ARDL Model as a Remedy for Spurious Regression: Problems, Performance and Prospects

Submitted by

Ghulam Ghouse

Reg. No. PIDE2015FPHDETS04

Supervised by

Dr. Saud Ahmed Khan

Co-Supervised by

Dr. Atiq Ur Rehman

Department of Econometrics and Statistics

Pakistan Institute of Development Economics

Islamabad, Pakistan

2019

ARDL Model as a Remedy for Spurious Regression: Problems, Performance and Prospects

Submitted by

Ghulam Ghouse

Reg. No. PIDE2015FPHDETS04

A Dissertation Submitted to the Pakistan Institute of Development Economics, Islamabad, in partial fulfillment of the requirements of the Degree of Doctor of Philosophy in Econometrics

Supervised by

Dr. Saud Ahmed Khan

Co-Supervised by

Dr. Atiq Ur Rehman

Department of Econometrics and Statistics

Pakistan Institute of Development Economics
Islamabad, Pakistan
2019

Pakistan Institute of Development Economics

Certificate of Approval

This is to certify that the research work presented in this thesis, entitled: **"ARDL Model as a Remedy for Spurious Regression: Problems, Performance and Prospects"** was conducted by Mr. Ghulam Ghouse under the supervision of Dr. Saud Ahmed Khan and Dr. Atiq ur Rehman. No part of this thesis has been submitted anywhere else for any other degree. This thesis is submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Econometrics from **Pakistan Institute of Development Economics, Islamabad.**

Student Name: **(Ghulam Ghouse)**
PIDE2015FPHDETS04

Examination Committee

Supervisor:
Dr. Saud Ahmed Khan
Assistant Professor
PIDE, Islamabad.

Co-supervisor:
Dr. Atiq ur Rehman
Associate Professor
The University of Azad Jammu & Kashmir, Muzaffarabad.

External Examiner:
Dr. Abdul Rashid
International Institute of Islamic Economics
International Islamic University, Islamabad

Internal Examiner:
Dr. Abdul Jalil
SBP Chair
PIDE, Islamabad

Head,
Department of Econometrics and Statistics:
Dr. Amena Urooj

Plagiarism Undertaking

I solemnly declare that research work presented in the thesis titled **"ARDL Model as a Remedy for Spurious Regression: Problems, Performance and Prospects"** is solely my research work with no significant contribution from any other person. Small contribution/help wherever taken has been duly acknowledged and that complete thesis has been written by me.

I understand the zero tolerance policy of the **HEC** and **Pakistan Institute of Development Economics, Islamabad** towards plagiarism. Therefore I as an Author of the above titled thesis declare that no portion of my thesis has been plagiarized and any material used as reference is properly cited.

I undertake that if I am found guilty of any formal plagiarism in the above titled thesis even after award of PhD degree, the University reserves the rights to withdraw/revoke my PhD degree and that HEC and the University has the right to publish my name on the HEC/University Website on which names of students are placed who submitted plagiarized thesis.

Students/Author Signature:______________

Mr. Ghulam Ghouse

PIDE2015FPHDETS04

Author's Declaration

I Mr. Ghulam Ghouse hereby state that my PhD thesis titled **"ARDL Model as a Remedy for Spurious Regression: Problems, Performance and Prospects"** is my own work and has not been submitted previously by me for taking any degree from **Pakistan Institute of Development Economics, Islamabad'** or anywhere else in the country/world.

At any time if my statement is found to be incorrect even after my Graduation the university has the right to withdraw my PhD degree.

Ghouse

(Mr. Ghulam Ghouse)

PIDE2015FPHDETS04

ALLAH

The Most Generous

The Most Merciful

The very first verse of the Qur'an revealed to the Prophet of Islam on the night of 27th of Ramadan in 611 AD reads:

"Recite: In the name of thy Lord who created man from a clot.

Recite: And thy Lord is the Most Generous Who taught by the

pen, taught man that which he knew not."

(Quran, Chapter 30, Section 96, Verse No:1-5)

GOLDEN SAYING OF

THE HOLY PROPHET

(Peace and Blessings of Allah be Upon Him)

Hazrat Abdullah bin 'Amr said:

"The Messenger of Allah came out of one of his apartments one day and entered the mosque, where he saw two circles, one reciting Qur'an and supplicating to Allah, and the other learning and teaching. The Prophet said: 'Both of them are good. These people are reciting the Qur'an and supplicating to Allah, and if He wills, He will give them, and if He wills, He will withhold from them. And these people are learning and teaching. Verily I have been sent as a teacher.' Then he sat down with them."

Sunan-Ibn-Majah

English reference: *Vol. 1, Book 1, Hadith 229*

Arabic reference: *Book 1, Hadith 234*

DEDICATED

TO

HAZRAT IMAM HUSSAIN (A.S)

AND HIS

DEVOTEE

MUHAMMAD AKRAM ***(FATHER)***

Acknowledgements

First of all, a special appreciation and thankfulness go to ***ALLAH*** almighty; without His blessings I would not be able to complete my research work. After that, I offer my admirations and respect from the core of my heart to the Holy Prophet ***MOHAMMAD*** (Peace be upon him) who urges his followers to "Seek knowledge from cradle to grave". I would like to pay my humble thank to my beloved ***PARENTS,*** who prayed for me a lot and always encouraged me and guide me in a proper way and advise me not to lose heart.

Being a researcher in the field of econometrics, this dissertation would not have been possible without the help, provision and patience of my dedicated supervisors ***Dr***. ***Saud Ahmed Khan and Dr. Atiq Ur Rehman*** who supported me and guided me right from the first day of my research. They made valuable and fruitful comments that helped me to improve my research. I deeply appreciate and recognize all that I have received under the mentorship of Legendry Sir ***Dr. Asad Zaman*** *Ex-VC,* PIDE. A special thanks to my external and internal research work reviewers, ***Dr. William Griffiths, Dr. Yilmaz Akdi, Dr. Abdul Rashid, and Dr. Abdul Jalil.***

I would never have been able to finish my dissertation without the kind leadership of my teachers, *Dr. Amna Urooj, Dr. Usman Mustafa, Dr. Ahsan Ul Haq Satti, Dr. Saqlain Raza, Dr. Hafza Hina, Prof. Muhammad Arshad, Prof. Muhammad Jamil.*

I want to thanks my dear family members for their assistance through out all the research work, *Zanib Bibi, Muhammad Arshad, Shahzad Muhammad, Robina Shahzad, Azam Ali, Naheed Azam, Mohammad Akbar, Mohammad Aslam, Suriya Aslam, Umma-e-Kalsoom, Shahid Aslam, Umair Aslam, Mazhar Ali Haider, Azhar Ali Haider, Amna*

Noor, Abdullah Azam, Barira Azam, Siam Raza, Umma-e-Amama, Sair Raza, Dua Fatima.

I also want to thanks my friend and class fellows for their kind support, *Adil Hussian Shahzada, Ehtisham Noor, Umar Latif, Farhan, Asad Ali Ashraf, Kashif Habib, Samina Khan, Shumiala, Shahid Fridi, Afzal Khan, Jawad, Sheikh Umair Nadeem, Muhammad Raza, Shahzad Iqbal, Syed Almas Shah, Rizwan Fazal, Shakeel Shahzad, Benish Rasheed, Aysha Tu Zahra, Imran, Atiq Ur Rehman, Asad Shahbaz, Hayat Khan, Tariq, Aslam, Noman Khan, Hanzla, Junaid Nasrullah, Hassam Shahid, Adeel Sheikh, Taqi Raza, Rizwan, Shafaqat, Zubair, Yasir, Yasir, Zulfiqar Ali, Asif Javed, Muhammad Ayub, and Anam Ali.*

I would also like to thanks all of my **Friends**, my **Class Fellows** who helped me to grasp some key concepts regarding subject knowledge during course work and morally supported me on regular basis throughout the session.

Ghulam Ghouse

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

ABSTRACT

The most important feature that directed to the development of new time series econometrics was the spurious regression. It is a phenomenon known to econometricians since the times of Yule (1926) who attributed this problem to missing variable. A spurious regression occurs when two independent series come up with significant regression results. For a long time, missing variables were considered as root cause of spurious regression. However, Granger and Newbold (1974) challenged this wisdom and presented unit root as one of the causes of spurious regression. The extensive literature considers the nonstationarity as the only cause of spurious regression. The researchers frequently employed unit root and co-integration procedures for the treatment of spurious regression in case of nonstationarity but these procedures are equally unreliable because of uncertainty about various specification decisions like choice of the deterministic part, structural breaks, choice of autoregressive, lag length and distribution of error term. On the other hand Granger et al. (2001) show that unit root is not the only reason for spurious regression. They show the possibility of spurious regression in stationary time series. Whereas unit root and cointegration are unable to deal with this problem because they deal only nonstationary series. Such amount of conventional econometric literature is inadequate to deal with the problem of spurious regression in stationary time series. The objective of this study is to provide an alternative solution of spurious regression for both stationary and nonstationary time series. So, this study makes two contributions in this particular setup. First, spurious regression occurs due to missing variable and can be avoided by including missing lag values. Therefore, an alternative way to look at the problem of spurious regression takes us back to the missing variable (lag values) which further leads to ARDL model. Second, it significantly reduces the probability of spurious regression in both stationary and nonstationary time series case. This study mainly focusing on Monte Carlo simulations and real data is also used for performance comparison of ARDL model and conventional procedures. Our results indicate that conventional methods are significantly suffering in size and there is power problems but the performance of ARDL in both cases is far better than conventional methods. ARDL model significantly reduced the probability of spurious regression in stationary and nonstationary time series case.

Key words: Spurious regression, Cointegration, Unit root and ARDL.

CHAPTER 1

INTRODUCTION

Time series econometrics experienced a revolution during last four decades. Econometricians and economists realized that inadequate consideration was being given to trending in time series in late seventies. Nelson and Plosser (1982) found that most of the time series are better characterized as nonstationary. The theory of nonstationary time series is remarkably different from the stationary time series which was used previously. The most important feature linked to nonstationary that led to development of new time series econometrics was spurious regression. The spurious regression occurs when two independent series produce significant regression results. Granger and Newbold (1974) argued that the spurious regression is due to nonstationary time series but they are not intended that it is only cause of spurious regression. Spurious regression has performed a vital role in the construction of contemporary time series econometrics and large number of tools of time series econometrics were devised to avoid the possibility of spurious regression.

The spurious regression can exist for many reasons (Aldrich, 1995). However, the widespread literature assumes the non-stationarity as the ‘only’ reason for spurious regression (Harris, 1995; Thomas, 1997; Gaughan, 2009; Song & Witt, 2012). A great number of studies are available on spurious regression that occurs in nonstationary time series. To deal with the problem of spurious regression, the most common situation is the use of unit root and co-integration testing.

Unfortunately, the non-stationarity is not the only cause of spurious regression in time series. Granger et al. (2001) have shown the possibility of spurious regression in stationary time series as well. In such case where spurious regression exists between stationary series, the unit root and cointegration procedures fail to deal with this problem. The cointegration analysis is the way to deal with spurious regression based on the assumption of nonstationarity. It means that literature on spurious regression is insufficient to deal with the problem of spurious regression in stationary time series.

Suppose that the spurious regression occurs due to non-stationarity while the unit root and cointegration testing are used as a remedy for this purpose, even then it is very hard to find reliable inference. There is no test of unit root with good size and power in small sample. The unit root and cointegration procedures involve many prior specification decisions e.g. lag length, trend and structural stability etc. If we do a data based decision making, it will involve a large battery of tests. Each test is having specific statistical error (type I and type II error). The cumulative probability of error in all tests needed for cointegration analysis leave the results of unit root test unreliable. Because, of these reasons, the literature is still developing after four decades without reaching any conclusion. It would be interesting to see the example of US, GNP which has been examined by large number of researchers. About 40 years of investigation of the series, the stationarity of this series is still undecided. The cointegration is a step that comes after unit root testing and therefore undecided with greater level of uncertainty.

An alternative way to look at the problem of spurious regression takes us back to missing variable which further leads as to ARDL. Suppose, we have two independent autoregressive nonstationary series:

$$y_t = y_{t-1} + \varepsilon_{yt} \tag{1.1}$$

$$x_t = x_{t-1} + \varepsilon_{xt} \tag{1.2}$$

where X_t and Y_t both are expressed by their own lag values. There is no third variable involved in the construction of both variables. Granger and Newbold (1974) shown the spurious regression by estimating of regression of the type:

$$y_t = a + \beta_1 x_t + \varepsilon_{yt} \tag{1.3}$$

but we know that true data generating process (DGP) of y_t contains lag of y_t which is missing in Granger and Newbold experiment, taking the lag into account of y_t we get:

$$y_t = a + \beta_1 x_t + \beta_2 y_{t-1} + \varepsilon_{yt} \tag{1.4}$$

which is an ARDL model. If we miss the y_{t-1} and estimate the equation (1.3) because of the missing variable, the coefficient of x_t is expected to be biased. But if we estimate equation (1.4), due to presence of right determinant of y_t estimate of equation (1.4), it is expected to be unbiased. It is observed in our study (chapter 5, page 49) that this kind of model significantly reduces the probability of spurious regression and the model works for both stationary and nonstationary series. The ARDL model also works for the correction of serially correlated errors. It means it can also be used for the correction of serial correlation of errors.

1.1 Gap of study

This study is to explore ARDL model as an alternative solution of problem of spurious regression in both stationary and non-stationary time series case. The ARDL had never been used before for the treatment of spurious regression. On the other hand, all the method which have been proposed before for treatment of spurious regression in case of nonstationary series are having size distortion and power loss problems due to cumulative errors of pre-specification decisions. Also, these procedures are unable to deal with the problem of spurious regression when the series are stationary because they only deal with nonstationary series.

After the study of Granger and Newbold (1974) most of the literature assumed that the spurious regression is only due to unit root. They offered cointegration procedure to tackle the problem of spurious regression. While after reviewing lot of literature we find even the series are nonstationary or stationary the spurious regression is because of relevant missing variables. So, for the treatment of spurious regression just introduces the lag value of dependent and independent variables in the model at the place of missing variable, it significantly reduces the probability of spurious regression, this further leads to ARDL model.

We investigate that, is it possible to use ARDL model to avoid the spurious regression by passing the very complicated and ambiguous unit root and cointegration analysis and associated specification decisions. The properties of ARDL model for avoiding of spurious regression shall be investigated via Monte Carlo Simulations, with various sample sizes and various specifications of deterministic components. The case of spurious regression in stationary time series shall also be considered since it has been shown by Granger et al. (2001) that spurious regression is also possible in stationary series.

1.2 Motivation

Since it was pre assumed that spurious regression occurs due to nonstationary, therefore numerous studies proposed different methods for the treatment of spurious regression in case of nonstationary time series. Granger et al. (2001) have proven that spurious regression can also exist in stationary time series. The unit root and cointegration procedures do not offer any solution to this problem.

Even in non-stationary world, the unit root and cointegration testing involve many specification decisions and output cannot be trusted because of multiple testing and huge cumulative error probabilities. On the other hand, the cointegration solutions deal with nonstationary cases and don't offer any solution for spurious regression in stationary series. The study explores an alternative solution that is expected to perform well for both stationary and nonstationary series.

1.3 Objective of Study

The study explores following objectives:

1. To investigate the performance of ARDL to avoid spurious regression in the following cases:

    i. Spurious regression in stationary time series.

    ii. Spurious regression in non-stationary time series.

    iii. Trend misspecification.

2. To evaluate the robustness of ARDL and cointegration methods.

3. To evaluate the forecast performance of ARDL by using real data and to compare it with commonly used cointegration methods.

4. To assess the performance of ARDL model and GARCH models for Volatility modelling.

1.4 Significance of Study

The spurious regression is an issue of great importance which leads to the development of new time series econometrics. Many researchers offered their methods with different specifications for the treatment of this problem. These procedures involve many phases for testing cointegration which leads to huge cumulative probability of error and unreliable output. There are number of studies indicating the possibility of spurious regression in stationary time series so in case unit root and cointegration can't offer any solution. However, it can be shown that spurious regression in nonstationary and stationary time series indicate an autoregressive problem and that autoregressive problem can be handled by using ARDL. This study analyzes the performance of ARDL to avoid spurious regression in different scenarios (specifications). If the performance of ARDL model is found good, then it will simplify the modern econometrics practice.

1.5 Outline of the Thesis

Chapter 2 provides a brief discussion on theoretical and empirical tools used for the handling of problem of spurious regression in time series econometrics literature. It offers comprehensive review of cointegration procedures used in the conventional time series econometrics and some are employed in this study.

Chapter 3 provides a brief discussion on theoretical background of problem of spurious regression in time series econometrics literature. It consists of brief discussion on spurious regression theories. It also contains comprehensive discussion on the causes of spurious regression. The asymptotic theory of spurious regression and other relevant information of spurious regression is also given in this chapter.

Chapter 4 contains discussion on methodologies which are being used for empirical analysis. The methodology is based on two components, first is the data generating process and second is the Monte Carlo simulations.

Chapter 5 contains discussion on empirical results which are obtained by using ARDL model and conventional econometric tools. First comparison is made between ARDL and OLS model in term of size and power. Second, comparison is made among ARDL and cointegration procedures in term of size and power under different specifications.

Chapter 6 encompasses conclusion and discussion of study. It also discusses the real application of this study in econometrics. The directions for future research have also been provided in last section of this chapter.

Chapter 7 provides a brief discussion on the comparison of forecasting between ARDL model and Engle-Granger and Johnson and Juselius procedures.

Chapter 8 consists on the comparison of Hendry ARDL model and Pesaran bound testing procedures on the basis of size and power. The comparison is also based on robustness to misspecification.

Chapter 9 based on the comparison ARDL model and GARCH type model in term of volatility modeling.

Chapter 10 provide on the conclusion, discussion and future recommendations.

CHAPTER 2

REVIEW OF LITERATURE

An immense amount of studies is available on spurious regression in time series econometric literature. In this chapter we briefly discuss the proposed theoretical and empirical methods for the treatment of spurious regression in literature. In classical econometrics literature, it was assumed that the spurious regression exists due to missing variables. For detail see (Section, 2.1). Granger and Newbold (1974) showed that if the series are nonstationary then the results would be significant, for further detail (Section, 2.1.1). However, Nelson and Plosser (1982) examined that most of the macroeconomic series of US economy are having unit root (Section, 2.1.2.3). Hendry (1980) and Plosser and Schwert (1978) argued that the spurious regression provides nonsense or invalid results (Section, 2.1.2.1). To avoid the problem of spurious regression caused by the non-stationarity, researchers frequently employed unit root and co-integration testing (Section 2.1.2.5). The unit root and cointegration testing involve many specification decisions which cut the reliability of results. The existing unit root and cointegration testing procedures do not provide any reasonable criteria regarding these specification decisions: choice of the deterministic part; the structural breaks; autoregressive lag length choice and innovation process distribution. For further detail see (Sections, 2.3.1 and 2.3.2). It is a common misconception that the spurious regression only prevails due to unit root. Nevertheless, the missing relevant variable is a major cause of spurious regression. Even it can be shown that the spurious regression in Granger and Newbold (1974) experiment was also due to missing variable, see (Section, 2.2). This review emphasizes to point out the unreliability of existing methods. The literature review is arranged as follows

2.1 Spurious Regression in Classical Econometrics

There is long historical debate on nonsense correlation (spurious regression) in econometrics literature. For example, we see back to the well-known study of Yule (1926). In his study, Yule found the presence of a strong correlation of 0.95 between mortality rate and proportion of marriages of the Church of England to all marriages during 1866 to 1911.Yule thought that the spurious regression is a consequence of missing relevant variables. Zeisel (1948) concluded that the key cause of spurious correlation is missing variable. Zeisel measured correlation between three variables as experiment, X is the married female employee's percentage in group, Y is the average absence of per employee per week and Z is average housework hours consumed per employee per week. High correlations have been found between X and Y, X and Z, but when Z held constant the correlation between X and Y was found close to zero. The correlation between X and Y is spurious and it's due to joint effect produced by the variation in Z. It is common that married female perform more housework hours and that is why more absences came into being. The spuriousness of this relation depends upon purpose if we want to estimate regression then it can be spurious. If the purpose is forecasting then it is not spurious.

Kendall and Lazarsfeld (1950) also found that that the missing relevant variable is the mian cause of spurious correlation like two variables X and Y are correlated due to intervening Z variable. Simon (1954) also supported the idea that the missing variable is a source of spurious correlation. Simon described that if we are uncertain that the perceived correlation is spurious, we have to introduce extra variable which could be observed in the genuine correlation.

2.1.1 Granger and Newbold's Experiment

Granger and Newbold (1974) showed that if two serially independent nonstationary series are regressed onto each other than the results would be significant. They suggested that if the time series are nonstationary, then the results would be significant. In their experiment they generated independent autoregressive series, where $X_t$and Y_t both are expressed by their own lag values:

$$y_t = y_{t-1} + \varepsilon_{yt} \tag{2.1}$$

$$x_t = x_{t-1} + \varepsilon_{xt} \tag{2.2}$$

There is no third variable involved in the construction of these two variables. They regressed x_t on y_t and y_t on x_t.. Since the two series are independent of each other, $\beta_1$and β_2, the coefficients of regressors should be insignificant. On contrary, they found that the probability of getting significant coefficient varies high and they also found that this probability increases with increase in sample size. This was contrary to the perception of Yule (1926) who thought that the spurious regression will reduce with large samples.

$$y_t = a_1 + \beta_1 x_t + \varepsilon_{yt} \tag{2.3}$$

$$x_t = a_2 + \beta_2 y_t + \varepsilon_{xt} \tag{2.4}$$

This alternative explanation of spurious regression become more popular in literature and other explanations went to the darkness.

2.1.2 Aftermath of Granger and Newbold's Experiment

After the Granger and Newbold's Experiment researcher considered nonstationarity as key cause of spurious regression. The research after Granger and Newbold (974) focused on finding the more details, what is spurious regression and what are its implications? The solution for spurious regression was also developed.

2.1.2.1 Why is Spurious Regression a Problem?

To find the relationship between the economic variables has been the core objective of economic studies. The spurious regression offers deceptive statistical evidence of strong relationship even though the variables are independent. There are many well-known examples of spurious regression like; Hendry (1980) demonstrated a spurious correlation between cumulative rainfall and price level in UK. He inspected that all these time series were nonstationary except unemployment rate. Plosser and Schwert (1978) claimed that, the regression without taking difference of nonstationary series most probably come up with invalid or nonsense results. The reasoning behind this claim is that if we run regression without taking difference of difference stationary series, the estimator properties and the distribution of test statistics are no more reliable. Phillips (1986) examined the asymptotic properties of least square regression model and endorsed the findings of Granger and Newbold (1974) simulation results, via theoretical calculations that the misspecification of level of series is the key element of spurious correlation.

2.1.2.2 Example of Spurious Regression in Classical Literature

Mostly, the nominal economic variables are correlated, even there is no relationship between them, and the mutual presence of price level in data series develops correlation between them. It was also shown that many time series are nonstationary and that's why the probability of spurious regression is very high. We are presenting here some examples of spurious regression from time series econometrics literature.

Chaouachi (2013) inspected that Dar et al. (2012) in their study provided spurious strong positive relationship among usage of nass chewing, hookah smoking and many other habits with oesophageal squamous cell carcinoma (ESCC) risk. They conducted a case control study in valley of Kashmir, India. They considered 702 historical cases of oesophageal squamous cell carcinoma (ESCC) and 1663 hospital based controls, exclusively matched to the cases for sex, age and residence district from Sep, 2008 to Jan, 2012. They used monthly data from Sep, 2008 to Jan, 2012. They concluded that nass chewing and hookah smoking are strongly positively associated with (ESCC) risk, which is based on severe misinterpretation. According to Chaouachi (2013) all the relevant studies showed that there is feeble or insignificant association among nass chewing, hookah smoking with (ESCC) risk. Chaouachi (2013) stated that Dar et al. (2012) came up with spurious results because they did not incorporate the very significant element which is filtering factor of water.

Roger and Jupp (2006) described an example of spurious positive relationship between human baby's birth and stork nesting in the sequence of spring, because these two variables are correlated to a third variable. According to the Roger and Jupp (2006) the sequence of Dutch statistics is showing a positive relationship between stork nesting in the sequence of spring and human baby' s birth at that time, it is due to that the both variables are associated to the state of weather. It means that both variables are independent, but they have relation with the state of weather. This shows that both variables are spuriously correlated because of third missing variable. According to the Hofer et al. (2004) this spurious correlation is due to lack of statistical information.

2.1.2.3 Nelson and Plosser Experiment and Implications

Nelson and Plosser (1982) examined that most of the macroeconomics series of U.S.A economy are having unit root. They employed Dickey Fuller test for unit root detection to fourteen historical macroeconomics series for U.S.A economy, including GNP, wage, employment, prices, stock prices and interest rate and they found that twelve out of fourteen series were having unit root. In fact Nelson and Plosser, (1982) study is a noteworthy contribution in time series econometric literature which enhanced the interest of researchers in unit root tests. It has fashioned the development in the unit root theory. On the other hand, Granger and Newbold (1974) showed in their well-known study that the nonstationarity is the key cause of spurious regression. After that for the treatment of spurious regression many researcher developed their unit root and cointegrating testing procedures. So, the idea of cointegration came into being in 1982 and was published in 1987 which is discussed in the next section.

2.1.2.4 Revival of Time Series-Cointegration

The theory of cointegration is a huge innovation in time series econometric literature. That's why it has attained the attention of the economists in the last decades. The unit root time series are cointegrated, if their linear combination is a stationary process. The cointegration analysis created hope for reliable inference in time series when series are nonstationary.

2.1.2.4.1 Granger Explanation of Cointegration

According to Granger (1981) the mechanism of cointegration is as follows, suppose we have equation:

$$\mathrm{D(B)y_t = mx_t + nz_t + h(B)\varepsilon_t} \qquad (2.5)$$

For convenience, initially we assumed no lag case:

$$\mathrm{m(M) = m \text{ and } n(B) = n} \qquad (2.6)$$

Where B is a backward lag operator$\mathrm{y_t}$, $\mathrm{x_t}$ and $\mathrm{z_t} \sim \mathrm{I(d)}$.

$\mathrm{d_y} > 0, h\mathrm{(B)\varepsilon_t}$ is I(d) and $\mathrm{Var(\varepsilon_t)} = 1$. The right hand side spectrum will be:

$$\frac{\{\mathrm{m^2 f_x(\omega) + n^2 f_z(\omega)}\} + \mathrm{mn[mr(\omega) + \overline{mn}(\omega)]\} + |h(z)|^2}}{2\pi} \qquad (2.7)$$

$mr(\omega)$ is cross spectrum between variables y_t and x_t. The special conditions are following:

(i) $f_x(\omega) = a^2 f_z(\omega)$, when (ω) is small, and so, $d_x = d_z$

(ii) $mn(\omega) = a f_z(\omega)$, when (ω) is small, and so the coherence $C(\omega)$= 1. The phase $\emptyset(\omega) = 0$ when (ω) is small.

Engle and Granger (1987) adopted the definition of cointegration form Granger (1981) and Granger and Weiss (1983) which is given as follows:

> "*The components of vector x_t are said to be cointegrated of order (d, b), if (i) all components of x_t are I(d); (ii) there exists a vector $a(\neq 0)$ $z_t = a'x_t \sim I(d, b)$, b>0. The vector a is called the cointegrated vector*."

In other words, the two unit root series like, X_t and Y_t are cointegrated if their linear combination Z_t is stationary.

2.1.2.5 Development of Cointegration Tests

A great number of tests were developed which can be categorized in the following classes

i. Residual based tests
ii. Tests with cointegration as null
iii. Multiple equation cointegration tests

2.1.2.5.1 Residual based Cointegration Tests

Engle and Granger (1987) introduced residual based tests for co-integration testing to avoid spurious regression in nonstationary series. According to Engle and Granger:

Consider a vector of time series $\mathrm{x_t}$ and each element of $\mathrm{x_t}$ achieves stationarity after differencing, but their linear combination $a'\mathrm{x_t}$ is stationary with cointegrating vector a, then the series said to be cointegrated.

Since a is quantified as only multiplicative constant and normalize the first variable in time series vector $\mathrm{x_t}$ to get coefficient 1. So, if we write a as $(1, -\delta')'$ and partition of $\mathrm{x_t}$ into ($\mathrm{x_{1t}}$, $\mathrm{x_{2t}}$):

$$\mathrm{x_{1t}} - \delta'\mathrm{x_{2t}} \tag{2.8}$$

then both have cointegration (long run) relationship. The residual based test use the equation:

$$\mathrm{x_{1t}} = \delta'\mathrm{x_{2t}} + \varepsilon_t \tag{2.9}$$

After that employ the unit root test on ε_t, if it is stationary, it shows variables are cointegrated. if ε_t has unit root then variables are not cointegrated. It means non-stationary time series are cointegrated if their linear combination is a stationary process. It is a residual based testing procedure. The first drawback of EG (Engle and Granger) cointegration test is that it only deals with one cointegrated vector. Second, it depends upon two step estimator, first step is to produce series of residuals and second, to check the stationarity of residuals series. Third, the major limitation is the distributions of the estimators are non-standard.

Phillips and Ouliaris (1990) proposed residual based tests under the null hypothesis of no cointegration in time series. In which the asymptotic distributions of residual based tests depend upon number of variables and deterministic trend terms. They compared the asymptotic properties of many residual based tests for cointegration. There are two Phillips and Ouliaris tests first $\widehat{Z_a}$ test and $\widehat{Z}_t$ test.

The residuals of cointegration regression are being used for the construction of $\widehat{Z_a}$ test. Consider an equation:

$$\mathrm{x}_{1\mathrm{t}} = \sum_{\mathrm{i}=1}^{\mathrm{m}} \widehat{\beta_{\mathrm{it}}} \mathrm{x}_{2\mathrm{it}} + \widehat{\mathrm{U}}_{\mathrm{t}} \tag{2.10}$$

where t = 1, ..., T. The $\mathrm{x}_{1\mathrm{t}}$ and $\mathrm{x}_{2\mathrm{it}}$ are integrated of order 1 and their residual series $\widehat{U_t}$ is stationary at level then the time series are cointegrated. If $\widehat{U_t}$ is stationary at integrated level 1 then the time series are not cointegrated.

Furthermore, regress :

$$\widehat{U_t} = \hat{a}\widehat{U_{t-1}} + \widehat{K_t} \tag{2.11}$$

now compute the $\widehat{Z_a}$

$$\widehat{Z_a} = \mathrm{T}(\hat{a} - 1) - (\tfrac{1}{2})\left(S_{Tl}^2 - S_k^2 (T^{-2} \textstyle\sum_2^T \widehat{U_{t}}_{t-1}^{2})^{-1}\right) \tag{2.12}$$

where,

$$S_k^2 = T^{-1} \sum_1^T \widehat{k_t}^2 \tag{2.13}$$

$$S_{Tl}^2 = T^{-1} \sum_1^T \widehat{k_t}^2 + 2T^{-1} \sum_{s=1}^{l} W_{sl} \sum_{t=s+1}^{T} \widehat{k_t} \widehat{k_{t-s}} \tag{2.14}$$

and

$$W_{sl} = 1 - s/(l+1)^4 \tag{2.15}$$

The spectral density matrix of K_t error is expressed as $f_{kk}(\lambda)$ under the null hypothesis of cointegration. Phillips and Ouliaris substantiated that for $f_{qq}(0) > 0$

$$\widehat{Z_a} = O_p(T) \tag{2.16}$$

and

$$\widehat{Z_t} = O_p(T^{1/2}) \tag{2.17}$$

They also proved the $\widehat{Z_t}$ test and augmented dickey fuller (ADF) test have same limiting distribution, that is why they have same critical values.

Engle and Yoo (1991) proposed three step procedure to evade the limitations of EG model, which is an extension of EG model. Engle and Yoo (EY) procedure confirms that the estimators yield the normal distribution. It is also only useful for one cointegrated vector. These residual base procedures have low power because in first step they use static regression and ignore the dynamic equation and use error dynamics (Kremers et al., 1992; Zivot, 1994; Banerjee, 1995).

First step is to estimate the cointegration regression and estimate the residual series:

$$\mathrm{x}_{1\mathrm{t}} = \theta\mathrm{x}_{2\mathrm{t}} + \varepsilon_t \tag{2.18}$$

Where

$$\hat{\varepsilon}_t = \mathrm{x}_{1\mathrm{t}} - \hat{\theta}\mathrm{x}_{2\mathrm{t}} \tag{2.19}$$

The second step is to estimate error correction model by using estimated residual series:

$$\Delta\mathrm{x}_{1\mathrm{t}} = \alpha\Delta\mathrm{x}_{2\mathrm{t}} - \beta\hat{\varepsilon}_{t-1} + v_t \tag{2.20}$$

$$\Delta\mathrm{x}_{1\mathrm{t}} = \alpha\Delta\mathrm{x}_{2\mathrm{t}} - \beta(\mathrm{x}_{1\mathrm{t}} - \hat{\theta}\mathrm{x}_{2\mathrm{t}})_{t-1} + v_t \tag{2.21}$$

The third step is to correct the errors:

$$v_t = \eta(-\beta\mathrm{x}_{2\mathrm{t}}) + \mu_t \tag{2.22}$$

Now the correction of estimator of first regression is so easy:

$$\theta_{cor} = \hat{\theta} + \eta \tag{2.23}$$

The corrected standard error for θ_{cor} are provided by the standard errors for η in last step of regression.

Now the problem is that how to estimate the long run equilibrium relationship parameters. For this purpose Kremers et al. (1992) presented an error correction mechanism (ECM). The residuals of equilibrium regression are used for error correction model. The error correction mechanism has been provided in last Engle and Yoo procedure.

2.1.2.5.2 Tests with Cointegration as Null

Park and Choi (1988) and Park (1990) proposed a cointegration test which can be performed under null hypothesis of cointegration or no cointegration. In this test they introduce superfluous (extra) regressor in the cointegration regression, after that run the test for coefficients of auxiliary superfluous (extra) regressors.

Consider following cointegration equation:

$$\mathrm{x_{1t}} = \delta'\mathrm{x_{2t}} + \theta_1'\mathrm{S_{1t}} + \theta_1'\mathrm{S_{1t}} + \varepsilon_t \qquad (2.24)$$

The $\mathrm{S_{1t}}$ extra variable shows the qnon-stationary deterministic functions vector which have higher integrated order as compare to other variables in cointegration regression equation. If we include drift 1 and trend t in equation, then $\mathrm{S_{1t}}$ might include regressors $\{t^2, \dots \dots, t^{q+1}\}$. The $\mathrm{S_{2t}}$ added variable shows the pnon-stationary variables vector integrated of order one. Park (1990) introduced the J_1 test under the null hypothesis of cointegration. This test is basically based on comparison of two residual sum of squares (RSS).

$$\mathrm{J_1} = \frac{\mathrm{RSS_1 - RSS_2}}{\mathrm{w}} \qquad (2.25)$$

The RSS_1 is the residual sum of square of simple cointegration equation after transforming the nuisance parameters for independence. The RSS_2 is residual sum of square from the transformed cointegration regression with superfluous independent variables. The w is a normalized variance corresponding to the test statistics. The Park indicates that the J_1 test under the null hypothesis has limiting Chi square $\mathcal{X}^2$ distribution and the degree of freedom is equal to the added independent variables. When the value of J_1 is high, it indicates that there is no cointegration or in other words rejection of null hypothesis of cointegration.

Leybourne and McCabe (1993) proposed a cointegration test (LBI) in favor of null hypothesis of cointegration versus the alternative hypothesis of no cointegration. They conducted a small Monte Carlo simulation experiment and concluded that test did not show any size distortion when sample size is 100 and 200. They did not compare LBI test with other test in term of power and robustness because of different type of null and alternative hypotheses.

Consider the cointegration regression:

$$\mathrm{x}_{1\mathrm{t}} = \theta' \mathrm{x}_{2\mathrm{t}} + \varepsilon_t \tag{2.26}$$

where

$$\varepsilon_t = \phi_t + \mu_t \qquad \mu_t \sim IN(0, \sigma^2) \tag{2.27}$$

$$\phi_t = \phi_{t-1} + v_t \qquad v_t \sim IN(0, \sigma_v^2) \tag{2.28}$$

The null hypothesis of cointegration and the alternative hypothesis of no cointegration are:

$$H_0 : \sigma_v^2 = 0 \tag{2.29}$$

$$H_1 : \sigma_v^2 > 0 \tag{2.30}$$

The test statistics of LBI test is following:

$$LBI = T^{-2}\widehat{\sigma_\varepsilon}^{2}\hat{\varepsilon}' V \hat{\varepsilon} \tag{2.31}$$

$\hat{\varepsilon}$ is the ordinary least square residual's vector under H_0 from cointegration equation and V is the with T x T dimension matrix with ijth elements. The Newey-West estimator are being used for the estimation of variance $\widehat{\sigma_\varepsilon}^{2}$.

Shin (1994) proposed cointegration test which is an extensive form of the KPSS test which is commonly used for unit root testing. In context of multivariate testing, the null hypothesis of cointegration versus alternative hypothesis of no cointegration is examined. Shin comprehensively used the procedure of parametric correction of quantification of cointegrating regression. The final correctly cointegration regression is:

$$\mathrm{x}_{1\mathrm{t}} = \theta' \mathrm{x}_{2\mathrm{t}} + \sum_{j=-k1}^{k2} c_j \, \Delta \mathrm{x}_{2\mathrm{t-j}} + \varepsilon_t \tag{2.32}$$

Now estimate the residuals and used it for the construction of test statistics:

$$C = T^{-2} \sum S_t^2 / \hat{\sigma}^2 \tag{2.33}$$

Where

$$S_t = \sum_{i=1}^{t} \hat{\varepsilon}_i^2 \tag{2.34}$$

The $\hat{\sigma}^2$ is the semi-parametric consistent estimator of long run variance of ε_t^2.

The asymptotic distribution of the shin test statics is defined as:

$$C \Rightarrow \int_0^1 P^2 \tag{2.35}$$

and

$$P = V_1 - \left(\int_0^r V_2'\right)\left(\int_0^1 V_2 V_2'\right)\left(\int_0^r V_2 \, dV_1\right) \quad (2.36)$$

The V_1 and V_2 are the standard Brownian motions which are independent and corresponding to scalar variable x_{1t} and n-vector variable x_{2t}. The limiting distribution of test statistics has been affected by inclusion of deterministic components. The asymptotic distribution of the test depends upon the n dimensions of the cointegration equation system.

Harris and Inder (1994) proposed same test but they used procedure of nonparametric correction of estimation of regression of cointegration. In both tests similar LM test statistic has been used.

$$x_{1t} = \gamma_0 + \theta' x_{2t} + \varepsilon_t \quad (2.37)$$

Now estimate residual $\hat{\varepsilon}_t$ and find out:

$$\hat{\pi}_t = [\hat{\varepsilon}_t, \Delta x_t'] \quad (2.38)$$

$$\hat{\Lambda} = \begin{bmatrix} \hat{\omega}_{11} & \hat{\omega}_{12} \\ \hat{\omega}_{21} & \hat{\omega}_{22} \end{bmatrix} \quad (2.39)$$

$$= \frac{1}{T}\left[\sum_{t=1}^{T} \hat{\pi}_t \hat{\pi}_t' + \sum_{k=1}^{l} w(k,l) \sum_{t=k+1}^{T} (\hat{\pi}_{t-k} \hat{\pi}_t') + (\hat{\pi}_t \hat{\pi}_{t-k}')\right] \quad (2.40)$$

$$\hat{\Gamma} = \begin{bmatrix} \hat{\varphi}_{11} & \hat{\varphi}_{12} \\ \hat{\varphi}_{21} & \hat{\varphi}_{22} \end{bmatrix} \quad (2.41)$$

$$= \frac{1}{T} \sum_{k=0}^{l} \sum_{t=k+1}^{T} \hat{\pi}_t \hat{\pi}_{t-k}' \quad (2.42)$$

$$w(k,l) = 1 - k/(l+1) \quad (2.43)$$

after that estimate:

$$\mathrm{x^+}_{1t} = \mathrm{x}_{1t} - \widehat{\omega}_{12}\widehat{\Lambda}_{22}^{-1}\Delta \mathrm{x}_{2t} \tag{2.44}$$

and

$$\varphi^+{}_{t} = \hat{\varphi}_{21} - \widehat{\Gamma}_{22}\widehat{\Lambda}_{22}^{-1}\widehat{\omega}_{21} \tag{2.45}$$

Now again estimate the cointegration regression with modified estimators:

$$\widehat{\theta}^+ = (\mathrm{x}'\mathrm{x})^{-1}(\mathrm{x}'\mathrm{y}^+ - \mathrm{e_k T}\widehat{\varphi}_{12}^+) \tag{2.46}$$

where

$$e_k = [0, I_k]' \tag{2.47}$$

estimate residual $\hat{\varepsilon}_t^+$

$$S_2^+ = \frac{T^{-2}\sum_{t=1}^{T}(L_t^+)^2}{\widehat{w}_{1.2}^2} \tag{2.48}$$

$$L_t^+ = \sum_{i=1}^{t}\hat{\varepsilon}_i^+ \tag{2.49}$$

$$\widehat{w}_{1.2}^2 = \widehat{w}_{11}^2 - \widehat{w}_{12}\widehat{\Lambda}_{22}^{-1}\widehat{w}_{21} \tag{2.50}$$

The number of regressors excluding intercept determine the critical values of this Harris and Inder test statistics. The asymptotic critical values can be found by employing Monte Carlo simulations.

2.1.2.5.3 Multiple Equation Cointegration Tests

When we have more than two variables then there is the possibility of more than one cointegrated vector. In this case these cointegration procedures do not provide any solution. So, to overcome this problem Johansen and Juselius (1990) introduced the multivariate cointegration test. The Johansen and Juselius (JJ) used two test statistics for detection of cointegration. First one is trace test with the null hypothesis that there are no more than “r” cointegrating vectors. The second is maximum eigen value test with the null hypothesis that there are no more than “r+1” cointegrating vectors against the alternative “r” cointegrating vectors.

Suppose we have vector of variables with each element have same order of integration:

$\mathrm{X_t} \sim \mathrm{I}(1)$

Here X_t denotes n x 1 vector of regressors. The JJ cointegration testing procedure starts from vector autoregressive (VAR) model and we have VAR model of order p:

$$\mathrm{X_t} = \gamma + \mathrm{A_1 X_{t-1}} + \mathrm{A_2 X_{t-2}} + \mathrm{A_3 X_{t-3}} + \cdots + \mathrm{A_p X_{t-p}} + \mathrm{\mu_t} \quad (2.51)$$

It can be written as:

$$\Delta X_t = \gamma + \Psi X_{t-1} + \sum_{i=1}^{p-1} \Phi_i \Delta X_{t-i} + \mu_t \quad (2.52)$$

where

$$\Psi = \sum_{i=1}^{p} A_i - \mathrm{I} \quad (2.53)$$

$$\Phi_i = -\sum_{j=i+1}^{p} A_j \quad (2.54)$$

If Ψ the coefficient matrix reduces its rank $r < n$, then there is n x r matrices of a and $\beta$and each of them has’ r’ rank such that $\Psi = a\beta'$while $\beta' X_t$ is stationary. In the error correction model a is representing the adjustment parameter and every column of the β is showing cointegrating vector. The r is representing the cointegration relationships. They used two likelihood ratio test statistics for cointegration.

Trace test:

$$Trace = -T\sum_{i=r+1}^{n} ln\,(1-\hat{\lambda}_i)` \quad (2.55)$$

Maximum eigenvalue test:

$$\mathrm{M.E} = -\mathrm{T}\ln(1-\hat{\lambda}_{r+1}) \quad (2.56)$$

The $\hat{\lambda}_i$ shows the i_{th} high canonical correlation and T represents the number of observations. The JJ test allows to find out more than one cointegrated vectors such that it is generally more applicable than EG and EY cointegration tests. We know that EG and EY single equation procedures ignore short run dynamics, when the relationships are estimated. But, the JJ procedure also considers the short run dynamics. Johansen (1992) and Perron and Campbell (1993) proposed an extensive form of JJ test which includes the trends for the treatment of stochastic cointegration in data series. Podivinsky (1990) explored that the asymptotic critical values are not applicable when the sample size is 100 or less than 100. The JJ test suffers in size and power problem when sample size is small. Phillips and Ouliaris (1988), Stock and Watson (1988) and Harris (1997) estimated cointegration by employing principle component analysis.

Pesaran et al. (1996) and Pesaran (1997) proposed a single equation ARDL (autoregressive distributed lag) approach for cointegration as an alternative of EG and EY. The first advantage is the ARDL cointegration approach provides explicit tests for the presence of a single cointegrating vector, instead of assuming uniqueness.

Suppose we have two variables x_{1t} and x_{2t} and both are first difference stationary, then the cointegration regression equation of ARDL is following:

$$\Delta x_{1t} = \theta_{10} + \theta_{11}x_{1t-1} + \theta_{12}x_{2t-1} + \sum_{i=1}^{p-1}\beta_i\,\Delta x_{1t-1} + \sum_{i=0}^{q-1}\delta_i\,\Delta x_{2t-1} + \varepsilon_{1t} \quad (2.57)$$

$$\Delta x_{2t} = \theta_{20} + \theta_{21}x_{2t-1} + \theta_{22}x_{2t-1} + \sum_{i=1}^{p-1}\beta_i\,\Delta x_{2t-1} + \sum_{i=0}^{q-1}\delta_i\,\Delta x_{1t-i} + \varepsilon_{2t} \quad (2.58)$$

The F test is employed to find out joint significance for short run and long run relationships. The hypotheses for long run relationship are following:

H_0: $\theta_{11} = \theta_{12} = 0$ (There is no long run relationship)

H_1: At least one of them is nonzero (There is long run relationship)

The hypotheses for short run relationship are following:

H_0: $\beta_i = \delta_i = 0$ (There is no short run relationship)

H_1: At least one of them is nonzero (There is short run relationship)

The F statistic (Wald test) for these hypotheses tested in each of the models can be denoted as:

$$F_{x1}(x_{1t} | x_{2t}) \tag{2.59}$$

$$F_{x2}(x_{2t} | x_{1t}) \tag{2.60}$$

The distribution of Wald test is non-standard asymptotically under the null of no cointegration. Pesaran and Shin (1995) revealed that asymptotically valid inference on short run and long run parameters could be made by employing ordinary least square estimations of ARDL model. So, the ARDL model order is properly augmented to grant for contemporary correlation among the stochastic elements of the data generating processes involved in estimation. Pesaran et al. (2001) provided critical values of two bounds upper and lower which are being used for cointegration. The lower bound considers variables are stationary and they have no long run relationship. The upper bound considers variables are difference stationary and they have long run relationship. When the F-stat values lines in upper bound critical region, then it rejects H_0. It means variables are cointegrated.

2.1.2.6 Problems in Cointegration Analysis

The cointegration testing involves many specification decisions which cut the reliability of results. The existing cointegration testing procedures do not provide any reasonable criteria regarding these specification decisions: choice of the deterministic part; the structural breaks; autoregressive lag length choice and innovation process distribution. For further detail (section, 2.3.2).

