. H., Hatekar, N., Fatima, S., Nurudeen, I., & Mohammad, S. (2022). Money Demand Function: A Not-So-Fond Farewell in the Light of Financial Development. *Journal of Economic Integration*, *37*(1), 93–120. https://www.jstor.org/stable/27107055?seq=1

Adil, M. H., Hatekar, N., & Sahoo, P. (2020). The Impact of Financial Innovation on the Money Demand Function : An Empirical Verification in India. *Margin—The Journal of Applied Economic Research*, *14*(1), 28–61. https://doi.org/10.1177/0973801019886479

Ahad, M. (2017). Financial Development and Money Demand Function: Cointegration, Causality and Variance Decomposition Analysis for Pakistan. *Global Business Review*, *18*(4), 811–824. https://doi.org/10.1177/0972150917692209

Ahmad, M., & Khan, A. H. (1990). A reexamination of the stability of the demand for money in pakistan. *Journal of Macroeconomics*, *12*(2), 307–321. https://doi.org/10.1016/0164-0704(90)90035-9

Ahmad, N., Naz, A., Naveed, A., & Jalil, A. (2007). Money Demand Function for Pakistan. *Forman Journal of Economic Studies*, *3*, 32–60.

Ahumada, H. (1992). A dynamic model of the demand for currency: Argentina 1977–1988. *Journal of Policy Modeling*, *14*(3), 335–361.

Akhtar, M. A. (1974). The Demand for Money in Pakistan. *The Pakistan*

Development Review, *13*(1), 40–54. https://ideas.repec.org/a/pid/journl/v13y1974i1p40-54.html

Andrews, D. W. K. (1993). Tests for Parameter Instability and Structural Change With Unknown Change Point. *Econometrica*, *61*(4), 821. https://doi.org/10.2307/2951764

Angrist, J. D., & Krueger, A. B. (1999). Empirical Strategies in Labor Economics. In O. Ashenfelter & D. Card (Eds.), *Handbook of Labor Economics, vol. 3A* (pp. 1277–1366). Amsterdam: Elsevier.

Angrist, J. D., & Krueger, A. B. (2001). Instrumental Variables and the Search for Identification: From Supply and Demand to Natural Experiments. *Journal of Economic Perspectives*, *15*(4), 69–85.

Anwar, S., & Asghar, N. (2012). Is Demand for Money Stable in Pakistan? *Pakistan Economic and Social Review*, *50*(1), 1–22. https://www.jstor.org/stable/24398814

Arrow, K. J. (1991). *Cowles in the History of Economic Thought. Cowles Fiftieth Anniversary*. , The Cowles Foundation, Yale University; New Haven.

Asad, M., Syed, S. H., & Hussain, I. (2011). Modeling Demand for Money in Pakistan: An ARDL Approach. *Forman Journal of Economic Studies*, *7*, 75–88. https://doi.org/10.1007/978-3-319-99555-7_2

Azeem, M. M., & Mohammad, A. (2015). Money and Physical Capital Relationship: McKinnon’s Complementarity Hypothesis on Turkey’s Economy. *Expert Journal of Finance*, *3*(1), 21–30. http://finance.expertjournals.com

Azim, P., Ahmed, N., Ullah, S., & Zakaria, M. (2010). Demand for Money in Pakistan: An ARDL Approach. *Global Journal of Management and Business Research*, *10*(9), 6–10.

Bahmani-Oskooee, M. (2001). How stable is M2 money demand function in Japan? *Japan and the World Economy*, *13*(4), 455–461. https://doi.org/10.1016/S0922-1425(01)00064-0

Bahmani-Oskooee, M., & Barry, M. P. (2000). Stability of the Demand for Money in an Unstable Country : Russia. *Journal of Post Keynesian Economics*, *22*(4), 619–629. https://www.jstor.org/stable/4538704

Bahmani-Oskooee, M., & Maki-Nayeri, M. (2018). Asymmetric Effects of Policy Uncertainty on the Demand for Money in the United States. *Journal of Risk and Financial Management*, *12*(1), 1–13. https://doi.org/10.3390/jrfm12010001

Bahmani, S. (2008). Stability of the Demand for Money in the Middle East Middle. *Emerging Markets Finance & Trade*, *44*(1), 62–83. https://www.jstor.org/stable/27750587

Bai, J. (1994). Least Squares Estimation of a Shift in a Linear Process. *Journal of*

Time Series Analysis, *15*, 453–472. https://doi.org/10.1111/j.1467-9892.1994.tb00204.x

Bai, J., & Perron, P. (1998). Estimating and Testing Linear Models With Multiple Structural Changes Econometrica. *Econometrica*, *66*, 47–78. https://doi.org/10.2307/2998540

Bai, J., & Perron, P. (2003). Computation and Analysis of Multiple Structural Change Models. *Journal of Applied Econometrics*, *18*(1), 1–22. https://doi.org/10.1002/jae.659

Bårdsen, G. (1992). Dynamic modeling of the demand for narrow money in Norway. *Journal of Policy Modeling*, *14*(3), 363–393. https://doi.org/10.1016/0161-8938(92)90005-W

Barnett, W. A., Ghosh, T., & Adil, M. H. (2022). Is money demand really unstable? Evidence from Divisia monetary aggregates. *Economic Analysis and Policy*, *74*, 606–622. https://doi.org/10.1016/J.EAP.2022.03.019

Barro, R. J. (1974). Are government bonds net wealth? Journal of Political Economy. *Journal of Political Economy*, *82*(6), 1095–1118. https://doi.org/10.1086/260266

Barro, R. J. (1989). The Ricardian Approach to Budget Deficits. *Journal of Economic Perspectives*, *3*, 37–54. https://doi.org/doi.org/10.1257/jep.3.2.37

Ben-Salha, O., & Jaidi, Z. (2014). Some new evidence on the determinants of money demand in developing countries - A case study of Tunisia. *Journal of Economic Asymmetries*, *11*, 30–45. https://doi.org/10.1016/j.jeca.2014.06.001

Bernanke, B. S., Boivin, J., & Eliasz, P. (2005). Measuring the Effects of Monetary Policy: A Factor-Augmented Vector Autoregressive (FAVAR) Approach. *Quarterly Journal of EconomicsEconomics*, *100*(1), 387–422.

Boswijk, H. P. (1991). *The LM-Test for Weak Exogeneity in Error Correction Models. Report AE 13/91. Institute of Actuarial Science and Economics, University of Amsterdam*.

Brown, R. L., Burbin, J., & Evans, J. M. (1975). Techniques for Testing the Constancy of Regression Relationships over Time. *Journal of the Royal Statistical Society . Series B (Methodological)*, *37*(2), 149–192. http://www.jstor.com/stable/2984889

Campos, J., & Ericsson, N. R. (1999). Constructive Data Mining: Modeling Consumers' Expenditure in Venezuela. *Econometrics Journal*, *2*(2), 226—240.

Canova, F. (1994). Statistical Inference in Calibrated Models. *Journal of Applied Econometrics*, *9*, 123–144. https://www.jstor.org/stable/2285227

Castle, J. L., Clements, M. P., & Hendry, D. F. (2011). *Forecasting by Factors, by Variables, by Both, or Neither?, mimeo, Institute for New Economic Thinking* *at the Oxford Martin School, University of Oxford, Oxford*.

Castle, J. L., Doornik, J. A., & Hendry, D. F. (2012). Model selection when there are multiple breaks. *Journal of Econometrics*, *169*(2), 239–246. https://doi.org/10.1016/J.JECONOM.2012.01.026

Castle, J. L., Doornik, J. A., Hendry, D. F., & Pretis, F. (2015). Detecting Location Shifts by Step-indicator Saturation. *Econometrics*, *3*(2), 240–264. https://doi.org/10.3390/econometrics03020240

Castle, J. L., Doornik, J. A., Hendry, D. F., & Pretis, F. (2019). *Trend Indicator Saturation. Working paper Oxford University Nuffield College*.

Castle, J. L., Hendry, D. F., & Martinez, A. B. (2017). Evaluating forecasts, narratives and policy using a test of invariance. *Econometrics*, *5*(3), 1–27. https://doi.org/10.3390/econometrics5030039

Chapman, D. A., & Ogaki, M. (1993). Cotrending and the stationarity of the real interest rate. *Economics Letters*, *42*(2–3), 133–138. https://doi.org/10.1016/0165-1765(93)90050-M

Charemza, W. W., & Deadman, D. F. (2003). *New Directions in Econometric Practice : General to Specific Modelling, Cointegration and Vector Autoregression* (2nd ed.). Cheltenham: Elgar.

Charemza, W. W., & Király, J. (1988). *A Simple Test for Structural Invariances and Policy Exogeneity* (Discussion Paper No. 95; Department of Economics, University of Leicester).

Charemza, W. W., & Király, J. (1990). Plans and Exogeneity : The Genetic-Teleological Dispute Revisited. *Oxford Economic Papers, New Series*, *42*(3), 562–573. https://www.jstor.org/stable/2663061

Cheung, Y. -W, & Lai, K. S. (1993). Finite-Sample Sizes of Johansen's Likelihood Ratio Tests for Cointegration. *Oxford Bulletin of Economics and Statistics*, *55*(3), 313–328. https://doi.org/10.1111/j.1468-0084.1993.mp55003003.x

Cho, H. C., & Ramirez, M. D. (2016). Money Demand in Korea : A Cointegration Analysis , 1973-2014. *Business and Economic Research*, *6*(1), 96–110. https://doi.org/10.5296/ber.v6i1.8950

Choo, H. G., & Kurita, T. (2015). A Monetary Perspective On Inflation Dynamics In Norway. *International Journal Of Economic Research*, *12*(3), 583–606.