2.2 Conceptual Flaws in Understating of Spurious Regression

It is a common misconception that the spurious regression only prevails due to unit root. Nevertheless, the missing relevant variable is a major cause of spurious regression. Yule (1926) first time anticipated that the nonsense correlations could prevail due to missing variable.

Simon (1954) argued that the missing variable is a cause of spurious correlation. Simon has described this problem in following tactic that if we are uncertain that the observed correlation is spurious, we should introduce another (extra) variable which may observe the true correlation. Frey (2002) argued that the spurious regression could probably be due to missing variable.

Even it can be shown that the spurious regression in Granger and Newbold (1978) experiment was also due to missing variable. In their experiment they generated independent autoregressive series like,X_t and Y_t. Where $X_t$and Y_t both are expressed by their own lag values:

$$y_t = y_{t-1} + \varepsilon_{yt} \tag{2.61}$$

$$x_t = x_{t-1} + \varepsilon_{xt} \tag{2.62}$$

There is no third variable involved in the construction of both variables. They regressed $X_t$on Y_t or vice versa without involving their lag values in regression analysis.

$$y_t = a + \beta_1 x_t + \varepsilon_{yt} \tag{2.63}$$

$$x_t = a + \beta_1 y_t + \varepsilon_{xt}. \tag{2.64}$$

They came up with spurious results due to missing variable because they did not include the lag values of variables as an independent variable. It is obvious that on determinant of Y_t that is $Y_{t-1}$is missing in equation (2.63) and similarly one determinant of X_t i.e. X_{t-1} is missing in equation (2.64). Taking these missing variables into account the equation shall become:

$$y_t = a + \beta_1 x_t + \beta_2 y_{t-1} + \varepsilon_{yt} \tag{2.65}$$

Therefore, equation (2.65) shall not have spurious regression, if our supposition of missing variable problem is true. It is shown in section (4) that it is actually true.

The stationary variables are also lag dependent at some extent that is why we would come up with spurious regression. For detail see, Granger et al. (2001) and Rehman and Malik (2014). The missing variable phenomena also solve the mystery observed by Granger et al. (2001) i.e. spurious regression in stationary series.

In our experiment which is explained in section 4, we generated independent autoregressive series like,X_t and Y_t. Where $X_t$and Y_t both are expressed by their own lag values:

$$y_t = \rho y_{t-1} + \varepsilon_{yt} \qquad \rho < 1 \tag{2.66}$$

$$x_t = \rho x_{t-1} + \varepsilon_{xt} \qquad \rho < 1 \tag{2.67}$$

There is no third variable involved in the construction of both variables. They regressed $X_t$on Y_t or vice versa without involving their lag values in regression analysis:

$$y_t = a + \beta_1 x_t + \varepsilon_{yt} \tag{2.68}$$

$$x_t = a + \beta_1 y_t + \varepsilon_{xt} \tag{2.69}$$

We came up with spurious results due to missing variable because we did not include the lag values of variables as an independent variable.

The reason behind the spurious regression is that when the potential variable is missing from the regression, then the irrelevant variable acts as a proxy of potential variable. It captures the effect of potential variables and then the results would become significant. If we start with ARDL model, it will overtake the problem of missing variable.

2.3 Problems in Prevailing Treatments

The most popular procedures to evade the spurious regression is unit root and cointegrating analysis. These methods are equally capricious because of a large number of specification decisions needed before application of unit root and cointegration tests like, choice of the deterministic part; the structural breaks; autoregressive lag length choice and innovation process distribution. For detail see section (2.3.1.1). The cointegration analysis which is employed as a tool to avoid spurious regression, also experience specification decisions problems. For detail see section (2.3.2). It involves unit root testing which is also unreliable. The tests of unit root are unreliable that is why it is very hard to conclude something reasonable. For detail see section (2.3.1).

2.3.1 Unit Root Testing

Granger and Newbold (1974) argued that unit root leads to spurious regression and Nelson and Plosser (1982) found that most of economic time series are in fact unit root Numerous financial and economic series exhibit nonstationary or trending behavior like, Stock prices, exchange rate, Gross Domestic Product (GDP) and many others. It is unlikely to get accurate results from trendy series. The most common procedures to avoid the spurious regression are unit root and cointegrating testing. These procedures are equally unreliable due to specification decisions. The cointegration analysis which is used as a tool to avoid spurious regression, suffer from numerous problems. It involves unit root testing and then testing for cointegration. The tests of unit root are so unreliable that is why it is very hard to conclude something reasonable. The United States (US) GNP is the series used by the large number of researchers as a guinea pig for the tests of unit root. However, nothing reasonable could be said about the unit root in series. Rehman and Zaman (2008) summarized findings of researchers in US GNP as follows:

"Trend Stationary: Perron (1989), Zivot and Andrews (1992), Diebold and Senhadji (1996), Papell and Prodan (2003),

Difference stationary: Nelson and Plosser (1982), Murray and Nelson (2002), Kilian and Ohanian (2002),

Don't know; Rudebusch (1993)".

2.3.1.1 Why Unit Root Tests are so Unreliable

The important task in econometrics is to determine the most suitable arrangement of trend in time series. There are two common procedures to eradicate the trend of data are regression with time trend and differencing. The unit root testing procedure offers an idea which can be adopted to render the time series stationarity. Besides, the precision and specification of unit root procedures are still a paradox, though, since mid-eighties the literature on unit root testing has been raised stormily.

Rehman and Zaman (2008) investigated that the two main causes for inadequate performance of unit root tests are observational equivalence and model misspecification. They mainly targeted four specification decisions: choice of the deterministic part; the structural breaks; autoregressive lag length choice and innovation process distribution, and examine their role in an inference from unit root tests. They explored that these specification decisions seriously affect the performance of unit root tests. They also investigated that the existing unit root tests do not provide any set criteria regarding these specification decisions. That is why, they came up with unreliable results.

DeJong et al. (1992) found that Choi and Philips (1991) and Philips and Perron (1988) unit root procedures suffer from size distortion and low power issues in the presence of moving average (MA). While, Augmented Dicky Fuller (ADF) behaved well. Schwert (2002) investigated that the Dicky Fuller (1979, 1981) is responsive to pure autoregressive process assumption. It means the data generating process of series is pure autoregressive (AR). When the moving average competent is involved in fundamental process, then the Dicky Fuller reported distribution and test statistic distribution can be quite different. Many other unit root tests are being proposed and to some extent they all are facing similar problems.

2.3.2 Problems with Cointegration Testing

Like unit root tests the cointegration testing also involves many specification decisions which cut the reliability of results. The existing cointegration testing procedures do not provide any reasonable criteria regarding these specification decisions, and that leads to their results unreliable.

For example, Lag length specification is a significant practical question about the application of any econometric analysis. Like, in case of unit root test, if the lag length is too short then the serial correlation remains in errors and the results will be biased. If the lag length is too large this will reduce the power of the test. In the same way, the cointegration tests are also very sensitive to lag length selection. Agunloye et al. (2014) explored that the Engle Granger (EG) cointegration test is extremely sensitive to lag length. Carrasco et al. (2009) examined that the lag length misspecification may significantly affect the cointegration results. In case of the under specification, it could undermine the cointegration results and in over specification, it may diminish the power of test. Similarly, trend specification is also a very significant issue in econometric literature.

Ahking (2002) explored that the deterministic linear time trend included in Johansen's cointegration test provides disproving results but after exclusion of deterministic linear time trend robust results are attained. He also suggested that great attention must be taken in trend specification in cointegration analysis. There are lot of studies available in literature on this issue but most of them provide with different results. Leybourne and Newbold (2003) used three cointegration tests for independent integrated series and each series has a structural break. They found cointegration among them until structural breaks are not properly treated. Choi et al. (2004) examined that the economic models for cointegration are often provided erroneous results. The main reason is the errors are unit root non-stationary owing one of the variables has non-stationary measurement error. They stated that "If the money demand function is stable in the long-run, we have a cointegrating regression when money is measured with a stationary measurement error but have a spurious regression when money is measured with a nonstationary measurement error".

CHAPTER 3

CONCEPTUAL FRAMEWORK

Spurious regression is an issue of great concern, especially when two economic time series are regressed without any theoretical basis and results are most expectedly significant. More or less all the nominal variables are correlated with each other, without having any theoretical link because they have dominant inflationary component. This component usually overshadows the true underlying relationship. This type of regression is known spurious regression which is due to missing variable but according to modern econometrics the mian reason is nonstationarity even though there is no missing variable. The focus of this chapter is to discuss different concepts regarding spurious regression. The chapter is arranged as follows

3.1 What is Spurious Regression?

The foremost problem with time series is that two independent variables can seem to highly significant than they are in true relation. The spurious regression happens when independent series come up with significant results.

3.1.1 Spurious Regression

There are many well-known definitions of spurious regression and some most famous are quoted here:

According to Yule (1926)

> *"We sometimes obtain between quantities varying with the time (time-variables) quite high correlations to which we cannot attach any physical significance whatever, although under the ordinary test the correlation would be held to be certainly "significant"".*

According to Granger and Newbold (1974)

> *"It is very common to see reported in applied econometric literature time series regression equations with an apparently high degree of fit, as measured by the coefficient of multiple correlation R^2 or the corrected coefficient $\bar{R}^2$, but with an extremely low value for the Durbin-Watson statistic".*

According to Phillips's (1986)

> *"A spurious regression occurs when a pair of independent series, but with strong temporal properties, is found apparently to be related according to standard inference in a Least Squares regression".*

3.1.1.1 Granger and Newbold Explanation of Spurious Regression

According to Granger and Newbold (1974), the spurious regression arises by relating the levels of independent nonstationary time series. They describe this phenomenon as following:

Let us suppose we have linear regression model with one dependent variable Y and X matrix containing one series and (K-1) independent variables. The u is an error term independently distributed.

$$Y = X\beta + u \tag{3.1}$$

where Y_{Tx1} vector consists on regressand observations, $X_{T\times K}$ matrix comprising 1's series and stochastic independent variables and β_{Kx1} vector of coefficients.

$$E(u) = 0 \tag{3.2}$$

$$E(uu') = \sigma^2 I \tag{3.3}$$

Then the joint significance test is employed to estimate the joint effect of all stochastic regressor on dependent variable. The null hypothesis is formed as

$$H_0: \beta_1 = \beta_2 = \cdots = \beta_{k-1} = 0 \qquad (3.4)$$

$$F = \frac{R^2}{1-R^2} \times \frac{T-K}{K-1} \qquad (3.5)$$

The $R^2$is coefficient of variation which describes that how much independent variables explain the dependent variable, K is number of regressors and T is number of observations. F test statistic compare with critical values of F distribution with "K-1" and "T-K" degrees of freedom by assuming normality. It is quite possible that there does prevail some β's.

$$Y - X\beta = u \qquad (3.6)$$

In case u satisfies the white noise conditions. However, suppose if Y_t's are not a white noise process then the null hypothesis of F test cannot be true and its tests are not suitable.

Suppose we run a regression with economic time series at levels, as we know that most of the economic series are nonstationary and highly correlated. In this case F test will not pursue Fisher's F distribution under the null of all the β's are equal to zero. Under this null hypothesis the residual series can be obtained as

$$Y_t - \beta_0 = u_t \qquad t = 1, 2, 3, \dots, T \qquad (3.7)$$

Now if some distributional problem involved, it can be attained by consideration following regression case:

$$Y_t = \beta_0 + \beta_1 X_t + u_t \qquad (3.8)$$

where it is supposed that both Y_t and X_t follow independent autoregressive process of order one.

$$Y_t = b_{10} + \rho_1 Y_{t-1} + u_{yt} \tag{3.9}$$

$$X_t = b_{20} + \rho_2 X_{t-1} + u_{xt} \tag{3.10}$$

In this case the coefficient of determination R^2 is equal to the ordinary correlation between X_t and Y_t which is provided by (Kendall, 1954):

$$\mathrm{Var(R)} = \frac{T^{-1}(1+\rho_1\rho_2)}{(1-\rho_1\rho_{2)}} \tag{3.11}$$

Since it is constrained that the value of "R" must lies in region -1 ≤ R^2 ≤+1 and if the variance of "R" larger than 1/3 then the distribution of "R" cannot be a unimodal at zero. The essential condition is $\rho_1\rho_2 > (T-3)/(T+3)$.

Let us take an example for better understanding of this phenomena which is adapted from Granger and Newbold (1974):

Suppose if T = 20 and $\rho_1 = \rho_2$, the distribution which has no single mode at origin will arise if $\rho_1 > 0.8$ or $\rho_1 = 0.9$ and $E(R^2) = 0.47$.

Thus the high value of R^2 cannot be considered as an indication of significant association between autocorrelated series. So the phenomena of spurious regression might arise because of running the regression between independent time series by relating their levels.

3.1.1.2 Asymptotic Theory of Spurious Regression by Philips

Asymptotic theory of spurious regression adapted from Philips (1986) based on the concept of Granger and Newbold (1974):

As we have seen in Granger and Newbold experiment, they used two independent first order autoregressive time seriesX_t and Y_t. They employed linear regression model as following:

$$Y_t = \hat{\beta}_0 + \hat{\beta}_1 X_t + \hat{u}_t \tag{3.12}$$

where

$$Y_t = b_{10} + \rho_1 Y_{t-1} + u_{yt} \tag{3.13}$$

$$X_t = b_{20} + \rho_2 X_{t-1} + u_{xt} \tag{3.14}$$

and

$$u_{yt} \sim iid\left(0, \sigma_y^2\right) \quad \text{and} \quad u_{xt} \sim iid(0, \sigma_x^2) \tag{3.15}$$

To make work more general, we relax Granger and Newbold's assumption on error terms and impose weaker assumption on error terms of autoregressive series. We introduce random n-vectors sequence $\{\zeta\}_1^{\infty}$which is defined on probability space (**Ω**, β and Y).

Suppose $G_t = \sum_{j=1}^{t} \zeta_j$ is a partial sum process and set $G_0 = 0$. So, we require following:

Assumption 1

(a) $E(\zeta_t) = 0$

(b) $\mathrm{Sup}_{j,t} E|\zeta_{it}|^{\beta+\varepsilon} < cc\ for\ some\ \beta > 2\ and\ also\ \varepsilon > 0$

(c) $\sum = \lim_{T\to\infty} T^{-1}\, E(G_T G_T')$ be present and positive definite

(d) $\{\zeta\}_1^{\infty}$ is strongly mixing with the mixing number a_m and sustaining $\sum_1^{\infty} a_m^{1-2/\beta} < \infty$.

Now if we set n = 2 and $\zeta'_t = (u_{yt}, u_{xt})$, then the conditions imposed in assumption 1 on the error terms of autoregressive time series are weak and they allow X_t and Y_t more general process of (order one). The differences are quite weakly dependent and innovations are probably heterogeneously distributed. This allows a wide variety of data generating mechanisms like, ARIMA (P, 1, q) model under general conditions on underlying innovations. Hence the condition (d) is only satisfied when mixing waning rate is $a_m = O(m^{-\lambda})$ for $(\lambda > \frac{\beta}{\beta-2})$. When β becomes close to 2 and the outlier probability grows [under condition (b)], the mixing waning rate increses outliers's effect, then required under condition (d) diminishes more rapidly.

Now if the $\{\zeta_t\}$ is weak stationary process then

$$\sum = E(\zeta_1\zeta_1') + \sum_{k=1}^{\infty} E\,(\zeta_1\zeta_k' + \zeta_k\zeta_1') \tag{3.16}$$

the convergence of this innovations series is inferred by the (d) mixing condition. Additionally, when $\zeta'_t = (u_{yt}, u_{xt})$ and u_{yt} and u_{xt} are independent, in the context of spurious regression, we have

$$\sum = \begin{bmatrix} \sigma_{u_y}^2 & 1 \\ 0 & \sigma_{u_x}^2 \end{bmatrix} \tag{3.17}$$

where

$$\sigma_y^2 = \lim_{T\to\infty} T^{-1}\, E(K_T^2) \tag{3.18}$$

$$\sigma_x^2 = \lim_{T\to\infty} T^{-1}\, E(L_T^2) \tag{3.19}$$

and

$$K_t = \sum_1^t u_{yj} \tag{3.20}$$

$$L_t = \sum_1^t u_{xj} \tag{3.21}$$

The ordinary t-test ratios for $\hat{\beta}_0$ and $\hat{\beta}_1$ are estimated as

$$t_{\beta_0} = \frac{\hat{\beta}_0}{S_{\hat{\beta}_0}} \tag{3.22}$$

$$t_{\beta_1} = \frac{\hat{\beta}_1}{S_{\hat{\beta}_1}} \tag{3.23}$$

where $S_{\hat{\beta}_0}$ and $S_{\hat{\beta}_1}$ are the standard error of estimated parameter $\hat{\beta}_0$ and $\hat{\beta}_1$ in equation (3.12). The DW is the ordinary Durban Watson test d-statistic and R^2 has been used for coefficient of determination. Whereas Box pierce test is defined as

$$Q_k = T \sum_{s=1}^{k} r_s^2 \tag{3.24}$$

where

$$r_s = \frac{\sum_{s+1}^{T} \hat{v}_t \hat{v}_{t-s}}{\sum_1^T \hat{v}_t^2} \tag{3.25}$$

Theorem 1 derived below provide the asymptotic theory of ordinary linear regression and some of its associated statics like t-statistics, R^2 and Q_k. The lemma 1 is used for the derivation of theorem 1.

However, Theorem 1 conditions (a) and (b) indicate that $\hat{\beta}_0$ and $\hat{\beta}_1$do not converge in the probability to constant as T approaches to ∞. Definitely, $\hat{\beta}_0$ distribution diverges and $\hat{\beta}_1$ follows non degenerate limiting distribution as T approaches to ∞. So, the uncertainty regarding the regression (3.12) that arises due to its spurious nature (because series are independent and autoregressive 3.13 and 3.14) perseveres asymptotically in these limiting distributions.

3.2 Spurious Regression is not just a Nonstationary Phenomena

Modern time series econometrics consider nonstationary key cause of spurious regression. It was Granger and Newbold (1974) who first time publicized that the mian reason of spurious regression is nonstationarity in time series even there is no missing variable. Thereafter, Nelson and Plosser (1982) claimed that most of the economic series are nonstationary in nature. The theory of nonstationary is unusually different from the stationary time series theory which was employed by the researchers for regression analysis. These finding changed the direction of time series econometrics that led to the development of modern time series econometrics. After that large setoff literature published on the concept of nonstationary, it was found that the mian cause of spurious regression was nonstationarity and proposed battery of tests of time series econometrics to avoid the possibility of spurious regression (see section 2 of the study in hand). For handling the problem that spurious regression produced by nonstationarity, researcher used unit root and cointegration procedures (see section 5 of the study in hand).

These proposed procedures are also facing size and power problems in small sample size. So, it is very hard to get reliable results from these tests because these procedures involve many prior specification decisions like lag length, trends and structural stability etc. There are not only one test which can provide good guidance regarding correct specification decisions, all available tests are also facing specific statistical error (type I and type II). Unit root testing is pre-testing before going for cointegration. The first step of cointegration testing is to test unit root, while the unit root testing has problems in specification decisions and it suffers in size and power problem, second step is to apply cointegration testing which is also facing same problems, so the cumulative probability of error in all procedures leave the results unreliable.

3.3 Spurious Regression is also a Stationary Phenomena

The spurious regression can also prevail in stationary series. Granger et al. (2001) indicted the possibility of spurious regression in stationary series. In this situation these procedures do not provide any solution. The cointegration is only way to hold the problem of spurious regression but it only works in case of nonstationarity. It means existing literature does not provide any solution for the remedy of spurious regression in stationary time series.

3.4 Missing Variable can Generate Spurious Regression

The spurious regression has long history but we start at least from Yule (1926). Spurious regression is a phenomenon known to econometrician and statisticians since Yule (1926). He was who attributed spurious regression to missing variable. Simon (1954) and Frey (2002) argued that the main reason of spurious regression is missing relevant variable.

It is a general misconception that spurious regression is due to nonstationarity. After reviewing a large number of studies, we come to conclusion that missing variable is the true cause of spurious regression even series is stationary or nonstationary. If we deeply analyze the spurious regression results, we find that Granger and Newbold study was also considering the missing variable. The mian purpose of their study is to show the nonstationary series can generate spurious regression, for this purpose they used autoregressive series and regressed them on each other; the regressions came up with significant results. It is because of missing variable that the true data generating process is autoregressive. If they include lag values of variables in regression, the results would be different and they came expectedly with insignificant results. We adopted the example from Zeisel (1948) for clarification of problem.

Example:X is the married female employee's percentage in group, Y is the average absence of per employee per week and Z is the average housework hours consumed per employee per week. High correlations have been found between X and Y and X and Z, but when Z held constant, the correlation between X and Y is going close to zero. The correlation between X and Y is spurious and it is due to joint effect produced by the variation in Z. It is common that married female spend more housework hours that is why more absence came into being.

CHAPTER 4

METHODOLOGY AND MODEL SPECIFICATION

4.1 What is ARDL Model?

In ARDL model, the dependent variable is expressed as a function of lag and current values of independent variable and its own lag value. Davidson et al. (1978) proposed ARDL methodology (DHSY hereafter) to model the UK consumption function. ARDL model normally starts from reasonably general and large dynamic model and progressively reduces its mass and alter variables by imposing linear and non-linear restrictions (Charemza and Deadman, 1997). Autoregressive distributed lag (ARDL) model is one of the most general dynamic unrestricted model in econometric literature. It has lag values of independent and dependent variables, therefore it should be able to tackle correlation problem. The ARDL model is a general model, that's why it could be possible to tackle many econometric problems like misspecification and come up with a most appropriate interpretable model.

The ARDL (1, 1) is the simplest form of ARDL model. Consideran ARDL (1, 1) model:

$$y_t = a + \beta_1 x_t + \beta_2 x_{t-1} + \beta_3 y_{t-1} + \varepsilon_{yt} \tag{4.1}$$

Hendry and Richard (1983), Hendry, Pagan and Sargan (1984) and Charemza and Deadman (1997) argued that by imposing restrictions we can find out at least ten most appropriate and economically interpretable models from ARDL (1, 1) model. We are giving hare some important cases of restriction:

1. $\beta_2 = \beta_3 = 0$ Static regression,
2. $\beta_1 = \beta_2 = 0$ First order autoregressive process,
3. $\beta_3 = 1,\ \beta_1 = -\beta_2$ Equation in first difference,
4. $\beta_2 = 0$ Partial adjustment equation

As discussed, the spurious regression is may be a consequence of missing variable. ARDL is a general specification taking into account the lag structure. Therefore, it could give better results.

4.1.1 Difference between ARDL model and ARDL cointegration test

Pesaran et al. (1996) and Pesaran (1997) proposed a single equation ARDL (autoregressive distributed lag) approach for cointegration as an alternative of Engle and Granger and Engle and Yoo procedures. The ARDL cointegration approach provides explicit tests for the presence of a single cointegrating vector, instead of assuming uniqueness. Suppose we have two variables x_{1t} and x_{2t} and the generalized ARDL model equation is following:

$$x_{1t} = \theta_{10} + \sum_{i=1}^{p} \beta_i \Delta x_{1t-i} + \sum_{i=0}^{q} \delta_i x_{2t-i} + \varepsilon_{1t} \qquad (4.2)$$

Equation 4.2 was proposed by Hendry for time series modeling and he stated that it provides convenient way of following General to Simple methodology. Hendry shows that numerous theoretical models can be driven from the equations similar to 4.2.

On the other hand, the equation proposed by Pesaran for testing long run relationship is

$$\Delta x_{1t} = \theta_{10} + \theta_{11} x_{1t-1} + \theta_{12} x_{2t-1} + \sum_{i=1}^{p-1} \beta_i \Delta x_{1t-i} + \sum_{i=1}^{q-1} \delta_i \Delta x_{2t-i} + \varepsilon_{1t} \qquad (4.3)$$

One can see that just like the simplifications proposed by Hendry (1978), the Pesaran model is also restricted version of generalized ARDL model.by imposing restrictions on unrestricted ARDL model as in Eq 4.2, the Pesaran cointegration regression equation can be derived.

$$\Delta x_{1t} = \theta_{10} + \theta_{11}x_{1t-1} + \theta_{12}x_{2t-1} + \sum_{i=1}^{p} \beta_i \Delta x_{1t-1} + \sum_{i=1}^{q} \delta_i \Delta x_{2t-1} + \varepsilon_{1t} \quad (4.4)$$

$$\Delta x_{2t} = \theta_{20} + \theta_{21}x_{2t-1} + \theta_{22}x_{2t-1} + \sum_{i=1}^{p} \beta_i \Delta x_{2t-1} + \sum_{i=1}^{q} \delta_i \Delta x_{1t-i} + \varepsilon_{2t} \quad (4.5)$$

The Pesaran Equations 4.5 and 4.5 can also be regarded as the Error Correction Version of unrestricted ARDL equation 4.2, because just like the error correction model, the equation contains all difference form variables and error correction term which mentioned without differencing. Pesaran argued that this model can be used for testing long run and short run relationships. The F test is employed to find out joint significance for short run and long run relationships. The hypotheses for long run and short run relationships are following:

H_0: $\theta_{11} = \theta_{12} = 0$ (There is no long run relationship)

H_1: At least one of them is nonzero (There is long run relationship)

The hypotheses for short run relationship are following:

H_0: $\beta_i = \delta_i = 0$ (There is no short run relationship)

H_1: At least one of them is nonzero (There is short run relationship)

The F statistic (Wald test) for these hypotheses tested in each of the models can be denoted as:

$$F_{x1}(x_{1t} | x_{2t}) \quad (4.6)$$

$$F_{x2}(x_{2t} | x_{1t}) \quad (4.7)$$

The distribution of Wald test is non-standard asymptotically under the null of no cointegration. Pesaran and Shin (1995) revealed that asymptotically valid inference on short run and long run parameters could be made by employing ordinary least square estimations of ARDL model. So, the ARDL model order is properly augmented to grant for contemporary correlation among the stochastic elements of the data generating processes involved in estimation.

As stated by Pesaran the beauty of Pesaran's model is that it can differentiate between genuine and spurious relationship without knowing about stationarity. Since Pesaran ECM version of ARDL is a restriction of the generalized ARDL model, therefore, the simple ARDL (DHSY version) should also be utilized to differentiate between genuine and spurious relationship. This study shows that actually it is possible to use unrestricted ARDL model to differentiate between genuine and spurious relationship.

In ARDL cointegration procedure, we used bound testing approach for long run relationship. ARDL cointegration procedure is only used for nonstationary series and involves prior specification decisions. ARDL cointegration procedure estimates short run effects by taking differencing in equation and long run effect through bound testing. Pesaran et al. (2001) provided critical values of two bounds, upper and lower which are being used for cointegration. The lower bound considers variables are stationary and they have no long run relationship. The upper bound considers variables are nonstationary and they have long run relationship. While the unrestricted ARDL model used least square regression and there is no need of special critical values and it also works in stationary time series case. The simple ARDL model provides only static relationships.

4.2 The Methodology

The Components of the methodology are as following:

I. Data generating process (DGP)
II. Testing and Simulations

4.2.1 Data Generating Process (DGP)

Let's, we have a data generating process

$$\begin{bmatrix} x_t \\ y_t \end{bmatrix} = \begin{bmatrix} \theta_1 & \theta_{12} \\ \theta_{21} & \theta_2 \end{bmatrix} \begin{bmatrix} x_{t-1} \\ y_{t-1} \end{bmatrix} + \begin{bmatrix} a_1 & a_2 \\ b_1 & b_2 \end{bmatrix} \begin{bmatrix} 1 \\ t \end{bmatrix} + \begin{bmatrix} \varepsilon_{xt} \\ \varepsilon_{yt} \end{bmatrix} \begin{bmatrix} \varepsilon_{xt} \\ \varepsilon_{yt} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix} \right) \quad (4.8)$$

We can rewrite it as for simplification of notation

$$X_t = AX_{t-1} + Bd + \varepsilon_t \qquad \varepsilon_t \sim N(0, \Sigma) \quad (4.9)$$

The data generating process equation (18) can generate data in quite large types of scenarios. Suppose, $\theta_{12} = \theta_{21} = 0$ and ρ =0, the data generating process will generate two independent series and it would be indication of spurious regression if the regression of x_t on y_t turns out to be significant. The matrix 'B' is nuisance parameter which does not determines the relation between X and Y, however, it can play significant role in determining size and power of the statistical tests. If A= 0, it indicates that there is no autocorrelation and cross autocorrelation in the series. If B= 0, it shows that series have no drift and trend. If A and B both are zero, it means that two series would be IID (identically independently distributed). The value of degree of association depends upon only $\sum$.

It could be possible to make many types of models from this data generating process by imposing restrictions. Some important scenarios which can be drawn from equation (4.8) are mentioned below:

S1. $\theta_{12} = \theta_{21} = 0, \rho = 0$

Assumption $\theta_{12} = \theta_{21} = 0$, implies that both X_t and Y_t are autoregressive without having any dependence on the lags of other variable. The $\rho = 0$ implies that the error terms added to each have no contemporaneous correlation. Therefor X_t and Y_t are neither serially nor contemporaneously dependent on each other.

S2.$\theta_{12} = \theta_{21} = 0, \rho = 0$, $a_1 = b_1 = 0$

Assumption $\theta_{12} = \theta_{21} = 0$, implies that both X_t and Y_t are autoregressive without having any dependence on the lags of other variable. The $\rho = 0$ implies that the error terms added to each have no contemporaneous correlation. Therefore X_t and Y_t are neither serially nor contemporaneously dependent on each other. This restriction $a_1 = b_1 = 0$ implies that both series are without drift term or series are having only linear trend

S3. $\theta_{12} = \theta_{21} = 0, \rho = 0 a_2 = b_2 = 0$

Assumption $\theta_{12} = \theta_{21} = 0$, implies that both X_t and Y_t are autoregressive without having any dependence on the lags of other variable. The $\rho = 0$ implies that the error terms added to each have no contemporaneous correlation. Therefor X_t and Y_t are neither serially nor contemporaneously dependent on each other. This restriction $a_1 = b_1 = 0$ implies that both series are with drift term or series are having no linear trend

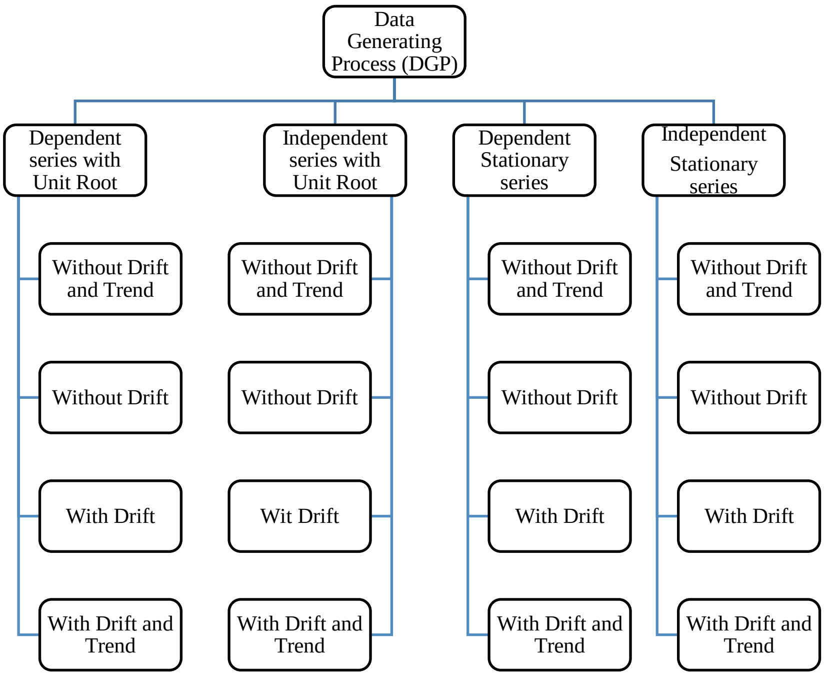
S4. $b_2 = a_2 = 0$

Assumption $b_2 = a_2 = 0$, implies that both X_t and Y_t are autoregressive having dependence on other variable. Therefor X_t and Y_t are serially or contemporaneously dependent on each other. This restriction $a_2 = b_2 = 0$ also implies that both series are with drift term or series are having no linear trend

S5. $a_1 = b_1 = 0$

Assumption $b_1 = a_1 = 0$, implies that both X_t and Y_t are autoregressive having dependence on other variable. Therefore X_t and Y_t are serially or contemporaneously dependent on each other. This restriction $a_1 = b_1 = 0$ also implies that both series are without drift term or series are having only linear trend

Organization Chart of data generating process and its different specification



We will investigate the performance of ARDL with following specifications of data generating process.

4.2.2 Testing and Simulation

We will evaluate the forecast performance of ARDL model through Monte Carlo simulations and compare size and power of conventional methods with ARDL model. We will compare ARDL size and power performance with commonly practiced Engle, Granger and Johansen and Juselius cointegration tests. We will check the robustness of Engle, Granger and Johansen and Juselius cointegration tests and ARDL model under different specifications, like exact, over and under specifications by estimating size and power. We will also evaluate the forecast performance of ARDL by taking real data.

CHAPTER 5

THE SPURIOUS REGRESSION WITH OLS, COINTEGRATION METHODS, AND ARDL

The mian objective of this study is to evaluate the performance of ARDL model to differentiate between genuine and spurious regression. If ARDL is actually able to differentiate then it should have following feature:

Suppose we have set of independent time series with no mutual relationship and we test the relationship using ARDL model, it should be able to find that there is no relationship. More technically suppose x_t and y_t are independent by construction and we estimate $x_t = \alpha_1 x_{t-1} + \beta_0 y_t + \beta_1 y_{t-1} + \varepsilon_t$ and we test the hypotheses$H_0(1): \beta_0 = 0, H_0(2): \beta_1 = 0$ and $H_0(3): (\beta_0, \beta_1) = 0$. Since in fact all three hypotheses are true, the possibility of rejection of the three hypotheses should not exceed the nominal size. If the probability is higher, then this size distortion can be regarded as spurious regression.

It is well known that OLS produces high probability of spurious regression and this probability increases with the increase in sample size. On the other hand since ARDL contains all the true determinants of regressand, we hope that ARDL would perform better. The size analysis is performed to quantify the distortion in probability of type I error. It can be expressed in following way:

$$\text{Size} = \text{Prob (reject } H_0 | \text{ when} H_0 \text{ is true)}$$

In this study size analysis is used to estimate the probability of spurious regression after employing conventional method and ARDL model with different specification. For this analysis, the independent autoregressive stationary and nonstationary time series are being generated with different specification; without drift and trend, without drift, with drift and with drift and trend. Suppose, two independent autoregressive series x and y are being generated, after that regress y on x, if the results are significant, it indicates regression is producing spurious results. All the results in tables 5.1, 5.2 and 5.3 came after 100,000 simulation. The data generating process for table 5.1 is discussed in chapter 4:

$$\begin{bmatrix} x_t \\ y_t \end{bmatrix} = \begin{bmatrix} \theta_1 & 0 \\ 0 & \theta_2 \end{bmatrix} \begin{bmatrix} x_{t-1} \\ y_{t-1} \end{bmatrix} + \begin{bmatrix} a_1 & a_2 \\ b_1 & b_2 \end{bmatrix} \begin{bmatrix} 1 \\ t \end{bmatrix} + \begin{bmatrix} \varepsilon_{xt} \\ \varepsilon_{yt} \end{bmatrix} \begin{bmatrix} \varepsilon_{xt} \\ \varepsilon_{yt} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right) \quad (5.1)$$

The two independent autoregressive stationary and non-stationary series are generated by using above given data generating process as it is apparent from equation (5.1). We estimate the following regression:

$$y_t = a + \beta_1 x_{t-1.} + \varepsilon_{yt} \quad (5.2)$$

The probability of getting significant β_1 would be the actual size and it is different from nominal size. It would be considered as probability of spurious regression. Below is the summary of actual empirical size and the probability of spurious regression given in table 5.1, 5.2 and 5.3.

5. 1 Size Analysis with Nonstationary Series

The data are generated with pre decided specifications and the probability of spurious regression is being tested by using classical methods and with ARDL model. The two independent autoregressive non-stationary series have been generated by using equation 5.1.

The figure 1given below is based on data of first panel of table 5.1. When autoregressive parameters $\theta_1 = \theta_2$ are equal to 1, it means series are nonstationary. Figure 1, shows the comparison among the probability of spurious regression with OLS and ARDL models. This comparison has been made on the different sample sizes. Using OLS, we got

66.6%, 78.2%, and 86.3% probability of spurious regression at 50, 100, 200 sample size respectively. It supported the argument of Granger and Newbold (1974) that as we increase the sample size in case of independent nonstationary series, the probability of spurious regression is also increasing. At sample size 50, ARDL (1, 1) reduced this probability from 66.2% to just 6.2%, and ARDL (2, 2) also reduced this probability from 17.2% to just 6.6%. It shows that the conventional OLS method can generate spurious regression in case of nonstationary time series and ARDL model has reduced the probability of spurious regression significantly.

Figure 5.1: Independent Non-Stationary Series without Drift and Trend

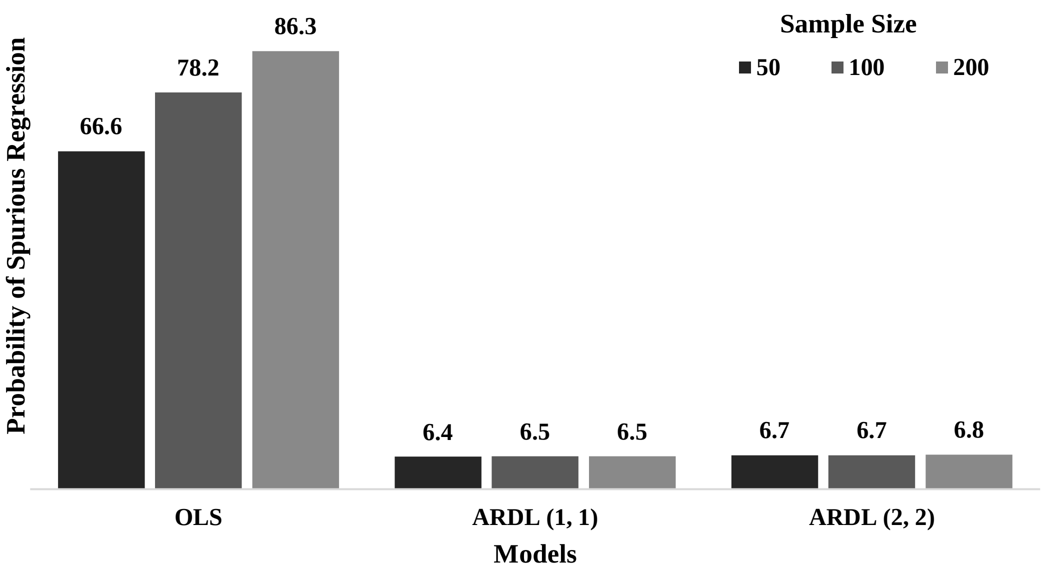


Figure 5.1 shows the probability of spurious regression on the basis of t-statistics for coefficients of independent variable by using Ordinary Least Square and Autoregressive distributed Lag models. The F-stats are used for joint significance of current and lag values of independent variable for ARDL models.

When we employed ARDL (1, 1), it reduced the probability of spurious regression from 66.6% to just 6.2% at sample size of 50. The ARDL (2, 2) also reduced this probability from 66.6% to 6.6% at sample size of 50. When we increase the sample size from 50 to 100 and 200 the probability of spurious regression remains same. It clearly shows that the probability of spurious regression with ARDL model is not increasing as sample size increases. It shows that the OLS can generate spurious regression in case of nonstationary time series and ARDL model has reduced the probability of spurious regression significantly. We use this figure only for first panel of table (5.1). Similarly, we can make figures for other panels in same way. The comparison of OLS and ARDL on the basis of probability of spurious regression under different specifications have been given below in table (5.1):

Table 5.1: Probability of Spurious Regression in Nonstationary with OLS and ARDL Models

	OLS	ARDL (1, 1)				ARDL (2, 2)					
	xt	**Xt**	**xt-1**	**yt-1**	**F-stat**	**Xt**	**xt-1**	**xt-2**	**yt-1**	**yt-2**	**F-stat**
N	$a_1 = b_1 = 0 = a_2 = b_2 = 0$										
50	66.6	6.6	6.6	100.0	6.4	7.3	6.2	7.0	100.0	7.0	6.7
100	78.2	6.1	6.5	99.7	6.5	7.2	6.1	7.0	100.0	7.3	6.7
200	86.3	6.0	6.6	95.8	6.5	7.3	6.1	6.9	100.0	6.9	6.8
N	$a_1 = b_1 = 0$										
50	100.0	94.9	81.8	100.0	80.4	75.4	7.8	36.2	100.0	56.0	55.3
100	100.0	93.0	80.0	100.0	81.5	75.2	8.1	35.1	100.0	58.3	56.6
200	100.0	93.1	81.1	100.0	81.0	76.8	7.9	35.6	100.0	60.2	56.4
n	$a_2 = b_2 = 0$										
50	100.0	6.1	8.0	100.0	7.0	8.3	6.3	8.2	100.0	6.4	7.5
100	100.0	6.1	6.4	100.0	6.2	7.5	6.5	8.6	100.0	6.6	7.4
200	100.0	6.3	7.2	100.0	6.4	7.6	6.6	8.4	100.0	6.5	6.6
N	$a_1 = b_1 = a_2 = b_2 \neq 0$										
50	100.0	95.9	83.1	100.0	81.7	76.8	7.6	36.7	100.0	55.9	56.6
100	100.0	93.5	83.4	100.0	80.9	76.3	7.4	33.2	100.0	56.1	56.3
200	100.0	94.1	83.5	100.0	82.3	77.1	8.3	35.8	100.0	55.6	57.2

The series have only autoregressive structure; this means the series has strong dependence on its own past. But the error terms of Series X are independent of the terms in Y. Therefore, X should not appear in the equation of Y, and if it appears, it indicates spurious regression. The series are generated by following equation (5.1) and results are summarized after estimating equation (5.2).

In first row of first panel of table (5.1), the results are indicating that when series are nonstationary, the autoregressive parameters θ_1=1 and θ_2=1having no drift and trend (a_1= b_1= 0 =a_2= b_2= 0) then after employing OLS, we get 66.6%, actual empirical size at sample size of 50. So on the basis of 5% nominal size, the probability of spurious regression is 61.6%. In ARDL models, F-test is being used to test the joint significance of current and lag values of independent variables, the F-stat value after employing ARDL (1, 1) model is found only 6.4% which shows only 1.4% probability of spurious regression on the basis of 5% nominal level of significance at sample size of 50. It means ARDL (1, 1) reduced the probability of spurious regression from 66.6 to only 6.4%. For ARDL (2, 2) model actual size is 6.8% when nominal size is 5%, which indicates that ARDL (2, 2) is 1.8% which is very minor and negligible at sample size of 50.

In second row of first panel of table (5.1), the results are indicating that by employing OLS, we get 78.2% actual empirical size at sample size of 100. So with 5% nominal size, the probability of spurious regression is 73.2%. It indicates that as we increase sample size from 50 to 100 the probability of spurious regression also increases. But the probability of rejection with F-stat employed to ARDL (1, 1) model is only 6.5% which shows 1.5% probability of spurious regression. It means ARDL (1, 1) reduced the probability of spurious regression from 66.6 to only 6.5%. For ARDL (2, 2) model actual size is 6.7% when nominal size is 5%, it indicates 1.7% probability of spurious regression.

Consider the third row of first panel in table 5.1, the results are indicating that after employing OLS, we got 86.3% actual empirical size at sample size of 200. So with 5% nominal size, the probability of spurious regression is 81.3%. It again indicates that as we increase sample size the probability of spurious also increases. But the probability of significance of irrelevant variable using F-stat employed to the zero to zeroth and first lag or x variable in ARDL (1, 1) model is only 6.5% which shows only 1.5% probability of spurious regression. ARDL (1, 1) model again reduced the probability of spurious regression from 86% to 6.5%. The actual size of ARDL (2, 2) model is 6.8%, so on the basis of 5% nominal size the probability of spurious regression after employing ARDL (2, 2) is 1.8% which is very minor and negligible at sample size of 200.

It shows that the conventional OLS method badly suffer with size problem when series are nonstationary without drift and trend and probability of spurious regression increases as sample size increases. It supported the conventional argument of Granger and Newbold (1974) regarding spurious regression that as we increase sample size in case of nonstationary series, the probability of spurious regression also increases. On contrary, ARDL model is not having size distortion problem on all sample sizes and size distortion not increases as we increase sample size. It clarifies that when series are nonstationary without having drift and trend ARDL works better than OLS.

In first row of second panel of table 5.1, the results are representing that when the nonstationary series has θ_1=1 and θ_2=1 having no drift a_1= b_1= 0 or having linear trend then after running OLS, we come up with 100% actual empirical size and the probability of spurious regression is 95% on basis of 5% nominal size at sample size of 50. The value of F-stat after employing ARDL (1, 1) model is 80.4% which shows 14.6% size reduction on the basis of 5% nominal size. Hence, the probability of spurious regression after employing ARDL (1, 1) is only 75.4% at sample size of 50. The ARDL (2, 2) model is also showing that for 55.9% actual size, the probability of spurious regression is 50.9% on the basis of 5% nominal size at sample size of 50. The probability of spurious regression with OLS is 100% on all the sample size of panel 2, panel 3 and 4. That is why, we cannot show the increasing probability of spurious regression with OLS on different sample size. Similar pattern has been found at remaining sample size and in other panels of table 5.1.

There is a special effect which we should consider, in case of nonstationary time series the ARDL model works very well but it is unable to reduce the probability of spurious regression significantly in presence of trend. Basically both models OLS and ARDL are specified because the series are having linear trend but models do not have linear trend term in their equations. On the other we can see that the OLS model completely failed to tackle this problem but ARDL model works well as compare to OLS. One the other, our data generating process in equation 5.1, generates first order autoregressive series AR(1) but we used second lag in ARDL (2, 2) model, so in case of over specification, ARDL (2, 2) significantly reduced the probability of spurious regression as compare to OLS and ARDL (1, 1). It indicates that in case of over specification ARDL also works well. These results indicate that the conventional method is suffering in size distortion problem, while ARDL models significantly reduce the probability of spurious regression in both stationary and nonstationary time series and have negligible size distortion.

5.2 Robustness of Size to Misspecification

In this analysis, we evaluate the robustness of conventional cointegration procedures Engle and Granger, Johansen and Juselius and ARDL model with different specifications on the basis of size analysis. The possible three specification cases which have been considered in this analysis are, under, exact and over specified regression. The Monte Carlo simulations has been used in this analysis. All the results in table 5.2 summarized after 100,000 times simulations. The series have been generated by using data generating process in equation 5.1. In this analysis only independent nonstationary series are used with autoregressive parameter specification θ_1=1 and θ_1=1.

Specification Cases			
		Data Generating Process	
		Drift	***Drift and Trend***
Test Equation	***Drift***	Exactly Specified	Under Specified
	Drift and Trend	Over Specified	Exactly Specified

In our analysis two cases of exact specification have been considered. First, when data generating process and test equation both contain drift term second, when data generating process and test equation both contain drift and trend term. The under specification means when data generating process contains drift and trend and test equation takes on drift and trend terms. The over specification generates, when data generating process contains drift and test equation takes drift and trend terms.

In fact, Regression analysis comprises three major stages, model specification, estimation of regression parameters and interpretation of estimated parameters. Thus first and crucial stage is the specification of regression equation. The reliability of estimated parameters and interpretation mainly rely on the correct specification of model. Consequently, misspecification can generate two types of errors. First when we include theoretically irrelevant variable(s) in regression equation and second, when we exclude theoretically relevant variable from regression equation. These specification errors can generate estimation and interpretation problems. Misspecification may produce any little problem when the independent variables are uncorrelated or orthogonal to each other. When we include or omit an orthogonal independent variable from regression equation, it will affect the standard errors of partial regression coefficients. The exclusion of relevant variable has serious issues, it will lead to size and power problems. In this analysis we compare the size of conventional cointegration procedures and ARDL model and understand which one is working well in these three type of specifications.

In this study size analysis is used to estimate the probability of spurious regression after employing conventional cointegration procedures and ARDL model with different specification. For this analysis, the independent autoregressive nonstationary time series are being generated by using data generating process in equation 5.1, and structure of equations can be seen in equation 5.2. All the results in table 5.2 have been summarized after 100,000 simulation.

Table 5.2 shows the size analysis of Engle and Granger, Johansen and Juselius cointegration tests and ARDL model with different specifications. In this analysis we estimate the size distortion under different specifications cases like, under, exact and over specification. Size distortion is a difference between nominal and actual level of significance when the time series are independent. At first, we discuss the correct (exact) specification cases. The results are following:

Table 5.2 Size Analysis under Different Specifications

	Engle Granger (EG) Cointegration Test		
		Data Generating Process	
		Drift	***Drift and Trend***
Test Equation	***Drift***	21.3	25.8
	Drift and Trend	15.7	20.1
	Johansen and Juselius (JJ) Cointegration Test		
		Data Generating Process	
		Drift	***Drift and Trend***
Test Equation	***Drift***	17	20
	Drift and Trend	8	19.5
	ARDL Model		
		Data Generating Process	
		Drift	***Drift and Trend***
Test Equation	***Drift***	6.1	87.7
	Drift and Trend	7	8.4

The first panel of table 5.2 given above describes the results of Engle and Granger cointegration test. The size in cases of exact specification is 21.3% in case of only drift and 20.1% in case of drift and trend. It shows the probabilities of spurious regression in case of both exact specification cases are 16.3% and 15.1% respectively on the basis of 5% nominal level of significance. The second panel of table 5.30 shows the results of Johansen and Juselius cointegration test with different specifications. The size in cases of exact specifications is 17% with only drift and 19.5% with drift and trend. It means the probabilities of spurious regression are 12% and 14.5% with drift and with drift and trend respectively. The third panel of table 5.30 illustrates the results of ARDL model. In case of correct specification with drift and drift and trend, the size distortions (probabilities of spurious regression) are very minor 1.1% and 3.4% respectively. The size of ARDL model with correct specifications are 6.1% with drift and 8.4% with drift and trend, which is negligible. The order of statistics of spuriousness in case of correct specification is given in following equation:

$$\text{Probability of spurious regression (EG} > JJ > ARDL)$$

These statistics clearly indicates that conventional cointegration procedures have huge probabilities of spurious regression even in correct specifications and ARDL model has very minor spurious regression probability which is theoretically negligible. It explains that ARDL works very better than conventional cointegration procedures.

Secondly, we consider the case of under specification. The first panel of table 5.2 which is showing the size results of Engle and Granger cointegration test, indicates that the size is 25%. It means there is 20% probability of spurious regression. From second panel of table 5.30, the size of Johansen and Juselius cointegration test in case of under specification is 19.5% showing that probability of spurious regression is 14.5%. The third panel of table 5.30 indicates that the probability of spurious regression is 82.7% which is too high. The order of statistics of spuriousness in case of under specification is given in the following equation:

$$\text{Probability of spurious regression (ARDL} > EG > JJ)$$

Thus, these results demonstrate that conventional cointegration procedures have huge probabilities of spurious regression even in correct specifications but ARDL model has very high spurious regression probability. It means in case of under specification it works worse than other conventional techniques.

Thirdly, we take the case of over specification. The first panel of table 5.2 which is showing the size results of Engle and Granger cointegration test indicates that the size is 15.7%. It means that there is 10.7% probability of spurious regression. From second panel of table 5.30, the size of Johansen and Juselius cointegration test in case of under specification is 8%, showing that probability of spurious regression is 3%. The third panel of table 5.2 indicates that the probability of spurious regression is 2%. The order of statistics of spuriousness in case of over specification is given in the following equation:

$$\text{Probability of spurious regression (EG} > JJ > ARDL)$$

Thus, these results validate that conventional cointegration procedure Engle and Granger has large probabilities of spurious regression even in correct specifications and ARDL model and Johansen and Juselius cointegration test have very minor spurious regression probability. But ARDL has less probability of spurious regression as compare to Johansen and Juselius cointegration test. It means that in case of under specification, the ARDL model works well than other conventional techniques. After size analysis we conclude that the ARDL model works better than other conventional cointegration techniques except under specification.

5.3 Size Analysis with Stationary Series

The non-stationarity is not the only cause of spurious regression in time series. Granger et al. (2001) have shown that the possibility of spurious regression in stationary time series. In this section we estimate the probability of spurious regression in stationary series with OLS and ARDL model.

Figure 5.2: Independent Stationary Series without Drift and Trend

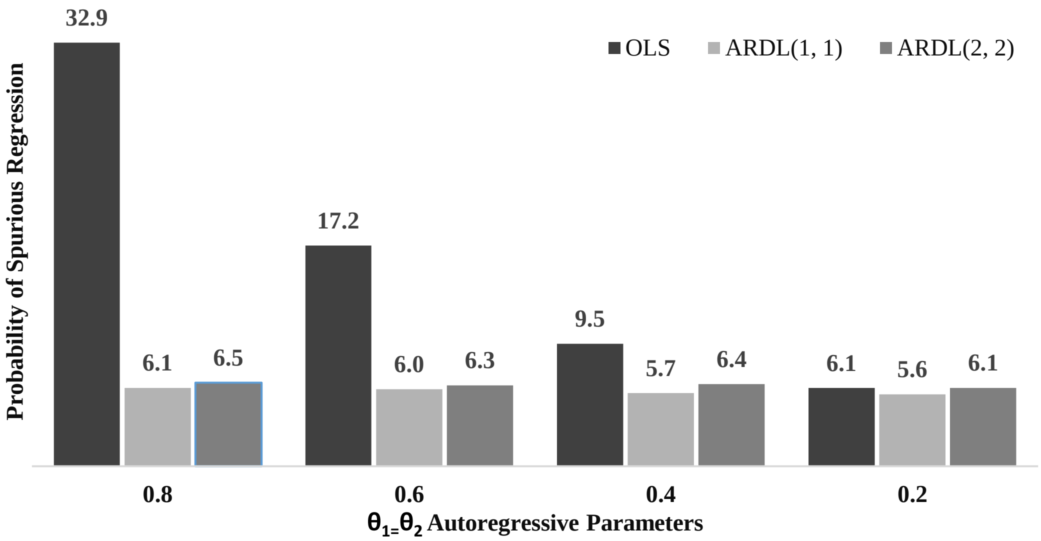


Figure 5.2 shows the probability of spurious regression on the basis of t-statistics for coefficients of independent variable by using Ordinary Least Square and Autoregressive distributed Lag models. The F test is used for joint significance of current and lag values of independent variable in ARDL models.

The figure 5.2 is based on results of first panel of table 5.3 at different values of autoregressive parameters $\theta_1 = \theta_2$. Figure 5.2 portrays the comparison of results of OLS and ARDL models. When the value of autoregressive parameters $\theta_1 = \theta_2$are 0.8, we got 37% probability of spurious regression after using OLS. While when we employed ARDL model, the probability reduced from 37% to 6.1% after ARDL (1, 1) and 6.5% after ARDL (2, 2). When the values of autoregressive parameters $\theta_1 = \theta_2$are 0.6 after using OLS, we got 17.2 probability of spurious regression but ARDL (1, 1) reduced this probability from 17.2% to just 6%, and ARDL (2, 2) reduced this probability from 17.2% to just 6.3%. It shows that the conventional method OLS can generate spurious regression even in case of stationary series and ARDL model has no size distortion. Similar, fashion has been found on other two values of autoregressive parameters $\theta_1 = \theta_2$. It means that in case of stationary series ARDL works very well as compare to conventional method. The detail results are given in following table 5.3:

Table 5.3: Probability of Spurious Regression in Stationary with OLS and ARDL Models

	OLS	ARDL (1, 1)				ARDL (2, 2)					
	Xt	xt	xt-1	yt-1	F-stat	Xt	xt-1	xt-2	yt-1	yt-2	F-stat
$\theta_1 = \theta_2$	$a_1 = b_1 = 0 = a_2 = b_2 = 0$										
0.8	32.9	6.2	6.2	99.7	6.1	7.1	6.2	7.1	99.6	7.2	6.5
0.6	17.2	6.0	6.0	95.8	6.0	7.0	6.3	6.9	94.9	6.7	6.3
0.4	11.5	5.8	5.9	95.9	5.7	6.8	6.4	6.8	68.3	6.8	6.4
0.2	9.1	5.8	5.9	21.1	5.6	6.8	6.7	6.9	22.5	6.7	6.5
	$a_1 = b_1 = 0$										
0.8	100.0	16.0	6.5	100.0	34.6	28.3	19.6	29.1	65.2	15.2	23.6
0.6	100.0	16.9	17.1	100.0	25.4	15.1	7.5	15.1	99.4	7.7	22.4
0.4	100.0	29.3	30.5	99.0	13.8	21.1	11.1	21.8	93.2	10.8	18.5
0.2	100.0	47.1	49.6	90.2	11.4	28.5	19.7	29.1	65.1	15.2	17.7
	$a_2 = b_2 = 0$										
0.8	100.0	6.2	6.6	100.0	6.1	7.0	6.1	7.6	99.9	6.5	7.1
0.6	90.0	6.0	6.2	99.4	6.1	7.0	6.2	7.0	95.6	6.4	6.8
0.4	47.8	5.9	6.0	80.8	5.9	6.7	6.4	7.1	68.3	6.8	6.4
0.2	47.9	6.0	6.0	25.2	5.6	6.7	6.7	6.8	22.3	6.9	6.4
	$a_1 = b_1 = a_2 = b_2 \neq 0$										
0.8	100.0	11.5	8.4	100.0	38.7	11.8	6.4	9.2	100.0	6.1	27.7
0.6	100.0	31.1	8.5	99.9	24.9	22.3	8.5	8.8	99.5	5.5	19.6
0.4	100.0	49.9	17.1	98.4	19.9	28.8	13.5	13.2	95.1	6.3	16.4
0.2	100.0	67.5	36.3	86.1	9.20	35.2	24.2	21.0	71.4	9.7	9.1

The series have autoregressive structure; this means the series has strong dependence on its own past. But the error terms of Series X are independent of the error terms in Y. Therefore, X should not appear in the equation of Y, and if it appears, it indicates spurious regression. The series are generated by following equation 5.1 and results are summarized after estimating equation 5.2.

In first row of first panel of table 5.3, the results are indicating that when series are stationary, the autoregressive parameters θ_1= 0.8 and θ_2= 0.8 having no drift and trend, a_1= b_1= 0 =a_2= b_2= 0, then after employing OLS we got 32.9%actual empirical size with 5% nominal size. So, the probability of spurious regression is 27.9%. On the other hand when we used ARDL (1, 1), the probability of spurious regression reduced from 27.9% to 1.1%, because the F-stat value of ARDL (1, 1) is only 6.1%. The ARDL (2, 2) model F-stat value is 6.5% which shows a huge reduction in probability of spurious regression from 27.9% to 1.5%.

In second row of first panel of table 5.3, when series are stationary θ_1= 0.6 and θ_2= 0.6 and having no drift and trend a_1= b_1= 0 =a_2= b_2= 0, then OLS shows 17.2%actual empirical size with 5% nominal size. So, the probability of spurious regression is 12.2%. On the other hand when we used ARDL (1, 1) the probability of spurious regression reduced from 12.2% to 1%. The ARDL (2, 2) model F-stat values are also showing a huge reduction in probability of spurious regression from 27.9% to 1.3%.

It shows that the conventional OLS method badly suffers in size problem even the series are stationary with no drift and trend. On contrary, ARDL model is not having size distortion problem in both cases. It indicates that when series are stationary without having drift and trend, ARDL works better than OLS and significantly reduce the probability of spurious regression. Similar pattern has been found when we changed the values of autoregressive parameters θ_1 and θ_2 from 0.6 to 0.4 and so on.

In first row of second panel of table 5.3, the results are indicating that the stationary series, θ_1=0.8 and θ_2=0.8 having no drift or having trend a_1= b_1= 0 then after running OLS, we came up with 100% actual empirical size and the probability of spurious regression is 95% on basis of 5% nominal size. The F-stat value after employing ARDL

(1, 1) model is showing only 34.6% actual size which shows only 29.6% size distortion on the basis of 5% nominal size. Hence, the probability of spurious regression after employing ARDL (1, 1) is only 29.6%. The ARDL (2, 2) model is also showing 23.6% actual size, the probability of spurious regression is 18.6% on the basis of 5% nominal size.

The second row of second panel of table 5.3, is showing that when we regressed stationary series, θ_1= 0.6 and θ_2= 0.6 with no drift a_1= b_1= 0, the OLS actual empirical size is 100%. On the basis of 5% nominal size, the probability of spurious regression is 95%. On contrary, when we used ARDL (1, 1) the actual empirical size is 25.4% which shows the probability of spurious regression is only 20.4% at 5% nominal size. When we employed ARDL (2, 2) we came with 22.4% actual empirical size and 17.4% probability of spurious regression. Same fashion has been found on other values of autoregressive parameters θ_1 and θ_2 like, 0.4 and so on.

There is a special effect which we should consider, In case of stationary time series the ARDL model works very well but it becomes unable to reduce the probability of spurious regression significantly in presence of trend. Basically both models OLS and ARDL are under specification because the series are having linear trend but models do not have linear trend term in their equations. On the other hand, we can see that the OLS model completely failed to tackle this problem but ARDL model works well as compare to OLS. One thing which is also very important, our data generating process in equation 5.1, generates first order autoregressive series AR(1) but we used second lag in ARDL (2, 2) model, so in case of over specification ARDL (2, 2) significantly reduced the probability of spurious regression as compare to OLS and ARDL (1, 1). It indicates that in case of over specification ARDL works well.

Similar fashion has been found in other panels of table 5.3. However, we can see that as we decrease the value of autoregressive parameters θ_1 and θ_2 like, 0.8, 0.6, and so on, there is decrease in lag dependency which reduces the probability of spurious regression. But when the time series contains linear trend, conventional method completely fails to reduce probability of spurious regression at all values of autoregressive correlation.

These results indicate that the conventional method is suffering in size distortion problem, while ARDL models significantly reduce the probability of spurious regression in stationary time series and have negligible size distortion.

CHAPTER 6

POWER ANALYSIS

The main objective of this study is to evaluate the performance of ARDL model to differentiate between genuine and spurious regression. If ARDL is actually becomes able to differentiate, then it would be the feature of ARDL.

Suppose we have set of dependent time series with mutual relationship and we test the relationship using ARDL model, it should be able to find that there is a relationship. More technically suppose X_t and Y_t are generated in such a way that either X_t is used in construction of Y_t or vice versa, and we estimate $X_t = \alpha_1 X_{t-1} + \beta_0 Y_t + \beta_1 Y_{t-1} + \varepsilon_t$ and we test the hypotheses$H_0(1): \beta_0 = 0, H_0(2): \beta_1 = 0$ and $H_0(3): (\beta_0, \beta_1) = 0$. Since all three hypotheses are not true, so they should be rejected.

Power analysis is executed to evaluate the probability of rejection the null hypothesis, when the alternative hypothesis is true. As the statistical power of test increases, the probability of type II error is decreased. It can be expressed in following way:

$$\text{Power} = \text{Prob (reject } H_0 | \text{ when} H_1 \text{ is true)}$$

In this study, we use power analysis to quantify the power of conventional method i.e., OLS and ARDL model with different specifications in different scenarios. The Monte Carlo simulations have been used in this analysis. All the results in the tables given below have been summarized after 100,000 times simulations.

Suppose, two dependent series x and y are being generated by using given data in equation 6.1, after that we regress y on x, if the regression results are significant, it reflects the power of regression model. All the results in tables from 6.1 to 6.28 composed after 100,000 simulation.

The data generating process is given in equation 6.1:

$$\begin{bmatrix} x_t \\ y_t \end{bmatrix} = \begin{bmatrix} \theta_1 & \theta_{12} \\ \theta_{21} & \theta_2 \end{bmatrix} \begin{bmatrix} x_{t-1} \\ y_{t-1} \end{bmatrix} + \begin{bmatrix} a_1 & a_2 \\ b_1 & b_2 \end{bmatrix} \begin{bmatrix} 1 \\ t \end{bmatrix} + \begin{bmatrix} \varepsilon_{xt} \\ \varepsilon_{yt} \end{bmatrix} \begin{bmatrix} \varepsilon_{xt} \\ \varepsilon_{yt} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix} \right) \quad (6.1)$$

We can rewrite it as for simplification of notation

$$X_t = AX_{t-1} + Bd + \varepsilon_t \varepsilon_t \sim N(0, \Sigma) \quad (6.2)$$

Three types of dependent series are being generated by using data generating process in equation 6.1.

i. Lag dependence between series,
ii. Contemporaneous and lag dependence between series,
iii. Contemporaneous dependence between series.

This data generating process can generate two dependent autoregressive stationary and non-stationary series.

The lag dependent series can be generated, if the cross correlation parameters found to be $\theta_{21} \neq 0$ and $\theta_{12} \neq$ and $\rho = 0$. But in our analysis we are regressing y on x, that is why for the problem of singular matrix we put θ_{12}= 0. It means y is a function of its own and lag value of x. The $\rho = 0$ implies that the error terms added to each series have no contemporaneous correlation. Therefore x and y are serially dependent but have no contemporaneous dependence on each other.

The contemporaneous and lag dependent series can be generated, if the cross correlation parameters become $\theta_{21} \neq 0$ and $\theta_{12} \neq 0$ and $\rho \neq 0$. But in our analysis, we are regressing y on x and that is why for the problem of singular matrix, we put θ_{12}= 0. It means y is a function of its own and lag value of x. The $\rho \neq 0$ implies that the error terms added to each series have contemporaneous correlation. Therefore x and y are serially and contemporaneously dependent on each other.

The contemporaneous dependent series can be generated, if the cross correlation parameters become $\theta_{21} = 0$ and $\theta_{12} = 0$ and $\rho \neq 0$. The $\rho \neq 0$ implies that the error terms added to each series have contemporaneous correlation. Therefore x and y are not serially but contemporaneously dependent on each other.

6.1 Power Analysis of Lag Dependent Series

The lag dependent series can be generated, if the cross correlation parameters are found to be $\theta_{21} \neq 0$ and $\theta_{12} \neq$ and $\rho = 0$. But in our analysis, we are regressing x on y and that is why for the problem of singular matrix we put θ_{12}= 0. After this restriction the structure of equations are given as follows:

$$x_t = a + \theta_1 x_{t-1} + \varepsilon_{xt} \tag{6.3}$$

$$y_t = a + \theta_2 y_{t-1} + \theta_{21} x_{t-1} + \varepsilon_{yt} \tag{6.4}$$

The series are non-stationary, if the own lag value parameters are θ_1= 1 and θ_2= 0.8, and the series are stationary, if the own lag value parameters are θ_1< 1 and θ_2< 1. When the matrix B = 0, then it means series are without drift and trend. If a_1= b_1= 0 in matrix B, then the series are without drift or with linear trend. When a_2= b_2= 0 in matrix B then the series are with drift or without linear trend. When a_1= b_1= a_2= b_2= 1 in matrix B, then the series are with drift and trend terms. When the $\rho = 0$, it means the error of x series has no relation with error of y series and when $\rho \neq 0$, it indicates that the error of x series correlated with error of y series which shows that there is contemporaneous dependence between the series. In this experiment, we used dependent series with different scenarios and with different values of parameters which are given in the following tables:

Table 6.1: Power Analysis of Lag Dependent Series without drift and trend3

	OLS	ARDL (1, 1)				ARDL (2, 2)					
	xt	**xt**	**xt-1**	**yt-1**	**F-stat**	**xt**	**xt-1**	**xt-2**	**yt-1**	**yt-2**	**F-stat**
θ_1=θ_2						θ_{21}= **0.8**					
1	89.9	95.9	6.7	100.0	93.9	7.0	96.3	7.3	100.0	7.8	87.6
0.8	82.1	91.5	6.0	100.0	85.6	6.9	98.2	7.6	100.0	6.6	90.4
0.6	67.5	48.8	6.0	100.0	56.4	6.9	99.1	7.2	96.2	6.4	96.3
0.4	37.3	18.4	5.9	92.6	19.9	6.7	99.6	7.0	68.4	6.5	90.5
0.2	12.4	8.4	5.9	36.4	11.3	6.9	99.8	6.7	22.0	6.7	90.3
						θ_{21}= **0.6**					
1	83.2	86.5	6.2	100.0	84.3	6.8	83.3	7.1	100.0	7.6	70.4
0.8	77.2	68.0	6.1	100.0	60.2	6.9	88.9	7.7	99.9	6.5	82.7
0.6	58.5	21.0	5.9	99.7	20.7	6.7	92.8	7.3	95.9	6.5	88.6
0.4	29.9	9.4	6.0	86.3	10.6	6.8	95.1	6.9	68.3	6.7	87.4
0.2	10.8	6.6	5.8	29.6	6.8	6.6	96.6	6.8	22.2	6.6	86.4
						θ_{21}= **0.4**					
1	81.8	67.1	6.1	100.0	74.6	6.8	52.4	7.4	100.0	7.6	67.3
0.8	68.1	21.8	6.1	100.0	30.6	7.0	59.7	7.8	99.9	6.4	53.4
0.6	44.8	8.5	6.0	98.9	21.4	7.0	66.1	7.1	95.6	6.7	60.6
0.4	21.3	6.7	5.9	78.1	11.9	6.9	71.7	6.8	68.1	6.7	63.5
0.2	8.8	6.1	6.0	25.1	7.2	6.8	75.0	6.8	22.2	6.7	63.9
						θ_{21}= **0.2**					
1	80.5	19.8	5.2	100.0	27.6	7.1	18.5	7.4	100.0	7.4	12.4
0.8	50.0	7.0	6.2	99.9	9.5	7.0	21.1	7.3	99.7	6.6	23.7
0.6	27.8	6.3	5.9	97.0	7.5	6.8	24.4	6.9	95.2	6.7	23.8
0.4	13.5	6.3	5.9	71.7	6.4	6.8	27.0	7.0	67.8	6.8	25.0
0.2	8.8	6.1	5.8	22.4	6.1	6.8	29.0	6.8	22.3	6.8	25.9

Table 6.2: Power Analysis of Lag Dependent Series without drift

	OLS	ARDL (1, 1)			ARDL (2, 2)				
	xt	xt	xt-1	yt-1	xt	xt-1	xt-2	yt-1	yt-2
$\theta_1 = \theta_2$					θ_{21} = **0.8**				
1	100.0	90.9	31.0	100.0	48.1	93.0	65.0	100.0	24.6
0.8	100.0	86.8	5.6	100.0	18.1	97.8	7.4	100.0	9.6
0.6	100.0	82.7	5.7	100.0	13.5	99.2	6.8	99.4	7.1
0.4	100.0	33.1	23.3	100.0	15.1	99.7	6.6	89.6	16.8
0.2	100.0	8.8	53.5	99.5	20.1	99.9	6.9	53.8	27.7
					θ_{21} = **0.6**				
1	100.0	84.1	26.2	100.0	48.4	75.9	63.8	100.0	24.8
0.8	100.0	82.1	27.4	100.0	18.2	88.6	8.1	100.0	10.1
0.6	100.0	42.9	6.7	100.0	13.2	93.8	6.9	99.4	7.5
0.4	100.0	10.7	27.6	100.0	15.5	97.1	7.3	90.5	17.7
0.2	100.0	6.6	56.8	99.0	21.2	99.0	8.2	55.0	27.1
					θ_{21} = **0.4**				
1	100.0	76.1	15.1	100.0	48.8	45.5	59.3	100.0	24.8
0.8	100.00	67.72	28.82	100.00	19.16	62.36	8.52	99.99	11.14
0.6	100.00	10.32	9.61	100.00	12.92	71.86	7.50	99.44	8.62
0.4	100.00	6.47	31.68	99.97	16.02	81.86	8.51	91.50	17.40
0.2	100.00	15.86	58.24	97.83	22.56	90.95	10.91	57.72	24.92
					θ_{21} = **0.2**				
1	100.0	21.9	10.2	100.0	48.4	17.1	49.7	100.0	25.2
0.8	100.0	9.4	26.3	100.0	20.2	26.1	8.6	100.0	13.2
0.6	100.0	7.7	14.2	100.0	13.6	31.5	9.4	99.5	8.9
0.4	100.0	17.6	33.7	99.8	18.1	43.5	12.9	92.5	15.1
0.2	100.0	34.3	56.4	95.4	24.9	60.7	17.1	61.2	20.8

Table 6.3: Power Analysis of Lag Dependent Series with drift

	OLS	ARDL (1, 1)			ARDL (2, 2)				
	xt	Xt	xt-1	yt-1	xt	xt-1	xt-2	y t-1	yt-2
$\theta_1=\theta_2$					$\theta_{21}=$ **0.8**				
1	92.2	94.3	7.7	100.0	6.8	96.2	7.3	100.0	7.8
0.8	96.7	96.7	6.0	100.0	6.8	98.1	7.7	100.0	7.1
0.6	82.0	53.2	5.9	100.0	7.0	99.1	7.4	96.5	0.0
0.4	46.3	16.7	5.8	94.8	6.9	99.6	7.0	68.0	6.6
0.2	14.8	7.1	5.9	37.9	6.8	99.8	6.8	22.2	6.8
					$\theta_{21}=$ **0.6**				
1	88.8	89.0	6.9	100.0	6.9	83.2	7.2	100.0	7.7
0.8	95.7	76.3	6.0	100.0	6.9	88.6	7.6	100.0	6.8
0.6	74.6	20.7	5.9	100.0	6.8	92.8	7.3	96.3	6.4
0.4	38.2	8.0	6.0	89.0	6.9	95.3	7.0	68.4	6.5
0.2	12.7	5.8	5.9	30.9	7.0	96.5	6.9	22.0	6.9
					$\theta_{21}=$ **0.4**				
1	88.1	57.6	4.6	100.0	6.9	52.5	7.0	100.0	7.6
0.8	93.0	28.8	6.1	100.0	6.7	59.6	7.5	100.0	6.7
0.6	61.9	6.7	6.0	99.6	6.7	66.6	7.2	95.8	6.5
0.4	27.4	5.5	5.9	81.6	6.8	71.7	6.9	68.4	6.7
0.2	9.9	5.7	5.7	26.0	6.6	75.2	6.8	22.5	6.7
					$\theta_{21}=$ **0.2**				
1	83.2	11.9	2.7	100.0	6.9	18.6	7.0	100.0	7.6
0.8	84.4	5.7	6.0	100.0	15.8	25.3	9.5	100.0	9.4
0.6	39.6	5.8	5.9	98.4	12.5	30.7	9.3	99.5	10.4
0.4	16.8	5.7	5.8	74.1	18.7	44.0	11.5	92.9	14.1
0.2	7.7	5.8	5.9	22.7	25.8	61.5	15.8	62.7	19.4

Table 6.4: Power Analysis of Lag Dependent Series with drift and trend

	OLS	ARDL (1, 1)			ARDL (2, 2)				
	xt	Xt	xt-1	yt-1	Xt	xt-1	xt-2	yt-1	yt-2
$\theta_1 = \theta_2$					θ_{21} = **0.8**				
1	97.2	95.9	7.1	100.0	44.0	93.1	57.6	100.0	22.0
0.8	100.0	99.9	17.9	100.0	15.0	97.8	8.0	100.0	8.0
0.6	100.0	84.2	7.6	100.0	11.2	99.2	6.8	99.2	9.3
0.4	100.0	32.6	23.3	100.0	14.9	99.8	6.7	90.0	17.1
0.2	100.0	7.8	51.6	99.5	21.2	99.9	6.6	55.6	25.8
					θ_{21} = **0.6**				
1	97.2	81.5	6.9	100.0	44.4	76.3	56.7	100.0	22.3
0.8	100.0	97.6	17.9	100.0	15.2	88.5	8.4	100.0	8.6
0.6	100.0	45.3	9.9	100.0	11.2	93.8	7.0	99.4	10.1
0.4	100.0	10.4	27.9	100.0	15.5	97.1	6.8	91.0	17.5
0.2	100.0	6.9	54.7	98.9	22.0	99.1	7.5	56.9	25.2
					θ_{21} = **0.4**				
1	96.1	67.1	6.1	100.0	44.6	45.6	52.4	100.0	22.3
0.8	100.0	73.7	17.6	100.0	15.2	62.5	9.3	100.0	8.8
0.6	100.0	11.3	13.3	100.0	11.4	70.6	7.5	99.5	11.0
0.4	100.0	6.8	31.2	100.0	16.7	81.9	8.2	92.0	16.8
0.2	100.0	17.5	56.2	97.7	23.6	91.4	10.1	59.4	23.0
					θ_{21} = **0.2**				
1	90.2	19.8	4.6	100.0	44.8	16.8	43.5	100.0	22.4
0.8	100.0	13.0	13.6	100.0	15.8	25.3	9.5	100.0	9.4
0.6	100.0	7.6	17.2	100.0	12.5	30.7	9.3	99.5	10.4
0.4	100.0	19.0	32.4	99.8	18.7	44.0	11.5	92.9	14.1
0.2	100.0	37.6	54.1	95.1	25.8	61.5	15.8	62.7	19.4

In first row of first panel of table 6.1, the results are indicating that when series are nonstationary, θ_1=1 and θ_2=0.8, having no drift and trend, a_1= b_1= 0 =a_2= b_2= 0 and θ_{21}=0.8, then the OLS power is 89.9%, which shows 5.1% power loss on the basis of 5% nominal size. In case of ARDL models, F-test are being used to test the joint significance of current and lag values of independent variable. The F-stat value after employing ARDL (1, 1) model is indicating that the power is ARDL (1, 1) model is 93.9% which shows 1.1% power loss at 5% nominal size. The power of ARDL (2, 2) is 87.6% and loss of power is 7.4 at 5% nominal size.

In second row of first panel of table 6.1, the results are representing that when series are stationary, θ_1= 0.8 and θ_2 = 0.8, having no drift and trend, a_1= b_1= 0 =a_2= b_2= 0 and θ_{21}=0.8, then the value of OLS power is 82.1%, which shows 12.9% power loss on the basis of 5% nominal size. In case of ARDL models F-test are being used to test the joint significance of current and lag values of independent variables. The F-stat value after employing ARDL (1, 1) model is indicating that the power ofARDL (1, 1) model is 85.6% which shows 14.4% power loss at 5% nominal size. The power of ARDL (2, 2) is 90.4% and loss of power is 4.6 at 5% nominal size.

In first row of second panel of table 6.1, the results are indicating that when series are nonstationary, θ_1=1 and θ_2=0.8, having no drift and trend, a_1= b_1= 0 =a_2= b_2= 0 and θ_{21}=0.6, then the OLS power is 83.2%, which shows 11.8% power loss on the basis of 5% nominal size. In case of ARDL models F-test are being used to test the joint significance of current and lag values of independent variables. The F-stat value after employing ARDL (1, 1) model is indicates that the power of ARDL (1, 1) model is 84.3% which shows 10.7% power loss at 5% nominal size. The power of ARDL (2, 2) is 70.4% and loss of power is 24.6 at 5% nominal size.

In second row of second panel of table 6.1, the results are representing that when there is stationary series, θ_1= 0.8 and θ_2 = 0.8, having no drift and trend, a_1= b_1= 0 =a_2= b_2= 0 and θ_{21}=0.6, then the OLS power is 77.2%, which shows 17.8% power loss on the basis of 5% nominal size. In case of ARDL models F-test are being used to test the joint significance of current and lag values of independent variables. The F-stat value after employing ARDL (1, 1) model is indicating that the power ofARDL (1, 1) model is 60.2% which shows 34.8% power loss at 5% nominal size. The power of ARDL (2, 2) is 82.7% and loss of power is 12.3% at 5% nominal size.

It shows that the conventional OLS method badly suffer in power problem when series are nonstationary and even they are stationary with no drift and trend. On contrary, ARDL model is not showing huge power in both cases. It clarifies that when series are stationary or nonstationary without having drift and trend ARDL works better than OLS.

In first row of first panel of table 6.2, the results show that when we regressed nonstationary series θ_1=1 and θ_2=0.8, without drift, a_1= b_1= 0 and θ_{21} = 0.8 by using OLS, the probability of rejection false null hypothesis (power) is 100%, which represents a misleading figure. As in table 5.1, the OLS has huge size distortion problem, specially, when series are with linear trend. That is why, it is showing 100% power which does not exist in reality. The F-test is used only in one case for displaying the joint significance of independent lag and current value. So, table 6.2, 6.3 and 6.4 have only t-stat values. After employing ARDL (1, 1) model, the power of current value of X is 90.9%. It means that there is only 4.1% power loss. The reason behind it might be we did not include linear trend in ARDL. If we include linear trend, it may provide more power. The figure of lag value of X is showing only 31% power which means 59% power loss. In ARDL (2, 2) model, the first lag value X showing 93.0% probability of rejecting the false null hypothesis. The powers of current and second lag values of X are 48.1% and 65%, and show 46.9% and 30% power loss respectively

As we know that Y value is determined through lag value of X, but the current value are more significant as compare to lag value of X. The reason is that there is multicollinearity effect. The current and lag values of X variable are collinear that is why the effect shifts into current value in ARDL(1, 1) and in first lag value of ARDL (2, 2).

In second row of first panel of table 6.2, the results illustrate that when we regressed stationary series θ_1=0.8 and θ_2=0.8, without drift, a_1= b_1= 0 and θ_{21} = 0.8 by using OLS, the probability of rejecting false null hypothesis (power) is 100%, which represents a misleading figure. As we observe in table 5.1, the OLS has huge size distortion problem, specially, when series are with linear trend. That is why, it is showing 100% power which does not exist in reality. After employing ARDL (1, 1) model, the power of current value of X is 86.8% which means only 8.6% power loss. The reason behind it is that we did not include linear trend in ARDL. If we include linear trend, it may provide more power. The figure of lag value of X is showing only 5.6% power which means 89.4% power loss. In ARDL (2, 2) model, the first lag value is X showing 97.0% probability of rejection the false null hypothesis. The powers of current and second lag values of X are 18.1% and 7.4%, and show 76.9% and 87.6% power loss respectively

Same pattern has been found on other values of θ_{21} like, 0.6 and so on. On all these point ARDL shows better performance as compare to OLS.

Though in some cases the values of parameter*s* θ_1 and θ_2, and θ_{21} and OLS shows more power, yet we cannot consider it because as we have seen in size analysis the OLS suffers badly in size problem and ARDL in all cases has less size problem. That is why, we cannot say that OLS has more power. In case without drift or with linear trend, and with drift and trend due to under specification, ARDL shows size problem but even in these cases OLS has more size distortion as compare to ARDL model.

There is another special effect which we should consider. In case of stationary and nonstationary time series the ARDL model works very well but it becomes unable to reduce the probability of spurious regression significantly in presence of trend. Basically both models OLS and ARDL are under specified because the series are having linear trend but models do not have linear trend term in their equations. On the other hand, we can see that the OLS model completely failed to tackle this problem but ARDL model works well as compare to OLS in size analysis. One thing which is also very important; our data generating process in equation 5.1, generates first order autoregressive series AR (1) but we used second lag in ARDL (2, 2) model. Thus in case of over specification, ARDL (2, 2) shows more power in case of stationary series as compare to ARDL (1, 1) and OLS. It also explores that the ARDL models performs better as compare to OLS in under specification and over specification.

Similarly results in tables 6.3 and 6.4 also show that two scenarios of lag dependent series with drift and with drift and trend depict the same fashion. So, the interpretations of these cases are approximately alike. The own lag values of Y are highly significant in all cases, but one thing which is necessary to consider is that as we reduce the value of autoregressive terms, the significance of lag values also decreases in case of ARDL and not in OLS.

6.2 Contemporaneous and Lag Dependence Between Series

The contemporaneous and lag dependent series are generated from equation 6.1, if the cross correlation parameters are $\theta_{21} \neq 0$ and $\theta_{12} \neq 0$ and $\rho \neq 0$. But in our analysis we are regressing y on x and that is why for the problem of singular matrix we put $\theta_{12}= 0$, see equation 6.3 and 6.4 in this regard. It means y is a function of its own and lag value of x. The $\rho \neq 0$ implies that the error terms added to each series have contemporaneous correlation. Therefore x and y are serially and contemporaneously dependent onto each other. In this experiment we summarized the results of serially and contemporaneously dependent series with different scenarios by using equation 6.1. The results are given in the following equation :

Table 1.5: Power Analysis of Contemporaneous and Lag Dependent Series without drift and trend with ρ = 1

	OLS	ARDL (1, 1)				ARDL (2, 2)					
	xt	xt	xt-1	yt-1	F-stat	xt	xt-1	xt-2	yt-1	yt-2	F-stat
						ρ = 1					
$\theta_1 = \theta_2$						θ_{21} = 0.8					
1	71.9	99.6	6.0	100.0	95.3	6.9	96.2	7.4	100.0	7.8	90.2
0.8	82.2	91.5	6.0	100.0	87.9	6.9	98.2	7.6	100.0	6.6	92.7
0.6	67.5	48.8	6.0	100.0	47.1	6.9	99.1	7.2	96.2	6.4	94.5
0.4	37.6	18.4	5.9	92.6	15.2	6.7	99.6	7.0	68.4	6.5	94.7
0.2	12.6	8.4	5.9	36.4	7.8	6.9	99.8	6.7	22.0	6.7	94.8
						θ_{21} = 0.6					
1	72.0	95.3	5.9	100.0	93.1	6.9	83.1	7.4	100.0	7.6	79.1
0.8	77.4	62.0	6.1	100.0	60.5	6.9	88.9	7.7	99.9	6.5	83.9
0.6	58.7	21.0	5.9	99.7	19.1	6.7	92.8	7.3	95.9	6.5	84.6
0.4	30.1	9.4	6.0	86.3	8.7	6.8	95.1	6.9	68.3	6.7	89.8
0.2	10.5	6.6	5.8	29.6	6.1	6.6	96.6	6.8	22.2	6.6	93.2
						θ_{21} = 0.4					
1	72.7	22.4	6.1	100.0	21.9	7.1	18.5	7.4	100.0	7.3	15.2
0.8	68.2	21.8	6.1	100.0	17.7	7.0	59.7	7.8	99.9	6.4	47.6
0.6	44.8	8.5	6.0	98.9	7.2	7.0	66.1	7.1	95.6	6.7	62.1
0.4	21.3	6.7	5.9	78.1	6.1	6.9	71.7	6.8	68.1	6.7	67.4
0.2	9.0	6.1	6.0	25.1	6.1	6.8	75.0	6.8	22.2	6.7	68.8
						θ_{21} = 0.2					
1	73.0	22.7	6.1	100.0	21.8	7.0	18.5	7.5	100.0	7.2	15.1
0.8	50.1	7.0	6.2	99.9	6.8	7.0	21.1	7.3	99.7	6.6	18.5
0.6	27.7	6.3	5.9	97.0	6.2	6.8	24.4	6.9	95.2	6.7	18.7
0.4	13.6	6.3	5.9	71.7	6.2	6.8	27.0	7.0	67.8	6.8	21.0
0.2	7.3	6.1	5.8	22.4	6.1	6.8	29.0	6.8	22.3	6.8	23.4

Table 6.6: Power Analysis of Contemporaneous and Lag Dependent Series without drift with ρ = 1

	OLS	ARDL (1, 1)			ARDL (2, 2)				
	xt	**xt**	**xt-1**	**yt-1**	**xt**	**xt-1**	**xt-2**	**yt-1**	**yt-2**
					ρ = 1				
$\theta_1 = \theta_2$					θ_{21} = **0.8**				
1	100.0	76.2	96.8	100.0	49.1	93.0	65.6	100.0	24.6
0.8	100.0	82.8	5.6	100.0	19.4	96.1	9.5	100.0	9.6
0.6	100.0	82.7	5.7	100.0	13.5	99.2	6.8	99.4	7.1
0.4	100.0	33.1	23.3	100.0	15.1	99.7	6.6	89.6	16.8
0.2	100.0	8.8	53.5	99.5	20.1	99.9	6.9	53.8	27.7
					θ_{21} = **0.6**				
1	100.0	54.1	96.4	100.0	49.2	76.1	64.2	100.0	24.7
0.8	100.0	96.7	27.4	100.0	19.7	87.4	9.3	100.0	10.1
0.6	100.0	42.9	6.7	100.0	13.2	93.8	6.9	99.4	7.5
0.4	100.0	10.7	27.6	100.0	15.5	97.1	7.3	90.5	17.7
0.2	100.0	6.6	56.8	99.0	21.2	99.0	8.2	55.0	27.1
					θ_{21} = **0.4**				
1	100.0	40.3	97.1	100.0	48.6	17.1	49.9	100.0	24.9
0.8	100.0	67.7	28.8	100.0	19.2	62.4	8.5	100.0	11.1
0.6	100.0	10.3	9.6	100.0	12.9	71.9	7.5	99.4	8.6
0.4	100.0	6.5	31.7	100.0	16.0	81.9	8.5	91.5	17.4
0.2	100.0	15.9	58.2	97.8	22.6	91.0	10.9	57.7	24.9
					θ_{21} = **0.2**				
1	100.0	39.9	97.2	100.0	48.9	16.9	49.6	100.0	24.9
0.8	100.0	9.4	26.3	100.0	20.2	26.1	8.6	100.0	13.2
0.6	100.0	7.7	14.2	100.0	13.6	31.5	9.4	99.5	8.9
0.4	100.0	17.6	33.7	99.8	18.1	43.5	12.9	92.5	15.1
0.2	100.0	34.3	56.4	95.4	24.9	60.7	17.1	61.2	20.8

Table 6.7: Power Analysis of Contemporaneous and Lag Dependent Series with drift with ρ = 1

WD	OLS	ARDL (1, 1)			ARDL (2, 2)				
	xt	xt	xt-1	yt-1	xt	xt-1	xt-2	yt-1	yt-2
					ρ = 1				
$\theta_1 = \theta_2$					θ_{21} = **0.8**				
1	100.0	99.8	5.8	100.0	6.9	96.3	7.4	100.0	7.7
0.8	96.7	96.7	6.0	100.0	6.8	98.1	7.7	100.0	7.1
0.6	82.0	53.2	5.9	100.0	7.0	99.1	7.4	96.5	6.6
0.4	46.4	16.7	5.8	94.8	6.9	99.6	7.0	68.0	6.6
0.2	14.9	7.1	5.9	37.9	6.8	99.8	6.8	22.2	6.8
					θ_{21} = **0.6**				
1	100.0	96.6	5.9	100.0	6.8	83.4	7.2	100.0	7.7
0.8	95.8	76.3	6.0	100.0	6.9	88.6	7.6	100.0	6.8
0.6	74.7	20.7	5.9	100.0	6.8	92.8	7.3	96.3	6.4
0.4	38.1	8.0	6.0	89.0	6.9	95.3	7.0	68.4	6.5
0.2	12.5	5.8	5.9	30.9	7.0	96.5	6.9	22.0	6.9
					θ_{21} = **0.4**				
1	100.0	26.1	5.8	100.0	6.8	18.7	7.0	100.0	7.7
0.8	93.2	28.8	6.1	100.0	6.7	59.6	7.5	100.0	6.7
0.6	61.8	6.7	6.0	99.6	6.7	66.6	7.2	95.8	6.5
0.4	27.6	5.5	5.9	81.6	6.8	71.7	6.9	68.4	6.7
0.2	10.0	5.7	5.7	26.0	6.6	75.2	6.8	22.5	6.7
					θ_{21} = **0.2**				
1	100.0	26.0	5.9	100.0	7.0	18.7	7.1	100.0	7.7
0.8	84.3	5.7	6.0	100.0	6.9	21.6	7.4	99.9	6.3
0.6	39.4	5.8	5.9	98.4	6.9	24.3	7.2	95.4	6.7
0.4	16.7	5.7	5.8	74.1	6.9	26.9	6.7	68.1	6.8
0.2	7.8	5.8	5.9	22.7	6.9	29.2	7.0	22.4	6.8

Table 6.8: Power Analysis of Contemporaneous and Lag Dependent Series with drift and trend with ρ = 1

WDT	OLS	ARDL (1, 1)			ARDL (2, 2)				
	xt	xt	xt-1	yt-1	xt	xt-1	xt-2	yt-1	yt-2
					ρ = 1				
$\theta_1 = \theta_2$					θ_{21} = **0.8**				
1	100.0	82.8	92.2	100.0	44.1	92.9	57.2	100.0	21.9
0.8	100.0	99.9	17.9	100.0	15.0	97.8	8.0	100.0	8.0
0.6	100.0	84.2	7.6	100.0	11.2	99.2	6.8	99.2	9.3
0.4	100.0	32.6	23.3	100.0	14.9	99.8	6.7	90.0	17.1
0.2	100.0	7.8	51.6	99.5	21.2	99.9	6.6	55.6	25.8
					θ_{21} = **0.6**				
1	100.0	34.1	92.2	100.0	44.3	76.1	56.8	100.0	22.1
0.8	100.0	97.6	17.9	100.0	15.2	88.5	8.4	100.0	8.6
0.6	100.0	45.3	9.9	100.0	11.2	93.8	7.0	99.4	10.1
0.4	100.0	10.4	27.9	100.0	15.5	97.1	6.8	91.0	17.5
0.2	100.0	6.9	54.7	98.9	22.0	99.1	7.5	56.9	25.2
					θ_{21} = **0.4**				
1	100.0	27.6	93.1	100.0	44.8	16.8	43.6	100.0	22.6
0.8	100.0	73.7	17.6	100.0	15.2	62.5	9.3	100.0	8.8
0.6	100.0	11.3	13.3	100.0	11.4	70.6	7.5	99.5	11.0
0.4	100.0	6.8	31.2	100.0	16.7	81.9	8.2	92.0	16.8
0.2	100.0	17.5	56.2	97.7	23.6	91.4	10.1	59.4	23.0
					θ_{21} = **0.2**				
1	100.0	27.8	93.1	100.0	44.9	17.0	43.6	100.0	22.3
0.8	100.0	13.0	13.6	100.0	15.8	25.3	9.5	100.0	9.4
0.6	100.0	7.6	17.2	100.0	12.5	30.7	9.3	99.5	10.4
0.4	100.0	19.0	32.4	99.8	18.7	44.0	11.5	92.9	14.1
0.2	100.0	37.6	54.1	95.1	25.8	61.5	15.8	62.7	19.4

In first row of first panel of table 6.5, the results are indicating that when series are nonstationary, θ_1=1 and θ_2=0.8, having no drift and trend, a_1= b_1= 0 =a_2= b_2= 0, θ_{21}=0.8 and covariance ρ=1, then the OLS power is 71.9%, which shows 23.1% power loss on the basis of 5% nominal size. In case of ARDL models F-test are being used to test the joint significance of current and lag values of independent variable. The F-stat value after employing ARDL (1, 1) model is indicating that the power of ARDL (1, 1) model is 95.3% which shows 0.0% power loss at 5% nominal size. The power of ARDL (2, 2) is 90.2% and loss of power is 4.8 at 5% nominal size.