Chow, G. C. (1960). Tests of Equality Between Sets of Coefficients in Two Linear Regressions. *Econometrica*, *28*(3), 591–605. https://doi.org/10.2307/1910133

Clements, M. P., & Hendry, D. F. (1999). *Forecasting Non-stationary Economic Time Series*. MIT Press Cambridge England.

Columba, F. (2009). Narrow money and transaction technology: New disaggregated evidence. *Journal of Economics and Business*, *61*(4), 312–325.

https://doi.org/10.1016/j.jeconbus.2009.01.001

Cooley, T. F., & LeRoy, S. F. (1981). Identification and Estimation of Money Demand. *American Economic Review, 71*(5), 824–844. https://www.jstor.org/stable/1803468

Crist, C. (1994). The Cowles Commission's Contributions to Econometrics at Chicago, 1939-1955. *Journal of Economic Literature*, *32,* 30–59.

Cuthbertson, K. (1991). The Encompassing Implications of Feedforward versus Feedback Mechanisms: A Reply to Professor Hendry. *Oxford Economic Papers*, *43*(2), 344–350.

Cuthbertson, Keith. (1988). Expectations, Learning and the Kalman Filter. *The Manchester School*, *56*(3), 223–246. https://doi.org/10.1111/j.1467-9957.1988.tb01330.x

Cuthbertson, Keith, & Taylor, M. P. (1990). Money demand, expectations, and the forward-looking model. *Journal of Policy Modeling*, *12*(2), 289–315. https://doi.org/10.1016/0161-8938(90)90032-A

Das, S., & Mandal, K. (2000). Money Demand in India : Testing Weak , Strong & Super Exogeneity. *Indian Economic Review*, *XXXV*(1), 1–19. http://www.jstor.org/stable/29794199

Davidson, R., & MacKinnon, J. G. (1981). Several Tests for Model Specification in the Presence of Alternative Hypotheses. *Econometrica*, *49*(3), 793. https://doi.org/10.2307/1911522

Davidson, R., & MacKinnon, J. G. (2004). *Econometric theory and methods*. Oxford University Press.

DeJong, D. N., Ingram, B. F., & Whiteman, C. H. (1996). A Bayesian Approach to Calibration. *Journal of Business and Economic Statistics*, *14*(1), 1–9. https://doi.org/10.1080/07350015.1996.10524625

Dickey, D. A., & Fuller, W. A. (1979). Distribution of the Estimators for Autoregressive Time Series With a Unit Root. *Journal of the American Statistical Association*, *74*(366), 427. https://doi.org/10.2307/2286348

Doan, T., Litterman, R., & Sims, C. A. (1984). Forecasting and Conditional Projection using Realistic Prior Distributions. *Econometric Reviews*, *3*(1), 1–100. https://doi.org/10.1080/07474938408800053

Doornik, J. A. (2007). *Autometrics. Working Paper, Economics Department, University of Oxford*.

Doornik, J. A. (2009a). Autometrics. In J. L. Castle & N. Shepard (Eds.), *The Methodology and Practice of Econometrics: A Festschrift in Honour of David F. Hendry, Chapter 4* (pp. 88–121). Oxford: Oxford University Press. https://doi.org/10.1093/acprof:oso/9780199237197.003.0004

Doornik, J. A. (2009b). *Econometric Model Selection With More Variables Than Observations. Working Paper.*

Doornik, J. A., & Hendry, D. F. (2007). *Empirical Econometric Modelling - PcGive 12: Volume I*. London: Timberlake Consultants Ltd.

Doornik, J. A., Hendry, D. F., & Pretis, F. (2013). *Step Indicator Saturation. Discussion Paper 658, Department of Economics, University of Oxford*.

Dou, X. (2019). The determinants of money demand in China. *Cogent Economics & Finance*, *6*(1), 1–17. https://doi.org/10.1080/23322039.2018.1564422

Drèze, J. (1962). The Bayesian approach to simultaneous equations estimation. O. N. R. Research Memorandum 67, Northwestern University. In D. Qin (Ed.), *The Rise of Econometrics (2013), vol. 3* (pp. 3–62). London: Routledge.

Dritsaki, C., & Dritsaki, M. (2020). The Long-run Money Demand Function : Empirical Evidence from Italy. *International Journal of Economics and Financial Issues*, *10*(1), 186–195. https://doi.org/https://doi.org/10.32479/ijefi.8943

Dritsaki, M., & Dritsaki, C. (2022). Long-Run Stability of Money Demand and Monetary Policy: The Case of South Korea. *Asian Economic and Financial Review*, *12*(5), 296–316. https://doi.org/10.55493/5002.V12I5.4482

Durbin, J. (1954). Errors in Variables. *Review of the International Statistical Institute, 22*(1), 23–32. http://www.jstor.org/stable/1401917

Ebadi, E. (2019). Does Government Spending Affect Money Demand in the United States? *The Economic Research Guardian*, *9*(1), 35–45.

Efron, B., Hastie, T., Johnstone, I., & Tibshirani, R. (2004). Least Angle Regression. *The Annals of Statistics*, *32*, 407–489.

Ehrlich, I. (1996). Crime, Punishment, and the Market for Offenses. *Journal of Economic Perspectives*, *10*(1), 43–67. https://doi.org/10.1257/jep.10.1.43

Ehrlich, I., & Gibbsons, J. C. (1977). On the Measurement of the Deterrent Effect of Capital Punishment and the Theory of Deterrence. *The Journal of Legal Studies*, *6*(1), 35–50. http://www.jstor.com/stable/724188

Emory, K. M., & Chang, C. P. (1996). Do Wages Help predict Inflation? *Economic Review*, *8*, 2–9, Federal Reserve Bank of Dallas, First Quarter.

Engle, R. F. (1980). *Exogeneity, Causality and Structural Invariance in Econometric Moddeling, CORE Discussion Paper, No. 8038*.

Engle, R. F., & Granger, C. W. J. (1987). Co-Integration and Error Correction : Representation , Estimation , and Testing. *Econometrica*, *55*(2), 251–276. http://www.jstor.com/stable/1913236

Engle, R. F., & Hendry, D. F. (1993). Testing superexogeneity and invariance in regression models. *Journal of Econometrics*, *56*, 119–139.

Engle, R. F., Hendry, D. F., & Richard, J. (1983). Exogeneity. *Econometrica*, *51*(2), 277–304. http://www.jstor.org/stable/1911990

Engle, R. F., & Kozicki, S. (1993). Testing for common features. *Journal of Business and Economic Statistics*, *11*(4), 369–380. https://doi.org/10.1080/07350015.1993.10509966

Epstein, R. J. (1987). *A History of Econometrics*. Amsterdam: North-Holland.

Ericsson, N. R. (1992). Cointegration, Exogeneity, and Policy Analysis: An Overview. *Journal of Policy Modeling*, *14*(3), 251–280.

Ericsson, N. R. (1999). Empirical modeling of money demand. *Money Demand in Europe*, 29–49. https://doi.org/10.1007/978-3-662-12539-7_2

Ericsson, N. R. (2004). The ET Interview: Professor David F. Hendry. *Econometric Theory*, *20*, 743–414. https://doi.org/I0.1017/S(1266466604704078

Ericsson, N. R. (2012). *Detecting Crises, Jumps, And Changes In Regime Revised: November 26, 2012*. 1–17.

Ericsson, N. R. (1991). Cointegration, exogeneity, and policy analysis: An overview. *International Finance Discussion Paper*, *415*, 1–31. https://doi.org/10.1016/0161-8938(92)90001-S

Ericsson, N. R., Hendry, D. F., & Mizon, G. E. (1998). Exogeneity, cointegration, and economic policy analysis. *Journal of Business and Economic Statistics*, *16*(4), 370–387. https://doi.org/10.1080/07350015.1998.10524778

Ericsson, N. R., & Irons, J. S. (1994). *Testing Exogeneity*. Oxford: Oxford University Press.

Ericsson, N. R., & Irons, J. S. (1995). The Lucas Critique in Practice: Theory Without Measurement. In K. D. Hoover (Ed.), *Macroeconometrics: Developments, Tensions and Prospect* (pp. 263–312). Boston, MA: Kluwer.

Ericsson, N. R., & Reisman, E. L. (2012). Evaluating a Global Vector Autoregression for Forecasting. *International Advances in Economic Research*, *18*(3), 247–258. https://doi.org/10.1007/S11294-012-9357-0

Faridi, M. Z., & Akhtar, M. H. (2013). An Estimation of Money Demand Function in Pakistan: Bound Testing Approach to Co-integration. *Pakistan Journal of Social Sciences*, *33*(1), 11–24.

Favero, C., & Hendry, D. F. (1992). Testing the Lucas critique: A Review. *Econometric Reviews*, *11*(3), 265–306. https://doi.org/10.1080/07474939208800238

Feldstein, M., & Horioka, C. (1980). Domestic Savings and International Capital Flows. *National Bureau of Economic Research Working Paper Series*, *310*.

Fisher, R. A. (1925a). Applications of 'Student's' Distribution. *Metron*, *5*, 90–104.

Fisher, R. A. (1925b). Statistical Methods for Research Workers. In *Oliver and Boyd*.

Fisher, R. A. (1925c). The influence of rainfall on the yield of wheat at Rothamsted. *Philos. Trans. Roy. Soc. London Ser. B*, *213*, 89–142.

Fisher, R. A. (1925d). Theory of Statistical Estimation. *Proc. Cambridge Philos. Soc.*, *22*, 700–725.

Fisher, W. D. (1962). Estimation in the Linear Decision Model. *International Economic Review*, *3*, 1–29.

Florens, J. P., Mouchart, M., & Richard, J. F. (1976). *Likelihood Analysis of Linear Models, CORE Discussion Paper No. 7619*.

Friedman, M. (1987). Quantity Theory of Money. *In J. Eatwell, M. Milgate, and P. Newman (Eds.) The New Palgrave: A Dictionary of Economics*, *4*(September), London: The Macmillan Press,3-20.

Frisch, R. (1933a). *Pitfalls in the Statistical Construction of Demand and Supply Curves*. Leipzig: Hans Buske.

Frisch, R. (1933b). Propagation Problems and Impulse Problems in Dynamic Economics. . Reprinted from Economic Essays in Honour of Gustav Cassel. *Universitetets Okonomiske Institutt, Oslo.*

Frisch, R. (1933c). Propagation Problems and Impulse Problems in Dynamic Economics. In *Economic Essays in Honor of Gustav Cassel* (pp. 171–205). London: Allen and Unwin.

Frisch, R. (1934). *Statistical Confluence Analysis by Means of Complete Regression Systems*. The University Institute of Economics; Oslo Norway.

Frisch, R., & Waugh, F. (1933). Partial Time Regressions as Compared with Individual Trends. *Econometrica*, *1*, 387–401.

Geweke, J., Koop, G., & Van Dijk, H. (Eds.). (2011). *The Oxford Handbook of Bayesian Econometrics*. Oxford: Oxford University Press.

Ghumro, N. H., & Karim, M. Z. A. (2017). The role of remittances in the stability of money demand in Pakistan: A cointegration analysis. *Economic Annals*, *LXII*(213), 45–65. https://doi.org/10.2298/EKA1713045G

Gilbert, C. L. (1986). Professor Hendry's Econometric Methodology. *Oxford Bulletin of Economics and Statistics*, *48*(3), 283–307.

Gilbert, C. L. (1989). LSE and The British Approach to Time Series Econometrics. *Oxford Economic Papers*, *41*(1), 108–128. https://www.jstor.org/stable/2663185

Goldfeld, S. M., & Sichel, D. E. (1990). The demand for money. In *Handbook of*

Monetary Economics: Vol. I (pp. 299–356). https://doi.org/10.1016/s1573-4498(05)80011-6

Gordon, R. J. (1976). Recent Developments in the Theory of Inflation and Unemployment. *Journal of Monetary Economics*, *2*(2), 185–219. https://doi.org/10.1016/0304-3932(76)90033-7

Granger, C. W. J. (1969). Investigating Causal Relations by Econometric Models and Cross-Spectral Methods. *Econometrica*, *37*(3), 424–438. https://doi.org/10.1017/ccol052179207x.002

Granger, C. W. J. (1980). Testing for causality: A Personal Viewpoint. *Journal of Economic Dynamics and Control*, *2*, 329–352. https://doi.org/10.1016/0165-1889(80)90069-X

Granger, C. W. J. (1981). Some properties of time series data and their use in econometric model specification. *Journal of Econometrics*, *16*(1), 121–130. https://doi.org/10.1016/0304-4076(81)90079-8

Granger, C. W. J. (1988). Some Recent Development in a Concept of Causality. *Journal of Econometrics*, *39*, 199–211. https://doi.org/10.1016/0304-4076(88)90045-0

Granger, C. W. J., & Newbold, P. (1974). Spurious Regressions in Econometrics. *Journal of Econometrics*, *2*, 111–120. https://doi.org/10.1002/9780470996249.ch27

Granger, C. W. J., & Teräsvirta, T. (1993). Modelling Nonlinear Economic Time Series. *OUP Catalogue, Oxford University Press*, 5. https://doi.org/10.1093/ACPROF:OSO/9780199587148.001.0001

Greene, W. H. (2003). Econometric Analysis. In *Journal of the American Statistical Association* (5th ed., Issue 428). Prentice Hall. https://www.jstor.org/stable/2291031?origin=crossref

Haavelmo, T. (1938). The Method of Supplementary Confluent Relations, Illustrated by a Study of Stock Prices. *Econometrica*, *6*(3), 203–218. https://doi.org/https://doi.org/10.2307/1907052

Haavelmo, T. (1943). The Statistical Implications of a System of Simultaneous Equations. *Econometrica*, *11*, 1–12.

Haavelmo, T. (1947). Methods of Measuring the Marginal Propensity to Consume. *Journal of the American Statistical Association*, *42*(237), 105–122. https://doi.org/10.1080/01621459.1947.10501917

Haberler, G. (1937). *Prosperity and Depression: A Theoretical Analysis of Cyclical Movements*. Geneva: League of Nations.

Hallam, D., & Zanoli, R. (1993). Error correction models and agricultural supply response. *European Review of Agricultural Economics*, *20*(2), 151–166. https://doi.org/10.1093/erae/20.2.151

Harbo, I., Johansen, S., Nielsen, B., & Rahbek, A. (1998). Asymptotic inference on cointegrating rank in partial systems. *Journal of Business and Economic Statistics*, *16*(4), 388–399. https://doi.org/10.1080/07350015.1998.10524779

Hatanaka, M., & Yamada, H. (2003). Co-trending: A Statistical System Analysis of Economic Trends. In *Co-trending: A Statistical System Analysis of Economic Trends*. Springer Japan. https://doi.org/10.1007/978-4-431-65912-9

Hausman, J. A. (1978). Specification Tests in Econometrics. *Econometrica*, *46*(6), 1251–1271.

Hendry, D. F. (1974). Stochastic Specification in an Aggregate Demand Model of the United Kingdom. *Econometrica*, *42*(3), 559—578.

Hendry, D. F. (1985). Monetary Economic Myth and Econometric Reality. *Oxford Review of Economic Policy*, *1*(1), 72–84. http://www.jstor.org/stable/23605727

Hendry, D. F. (1988). The Encompassing Implications of Feedback versus Feedforward Mechanisms in Econometrics. *Oxford Economic Papers*, *40*(1), 132–149.

Hendry, D. F. (1992). An Econometric Analysis of Expenditure in the United Kindom. *Journal of Policy Modeling*, *14*(3), 281–311.

Hendry, D. F. (1995). *Dynamic Econometrics*. (New York : Oxford University Press).

Hendry, D. F. (1999). An Econometric Analysis of US Food Expenditure, 1931-1989. In J. R. Magnus & M. S. Morgan (Eds.), *Methodology and Tacit Knowledge: Two Experiments in Econometrics, Chapter 17* (pp. 341–361). Chicester: John Wiley & Sons.

Hendry, D. F. (2000). Econometrics: Alchemy or Science? In *Econometrics: Alchemy or Science?* (New Edition). Oxford University Press. https://doi.org/10.1093/0198293542.001.0001

Hendry, D. F. (2003). J. Denis Sargan and the Origins of LSE Econometric Methodology. *Econometric Theory*, *19*(3), 458–480. http://www.jstor.org/stable/3533616

Hendry, D. F. (2004). Causality and Exogeneity in Non-Stationary Economic Time Series. In B. H. Baltagi & E. Sadka (Eds.), *Contributions to Economic Analysis* (pp. 21–48). New York: Emerald Insight.

Hendry, D. F., & Doornik, J. A. (2014). Empirical Model Discovery and Theory Evaluation. In *Empirical Model Discovery and Theory Evaluation*. The MIT Press. https://doi.org/10.7551/MITPRESS/9780262028356.001.0001

Hendry, D. F., & Ericsson, N. R. (1991a). An Econometric Analysis of U.K. Money Demand in Monetary Trends in the United States and the United Kingdom by Milton Friedman and Anna J. Schwartz. *The American Economic Review*, *81*(1), 8–38. http://www.jstor.org/stable/2006786

Hendry, D. F., & Ericsson, N. R. (1991b). An Econometric Analysis of UK Money Demand In Monetary Trends in the United States and the United Kingdom by Milton Friedman and Anna J. Schwartz. *International Finance Discussion Paper*, *81*(355), 8–38. https://doi.org/10.17016/ifdp.1989.355

Hendry, D. F., & Ericsson, N. R. (1991c). Modeling the demand for narrow money in the United Kingdom and the United States. *European Economic Review*, *35*(4), 833–881. https://doi.org/European Economic Review

Hendry, D. F., Johansen, S., & Santos, C. (2008). Automatic selection of indicators in a fully saturated regression. *Computational Statistics*, *23*(2), 337–339. https://doi.org/10.1007/s00180-008-0112-1

Hendry, D. F., & Krolzig, H.-M. (2005). The Properties of Automatic Gets Modelling. *The Economic Journal*, *115*(502), C32–C61. https://doi.org/10.1111/J.0013-0133.2005.00979.X

Hendry, D. F., & Krolzig, H. M. (1999). Improving on 'Data Mining Reconsidered' by K.D. Hoover and S.J. Perez. *Econometrics Journal*, *2*, 202–219.