In second row of first panel of table 6.5, the results are representing that when series are stationary, θ_1= 0.8 and θ_2 = 0.8, having no drift and trend, a_1= b_1= 0 =a_2= b_2= 0, θ_{21}=0.8 and ρ= 1, then the OLS power is 82.2%, which shows 12.8% power loss on the basis of 5% nominal size. In case of ARDL models, F-test is being used to test the joint significance of current and lag values of independent variable. The F-stat value after employing ARDL (1, 1) model is indicating that the power of ARDL (1, 1) model is 87.9% which shows 6.9% power loss at 5% nominal size. The power of ARDL (2, 2) is 92.7% and loss of power is 2.3% at 5% nominal size.

In first row of second panel of table 6.5, the results are indicating that when series are nonstationary, θ_1=1 and θ_2=0.8, having no drift and trend, a_1= b_1= 0 =a_2= b_2= 0, θ_{21}=0.6 and ρ= 1, then the OLS power is 72.2%, which shows 22.8% power loss on the basis of 5% nominal size. In case of ARDL models F-test are being used to test the joint significance of current and lag values of independent variable. The F-stat value after employing ARDL (1, 1) model is indicating that the power ofARDL (1, 1) model is 93.1% which shows 1.9% power loss at 5% nominal size. The power of ARDL (2, 2) is 79.1% and loss of power is 15.9 at 5% nominal size.

In second row of second panel of table 6.5, the results are representing that when series are stationary, θ_1= 0.8 and θ_2 = 0.8, having no drift and trend, a_1= b_1= 0 =a_2= b_2= 0, θ_{21}=0.6 and ρ= 1, then the OLS power is 77.4%, which shows 17.6% power loss on the basis of 5% nominal size. In case of ARDL models F-test are being used to test the joint significance of current and lag values of independent variable. The F-stat value after employing ARDL (1, 1) model is indicating that the power of ARDL (1, 1) model is 60.5% which shows 34.5% power loss at 5% nominal size. The power of ARDL (2, 2) is 83.9% and loss of power is 11.1% at 5% nominal size.

It shows that the conventional OLS method badly suffers in power problem when series are nonstationary even they are stationary with no drift and trend. On contrary, ARDL model is not showing huge power in both cases. It clarifies that when series are stationary or nonstationary without having drift and trend, ARDL works better than OLS.

In first row of first panel of table 6.6, the results show that when we regressed nonstationary series θ_1=1 and θ_2=0.8, without drift, a_1= b_1= 0, θ_{21} = 0.8 and ρ= 1 by using OLS, the probability of rejection of false null hypothesis (power) is 100%, which represents a misleading figure. Because as seen above in table 5.1, the OLS has huge size distortion problem, specially, when series are with linear trend. That is why, it is showing 100% power which does not exist in reality. The F-test used only in one case for displaying the joint significance of independent lag and current value. So, table 6.6, 5.8 and 5.9 have only t-stat values. After employing ARDL (1, 1) model the power of current value of x is 76.2%, which shows only 18.8% power loss. The reason behind it is that we did not include linear trend in ARDL, if we include linear trend in it. The numeral of lag value of x is showing only 96.8% power which means 0.0% power loss. In ARDL (2, 2) model the first lag value of x showing 93.0% probability of rejection the false null hypothesis. The powers of current and second lag values of x are 49.1% and 65.6%, which show 45.9% and 29.4% power loss respectively

As we know that y value is determined through lag value of x, but the first lag value are more significant as compare to current value of x. The reason is that there is multicollinearity effect, the current and lag values of x variable are collinear that is why the effect shifts into lag value in ARDL(1, 1) and in lag value in ARDL (2, 2).

In second row of first panel of table 6.6, the results illustrate that when we regressed stationary series θ_1=0.8 and θ_2=0.8, without drift, a_1= b_1= 0, θ_{21} = 0.8 and ρ= 1 by using OLS, the probability of rejection of false null hypothesis (power) is 100%, which represents a misleading figure. Because as we see above in table 5.1, the OLS has huge size distortion problem, specially, when series are with linear trend. That is why, it is showing 100% power which does not exist in reality. After employing ARDL (1, 1) model, the power of current value of x is 82.2%, which means that there is only 12.8% power loss, the reason behind it is that we did not include linear trend in ARDL. If we include linear trend, it may provide more power. The figure of lag value of x is showing only 5.6% power which means 89.4% power loss. In ARDL (2, 2) model, the first lag value of x is showing 96.1% probability of rejection of the false null hypothesis. The powers of current and second lag values of x are 19.4% and 96.1%, which show 75.6% and 0.0% power loss respectively

Same pattern has been found on other values of θ_{21} like, 0.6 and so on. On all these points ARDL shows better performance as compare to OLS.

On some values of parametersθ_1 and θ_2, and θ_{21} and OLS shows more power but we cannot consider it because as we have seen in size analysis the OLS suffers badly in size problem while ARDL in all cases has less size problem. In case without drift or with linear trend, and with drift and trend due to under specification, ARDL shows size problem but even in these cases OLS has more size distortion as compare to ARDL model.

There is another special effect which we should consider, In case of stationary and nonstationary time series the ARDL model works very well but it becomes unable to reduce the probability of spurious regression significantly in presence of trend. Basically both models OLS and ARDL are under specified because the series are having linear trend but models do not have linear trend term in their equations. On the other hand, we can see that the OLS model completely failed to tackle this problem but ARDL model works well as compare to OLS in size analysis. There is an important point, our data generating process in equation 6.1, generates first order autoregressive series AR(1) but we used second lag in ARDL (2, 2) model, so in case of over specification ARDL (2, 2) shows more power in case of stationary series as compare to ARDL (1, 1) and OLS. It also explores that the ARDL models performs better as compare to OLS in under specification and over specification.

Similarly table 6.7 and 6.8 also display the results of next two scenarios of lag and contemporaneous dependent series with drift and with drift and trend. So the interpretations of these cases are approximately alike, that is why we are interpreting them here. The lag values of y itself are highly significant in all cases, but one thing which is necessary is that as we reduce the value of autoregressive terms, the lag significance values also decreases in case of ARDL not in OLS.

Table 6.9: Power Analysis of Contemporaneous and Lag Dependent Series without drift and trend with ρ = 0.8

	OLS	ARDL (1, 1)				ARDL (2, 2)					
	xt	xt	xt-1	yt-1	F-stat	xt	xt-1	xt-2	yt-1	yt-2	F-stat
						ρ = 0.8					
$\theta_1 = \theta_2$						θ_{21} = **0.8**					
1	71.5	99.6	5.9	100.0	91.2	6.9	96.2	7.4	100.0	7.8	87.3
0.8	80.1	91.5	6.0	100.0	80.9	6.9	98.2	7.6	100.0	6.6	75.4
0.6	59.5	48.8	6.0	100.0	49.7	6.9	99.1	7.2	96.2	6.4	40.8
0.4	37.3	18.4	5.9	92.6	21.5	6.7	99.6	7.0	68.4	6.5	23.7
0.2	12.4	8.4	5.9	36.4	11.9	6.9	99.8	6.7	22.0	6.7	20.8
						θ_{21} = **0.6**					
1	73.1	95.1	5.9	100.0	90.3	6.9	83.2	7.3	100.0	7.8	78.1
0.8	77.3	62.0	6.1	100.0	53.4	6.9	88.9	7.7	99.9	6.5	45.7
0.6	58.5	41.0	5.9	99.7	30.6	6.7	92.8	7.3	95.9	6.5	30.0
0.4	29.9	9.4	6.0	86.3	10.9	6.8	95.1	6.9	68.3	6.7	16.4
0.2	10.8	6.6	5.8	29.6	8.6	6.6	96.6	6.8	22.2	6.6	12.4
						θ_{21} = **0.4**					
1	73.0	22.6	6.0	100.0	29.9	7.1	18.7	7.4	100.0	7.3	23.0
0.8	68.1	21.8	6.1	100.0	25.8	7.0	59.7	7.8	99.9	6.4	20.9
0.6	44.8	8.5	6.0	98.9	14.9	7.0	66.1	7.1	95.6	6.7	19.0
0.4	21.3	6.7	5.9	78.1	8.7	6.9	71.7	6.8	68.1	6.7	18.3
0.2	8.8	6.1	6.0	25.1	5.8	6.8	75.0	6.8	22.2	6.7	16.1
						θ_{21} = **0.2**					
1	72.8	22.4	6.0	100.0	25.4	6.9	18.6	7.4	100.0	7.2	18.3
0.8	50.0	7.0	6.2	99.9	10.4	7.0	21.1	7.3	99.7	6.6	11.3
0.6	27.8	6.3	5.9	97.0	8.4	6.8	24.4	6.9	95.2	6.7	10.5
0.4	13.5	6.3	5.9	71.7	6.2	6.8	27.0	7.0	67.8	6.8	9.0
0.2	8.8	6.1	5.8	22.4	6.1	6.8	29.0	6.8	22.3	6.8	9.6

Table 6.10: Power Analysis of Contemporaneous and Lag Dependent Series without drift with ρ = 0.8

	OLS	ARDL (1, 1)			ARDL (2, 2)				
	xt	**xt**	**xt-1**	**yt-1**	**xt**	**xt-1**	**xt-2**	**yt-1**	**yt-2**
	ρ = 0.8								
$\theta_1 = \theta_2$	$\theta_{21} = 0.8$								
1	100.0	77.0	95.7	100.0	43.9	91.0	62.6	100.0	24.7
0.8	100.0	81.3	9.2	100.0	17.8	95.8	8.5	100.0	9.6
0.6	100.0	82.7	5.7	100.0	13.5	99.2	6.8	99.4	7.1
0.4	100.0	33.1	23.3	100.0	15.1	99.7	6.6	89.6	16.8
0.2	100.0	8.8	53.5	99.5	20.1	99.9	6.9	53.8	27.7
	$\theta_{21} = 0.6$								
1	100.0	50.1	94.6	100.0	48.4	76.1	64.5	100.0	24.8
0.8	100.0	72.4	26.1	100.0	16.3	83.5	8.3	100.0	10.1
0.6	100.0	42.9	6.7	100.0	13.2	93.8	6.9	99.4	7.5
0.4	100.0	10.7	27.6	100.0	15.5	97.1	7.3	90.5	17.7
0.2	100.0	6.6	56.8	99.0	21.2	99.0	8.2	55.0	27.1
	$\theta_{21} = 0.4$								
1	100.0	40.3	93.2	100.0	48.2	17.1	49.6	100.0	24.7
0.8	100.0	67.7	28.8	100.0	19.2	62.4	8.5	100.0	11.1
0.6	100.0	10.3	9.6	100.0	12.9	71.9	7.5	99.4	8.6
0.4	100.0	6.5	31.7	100.0	16.0	81.9	8.5	91.5	17.4
0.2	100.0	15.9	58.2	97.8	22.6	91.0	10.9	57.7	24.9
	$\theta_{21} = 0.2$								
1	100.0	40.3	91.6	100.0	48.6	17.0	49.8	100.0	24.5
0.8	100.0	9.4	26.3	100.0	20.2	26.1	8.6	100.0	13.2
0.6	100.0	7.7	14.2	100.0	13.6	31.5	9.4	99.5	8.9
0.4	100.0	17.6	33.7	99.8	18.1	43.5	12.9	92.5	15.1
0.2	100.0	34.3	56.4	95.4	24.9	60.7	17.1	61.2	20.8

Table 6.11: Power Analysis of Contemporaneous and Lag Dependent Series with drift with ρ = 0.8

	OLS	ARDL (1, 1)			ARDL (2, 2)				
	xt	xt	xt-1	yt-1	xt	xt-1	xt-2	yt-1	yt-2
	ρ = 0.8								
$\theta_1 = \theta_2$	θ_{21} = **0.8**								
1	100.0	99.8	6.0	100.0	6.9	96.3	7.3	100.0	7.8
0.8	96.7	96.7	6.0	100.0	6.8	98.1	7.7	100.0	7.1
0.6	82.0	53.2	5.9	100.0	7.0	99.1	7.4	96.5	0.0
0.4	46.3	16.7	5.8	94.8	6.9	99.6	7.0	68.0	6.6
0.2	14.8	7.1	5.9	37.9	6.8	99.8	6.8	22.2	6.8
	θ_{21} = **0.6**								
1	100.0	96.6	5.9	100.0	6.9	83.1	7.2	100.0	7.9
0.8	95.7	76.3	6.0	100.0	6.9	88.6	7.6	100.0	6.8
0.6	74.6	20.7	5.9	100.0	6.8	92.8	7.3	96.3	6.4
0.4	38.2	8.0	6.0	89.0	6.9	95.3	7.0	68.4	6.5
0.2	12.7	5.8	5.9	30.9	7.0	96.5	6.9	22.0	6.9
	θ_{21} = **0.4**								
1	100.0	25.8	5.8	100.0	7.0	18.4	6.9	100.0	7.9
0.8	93.0	28.8	6.1	100.0	6.7	59.6	7.5	100.0	6.7
0.6	61.9	6.7	6.0	99.6	6.7	66.6	7.2	95.8	6.5
0.4	27.4	5.5	5.9	81.6	6.8	71.7	6.9	68.4	6.7
0.2	9.9	5.7	5.7	26.0	6.6	75.2	6.8	22.5	6.7
	θ_{21} = **0.2**								
1	100.0	26.0	5.9	100.0	6.7	18.4	6.9	100.0	7.7
0.8	84.4	5.7	6.0	100.0	15.8	25.3	9.5	100.0	9.4
0.6	39.6	5.8	5.9	98.4	12.5	30.7	9.3	99.5	10.4
0.4	16.8	5.7	5.8	74.1	18.7	44.0	11.5	92.9	14.1
0.2	7.7	5.8	5.9	22.7	25.8	61.5	15.8	62.7	19.4

Table 6.12: Power Analysis of Contemporaneous and Lag Dependent Series with drift and trend with ρ = 0.8

	OLS	ARDL (1, 1)			ARDL (2, 2)				
	xt	**xt**	**xt-1**	**yt-1**	**xt**	**xt-1**	**xt-2**	**yt-1**	**yt-2**
	ρ = 0.8								
$\theta_1 = \theta_2$	$\theta_{21} = 0.8$								
1	100.0	82.8	92.3	100.0	44.0	93.1	57.6	100.0	22.1
0.8	100.0	99.9	17.9	100.0	15.0	97.8	8.0	100.0	8.0
0.6	100.0	84.2	7.6	100.0	11.2	99.2	6.8	99.2	9.3
0.4	100.0	32.6	23.3	100.0	14.9	99.8	6.7	90.0	17.1
0.2	100.0	7.8	51.6	99.5	21.2	99.9	6.6	55.6	25.8
	$\theta_{21} = 0.6$								
1	100.0	33.9	92.2	100.0	44.3	76.4	56.6	100.0	22.4
0.8	100.0	97.6	17.9	100.0	15.2	88.5	8.4	100.0	8.6
0.6	100.0	45.3	9.9	100.0	11.2	93.8	7.0	99.4	10.1
0.4	100.0	10.4	27.9	100.0	15.5	97.1	6.8	91.0	17.5
0.2	100.0	6.9	54.7	98.9	22.0	99.1	7.5	56.9	25.2
	$\theta_{21} = 0.4$								
1	100.0	27.6	93.0	100.0	44.7	16.9	43.9	100.0	22.5
0.8	100.0	73.7	17.6	100.0	15.2	62.5	9.3	100.0	8.8
0.6	100.0	11.3	13.3	100.0	11.4	70.6	7.5	99.5	11.0
0.4	100.0	6.8	31.2	100.0	16.7	81.9	8.2	92.0	16.8
0.2	100.0	17.5	56.2	97.7	23.6	91.4	10.1	59.4	23.0
	$\theta_{21} = 0.2$								
1	100.0	27.6	93.1	100.0	44.5	17.2	44.0	100.0	22.3
0.8	100.0	37.6	54.1	100.0	15.8	25.3	9.5	100.0	9.4
0.6	100.0	19.0	32.4	100.0	12.5	30.7	9.3	99.5	10.4
0.4	100.0	13.7	17.5	99.8	18.7	44.0	11.5	92.9	14.1
0.2	100.0	7.6	13.6	95.1	25.8	61.5	15.8	62.7	19.4

In first row of first panel of table 6.9, the results are indicating that when series are nonstationary, θ_1=1 and θ_2=0.8, having no drift and trend, a_1= b_1= 0 =a_2= b_2= 0, θ_{21}=0.8 and covariance ρ=0.8, then the OLS power is 71.5%, which shows 23.5% power loss on the basis of 5% nominal size. In case of ARDL models F-test are being used to test the joint significance of current and lag values of independent variable. The F-stat value after employing ARDL (1, 1) model is indicating that the power of ARDL (1, 1) model is 91.2% which shows 3.8% power loss at 5% nominal size. The power of ARDL (2, 2) is 87.3% and loss of power is 7.7 at 5% nominal size.

In second row of first panel of table 6.9, the results are representing that when series are stationary, θ_1= 0.8 and θ_2 = 0.8, having no drift and trend, a_1= b_1= 0 =a_2= b_2= 0, θ_{21}=0.8 and ρ= 0.8, then the OLS power is 80.1%, which shows 14.9% power loss on the basis of 5% nominal size. In case of ARDL models F-test is being used to test the joint significance of current and lag values of independent variable. The F-stat value after employing ARDL (1, 1) model is indicating that the power of ARDL (1, 1) model is 80.9% which shows 14.1% power loss at 5% nominal size. The power of ARDL (2, 2) is 75.1% and loss of power is 19.9% at 5% nominal size.

In first row of second panel of table 6.9, the results are indicating that when series are nonstationary, θ_1=1 and θ_2=0.8, having no drift and trend, a_1= b_1= 0 =a_2= b_2= 0, θ_{21}=0.6 and ρ= 0.8, then the OLS power is 73.1%, which shows 21.9% power loss on the basis of 5% nominal size. In case of ARDL models F-test are being used to test the joint significance of current and lag values of independent variable. The F-stat value after employing ARDL (1, 1) model is indicating that the power of ARDL (1, 1) model is 90.3% which shows 4.7% power loss at 5% nominal size. The power of ARDL (2, 2) is 78.1% and loss of power is 16.9 at 5% nominal size.

In second row of second panel of table 6.9, the results are representing that when series are stationary, θ_1= 0.8 and θ_2 = 0.8, having no drift and trend, a_1= b_1= 0 =a_2= b_2= 0, θ_{21}=0.6 and ρ= 0.8 then the OLS power is 77.2%, which shows 17.8% power loss on the basis of 5% nominal size. In case of ARDL models F-test are being used to test the joint significance of current and lag values of independent variable. The F-stat value after employing ARDL (1, 1) model is indicating that the power of ARDL (1, 1) model is 53.4% which shows 41.6% power loss at 5% nominal size. The power of ARDL (2, 2) is 45.7% and loss of power is 11.1% at 5% nominal size.

It shows that the conventional OLS method badly suffers in power problem when series are nonstationary even they are stationary with no drift and trend. On contrary ARDL model is not showing huge power in both cases. It clarifies that when series are stationary or nonstationary without having drift and trend ARDL works better than OLS.

In first row of first panel of table 6.10, the results show that when we regressed nonstationary seriesθ_1=1 and θ_2=0.8, without drift, a_1= b_1= 0, θ_{21} = 0.8 and ρ= 0.8 by using OLS, the probability of rejection of false null hypothesis (power) is 100%, which represents a misleading figure. Because as seen above in table 5.1 the OLS has huge size distortion problem, specially, when series are with linear trend. That is why it is showing 100% power which does not exist in reality. The F-test used only in one case for displaying the joint significance of independent lag and current value. So, table 6.10, 5.12 and 5.13 have only t-stat values. After employing ARDL (1, 1) model the power of current value of x is 50.1%, which shows only 44.9% power loss. The reason behind it is that we did not include linear trend in ARDL if we include linear trend in it may provide more power. The numeral of lag value of x is showing only 94.1% power which means 0.9% power loss. In ARDL (2, 2) model the first lag value of x showing 91.0%

probability of rejection of the false null hypothesis. The powers of current and second lag values of x are 43.9% and 62.6%, which show 51.1% and 32.4% power loss respectively

As we know that y value is determined through lag value of x, but the first lag value are more significant as compare to current value of x. The reason is that there is multicollinearity effect, the current and lag values of x variable are collinear that is why the effect shifts into lag value in ARDL(1, 1) and in lag value in ARDL (2, 2).

In second row of first panel of table 6.10, the results illustrate that when we regressed stationary series θ_1=0.8 and θ_2=0.8, without drift, a_1= b_1= 0, θ_{21} = 0.8 and ρ= 0.8 by using OLS, the probability of rejection of false null hypothesis (power) is 100%, which represents a misleading figure. Because as we see in table 5.1, the OLS has huge size distortion problem, specially, when series are with linear trend. That is why, it is showing 100% power which does not exist in reality. After employing ARDL (1, 1) model, the power of current value of x is 81.3%, which means only 13.7% power loss, the reason is that we did not include linear trend in ARDL. If we include linear trend, it may provide more power. The figure of lag value of x is showing only .9.2% power which means 85.8% power loss. In ARDL (2, 2), model the first lag value of x is showing 95.8% probability of rejection the false null hypothesis. The powers of current and second lag values of x are 17.8% and 8.5%, which shows 77.2% and 86.5% power loss respectively

Same pattern has been found on other values of θ_{21} like, 0.6 and so on. On all these points ARDL shows better performance as compare to OLS.

On some values of parameter*s* θ_1 and θ_2, and θ_{21} and OLS shows more power but we cannot consider it because as we have seen in size analysis the OLS suffers badly in

size problem while ARDL in all cases has less size problem. In case without drift or with linear trend, and with drift and trend due to under specification, ARDL shows size problem but even in these cases OLS has more size distortion as compare to ARDL model.

There is another special effect which we should consider, In case of stationary and nonstationary time series the ARDL model works very well but it becomes unable to reduce the probability of spurious regression significantly in presence of trend. Basically both models OLS and ARDL are under specified because the series are having linear trend but models do not have linear trend term in their equations. On the other hand, we can see that the OLS model completely failed to tackle this problem but ARDL model works well as compare to OLS in size analysis. There is an important point, our data generating process in equation 6.1, generates first order autoregressive series AR(1) but we used second lag in ARDL (2, 2) model, so in case of over specification ARDL (2, 2) shows more power in case of stationary series as compare to ARDL (1, 1) and OLS. It also explores that the ARDL models perform better than OLS in under specification and over specification.

Similarly table 6.11 and 6.12 results also display the results of next two scenarios of lag and contemporaneous dependent series with drift and with drift and trend. So, the interpretations of these cases are approximately alike that is why we are interpreting them here. The lag values of y itself are highly significant in all cases, but one thing which is necessary is that as we reduce the value of autoregressive terms, the lag significance is also going to decrease in case of ARDL not in OLS.

Table 6.13: Power Analysis of Contemporaneous and Lag Dependent Series without drift and trend with ρ = 0.6

	OLS	ARDL (1, 1)				ARDL (2, 2)					
	xt	xt	xt-1	yt-1	F-stat	xt	xt-1	xt-2	yt-1	yt-2	F-stat
						ρ = 0.6					
$\theta_1=\theta_2$						θ_{21} = **0.8**					
1	72	99.6	5.8	100	93.2	6.8	96.3	7.4	100	7.9	91.4
0.8	82.2	91.5	6	100	85.7	6.9	98.2	7.6	100	6.6	90.1
0.6	66.3	48.8	6	100	45.3	6.9	99.1	7.2	96.2	6.4	94.5
0.4	37.6	18.4	5.9	92.6	17.1	6.7	99.6	7	68.4	6.5	94.7
0.2	12.6	8.4	5.9	36.4	7.6	6.9	99.8	6.7	22	6.7	95.3
						θ_{21} = **0.6**					
1	72.4	95.1	5.9	100	90.1	6.9	83.1	7.4	100	7.7	79.6
0.8	71.4	62	6.1	100	61	6.9	88.9	7.7	99.9	6.5	81.2
0.6	58.7	21	5.9	99.7	18.9	6.7	92.8	7.3	95.9	6.5	87.6
0.4	30.1	9.4	6	86.3	7.6	6.8	95.1	6.9	68.3	6.7	91.3
0.2	10.5	6.6	5.8	29.6	6.3	6.6	96.6	6.8	22.2	6.6	91.5
						θ_{21} = **0.4**					
1	73.1	22.4	6	100	20.9	7	18.5	7.2	100	7.2	12.3
0.8	68.2	21.8	6.1	100	19.7	7	59.7	7.8	99.9	6.4	43.2
0.6	44.8	8.5	6	98.9	7.2	7	66.1	7.1	95.6	6.7	56.9
0.4	21.3	6.7	5.9	78.1	6.3	6.9	71.7	6.8	68.1	6.7	59.4
0.2	9	6.1	6	25.1	5.9	6.8	75	6.8	22.2	6.7	69.7
						θ_{21} = **0.2**					
1	72.9	22.2	6.1	100	19.8	7.1	18.7	7.4	100	7.3	13.2
0.8	50.1	7	6.2	99.9	6.8	7	21.1	7.3	99.7	6.6	18.9
0.6	27.7	6.3	5.9	97	6.1	6.8	24.4	6.9	95.2	6.7	19.8
0.4	13.6	6.3	5.9	71.7	6.1	6.8	27	7	67.8	6.8	20.1
0.2	7.3	6.1	5.8	22.4	5.7	6.8	29	6.8	22.3	6.8	23.5

Table 6.14: Power Analysis of Contemporaneous and Lag Dependent Series without drift with ρ = 0.6

WoD	OLS	ARDL (1, 1)			ARDL (2, 2)				
	xt	xt	xt-1	yt-1	Xt	xt-1	xt-2	yt-1	yt-2
					ρ = 0.6				
$\theta_1 = \theta_2$					θ_{21} = 0.8				
1	100.0	75.8	96.7	100.0	48.2	92.8	65.0	100.0	24.6
0.8	100.0	82.8	5.6	100.0	19.7	95.1	7.6	100.0	9.6
0.6	100.0	82.7	5.7	100.0	13.5	99.2	6.8	99.4	7.1
0.4	100.0	33.1	23.3	100.0	15.1	99.7	6.6	89.6	16.8
0.2	100.0	8.8	53.5	99.5	20.1	99.9	6.9	53.8	27.7
					θ_{21} = 0.6				
1	100.0	23.9	96.9	100.0	48.4	75.8	63.9	100.0	24.8
0.8	100.0	95.9	27.4	100.0	18.2	88.6	8.1	100.0	10.1
0.6	100.0	42.9	6.7	100.0	13.2	93.8	6.9	99.4	7.5
0.4	100.0	10.7	27.6	100.0	15.5	97.1	7.3	90.5	17.7
0.2	100.0	6.6	56.8	99.0	21.2	99.0	8.2	55.0	27.1
					θ_{21} = 0.4				
1	100.0	40.2	97.1	100.0	48.5	16.9	49.7	100.0	25.0
0.8	100.0	67.7	28.8	100.0	19.2	62.4	8.5	100.0	11.1
0.6	100.0	10.3	9.6	100.0	12.9	71.9	7.5	99.4	8.6
0.4	100.0	6.5	31.7	100.0	16.0	81.9	8.5	91.5	17.4
0.2	100.0	15.9	58.2	97.8	22.6	91.0	10.9	57.7	24.9
					θ_{21} = 0.2				
1	100.0	40.1	97.2	100.0	48.4	17.0	49.8	100.0	25.1
0.8	100.0	9.4	26.3	100.0	20.2	26.1	8.6	100.0	13.2
0.6	100.0	7.7	14.2	100.0	13.6	31.5	9.4	99.5	8.9
0.4	100.0	17.6	33.7	99.8	18.1	43.5	12.9	92.5	15.1
0.2	100.0	34.3	56.4	95.4	24.9	60.7	17.1	61.2	20.8

Table 6.15: Power Analysis of Contemporaneous and Lag Dependent Series with drift with ρ = 0.6

WD	OLS	ARDL (1, 1)			ARDL (2, 2)				
	xt	xt	xt-1	yt-1	Xt	xt-1	xt-2	yt-1	yt-2
					ρ = 0.6				
$\theta_1 = \theta_2$					θ_{21} = **0.8**				
1	100.0	99.8	6.0	100.0	6.8	96.4	7.3	100.0	7.7
0.8	96.7	96.7	6.0	100.0	6.8	98.1	7.7	100.0	7.1
0.6	74.9	53.2	5.9	100.0	7.0	99.1	7.4	96.5	6.6
0.4	46.4	16.7	5.8	94.8	6.9	99.6	7.0	68.0	6.6
0.2	14.9	7.1	5.9	37.9	6.8	99.8	6.8	22.2	6.8
					θ_{21} = **0.6**				
1	100.0	96.7	6.0	100.0	6.9	83.3	7.2	100.0	7.7
0.8	95.8	76.3	6.0	100.0	6.9	88.6	7.6	100.0	6.8
0.6	74.7	20.7	5.9	100.0	6.8	92.8	7.3	96.3	6.4
0.4	38.1	8.0	6.0	89.0	6.9	95.3	7.0	68.4	6.5
0.2	12.5	5.8	5.9	30.9	7.0	96.5	6.9	22.0	6.9
					θ_{21} = **0.4**				
1	100.0	25.9	5.9	100.0	7.0	18.7	7.1	100.0	7.7
0.8	93.2	28.8	6.1	100.0	6.7	59.6	7.5	100.0	6.7
0.6	61.8	6.7	6.0	99.6	6.7	66.6	7.2	95.8	6.5
0.4	27.6	5.5	5.9	81.6	6.8	71.7	6.9	68.4	6.7
0.2	10.0	5.7	5.7	26.0	6.6	75.2	6.8	22.5	6.7
					θ_{21} = **0.2**				
1	100.0	26.2	6.0	100.0	6.9	18.7	7.1	100.0	7.6
0.8	84.3	5.7	6.0	100.0	6.9	21.6	7.4	99.9	6.3
0.6	39.4	5.8	5.9	98.4	6.9	24.3	7.2	95.4	6.7
0.4	16.7	5.7	5.8	74.1	6.9	26.9	6.7	68.1	6.8
0.2	7.8	5.8	5.9	22.7	6.9	29.2	7.0	22.4	6.8

Table 6.16: Power Analysis of Contemporaneous and Lag Dependent Series with drift and trend with ρ = 0.6

	OLS	ARDL (1, 1)			ARDL (2, 2)				
	xt	**xt**	**xt-1**	**yt-1**	**Xt**	**xt-1**	**xt-2**	**yt-1**	**yt-2**
					ρ = 0.6				
$\theta_1 = \theta_2$					θ_{21} = **0.8**				
1	100.0	82.8	92.0	100.0	43.9	93.1	57.6	100.0	21.9
0.8	100.0	99.9	17.9	100.0	15.0	97.8	8.0	100.0	8.0
0.6	100.0	84.2	7.6	100.0	11.2	99.2	6.8	99.2	9.3
0.4	100.0	32.6	23.3	100.0	14.9	99.8	6.7	90.0	17.1
0.2	100.0	7.8	51.6	99.5	21.2	99.9	6.6	55.6	25.8
					θ_{21} = **0.6**				
1	100.0	33.9	92.3	100.0	44.3	76.3	56.7	100.0	22.3
0.8	100.0	97.6	17.9	100.0	15.2	88.5	8.4	100.0	8.6
0.6	100.0	45.3	9.9	100.0	11.2	93.8	7.0	99.4	10.1
0.4	100.0	10.4	27.9	100.0	15.5	97.1	6.8	91.0	17.5
0.2	100.0	6.9	54.7	98.9	22.0	99.1	7.5	56.9	25.2
					θ_{21} = **0.4**				
1	100.0	27.7	93.0	100.0	44.5	17.0	44.0	100.0	22.4
0.8	100.0	73.7	17.6	100.0	15.2	88.5	8.4	100.0	8.6
0.6	100.0	11.3	13.3	100.0	11.2	93.8	7.0	99.4	10.1
0.4	100.0	6.8	31.2	100.0	15.5	97.1	6.8	91.0	17.5
0.2	100.0	17.5	56.2	97.7	22.0	99.1	7.5	56.9	25.2
					θ_{21} = **0.2**				
1	100.0	27.4	93.1	100.0	44.6	17.0	43.9	100.0	22.4
0.8	100.0	13.0	13.6	100.0	22.0	99.1	7.5	56.9	25.2
0.6	100.0	7.6	17.2	100.0	11.4	70.6	7.5	99.5	11.0
0.4	100.0	19.0	32.4	99.8	16.7	81.9	8.2	92.0	16.8
0.2	100.0	37.6	54.1	95.1	23.6	91.4	10.1	59.4	23.0

In first row of first panel of table 6.13, the results are indicating that when series are nonstationary, θ_1=1 and θ_2=0.8, having no drift and trend, a_1= b_1= 0 =a_2= b_2= 0, θ_{21}=0.8 and covariance ρ=0.6, then the OLS power is 72.0%, which shows 23.0% power loss on the basis of 5% nominal size. In case of ARDL models F-test are being used to test the joint significance of current and lag values of independent variable. The F-stat value after employing ARDL (1, 1) model is indicating that the power of ARDL (1, 1) model is 93.2% which shows 1.8% power loss at 5% nominal size. The power of ARDL (2, 2) is 91.4% and loss of power is 3.6% at 5% nominal size.

In second row of first panel of table 6.13, the results are representing that when series are stationary, θ_1= 0.8 and θ_2 = 0.8, having no drift and trend, a_1= b_1= 0 =a_2= b_2= 0, θ_{21}=0.8 and ρ= 0.6, then the OLS power is 82.2%, which shows 12.8% power loss on the basis of 5% nominal size. In case of ARDL models F-test is being used to test the joint significance of current and lag values of independent variable. The F-stat value after employing ARDL (1, 1) model is indicating that the power of ARDL (1, 1) model is 85.7% which shows 9.3% power loss at 5% nominal size. The power of ARDL (2, 2) is 90.1% and loss of power is 4.9% at 5% nominal size.

In first row of second panel of table 6.13, the results are indicating that when series are nonstationary, θ_1=1 and θ_2=0.8, having no drift and trend, a_1= b_1= 0 =a_2= b_2= 0, θ_{21}=0.6 and ρ= 0.6, then the OLS power is 72.4%, which shows 22.6% power loss on the basis of 5% nominal size. In case of ARDL models F-test are being used to test the joint significance of current and lag values of independent variable. The F-stat value after employing ARDL (1, 1) model is indicating that the power of ARDL (1, 1) model is 90.1% which shows 4.9% power loss at 5% nominal size. The power of ARDL (2, 2) is 79.6% and loss of power is 15.4 at 5% nominal size.

In second row of second panel of table 6.13, the results are representing that when series are stationary, θ_1= 0.8 and θ_2 = 0.8, having no drift and trend, a_1= b_1= 0 =a_2= b_2= 0, θ_{21}=0.6 and ρ= 0.6 then the OLS power is 71.4%, which shows 23.6% power loss on the basis of 5% nominal size. In case of ARDL models F-test are being used to test the joint significance of current and lag values of independent variable. The F-stat value after employing ARDL (1, 1) model is indicating that the power of ARDL (1, 1) model is 61.0% which shows 34.0% power loss at 5% nominal size. The power of ARDL (2, 2) is 81.2% and loss of power is 13.8% at 5% nominal size.

It shows that the conventional OLS method badly suffers in power problem when series are nonstationary even they are stationary with no drift and trend. On contrary ARDL model is not showing huge power in both cases. It clarifies that when series are stationary or nonstationary without having drift and trend ARDL works better than OLS.

In first row of first panel of table 6.14, the results show that when we regressed nonstationary seriesθ_1=1 and θ_2=0.8, without drift, a_1= b_1= 0, θ_{21} = 0.8 and ρ= 0.6 by using OLS, the probability of rejection of false null hypothesis (power) is 100%, which represents a misleading figure. Because as seen above in table 5.1 the OLS has huge size distortion problem, specially, when series are with linear trend. That is why it is showing 100% power which does not exist in reality. The F-test used only in one case for displaying the joint significance of independent lag and current value. So, table 6.14, 6.15 and 6.16 have only t-stat values. After employing ARDL (1, 1) model the power of current value of x is 75.1%, which shows only 19.9% power loss. The reason behind it is that we did not include linear trend in ARDL if we include linear trend in it may provide more power. The numeral of lag value of x is showing only 96.7% power which means 0% power loss. In ARDL (2, 2) model the first lag value of x showing 92.8%

probability of rejection of the false null hypothesis. The powers of current and second lag values of x are 48.2% and 65.0%, which show 46.8% and 30.0% power loss respectively

As we know that y value is determined through lag value of x, but the first lag value are more significant as compare to current value of x. The reason is that there is multicollinearity effect, the current and lag values of x variable are collinear that is why the effect shifts into lag value in ARDL(1, 1) and in lag value in ARDL (2, 2).

In second row of first panel of table 6.14, the results illustrate that when we regressed stationary series θ_1=0.8 and θ_2=0.8, without drift, a_1= b_1= 0, θ_{21} = 0.8 and ρ= 0.6 by using OLS, the probability of rejection of false null hypothesis (power) is 100%, which represents a misleading figure. Because as we see in table 5.1, the OLS has huge size distortion problem, specially, when series are with linear trend. That is why, it is showing 100% power which does not exist in reality. After employing ARDL (1, 1) model, the power of current value of x is 82.8%, which means only 12.2% power loss, the reason is that we did not include linear trend in ARDL. If we include linear trend, it may provide more power. The figure of lag value of x is showing only 5.6% power which means 89.4% power loss. In ARDL (2, 2), model the first lag value of x is showing 95.1% probability of rejection the false null hypothesis. The powers of current and second lag values of x are 19.7% and 7.6%, which shows 75.3% and 87.4% power loss respectively

Same pattern has been found on other values of θ_{21} like, 0.6 and so on. On all these points ARDL shows better performance as compare to OLS.

On some values of parameter$s$ θ_1 and θ_2, and θ_{21} and OLS shows more power but we cannot consider it because as we have seen in size analysis the OLS suffers badly in

size problem while ARDL in all cases has less size problem. In case without drift or with linear trend, and with drift and trend due to under specification, ARDL shows size problem but even in these cases OLS has more size distortion as compare to ARDL model.

There is another special effect which we should consider, In case of stationary and nonstationary time series the ARDL model works very well but it becomes unable to reduce the probability of spurious regression significantly in presence of trend. Basically both models OLS and ARDL are under specified because the series are having linear trend but models do not have linear trend term in their equations. On the other hand, we can see that the OLS model completely failed to tackle this problem but ARDL model works well as compare to OLS in size analysis. There is an important point, our data generating process in equation 6.1, generates first order autoregressive series AR(1) but we used second lag in ARDL (2, 2) model, so in case of over specification ARDL (2, 2) shows more power in case of stationary series as compare to ARDL (1, 1) and OLS. It also explores that the ARDL models perform better than OLS in under specification and over specification.

Similarly table 6.15 and 6.16 results also display the results of next two scenarios of lag and contemporaneous dependent series with drift and with drift and trend. So, the interpretations of these cases are approximately alike that is why we are interpreting them here. The lag values of y itself are highly significant in all cases, but one thing which is necessary is that as we reduce the value of autoregressive terms, the lag significance is also going to decrease in case of ARDL not in OLS.

Table 6.17: Power Analysis of Contemporaneous and Lag Dependent Series without drift and trend with ρ = 0.4

	OLS	ARDL (1, 1)				ARDL (2, 2)					
	xt	xt	xt-1	yt-1	F-stat	xt	xt-1	xt-2	yt-1	yt-2	F-stat
						$\rho = 0.4$					
$\theta_1 = \theta_2$						$\theta_{21} = 0.8$					
1	71.4	99.6	6	100	91.5	6.8	96.3	7.4	100	7.7	87.3
0.8	81.6	91.3	6	100	82.6	6.8	98.1	7.7	100	6.8	84.7
0.6	63.9	48.6	6	100	43.9	6.7	99.1	7.4	96.2	6.4	91.2
0.4	37.2	18.5	5.9	92.7	15.4	6.8	99.6	7	68.1	6.5	91.6
0.2	12.3	8.3	6	36.1	7.8	6.8	99.8	6.9	22.1	6.7	92.6
						$\theta_{21} = 0.6$					
1	70.3	95.1	5.9	100	82.4	6.9	83.4	7.4	100	7.7	76.5
0.8	75.0	61.4	6.1	100	58.7	7.1	88.7	7.8	99.9	6.5	78.3
0.6	58.6	21	6	99.7	18.9	6.8	92.8	7.3	95.8	6.4	78.9
0.4	30.1	9.6	6	86.3	7.8	6.7	95.2	6.9	68.1	6.7	80.7
0.2	10.7	6.5	5.9	29.8	6.5	6.8	96.6	6.8	22.3	6.8	81.4
						$\theta_{21} = 0.4$					
1	73.3	22.5	6.1	100	18.7	7	18.6	7.4	100	7.2	15.7
0.8	67.9	21.6	6.1	100	17.6	7.1	59.7	7.7	99.9	6.5	48.7
0.6	44.6	8.4	5.9	98.8	6.5	6.9	66.3	7.2	95.5	6.5	54.6
0.4	21.4	6.6	6.1	78.5	6.1	6.8	71.5	6.9	68.2	6.5	57.2
0.2	8.8	6.3	5.9	25.1	5.9	6.9	75	6.9	22.3	6.8	65.4
						$\theta_{21} = 0.2$					
1	73	22.5	6	100	18.5	7.2	18.7	7.4	100	7.3	15.9
0.8	50.1	6.8	6.1	99.9	6.5	7.1	21.3	7.5	99.7	6.8	18.4
0.6	27.4	6.2	6.1	97	6.2	7	24.5	7	95.2	6.8	19.9
0.4	13.4	6.1	6	71.9	6.1	6.9	27	6.7	68	6.6	21.6
0.2	7.4	6.1	5.8	22.2	6.1	6.7	28.7	6.8	22.5	6.8	23.8

Table 6.18: Power Analysis of Contemporaneous and Lag Dependent Series without drift with ρ = 0.4

	OLS	ARDL (1, 1)			ARDL (2, 2)				
	Xt	Xt	xt-1	yt-1	xt	xt-1	xt-2	yt-1	yt-2
					$\rho = 0.4$				
$\theta_1 = \theta_2$					θ_{21} = 0.8				
1	100.0	74.6	93.1	100.0	47.0	93.1	64.8	100.0	24.6
0.8	100.0	81.6	5.6	100.0	17.9	94.7	7.5	100.0	9.4
0.6	100.0	82.0	5.6	100.0	13.4	99.2	6.8	99.3	7.1
0.4	100.0	33.1	23.0	100.0	14.9	99.7	6.7	89.5	17.2
0.2	100.0	8.8	53.8	99.5	20.5	99.9	6.9	53.8	27.6
					θ_{21} = 0.6				
1	100.0	24.1	95.6	100.0	46.9	76.3	62.4	100.0	24.7
0.8	100.0	91.7	27.2	100.0	18.5	86.9	7.1	100.0	10.0
0.6	100.0	43.2	6.7	100.0	13.1	93.8	6.9	99.4	7.7
0.4	100.0	10.8	27.8	100.0	15.2	97.1	7.0	90.5	17.8
0.2	100.0	6.7	56.7	98.9	21.0	98.9	8.1	55.4	27.0
					θ_{21} = 0.4				
1	100.0	40.3	97.2	100.0	48.4	17.1	49.8	100.0	24.8
0.8	100.0	67.8	29.0	100.0	19.0	62.3	8.3	100.0	11.1
0.6	100.0	10.3	9.8	100.0	13.1	71.5	7.7	99.5	8.5
0.4	100.0	6.5	31.9	100.0	16.1	81.6	8.5	91.6	17.4
0.2	100.0	16.0	58.2	97.8	22.7	90.9	10.8	57.7	24.8
					θ_{21} = 0.2				
1	100.0	40.6	97.1	100.0	48.5	16.8	49.5	100.0	25.0
0.8	100.0	9.2	26.6	100.0	20.6	25.8	8.6	100.0	13.3
0.6	100.0	7.6	14.4	100.0	13.4	31.3	9.4	99.4	9.2
0.4	100.0	17.7	33.6	99.8	18.1	43.6	12.7	92.7	15.1
0.2	100.0	34.4	56.3	95.5	24.9	60.5	16.9	61.0	20.8

Table 6.19: Power Analysis of Contemporaneous and Lag Dependent Series with drift with ρ = 0.4

	OLS	ARDL (1, 1)			ARDL (2, 2)				
	Xt	xt	xt-1	yt-1	xt	xt-1	xt-2	yt-1	yt-2
					$\rho = 0.4$				
$\theta_1 = \theta_2$					$\theta_{21} = 0.8$				
1	100.0	99.8	5.9	100.0	6.7	96.4	7.3	100.0	7.8
0.8	96.8	96.8	5.9	100.0	6.7	98.2	7.4	100.0	7.0
0.6	75.0	53.4	6.0	100.0	6.9	99.1	7.3	96.5	6.3
0.4	46.5	16.8	5.9	94.8	6.9	99.5	7.1	68.5	6.6
0.2	15.1	7.0	5.8	38.0	6.7	99.8	6.7	22.1	6.7
					$\theta_{21} = 0.6$				
1	100.0	96.7	5.8	100.0	6.8	83.4	7.2	100.0	7.8
0.8	95.7	76.3	5.9	100.0	6.9	88.9	7.6	100.0	6.9
0.6	75.0	20.6	5.9	100.0	6.9	92.6	7.2	96.3	6.4
0.4	38.6	7.8	5.9	89.1	6.9	95.3	6.9	68.6	6.7
0.2	12.7	5.8	5.9	30.9	6.7	96.5	6.8	22.1	6.6
					$\theta_{21} = 0.4$				
1	100.0	25.9	5.8	100.0	6.9	18.7	7.0	100.0	7.7
0.8	93.2	28.9	6.0	100.0	7.0	59.6	7.5	100.0	6.5
0.6	61.8	6.6	5.8	99.7	6.9	66.3	7.1	95.8	6.3
0.4	27.6	5.5	5.9	81.3	6.8	71.6	6.9	68.2	6.6
0.2	10.1	5.6	5.9	25.9	6.8	75.0	6.9	22.1	6.8
					$\theta_{21} = 0.2$				
1	100.0	25.9	6.0	100.0	6.9	18.6	7.0	100.0	7.6
0.8	84.4	5.8	6.1	100.0	6.9	21.5	7.4	99.9	6.4
0.6	39.8	5.9	6.0	98.4	6.9	24.3	7.1	95.4	6.7
0.4	16.3	5.8	6.0	74.2	6.8	26.8	6.8	68.2	6.9
0.2	7.8	5.8	5.9	22.8	6.9	28.8	6.8	22.3	6.8

Table 6.20: Power Analysis of Contemporaneous and Lag Dependent Series with drift and trend with ρ = 0.4

	OLS	ARDL (1, 1)			ARDL (2, 2)				
	Xt	**xt**	**xt-1**	**yt-1**	**xt**	**xt-1**	**xt-2**	**yt-1**	**yt-2**
					ρ = 0.4				
$\theta_1 = \theta_2$					θ_{21} = **0.8**				
1	100.0	82.6	92.1	100.0	44.1	93.1	57.3	100.0	22.1
0.8	100.0	99.9	18.0	100.0	14.8	97.8	8.1	100.0	8.1
0.6	100.0	84.1	7.8	100.0	11.3	99.2	6.8	99.2	9.1
0.4	100.0	32.3	23.3	100.0	15.2	99.7	6.6	89.8	17.0
0.2	100.0	7.9	51.5	99.5	21.4	99.9	6.7	55.7	25.9
					θ_{21} = **0.6**				
1	100.0	33.8	92.2	100.0	44.6	76.3	57.1	100.0	22.3
0.8	100.0	97.7	17.7	100.0	15.0	88.8	8.4	100.0	8.2
0.6	100.0	45.4	9.8	100.0	11.3	93.8	6.9	99.4	10.3
0.4	100.0	10.2	27.4	100.0	15.5	97.2	6.8	90.8	17.6
0.2	100.0	6.8	54.5	98.9	22.3	99.0	7.5	57.0	25.2
					θ_{21} = **0.4**				
1	100.0	27.5	93.1	100.0	44.9	16.9	43.7	100.0	22.4
0.8	100.0	73.5	17.5	100.0	15.3	62.3	8.9	100.0	8.8
0.6	100.0	11.3	13.5	100.0	11.7	70.9	7.6	99.4	10.9
0.4	100.0	6.7	31.4	100.0	16.6	82.3	8.1	91.9	16.8
0.2	100.0	17.6	56.0	97.7	23.6	91.3	9.8	59.5	23.2
					θ_{21} = **0.2**				
1	100.0	27.2	93.1	100.0	44.5	17.1	44.0	100.0	22.1
0.8	100.0	13.2	13.9	100.0	15.9	25.2	9.3	100.0	9.6
0.6	100.0	7.4	17.1	100.0	12.4	30.8	9.3	99.5	10.6
0.4	100.0	19.0	32.2	99.8	18.9	44.1	11.6	92.8	14.2
0.2	100.0	37.7	53.8	95.2	26.0	61.4	16.0	62.6	19.3

In first row of first panel of table 6.17, the results are indicating that when series are nonstationary, θ_1=1 and θ_2=0.8, having no drift and trend, a_1= b_1= 0 =a_2= b_2= 0, θ_{21}=0.8 and covariance ρ=0.4, then the OLS power is 71.4%, which shows 23.6% power loss on the basis of 5% nominal size. In case of ARDL models F-test are being used to test the joint significance of current and lag values of independent variable. The F-stat value after employing ARDL (1, 1) model is indicating that the power of ARDL (1, 1) model is 91.5% which shows 3.5% power loss at 5% nominal size. The power of ARDL (2, 2) is 87.3% and loss of power is 7.7% at 5% nominal size.