Hendry, D. F., & Krolzig, H. M. (2003). New developments in Automatic General-to-Specific Modelling. In B. P. Stigum (Ed.), *Econometrics and the Philosophy of Economics* (pp. 379–419). Princeton: Princeton University Press.

Hendry, D. F., & Massmann, M. (2007). Co-Breaking: Recent Advances and a Synopsis of the Literature. *Journal of Business & Economic Statistics*, *25*(1), 33–51. https://www.jstor.org/stable/27638905

Hendry, D. F., & Mizon, G. E. (1993). Evaluating Dynamic Econometric Models by Encompassing the VAR. In P. C. B. Phillips (Ed.), *Models, Methods and Applications of Econometrics* (pp. 272–300). Oxford, UK. Blackwell.

Hendry, D. F., & Mizon, G. E. (1998). Exogeneity, causality, and co-breaking in economic policy analysis of a small econometric model of money in the UK. *Empirical Economics*, *23*(3), 267–294. https://doi.org/10.1007/BF01294408

Hendry, D. F., & Mizon, G. E. (2000). Reformulating Empirical Macroeconometric Modelling. *Oxford Review of Economic Policy*, *16*(4), 138–159.

Hendry, D. F., & Mizon, G. E. (2011). Econometric Modelling of Time Series with Outlying Observations. *Journal of Time Series Econometrics*, *3*(1). https://doi.org/10.2202/1941-1928.1100

Hendry, D. F., Muellbauer, J. N. J., & Murphy, T. A. (1990). The econometrics of DHSY. In J. D. Hey & D. Winch (Eds.), *A Century of Economics* (pp. 298–334). Oxford: Basil Blackwell.

Hendry, D. F., Pagan, A. R., & Sargan, D. J. (1984). Dynamic Specification. In Z. Griliches and M. D. Intriligator (Ed.), *Handbook of Econometrics, vol. 2* (pp. 1023–1100). Amsterdam: North-Holland.

Hendry, D. F., & Pretis, F. (2012). *Step-Indicator Saturation, mimeo, Economics*

Department and Institute for New Economic Thinking at the Oxford Martin School, Oxford University, Oxford, September.

Hendry, D. F., & Richard, J. F. (1989). Recent Developments in the Theory of Encompassing. In B. Cornet & H. Tulkens (Eds.), *Contributions to Operations Research and Econometrics: The XX anniversary of CORE* (pp. 393–440). Cambridge, MA: MIT Press.

Hendry, D. F., & Santos, C. (2005). Regression models with Data-Based Indicator Variables. *Oxford Bulletin of Economics and Statistics*, *67,* 571–595.

Hendry, D. F., & Santos, C. (2006). *Automatic Tests of Super Exogeneity, Economics Department; Oxford University*. 1–44. https://pdfs.semanticscholar.org/ed97/95b2b98fdea4371e295c3826f2badcec783d.pdf

Hendry, D. F., & Santos, C. (2010). An Automatic Test of Super Exogeneity. In M. W. Watson, T. Bollerslev, & J. Russell (Eds.), *Volatility and Time Series Econometrics* (pp. 164–193). Oxford University Press: Oxford, UK.

Hendry, D. F., & Ungern-Sternberg, T. Von. (1981). Liquidity and inflation effects on consumers' expenditure. In *Essays in the Theory and Measurement of Consumer Behaviour: In Honour of Sir Richard Stone* (pp. 237–260). Cambridge University Press. https://doi.org/10.1017/CBO9780511984082.013

Hess, G. D., & Schweitzer, M. E. (2000). *Does wage inflation cause price inflation?, Policy Discussion Paper No. 1, Federal Reserve Bank of Cleveland, Cleveland, OH.*

Hoover, K. D. (1990). The Logic of Causal Inference: Econometrics and the Conditional Analysis of Causality. *Economics and Philosophy*, *6*(2), 207–234.

Hoover, K. D. (2001). Causlity in Macroeconomics. In *The Methodology of Empirical Macroeconomics* (pp. 58–89). Cambridge University Press.

Hoover, K. D. (2006). Causality in Economics and Econometrics. In *Palgrave Macmillan (eds) The New Palgrave Dictionary of Economics*. Palgrave Macmillan, London. https://doi.org/https://doi.org/10.1057/978-1-349-95121-5_2227-1

Hoover, K. D., & Perez, S. J. (1999). Data mining reconsidered: encompassing and the general-to-specific approach to specification search. *Econometrics Journal*, *2*, 167–191.

Hossain, A. (1994). The Seaerch for a Stable Money Demand Function for Pakistan: An Application of the Method of Cointegration. *The Pakistan Development Review*, *34*(4), Part II. 969-983.

Hossain, A. (2006). The Money Demand Behaviour in Bangladesh, 1973-2003 : An Application of the Cointegration and Error-Correction Methods. *Indian Economic Review*, *41*(1), 55–80. http://www.jstor.com/stable/29793854

Hubrich, K., Lütkepohl, H., & Saikkonen, P. (2001). A review of systems cointegration tests. *Econometric Reviews*, *20*(3), 247–318. https://doi.org/10.1081/ETC-100104936

Hunter, J. (1992). Tests of Cointegrating Exogeneity for PPP and Uncovered Interest Rate Parity in the United Kingdom. *Journal of Policy Modeling*, *14*(4), 453–463. https://doi.org/10.1016/0161-8938(92)90016-6

Hye, Q. M. A., Wasti, S. K. A., Khatoon, N., & Imran, K. (2009). Relationship Between Stock Prices, Exchange Rate and the Demand for Money in India. *Economics, Management, and Financial Markets*, *3*, 89–96. http://www.eurojournals.com/MEFE.htm%0ARelationship

Iftekhar, U., Mammoon, D., & Hassan, M. S. (2016). Revisiting Determinants of Money Demand Function in Pakistan. *Journal of Economics Bibliography*, *3*(4), 559–569.

ISHII, T. (2022). Monetary policy framework changes and the money demand function. *Journal of Economics and Political Economy*, *9*(2), 191–207. https://doi.org/10.1453/JEPE.V9I2.2336

Jansen, E. S., & Teräsvirta, T. (1996). Testing parameter constancy and super exogeneity in econometric equations. *Oxford Bulletin of Economics and Statistics*, *58*(4), 735–763. https://doi.org/10.1111/j.1468-0084.1996.mp58004008.x

Jawad, M. (2014). *Modeling the Impact of Policy Environment on Inflows of Worker's Remittances in Pakistan: A Multivariate Analysis*. MPhil Dissertation submitted at Pakistan Institute of Development Economics.

Jawad, M., Kemal, A., & Saleh, M. (2022). Testing Exogeneity of Savings in Feldstein-Horioka Equation: The Case of Pakistan. *Presented at Conference on Public Economic Theory, Aix Marsielle University, France*.

Jiao, X., & Pretis, F. (2018). Testing the Presence of Outliers to Assess Misspecification in Regression Models. *SSRN Electronic Journal*, 1–72. https://doi.org/10.2139/ssrn.3217213

Johansen, S. (1988). Statistical analysis of cointegration vectors. *Journal of Economic Dynamics and Control*, *12*(2–3), 231–254. https://doi.org/10.1016/0165-1889(88)90041-3

Johansen, S. (1991). Estimation and Hypothesis Testing of Cointegration Vectors in Gaussian Vector Autoregressive Models. *Econometrica*, *59*(6), 1551–1580. https://doi.org/10.2307/2938278

Johansen, S. (1992a). Cointegration in partial systems and the efficiency of single-equation analysis. *Journal of Econometrics*, *52*(3), 389–402. https://doi.org/10.1016/0304-4076(92)90019-N

Johansen, S. (1992b). Testing weak exogeneity and the order of cointegration in UK money demand data. *Journal of Policy Modeling*, *14*(3), 313–334.

https://doi.org/10.1016/0161-8938(92)90003-U

Johansen, S. (1995). Identifying restrictions of linear equations with applications to simultaneous equations and cointegration. *Journal of Econometrics*, *69*(1), 111–132. https://doi.org/10.1016/0304-4076(94)01664-L

Johansen, S. (1996). *Likelihood-Based Inference in Cointegrated Vector Autoregressive Models*. Oxford University Press.

Johansen, S., & Juselius, K. (1990). Maximum Likelihood Estimation and Inference on Cointegration — With Applications to The Demand for Money. *Oxford Bulletin of Economics and Statistics*, *52*(2), 169–210. https://doi.org/0.1111/j.1468-0084.1990.mp52002003.x

Johansen, S., & Nielsen, B. (2008). An Analysis of the Indicator Saturation Estimator as a Robust Regression Estimator. *Univ. of Copenhagen Dept. of Economics*, *2461*(08–03), 1–35. https://doi.org/10.2139/ssrn.1129770

Judd, J. P., & Scadding, J. L. (1982). The Search for a Stable Money Demand Function: A Survey of the Post-1973 Literature. *Journal of Economic Literature*, *20*(3), 993–1023.

Juselius, K. (2006). *The Cointegrated VAR Model : Methodology and Applications*. Oxford University Press.

Kaplan, D. (2004). On Exogeneity. In K. David (Ed.), *The SAGE Handbook of Quantitative Methodology for the Social Sciences* (pp. 409–424). SAGE Publications.

Kejriwal, M., & Perron, P. (2008). *Testing for Multiple Structural Changes in Cointegrated Regression Models* (Issue 1276).

Kejriwal, M., & Perron, P. (2010). Testing for multiple structural changes in cointegrated regression models. *Journal of Business and Economic Statistics*, *28*(4), 503–522. https://doi.org/10.1198/jbes.2009.07220

Khan, A. H. (1980). The Demand for Money in Pakistan: Some Further Results. *The Pakistan Development Review*, *19*(1), 25–50. https://www.jstor.org/stable/41258469

Khan, A. H., & Hossain, A. (1994). Financial Liberalisation and the Demand for Money in Pakistan [with Comments]. *Pakistan Development Review*, *33*(4, Papers and Proceedings PART II Tenth Annual General Meeting of the Pakistan Society Development Economists Islamabad), 997–1010.

Khan, I. N. (1992). The Demand for Money in Pakistan & India. *Pakistan Economic and Social Review*, *30*(2), 181–190. https://www.jstor.org/stable/25825083

Khan, M. A., Arby, M. F., & Lodhi, M. A. K. (2000). Disaggregated Approach for Modeling Money Demand in Pakistan. *Pakistan Journal of Applied Economics*, *16*(1&2), 65–77.

Khan, R. E. A., & Hye, Q. M. A. (2013a). Financial Liberalization and Demand for Money: A Case of Pakistan. *The Journal of Developing Areas*, *47*(2), 175–198. https://doi.org/10.1353/jda.2013.0034

Khan, R. E. A., & Hye, Q. M. A. (2013b). Financial Liberalization and Demand for Money: A Case of Pakistan. *The Journal of Developing Areas*, *47*(2), 175–198.

Kloek, T., & van Dijk, H. K. (1978). Bayesian Estimates of Equation System Parameters: An Application of Integration by Monte Carlo. *Econometrica*, *46*(1), 1–19. https://doi.org/10.2307/1913641

Kocaaslan, O. K. (2013). The causal link between energy and output growth: Evidence from Markov switching Granger causality. *Energy Policy*, *63*, 1196–1206. https://doi.org/10.1016/j.enpol.2013.08.086

Kónya, L., & Abdullaev, B. (2015). Does Ricardian equivalence hold in Australia? A revision based on testing super exogeneity with impulse-indicator saturation. *Empirical Economics*, *49*(2), 423–448. https://doi.org/10.1007/s00181-014-0876-9

Koop, G. (1994). Recent Progress in Applied Bayesian Econometrics. *Journal of Economic Survey*, *8*, 1–34.

Koop, G., Poirier, D. J., & Tobias, J. L. (2007). *Bayesian Econometric Methods*. Cambridge: Cambridge University Press.

Koopmans, T. C. (1937). *Linear Regression Analysis of Economic Time Series, Bohn, Haarlem, Netherlands*.

Koopmans, T. C. (1945). Statistical Estimation of Simultaneous Economic Relations. *Journal of the American Statistical Association*, *40*(232), 448–466. https://doi.org/10.1080/01621459.1945.10500746

Koopmans, T. C. (1947). Measurement without Theory. *Review of Economics and Statistics*, *29*, 161–172. https://doi.org/10.2307/1928627

Koopmans, T. C. (1950). When is an Equation System Complete for Statistical Purposes? In *Statistical Inference in Dynamic Economic Models, T. C. Koopmans (ed.), Cowles Commission Monograph 10* (pp. 393--409). Wiley, New York.

Koopmans, T. C., & Hood, W. C. (1953a). *Studies in Econometric Method, Cowles Commission Monograph 14* (W. Hood & T. Koopmans (Eds.)). New Haven: Yale University Press.

Koopmans, T. C., & Hood, W. C. (1953b). The Estimation of Simultanous Linear Economic Relationships. In *Chapter VI of Hood & Koopmans (1953)*.

Koopmans, T. C., Rubin, H., & Leipnik, R. B. (1950). *Measuring the Equation Systems of Dynamic Economics. Chapter II of Koopmans (1950)*.

Kraft, J., & Kraft, A. (1978). On the Relationship Between Energy and GNP. *Journal of Energy and Development*, *3*(2), 401–403. https://www.jstor.org/stable/24806805

Krolzig, H. M., & Hendry, D. F. (2001). Computer Automation of General-to-Specific Model Selection. *Journal of Economic Dynamics and Control*, *25*, 831–866.

Krolzig, & Toro. (2002). Testing for Super-Exogeneity in the Presence of Common Deterministic Shifts. *Annales d'Économie et de Statistique*, *67/68*, 41. https://doi.org/10.2307/20076342

Kumar, P. (2014). The Determinants and Stability of Money Demand in India. *Online International Interdisciplinary Research Journal*, *V*, 35–54.

Kurita, T. (2007). A dynamic econometric system for the real yen-dollar rate. *Empirical Economics*, *33*(1), 115–149. https://doi.org/10.1007/s00181-006-0096-z

Kurita, T. (2010). Co-breaking, cointegration, and weak exogeneity: Modelling aggregate consumption in Japan. *Economic Modelling*, *27*(2), 574–584. https://doi.org/10.1016/J.ECONMOD.2009.11.012

Kurita, T., & Nielsen, B. (2018). *Partial cointegrated vector autoregressive models with structural breaks in deterministic terms. Discussion Paper 2018-W03, Nuffield College, Oxford University*. https://www.nuffield.ox.ac.uk/economics/Papers/2018/2018W03

Leamer, E. E. (1983a). Let's Take the Con Out of Econometrics. *The American Economic Review*, *73*(1), 31–43. http://www.jstor.org/stable/1803924

Leamer, E. E. (1983b). Model Choice and Specifi cation Analysis. In Z. Griliches & M. D. Intriligator (Eds.), *Handbook of Econometrics, vol. 1* (pp. 285–330). Amsterdam: North-Holland.

Leamer, E. E. (1985). Sensitivity Analyses Would Help. *American Economic Review*, *75*, 308–313.

Lee, C. C. (2005). Energy consumption and GDP in developing countries: A cointegrated panel analysis. *Energy Economics*, *27*, 415–427.

Leiva, B., & Rubio-Varas, M. (2020). The Energy and Gross Domestic Product Causality Nexus in Latin America 1900-2010. *International Journal of Energy Economics and Policy*, *10*(1), 423–435. https://doi.org/10.32479/ijeep.8670

Linde, J. (2001). Testing for the Lucas Critique: A Quantitative Investigation. *American Economic Review*, *91*(4), 986–1005. https://doi.org/10.1257/aer.91.4.986

Ljung, G. M., & Box, G. E. P. (1978). On a measure of lack of fit in time series models. *Biometrika*, *65*(2), 297–303. https://doi.org/10.1093/biomet/65.2.297

Lovell, M. C. (1983). Data Mining. *The Review of Economics and Statistics*, *65*(1), 1. https://doi.org/10.2307/1924403

Lubrano, M., Pierse, R. G., & Richard, J. F. (1986). Stability of a U. K. Money Demand Equation: A Bayesian Approach to Testing Exogeneity. *The Review of Economics Studies*, *53*(4), 603–634. https://doi.org/10.2307/2297608

Lucas, R. E. (1976). Econometric Policy Evaluation: A Critique. *Carnegie-Rochester Conference Series on Public Policy*, *1*, 19–46.

Lutkepohl, H. (1993). *Introduction to Multiple Time Series* (Second). Springer-Verlag.

MacKinnon, J. G. (1996). Numerical Distribution Functions for Unit Root and Cointegration Tests. *Journal of Applied Econometrics*, *11*(6), 601–618. http://www.jstor.com/stable/2285154

Magnusson, L. M., & Mavroeidis, S. (2014). Identification Using Stability Restrctions. *Econometrica*, *82*(5), 1799–1851. https://www.jstor.org/stable/24029294?seq=19

Mangla, I. U. (1979). An Annual Money Demand Function for Pakistan: Some Further Results. *The Pakistan Development Review*, *18*(1), 21–33. https://doi.org/10.30541/v18i1pp.21-33

Marschak, J. (1942). Identity and Stability in Economics: A Survey. *Econometrica*, *10*, 61–74.

Marschak, J. (1954). *Probability in the Social Sciences* (Cowles Commission Papers, New Series 82).

Masih, A. M. M., & Masih, R. (1996). Energy consumption, real income and temporal causality: Results from a multi-country study based on cointegration and error-correction modelling techniques. *Energy Economics*, *18*(3), 165–183. https://doi.org/10.1016/0140-9883(96)00009-6

McAleer, M. (1995). The significance of testing empirical non-nested models. *Journal of Econometrics*, *67*(1), 149–171. https://doi.org/10.1016/0304-4076(94)01631-9

Mehra, Y. (2000). Wage-Price Dynamics: Are they Consistent with Cost Push? *Federal Reserve Bank of Richmond Economic Quarterly*, *86*, 27–43.

Mizon, G. E. (1995). Progressive Modelling of Macroeconomic Time Series: The LSE Methodology. In K. D. Hoover (Ed.), *Macroeconometrics: Developments, Tensions, and Prospects* (pp. 107–170). Boston: Kluwer Academic Publishers.