In second row of first panel of table 6.17, the results are representing that when series are stationary, θ_1= 0.8 and θ_2 = 0.8, having no drift and trend, a_1= b_1= 0 =a_2= b_2= 0, θ_{21}=0.8 and ρ= 0.4, then the OLS power is 81.6%, which shows 13.4% power loss on the basis of 5% nominal size. In case of ARDL models F-test is being used to test the joint significance of current and lag values of independent variable. The F-stat value after employing ARDL (1, 1) model is indicating that the power of ARDL (1, 1) model is 82.6% which shows 12.4% power loss at 5% nominal size. The power of ARDL (2, 2) is 84.7% and loss of power is 10.3% at 5% nominal size.

In first row of second panel of table 6.17, the results are indicating that when series are nonstationary, θ_1=1 and θ_2=0.8, having no drift and trend, a_1= b_1= 0 =a_2= b_2= 0, θ_{21}=0.6 and ρ= 0.4, then the OLS power is 70.3%, which shows 24.7% power loss on the basis of 5% nominal size. In case of ARDL models F-test are being used to test the joint significance of current and lag values of independent variable. The F-stat value after employing ARDL (1, 1) model is indicating that the power of ARDL (1, 1) model is 82.4% which shows 12.6% power loss at 5% nominal size. The power of ARDL (2, 2) is 76.5% and loss of power is 18.5% at 5% nominal size.

In second row of second panel of table 6.17, the results are representing that when series are stationary, θ_1= 0.8 and θ_2 = 0.8, having no drift and trend, a_1= b_1= 0 =a_2= b_2= 0, θ_{21}=0.6 and ρ= 0.4 then the OLS power is 75.0%, which shows 20.0% power loss on the basis of 5% nominal size. In case of ARDL models F-test are being used to test the joint significance of current and lag values of independent variable. The F-stat value after employing ARDL (1, 1) model is indicating that the power of ARDL (1, 1) model is 58.7% which shows 36.3% power loss at 5% nominal size. The power of ARDL (2, 2) is 78.3% and loss of power is 16.7% at 5% nominal size.

It shows that the conventional OLS method badly suffers in power problem when series are nonstationary even they are stationary with no drift and trend. On contrary ARDL model is not showing huge power in both cases. It clarifies that when series are stationary or nonstationary without having drift and trend ARDL works better than OLS.

In first row of first panel of table 6.18, the results show that when we regressed nonstationary seriesθ_1=1 and θ_2=0.8, without drift, a_1= b_1= 0, θ_{21} = 0.8 and ρ= 0.4 by using OLS, the probability of rejection of false null hypothesis (power) is 100%, which represents a misleading figure. Because as seen above in table 5.1 the OLS has huge size distortion problem, specially, when series are with linear trend. That is why it is showing 100% power which does not exist in reality. The F-test used only in one case for displaying the joint significance of independent lag and current value. So, table 6.18, 6.19 and 6.20 have only t-stat values. After employing ARDL (1, 1) model the power of current value of x is 74.6%, which shows only 20.4% power loss. The reason behind it is that we did not include linear trend in ARDL if we include linear trend in it may provide more power. The numeral of lag value of x is showing only 93.1% power which means 1.9% power loss. In ARDL (2, 2) model the first lag value of x showing 93.1%

probability of rejection of the false null hypothesis. The powers of current and second lag values of x are 47.0% and 64.8%, which show 48.0% and 30.2% power loss respectively

As we know that y value is determined through lag value of x, but the first lag value are more significant as compare to current value of x. The reason is that there is multicollinearity effect, the current and lag values of x variable are collinear that is why the effect shifts into lag value in ARDL(1, 1) and in lag value in ARDL (2, 2).

In second row of first panel of table 6.18, the results illustrate that when we regressed stationary series θ_1=0.8 and θ_2=0.8, without drift, a_1= b_1= 0, θ_{21} = 0.8 and ρ= 0.4 by using OLS, the probability of rejection of false null hypothesis (power) is 100%, which represents a misleading figure. Because as we see in table 5.1, the OLS has huge size distortion problem, specially, when series are with linear trend. That is why, it is showing 100% power which does not exist in reality. After employing ARDL (1, 1) model, the power of current value of x is 81.6%, which means only 13.4% power loss, the reason is that we did not include linear trend in ARDL. If we include linear trend, it may provide more power. The figure of lag value of x is showing only 5.6% power which means 89.4% power loss. In ARDL (2, 2), model the first lag value of x is showing 94.7% probability of rejection the false null hypothesis. The powers of current and second lag values of x are 17.9% and 7.5%, which shows 77.1% and 87.5% power loss respectively

Same pattern has been found on other values of θ_{21} like, 0.6 and so on. On all these points ARDL shows better performance as compare to OLS.

On some values of parameter*s* θ_1 and θ_2, and θ_{21} and OLS shows more power but we cannot consider it because as we have seen in size analysis the OLS suffers badly in

size problem while ARDL in all cases has less size problem. In case without drift or with linear trend, and with drift and trend due to under specification, ARDL shows size problem but even in these cases OLS has more size distortion as compare to ARDL model.

There is another special effect which we should consider, In case of stationary and nonstationary time series the ARDL model works very well but it becomes unable to reduce the probability of spurious regression significantly in presence of trend. Basically both models OLS and ARDL are under specified because the series are having linear trend but models do not have linear trend term in their equations. On the other hand, we can see that the OLS model completely failed to tackle this problem but ARDL model works well as compare to OLS in size analysis. There is an important point, our data generating process in equation 6.1, generates first order autoregressive series AR(1) but we used second lag in ARDL (2, 2) model, so in case of over specification ARDL (2, 2) shows more power in case of stationary series as compare to ARDL (1, 1) and OLS. It also explores that the ARDL models perform better than OLS in under specification and over specification.

Similarly table 6.19 and 6.20 results also display the results of next two scenarios of lag and contemporaneous dependent series with drift and with drift and trend. So, the interpretations of these cases are approximately alike that is why we are interpreting them here. The lag values of y itself are highly significant in all cases, but one thing which is necessary is that as we reduce the value of autoregressive terms, the lag significance is also going to decrease in case of ARDL not in OLS.

Table 6.21: Power Analysis of Contemporaneous and Lag Dependent Series without drift and trend with ρ = 0.2

	OLS	ARDL (1, 1)				ARDL (2, 2)					
	xt	**xt**	**xt-1**	**yt-1**	**F-stat**	**xt**	**xt-1**	**xt-2**	**yt-1**	**yt-2**	**F-stat**
						ρ = 0.2					
$\theta_1 = \theta_2$						$\theta_{21} = 0.8$					
1	70.8	82.6	6	100	91.6	6.8	96.3	7.3	100	7.6	86.3
0.8	80.2	86.2	6	100	81.3	6.8	98.1	7.7	100	6.8	85.2
0.6	62.5	48.8	6	99.9	43.4	6.7	99.1	7.4	96.2	6.4	91.2
0.4	37.6	18.5	5.9	92.7	15.4	6.8	99.6	7	68.1	6.5	91.6
0.2	12.6	8.3	6	36.1	7.8	6.8	99.8	6.9	22.1	6.7	94.8
						$\theta_{21} = 0.6$					
1	69.9	94.6	6	100	85.4	6.8	83.3	7.3	100	7.7	75.1
0.8	73.3	61.7	6.1	100	55.7	7.1	88.7	7.8	99.9	6.5	76.5
0.6	58.7	21	5.9	99.7	18.4	6.8	92.8	7.3	95.8	6.4	79.2
0.4	30.1	9.5	6	86.2	7.9	6.7	95.2	6.9	68.1	6.7	80.7
0.2	10.5	6.7	5.8	29.6	6.4	6.8	96.6	6.8	22.3	6.8	85.6
						$\theta_{21} = 0.4$					
1	72.9	22.6	6.1	100	18.4	7	18.7	7.5	100	7.2	15.7
0.8	68.2	21.4	6	100	17.6	7.1	59.7	7.7	99.9	6.5	48.7
0.6	44.8	8.6	5.9	98.8	7.3	6.9	66.3	7.2	95.5	6.5	53.6
0.4	21.3	6.6	5.9	78.3	6.1	6.8	71.5	6.9	68.2	6.5	57.2
0.2	9	6.1	5.9	25.1	5.8	6.9	75	6.9	22.3	6.8	68.4
						$\theta_{21} = 0.2$					
1	73	22.5	6	100	17.5	7.1	18.6	7.6	100	7.2	15.9
0.8	50.1	7	6	99.9	6.5	7.1	21.3	7.5	99.7	6.8	17.3
0.6	27.7	6.2	5.9	96.9	6.2	7	24.5	7	95.2	6.8	19.7
0.4	13.6	6.2	5.8	71.7	6.1	6.9	27	6.7	68	6.6	21.6
0.2	7.3	6.1	5.9	22	6.1	6.7	28.7	6.8	22.5	6.8	24.2

Table 6.22: Power Analysis of Contemporaneous and Lag Dependent Series without drift with ρ = 0.2

	OLS	ARDL (1, 1)			ARDL (2, 2)				
	xt	xt	xt-1	yt-1	xt	xt-1	xt-2	yt-1	yt-2
	ρ = 0.2								
$\theta_1 = \theta_2$	θ_{21} = **0.8**								
1	100.0	73.0	92.7	100.0	48.0	92.5	65.0	100.0	24.5
0.8	100.0	80.7	5.7	100.0	16.7	95.4	7.5	100.0	9.4
0.6	100.0	82.9	5.6	100.0	13.4	99.2	6.8	99.3	7.1
0.4	100.0	33.4	23.2	100.0	14.9	99.7	6.7	89.5	17.2
0.2	100.0	8.8	53.5	99.6	20.5	99.9	6.9	53.8	27.6
	θ_{21} = **0.6**								
1	100.0	23.8	94.8	100.0	48.6	75.9	64.1	100.0	24.7
0.8	100.0	91.3	27.4	100.0	18.5	88.5	7.9	100.0	10.0
0.6	100.0	43.4	6.8	100.0	13.1	93.8	6.9	99.4	7.7
0.4	100.0	10.8	27.6	100.0	15.2	97.1	7.0	90.5	17.8
0.2	100.0	6.5	56.6	99.0	21.0	98.9	8.1	55.4	27.0
	θ_{21} = **0.4**								
1	100.0	40.2	97.1	100.0	48.7	17.1	49.8	100.0	25.0
0.8	100.0	67.6	28.7	100.0	19.0	62.3	8.3	100.0	11.1
0.6	100.0	10.5	9.7	100.0	13.1	71.5	7.7	99.5	8.5
0.4	100.0	6.5	31.8	100.0	16.1	81.6	8.5	91.6	17.4
0.2	100.0	15.7	58.0	97.9	22.7	90.9	10.8	57.7	24.8
	θ_{21} = **0.2**								
1	100.0	40.3	97.2	100.0	48.6	17.0	49.5	100.0	25.1
0.8	100.0	9.2	26.5	100.0	20.6	25.8	8.6	100.0	13.3
0.6	100.0	7.6	14.3	100.0	13.4	31.3	9.4	99.4	9.2
0.4	100.0	17.6	33.6	99.8	18.1	43.6	12.7	92.7	15.1
0.2	100.0	34.4	56.5	95.5	24.9	60.5	16.9	61.0	20.8

Table 6.23: Power Analysis of Contemporaneous and Lag Dependent Series with drift with ρ = 0.2

WD	OLS	ARDL (1, 1)			ARDL (2, 2)				
	xt	xt	xt-1	yt-1	xt	xt-1	xt-2	yt-1	yt-2
					$\rho = 0.2$				
$\theta_1 = \theta_2$					$\theta_{21} = 0.8$				
1	100.0	99.8	5.9	100.0	6.8	96.3	7.2	100.0	7.8
0.8	96.7	96.7	5.9	100.0	6.7	98.2	7.4	100.0	7.0
0.6	74.9	53.1	6.0	100.0	6.9	99.1	7.3	96.5	6.3
0.4	46.4	16.7	6.0	94.9	6.9	99.5	7.1	68.5	6.6
0.2	14.9	7.2	5.9	37.9	6.7	99.8	6.7	22.1	6.7
					$\theta_{21} = 0.6$				
1	100.0	96.7	5.8	100.0	6.9	83.1	7.1	100.0	7.7
0.8	95.8	76.1	6.0	100.0	6.9	88.9	7.6	100.0	6.9
0.6	74.7	20.8	5.8	100.0	6.9	92.6	7.2	96.3	6.4
0.4	38.1	7.8	6.0	89.1	6.9	95.3	6.9	68.6	6.7
0.2	12.5	5.9	5.8	30.9	6.7	96.5	6.8	22.1	6.6
					$\theta_{21} = 0.4$				
1	100.0	26.2	5.9	100.0	7.1	18.8	7.1	100.0	7.7
0.8	93.2	28.9	6.0	100.0	7.0	59.6	7.5	100.0	6.5
0.6	61.8	6.8	5.8	99.7	6.9	66.3	7.1	95.8	6.3
0.4	27.6	5.7	5.9	81.6	6.8	71.6	6.9	68.2	6.6
0.2	10.0	5.7	6.0	26.0	6.8	75.0	6.9	22.1	6.8
					$\theta_{21} = 0.2$				
1	100.0	26.0	6.0	100.0	7.0	18.6	6.9	100.0	7.6
0.8	84.3	5.8	6.1	100.0	6.9	21.5	7.4	99.9	6.4
0.6	39.4	5.7	6.0	98.5	6.9	24.3	7.1	95.4	6.7
0.4	16.7	5.8	5.8	74.2	6.8	26.8	6.8	68.2	6.9
0.2	7.8	5.8	5.8	22.8	6.9	28.8	6.8	22.3	6.8

Table 6.24: Power Analysis of Contemporaneous and Lag Dependent Series with drift and trend with ρ = 0.2

	OLS	ARDL (1, 1)			ARDL (2, 2)				
	xt	**xt**	**xt-1**	**yt-1**	**xt**	**xt-1**	**xt-2**	**yt-1**	**yt-2**
					$\rho = 0.2$				
$\theta_1 = \theta_2$					**$\theta_{21} = 0.8$**				
1	100.0	82.7	92.2	100.0	44.4	92.9	57.4	100.0	22.0
0.8	100.0	99.9	18.0	100.0	14.8	97.8	8.1	100.0	8.1
0.6	100.0	83.6	7.7	100.0	11.3	99.2	6.8	99.2	9.1
0.4	100.0	32.4	23.6	100.0	15.2	99.7	6.6	89.8	17.0
0.2	100.0	7.8	51.4	99.5	21.4	99.9	6.7	55.7	25.9
					$\theta_{21} = 0.6$				
1	100.0	34.0	92.2	100.0	44.5	76.3	56.8	100.0	22.5
0.8	100.0	97.7	18.0	100.0	15.0	88.8	8.4	100.0	8.2
0.6	100.0	45.1	9.9	100.0	11.3	93.8	6.9	99.4	10.3
0.4	100.0	10.4	27.5	100.0	15.5	97.2	6.8	90.8	17.6
0.2	100.0	6.9	54.8	98.9	22.3	99.0	7.5	57.0	25.2
					$\theta_{21} = 0.4$				
1	100.0	27.5	93.2	100.0	44.6	17.2	43.9	100.0	22.5
0.8	100.0	73.7	17.5	100.0	15.3	62.3	8.9	100.0	8.8
0.6	100.0	11.2	13.5	100.0	11.7	70.9	7.6	99.4	10.9
0.4	100.0	6.7	31.0	100.0	16.6	82.3	8.1	91.9	16.8
0.2	100.0	17.6	55.9	97.7	23.6	91.3	9.8	59.5	23.2
					$\theta_{21} = 0.2$				
1	100.0	27.6	93.1	100.0	44.8	16.8	43.7	100.0	22.6
0.8	100.0	13.2	13.8	100.0	15.9	25.2	9.3	100.0	9.6
0.6	100.0	7.6	17.2	100.0	12.4	30.8	9.3	99.5	10.6
0.4	100.0	19.2	32.0	99.8	18.9	44.1	11.6	92.8	14.2
0.2	100.0	37.7	54.0	95.2	26.0	61.4	16.0	62.6	19.3

In first row of first panel of table 6.21, the results are indicating that when series are nonstationary, θ_1=1 and θ_2=0.8, having no drift and trend, a_1= b_1= 0 =a_2= b_2= 0, θ_{21}=0.8 and covariance ρ=0.2, then the OLS power is 70.8%, which shows 24.2% power loss on the basis of 5% nominal size. In case of ARDL models F-test are being used to test the joint significance of current and lag values of independent variable. The F-stat value after employing ARDL (1, 1) model is indicating that the power of ARDL (1, 1) model is 91.6% which shows 3.4% power loss at 5% nominal size. The power of ARDL (2, 2) is 86.3% and loss of power is 8.7% at 5% nominal size.

In second row of first panel of table 6.21, the results are representing that when series are stationary, θ_1= 0.8 and θ_2 = 0.8, having no drift and trend, a_1= b_1= 0 =a_2= b_2= 0, θ_{21}=0.8 and ρ= 0.2, then the OLS power is 80.2%, which shows 14.8% power loss on the basis of 5% nominal size. In case of ARDL models F-test is being used to test the joint significance of current and lag values of independent variable. The F-stat value after employing ARDL (1, 1) model is indicating that the power of ARDL (1, 1) model is 81.3% which shows 12.7% power loss at 5% nominal size. The power of ARDL (2, 2) is 86.3% and loss of power is 8.7% at 5% nominal size.

In first row of second panel of table 6.21, the results are indicating that when series are nonstationary, θ_1=1 and θ_2=0.8, having no drift and trend, a_1= b_1= 0 =a_2= b_2= 0, θ_{21}=0.6 and ρ= 0.2, then the OLS power is 69.9%, which shows 25.1% power loss on the basis of 5% nominal size. In case of ARDL models F-test are being used to test the joint significance of current and lag values of independent variable. The F-stat value after employing ARDL (1, 1) model is indicating that the power of ARDL (1, 1) model is 85.4% which shows 9.6% power loss at 5% nominal size. The power of ARDL (2, 2) is 75.1% and loss of power is 19.9% at 5% nominal size.

In second row of second panel of table 6.21, the results are representing that when series are stationary, θ_1= 0.8 and θ_2 = 0.8, having no drift and trend, a_1= b_1= 0 =a_2= b_2= 0, θ_{21}=0.6 and ρ= 0.4 then the OLS power is 73.3%, which shows 12.7% power loss on the basis of 5% nominal size. In case of ARDL models F-test are being used to test the joint significance of current and lag values of independent variable. The F-stat value after employing ARDL (1, 1) model is indicating that the power of ARDL (1, 1) model is 85.4% which shows 9.6% power loss at 5% nominal size. The power of ARDL (2, 2) is 75.1% and loss of power is 19.9% at 5% nominal size.

It shows that the conventional OLS method badly suffers in power problem when series are nonstationary even they are stationary with no drift and trend. On contrary ARDL model is not showing huge power in both cases. It clarifies that when series are stationary or nonstationary without having drift and trend ARDL works better than OLS.

In first row of first panel of table 6.22, the results show that when we regressed nonstationary seriesθ_1=1 and θ_2=0.8, without drift, a_1= b_1= 0, θ_{21} = 0.8 and ρ= 0.2 by using OLS, the probability of rejection of false null hypothesis (power) is 100%, which represents a misleading figure. Because as seen above in table 5.1 the OLS has huge size distortion problem, specially, when series are with linear trend. That is why it is showing 100% power which does not exist in reality. The F-test used only in one case for displaying the joint significance of independent lag and current value. So, table 6.22, 6.23 and 6.24 have only t-stat values. After employing ARDL (1, 1) model the power of current value of x is 73.0%, which shows only 22.0% power loss. The reason behind it is that we did not include linear trend in ARDL if we include linear trend in it may provide more power. The numeral of lag value of x is showing only 92.7% power which means 2.3% power loss. In ARDL (2, 2) model the first lag value of x showing 92.5%

probability of rejection of the false null hypothesis. The powers of current and second lag values of x are 48.0% and 64.8%, which show 47.0% and 30.2% power loss respectively

As we know that y value is determined through lag value of x, but the first lag value are more significant as compare to current value of x. The reason is that there is multicollinearity effect, the current and lag values of x variable are collinear that is why the effect shifts into lag value in ARDL(1, 1) and in lag value in ARDL (2, 2).

In second row of first panel of table 6.18, the results illustrate that when we regressed stationary series θ_1=0.8 and θ_2=0.8, without drift, a_1= b_1= 0, θ_{21} = 0.8 and ρ= 0.2 by using OLS, the probability of rejection of false null hypothesis (power) is 100%, which represents a misleading figure. Because as we see in table 5.1, the OLS has huge size distortion problem, specially, when series are with linear trend. That is why, it is showing 100% power which does not exist in reality. After employing ARDL (1, 1) model, the power of current value of x is 80.7%, which means only 13.4% power loss, the reason is that we did not include linear trend in ARDL. If we include linear trend, it may provide more power. The figure of lag value of x is showing only 5.7% power which means 89.3% power loss. In ARDL (2, 2), model the first lag value of x is showing 89.3% probability of rejection the false null hypothesis. The powers of current and second lag values of x are 16.7% and 7.5%, which shows 78.3% and 87.5% power loss respectively

Same pattern has been found on other values of θ_{21} like, 0.6 and so on. On all these points ARDL shows better performance as compare to OLS.

On some values of parameter*s* θ_1 and θ_2, and θ_{21} and OLS shows more power but we cannot consider it because as we have seen in size analysis the OLS suffers badly in

size problem while ARDL in all cases has less size problem. In case without drift or with linear trend, and with drift and trend due to under specification, ARDL shows size problem but even in these cases OLS has more size distortion as compare to ARDL model.

There is another special effect which we should consider, In case of stationary and nonstationary time series the ARDL model works very well but it becomes unable to reduce the probability of spurious regression significantly in presence of trend. Basically both models OLS and ARDL are under specified because the series are having linear trend but models do not have linear trend term in their equations. On the other hand, we can see that the OLS model completely failed to tackle this problem but ARDL model works well as compare to OLS in size analysis. There is an important point, our data generating process in equation 6.1, generates first order autoregressive series AR(1) but we used second lag in ARDL (2, 2) model, so in case of over specification ARDL (2, 2) shows more power in case of stationary series as compare to ARDL (1, 1) and OLS. It also explores that the ARDL models perform better than OLS in under specification and over specification.

Similarly table 6.23 and 6.24 results also display the results of next two scenarios of lag and contemporaneous dependent series with drift and with drift and trend. So, the interpretations of these cases are approximately alike that is why we are interpreting them here. The lag values of y itself are highly significant in all cases, but one thing which is necessary is that as we reduce the value of autoregressive terms, the lag significance is also going to decrease in case of ARDL not in OLS.

6.3 Contemporaneous Dependence Between the Series

The contemporaneous dependent series are generated by using equation 5.6, the cross correlation parameters $\theta_{21} = 0$ and $\theta_{12} = 0$ and $\rho \neq 0$. The $\rho \neq 0$ implies that the error terms added to each have contemporaneous correlation. Therefor X and Yare not serially but contemporaneously dependent onto each other.

The series are non-stationary, if the own lag value parameters are $\theta_1 = 1$ and $\theta_2 = 0.8$, and the series are stationary, if the own lag value parameters are $\theta_1 < 1$ and $\theta_2 < 1$. When the matrix B = 0 then it means series are without drift and trend. If $a_1 = b_1 = 0$ in matrix B then the series are without drift or with linear trend. When $a_2 = b_2 = 0$ in matrix B then the series are with drift or without linear trend. When $a_1 = b_1 = a_2 = b_2 = 1$ in matrix B then the series are with drift and trend terms. when $\rho \neq 0$, it indicates that the error of X series correlated with error of Y series, means there is contemporaneous dependence between the series. In this experiment we used dependent series with different scenarios and with different values of parameters which are following:

Table 6.25: Power Analysis of Contemporaneous Dependent Series without drift and trend

	OLS	ARDL (1, 1)				ARDL (2, 2)					
	xt	xt	xt-1	yt-1	F-stat	xt	xt-1	xt-2	yt-1	yt-2	F-stat
$\theta_1 = \theta_2$						$\rho = 1$					
1	76.5	90.6	78.2	100	87.5	89.6	54.2	7.4	100	7.9	82.3
0.8	77.2	89.6	59.5	100	81.2	88.8	47.4	7.6	100	6.6	80.2
0.6	85.1	89.6	37.4	96.7	78.9	86.3	31.2	7.2	96.2	6.5	76.8
0.4	88.5	89.3	22.4	70.3	78.3	85.4	19.1	7.1	68.4	6.5	68.4
0.2	90.9	89.8	8.8	23.5	76.7	81.2	7.9	6.7	22	6.7	56.7
						$\rho = 0.8$					
1	72.1	79.8	61.2	100	74.6	75.1	41.4	7.5	100	7.8	64.9
0.8	67.3	75.4	45.2	99.8	69.7	73.2	39.2	7.6	100	6.6	56.7
0.6	71.4	76.5	26.9	95.2	68.5	71.4	31.2	7.1	95.9	6.4	55.7
0.4	76.5	78.4	16.9	69.9	64.3	70.9	18.6	7	67.5	6.5	54.3
0.2	76.6	75.9	7.1	22.3	62.1	68.7	7.6	6.6	22.1	6.7	42.6
						$\rho = 0.6$					
1	67.1	73.5	45.1	100	69.6	52.5	29.7	7.5	100	7.8	40.6
0.8	55.6	50.6	29.5	99.5	46.3	52.3	27.5	7.2	100	6.6	39.5
0.6	52.6	54.1	16.2	96.1	46.2	51.8	27.3	7	95.1	6.4	39.2
0.4	52.1	53.8	12.5	66.4	43.1	51.4	15.8	7.1	67.1	6.5	32.1
0.2	54.1	54.7	7.2	19.1	39.8	52.2	8.1	6.3	21.8	6.7	29.7
						$\rho = 0.4$					
1	58.1	70.5	23.3	100	62.1	50.6	28.5	7.4	100	7.3	39.7
0.8	44.8	29.1	16.5	99.4	23.6	49.9	27.4	7.2	99.8	6.2	33.2
0.6	34.5	30.9	10.6	95.4	24.3	49.2	26.9	7.2	94.3	6.3	32.6
0.4	31.5	28.4	8.6	70.8	19.7	47.3	13.1	7	69.1	6.5	29.8
0.2	33.8	31.4	6.6	20.3	21.9	46.1	7.9	5.9	20.1	6.7	21.6
						$\rho = 0.2$					
1	53.4	70.8	21.5	100	61.4	50.2	23.6	7.3	100	7.2	39.4
0.8	39.5	40.1	7.7	99.7	35.7	37.7	23.4	7.1	97.6	6.6	31.5
0.6	25.1	10.4	8.7	95.1	8.6	36.9	19.7	7	91.2	6.3	28.6
0.4	15.1	13.6	6.8	67.4	8.9	31.1	12.6	7	62.6	6.3	21.4
0.2	12.7	11.3	4.6	20.3	7.5	28	6.5	5.7	20.7	6.1	17.6

Table 6.26: Power Analysis of Contemporaneous Dependent Series without drift

WOD	OLS	ARDL (1, 1)			ARDL (2, 2)				
	xt	xt	xt-1	yt-1	xt	xt-1	xt-2	yt-1	yt-2
$\theta_1 = \theta_2$					**ρ = 1**				
1	100	99.6	90.6	100	90.5	51.2	7.8	100	8.4
0.8	100	97.3	64.1	99.3	87.8	45.1	7.5	100	7.6
0.6	100	98.5	53.6	98.7	86.3	33.8	7.4	95.1	6.5
0.4	100	97.8	21.3	92.8	85.4	19.1	7.1	69.2	6.8
0.2	100	95.3	9.8	87.7	78.2	7.9	7	24.6	6.7
					ρ = 0.8				
1	100	99.1	95.2	100	76.1	43.4	7.5	100	7.8
0.8	100	96.2	75.4	98.1	73.2	39.2	7.6	100	6.6
0.6	100	95.3	76.5	92.6	72.6	31.2	7.1	94.7	6.4
0.4	100	94.4	78.4	85.9	71	19.6	7	68.3	6.8
0.2	100	93.7	8.5	79.3	70.7	7.6	6.6	22.1	6.7
					ρ = 0.6				
1	100	98.4	94.2	100	54.1	30.4	7.5	100	7.8
0.8	100	95.5	50.6	97.1	53	27.5	7.2	100	6.6
0.6	100	95.1	54.1	92.1	52.5	28.1	7	95.1	6.5
0.4	100	93.9	53.8	81.2	52.1	17.5	7.1	67.1	6.5
0.2	100	93.1	15.1	78.7	54.1	8.1	6.3	21.8	6.8
					ρ = 0.4				
1	100	97.9	91.5	100	51.5	28.5	7.4	100	7.1
0.8	100	94.9	49.1	96.2	50.2	27.4	7.2	99.8	6.2
0.6	100	93.7	33.9	90.1	49.1	26.9	7.2	94.3	6.8
0.4	100	91.5	26.4	76.8	47.6	13.1	7	69.1	6.5
0.2	100	88.4	21.4	68.3	45.1	7.9	5.9	20.1	6.7
					ρ = 0.2				
1	100	97.3	88.8	100	50.2	23.6	7.3	100	7.2
0.8	100	94.7	44.1	93.4	37.9	23.4	7.1	96.3	6.8
0.6	100	89.6	25.4	87.1	36.9	19.8	7	93.7	6.4
0.4	100	81.4	15.2	69.9	31.1	12.6	7	65.1	6.3
0.2	100	74.7	13.1	62.3	30	7.5	5.7	19.8	6.2

Table 6.27: Power Analysis of Contemporaneous Dependent Series with drift

WD	OLS	ARDL (1, 1)			ARDL (2, 2)				
	Xt	xt	xt-1	yt-1	xt	xt-1	xt-2	yt-1	yt-2
$\theta_1 = \theta_2$					ρ = 1				
1	100	95.8	69.2	100	94.6	56.2	7.5	100	7.6
0.8	97.3	95.3	59.5	100	87.4	47.5	7.6	100	6.6
0.6	96.9	93.8	37.4	96.7	86.2	31.7	7.2	96.5	6.5
0.4	95.3	93.6	22.4	70.3	85.4	19.3	7.1	68.4	6.5
0.2	93.2	92.3	9.4	20.5	84.2	10.4	6.7	22.3	6.7
					ρ = 0.8				
1	98.2	85.6	53.7	100	83.1	39.2	7.5	100	7.8
0.8	94.2	83.2	45.2	99.8	81.2	19.2	7.6	100	6.6
0.6	86.5	82.4	26.9	95.2	77.4	18.6	7.1	95.9	6.4
0.4	82.1	79.1	16.9	69.9	76.1	11.2	7	67.5	6.5
0.2	78.1	78.5	7.1	23.3	75.2	10.2	6.6	21.4	6.7
					ρ = 0.6				
1	97.9	62.4	32.2	100	72.5	29.7	7.5	100	7.8
0.8	82.1	50.6	29.5	99.5	62.1	27.5	7.2	100	6.6
0.6	76.4	54.1	16.2	96.1	61	27.3	7	95.8	6.4
0.4	69.3	53.8	12.5	66.4	57.3	15.8	7.1	65.6	6.5
0.2	57.9	54.7	7.2	19.1	52.2	8.1	6.3	20.1	6.7
					ρ = 0.4				
1	97.5	70.5	23.3	100	50.6	28.5	7.4	100	7.3
0.8	81.3	49.1	16.5	99.4	49.9	27.4	7.2	99.8	6.2
0.6	44.5	30.9	10.6	95.4	49.2	26.9	7.2	94.3	6.3
0.4	39.3	28.4	8.6	70.8	47.3	13.1	7	69.1	6.5
0.2	32.1	58	6.2	22.3	46.1	7.9	5.9	20.1	6.7
					ρ = 0.2				
1	97.4	70.8	21.5	100	50.2	23.6	7.3	100	7.2
0.8	78.7	40.1	7.7	99.7	37.7	23.4	7.1	97.6	6.6
0.6	57.5	10.4	8.7	95.1	36.9	19.7	7	91.2	6.3
0.4	31.6	13.6	6.8	67.4	31.1	12.6	7	62.6	6.3
0.2	13.4	13.8	5.5	19.3	28	6.5	5.7	20.7	6.1

Table 6.28: Power Analysis of Contemporaneous Dependent Series with drift and trend

WDT	OLS	ARDL (1, 1)			ARDL (2, 2)				
	xt	xt	xt-1	yt-1	xt	xt-1	xt-2	yt-1	yt-2
$\theta_1 = \theta_2$					ρ = 1				
1	100	98.8	90.7	100	89.8	54.2	7.4	100	8.5
0.8	100	98.6	64.3	99.2	87.9	47.4	7.6	100	7.3
0.6	100	97.9	53.9	98.6	86.7	31.2	7.2	99.8	6.5
0.4	100	96.3	21.3	91.6	84.9	19.1	7.1	87.4	6.2
0.2	100	95.1	11.8	87.1	80.2	7.9	6.7	722	6.1
					ρ = **0.8**				
1	100	97.8	95.7	100	75.1	41.4	7.5	100	7.6
0.8	100	96.6	76.2	98.1	73.2	39.2	7.6	100	6.4
0.6	100	95.9	76.5	92.9	71.4	31.2	7.1	95.9	6.2
0.4	100	94.5	75.1	86.3	70.9	18.6	7	67.5	6.9
0.2	100	94.7	10.5	71.6	68.7	7.6	6.6	51.1	6.1
					ρ = **0.6**				
1	100	97.5	76.5	100	52.5	29.7	7.5	100	7.5
0.8	100	94.3	52.4	98.2	52.3	27.8	7.2	97.4	6.8
0.6	100	94.1	54.1	96.1	51.8	27.3	7	95.1	6.1
0.4	100	93.8	52.4	68.4	51.4	15.8	7.1	67.1	5.9
0.2	100	92.6	51.3	17.1	52.2	8.5	6.3	31.5	6
					ρ = **0.4**				
1	100	95.1	73.5	100	51.6	28.5	7.4	100	7.3
0.8	100	94.6	32.6	97.3	49.7	27.4	7.2	96.1	6.2
0.6	100	92.7	30.4	95.4	46.5	26.7	7.2	93.8	6.3
0.4	100	91.4	29.4	72.8	44.3	13.2	7	64.2	6.5
0.2	100	86.8	31.3	25.4	46.8	7.2	5.9	20	6.7
					ρ = **0.2**				
1	100	94.3	70.8	100	50.6	23.6	7.3	100	7.2
0.8	100	93.9	42.1	95.3	38.7	23.4	7.1	94.3	6.6
0.6	100	89.4	10.4	94.2	36.9	20.7	7	92.4	6
0.4	100	82.5	13.7	69.8	29.8	12.6	7	63.6	5.9
0.2	100	76.8	14.3	22.1	24.3	5.3	5.7	19.3	6.1

In first row of first panel of table 6.25, the results are indicating that when series are nonstationary, θ_1=1 and θ_2=0.8, having no drift and trend, a_1= b_1= 0 =a_2= b_2= 0, θ_{21}=0 and covariance ρ=1, then the OLS power is 76.5%, which shows 18.5% power loss on the basis of 5% nominal size. In case of ARDL models F-test are being used to test the joint significance of current and lag values of independent variable. The F-stat value after employing ARDL (1, 1) model is indicating that the power of ARDL (1, 1) model is 87.5% which shows 7.5% power loss at 5% nominal size. The power of ARDL (2, 2) is 82.3% and loss of power is 12.7% at 5% nominal size.

In second row of first panel of table 6.25, the results are representing that when series are stationary, θ_1= 0.8 and θ_2 = 0.8, having no drift and trend, a_1= b_1= 0 =a_2= b_2= 0, θ_{21}=0 and ρ= 1, then the OLS power is 77.2%, which shows 17.8% power loss on the basis of 5% nominal size. In case of ARDL models F-test is being used to test the joint significance of current and lag values of independent variable. The F-stat value after employing ARDL (1, 1) model is indicating that the power of ARDL (1, 1) model is 81.2% which shows 13.8% power loss at 5% nominal size. The power of ARDL (2, 2) is 80.2% and loss of power is 14.8% at 5% nominal size.

In first row of second panel of table 6.25, the results are indicating that when series are nonstationary, θ_1=1 and θ_2=0.8, having no drift and trend, a_1= b_1= 0 =a_2= b_2= 0, θ_{21}=0 and ρ= 0.8, then the OLS power is 72.1%, which shows 22.9% power loss on the basis of 5% nominal size. In case of ARDL models F-test are being used to test the joint significance of current and lag values of independent variable. The F-stat value after employing ARDL (1, 1) model is indicating that the power of ARDL (1, 1) model is 74.6% which shows 20.4% power loss at 5% nominal size. The power of ARDL (2, 2) is 64.9% and loss of power is 30.1% at 5% nominal size.

In second row of second panel of table 6.25, the results are representing that when series are stationary, θ_1= 0.8 and θ_2 = 0.8, having no drift and trend, a_1= b_1= 0 =a_2= b_2= 0, θ_{21}=0 and ρ= 0.8 then the OLS power is 67.3%, which shows 27.7% power loss on the basis of 5% nominal size. In case of ARDL models F-test are being used to test the joint significance of current and lag values of independent variable. The F-stat value after employing ARDL (1, 1) model is indicating that the power of ARDL (1, 1) model is 69.7% which shows 25.3% power loss at 5% nominal size. The power of ARDL (2, 2) is 56.7% and loss of power is 38.3% at 5% nominal size.

It shows that the conventional OLS method badly suffers in power problem when series are nonstationary even they are stationary with no drift and trend. On contrary ARDL model is not showing huge power in both cases. It clarifies that when series are stationary or nonstationary without having drift and trend ARDL works better than OLS.

In first row of first panel of table 6.26, the results show that when we regressed nonstationary seriesθ_1=1 and θ_2=0.8, without drift, a_1= b_1= 0, θ_{21} = 0 and ρ= 1 by using OLS, the probability of rejection of false null hypothesis (power) is 100%, which represents a misleading figure. Because as seen above in table 5.1 the OLS has huge size distortion problem, specially, when series are with linear trend. That is why it is showing 100% power which does not exist in reality. The F-test used only in one case for displaying the joint significance of independent lag and current value. So, table 6.26, 6.27 and 6.28 have only t-stat values. After employing ARDL (1, 1) model the power of current value of x is 99.6%, which shows only 0% power loss. The reason behind it is that we did not include linear trend in ARDL if we include linear trend in it may provide more power. The numeral of lag value of x is showing only 90.6% power which means 4.4% power loss. In ARDL (2, 2) model the first lag value of x showing 51.2%

probability of rejection of the false null hypothesis. The powers of current and second lag values of x are 90.5% and 7.8%, which show 4.5% and 87.2% power loss respectively

As we know that y value is determined through lag value of x, but the first lag value are more significant as compare to current value of x. The reason is that there is multicollinearity effect, the current and lag values of x variable are collinear that is why the effect shifts into lag value in ARDL(1, 1) and in lag value in ARDL (2, 2).

In second row of first panel of table 6.26, the results illustrate that when we regressed stationary series θ_1=0.8 and θ_2=0.8, without drift, a_1= b_1= 0, θ_{21} = 0 and ρ= 1 by using OLS, the probability of rejection of false null hypothesis (power) is 100%, which represents a misleading figure. Because as we see in table 5.1, the OLS has huge size distortion problem, specially, when series are with linear trend. That is why, it is showing 100% power which does not exist in reality. After employing ARDL (1, 1) model, the power of current value of x is 97.3%, which means only 0% power loss, the reason is that we did not include linear trend in ARDL. If we include linear trend, it may provide more power. The figure of lag value of x is showing only 64.1% power which means 30.9% power loss. In ARDL (2, 2), model the first lag value of x is showing 87.7% probability of rejection the false null hypothesis. The powers of current and second lag values of x are 45.1% and 7.5%, which shows 49.9% and 87.5% power loss respectively

Same pattern has been found on other values of θ_{21} like, 0.6 and so on. On all these points ARDL shows better performance as compare to OLS.

On some values of parameter$s$ θ_1 and θ_2, and θ_{21} and OLS shows more power but we cannot consider it because as we have seen in size analysis the OLS suffers badly in

size problem while ARDL in all cases has less size problem. In case without drift or with linear trend, and with drift and trend due to under specification, ARDL shows size problem but even in these cases OLS has more size distortion as compare to ARDL model.

There is another special effect which we should consider, In case of stationary and nonstationary time series the ARDL model works very well but it becomes unable to reduce the probability of spurious regression significantly in presence of trend. Basically both models OLS and ARDL are under specified because the series are having linear trend but models do not have linear trend term in their equations. On the other hand, we can see that the OLS model completely failed to tackle this problem but ARDL model works well as compare to OLS in size analysis. There is an important point, our data generating process in equation 6.1, generates first order autoregressive series AR(1) but we used second lag in ARDL (2, 2) model, so in case of over specification ARDL (2, 2) shows more power in case of stationary series as compare to ARDL (1, 1) and OLS. It also explores that the ARDL models perform better than OLS in under specification and over specification.

Similarly table 6.27 and 6.28 results also display the results of next two scenarios of lag and contemporaneous dependent series with drift and with drift and trend. So, the interpretations of these cases are approximately alike that is why we are interpreting them here. The lag values of y itself are highly significant in all cases, but one thing which is necessary is that as we reduce the value of autoregressive terms, the lag significance is also going to decrease in case of ARDL not in OLS.

6.4 Robustness of Power to Misspecification

In this analysis we evaluate the robustness of conventional cointegration procedures Engle and Granger, Johansen and Juselius and ARDL model with different specifications on the basis of power analysis. The possible three specification cases which have been considered in this analysis are under, exact and over specified regression. The Monte Carlo simulations have been used in this analysis. All the results in tables given below have been summarized after 100,000 times simulations. The series have been generated by using data generating process in equation 5.6. In this analysis only nonstationary series are used and with autoregressive parameter specification θ_1=1 and θ_1=0.8.

Specification Cases

		Data Generating Process	
		Drift	***Drift and Trend***
Test Equation	***Drift***	Exactly Specified	Under Specified
	Drift and Trend	Over Specified	Exactly Specified

In our analysis two cases of exact specification have been considered. First, when data generating process and test equation both contain drift term second, when data generating process and test equation both contain drift and trend term. The under specification means when data generating process contains drift and trend and test equation takes on drift and trend terms. The over specification generates when data generating process contains drift and test equation takes drift and trend terms.

In fact, Regression analysis comprises three major stages, model specification, estimation of regression parameters and interpretation of estimated parameters. Thus first and crucial stage is the specification of regression equation. The reliability of estimated parameters and interpretation mainly rely on the correct specification of model. Consequently, misspecification can generate two types of errors. First when we include theoretically irrelevant variable(s) in regression equation and second, when we exclude theoretically relevant variable from regression equation. These specification errors can generate estimation and interpretation problems. Misspecification may produce any little problem when the independent variables are uncorrelated or orthogonal to each other. When we include or omit an orthogonal independent variable from regression equation, it will affect the standard errors of partial regression coefficients. The exclusion of relevant variable has serious issues, it will lead to size and power problems. In this analysis we compare the power of conventional cointegration procedures and ARDL model and see which one is working well in these three types of specifications.

In this analysis we estimate the power of conventional cointegration procedures and ARDL model because of series dependent. The powers of conventional cointegration procedures and ARDL model are measured by taking different specifications and different values of. θ_2. We used three types of dependent series in this analysis

i. Lag dependent series
ii. Contemporaneous and lag dependent series
iii. Contemporaneous dependent series

The results are summarized in given below tables after 100,000 simulations. Different specifications of regression equations have been taken as follows:

i. Exactly specified
ii. Under specified
iii. Over specified

The description of these specifications has been given in chapter 5.