Moinuddin. (2009). Choice of Monetary Policy Regime : Should the SBP Adopt Inflation Targeting ? *SBP-Research Bulletin*, *5*(1), 1–30. http://www.sbp.org.pk/research/bulletin/2009/vol5/ChoiceofMonetaryPolicyRegime

Moore, H. (1914). *Economic Cycles: Their Law and Cause*. New York: MacMillan.

Nduka, E. K. (2014). Structural Breaks and the Long-Run Stability of Demand for Real Broad Money Function in Nigeria: A Gregory-Hansen Approach. *The Economics and Finance Letters*, *1*(4), 76–89. https://doi.org/10.18488/journal.29/2014.1.4/29.4.76.89

Neftçi, S., & Sargent, T. J. (1978). A Little bit of Evidence on the Natural Rate Hypothesis from the U. S. *Journal of Monetary Economics*, *4*(2), 315–319. https://doi.org/10.1016/0304-3932(78)90009-0

Nel, H., Blaauw, D., & Preitorius, A. (2020). Investigsting the Hungrian Money Demand Function: Possible Implications for Monetary Policy. *International Journal of Economics and Finance Studies*, *12*(1), 71–87. https://doi.org/10.34109/ijefs.202012105

Newey, W. K., & West, K. D. (1987). A Simple , Positive Semi-Definite , Heteroskedasticity and Autocorrelation Consistent Covariance Matrix. *Econometrica*, *55*(3), 703–708. http://www.jstor.com/stable/1913610

Nisar, S., & Naheed, A. (1983). Interest Rates in Pakistan. *The Pakistan Development Review*, *XXII*(2), 97–115.

Nourzad, F. (2012). Assessing the Predictive Power of Labour-Market Indicators of Inflation. *Applied Economics Letters*, *19*(10), 981–985. https://doi.org/10.1080/13504851.2011.610732

Nymoen, R. (1992). Finnish manufacturing wages 1960-1987: Real-wage fexiblity and hysteresis. *Journal of Policy Modeling, 14*, 429–451.

Nymoen, R. (2019). *Dynamic Econometrics for Empirical Macroeconomic Modelling*. World Scientific Pub Co Inc. https://www.abebooks.com/Dynamic-Econometrics-Empirical-Macroeconomic-Modelling-Nymoen/30364157342/bd

Nyong, M. O. (2014). The Demand for Money, Structural Breaks and Monetary Policy in the Gambia. *Developing Country Studies*, *4*(19), 93–106.

Odularu, G. O., & Okunrinboye, O. A. (2009). Empirical modelling of the impact of financial innovation on the demand for money in Nigeria. *African Journal of Business Management*, *3*(2), 39–51. http://www.academicjournals.org/AJBM

Ogbonna, B. C. (2015). Exchange Rate and Demand for Money in Nigeria. *Research in Applied Economics*, *7*(2), 21. https://doi.org/10.5296/rae.v7i2.7916

Omer, M. (2010). Stability of Money Demand Function in Pakistan. *SBP Working Paper Series*, *8*, 1–30.

Orcutt, G. H. (1952a). Actions, Consequences, and Causal Relations. *Review of Economics and Statistics, 34*, 305–313.

Orcutt, G. H. (1952b). Review of Cowles Commission Monograph 10. *American*

Economic Review, *42*, 165–169.

Orcutt, G. H. (1952c). Toward Partial Redirection of Econometrics. *Review of Economics and Statistics*, *34*(3), 195–200.

Orcutt, G. H., & Irwin, J. O. (1948). A study of the autoregressive nature of the time series used for Tinbergen's model of the economic system of the United States 1919-1932. *Journal of the Royal Statistical Society Series B*, *10*(1), 1–53. https://www.jstor.org/stable/2983795

Pagan, A. (1987). Three Econometric Methodologies: A Critical Appraisal. *Journal of Economic Surveys*, *1*(1), 3–24.

Pagan, A. (1995). Three econometric methodologies: An Update. In L. Oxley, D. A. R. Goerge, C. L. Roberts, & S. Sayer (Eds.), *Surveys in Econometrics* (pp. 30–41). Oxford: Blackwell.

Pearl, J. (2000). Exogeneity and Superexogeneity: A No-tear Perspective. *Technical Report R-278, September*, 1–7.

Pearl, J. (2010). The Foundations of Causal Inference. *Sociological Methodology*, *40*(1), 75–149. https://doi.org/10.1111/j.1467-9531.2010.01228.x

Perez, S. J. (2002). Monetary Policy Does Matter: Control Causality and Superexogeneity. *Oxford Bulletin of Economics and Statistics*, *64*(5), 473–486. https://doi.org/10.1111/1468-0084.00273

Phillips, P. C. B., & Sargan, D. J. (1985). ET Interviews: Professor J. D. Sargan. *Econometric Theory*, *1*, 119–139.

Phillips, P., & Perron, P. (1988). Testing for a Unit Root in Time Series Regression. *Biometrika*, *75*(2), 335–346. http://www.jstor.com/stable/2336182

Poirier, D. J. (1988). Frequentist and Subjectivist Perspectives on the Problems of Model Building in Economics. *Journal of Economic Perspectives*, *2*, 121–144.

Pradhan, B. K., & Subramanian, A. (2003). On the stability of demand for money in a developing economy Some empirical issues. *Journal of Development Economics*, *72*(1), 335–351. https://doi.org/10.1016/S0304-3878(03)00080-4

Press, S. J. (1980). Bayesian Computer Programs. In A. Zellner (Ed.), *Bayesian Analysis in Econometrics and Statistics* (pp. 429–442). Amsterdam: North-Holland.

Pretis, F. (2017). Exogeneity in Climate Econometrics. *SSRN Electronic Journal*, *September*, 1–30. https://doi.org/10.2139/ssrn.2898203

Pretis, F. (2021). Exogeneity in climate econometrics. *Energy Economics*, *96*, 1–27. https://doi.org/10.1016/j.eneco.2021.105122

Psaradakis, Z., & Sola, M. (1996). On the power of tests for superexogeneity and structural invariance. *Journal of Econometrics*, *72*(1–2), 151–175.

https://doi.org/10.1016/0304-4076(94)01718-2

Qayyum, A. (1998). Error Correction Model of the Demand for Money in Pakistan. *The Kashmir Economic Review*, *VI*(No. 1&2), 1994–1998.

Qayyum, A. (2001). Sectoral analysis of the demand for real money balances in Pakistan. *Pakistan Development Review*, *40*(4 PART II), 953–966. https://doi.org/10.30541/v40i4iipp.953-966

Qayyum, A. (2005a). Modelling the demand for money in Pakistan. *Pakistan Development Review*, *44*(3), 233–252. https://doi.org/10.30541/v44i3pp.233-252

Qayyum, A. (2005b). Modelling the Demand for Money in Pakistan. *Pakistan Development Review*, *44*(3), 233–252. https://doi.org/10.30541/v44i3pp.233-252

Qayyum, A., & Azid, T. (2000). *Demand for Real Money Balances by the Business Sector : An Econometric Investigation [with Comments]*. *39*(4), Part II, 857-873. https://www.jstor.org/stable/41260302

Qayyum, A., & Khan, M. A. (2003). Capital Flows and Money Supply : The Degree of Sterilisation in Pakistan. *The Pakistan Development Review*, *42*(4, Part II), 975–985. https://www.jstor.org/stable/41260449

Ramachandran, M. (2004). Do broad money, output, and prices stand for a stable relationship in India? *Journal of Policy Modeling*, *26*(8–9), 983–1001. https://doi.org/10.1016/j.jpolmod.2004.08.008

Rasasi, M. H. Al. (2020). Assessing the Stability of Money Demand Function in Saudi Arabia. *International Journal of Economics and Financial Research*, *6*(2), 22–28. https://doi.org/https://doi.org/10.32861/ijefr.62.22.28

Rathnasiri, R. A. (2021). The Stability of Money Demand Function in Sri Lanka: A Bound Cointegration Approach. *Wayamba Journal of Management*, *12*(1), 1. https://doi.org/10.4038/wjm.v12i1.7514

Reade, J. J., & Volz, U. (2011). From the General to the Specific—Modelling Inflation in China. *Applied Economics Quarterly*, *57*(1), 27–44. https://doi.org/10.3790/AEQ.57.1.27

ReiersØl, O. (1945). Confluence Analysis by Means of Instrumental Sets of Variables. *Arkiv for Mathematik, Astronomi, Och Fysik*, *32*(A), 1–11.

Richard, J. F. (1980). Models with Several Regimes and Changes in Exogeneity. *Review of Economic Studies*, *47*, 1–20.

Richard, J. F. (1995). Bayesian Model Selection and Prediction with Empirical Applications Discussion. *Journal of Econometrics*, *69*(1), 337–349. https://doi.org/10.1016/0304-4076(94)01674-O

Robbins, L. (1932). *An Essay on the Nature and Significance of Economic Science*.

London: Macmillan.

Rodríguez-Caballero, C. V., & Ventosa-Santaulària, D. (2017). Energy-growth long-term relationship under structural breaks. Evidence from Canada, 17 Latin American economies and the USA. *Energy Economics*, *61*, 121–134. https://doi.org/10.1016/j.eneco.2016.10.026

Rothenberg, T. J. (1963). *A Bayesian Analysis of Simultaneous Equations Systems* (Econometric Institute Report 6315, Netherlands School of Economics).