Table 6.29: Power Analysis of Engle and Granger Cointegration Test by using Lag dependent Series at Different Specifications

Engle Granger (EG) Cointegration Test			
		$\theta_2 = 0.8$	
		Data Generating Process	
		Drift	***Drift and Trend***
Test Equation	***Drift***	24.1	30.5
	Drift and Trend	10.1	19.1
		$\theta_2 = 0.6$	
		Data Generating Process	
		Drift	***Drift and Trend***
Test Equation	***Drift***	60.1	50.2
	Drift and Trend	70.8	51.6
		$\theta_2 = 0.4$	
		Data Generating Process	
		Drift	***Drift and Trend***
Test Equation	***Drift***	96.3	64.7
	Drift and Trend	97.6	63.8
		$\theta_2 = 0.2$	
		Data Generating Process	
		Drift	***Drift and Trend***
Test Equation	***Drift***	99.9	98.1
	Drift and Trend	98.7	99.9

Table 6.30: Power Analysis of Johansen and Juselius Cointegration Test by using Lag dependent Series at Different Specifications

Johansen and Juselius (JJ) Cointegration Test			
		θ_2= 0.8	
		Data Generating Process	
		Drift	***Drift and Trend***
Test Equation	***Drift***	38.4	48.7
	Drift and Trend	22.2	25.1
		θ_2= 0.6	
		Data Generating Process	
		Drift	***Drift and Trend***
Test Equation	***Drift***	64.7	51.3
	Drift and Trend	44.8	60.6
		θ_2= 0.4	
		Data Generating Process	
		Drift	***Drift and Trend***
Test Equation	***Drift***	81.3	85.3
	Drift and Trend	78.4	99.2
		θ_2= 0.2	
		Data Generating Process	
		Drift	***Drift and Trend***
Test Equation	***Drift***	91.5	99.9
	Drift and Trend	98.1	100

Table 6.31: Power Analysis of ARDL Model by using Lag dependent Series at Different Specifications

ARDL Model			
		$\theta_2 = 0.8$	
		Data Generating Process	
		Drift	*Drift and Trend*
Test Equation	*Drift*	98.1	98.7
	Drift and Trend	99	99.9
		$\theta_2 = 0.6$	
		Data Generating Process	
		Drift	*Drift and Trend*
Test Equation	*Drift*	99.3	99
	Drift and Trend	99	99.9
		$\theta_2 = 0.4$	
		Data Generating Process	
		Drift	*Drift and Trend*
Test Equation	*Drift*	99.5	99
	Drift and Trend	99	100
		$\theta_2 = 0.2$	
		Data Generating Process	
		Drift	*Drift and Trend*
Test Equation	*Drift*	100	99
	Drift and Trend	99	100

Table 6.29, shows the results of power of Engle and Granger cointegration test under different specifications at different values of autoregressive parameter θ_2. At first we consider correct specification with drift and drift and trend cases. The first panel of

table 6.29, describes the results of Engle and Granger cointegration test when θ_2=0.8. The power in case of correct specification is 24.1% with drift. It shows 70.9% power loss when autoregressive parameter θ_2= 0.8 and x lag value coefficient θ_{21}= 0.8. The power under correct specification is 24.1% with drift and trend. It shows 75.9% power loss when lag value parameter of y is θ_2= 0.8 and x lag value coefficient is θ_{21}= 0.8. Table 6.30 describes the results of power of Johansen and Juselius cointegration test under different specifications at different values of autoregressive parameter θ_2. Firstly, we consider correct specification with drift and drift and trend cases. The first panel of table 6.30, refers the results of Johansen and Juselius cointegration test when θ_2=0.8. The power in case of correct specification is 38.4% with drift. It shows 56.6% power loss when θ_2= 0.8 and x lag value coefficient is θ_{21}= 0.8. The power under correct specification is 25.1% with drift and trend. It shows 74.9% power loss when θ_2= 0.8 andθ_{21}= 0.8. Table 6.31, describes the results of power of ARDL model under different specifications at different values of autoregressive parameter θ_2. Firstly, we consider correct specification with drift and drift and trend cases. The first panel of table 6.31, refers the results of ARDL model when θ_2=0.8. The power in case of correct specification is 98.1% with drift. It shows 0% power loss when θ_2= 0.8 and θ_{21}= 0.8. The power under correct specification is 99.9% with drift and trend. It shows 0% power loss at θ_2= 0.8 and θ_{21}= 0.8.

The second panel of table 6.29, illustrates the power results of Engle and Granger cointegration test when θ_2=0.6. The power in case of correct specification is 60.1% with drift. It shows 34.9% power loss when θ_2= 0.6 andθ_{21}= 0.8. The power under correct specification is 51.6% with drift and trend, it shows 43.4% power loss when θ_2= 0.8 andθ_{21}= 0.8. The second panel of table 6.30 given above explains the power results of Johansen and Juselius cointegration test when θ_2=0.6. The power in case of correct specification is 64.7% with drift, it shows 30.3% power loss when θ_2= 0.6 andθ_{21}= 0.8. The power under correct specification is 60.6% with drift and trend, it shows 34.4% power loss when θ_2= 0.8 andθ_{21}= 0.8. The second panel of table 6.31 given above describes the power results of ARDL model when θ_2=0.6. The power in case of correct specification is 99.3% with drift, it shows 0% power loss when θ_2= 0.6 and θ_{21}= 0.8. The power under correct specification is 99.9% with drift and trend, it shows 0% power loss when θ_2= 0.8 andθ_{21}= 0.8. The order of statistics of power in case of correct specification is following:

$$\text{Power (ARDL} > JJ > EG)$$

Thus, these results validate that conventional cointegration procedures are having very low power when checked at θ_2= 0.8 andθ_{21}= 0.8. On contrast, the ARDL model has no power loss compare to Johansen and Juselius cointegration tests. It means in case of correct specification, the ARDL model works well than other conventional techniques.

Similarly, when the values of autoregressive parameters are θ_2 = 0.4 and 0.2, same pattern has been found. It clearly indicates that in correct specification cases the ARDL model have huge power while conventional cointegration procedures are having huge power loss. We cannot compare the power of these tests and ARDL model because as we have seen in size analysis these cointegration procedure suffer in size distortion problem even in case of correct specification.

Secondly, we consider the case of under specification. Table 6.29, shows the results of power of Engle and Granger cointegration test under different specifications at different values of autoregressive parameter θ_2. The first panel of table 6.29, describes the results of Engle and Granger cointegration test when θ_2=0.8. The power in case of under specification is 30.5% with drift. it shows 64.5% power loss when θ_2= 0.8 andθ_{21}= 0.8.

Table 6.30, shows the results of power of Johansen and Juselius cointegration tests under different specifications at different values of autoregressive parameter θ_2. The first panel of table 6.30, describes the results of Johansen and Juselius cointegration test when θ_2=0.8. The power in case of under specification is 48.7% with drift. It shows 46.3% power loss when θ_2= 0.8 andθ_{21}= 0.8. Table 6.31, shows the results of power of ARDL model under different specifications at different values of autoregressive parameter θ_2. The first panel of table 6.31 describes the results of ARDL model when θ_2=0.8. The power in case of under specification is 98.7% with drift. it shows 0% power loss when θ_2= 0.8 andθ_{21}= 0.8. Table 6.29, shows the results of power of Engle and Granger cointegration test under different specifications at different values of autoregressive parameter θ_2. The second panel of table 6.29, describes the results of Engle and Granger cointegration test when θ_2=0.6. The power in case of under specification is 50.2%, it shows 44.8% power loss when θ_2= 0.6 andθ_{21}= 0.8. Table 6.30, shows the results of power of Johansen and Juselius cointegration test under different specifications at different values of autoregressive parameter θ_2. The second panel of table 6.30, describes the results of Johansen and Juselius cointegration test when θ_2=0.6. The power in case of under specification is 51.3% with drift. It shows 43.7% power loss when θ_2= 0.6 andθ_{21}= 0.8. Table 6.31 shows the results of power of ARDL model under different specifications at different values of autoregressive parameter θ_2. The first panel of table 6.31, describes the results of ARDL model when θ_2=0.6. The power in case of under specification is 99.0% with drift. It shows 0% power loss when θ_2= 0.6 andθ_{21}= 0.8.

Thus, these results validate that conventional cointegration procedure Engle and Granger Johansen and Juselius cointegration test are having very low power when θ_2= 0.8 andθ_{21}= 0.8. On the other hand, the ARDL model shows no power loss compare to

Johansen and Juselius cointegration test. It does not mean in case of under specification the ARDL model works well than other conventional techniques.

In case of under specification, ARDL model badly suffers in size distortion problem as we have seen in size analysis. So, we cannot say that ARDL has power on the basis of these statistics that is in under specification according to our estimation the Johansen and Juselius cointegration test works well. Because it has minimum size distortion under specification. The order of power of ARDL in case of under specification is given in the following equation:

$$\text{Power } (\text{JJ} > EG > ARDL)$$

Similarly at all the values of autoregressive parameter θ_2 = 0.4 and 0.2 same pattern has been found.

At third we consider the case of over specification. Table 6.29, shows the results of power of Engle and Granger cointegration test over different specifications at different values of autoregressive parameter θ_2. The first panel of table 6.29, describes the results of Engle and Granger cointegration test when θ_2=0.8. The power in case of over specification is 24.1%. It shows 70.9% power loss when θ_2=0.8 andθ_{21}= 0.8. Table 6.30, shows that the results of power of Johansen and Juselius cointegration test over different specifications at different values of autoregressive parameter θ_2. The first panel of table 6.30, describes the results of Johansen and Juselius cointegration test when θ_2=0.8. The power in case of under specification is 38.4% with drift. It shows 56.6% power loss when θ_2= 0.8 andθ_{21}= 0.8. Table 6.31, shows the results of power of ARDL model under different specifications at different values of autoregressive parameter θ_2. The first panel of table 6.31, describes the results of ARDL model when θ_2=0.8. The power in case of over specification is 98.1% with drift. It shows 0% power loss when θ_2= 0.8 andθ_{21}= 0.8. The second panel of table 6.29 describes the results of Engle and Granger cointegration test when θ_2=0.6. The power in case of over specification is 60.1%. It shows 34.9% power loss when θ_2= 0.6 andθ_{21}= 0.8. The second panel of table 6.30 describes the results of Johansen and Juselius cointegration test when θ_2=0.6. The power in case of under specification is 64.7% with drift, it shows 30.3% power loss when θ_2= 0.6 andθ_{21}= 0.8. The first panel of table 6.31, describes the results of ARDL model when θ_2=0.6. The power in case of under specification is 99.3% with drift. It shows 0% power loss when θ_2= 0.6 andθ_{21}= 0.8.

Thus, these results validate that conventional cointegration procedure Engle and Granger Johansen and Juselius cointegration test are having very low power when we check them at at θ_2= 0.8 and 0.6, andθ_{21}= 0.8. On contrast, the ARDL model shows no power loss as compare to Johansen and Juselius cointegration test. It means in case of over specification, the ARDL model works well than other conventional techniques. In case of over specification ARDL model does not suffer in size distortion problem as we have seen in size analysis. So, we can say that ARDL has more power in case of over specification. The power of ARDL is given as follows;

$$\text{Power } (\text{ARDL} > JJ > EG)$$

Similarly at all the values of autoregressive parameter θ_2 = 0.4 and 0.2 same pattern has been found. It clearly indicates that in over specification ARDL works good as compare to other techniques.

There is a special effect which needs to be analyzed. If we decrease the value of autoregressive parameter θ_2 from 0.8 to 0.6 and so on, the powers of these procedures are increasing. The reason behind this is that power of test depends upon relationship between y and x, in presence of high autoregressive parameter value. That is why as we decrease the value of lag of y the relationship between y and x, gets stronger. Other reason is that in this analysis we only decrease the value of autoregressive parameter θ_2 from 0.8 to 0.2 but we do not change parameter value of lag of x. θ_{21}=0.8. Due to this reason lag of x. gets strong correlation as we decrease θ_2 value. In other words, when decrease θ_2 value θ_{21} remians same, then y depends more on x. as compare to its own lag.

Table 6.32: Power Analysis of Engle and Granger Cointegration Test by using Lag and Contemporaneous dependent Series at Different Specifications when $\rho = 1$

Engle Granger (EG) Cointegration Test			
		$\rho = 1$	
		$\theta_2 = 0.8$	
		Data Generating Process	
		Drift	***Drift and Trend***
Test Equation	***Drift***	53.2	56.4
	Drift and Trend	50.5	51.6
		$\theta_2 = 0.6$	
		Data Generating Process	
		Drift	***Drift and Trend***
Test Equation	***Drift***	61.2	64.7
	Drift and Trend	71.2	55.4
		$\theta_2 = 0.4$	
		Data Generating Process	
		Drift	***Drift and Trend***
Test Equation	***Drift***	91.9	80.3
	Drift and Trend	96.7	60.1
		$\theta_2 = 0.2$	
		Data Generating Process	
		Drift	***Drift and Trend***
Test Equation	***Drift***	99.8	99.7
	Drift and Trend	99.9	99

Table 6.33: Power Analysis of Johansen and Juselius Cointegration Test by using Lag and Contemporaneous dependent Series at Different Specifications when $\rho = 1$

Johansen and Juselius (JJ) Cointegration Test			
		$\rho = 1$	
		$\theta_2 = 0.8$	
		Data Generating Process	
		Drift	***Drift and Trend***
Test Equation	***Drift***	54.6	62.3
	Drift and Trend	53.8	52.1
		$\theta_2 = 0.6$	
		Data Generating Process	
		Drift	***Drift and Trend***
Test Equation	***Drift***	65.2	56.9
	Drift and Trend	74.8	58.3
		$\theta_2 = 0.4$	
		Data Generating Process	
		Drift	***Drift and Trend***
Test Equation	***Drift***	94.1	65.8
	Drift and Trend	98.2	87.3
		$\theta_2 = 0.2$	
		Data Generating Process	
		Drift	***Drift and Trend***
Test Equation	***Drift***	100	99
	Drift and Trend	99	99

Table 6.34: Power Analysis of ARDL Model by using Lag and Contemporaneous dependent Series at Different Specifications when $\rho = 1$

		ARDL Model	
		$\rho = 1$	
		$\theta_2 = 0.8$	
		Data Generating Process	
		Drift	***Drift and Trend***
Test Equation	***Drift***	99.9	99
	Drift and Trend	99	99
		$\theta_2 = 0.6$	
		Data Generating Process	
		Drift	***Drift and Trend***
Test Equation	***Drift***	99.9	99
	Drift and Trend	99	99
		$\theta_2 = 0.4$	
		Data Generating Process	
		Drift	***Drift and Trend***
Test Equation	***Drift***	100	99
	Drift and Trend	99	100
		$\theta_2 = 0.2$	
		Data Generating Process	
		Drift	***Drift and Trend***
Test Equation	***Drift***	100	99
	Drift and Trend	99	100

Table 6.32, shows the results of power of Engle and Granger cointegration test under different specifications at different values of autoregressive parameter θ_2 when

covariance $\rho = 1$. At first we consider correct specification with drift and drift and trend cases. The first panel of table 6.32, describes the results of Engle and Granger cointegration test when θ_2=0.8. The power in case of correct specification is 53.2% with drift. It shows 41.5% power loss when autoregressive parameter θ_2= 0.8 and x lag value coefficient θ_{21}= 0.8. The power under correct specification is 51.6% with drift and trend. It shows 43.4% power loss when lag value parameter of y is θ_2= 0.8 and x lag value coefficient is θ_{21}= 0.8. Table 6.33 describes the results of power of Johansen and Juselius cointegration test under different specifications at different values of autoregressive parameter θ_2. Firstly, we consider correct specification with drift and drift and trend cases. The first panel of table 6.33, refers the results of Johansen and Juselius cointegration test when θ_2=0.8. The power in case of correct specification is 54.6% with drift. It shows 40.4% power loss when θ_2= 0.8 and x lag value coefficient is θ_{21}= 0.8. The power under correct specification is 52.2% with drift and trend. It shows 42.8% power loss when θ_2= 0.8 andθ_{21}= 0.8. Table 6.34, describes the results of power of ARDL model under different specifications at different values of autoregressive parameter θ_2. Firstly, we consider correct specification with drift and drift and trend cases. The first panel of table 6.34, refers the results of ARDL model when θ_2=0.8. The power in case of correct specification is 99.9% with drift. It shows 0% power loss when θ_2= 0.8 and θ_{21}= 0.8. The power under correct specification is 99.9% with drift and trend. It shows 0% power loss at θ_2= 0.8 and θ_{21}= 0.8.

The second panel of table 6.32, illustrates the power results of Engle and Granger cointegration test when θ_2=0.6. The power in case of correct specification is 61.2% with drift. It shows 33.8% power loss when θ_2= 0.6 andθ_{21}= 0.8. The power under correct specification is 55.4% with drift and trend, it shows 39.6% power loss when θ_2= 0.8 and θ_{21}= 0.8. The second panel of table 6.33 given above explains the power results of Johansen and Juselius cointegration test when θ_2=0.6. The power in case of correct specification is 65.2% with drift, it shows 29.8% power loss when θ_2= 0.6 and θ_{21}= 0.8. The power under correct specification is 58.3% with drift and trend, it shows 36.7% power loss when θ_2= 0.8 and θ_{21}= 0.8. The second panel of table 6.34 given above describes the power results of ARDL model when θ_2=0.6. The power in case of correct specification is 99.9% with drift, it shows 0% power loss when θ_2= 0.6 and θ_{21}= 0.8. The power under correct specification is 99.0% with drift and trend, it shows 0% power loss when θ_2= 0.8 andθ_{21}= 0.8. The order of statistics of power in case of correct specification is following:

$$\text{Power} \ \ (\text{ARDL} > JJ > EG)$$

Thus, these results validate that conventional cointegration procedures are having very low power when checked at θ_2= 0.8 andθ_{21}= 0.8. On contrast, the ARDL model has no power loss compare to Johansen and Juselius cointegration tests. It means in case of correct specification, the ARDL model works well than other conventional techniques.

Similarly, when the values of autoregressive parameters are θ_2 = 0.4 and 0.2, same pattern has been found. It clearly indicates that in correct specification cases the ARDL model have huge power while conventional cointegration procedures are having huge power loss. We cannot compare the power of these tests and ARDL model because as we have seen in size analysis these cointegration procedure suffer in size distortion problem even in case of correct specification.

Secondly, we consider the case of under specification. Table 6.32, shows the results of power of Engle and Granger cointegration test under different specifications at different values of autoregressive parameter θ_2. The first panel of table 6.32, describes the results of Engle and Granger cointegration test when θ_2=0.8. The power in case of under specification is 56.4% with drift. It shows 38.6% power loss when θ_2= 0.8 and θ_{21}= 0.8. Table 6.33, shows the results of power of Johansen and Juselius cointegration tests under different specifications at different values of autoregressive parameter θ_2. The first panel of table 6.33, describes the results of Johansen and Juselius cointegration test when θ_2=0.8. The power in case of under specification is 62.3% with drift. It shows 32.7% power loss when θ_2= 0.8 andθ_{21}= 0.8. Table 6.34, shows the results of power of ARDL model under different specifications at different values of autoregressive parameter θ_2. The first panel of table 6.34 describes the results of ARDL model when θ_2=0.8. The power in case of under specification is 99.9% with drift. It shows 0% power loss when θ_2= 0.8 andθ_{21}= 0.8. Table 6.32, shows the results of power of Engle and Granger cointegration test under different specifications at different values of autoregressive parameter θ_2. The second panel of table 6.32, describes the results of Engle and Granger cointegration test when θ_2=0.6. The power in case of under specification is 64.7%, it shows 30.3% power loss when θ_2= 0.6 andθ_{21}= 0.8. Table 6.33, shows the results of power of Johansen and Juselius cointegration test under different specifications at different values of autoregressive parameter θ_2. The second panel of table 6.33, describes the results of Johansen and Juselius cointegration test when θ_2=0.6. The power in case of under specification is 56.9% with drift. It shows 38.1% power loss when θ_2= 0.6 andθ_{21}= 0.8. Table 6.34 shows the results of power of ARDL model under different specifications at different values of autoregressive parameter θ_2. The first panel of table 6.34, describes the results of ARDL model when θ_2=0.6. The power in case of under specification is 99.0% with drift. It shows 0% power loss when θ_2= 0.6 and θ_{21}= 0.8.

Thus, these results validate that conventional cointegration procedure Engle and Granger Johansen and Juselius cointegration test are having very low power when θ_2=

0.8 and θ_{21}= 0.8. On the other hand, the ARDL model shows no power loss compare to Johansen and Juselius cointegration test. It does not mean in case of under specification the ARDL model works well than other conventional techniques.

In case of under specification, ARDL model badly suffers in size distortion problem as we have seen in size analysis. So, we cannot say that ARDL has power on the basis of these statistics that is in under specification according to our estimation the Johansen and Juselius cointegration test works well. Because it has minimum size distortion under specification. The order of power of ARDL in case of under specification is given in the following equation:

$$\text{Power } (\text{JJ} > EG > ARDL)$$

Similarly at all the values of autoregressive parameter θ_2 = 0.4 and 0.2 same pattern has been found.

At third we consider the case of over specification. Table 6.32, shows the results of power of Engle and Granger cointegration test over different specifications at different values of autoregressive parameter θ_2. The first panel of table 6.32, describes the results of Engle and Granger cointegration test when θ_2=0.8. The power in case of over specification is 50.5%. It shows 44.5% power loss when θ_2=0.8 andθ_{21}= 0.8. Table 6.33, shows that the results of power of Johansen and Juselius cointegration test over different specifications at different values of autoregressive parameter θ_2. The first panel of table 6.33, describes the results of Johansen and Juselius cointegration test when θ_2=0.8. The power in case of under specification is 53.8% with drift and trend. It shows 41.2% power loss when θ_2= 0.8 and θ_{21}= 0.8. Table 6.34, shows the results of power of ARDL model under different specifications at different values of autoregressive parameter θ_2. The first panel of table 6.34, describes the results of

ARDL model when θ_2=0.8. The power in case of over specification is 99.0% with drift and trend. It shows 0% power loss when θ_2= 0.8 andθ_{21}= 0.8. The second panel of table 6.32 describes the results of Engle and Granger cointegration test when θ_2=0.6. The power in case of over specification is 71.2%. It shows 23.8% power loss when θ_2= 0.6 and θ_{21}= 0.8. The second panel of table 6.33 describes the results of Johansen and Juselius cointegration test when θ_2=0.6. The power in case of under specification is 74.8% with drift, it shows 20.2% power loss when θ_2= 0.6 and θ_{21}= 0.8. The second panel of table 6.34, describes the results of ARDL model when θ_2=0.6. The power in case of under specification is 99.3% with drift. It shows 0% power loss when θ_2= 0.6 and θ_{21}= 0.8.

Thus, these results validate that conventional cointegration procedure Engle and Granger Johansen and Juselius cointegration test are having very low power when we check them at θ_2= 0.8 and 0.6, and θ_{21}= 0.8. On contrast, the ARDL model shows no power loss as compare to Johansen and Juselius cointegration test. It means in case of over specification, the ARDL model works well than other conventional techniques. In case of over specification ARDL model does not suffer in size distortion problem as we have seen in size analysis. So, we can say that ARDL has more power in case of over specification. The power of ARDL is given as follows;

$$\text{Power}\ \ (\text{ARDL} > JJ > EG)$$

Similarly at all the values of autoregressive parameter θ_2 = 0.4 and 0.2 same pattern has been found. It clearly indicates that in over specification ARDL works good as compare to other techniques.

There is a special effect which needs to be analyzed. If we decrease the value of autoregressive parameter θ_2 from 0.8 to 0.6 and so on, the powers of these procedures are increasing. The reason behind this is that power of test depends upon relationship between y and x, in presence of high autoregressive parameter value. That is why as we decrease the value of lag of y the relationship between y and x, gets stronger. Other reason is that in this analysis we only decrease the value of autoregressive parameter θ_2 from 0.8 to 0.2 but we do not change parameter value of lag of x. θ_{21}=0.8. Due to this reason lag of x. gets strong correlation as we decrease θ_2 value. In other words, when decrease θ_2 value θ_{21} remians same, then y depends more on x. as compare to its own lag.

Table 6.35: Power Analysis of Engle and Granger Cointegration Test by using Lag and Contemporaneous dependent Series at Different Specifications when $\rho = 0.8$

	Engle Granger (EG) Cointegration Test		
		$\rho = 0.8$	
		θ_2= 0.8	
		Data Generating Process	
		Drift	***Drift and Trend***
Test Equation	***Drift***	40.7	53.3
	Drift and Trend	32.6	36.3
		θ_2= 0.6	
		Data Generating Process	
		Drift	***Drift and Trend***
Test Equation	***Drift***	59.7	62.6
	Drift and Trend	68.5	55.3
		θ_2= 0.4	
		Data Generating Process	
		Drift	***Drift and Trend***
Test Equation	***Drift***	91.7	78.4
	Drift and Trend	96.6	59.6
		θ_2= 0.2	
		Data Generating Process	
		Drift	***Drift and Trend***
Test Equation	***Drift***	99.98	99.97
	Drift and Trend	99.99	99.98

Table 6.36: Power Analysis of Johansen and Juselius Cointegration Test by using Lag and Contemporaneous dependent Series at Different Specifications when $\rho = 0.8$

Johansen and Juselius (JJ) Cointegration Test			
		$\rho = 0.8$	
		$\theta_2 = 0.8$	
		Data Generating Process	
		Drift	***Drift and Trend***
Test Equation	***Drift***	41.6	50.2
	Drift and Trend	37.7	39.9
		$\theta_2 = 0.6$	
		Data Generating Process	
		Drift	***Drift and Trend***
Test Equation	***Drift***	63.5	56.4
	Drift and Trend	72.1	57.1
		$\theta_2 = 0.4$	
		Data Generating Process	
		Drift	***Drift and Trend***
Test Equation	***Drift***	93.2	66.4
	Drift and Trend	98	94.3
		$\theta_2 = 0.2$	
		Data Generating Process	
		Drift	***Drift and Trend***
Test Equation	***Drift***	100	99
	Drift and Trend	99	99

Table 6.37: Power Analysis of ARDL Model by using Lag and Contemporaneous dependent Series at Different Specifications when $\rho = 0.8$

ARDL Model			
		$\rho = 0.8$	
		$\theta_2 = 0.8$	
		Data Generating Process	
		Drift	***Drift and Trend***
Test Equation	***Drift***	99	99
	Drift and Trend	99	99
		$\theta_2 = 0.6$	
		Data Generating Process	
		Drift	***Drift and Trend***
Test Equation	***Drift***	100	99
	Drift and Trend	99	99
		$\theta_2 = 0.4$	
		Data Generating Process	
		Drift	***Drift and Trend***
Test Equation	***Drift***	100	99
	Drift and Trend	99	100
		$\theta_2 = 0.2$	
		Data Generating Process	
		Drift	***Drift and Trend***
Test Equation	***Drift***	100	99
	Drift and Trend	99	100

Table 6.35, shows the results of power of Engle and Granger cointegration test under different specifications at different values of autoregressive parameter θ_2 when covariance $\rho = 0.8$. At first we consider correct specification with drift and drift and

trend cases. The first panel of table 6.35, describes the results of Engle and Granger cointegration test when θ_2=0.8. The power in case of correct specification is 40.7% with drift. It shows 54.3% power loss when autoregressive parameter θ_2= 0.8 and x lag value coefficient θ_{21}= 0.8. The power under correct specification is 36.3% with drift and trend. It shows 58.7% power loss when lag value parameter of y is θ_2= 0.8 and x lag value coefficient is θ_{21}= 0.8. Table 6.36 describes the results of power of Johansen and Juselius cointegration test under different specifications at different values of autoregressive parameter θ_2. Firstly, we consider correct specification with drift and drift and trend cases. The first panel of table 6.36, refers the results of Johansen and Juselius cointegration test when θ_2=0.8. The power in case of correct specification is 41.6% with drift. It shows 44.4% power loss when θ_2= 0.8 and x lag value coefficient is θ_{21}= 0.8. The power under correct specification is 39.9% with drift and trend. It shows 55.1% power loss when θ_2= 0.8 andθ_{21}= 0.8. Table 6.37, describes the results of power of ARDL model under different specifications at different values of autoregressive parameter θ_2. Firstly, we consider correct specification with drift and drift and trend cases. The first panel of table 6.37, refers the results of ARDL model when θ_2=0.8. The power in case of correct specification is 99.9% with drift. It shows 0% power loss when θ_2= 0.8 and θ_{21}= 0.8. The power under correct specification is 99.9% with drift and trend. It shows 0% power loss at θ_2= 0.8 and θ_{21}= 0.8.

The second panel of table 6.35, illustrates the power results of Engle and Granger cointegration test when θ_2=0.6. The power in case of correct specification is 59.7% with drift. It shows 35.3% power loss when θ_2= 0.6 andθ_{21}= 0.8. The power under correct specification is 55.3% with drift and trend, it shows 39.7% power loss when θ_2= 0.8 and θ_{21}= 0.8. The second panel of table 6.36 given above explains the power results of Johansen and Juselius cointegration test when θ_2=0.6. The power in case of

correct specification is 63.5% with drift, it shows 31.5% power loss when θ_2= 0.6 and θ_{21}= 0.8. The power under correct specification is 57.1% with drift and trend, it shows 37.9% power loss when θ_2= 0.8 and θ_{21}= 0.8. The second panel of table 6.37 given above describes the power results of ARDL model when θ_2=0.6. The power in case of correct specification is 99.9% with drift, it shows 0% power loss when θ_2= 0.6 and θ_{21}= 0.8. The power under correct specification is 99.0% with drift and trend, it shows 0% power loss when θ_2= 0.8 andθ_{21}= 0.8. The order of statistics of power in case of correct specification is following:

$$\text{Power } (\text{ARDL} > JJ > EG)$$

Thus, these results validate that conventional cointegration procedures are having very low power when checked at θ_2= 0.8 and θ_{21}= 0.8. On contrast, the ARDL model has no power loss compare to Johansen and Juselius cointegration tests. It means in case of correct specification, the ARDL model works well than other conventional techniques.

Similarly, when the values of autoregressive parameters are θ_2 = 0.4 and 0.2, same pattern has been found. It clearly indicates that in correct specification cases the ARDL model have huge power while conventional cointegration procedures are having huge power loss. We cannot compare the power of these tests and ARDL model because as we have seen in size analysis these cointegration procedure suffer in size distortion problem even in case of correct specification.

Secondly, we consider the case of under specification. Table 6.35, shows the results of power of Engle and Granger cointegration test under different specifications at different values of autoregressive parameter θ_2. The first panel of table 6.35, describes the results of Engle and Granger cointegration test when θ_2=0.8. The power in case of under specification is 53.3% with drift. It shows 41.7% power loss when θ_2= 0.8 and θ_{21}=

0.8. Table 6.36, shows the results of power of Johansen and Juselius cointegration tests under different specifications at different values of autoregressive parameter θ_2. The first panel of table 6.36, describes the results of Johansen and Juselius cointegration test when θ_2=0.8. The power in case of under specification is 50.2% with drift. It shows 44.8% power loss when θ_2= 0.8 andθ_{21}= 0.8. Table 6.37, shows the results of power of ARDL model under different specifications at different values of autoregressive parameter θ_2. The first panel of table 6.37 describes the results of ARDL model when θ_2=0.8. The power in case of under specification is 99.0% with drift. It shows 0% power loss when θ_2= 0.8 andθ_{21}= 0.8. Table 6.35, shows the results of power of Engle and Granger cointegration test under different specifications at different values of autoregressive parameter θ_2. The second panel of table 6.35, describes the results of Engle and Granger cointegration test when θ_2=0.6. The power in case of under specification is 62.6%, it shows 32.4% power loss when θ_2= 0.6 and θ_{21}= 0.8. Table 6.36, shows the results of power of Johansen and Juselius cointegration test under different specifications at different values of autoregressive parameter θ_2. The second panel of table 6.36, describes the results of Johansen and Juselius cointegration test when θ_2=0.6. The power in case of under specification is 56.4% with drift. It shows 37.6% power loss when θ_2= 0.6 andθ_{21}= 0.8. Table 6.37 shows the results of power of ARDL model under different specifications at different values of autoregressive parameter θ_2. The first panel of table 6.37, describes the results of ARDL model when θ_2=0.6. The power in case of under specification is 99.0% with drift. It shows 0% power loss when θ_2= 0.6 and θ_{21}= 0.8.

Thus, these results validate that conventional cointegration procedure Engle and Granger Johansen and Juselius cointegration test are having very low power when θ_2= 0.8 and θ_{21}= 0.8. On the other hand, the ARDL model shows no power loss compare

to Johansen and Juselius cointegration test. It does not mean in case of under specification the ARDL model works well than other conventional techniques.

In case of under specification, ARDL model badly suffers in size distortion problem as we have seen in size analysis. So, we cannot say that ARDL has power on the basis of these statistics that is in under specification according to our estimation the Johansen and Juselius cointegration test works well. Because it has minimum size distortion under specification. The order of power of ARDL in case of under specification is given in the following equation:

$$\text{Power} \quad (\text{JJ} > EG > ARDL)$$

Similarly at all the values of autoregressive parameter θ_2 = 0.4 and 0.2 same pattern has been found.

At third we consider the case of over specification. Table 6.35, shows the results of power of Engle and Granger cointegration test over different specifications at different values of autoregressive parameter θ_2. The first panel of table 6.35, describes the results of Engle and Granger cointegration test when θ_2=0.8. The power in case of over specification is 32.6%. It shows 62.4% power loss when θ_2=0.8 and θ_{21}= 0.8. Table 6.36, shows that the results of power of Johansen and Juselius cointegration test over different specifications at different values of autoregressive parameter θ_2. The first panel of table 6.36, describes the results of Johansen and Juselius cointegration test when θ_2=0.8. The power in case of under specification is 37.7% with drift and trend. It shows 57.3% power loss when θ_2= 0.8 and θ_{21}= 0.8. Table 6.37, shows the results of power of ARDL model under different specifications at different values of autoregressive parameter θ_2. The first panel of table 6.37, describes the results of ARDL model when θ_2=0.8. The power in case of over specification is 99.0% with drift

and trend. It shows 0% power loss when θ_2= 0.8 and θ_{21}= 0.8. The second panel of table 6.35 describes the results of Engle and Granger cointegration test when θ_2=0.6. The power in case of over specification is 68.5%. It shows 26.5% power loss when θ_2= 0.6 and θ_{21}= 0.8. The second panel of table 6.36 describes the results of Johansen and Juselius cointegration test when θ_2=0.6. The power in case of under specification is 74.1% with drift, it shows 20.9% power loss when θ_2= 0.6 and θ_{21}= 0.8. The second panel of table 6.37, describes the results of ARDL model when θ_2=0.6. The power in case of under specification is 99.0% with drift. It shows 0% power loss when θ_2= 0.6 and θ_{21}= 0.8.

Thus, these results validate that conventional cointegration procedure Engle and Granger Johansen and Juselius cointegration test are having very low power when we check them at θ_2= 0.8 and 0.6, and θ_{21}= 0.8. On contrast, the ARDL model shows no power loss as compare to Johansen and Juselius cointegration test. It means in case of over specification, the ARDL model works well than other conventional techniques. In case of over specification ARDL model does not suffer in size distortion problem as we have seen in size analysis. So, we can say that ARDL has more power in case of over specification. The power of ARDL is given as follows;

$$\text{Power} \quad (\text{ARDL} > JJ > EG)$$

Similarly at all the values of autoregressive parameter θ_2 = 0.4 and 0.2 same pattern has been found. It clearly indicates that in over specification ARDL works good as compare to other techniques.

There is a special effect which needs to be analyzed. If we decrease the value of autoregressive parameter θ_2 from 0.8 to 0.6 and so on, the powers of these procedures are increasing. The reason behind this is that power of test depends upon relationship

between y and x, in presence of high autoregressive parameter value. That is why as we decrease the value of lag of y the relationship between y and x, gets stronger. Other reason is that in this analysis we only decrease the value of autoregressive parameter θ_2 from 0.8 to 0.2 but we do not change parameter value of lag of x. θ_{21}=0.8. Due to this reason lag of x. gets strong correlation as we decrease θ_2 value. In other words, when decrease θ_2 value θ_{21} remians same, then y depends more on x. as compare to its own lag.

Table 6.38: Power Analysis of Engle and Granger Cointegration Test by using Lag and Contemporaneous dependent Series at Different Specifications when $\rho = 0.6$

Engle Granger (EG) Cointegration Test			
		$\rho = 0.6$	
		$\theta_2 = 0.8$	
		Data Generating Process	
		Drift	***Drift and Trend***
Test Equation	***Drift***	34.1	52.8
	Drift and Trend	32.4	35.1
		$\theta_2 = 0.6$	
		Data Generating Process	
		Drift	***Drift and Trend***
Test Equation	***Drift***	59.1	60.3
	Drift and Trend	68.4	55
		$\theta_2 = 0.4$	
		Data Generating Process	
		Drift	***Drift and Trend***
Test Equation	***Drift***	91.3	76.4
	Drift and Trend	96.1	60
		$\theta_2 = 0.2$	
		Data Generating Process	
		Drift	***Drift and Trend***
Test Equation	***Drift***	99.8	99.6
	Drift and Trend	99.9	99.5

Table 6.39: Power Analysis of Johansen and Juselius Cointegration Test by using Lag and Contemporaneous dependent Series at Different Specifications when $\rho = 0.6$

Johansen and Juselius (JJ) Cointegration Test			
		$\rho = 0.6$	
		$\theta_2 = 0.8$	
		Data Generating Process	
		Drift	***Drift and Trend***
Test Equation	***Drift***	40.5	48.2
	Drift and Trend	36.3	38.8
		$\theta_2 = 0.6$	
		Data Generating Process	
		Drift	***Drift and Trend***
Test Equation	***Drift***	61.7	55.6
	Drift and Trend	71.2	56.8
		$\theta_2 = 0.4$	
		Data Generating Process	
		Drift	***Drift and Trend***
Test Equation	***Drift***	93.3	66.2
	Drift and Trend	97.7	94
		$\theta_2 = 0.2$	
		Data Generating Process	
		Drift	***Drift and Trend***
Test Equation	***Drift***	100	99
	Drift and Trend	99	98

Table 6.40 Power Analysis of ARDL Model by using Lag and Contemporaneous dependent Series at Different Specifications when $\rho = 0.6$

ARDL Model			
		$\rho = 0.6$	
		θ_2= 0.8	
		Data Generating Process	
		Drift	***Drift and Trend***
Test Equation	***Drift***	99	99
	Drift and Trend	99	99
		θ_2= 0.6	
		Data Generating Process	
		Drift	***Drift and Trend***
Test Equation	***Drift***	100	99
	Drift and Trend	99	99
		θ_2= 0.4	
		Data Generating Process	
		Drift	***Drift and Trend***
Test Equation	***Drift***	100	99
	Drift and Trend	99	100
		θ_2= 0.2	
		Data Generating Process	
		Drift	***Drift and Trend***
Test Equation	***Drift***	100	99
	Drift and Trend	99	100

Table 6.38, shows the results of power of Engle and Granger cointegration test under different specifications at different values of autoregressive parameter θ_2 when covariance $\rho = 0.6$. At first we consider correct specification with drift and drift and

trend cases. The first panel of table 6.38, describes the results of Engle and Granger cointegration test when θ_2=0.8. The power in case of correct specification is 34.1% with drift. It shows 60.9% power loss when autoregressive parameter θ_2= 0.8 and x lag value coefficient θ_{21}= 0.8. The power under correct specification is 35.1% with drift and trend. It shows 59.9% power loss when lag value parameter of y is θ_2= 0.8 and x lag value coefficient is θ_{21}= 0.8. Table 6.39 describes the results of power of Johansen and Juselius cointegration test under different specifications at different values of autoregressive parameter θ_2. Firstly, we consider correct specification with drift and drift and trend cases. The first panel of table 6.39, refers the results of Johansen and Juselius cointegration test when θ_2=0.8. The power in case of correct specification is 40.5% with drift. It shows 54.5% power loss when θ_2= 0.8 and x lag value coefficient is θ_{21}= 0.8. The power under correct specification is 38.8% with drift and trend. It shows 56.2% power loss when θ_2= 0.8 andθ_{21}= 0.8. Table 6.40, describes the results of power of ARDL model under different specifications at different values of autoregressive parameter θ_2. Firstly, we consider correct specification with drift and drift and trend cases. The first panel of table 6.40, refers the results of ARDL model when θ_2=0.8. The power in case of correct specification is 99.0% with drift. It shows 0% power loss when θ_2= 0.8 and θ_{21}= 0.8. The power under correct specification is 99.0% with drift and trend. It shows 0% power loss at θ_2= 0.8 and θ_{21}= 0.8.

The second panel of table 6.38, illustrates the power results of Engle and Granger cointegration test when θ_2=0.6. The power in case of correct specification is 59.1% with drift. It shows 35.9% power loss when θ_2= 0.6 and θ_{21}= 0.8. The power under correct specification is 55.0% with drift and trend, it shows 40.0% power loss when θ_2= 0.8 and θ_{21}= 0.8. The second panel of table 6.39 given above explains the power results of Johansen and Juselius cointegration test when θ_2= 0.6. The power in case of

correct specification is 61.7% with drift, it shows 33.3% power loss when θ_2= 0.6 and θ_{21}= 0.8. The power under correct specification is 56.8% with drift and trend, it shows 38.2% power loss when θ_2= 0.8 and θ_{21}= 0.8. The second panel of table 6.40 given above describes the power results of ARDL model when θ_2=0.6. The power in case of correct specification is 99.0% with drift, it shows 0% power loss when θ_2= 0.6 and θ_{21}= 0.8. The power under correct specification is 99.0% with drift and trend, it shows 0% power loss when θ_2= 0.8 andθ_{21}= 0.8. The order of statistics of power in case of correct specification is following:

$$\text{Power} \quad (\text{ARDL} > JJ > EG)$$

Thus, these results validate that conventional cointegration procedures are having very low power when checked at θ_2= 0.8 and θ_{21}= 0.8. On contrast, the ARDL model has no power loss compare to Johansen and Juselius cointegration tests. It means in case of correct specification, the ARDL model works well than other conventional techniques.

Similarly, when the values of autoregressive parameters are θ_2 = 0.4 and 0.2, same pattern has been found. It clearly indicates that in correct specification cases the ARDL model have huge power while conventional cointegration procedures are having huge power loss. We cannot compare the power of these tests and ARDL model because as we have seen in size analysis these cointegration procedure suffer in size distortion problem even in case of correct specification.

Secondly, we consider the case of under specification. Table 6.38, shows the results of power of Engle and Granger cointegration test under different specifications at different values of autoregressive parameter θ_2. The first panel of table 6.38, describes the results of Engle and Granger cointegration test when θ_2=0.8. The power in case of under specification is 52.8% with drift. It shows 42.2% power loss when θ_2= 0.8 and θ_{21}=

0.8. Table 6.39, shows the results of power of Johansen and Juselius cointegration tests under different specifications at different values of autoregressive parameter θ_2. The first panel of table 6.39, describes the results of Johansen and Juselius cointegration test when θ_2=0.8. The power in case of under specification is 48.2% with drift. It shows 46.8% power loss when θ_2= 0.8 andθ_{21}= 0.8. Table 6.40, shows the results of power of ARDL model under different specifications at different values of autoregressive parameter θ_2. The first panel of table 6.40 describes the results of ARDL model when θ_2=0.8. The power in case of under specification is 99.0% with drift. It shows 0% power loss when θ_2= 0.8 andθ_{21}= 0.8. Table 6.38, shows the results of power of Engle and Granger cointegration test under different specifications at different values of autoregressive parameter θ_2. The second panel of table 6.38, describes the results of Engle and Granger cointegration test when θ_2=0.6. The power in case of under specification is 60.3%, it shows 34.7% power loss when θ_2= 0.6 and θ_{21}= 0.8. Table 6.39, shows the results of power of Johansen and Juselius cointegration test under different specifications at different values of autoregressive parameter θ_2. The second panel of table 6.39, describes the results of Johansen and Juselius cointegration test when θ_2=0.6. The power in case of under specification is 55.6% with drift. It shows 44.4% power loss when θ_2= 0.6 andθ_{21}= 0.8. Table 6.40 shows the results of power of ARDL model under different specifications at different values of autoregressive parameter θ_2. The first panel of table 6.40, describes the results of ARDL model when θ_2=0.6. The power in case of under specification is 99.0% with drift. It shows 0% power loss when θ_2= 0.6 and θ_{21}= 0.8.

Thus, these results validate that conventional cointegration procedure Engle and Granger Johansen and Juselius cointegration test are having very low power when θ_2= 0.8 and θ_{21}= 0.8. On the other hand, the ARDL model shows no power loss compare

to Johansen and Juselius cointegration test. It does not mean in case of under specification the ARDL model works well than other conventional techniques.

In case of under specification, ARDL model badly suffers in size distortion problem as we have seen in size analysis. So, we cannot say that ARDL has power on the basis of these statistics that is in under specification according to our estimation the Johansen and Juselius cointegration test works well. Because it has minimum size distortion under specification. The order of power of ARDL in case of under specification is given in the following equation:

$$\text{Power} \quad (\text{JJ} > EG > ARDL)$$

Similarly at all the values of autoregressive parameter θ_2 = 0.4 and 0.2 same pattern has been found.

At third we consider the case of over specification. Table 6.38, shows the results of power of Engle and Granger cointegration test over different specifications at different values of autoregressive parameter θ_2. The first panel of table 6.38, describes the results of Engle and Granger cointegration test when θ_2=0.8. The power in case of over specification is 32.4%. It shows 62.6% power loss when θ_2=0.8 and θ_{21}= 0.8. Table 6.39, shows that the results of power of Johansen and Juselius cointegration test over different specifications at different values of autoregressive parameter θ_2. The first panel of table 6.39, describes the results of Johansen and Juselius cointegration test when θ_2=0.8. The power in case of under specification is 36.3% with drift and trend. It shows 58.7% power loss when θ_2= 0.8 and θ_{21}= 0.8. Table 6.40, shows the results of power of ARDL model under different specifications at different values of autoregressive parameter θ_2. The first panel of table 6.40, describes the results of ARDL model when θ_2=0.8. The power in case of over specification is 99.0% with drift

and trend. It shows 0% power loss when θ_2= 0.8 and θ_{21}= 0.8. The second panel of table 6.38 describes the results of Engle and Granger cointegration test when θ_2=0.6. The power in case of over specification is 68.4%. It shows 26.6% power loss when θ_2= 0.6 and θ_{21}= 0.8. The second panel of table 6.39 describes the results of Johansen and Juselius cointegration test when θ_2=0.6. The power in case of under specification is 71.2% with drift, it shows 23.8% power loss when θ_2= 0.6 and θ_{21}= 0.8. The second panel of table 6.40, describes the results of ARDL model when θ_2=0.6. The power in case of under specification is 100.0% with drift. It shows 0% power loss when θ_2= 0.6 and θ_{21}= 0.8.