Sachsida, A., & Cardoso de Mendonça, M. J. (2006). Domestic Saving And Investment Revised: Can The Feldstein- Horioka Equation Be Used For Policy Analysis? *TEXTO PARA DISCUSSÃO N° 1158*, 1–32.

Sachsida, A., & Teixeira, J. R. (2000). Ricardian Equivalence, Lucas Critic and Exogeneity Tests. *SSRN Electronic Journal*. https://doi.org/10.2139/SSRN.293190

Sachsida, A., Walnut, F. C., & Cardoso de Mendonça, M. J. (2010). Ricardian Equivalence and Lucas Critique: An Alternative Test of Ricardian Equivalence Using Super Exogeneity Tests in Simulated Series. *Revista EconomiA*, *11*(4), 15–26.

Sargan, D. J. (1958). The Estimation of Economic Relationships Using Instrumental Variables. *Econometrica*, *26*, 393–415.

Sargan, D. J. (1959). The estimation of relationships with autocorrelated residuals by the use of instrumental variables. *Journal of the Royal Statistical Society, Series B*, *21*, 91–105.

Sargan, D. J. (1964). Wages and Prices in the United Kingdom: A Study in Econometric Methodology. In *Econometric Analysis for National Economic Planning. Edited by P. E. Hart. London: Butterworths* (pp. 25–54).

Sargent, T. J., & Sims, C. A. (1977). Business Cycle Modelling without Pretending to have too much a priori Economic Theory. In *New Methods in Business Cycle Research: Proceedings from a Conference, Federal Reserve Bank of Minneapolis* (pp. 45–109).

Sarwar, H., Hussain, Z., & Sarwar, M. (2010). Money Demand Function For Pakistan (Divisia Approach). *Pakistan Economic and Social Review*, *48*(1), 1–20.

Sarwar, H., Sarwar, M., & Waqas, M. (2013). Stability of Money Demand Function in Pakistan. *Economic And Business Review*, *15*(3), 197–212.

Schrooten, M., & Stephan, S. (2005). Private savings and transition. *Economics of Transition*, *13*(2), 287–309.

Schultz, H. (1928). *Statistical Laws of Demand and Supply with Special Application to Sugar*. Chicago: University of Chicago Press.

Seaks, T. G., & Layson, S. K. (1983). Box-Cox Estimation with Standard

Econometric Problems. *The Review of Economics and Statistics*, *65*(1), 160–164. http://www.jstor.com/stable/1924424

Shafiq, H. & Malik, W. S. (2018). Money Demand Function Revisited : the Role of Asset Prices in Pakistan. *Pakistan Economic and Social Review, 56*(1), 67–92.

Shiller, R. J. (1973). A Distributed Lag Estimator Derived from Smoothness Priors. *Econometrica*, *41*(4), 775–788. https://doi.org/10.2307/1914096

Simon, H. A. (1953). Causal Order and Identifiability. In *Hood and Koopmans (1953)* (pp. 49–74).

Simon, H. A. (1954). Spurious Correlation: A Causal Interpretation. *Journal of the American Statistical Association*, *49*(267), 467–479. https://doi.org/10.2307/2281124

Sims, C. A. (1980a). Comparison of Interwar and Postwar Business Cycles: Monetarism Reconsidered. *The Americam Economic Review*, *70*(2), 250–257. http://www.jstor.org/stable/1815476

Sims, C. A. (1980b). Macroeconomics and Reality. *Econometrica*, *48*(1), 1. https://doi.org/10.2307/1912017

Sims, C. A., & Zha, T. (1998). Bayesian Methods for Dynamic Multivariate Models. *International Economic Review*, *39*(4), 949–968. https://doi.org/10.2307/2527347

Spanos, A. (1986). *Statistical Foundations of Econometric Modeling*. Cambridge University Press.

Sriram, S. S. (1999). Survey of Literature on Demand for Money: Theoretical and Empirical Work with Special Reference to Error-Correction Models. In *IMF Working Paper* (WP/99/64; Vol. 1).

Stock, J. H., & Watson, M. W. (2002). Macroeconomic Forecasting Using Diffusion Indexes. *Journal of Business and Economic Statistics*, *20*(2), 147–162. https://doi.org/https://doi.org/10.1198/073500102317351921

Stock, J. H., & Watson, M. W. (2005). *Implications of Dynamic Factor Models for VAR Analysis, NBER Working Paper No. 11467, National Bureau of Economic Research, Cambridge, Massachusetts.*

Sucarrat, G., Ptretis, F., & Reade, J. (2020). *Package ‘ gets ’* (0.23). 1–76. https://cran.r-project.org/package=gets, http://www.sucarrat.net/R/gets

Suliman, Z. S., & Dafaalla, H. A. (2011). An econometric analysis of money demand function in Sudan, 1960 to 2010. *Journal of Economics and International Finance*, *3*(16), 793–800. https://doi.org/10.5897/jeif11.122

Thornber, H. (1967). Finite sample Monte Carlo studies: An autoregressive illustration. *Journal of the American Statistical Association*, *62*(319), 801–818. https://doi.org/10.2307/2283672

Tibshirani, R. (1996). Regression Shrinkage and Selection via the Lasso. *Journal of the Royal Statistical Society. Series B (Methodological)*, *58*(1), 267–288. http://www.jstor.org/stable/2346178

Tinbergen, J. (1940). On a Method of Statistical Business-Cycle Research; A Reply. *The Economic Journal*, *50*(197), 141–154.

Togay, S., & Kose, N. (2013). Money-Price Relationships Under a Currency Board System: The Case of Argentina. *Journal of Applied Economics*, *26*(2), 373–390.

Urbain, J. P. (1992). On Weak Exogeneity in Error Correction Models. *Oxford Bulletin of Economics and Statistics*, *54*(2), 187–207. https://ideas.repec.org/p/fth/lieged/9103.html

Van Dijk, H. K. (2003). On Bayesian Structural inference in a Simultaneous Equation Model. In B. P. Stigum (Ed.), *Econometrics and Philosophy of Economics* (pp. 642–682). Princeton, NJ: Princeton University Press.

Walker, A. J., Pretis, F., Powell-Smith, A., & Goldacre, B. (2019). Variation in responsiveness to warranted behaviour change among NHS clinicians: novel implementation of change detection methods in longitudinal prescribing data. *British Medical Journal*, *367*. https://doi.org/10.1136/BMJ.L5205

Wold, H. O. A. (1960). A generalization of causal chain models. *Econometrica*, *28*(2), 443–463. https://doi.org/0012-9682(196004)28:2<443:AGOCCM>2.0.CO;2-J

Wold, H. O. A. (Ed.). (1964). *Econometric Model Building: Essays on the Causal Chain Approach*. Amsterdam: North-Holland.

Wolde-Rufael, Y. (2004). Disaggregated Industrial Energy Consumption and GDP: The Case of Shanghai. *Energy Economics*, *26*, 69–75.

Wu, D. (1973). Alternative Tests of Independence between Stochastic Regressors and Disturbances: Finite Sample Results. *Econometrica*, *42*(3), 529–546. http://www.jstor.org/stable/1911789

Zellner, A. (1971). *An Introduction to Bayesian Inference in Econometrics*. New York: Wiley.

Zellner, A. (1979). Causality and Econometrics. In K. Brunner & H. M. Allan (Eds.), *Three Aspects of Policy Making: Knowledge, Data and Institutions, Carnegie-Rochester Conference Series on Public Policy, vol. 10. Amsterdam, NorthHolland* (pp. 9–54).

Zellner, A. (1984). *Basic Issues in Econometrics*. Chicago: University of Chicago Press.

Zellner, A. (2008). Bayesian Econometrics: Past, Present and Future. In and D. T. S. Chib, W.Griffiths, G. Koop (Ed.), *Bayesian Econometrics, vol. 23 of the series: Advances in Econometrics*.

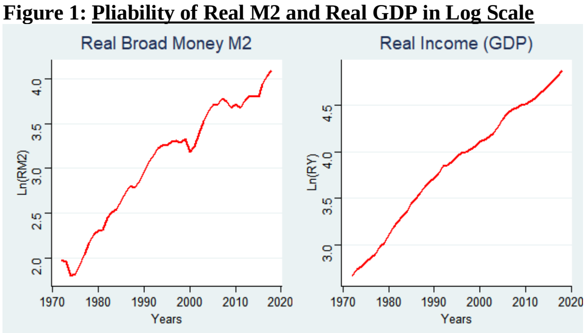
Zellner, A., Bauwens, L., & Van Dijk, H. K. (1988). Bayesian Specification Analysis and Estimation of Simultaneous Equation Models using Monte Carlo Methods. *Journal of Econometrics*, *38*, 39–72.