Thus, these results validate that conventional cointegration procedure Engle and Granger Johansen and Juselius cointegration test are having very low power when we check them at θ_2= 0.8 and 0.6, and θ_{21}= 0.8. On contrast, the ARDL model shows no power loss as compare to Johansen and Juselius cointegration test. It means in case of over specification, the ARDL model works well than other conventional techniques. In case of over specification ARDL model does not suffer in size distortion problem as we have seen in size analysis. So, we can say that ARDL has more power in case of over specification. The power of ARDL is given as follows;

$$\text{Power } (\text{ARDL} > JJ > EG)$$

Similarly at all the values of autoregressive parameter θ_2 = 0.4 and 0.2 same pattern has been found. It clearly indicates that in over specification ARDL works good as compare to other techniques.

There is a special effect which needs to be analyzed. If we decrease the value of autoregressive parameter θ_2 from 0.8 to 0.6 and so on, the powers of these procedures are increasing. The reason behind this is that power of test depends upon relationship

between y and x, in presence of high autoregressive parameter value. That is why as we decrease the value of lag of y the relationship between y and x, gets stronger. Other reason is that in this analysis we only decrease the value of autoregressive parameter θ_2 from 0.8 to 0.2 but we do not change parameter value of lag of $x_.$ θ_{21}=0.8. Due to this reason lag of $x_.$ gets strong correlation as we decrease θ_2 value. In other words, when decrease θ_2 value θ_{21} remians same, then y depends more on $x_.$ as compare to its own lag.

Table 6.41: Power Analysis of Engle and Granger Cointegration Test by using Lag and Contemporaneous dependent Series at Different Specifications when $\rho = 0.4$

Engle Granger (EG) Cointegration Test			
		$\rho = 0.4$	
		$\theta_2 = 0.8$	
		Data Generating Process	
		Drift	***Drift and Trend***
Test Equation	***Drift***	33.3	51.6
	Drift and Trend	32.2	33.1
		$\theta_2 = 0.6$	
		Data Generating Process	
		Drift	***Drift and Trend***
Test Equation	***Drift***	58.3	58.6
	Drift and Trend	67.8	54.5
		$\theta_2 = 0.4$	
		Data Generating Process	
		Drift	***Drift and Trend***
Test Equation	***Drift***	90.9	75.8
	Drift and Trend	96.4	58.1
		$\theta_2 = 0.2$	
		Data Generating Process	
		Drift	***Drift and Trend***
Test Equation	***Drift***	98.9	99
	Drift and Trend	98	99

Table 6.42: Power Analysis of Johansen and Juselius Cointegration Test by using Lag and Contemporaneous dependent Series at Different Specifications when $\rho = 0.4$

Johansen and Juselius (JJ) Cointegration Test			
		$\rho = 0.4$	
		$\theta_2 = 0.8$	
		Data Generating Process	
		Drift	*Drift and Trend*
Test Equation	*Drift*	39.7	45.8
	Drift and Trend	34.9	37.7
		$\theta_2 = 0.6$	
		Data Generating Process	
		Drift	*Drift and Trend*
Test Equation	*Drift*	60.8	54.4
	Drift and Trend	70.6	55.9
		$\theta_2 = 0.4$	
		Data Generating Process	
		Drift	*Drift and Trend*
Test Equation	*Drift*	92.4	66
	Drift and Trend	97.3	89
		$\theta_2 = 0.2$	
		Data Generating Process	
		Drift	*Drift and Trend*
Test Equation	*Drift*	99	98
	Drift and Trend	99	99

Table 6.43: Power Analysis of ARDL Model by using Lag and Contemporaneous dependent Series at Different Specifications when $\rho = 0.4$

ARDL Model			
		$\rho = 0.4$	
		$\theta_2 = 0.8$	
		Data Generating Process	
		Drift	***Drift and Trend***
Test Equation	***Drift***	99	99
	Drift and Trend	99	99
		$\theta_2 = 0.6$	
		Data Generating Process	
		Drift	***Drift and Trend***
Test Equation	***Drift***	100	99
	Drift and Trend	99	99
		$\theta_2 = 0.4$	
		Data Generating Process	
		Drift	***Drift and Trend***
Test Equation	***Drift***	100	99
	Drift and Trend	99	100
		$\theta_2 = 0.2$	
		Data Generating Process	
		Drift	***Drift and Trend***
Test Equation	***Drift***	100	99
	Drift and Trend	99	100

Table 6.41, shows the results of power of Engle and Granger cointegration test under different specifications at different values of autoregressive parameter θ_2 when covariance $\rho = 0.4$. At first we consider correct specification with drift and drift and

trend cases. The first panel of table 6.41, describes the results of Engle and Granger cointegration test when θ_2=0.8. The power in case of correct specification is 33.3% with drift. It shows 61.7% power loss when autoregressive parameter θ_2= 0.8 and x lag value coefficient θ_{21}= 0.8. The power under correct specification is 33.1% with drift and trend. It shows 61.9% power loss when lag value parameter of y is θ_2= 0.8 and x lag value coefficient is θ_{21}= 0.8. Table 6.42 describes the results of power of Johansen and Juselius cointegration test under different specifications at different values of autoregressive parameter θ_2. Firstly, we consider correct specification with drift and drift and trend cases. The first panel of table 6.42, refers the results of Johansen and Juselius cointegration test when θ_2=0.8. The power in case of correct specification is 39.7% with drift. It shows 55.3% power loss when θ_2= 0.8 and x lag value coefficient is θ_{21}= 0.8. The power under correct specification is 37.7% with drift and trend. It shows 56.3% power loss when θ_2= 0.8 and θ_{21}= 0.8. Table 6.43, describes the results of power of ARDL model under different specifications at different values of autoregressive parameter θ_2. Firstly, we consider correct specification with drift and drift and trend cases. The first panel of table 6.43, refers the results of ARDL model when θ_2=0.8. The power in case of correct specification is 99.0% with drift. It shows 0% power loss when θ_2= 0.8 and θ_{21}= 0.8. The power under correct specification is 99.0% with drift and trend. It shows 0% power loss at θ_2= 0.8 and θ_{21}= 0.8.

The second panel of table 6.41, illustrates the power results of Engle and Granger cointegration test when θ_2=0.6. The power in case of correct specification is 58.3% with drift. It shows 36.7% power loss when θ_2= 0.6 and θ_{21}= 0.8. The power under correct specification is 54.5% with drift and trend, it shows 40.5% power loss when θ_2= 0.8 and θ_{21}= 0.8. The second panel of table 6.42 given above explains the power results of Johansen and Juselius cointegration test when θ_2= 0.6. The power in case of

correct specification is 60.8% with drift, it shows 34.2% power loss when θ_2= 0.6 and θ_{21}= 0.8. The power under correct specification is 55.9% with drift and trend, it shows 39.1% power loss when θ_2= 0.8 and θ_{21}= 0.8. The second panel of table 6.43 given above describes the power results of ARDL model when θ_2=0.6. The power in case of correct specification is 99.0% with drift, it shows 0% power loss when θ_2= 0.6 and θ_{21}= 0.8. The power under correct specification is 99.0% with drift and trend, it shows 0% power loss when θ_2= 0.8 andθ_{21}= 0.8. The order of statistics of power in case of correct specification is following:

$$\text{Power } (\text{ARDL} > JJ > EG)$$

Thus, these results validate that conventional cointegration procedures are having very low power when checked at θ_2= 0.8 and θ_{21}= 0.8. On contrast, the ARDL model has no power loss compare to Johansen and Juselius cointegration tests. It means in case of correct specification, the ARDL model works well than other conventional techniques.

Similarly, when the values of autoregressive parameters are θ_2 = 0.4 and 0.2, same pattern has been found. It clearly indicates that in correct specification cases the ARDL model have huge power while conventional cointegration procedures are having huge power loss. We cannot compare the power of these tests and ARDL model because as we have seen in size analysis these cointegration procedure suffer in size distortion problem even in case of correct specification.

Secondly, we consider the case of under specification. Table 6.41, shows the results of power of Engle and Granger cointegration test under different specifications at different values of autoregressive parameter θ_2. The first panel of table 6.41, describes the results of Engle and Granger cointegration test when θ_2=0.8. The power in case of under specification is 51.6% with drift. It shows 38.4% power loss when θ_2= 0.8 and θ_{21}=

0.8. Table 6.42, shows the results of power of Johansen and Juselius cointegration tests under different specifications at different values of autoregressive parameter θ_2. The first panel of table 6.42, describes the results of Johansen and Juselius cointegration test when θ_2=0.8. The power in case of under specification is 45.8% with drift. It shows 49.2% power loss when θ_2= 0.8 andθ_{21}= 0.8. Table 6.43, shows the results of power of ARDL model under different specifications at different values of autoregressive parameter θ_2. The first panel of table 6.43 describes the results of ARDL model when θ_2=0.8. The power in case of under specification is 99.0% with drift. It shows 0% power loss when θ_2= 0.8 andθ_{21}= 0.8. Table 6.41, shows the results of power of Engle and Granger cointegration test under different specifications at different values of autoregressive parameter θ_2. The second panel of table 6.41, describes the results of Engle and Granger cointegration test when θ_2=0.6. The power in case of under specification is 58.6%, it shows 36.4% power loss when θ_2= 0.6 and θ_{21}= 0.8. Table 6.42, shows the results of power of Johansen and Juselius cointegration test under different specifications at different values of autoregressive parameter θ_2. The second panel of table 6.42, describes the results of Johansen and Juselius cointegration test when θ_2=0.6. The power in case of under specification is 54.4% with drift. It shows 40.6% power loss when θ_2= 0.6 and θ_{21}= 0.8. Table 6.43 shows the results of power of ARDL model under different specifications at different values of autoregressive parameter θ_2. The first panel of table 6.43, describes the results of ARDL model when θ_2=0.6. The power in case of under specification is 99.0% with drift. It shows 0% power loss when θ_2= 0.6 and θ_{21}= 0.8.

Thus, these results validate that conventional cointegration procedure Engle and Granger Johansen and Juselius cointegration test are having very low power when θ_2= 0.8 and θ_{21}= 0.8. On the other hand, the ARDL model shows no power loss compare

to Johansen and Juselius cointegration test. It does not mean in case of under specification the ARDL model works well than other conventional techniques.

In case of under specification, ARDL model badly suffers in size distortion problem as we have seen in size analysis. So, we cannot say that ARDL has power on the basis of these statistics that is in under specification according to our estimation the Johansen and Juselius cointegration test works well. Because it has minimum size distortion under specification. The order of power of ARDL in case of under specification is given in the following equation:

$$\text{Power } (\text{JJ} > EG > ARDL)$$

Similarly at all the values of autoregressive parameter θ_2 = 0.4 and 0.2 same pattern has been found.

At third we consider the case of over specification. Table 6.41, shows the results of power of Engle and Granger cointegration test over different specifications at different values of autoregressive parameter θ_2. The first panel of table 6.41, describes the results of Engle and Granger cointegration test when θ_2=0.8. The power in case of over specification is 32.3%. It shows 62.7% power loss when θ_2=0.8 and θ_{21}= 0.8. Table 6.42, shows that the results of power of Johansen and Juselius cointegration test over different specifications at different values of autoregressive parameter θ_2. The first panel of table 6.42, describes the results of Johansen and Juselius cointegration test when θ_2=0.8. The power in case of under specification is 34.9% with drift and trend. It shows 60.1% power loss when θ_2= 0.8 and θ_{21}= 0.8. Table 6.43, shows the results of power of ARDL model under different specifications at different values of autoregressive parameter θ_2. The first panel of table 6.40, describes the results of ARDL model when θ_2=0.8. The power in case of over specification is 99.0% with drift

and trend. It shows 0% power loss when θ_2= 0.8 and θ_{21}= 0.8. The second panel of table 6.41 describes the results of Engle and Granger cointegration test when θ_2=0.6. The power in case of over specification is 67.8%. It shows 27.2% power loss when θ_2= 0.6 and θ_{21}= 0.8. The second panel of table 6.42 describes the results of Johansen and Juselius cointegration test when θ_2=0.6. The power in case of under specification is 70.6% with drift, it shows 24.4% power loss when θ_2= 0.6 and θ_{21}= 0.8. The second panel of table 6.43, describes the results of ARDL model when θ_2=0.6. The power in case of under specification is 99.0% with drift. It shows 0% power loss when θ_2= 0.6 and θ_{21}= 0.8.

Thus, these results validate that conventional cointegration procedure Engle and Granger Johansen and Juselius cointegration test are having very low power when we check them at θ_2= 0.8 and 0.6, and θ_{21}= 0.8. On contrast, the ARDL model shows no power loss as compare to Johansen and Juselius cointegration test. It means in case of over specification, the ARDL model works well than other conventional techniques. In case of over specification ARDL model does not suffer in size distortion problem as we have seen in size analysis. So, we can say that ARDL has more power in case of over specification. The power of ARDL is given as follows;

$$\text{Power (ARDL} > JJ > EG)$$

Similarly at all the values of autoregressive parameter θ_2 = 0.4 and 0.2 same pattern has been found. It clearly indicates that in over specification ARDL works good as compare to other techniques.

There is a special effect which needs to be analyzed. If we decrease the value of autoregressive parameter θ_2 from 0.8 to 0.6 and so on, the powers of these procedures are increasing. The reason behind this is that power of test depends upon relationship

between y and x, in presence of high autoregressive parameter value. That is why as we decrease the value of lag of y the relationship between y and x, gets stronger. Other reason is that in this analysis we only decrease the value of autoregressive parameter θ_2 from 0.8 to 0.2 but we do not change parameter value of lag of x. θ_{21}=0.8. Due to this reason lag of x. gets strong correlation as we decrease θ_2 value. In other words, when decrease θ_2 value θ_{21} remians same, then y depends more on x. as compare to its own lag.

Table 6.44: Power Analysis of Engle and Granger Cointegration Test by using Lag and Contemporaneous dependent Series at Different Specifications when $\rho = 0.2$

Engle Granger (EG) Cointegration Test			
		$\rho = 0.2$	
		$\theta_2 = 0.8$	
		Data Generating Process	
		Drift	***Drift and Trend***
Test Equation	***Drift***	32.1	50.6
	Drift and Trend	32.3	33
		$\theta_2 = 0.6$	
		Data Generating Process	
		Drift	***Drift and Trend***
Test Equation	***Drift***	58.0	56.4
	Drift and Trend	66.2	53.1
		$\theta_2 = 0.4$	
		Data Generating Process	
		Drift	***Drift and Trend***
Test Equation	***Drift***	90.0	74.3
	Drift and Trend	97.9	57.2
		$\theta_2 = 0.2$	
		Data Generating Process	
		Drift	***Drift and Trend***
Test Equation	***Drift***	99	99
	Drift and Trend	98	99

Table 6.45 Power Analysis of Johansen and Juselius Cointegration Test by using Lag and Contemporaneous dependent Series at Different Specifications when $\rho = 0.2$

Johansen and Juselius (JJ) Cointegration Test			
		$\rho = 0.2$	
		$\theta_2 = 0.8$	
		Data Generating Process	
		Drift	***Drift and Trend***
Test Equation	***Drift***	38.7	43.9
	Drift and Trend	34	30.1
		$\theta_2 = 0.6$	
		Data Generating Process	
		Drift	***Drift and Trend***
Test Equation	***Drift***	60.1	52.5
	Drift and Trend	70.3	54.6
		$\theta_2 = 0.4$	
		Data Generating Process	
		Drift	***Drift and Trend***
Test Equation	***Drift***	92.2	66.4
	Drift and Trend	96.3	90
		$\theta_2 = 0.2$	
		Data Generating Process	
		Drift	***Drift and Trend***
Test Equation	***Drift***	100	99
	Drift and Trend	99	99

Table 6.46: Power Analysis of ARDL Model by using Lag and Contemporaneous dependent Series at Different Specifications when $\rho = 0.2$

ARDL Model			
		$\rho = 0.2$	
		$\theta_2 = 0.8$	
		Data Generating Process	
		Drift	***Drift and Trend***
Test Equation	***Drift***	99	99
	Drift and Trend	99	99
		$\theta_2 = 0.6$	
		Data Generating Process	
		Drift	***Drift and Trend***
Test Equation	***Drift***	100	99
	Drift and Trend	99	99
		$\theta_2 = 0.4$	
		Data Generating Process	
		Drift	***Drift and Trend***
Test Equation	***Drift***	100	99
	Drift and Trend	99	100
		$\theta_2 = 0.2$	
		Data Generating Process	
		Drift	***Drift and Trend***
Test Equation	***Drift***	100	99
	Drift and Trend	99	100

Table 6.44, shows the results of power of Engle and Granger cointegration test under different specifications at different values of autoregressive parameter θ_2 when covariance $\rho = 0.2$. At first we consider correct specification with drift and drift and

trend cases. The first panel of table 6.44, describes the results of Engle and Granger cointegration test when θ_2=0.8. The power in case of correct specification is 32.1% with drift. It shows 62.9% power loss when autoregressive parameter θ_2= 0.8 and x lag value coefficient θ_{21}= 0.8. The power under correct specification is 33.0% with drift and trend. It shows 62.0% power loss when lag value parameter of y is θ_2= 0.8 and x lag value coefficient is θ_{21}= 0.8. Table 6.45 describes the results of power of Johansen and Juselius cointegration test under different specifications at different values of autoregressive parameter θ_2. Firstly, we consider correct specification with drift and drift and trend cases. The first panel of table 6.45, refers the results of Johansen and Juselius cointegration test when θ_2=0.8. The power in case of correct specification is 38.7% with drift. It shows 56.3% power loss when θ_2= 0.8 and x lag value coefficient is θ_{21}= 0.8. The power under correct specification is 30.1% with drift and trend. It shows 64.9% power loss when θ_2= 0.8 and θ_{21}= 0.8. Table 6.46, describes the results of power of ARDL model under different specifications at different values of autoregressive parameter θ_2. Firstly, we consider correct specification with drift and drift and trend cases. The first panel of table 6.46, refers the results of ARDL model when θ_2=0.8. The power in case of correct specification is 99.0% with drift. It shows 0% power loss when θ_2= 0.8 and θ_{21}= 0.8. The power under correct specification is 99.0% with drift and trend. It shows 0% power loss at θ_2= 0.8 and θ_{21}= 0.8.

The second panel of table 6.44, illustrates the power results of Engle and Granger cointegration test when θ_2=0.6. The power in case of correct specification is 58.0% with drift. It shows 37.0% power loss when θ_2= 0.6 and θ_{21}= 0.8. The power under correct specification is 53.1% with drift and trend, it shows 41.9% power loss when θ_2= 0.8 and θ_{21}= 0.8. The second panel of table 6.45 given above explains the power results of Johansen and Juselius cointegration test when θ_2= 0.6. The power in case of

correct specification is 60.1% with drift, it shows 34.9% power loss when θ_2= 0.6 and θ_{21}= 0.8. The power under correct specification is 54.6% with drift and trend, it shows 40.4% power loss when θ_2= 0.8 and θ_{21}= 0.8. The second panel of table 6.46 given above describes the power results of ARDL model when θ_2=0.6. The power in case of correct specification is 99.0% with drift, it shows 0% power loss when θ_2= 0.6 and θ_{21}= 0.8. The power under correct specification is 99.0% with drift and trend, it shows 0% power loss when θ_2= 0.8 andθ_{21}= 0.8. The order of statistics of power in case of correct specification is following:

$$\text{Power } (\text{ARDL} > JJ > EG)$$

Thus, these results validate that conventional cointegration procedures are having very low power when checked at θ_2= 0.8 and θ_{21}= 0.8. On contrast, the ARDL model has no power loss compare to Johansen and Juselius cointegration tests. It means in case of correct specification, the ARDL model works well than other conventional techniques.

Similarly, when the values of autoregressive parameters are θ_2 = 0.4 and 0.2, same pattern has been found. It clearly indicates that in correct specification cases the ARDL model have huge power while conventional cointegration procedures are having huge power loss. We cannot compare the power of these tests and ARDL model because as we have seen in size analysis these cointegration procedure suffer in size distortion problem even in case of correct specification.

Secondly, we consider the case of under specification. Table 6.44, shows the results of power of Engle and Granger cointegration test under different specifications at different values of autoregressive parameter θ_2. The first panel of table 6.44, describes the results of Engle and Granger cointegration test when θ_2=0.8. The power in case of under specification is 50.6% with drift. It shows 44.4% power loss when θ_2= 0.8 and θ_{21}=

0.8. Table 6.45, shows the results of power of Johansen and Juselius cointegration tests under different specifications at different values of autoregressive parameter θ_2. The first panel of table 6.45, describes the results of Johansen and Juselius cointegration test when θ_2=0.8. The power in case of under specification is 43.9% with drift. It shows 51.1% power loss when θ_2= 0.8 andθ_{21}= 0.8. Table 6.46, shows the results of power of ARDL model under different specifications at different values of autoregressive parameter θ_2. The first panel of table 6.46 describes the results of ARDL model when θ_2=0.8. The power in case of under specification is 99.0% with drift. It shows 0% power loss when θ_2= 0.8 andθ_{21}= 0.8. Table 6.44, shows the results of power of Engle and Granger cointegration test under different specifications at different values of autoregressive parameter θ_2. The second panel of table 6.44, describes the results of Engle and Granger cointegration test when θ_2=0.6. The power in case of under specification is 56.4%, it shows 38.6% power loss when θ_2= 0.6 and θ_{21}= 0.8. Table 6.45, shows the results of power of Johansen and Juselius cointegration test under different specifications at different values of autoregressive parameter θ_2. The second panel of table 6.45, describes the results of Johansen and Juselius cointegration test when θ_2=0.6. The power in case of under specification is 52.5% with drift. It shows 42.5% power loss when θ_2= 0.6 and θ_{21}= 0.8. Table 6.46 shows the results of power of ARDL model under different specifications at different values of autoregressive parameter θ_2. The first panel of table 6.46, describes the results of ARDL model when θ_2= 0.6. The power in case of under specification is 99.0% with drift. It shows 0% power loss when θ_2= 0.6 and θ_{21}= 0.8.

Thus, these results validate that conventional cointegration procedure Engle and Granger Johansen and Juselius cointegration test are having very low power when θ_2= 0.8 and θ_{21}= 0.8. On the other hand, the ARDL model shows no power loss compare

to Johansen and Juselius cointegration test. It does not mean in case of under specification the ARDL model works well than other conventional techniques.

In case of under specification, ARDL model badly suffers in size distortion problem as we have seen in size analysis. So, we cannot say that ARDL has power on the basis of these statistics that is in under specification according to our estimation the Johansen and Juselius cointegration test works well. Because it has minimum size distortion under specification. The order of power of ARDL in case of under specification is given in the following equation:

$$\text{Power} \quad (\text{JJ} > EG > ARDL)$$

Similarly at all the values of autoregressive parameter θ_2 = 0.4 and 0.2 same pattern has been found.

At third we consider the case of over specification. Table 6.44, shows the results of power of Engle and Granger cointegration test over different specifications at different values of autoregressive parameter θ_2. The first panel of table 6.44, describes the results of Engle and Granger cointegration test when θ_2=0.8. The power in case of over specification is 32.3%. It shows 62.7% power loss when θ_2=0.8 and θ_{21}= 0.8. Table 6.45, shows that the results of power of Johansen and Juselius cointegration test over different specifications at different values of autoregressive parameter θ_2. The first panel of table 6.45, describes the results of Johansen and Juselius cointegration test when θ_2=0.8. The power in case of under specification is 34.0% with drift and trend. It shows 61.0% power loss when θ_2= 0.8 and θ_{21}= 0.8. Table 6.46, shows the results of power of ARDL model under different specifications at different values of autoregressive parameter θ_2. The first panel of table 6.46, describes the results of ARDL model when θ_2=0.8. The power in case of over specification is 99.0% with drift

and trend. It shows 0% power loss when θ_2= 0.8 and θ_{21}= 0.8. The second panel of table 6.44 describes the results of Engle and Granger cointegration test when θ_2=0.6. The power in case of over specification is 66.3%. It shows 28.7% power loss when θ_2= 0.6 and θ_{21}= 0.8. The second panel of table 6.45 describes the results of Johansen and Juselius cointegration test when θ_2=0.6. The power in case of under specification is 70.3% with drift, it shows 24.7% power loss when θ_2= 0.6 and θ_{21}= 0.8. The second panel of table 6.46, describes the results of ARDL model when θ_2= 0.6. The power in case of under specification is 99.0% with drift. It shows 0% power loss when θ_2= 0.6 and θ_{21}= 0.8.

Thus, these results validate that conventional cointegration procedure Engle and Granger Johansen and Juselius cointegration test are having very low power when we check them at θ_2= 0.8 and 0.6, and θ_{21}= 0.8. On contrast, the ARDL model shows no power loss as compare to Johansen and Juselius cointegration test. It means in case of over specification, the ARDL model works well than other conventional techniques. In case of over specification ARDL model does not suffer in size distortion problem as we have seen in size analysis. So, we can say that ARDL has more power in case of over specification. The power of ARDL is given as follows;

$$\text{Power } (\text{ARDL} > JJ > EG)$$

Similarly at all the values of autoregressive parameter θ_2 = 0.4 and 0.2 same pattern has been found. It clearly indicates that in over specification ARDL works good as compare to other techniques.

There is a special effect which needs to be analyzed. If we decrease the value of autoregressive parameter θ_2 from 0.8 to 0.6 and so on, the powers of these procedures are increasing. The reason behind this is that power of test depends upon relationship

between y and x, in presence of high autoregressive parameter value. That is why as we decrease the value of lag of y the relationship between y and x, gets stronger. Other reason is that in this analysis we only decrease the value of autoregressive parameter θ_2 from 0.8 to 0.2 but we do not change parameter value of lag of x. θ_{21}=0.8. Due to this reason lag of x. gets strong correlation as we decrease θ_2 value. In other words, when decrease θ_2 value θ_{21} remians same, then y depends more on x. as compare to its own lag.

Table 6.47: Power Analysis of Engle and Granger Cointegration Test by using Contemporaneous dependent Series at Different Specifications

Engle Granger (EG) Cointegration Test			
		$\rho = 1$	
		Data Generating Process	
		Drift	***Drift and Trend***
Test Equation	***Drift***	99.9	99.8
	Drift and Trend	99.8	99.5
		$\rho = 0.8$	
		Data Generating Process	
		Drift	***Drift and Trend***
Test Equation	***Drift***	76.8	82.3
	Drift and Trend	78.3	64.3
		$\rho = 0.6$	
		Data Generating Process	
		Drift	***Drift and Trend***
Test Equation	***Drift***	68.3	76.3
	Drift and Trend	71.2	61.4
		$\rho = 0.4$	
		Data Generating Process	
		Drift	***Drift and Trend***
Test Equation	***Drift***	48.7	50.9
	Drift and Trend	41.9	34.7
		$\rho = 0.2$	
		Data Generating Process	
		Drift	***Drift and Trend***
Test Equation	***Drift***	24.5	38.1
	Drift and Trend	23.4	25.2

Table 6.48: Power Analysis of Johansen and Juselius Cointegration Test by using Contemporaneous dependent Series at Different Specifications

Johansen and Juselius (JJ) Cointegration Test			
		$\rho = 1$	
		Data Generating Process	
		Drift	***Drift and Trend***
Test Equation	***Drift***	93	99.9
	Drift and Trend	95.4	100
		$\rho = 0.8$	
		Data Generating Process	
		Drift	***Drift and Trend***
Test Equation	***Drift***	81.5	83.2
	Drift and Trend	80.6	88.2
		$\rho = 0.6$	
		Data Generating Process	
		Drift	***Drift and Trend***
Test Equation	***Drift***	76.3	79.4
	Drift and Trend	77.1	81.8
		$\rho = 0.4$	
		Data Generating Process	
		Drift	***Drift and Trend***
Test Equation	***Drift***	50.3	51.3
	Drift and Trend	50.6	48.9
		$\rho = 0.2$	
		Data Generating Process	
		Drift	***Drift and Trend***
Test Equation	***Drift***	32.1	34.2
	Drift and Trend	28.3	38.9

Table 6.49: Power Analysis of ARDL Model by using Contemporaneous dependent Series at Different Specifications

ARDL Model			
		$\rho = 1$	
		Data Generating Process	
		Drift	***Drift and Trend***
Test Equation	***Drift***	90.3	99
	Drift and Trend	93.6	100
		$\rho = 0.8$	
		Data Generating Process	
		Drift	***Drift and Trend***
Test Equation	***Drift***	88.7	99
	Drift and Trend	94.3	100
		$\rho = 0.6$	
		Data Generating Process	
		Drift	***Drift and Trend***
Test Equation	***Drift***	81.5	99
	Drift and Trend	93.1	100
		$\rho = 0.4$	
		Data Generating Process	
		Drift	***Drift and Trend***
Test Equation	***Drift***	80.6	99
	Drift and Trend	77.2	98
		$\rho = 0.2$	
		Data Generating Process	
		Drift	***Drift and Trend***
Test Equation	***Drift***	56.3	99
	Drift and Trend	67.9	60.2

Table 6.47, shows the results of power of Engle and Granger cointegration test under different specifications at different values of covariance ρ when covariance ρ. At first we consider correct specification with drift and drift and trend cases. The first panel of

table 6.47, describes the results of Engle and Granger cointegration test when ρ = 1. The power in case of correct specification is 99.9% with drift. It shows 0% power loss when ρ = 1. The power under correct specification is 99.5% with drift and trend. It shows 0% power loss. Table 6.48 describes the results of power of Johansen and Juselius cointegration test under different specifications at different values of ρ. Firstly, we consider correct specification with drift and drift and trend cases. The first panel of table 6.48, refers the results of Johansen and Juselius cointegration test when ρ = 1. The power in case of correct specification is 93.0% with drift. It shows 2.0% power loss. The power under correct specification is 100% with drift and trend. It shows 0% power loss. Table 6.49, describes the results of power of ARDL model under different specifications at different values of ρ. Firstly, we consider correct specification with drift and drift and trend cases. The first panel of table 6.49, refers the results of ARDL model when ρ = 1. The power in case of correct specification is 90.3% with drift. It shows 4.7% power loss. The power under correct specification is 99.0% with drift and trend. It shows 0% power loss.

The second panel of table 6.47, illustrates the power results of Engle and Granger cointegration test when ρ = 0.8. The power in case of correct specification is 76.8% with drift. It shows 18.2% power loss. The power under correct specification is 64.3% with drift and trend, it shows 30.7% power loss. The second panel of table 6.48 given above explains the power results of Johansen and Juselius cointegration test when ρ = 0.8. The power in case of correct specification is 81.5% with drift, it shows 14.5% power loss. The power under correct specification is 88.2% with drift and trend, it shows 6.8% power loss. The second panel of table 6.49 given above describes the power results of ARDL model when ρ = 0.8. The power in case of correct specification is 88.7% with drift, it shows 6.3% power loss. The power under correct specification is

100.0% with drift and trend, it shows 0% power loss. The order of statistics of power in case of correct specification is following:

$$\text{Power (ARDL} > JJ > EG)$$

Thus, these results validate that conventional cointegration procedures are having very high power at different values of ρ. On contrast, the ARDL model has no power loss compare to Johansen and Juselius and Engle and Granger cointegration tests, because these procedures are having severe size problem. It means in case of correct specification, the ARDL model works well than other conventional techniques.

Similarly, when the values of autoregressive parameters are $\rho = 0.6$ and so on, same pattern has been found. It clearly indicates that in correct specification cases the ARDL model have huge power while conventional cointegration procedures are having huge power loss. We cannot compare the power of these tests and ARDL model because as we have seen in size analysis these cointegration procedure suffer in size distortion problem even in case of correct specification.

Secondly, we consider the case of under specification. Table 6.47, shows the results of power of Engle and Granger cointegration test under different specifications at different values of ρ. The first panel of table 6.47, describes the results of Engle and Granger cointegration test when $\rho = 1$. The power in case of under specification is 99.8% with drift. It shows 0% power loss. Table 6.48, shows the results of power of Johansen and Juselius cointegration tests under different specifications at different values of ρ. The first panel of table 6.48, describes the results of Johansen and Juselius cointegration test when $\rho = 1$. The power in case of under specification is 99.0% with drift. It shows 0% power loss. Table 6.49, shows the results of power of ARDL model under different specifications at different values of ρ. The first panel of table 6.49 describes the results of ARDL model when $\rho = 1$. The power in case of under specification is 99.0% with drift. It shows 0% power loss when $\theta_2 = 0.8$ and $\theta_{21} = 0.8$. Table 6.47, shows the results of power of Engle and Granger cointegration test under different specifications at different values of ρ. The second panel of table 6.47, describes the results of Engle and Granger cointegration test when $\rho = 0.8$. The power in case of under specification is 82.3%, it shows 12.7% power loss. Table 6.48, shows the results of power of Johansen and Juselius cointegration test under different specifications at different values of ρ. The second panel of table 6.48, describes the results of Johansen and Juselius cointegration test when $\rho = 0.8$. The power in case of under specification is 83.2% with drift. It shows 11.8% power loss. Table 6.49 shows the results of power of ARDL model under different specifications at different values of ρ. The first panel of table 6.49, describes the results of ARDL model when $\rho = 0.8$. The power in case of under specification is 99.0% with drift. It shows 0% power loss.

Thus, these results validate that conventional cointegration procedure Engle and Granger Johansen and Juselius cointegration test are having very high power. On the other hand, the ARDL model shows no power loss compare to Johansen and Juselius and Engle and Granger cointegration tests because they suffer in size problem. It does not mean in case of under specification the ARDL model works well than other conventional techniques.

In case of under specification, ARDL model badly suffers in size distortion problem as we have seen in size analysis. So, we cannot say that ARDL has power on the basis of these statistics that is in under specification according to our estimation the Johansen and Juselius cointegration test works well. Because it has minimum size distortion under specification. The order of power of ARDL in case of under specification is given in the following equation:

$$\text{Power} \quad (\text{JJ} > EG > ARDL)$$

Similarly at all the values of ρ same pattern has been found.

At third we consider the case of over specification. Table 6.47, shows the results of power of Engle and Granger cointegration test over different specifications at different values of ρ. The first panel of table 6.47, describes the results of Engle and Granger cointegration test when $\rho = 1$. The power in case of over specification is 99.8%. It shows 0% power loss. Table 6.48, shows that the results of power of Johansen and Juselius cointegration test over different specifications at different values of ρ. The first panel of table 6.48, describes the results of Johansen and Juselius cointegration test when ρ. The power in case of under specification is 95.4% with drift and trend. It shows 0% power loss. Table 6.49, shows the results of power of ARDL model under different specifications at different values of ρ. The first panel of table 6.46, describes the results of ARDL model when $\rho = 1$. The power in case of over specification is 93.4% with drift and trend. It shows 1.6% power loss. The second panel of table 6.47 describes the results of Engle and Granger cointegration test when $\rho = 0.8$. The power in case of over specification is 78.3%. It shows 16.7% power loss. The second panel of table 6.48 describes the results of Johansen and Juselius cointegration test when $\rho = 0.8$. The power in case of under specification is 80.6% with drift, it shows 14.4% power loss. The second panel of table 6.49, describes the results of ARDL model when $\rho = 0.8$. The power in case of under specification is 94.3% with drift. It shows 0.7% power loss.

Thus, these results validate that conventional cointegration procedure Engle and Granger and Johansen and Juselius cointegration test are having very low power. On contrast, the ARDL model shows no power loss as compare to Johansen and Juselius cointegration test. It means in case of over specification, the ARDL model works well than other conventional techniques. In case of over specification ARDL model does not suffer in size distortion problem as we have seen in size analysis. So, we can say that ARDL has more power in case of over specification. The power of ARDL is given as follows;

$$\text{Power} \quad (\text{ARDL} > JJ > EG)$$

Similarly at all the values of ρ same pattern has been found. It clearly indicates that in over specification ARDL works good as compare to other techniques.

There is a special effect which needs to be analyzed. If we decrease the value of ρ from 1 to 0.8 and so on, the powers of these procedures are decreasing. The reason behind this is that power of test depends upon relationship between y and x, in presence of covariance term ρ. That is why as we decrease the value of ρ the relationship between y and x, gets weaker.

CHAPTER 7

FORECASTING PERFORMANCE OF ARDL ON REAL DATA AND COMPARISON WITH COINTEGRATION TESTS AND OLS

In this chapter we check the forecast performance of ARDL model and commonly used conventional cointegration procedures, Engle and Granger (EG), and Johansen and Juselius (JJ) cointegration tests. The forecast performance is tested on the basis of real data. The real data consist of gross domestic product (GDP, at constant LCU) and household final consumption expenditures (HFC, at constant LCU) for the period of 1960 to 2016. The data based on ten lower middle income countries Pakistan, Bangladesh, India, Sri-Lanka, Indonesia, Bolivia, Cameroon, Morocco, Nicaragua and Philippines. In this analysis we used same country series for forecasting, the dependent variable is gross domestic product (GDP, at constant LCU) and independent variable is household final consumption expenditures (HFC, at constant LCU) in all cases.

In this analysis Root Mean Square Error (RMSE) has been used to compare the forecast performance of ARDL model and conventional procedures. The figure 3, is showing the RMSE statistics obtained after forecasting through these procedures:

Figure 7.1: The Root Mean Square Error (RMSE) after Forecasting

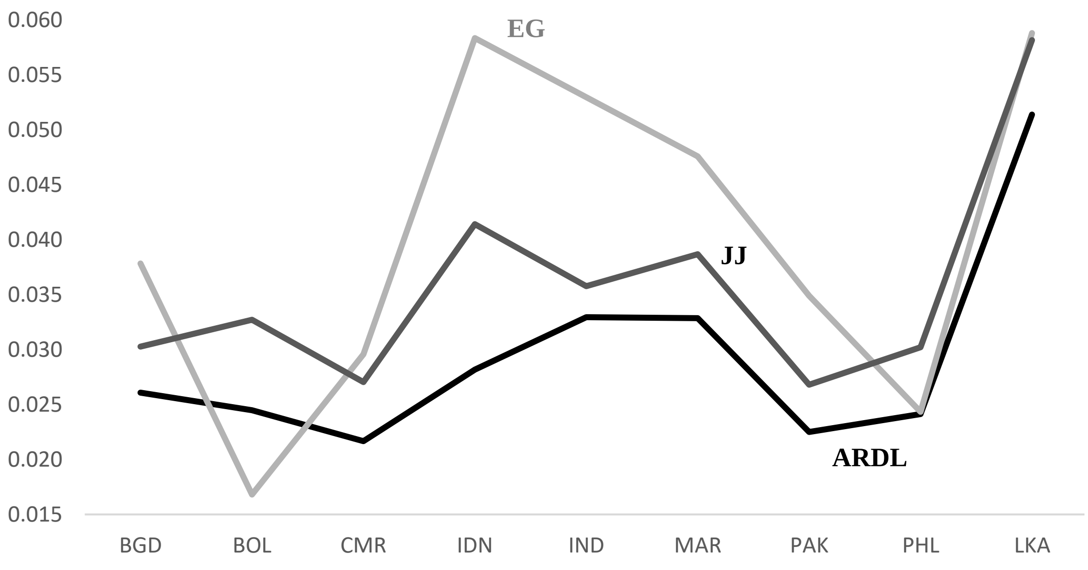


Figure 7.1 Represents the RMSE statistics obtained after forecasting. These statistics are taken from table 30 given below.

Figure 7.1 shows that the forecast performance of ARDL model is better as compare to conventional EG and JJ procedures. The RMSE statistics for ARDL model in all cases remains smaller from the RMSE for EG and JJ procedures except only one case of Bolivia (BOL). Figure 7.1 also shows that performance of JJ cointegration procedure is better than EG cointegration test. The RMSE statistics are given below in table 30:

Table 7.1: The results of Root Mean Square Error (RMSE) after Forecasting

Countries	ARDL	EG	JJ
Bangladesh (BGD)	0.02607	0.03786	0.03027
Bolivia (BOL)	0.02450	0.01681	0.03273
Cameroon (CMR)	0.02167	0.02958	0.02704
Indonesia (IDN)	0.02818	0.05836	0.04142
India (IND)	0.03295	0.05298	0.03578
Morocco (MAR)	0.03286	0.04758	0.03869
Nicaragua (NIC)	0.04923	0.13033	0.17397
Pakistan (PAK)	0.02251	0.03491	0.02681
Philippines (PHL)	0.02415	0.02434	0.03023
Sri Lanka (LKA)	0.05140	0.05882	0.05817

Table 7.1 illustrates the RMSE statistics of ARDL, EG and JJ procedures. These RMSE are calculated after forecasting. RMSE indicates the deviation of forecasted values from actual values. That is why the smaller value of RMSE shows less deviation from actual values and higher value shows higher variation. The results in table 7.1 clearly indicate that the RMSE statistics of ARDL remains smaller in all cases apart from Bolivia (BOL) case. In case of EG the second row is having circle which indicates that in this particular case EG performs well as compare to ARDL model. In other cases ARDL r performs better as compare to EG. While in case of JJ, rectangles are indicting that EG performs better than JJ procedure in only three cases. The overall condition is that ARDL model performs well as compare to conventional cointegration procedures EG and JJ, and JJ

cointegration procedure performs well as compare to EG. On the basis of this analysis, we can express performance of these procedures as:

$$\text{Forecast Performance } (\text{ARDL} > JJ > EG)$$

The results are theoretically admissible because, ARDL model is having both contemporaneous and lag values of independent variable. While JJ procedure contains only lag values of independent variable and EG procedure is based on static function. That is why, ARDL has more power to explain the relations instead of these conventional procedures.

7.1 Measuring the Probability of Spurious Regression

The size analysis is performed to measure the probability of spurious regression. After running regression between independent series if we got significant results, it counts as spurious regression. In this analysis we compare the ARDL model with conventional cointegration procedures Engle and Granger (EG), Johansen and Juselius (JJ) and ordinary least square (OLS) on the basis of real data.

7.1.1 Comparison between ARDL model and conventional cointegration procedures EG and JJ

In this analysis we used data given above of gross domestic product (GDP, at constant LCU) and Household Final consumption expenditures (HFC, at constant LCU) in all cases. But for this analysis, we take gross domestic product (GDP, at constant LCU) as dependent variable and Household Final consumption expenditures (HFC, at constant LCU) series of all other countries as independent variable. It means, we used statistically independent series because HFC of one country has no relation with GDP of any other country. After running regression if we got significant results, it counts as spurious regression. Same procedure was done with series of all countries to estimate the probability of spurious regression. The results are given in followingtables:

Table 7.2: The Probability of Spurious Regression by using Engle and Granger Cointegration

Household Final Expenditure (at, constant LCU)	Gross Domestic Product (at, constant LCU)									
	BGD	BOL	CMR	IDN	IND	LKA	MAR	NIC	PAK	PHL
BGD	1	1	0	0	0	0	0	0	0	0
BOL	1	1	0	0	1	0	0	0	0	0
CMR	0	0	1	1	0	1	1	0	1	1
IDN	0	0	0	1	0	0	0	0	1	0
IND	1	1	0	0	1	0	0	0	0	0
LKA	0	0	0	0	0	1	1	0	1	0
MAR	0	0	0	1	0	0	1	0	1	0
NIC	0	0	0	0	0	0	0	1	0	0
PAK	0	0	1	1	0	1	1	0	1	0
PHL	0	1	0	0	0	0	0	0	0	1
Total	2	3	1	3	1	2	3	0	4	1

Table 7.2 shows the probability of spurious regression after employing EG cointegration procedure. We used two Step procedure. In first step simple **OLS** regression is employed and generate a **residual** fromby using parametersof OLS regression. In second step we tested the **stationary of residual** series. If seriesis stationary it means series are cointegrated. **1's** are showing series are cointegrated and **0's** mean notcointegrated. Residual analysis is also employed for validation of results.

The table 7.2 shows the results of Engle and Granger cointegration test. In this matrix "1" means the statistically independent variables are cointegrated and "0" means variables are not cointegrated. In this analysis, we are ignoring the diagonal 1's because these are the relationship between same country series which means between dependent series. After employing EG cointegration test on independent series we got 20 significant relations out of 90 regression. It means the probability of spurious regression after employing EG procedure is 22.2%. It shows 15.2% size distortion on the basis of 5% level of significance. It indicates that EG procedure is also suffering in size distortion problem.

Table 7.3: The Probability of Spurious Regression by using Johansen and Juselius Cointegration

Gross Domestic Product (at, constant LCU)

Household Final Expenditure (at, constant LCU)		BGD	BOL	CMR	IDN	IND	LKA	MAR	NIC	PAK	PHL
	BGD	1	1	0	0	1	0	0	1	1	0
	BOL	1	1	0	0	1	0	0	0	0	0
	CMR	0	0	1	0	1	0	0	0	1	0
	IDN	0	0	0	1	1	0	1	0	1	0
	IND	1	1	1	1	1	0	0	0	1	1
	LKA	0	1	0	0	1	1	1	0	1	0
	MAR	0	0	1	0	1	0	1	0	1	1
	NIC	0	0	0	0	1	0	0	1	0	1
	PAK	1	0	0	0	1	0	1	0	1	0
	PHL	1	0	1	0	1	0	0	0	0	1
	Total	4	3	3	1	9	0	3	1	6	3

Table 7.3 shows the probability of spurious regression after employing **JJ** cointegration procedure. We took three steps in this procedure. In first step **VAR** model is being used. In second step we used the **lag selection criteria** for lag selection. In third step we employed **JJ** procedure and take decision on thebasis of **Unrestricted Cointegration Rank Test (Trace)** statistics. **1's** are showing series are cointegrated and **0's** mean not cointegrated. Residual analysis are also employed for validation of results.

The table 7.3 demonstrates the results of Johansen and Juselius cointegration procedure. The "1" means the statistically independent variables are cointegrated and "0" means variables are not cointegrated. In this analysis we are ignoring the diagonal 1's because these are the relationship between same country series, means between dependent series. After employing JJ cointegration test on independent series we got 33 significant relations out of 90 regressions. It means the probability of spurious regression after employing JJ procedure is 33.67%. It shows 28.67% size distortion on the basis of 5% level of significance. It indicates that JJ procedure is also suffering in size distortion problem.

Table 7.4: The Probability of Spurious Regression by using ARDL Model

Household Final Expenditure (at, constant LCU)	Gross Domestic Product (at, constant LCU)										
		BGD	BOL	CMR	IDN	IND	LKA	MAR	NIC	PAK	PHL
	BGD	1	0	0	0	0	0	0	1	1	0
	BOL	1	1	0	0	0	0	0	0	0	0
	CMR	0	0	1	0	0	0	0	0	1	0
	IDN	0	0	0	1	0	0	0	0	0	0
	IND	0	1	0	0	1	0	0	0	0	0
	LKA	0	0	0	0	0	1	0	0	0	0
	MAR	0	0	0	0	0	0	1	0	1	0
	NIC	0	0	0	0	0	0	0	1	0	1
	PAK	0	0	0	0	0	0	0	0	1	0
	PHL	0	0	0	0	0	0	0	0	0	1
	Total	1	1	0	0	0	0	0	1	3	1

Table 7.4 shows the probability of spurious regression after employing ARDL Model. We Used **F-stat** to check the joint significance of lag and current values of independent variable. All the decision are taken on the basis of **F-stat**. **1's** are showing series are cointegrated and **0's** mean not cointegrated. Residual analysis are also employed for validation of results.