Zellner, A., & Geisel, M. S. (1970). Analysis of distributed lag models with applications to consumption function estimation. *Econometrica*, *38*(6), 865–888. https://doi.org/10.2307/1909697

Zou, H. (2006). The Adaptive Lasso and Its Oracle Properties. *Journal of the American Statistical Association*, *101*(476), 1418–1429. https://doi.org/10.1198/016214506000000735

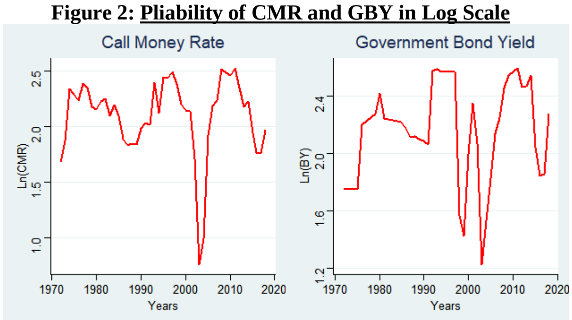
APPENDIX A

Figure 1: Pliability of Real M2 and Real GDP in Log Scale



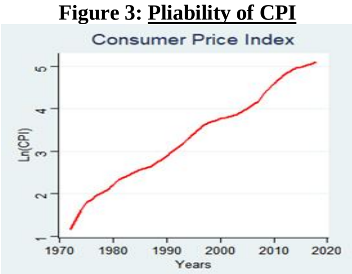
Source: SBP and WDI

Figure 2: Pliability of CMR and GBY in Log Scale



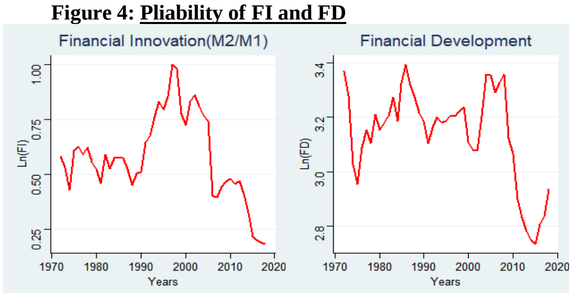
Source: IFS and SBP

Figure 3: Pliability of CPI



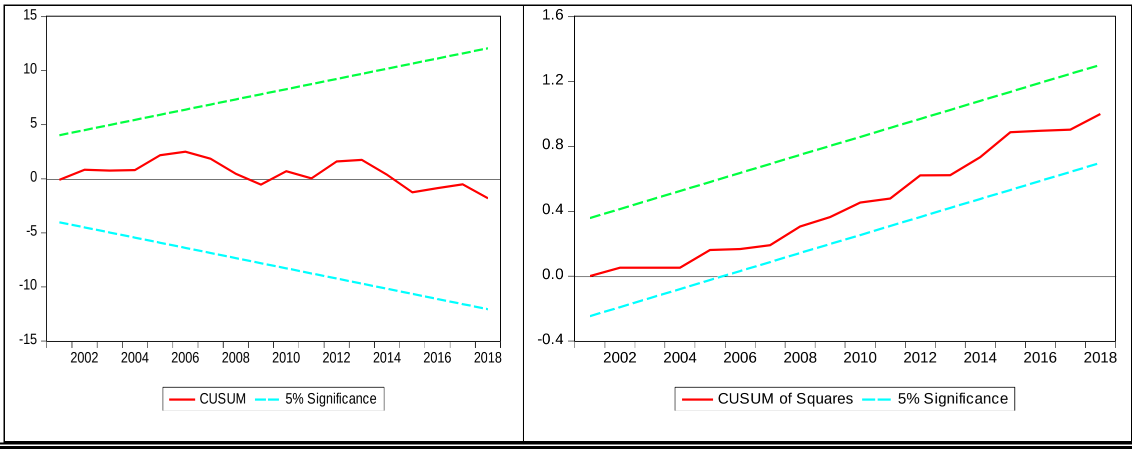
Source: WDI

Figure 4: Pliability of FI and FD



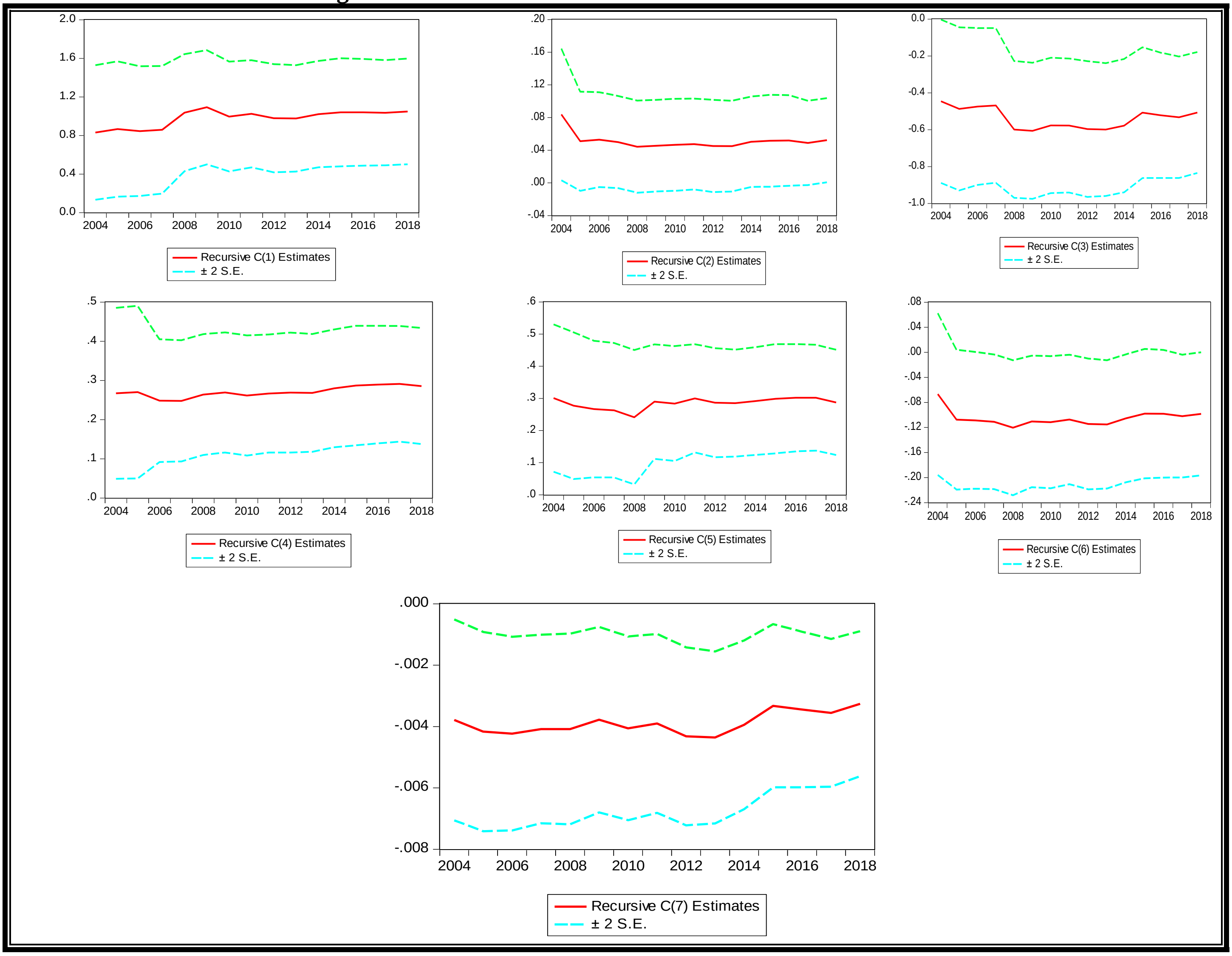
Source: SBP and WDI

Figure 5: CUSUM and CUSUM Sq. Plot with 5 % Significance Level Critical Bands



Source: Author's own estimations

Figure 6: Plot of Estimated Coefficient with 2 S.E. Bands



Generating Indices:

I_iis = 0.05* iis1978_y+0.05*iis1980_y+0.03*iis1985_y+ 0.03*iis1992_y-0.03* iis1993_y-0.03*iis1997_y+0.03*iis1976_i+0.08* iis2008_i+0.07* iis1975_fi-0.12* iis1997_fi-0.17* iis1999_fi-0.28* iis2006_fi+0.10*iis1976_fd+0.12* iis1985_fd-0.15*iis2009_fd

I_sis=-0.03* sis1978_y+0.05* sis1979_y-0.03*sis1981_y- 0.001*sis1993_y-0.01* sis1994_y-0.02* sis1997_y+0.02*sis2003_y- 0.02*sis2008_y-0.002* sis2013_y +0.05* sis1976_i-0.10* sis1977_i- 0.04*sis2008_i+0.008* sis2009_i 0.06*sis2000_fi+0.06*sis2006_fi-0.03 *sis2007_fi+0.001*sis1975_fd-0.16* sis2009_fd+0.11*sis2010_fd

I_tis= 0.04*tis1978_y-0.10*tis1979_y+0.12*tis1980_y-0.10*tis1981_y +0.03*tis1982_y-0.04*tis1993_y+ 0.05* tis1994_y-0.05* tis1997_y+0.04* tis1998_y-0.02* tis2006_y+0.02* tis2009_y-0.09*tis1975_i+0.06* tis1977_i +0.02*tis1978_i+0.10*tis2008_i-0.14* tis2009_i+0.03*tis2010_i+0.12* tis1997_fi -0.26* tis1998_fi+0.25* tis2000_fi-0.15* tis2002_fi-0.16* tis2006_fi +0.32*tis2007_fi-0.14* tis2009_fi+0.05*tis2016_fi+0.12* tis1975_fd-0.11* tis1977_fd-0.18*tis2009_fd+0.20*tis2010_fd

Note: For iterated index, these coefficients are being replaced by the corresponding value of the variable at the break date.