The table 7.4 represents the results of ARDL model. The "1" means the statistically independent variables are cointegrated and "0" means variables are not cointegrated. In this analysis we are ignoring the diagonal 1's because these are the relationship between same country series, means between dependent series. After employing ARDL model on independent series we got 7 significant relations out of 90 regression. It means the probability of spurious regression after employing ARDL model is 7.78%. It shows 2.78% size distortion on the basis of 5% level of significance which is negligible.

This analysis indicates that conventional cointegration procedures EG and JJ are suffering in size distortion problem while ARDL model tackle this problem by including lag values. It means the major cause of spurious regression is missing lag dynamics and by including the lag values, we can overcome the problem of spurious regression. It has theoretical justification, when we regress independent series, the independent variable starts working as a proxy of relevant variable and captures the effect of relevant variable that is why it becomes significant. But when we introduce the lag value of dependent variable as independent variable which is potential determinant, it captures the effect and irrelevant variable becomes insignificant.

7.2 Comparison between OLS and ARDL Model on the Basis of Real Data

In this section we present inferences based on real data. The real data are based on Gross domestic product of thirty seven countries Albania, Antigua and Barbuda, Argentina, Austria, Bahamas, Bahrain, Barbados, Belgium, Botswana, Brazil, Brunei Darussalam, Cabo Verde, Canada, Iraq Comoros, Congo, Costa Rica, Denmark, Dominica, El Salvador, Fiji, Finland, France, Gabon, Gambia, Germany, Grenada, Guinea-Bissau, Guyana, Honduras, Hong Kong, Iceland, Ireland, Israel, Italy, Kiribati and Luxembourg from 1980 to 2014. We employed the ADF unit root test and came to know all the series are stationary at first difference. All the series are statistically independent of each other. We regress all series on each other by employing ordinary least square (OLS) and ARDL models, the results are given in table 7.5.

Table 7.5: Comparison between OLS and ARDL Models in Case of Real Data

	OLS	ARDL (1, 1)			ARDL (2, 2)			
	Xt	***xt-1***	***xt***	***F-stat***	***xt-2***	***xt-1***	***xt***	***F-stat***
Percentile 5	8.431	-4.288	-1.094	0.241	-1.062	-3.973	-0.639	0.181
Percentile 25	12.457	-2.224	0.358	1.515	-0.092	-1.998	0.486	1.120
Percentile 50	17.103	-0.933	1.615	3.527	0.561	-0.903	1.509	2.440
Percentile 75	24.898	0.185	2.844	6.802	1.343	-0.154	2.835	4.720
Percentile 95	42.320	1.430	4.834	18.233	2.658	0.879	4.937	10.817
Positive significant	930	15	391	474	108	4	386	384
Negative significant	0	269	12	0	9	232	2	0
Total significant	930	284	403	474	117	236	388	384
	Percentages							
	100	26.667	43.333	50.968	12.581	25.376	41.720	41.290

The asymptotic significance value of t-statistics at 5% level of significance is 1.96, while The asymptotic significance value for ARDL (1, 1) of F-stat is 3.3158 and for ARDL (2, 2) is 2.9466.

We used percentiles to explain such a huge amount of results in simplest form, detail estimations are given below in appendix. The 5^{th} percentile is indicating that 5% values are equal or less than of this value or 95% values are greater than this value. We used t-statistics for OLS and F-stat for ARDL model to check the joint significance of all lagged values of independent variable. In case of OLS, the value of 5^{th}percentile is 8.431 which shows that 5% value are less than this value. It indicates that 95% t-statistics values are greater than this value. The 25^{th}, 50^{th} and 75^{th} percentiles are equal to 1^{st}, 2^{nd} and 3^{rd} quartiles. The 25^{th} percentile or 1^{st} quartile shows that 25% values are less than or 75% values are greater than this value. In the same way we can interpret 50^{th} and 75^{th} percentiles. The values of OLS t-statistics are 12.457, 17.103 and 24.898 for 1^{st}, 2^{nd} and 3^{rd} quartiles respectively. In the last panel of table the total percentages are given, we can see that the values of OLS t-statistics are 100% significant. It indicates that the probability of spurious regression is 100%.

In case of ARDL (1, 1), we used F-stat for joint significance, the value of F-stat at 5^{th} percentile is 0.241, and it means all values in 5% of F-stat are insignificant. The value first quartile of F-stat is 1.515 which means that 25% of F-stat values are less than those values. The value of 2^{nd} quartile of F-stat is 3.527 which indicates that 50% value of F-stat are less than those values. Th50th percentile or 2^{nd} quartiles are also considered as median of data. It indicates that 50% value of F-stat are less that asymptotic significance value which is 3.3158. The value of third quartile is 6.802 and the value of 95^{th} percentile is 18.233. The total percentage of significant values is 50.968. In other words out of 933 regressions only 474 times regression came up with spurious results. It clearly indicates that the ARDL (1, 1) model reduced the probability of spurious regression from 100% to 51 %. There is more than 49 percent reduction in probability of spurious regression only by using ARDL (1, 1) model. The ARDL (2, 2) model is used to check that how ARDL model behaves in over specification case.

In case of ARDL (2, 2) the value of F-stat at 5^{th} percentile is 0.181, and it means all values in 5% of F-stat are less than critical value. The value of 25^{th} percentile F-stat is 1.120 which means that 25% of F-stat values are less than these values. The value of 50^{th} percentile F-stat is 2.2440 which indicates that 50% of F-stat values are less than this values. It indicates that 50% value of of F-stat are less than asymptotic significance value which 2.9466. The value of 75^{th} percentile is 4.720 and the value of 95^{th} percentile is 10.817. The total percentage of significant values is 41.290. In other words out of 933 regression only 384 time regression came up with spurious results. It clearly indicates that the ARDL (2, 2) model reduced the probability of spurious regression from 100% to 41 %. It also shows that in case of over specification ARDL model also works better and reduces the probability of spurious regression. The lag values of dependent variables are significant in all cases. For further detail see, appendix.

7.2.1 Comparison between OLS and ARDL Model on the Basis of Real Data with Residual Analysis

The real data is based on Gross domestic product of thirty seven countries from 1980 to 2014. We employed the ADF unit root test and came to know that all the series are stationary at first difference. All the series are statistically independent of each other. We regress all other series on Albania and found that all the regression came up with significant results. Even though all series are independent of each other. As we can see in table 7.6 which consists of OLS regression results. All the GDP series are having statistically significant relations. The P-values indicate that all the relation are highly significant even at 5% level of significance.

The table 7.8, shows the residual analysis of linear regression model. It shows that all the results of autocorrelation are significant at 5% level of significance. While the results of LM test for heteroscedasticity are also significant, expect 15 cases. It means out of 36 regression only 15 regression residuals are facing heteroscedasticity problem.

The7.7, shows the results of ARDL model which significantly reduced the probability of spurious regression from 100% to approximately 5%. It also rejects the common misconception about the spurious regression that it commonly prevails due to unit root. Nevertheless, the missing relevant variable is a major cause of spurious regression. As we introduced the lag values the probability of spurious regression reduced significantly. We used F-test to check the joint significance of lag and current values of independent variable. The P-values indicates that all the relation are insignificant even at 5% level of significance except 2 cases. The table 7.9 presents the residual analysis of ARDL model. As we can see that the autocorrelation test are insignificant at 5% except Argentina and Brunei Darussalam, they are insignificant at 1%. The hetroscedasticity test statistics are insignificant at 5% except Argentina, Canada but in case of Canada it is insignificant at 1%. It indicates that all the regression are econometrically valid except 2 or 3 cases and ARDL can be used as an alternative tool for the reduction of spurious regression instead of conventional methods.

Table 7.6: Results after running Simple Linear Regression Model

Countries	ATG	ARG	AUT	BHS	BHR	BRB	BEL	BWA	BRA	BRN	CPV	CAN
Co-eff	173.456	0.845535	2.79251	115.61	55.0787	1176.54	2.44742	6.94373	0.374535	89.9904	3.37695	0.355487
P-value	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]
Countries	COM	COG	CRI	DNK	DMA	SLV	FJI	FIN	FRA	GAB	GMB	DEU
Co-effi	9.22622	0.558613	0.245078	0.299897	682.991	69.922	158.877	4.29561	0.278377	0.127738	32.1748	0.20898
P-value	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]
Countries	GRD	GNB	GUY	HND	HKG	ISL	IRQ	IRL	ISR	ITA	KIR	LUX
Co-eff	324.926	1.98374	2.03643	3.9283	0.2911	0.374548	0.0050	2.78307	0.630868	0.319785	5020.83	13.7727
P-value	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]

Table 7.7: Results after employing ARDL model

Countries	ATG	ARG	AUT	BHS	BHR	BRB	BEL	BWA	BRA	BRN	CPV	CAN
F-stat	2.2113	2.3136	1.9177	3.3884	2.3568	0.97170	1.5636	1.2890	2.6769	2.9427	2.5011	1.7673
P-value	[0.1271]	[0.0984]	[0.1515]	[0.024]*	[0.0949]	[0.4211]	[0.2220]	[0.2991]	[0.0679]	[0.0517]	[0.0692]	[0.1781]
Countries	COM	COG	CRI	DNK	DMA	SLV	FJI	FIN	FRA	GAB	GMB	DEU
F-stat	2.5938	1.0733	2.4079	0.55250	1.3533	2.5329	3.9684	2.7890	2.4591	0.75471	1.7834	1.2943
P-value	[0.0741]	[0.3776]	[0.0900]	[0.6510]	[0.2789]	[0.0789]	[0.018]*	[0.0605]	[0.0905]	[0.4795]	[0.1751]	[0.2900]
Countries	GRD	GNB	GUY	HND	HKG	ISL	IRQ	IRL	ISR	ITA	KIR	LUX
F-stat	2.5668	1.8490	2.7830	2.2760	1.4923	2.1955	2.1649	2.8124	2.2770	0.19603	2.5335	3.0034
P-value	[0.0947]	[0.1631]	[0.0609]	[0.1034]	[0.2399]	[0.1301]	[0.1163]	[0.0591]	[0.1033]	[0.8231]	[0.0666]	[0.0658]

The coefficient values are given in table 1 and 2. The P values are in square brackets. The table 2 consists on the F-stat coefficient value which is used to check the joint significance of independent variable and its lag values. Under null hypothesis H0: restrictions are valid. * shows the values which are significant at less than 5% level of significance.

Table 7.8: Residual Analysis after simple linear regression Model

Countries	ATG	ARG	AUT	BHS	BHR	BRB	BEL	BWA	BRA	BRN	CPV	CAN
AR (1-2)	108.46	37.166	74.421	50.957	44.826	28.088	58.430	47.607	46.912	70.425	42.454	93.299
P-value	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]
Hetro test	4.8584	11.807	3.0080	3.4207	10.664	1.8093	4.6607	6.8516	1.5076	0.38325	12.618	0.42721
P-value	[0.0144]	[0.0002]	[0.0650]	[0.0464]	[0.0003]	[0.1818]	[0.0176]	[0.0037]	[0.2383]	[0.6850]	[0.0001]	[0.6564]
Countries	COM	COG	CRI	DNK	DMA	SLV	FJI	FIN	FRA	GAB	GMB	DEU
AR (1-2)	23.463	27.073	48.132	192.11	49.202	92.324	70.445	52.093	179.65	108.46	37.166	176.23
P-value	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]
Hetro test	3.9430	0.47118	12.139	2.6543	6.0150	1.4328	0.93896	0.56898	1.9298	4.8584	11.807	3.0440
P-value	[0.0306]	[0.6290]	[0.0001]	[0.0874]	[0.0065]	[0.2550]	[0.4026]	[0.5723]	[0.1634]	[0.0144]	[0.0002]	[0.0631]
Countries	GRD	GNB	GUY	HND	HKG	ISL	IRQ	IRL	ISR	ITA	KIR	LUX
AR (1-2)	51.204	44.374	60.715	38.748	43.418	46.786	17.337	70.961	56.707	271.27	36.165	55.628
P-value	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]
Hetro test	0.84706	1.8925	1.7900	9.6925	6.4124	0.32064	8.8652	0.0082	9.2912	4.0579	0.54471	4.9846
P-value	[0.4390]	[0.1688]	[0.1849]	[0.0006]	[0.0049]	[0.7282]	[0.0010]	[0.9917]	[0.0008]	[0.0279]	[0.5858]	[0.0138]

Table 7.9: Residual Analysis after ARDL Model

Countries	ATG	ARG	AUT	BHS	BHR	BRB	BEL	BWA	BRA	BRN	CPV	CAN
AR (1-2)	3.2957	4.8584	2.8581	1.8220	1.9423	1.5511	4.0584	2.4124	2.5211	3.5946	2.0736	0.91149
P-value	[0.0530]	[0.0144]	[0.0770]	[0.1834]	[0.1653]	[0.2325]	[0.1144]	[0.1110]	[0.1014]	[0.0431]	[0.1477]	[0.4154]
Hetro test	0.32156	173.456	1.7750	1.7026	1.4521	1.0165	173.456	1.3759	1.9732	0.83462	0.74033	2.5478
P-value	[0.9195]	(0.000)	[0.1287]	[0.1461]	[0.2257]	[0.4624]	(0.8006)	[0.2572]	[0.0911]	[0.6020]	[0.6804]	[0.0341]
Countries	COM	COG	CRI	DNK	DMA	SLV	FJI	FIN	FRA	GAB	GMB	DEU
AR (1-2)	2.6788	2.5176	2.2340	1.6776	2.4530	2.6688	1.9338	2.9251	4.1468	3.2957	2.0250	2.4418
P-value	[0.0891]	[0.1017]	[0.1289]	[0.2080]	[0.1073]	[0.0898]	[0.1665]	[0.0730]	[0.5284]	[0.0530]	[0.1539]	[0.1067]
Hetro test	2.1383	0.85294	1.4666	0.90939	1.3667	1.4615	2.6555	2.0190	3.0658	0.32156	2.4831	2.1177
P-value	[0.0684]	[0.5871]	[0.2201]	[0.5422]	[0.2612]	[0.2221]	[0.0285]	[0.0841]	[0.2147]	[0.9195]	[0.0380]	[0.0869]
Countries	GRD	GNB	GUY	HND	HKG	ISL	IRQ	IRL	ISR	ITA	KIR	LUX
AR (1-2)	3.0651	3.3840	3.0038	3.3708	2.2546	3.1490	1.0909	2.4140	1.6316	3.2739	0.92427	3.2180
P-value	[0.0638]	[0.0507]	[0.0670]	[0.1499]	[0.1267]	[0.0596]	[0.3520]	[0.1109]	[0.2166]	[0.0539]	[0.4117]	[0.0564]
Hetro test	0.82311	1.2434	2.2719	0.26387	1.1274	0.60599	0.68400	1.6800	1.9253	1.5242	2.0197	1.3186
P-value	[0.5628]	[0.3214]	[0.0691]	[0.9486]	[0.3885]	[0.7231]	[0.7276]	[0.1520]	[0.0990]	[0.2112]	[0.0848]	[0.2857]

AR null hypothesis H0: There is autocorrelation. LM test for Hetroskedastic with null hypothesis H0: There is no hetroskedasticity

CHAPTER 8

PESARAN'S BOUND TESTING VERSUS UNRISTRICTED ARDL APPROACH`

The Pesaran (1998) has introduced a method for testing long run relationship based on ECM version of Hendry ARDL model. The method has become so popular that most people cannot differentiate between Pesaran ARDL model and the parent Hendry (1978) ARDL model, whereas, in fact, the later is known to econometricians long before the Pesaran version of ARDL. Pesaran's equation is a restricted version of Hendry equation. Pesaran et al. (1996) and Pesaran (1997) proposed a single equation ARDL (autoregressive distributed lag) approach for testing long run as an alternative of Engle and Granger and Engle and Yoo procedures. The Pesaran ARDL approach provides explicit tests for the presence of long run relationship, instead of assuming uniqueness. Suppose we have two variables y_t and x_t and the generalized Hendry ARDL model equation is following:

$$y_t = \beta_0 + \sum_{i=1}^{p} \beta_i \, dy_{t-i} + \sum_{i=0}^{q} \delta_i \, x_{t-i} + \varepsilon_{1t} \tag{8.1}$$

Equation 8.1 was proposed by Hendry for time series modeling and he stated that it provides convenient way of following General to Simple methodology. Hendry shows that numerous theoretical models can be driven from the equations similar to 8.1. On the other hand, the equation proposed by Pesaran for testing long run relationship is

$$dy_t = \theta_{10} + \theta_{11} y_{t-1} + \theta_{12} x_{t-1} + \sum_{i=1}^{p} \beta_i \, dy_{t-i} + \sum_{i=0}^{q} \delta_i \, dx_{t-i} + \varepsilon_{1t} \tag{8.2}$$

One can see that it is just like the a re-specification of equation 8.1 proposed by Hendry (1978), the Pesaran model is also restricted version of Hendry ARDL model by imposing restrictions on Hendry ARDL model as in equation 8.1. The Pesaran regression equations can be derived as:

$$dy_t = \theta_{10} + \theta_{11}y_{t-1} + \theta_{12}x_{t-1} + \sum_{i=1}^{p} \beta_i \, dy_{t-i} + \sum_{i=0}^{q} \delta_i \, dx_{t-i} + \varepsilon_{1t} \qquad (8.3)$$

$$dx_t = \theta_{10} + \theta_{11}y_{t-1} + \theta_{12}x_{t-1} + \sum_{i=1}^{p} \beta_i \, dx_{t-i} + \sum_{i=0}^{q} \delta_i \, dy_{t-i} + \varepsilon_{1t} \qquad (8.4)$$

The Pesaran equations 8.3 and 8.4 are similar to Error Correction Version of ARDL equation 8.1, because just like the error correction model, the equation contains all difference form variables and error correction term which mentioned without differencing. Pesaran argued that this model can be used for testing long run and short run relationships. The F test is employed to find out joint significance for short run and long run relationships. The hypotheses for long run and short run relationships are following:

$H_0 = \theta_{11} = \theta_{12} = 0$ There is no long run relationship

$H_1 = \theta_{11} = \theta_{12} \neq 0$ There is long run relationship

The hypotheses for short run relationship are following:

$H_0 = \beta_i = \delta_i = 0$ There is no short run relationship

$H_1 = \beta_i = \delta_i \neq 0$ There is short run relationship

The F statistic (Wald test) for these hypotheses tested in each of the models can be denoted as

$$F_{x1}(x_{1t} \,|\, x_{2t}) \qquad (8.5)$$

$$F_{x2}(x_{2t} \,|\, x_{1t}) \qquad (8.6)$$

The distribution of Wald test is non-standard asymptotically under the null of no cointegration. Pesaran and Shin (1995) revealed that asymptotically valid inference on short run and long run parameters could be made by employing ordinary least square estimations of ARDL model. So, the ARDL model order is properly augmented to grant for contemporary correlation among the stochastic elements of the data generating processes involved in estimation.

As stated by Pesaran the beauty of Pesaran's model is that it can differentiate between genuine and spurious relationship without knowing about stationarity. Since Pesaran ECM version of ARDL is a restriction of the generalized ARDL model, thus, the simple ARDL (DHSY version) should also be utilized to differentiate between genuine and spurious relationship. This study shows that actually it is possible to use unrestricted ARDL model to differentiate between genuine and spurious relationship. Pesaran has devised two set of critical values, first applicable when the series are stationery and second applicable when the series are nonstationary. The Pesaran ARDL procedure is only used for nonstationary series and involves prior specification decisions. Pesaran ARDL model procedure estimates short run effects by taking differencing in equation and long run effect through bound testing. Pesaran et al. (2001) provided critical values of two bounds, upper and lower which are being used for cointegration. The lower bound considers variables are stationary and they have no long run relationship. The upper bound considers variables are nonstationary and they have long run relationship. While the Hendry ARDL model used least square regression and there is no need of special critical values and it also works in stationary time series case. The simple ARDL model provides only static relationships.

However the Pesaran equation is based on difference form variables and if the original series are stationary, the differencing will produce negative moving average (NMA) which will yield the power problem.

8.1 Probability of Spurious Regression with Pesaran's Bound Testing and Unristricted ARDL Equation

The size analysis is performed to quantify the distortion in probability of type I error. It can be expressed in following way:

$$\text{Size} = \text{Prob (reject } H_0 | \text{ when } H_0 \text{ is true)}$$

The size distortion can be regarded as probability of spurious regression because size is the probability of getting significant coefficients when actually there is no relationship. The size is measured after employing Pesaran's bound testing and Hendry ARDL model with different specifications. For this analysis, the independent autoregressive stationary and nonstationary time series are being generated by following equation 5.1 with different specifications i.e. without drift and trend, without drift, with drift and with drift and trend. Since Pesaran's equation is restricted version, it cannot supersede the Hendry version. This is intuition but the exact results are summarized in table 8.1 from Monte Carlo simulations. These results are summarized after 100,000 simulations:

The figure 8.1 given below is based on results of table 8.1. When autoregressive parameters $\theta_1 = \theta_2$ are equal to 1, it means series are nonstationary. Figure 8.1, shows the comparison among the probability of spurious regression with Pesaran ARDL model and ARDL model. This comparison has been made on the different specifications.

Figure 8.1: Size of Pesaran ARDL Version and Hendry ARDL model Version for Testing Long Run Relationship.

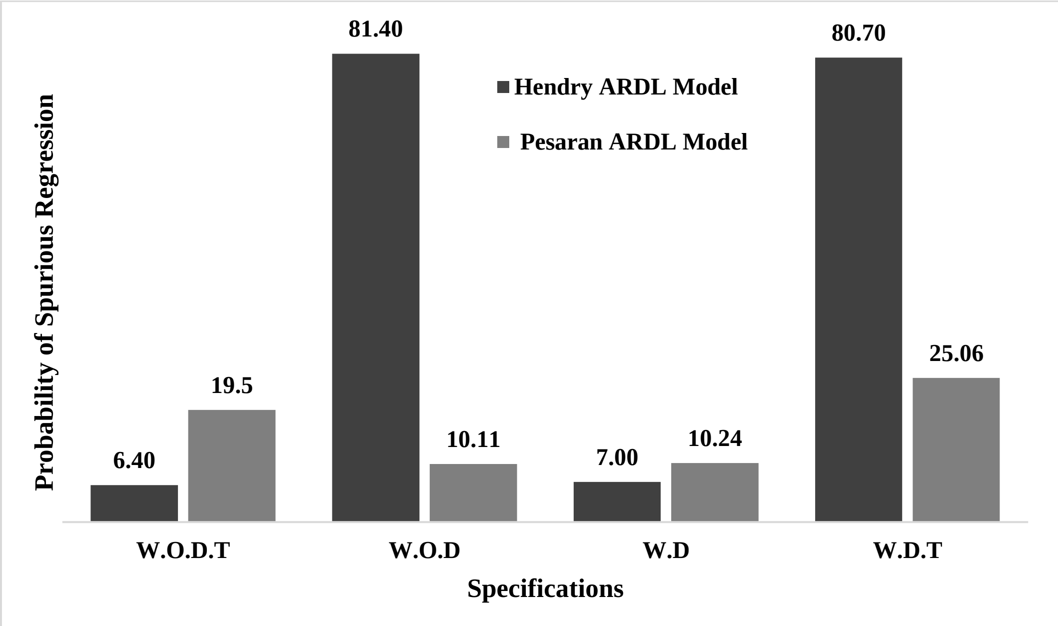


Figure 8.1 shows the probability of spurious regression on the basis of Pesaran's cointegration equation and Autoregressive Distributed Lag models. The F test is used for joint significance of current and lag values of independent variable in ARDL models.

In first case, when series have no drift and trend we got 19.5% probability of spurious regression by employing bond testing and only 6.40% with Hendry ARDL model. It shows that in case of without drift and trend Pesaran ARDL model procedure suffers in size problem while ARDL has no size distortion in this case. In second case, when series have linear trend we got 10.11% probability of spurious regression by using Pesaran ARDL model and 81.40% with Hendry ARDL model. In this case Hendry ARDL model performs badly due to under specification. In third case when series have drift only, we got 10.24% probability of spurious regression by using Pesaran ARDL model and 7.00% with Hendry ARDL model. In this case Hendry ARDL model perform very well as compare to Pesaran ARDL model because in this case Pesaran ARDL model has more size distortion as compare to Hendry ARDL model. In fourth case when series have drift and linear trend, we got 25.06% probability of spurious regression by using Pesaran ARDL model and 80.70% with Hendry ARDL model. In this case Hendry ARDL model performs due to under specification. All these results show that Hendry ARDL model has good size in case of exact and under specification and badly perform in case of trend misspecification. While Pesaran ARDL model performs good in trend misspecification case due to difference term and it has huge size distortion in all other cases as compare to Hendry ARDL model.

The figure 8.2 is based on the results of table 8.1. When autoregressive parameters $\theta_1 = \theta_2$ are equal to 0.8, means series are stationary. Figure 8.2, shows the comparison among the probability of spurious regression with Pesaran ARDL model and Hendry ARDL model. This comparison has been made on different specifications.

In first case when series have no drift and trend we got 42.80% probability of spurious regression by employing bond testing and only 6.10% with Hendry ARDL model. It shows that in case of without drift and trend Pesaran ARDL model procedure suffers in huge size problem while ARDL has no size distortion in this case. In second case when series have linear trend we got 9.75% probability of spurious regression by using Pesaran ARDL model and 34.60% with Hendry ARDL model. In this case Hendry ARDL model performs badly due to under specification and Pesaran ARDL model has less size distortion due to difference terms.

Figure 8.2: Size of Pesaran ARDL Model and Hendry ARDL Model with Stationary Series at different Specification when $\theta_1 = \theta_2 = 0.8$

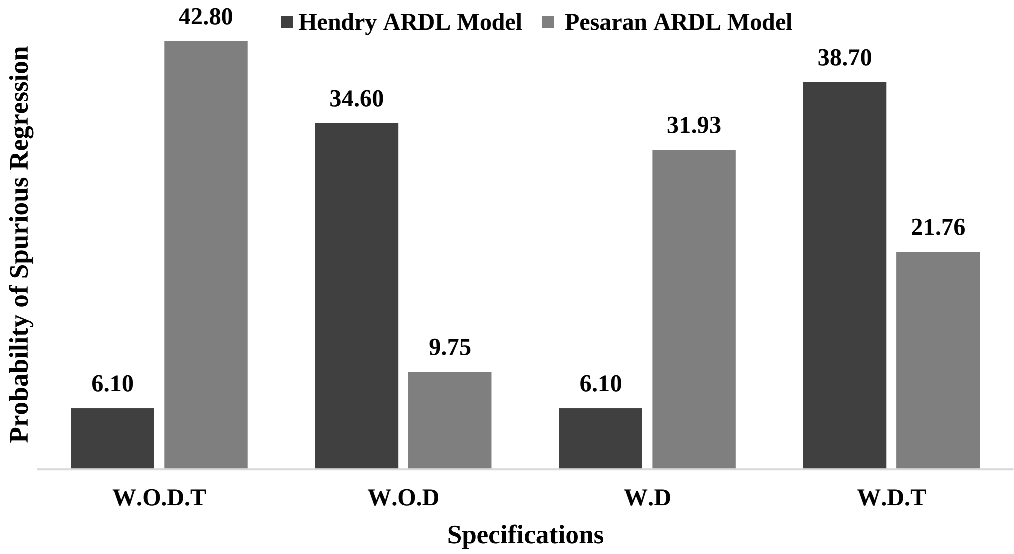


Figure 8.2 shows the probability of spurious regression on the basis of Pesaran's cointegration equation and Autoregressive Distributed Lag models. The F test is used for joint significance of current and lag values of independent variable in ARDL models.

In third case when series have drift only we got 31.93% probability of spurious regression by using Pesaran ARDL model and 6.10% with Hendry ARDL model. In this case Hendry ARDL model perform very well as compare to Pesaran ARDL model because in this case Pesaran ARDL model has more size distortion as compare to Hendry ARDL model. In forth case when series have drift and linear trend we got 21.76% probability of spurious regression by using Pesaran ARDL model and 38.70% with Hendry ARDL model. In this case Hendry ARDL model performs badly due to under specification. All these results show that Hendry ARDL model has good size in case of exact and under specification and badly perform in case of trend misspecification. While Pesaran ARDL model performs good in trend misspecification case due to difference term and it has huge size distortion in all other cases as compare to Hendry ARDL model. The complete results of size analysis are given in table 8.1:

Table 8.1: Probability of Spurious Regression with Pesaran ARDL model and Hendry ARDL models

	Hendry ARDL (1, 1)				Pesaran ARDL model
	xt	xt-1	yt-1	F-stat	F-stat
$\theta_1 = \theta_2$	$a_1 = b_1 = 0 = a_2 = b_2 = 0$ (no drift, no trend)				
1	6.6	6.6	100.0	6.4	19.5
0.8	6.2	6.2	99.7	6.1	42.8
0.6	6.0	6.0	95.8	6.0	79.9
0.4	5.8	5.9	95.9	5.7	95.8
0.2	5.8	5.9	21.1	5.6	99.3
	$a_1 = b_1 = 0$ (no drift)				
1	94.9	81.8	100.0	80.4	10.11
0.8	16.0	6.5	100.0	34.6	9.75
0.6	16.9	17.1	100.0	25.4	7.80
0.4	29.3	30.5	99.0	13.8	15.80
0.2	47.1	49.6	90.2	11.4	28.11
	$a_2 = b_2 = 0$ (with drift)				
1	6.1	8.0	100.0	7.0	10.24
0.8	6.2	6.6	100.0	6.1	31.93
0.6	6.0	6.2	99.4	6.1	68.47
0.4	5.9	6.0	80.8	5.9	90.48
0.2	6.0	6.0	25.2	5.6	97.92
	$a_1 = b_1 = a_2 = b_2 \neq 0$ (with drift and trend)				
1	95.9	83.1	100.0	81.7	25.06
0.8	11.5	8.4	100.0	38.7	21.76
0.6	31.1	8.5	99.9	24.9	22.62
0.4	49.9	17.1	98.4	19.9	39.35
0.2	67.5	36.3	86.1	9.20	57.06

The series have only autoregressive structure; this means the series has strong dependence on its own past. But the error terms of Series x are independent of the terms in y. Therefore, x should not appear in the equation of y, and if it appears, it indicates spurious regression. In first row of first panel of table 8.1, the results are indicating that when series are nonstationary, the autoregressive parameters θ_1=1 and θ_2=1 having no drift and trend (a_1= b_1= 0 =a_2= b_2= 0) then after employing Pesaran ARDL model, we get 19.5%, actual empirical size at sample size of 50. So on the basis of 5% nominal size, the probability of spurious regression is 14.5%. In Hendry ARDL models, F-test is being used to test the joint significance of current and lag values of independent variables, the F-stat value after employing Hendry ARDL (1, 1) model is found only 6.4% which shows only 1.4% probability of spurious regression on the basis of 5% nominal level of significance at sample size of 50. It means Hendry ARDL (1, 1) reduced the probability of spurious regression from 19.5% to only 6.4%. It shows that Pesaran ARDL model with nonstationary series having no drift and trend produce huge size distortion and Hendry ARDL model has no size distortion in this case.

In second row, of first panel of table 8.1, the results are indicating that by employing Pesaran ARDL model, we get 42.8% actual empirical. So with 5% nominal size, the probability of spurious regression is 37.8%. The F-stat value after employing ARDL (1, 1) model is found only 6.1% which shows only 1.1% probability of spurious regression. In indicates that the Pesaran ARDL model has huge size distortion with stationary time series while Hendry ARDL model has no size problem in this case. It means Pesaran ARDL model has huge size distortion in this case with both stationary and nonstationary time series and Hendry ARDL model has no size problem in this case. The results of first panel of table 8.1 illustrate as we reduce the values of autoregressive parameters from 0.8 to 0.6 and so on, the Pesaran ARDL model got more size distortion. It indicates that as we move faraway from unit root the Pesaran ARDL model performs badly.

In first row, of second panel, of table 8.1, the results are indicating that when series are nonstationary, the autoregressive parameters θ_1=1 and θ_2=1 having no drift (a_1= b_1= 0) then after employing Pesaran ARDL model, we get 10.11%, actual empirical size. So on the basis of 5% nominal size, the probability of spurious regression is 5.11%. The F-stat value after employing Hendry ARDL (1, 1) model is found only 81.4% which shows only 75.4% probability of spurious regression on the basis of 5% nominal level of significance. It indicates that in case of linear trend misspecification Hendry ARDL model has huge size while Pesaran ARDL model has less size distortion due to difference terms.

In second row, of second panel, of table 8.1 when the series are stationary, the results are indicating that by employing Pesaran ARDL model, we get 9.75% actual empirical. So with 5% nominal size, the probability of spurious regression is 4.75%. The F-stat value after employing Hendry ARDL (1, 1) model is found 34.6% which shows only 29.6% probability of spurious regression. In indicates that the Pesaran ARDL model has size distortion with stationary time series while Hendry ARDL model has huge size problem in this case due to trend misspecification. The results of second panel of table 8.1 illustrate as we reduce the values of autoregressive parameters from 0.8 to 0.6 and so on, the Pesaran ARDL model got more size distortion. It indicates that as we move faraway from unit root the Pesaran ARDL model performs badly. When the values of autoregressive parameters are 0.4 and 0.2 the probability of spurious regression with Pesaran ARDL model are more than the Hendry ARDL model. It shows that Pesaran ARDL model has huge size distortion with stationary time series but with nonstationary time series Pesaran ARDL model has less size distortion in case of trend misspecification. The next two panel has same fashion that is why we are not interpreting them here.

8.2 Robustness of Size to Misspecification

In this analysis, we evaluate the robustness of Pesaran ARDL model and Hendry ARDL model with different specifications on the basis of size analysis. The possible three specification cases which have been considered in this analysis are, under, exact and over specified regression. The Monte Carlo simulations have been used in this analysis. All the results in table 8.2 summarized after 100,000 times simulations. The series have been generated by using data generating process in equation 5.1. In this analysis only independent nonstationary series are used with autoregressive parameter specification θ_1=1 and θ_1=1.

Specification Cases

		Data Generating Process	
		Drift	***Drift and Trend***
Test Equation	***Drift***	Exactly Specified	Under Specified
	Drift and Trend	Over Specified	Exactly Specified

The results are summarized in table 8.2 given below. The first panel of table 8.2 given above describes the results of Pesaran ARDL model. The size in cases of exact specification is 10.24% in case of only drift and 4.2% in case of drift and trend. It shows the probabilities of spurious regression in case of both exact specification cases are 5.24% and 0% respectively on the basis of 5% nominal level of significance. The second panel of table 8.2 illustrates the results of Hendry ARDL model. In case of correct specification with drift and drift and trend, the size distortions (probabilities of spurious regression) are very minor 1.1% and 3.4% respectively. The size of Hendry ARDL model with correct specifications are 6.1% with drift and 8.4% with drift and trend, which is negligible. The order of statistics of spuriousness in case of correct specification is given in following equation:

$$Probability\ of\ spurious\ regression\ (Pesaran\ ARDL\ Model > Hendry\ ARDL\ Model)$$

Table 8.2 Size Analysis under Different Specifications

		Pesaran ARDL model Test	
		Data Generating Process	
		Drift	***Drift and Trend***
Test Equation	***Drift***	10.24	25.06
	Drift and Trend	8.37	4.42
		Hendry ARDL model	
		Data Generating Process	
		Drift	***Drift and Trend***
Test Equation	***Drift***	6.1	87.7
	Drift and Trend	7	8.4

These statistics clearly indicates that Pesaran ARDL model has probabilities of spurious regression even in correct specifications and Hendry ARDL model has very minor spurious regression probability which is theoretically negligible.

Secondly, we consider the case of under specification. The first panel of table 8.2 which is showing the size results of Pesaran ARDL model, indicates that the size is 25.06%. It means there is 20.06% probability of spurious regression. The second panel of table 8.2 indicates that the probability of spurious regression is 87.7% which is too high. The order of statistics of spuriousness in case of under specification is given in the following equation:

$$Probability\ of\ spurious\ regression\ (\text{Hendry}\ ARDL\ Model > Pesaran\ ARDL\ Model)$$

Thus, these results demonstrate that Pesaran ARDL model has huge probability of spurious regression even in under specification but Hendry ARDL model has very high spurious regression probability. It means in case of under specification Hendry ARDL model works worse than other Pesaran ARDL model but both procedures are suffering in huge size distortion problem.

Thirdly, we take the case of over specification. The first panel of table 8.2 which is showing the size results of Pesaran ARDL model indicates that the size is 8.37%. It means that there is 3.37% probability of spurious regression. The second panel of table 8.2 indicates that the probability of spurious regression is 7% with Hendry ARDL model. The order of statistics of spuriousness in case of over specification is given in the following equation:

$$Probability\ of\ spurious\ regression\ \ (Pesran\ ARDL\ > Hendry\ ARDL\ Model)$$

Thus, these results validate that Pesaran ARDL model suffer in size distortion problem even in correct specifications and Hendry ARDL model has very minor spurious regression probability. After size analysis we conclude that the Hendry ARDL model works better than other Pesaran ARDL model except under specification. However, in case of trend misspecification Pesaran ARDL model works better than Hendry ARDL model with nonstationary time series.

8.3 Power Analysis

Power analysis is performed to assess the probability of rejection the null hypothesis, when the alternative hypothesis is true. As the statistical power of test increases, the probability of type II error is decreased. It can be expressed in following way:

$$\text{Power} = \text{Prob (reject } \mathrm{H_0}| \text{ when} \mathrm{H_1} \text{ is true)}$$

In this study, we employ power analysis by following equation 6.1 to measure the power of Pesaran ARDL model and Hendry ARDL model with different specifications in different scenarios. The Monte Carlo simulations have been used in this analysis. All the results in the tables given below have been summarized after 100,000 times simulations.

It is important thing that as Pesaran model suffers in size distortion problem, it is generally accepted that power cannot be compared for when size is not same and the test with no size distortion becomes preferable. The size analysis shows that Pesaran ARDL model and Hendry ARDL model have size distortion in trend misspecification but Hendry ARDL model has more size distortion in this case. In case of without drift and trend time series the Hendry ARDL model has no size distortion while Pesaran ARDL model has size distortion. In case with drift time series the Hendry ARDL model has no size distortion while Pesaran ARDL model has size distortion.

Another important thing is that the Pesaran ARDL model has huge size distortion with stationary time series and it is going to be high when we move faraway from unit root. While Hendry ARDL model perform well in both cases. This concludes that Pesaran ARDL model cannot be used as a remedy of spurious regression with stationary time series. The reason behind this size distortion is negative moving average (NMA). It also has size distortions in nonstationary case.

In first row of first panel of table 8.3, the results are indicating that when series are nonstationary, θ_1=1 and θ_2=0.8, having no drift and trend, a_1= b_1= 0 =a_2= b_2= 0 and θ_{21}=0.8, then the F-stat value after employing Hendry ARDL (1, 1) model is indicating that the power of Hendry ARDL (1, 1) model is 93.9% which shows 1.1% power loss at 5% nominal size. While the power of Pesaran ARDL model procedure is 99.7%

which shows no power loss. But it is not the real power because Pesaran ARDL model has size distortion.

Table 8.3: Power Analysis of Lag Dependent Series without Drift and Trend

	Hendry ARDL (1, 1)				Pesaran ARDL model
	xt	**xt-1**	**yt-1**	**F-stat**	**F-stat**
$\theta_1=\theta_2$			$\theta_{21}=$ **0.8**		
1	95.9	6.7	100.0	93.9	99.7
0.8	91.5	6.0	100.0	85.6	99.23
0.6	48.8	6.0	100.0	56.4	98.47
0.4	18.4	5.9	92.6	19.9	99.5
0.2	8.4	5.9	36.4	11.3	99.83
			$\theta_{21}=$ **0.6**		
1	86.5	6.2	100.0	84.3	99.25
0.8	68.0	6.1	100.0	60.2	94.47
0.6	21.0	5.9	99.7	20.7	95.13
0.4	9.4	6.0	86.3	10.6	98.07
0.2	6.6	5.8	29.6	6.8	99.95
			$\theta_{21}=$ **0.4**		
1	67.1	6.1	100.0	74.6	95.7
0.8	21.8	6.1	100.0	30.6	79.02
0.6	8.5	6.0	98.9	21.4	87.11
0.4	6.7	5.9	78.1	11.9	96.16
0.2	6.1	6.0	25.1	7.2	99.06
			$\theta_{21}=$ **0.2**		
1	19.8	5.2	100.0	27.6	57.53
0.8	7.0	6.2	99.9	9.5	48.77
0.6	6.3	5.9	97.0	7.5	74.46
0.4	6.3	5.9	71.7	6.4	92.15
0.2	6.1	5.8	22.4	6.1	98.27

Table 8.4: Power Analysis of Lag Dependent Series without drift

	Hendry ARDL (1, 1)			Pesaran ARDL model
	xt	xt-1	yt-1	F-stat
$\theta_1 = \theta_2$		$\theta_{21} = 0.8$		
1	90.9	31.0	100.0	97.87
0.8	86.8	5.6	100.0	99.8
0.6	82.7	5.7	100.0	87.56
0.4	33.1	23.3	100.0	71.38
0.2	8.8	53.5	99.5	70.43
		$\theta_{21} = 0.6$		
1	84.1	26.2	100.0	97.86
0.8	82.1	27.4	100.0	97.61
0.6	42.9	6.7	100.0	73.41
0.4	10.7	27.6	100.0	60.57
0.2	6.6	56.8	99.0	61.33
		$\theta_{21} = 0.4$		
1	76.1	15.1	100.0	97.98
0.8	67.72	28.82	100.00	99.16
0.6	10.32	9.61	100.00	46.01
0.4	6.47	31.68	99.97	36.86
0.2	15.86	58.24	97.83	44.72
		$\theta_{21} = 0.2$		
1	21.9	10.2	100.0	98.01
0.8	9.4	26.3	100.0	92.32
0.6	7.7	14.2	100.0	18.77
0.4	17.6	33.7	99.8	21.55
0.2	34.3	56.4	95.4	33.27

Table 8.5: Power Analysis of Lag Dependent Series with drift

	Hendry ARDL (1, 1)			Pesaran ARDL model
	xt	xt-1	yt-1	F-stat
θ_1=θ_2			θ_{21}= **0.8**	
1	94.3	7.7	100.0	99.9
0.8	96.7	6.0	100.0	99.03
0.6	53.2	5.9	100.0	98.59
0.4	16.7	5.8	94.8	99.33
0.2	7.1	5.9	37.9	99.97
			θ_{21}= **0.6**	
1	89.0	6.9	100.0	99.86
0.8	76.3	6.0	100.0	95.82
0.6	20.7	5.9	100.0	97.23
0.4	8.0	6.0	89.0	98.13
0.2	5.8	5.9	30.9	99.91
			θ_{21}= **0.4**	
1	57.6	4.6	100.0	99.15
0.8	28.8	6.1	100.0	97.35
0.6	6.7	6.0	99.6	97.13
0.4	5.5	5.9	81.6	98.7
0.2	5.7	5.7	26.0	99.64
			θ_{21}= **0.2**	
1	11.9	2.7	100.0	92.01
0.8	5.7	6.0	100.0	52.41
0.6	5.8	5.9	98.4	76.51
0.4	5.7	5.8	74.1	94.32
0.2	5.8	5.9	22.7	99.2

Table 8.6: Power Analysis of Lag Dependent Series with drift

	Hendry ARDL (1, 1)			Pesaran ARDL model
	xt	xt-1	yt-1	F-stat
$\theta_1=\theta_2$		θ_{21}= 0.8		
1	95.9	7.1	100.0	98.01
0.8	99.9	17.9	100.0	99.68
0.6	84.2	7.6	100.0	88.14
0.4	32.6	23.3	100.0	71.85
0.2	7.8	51.6	99.5	71.8
		θ_{21}= 0.6		
1	81.5	6.9	100.0	98.28
0.8	97.6	17.9	100.0	99.57
0.6	45.3	9.9	100.0	70.45
0.4	10.4	27.9	100.0	55.49
0.2	6.9	54.7	98.9	60.29
		θ_{21}= 0.4		
1	67.1	6.1	100.0	98.9
0.8	73.7	17.6	100.0	99.54
0.6	11.3	13.3	100.0	50.04
0.4	6.8	31.2	100.0	41.67
0.2	17.5	56.2	97.7	45.41
		θ_{21}= 0.2		
1	19.8	4.6	100.0	98.5
0.8	13.0	13.6	100.0	92.45
0.6	7.6	17.2	100.0	16.04
0.4	19.0	32.4	99.8	21.75
0.2	37.6	54.1	95.1	33.73

In second row of first panel of table 8.3, the results are representing that when series are stationary, θ_1= 0.8 and θ_2 = 0.8, having no drift and trend, a_1= b_1= 0 =a_2= b_2= 0 and θ_{21}=0.8, then the F-stat value after employing Hendry ARDL (1, 1) model is showing that the power of Hendry ARDL (1, 1) model is 85.6% which shows 14.4% power loss at 5% nominal size. In first row of second panel of table 8.3, the results are indicating that when series are nonstationary, θ_1=1 and θ_2=0.8, having no drift and trend, a_1= b_1= 0 =a_2= b_2= 0 and θ_{21}=0.6, then the F-stat value after employing Hendry ARDL (1, 1) model is indicates that the power of Hendry ARDL (1, 1) model is 84.3% which shows 10.7% power loss at 5% nominal size. The power of Pesaran ARDL model 99.25 which shows 0% power loss. In second row of second panel of table 6.1, the results are representing that when there is stationary series, θ_1= 0.8 and θ_2 = 0.8, having no drift and trend, a_1= b_1= 0 =a_2= b_2= 0 and θ_{21}=0.6, then the F-stat value after employing Hendry ARDL (1, 1) model is indicating that the power of Hendry ARDL (1, 1) model is 60.2% which shows 34.8% power loss at 5% nominal size.

The Pesaran ARDL model test is showing no power loss on all values of autoregressive parameters which are misleading due to size distortion. While Hendry ARDL model has no size distortion in without drift and trend specifications. That is why it has real powers on all the values of which are more than Pesaran ARDL model test power. It clarifies that when series are stationary or nonstationary without having drift and trend Hendry ARDL works better than Pesaran ARDL model.

In first row of first panel of table 8.4, the results show that when we regressed nonstationary series θ_1=1 and θ_2=0.8, without drift, a_1= b_1= 0 and θ_{21} = 0.8. The F-test is used only in one case for displaying the joint significance of independent lag and current value. So, table 8.4, 8.5, and 8.6 have only t-stat values. After employing Hendry ARDL (1, 1) model, the power of current value of x is 90.9%. It means that there is only 4.1% power loss. The reason behind it might be we did not include linear trend in Hendry ARDL model, if we include linear trend, it may provide more power. The figure of lag value of x is showing only 31% power which means 59% power loss.

As we know that y value is determined through lag value of x, but the current value are more significant as compare to lag value of x. The reason is that there is multicollinearity effect. The current and lag values of x variable are collinear that is why the effect shifts into current value in Hendry ARDL model (1, 1). The power of Pesaran ARDL model without drift or time series with linear trend is 97.7% which shows 0% power loss.

In second row of first panel of table 8.4, the results illustrate that when we regressed stationary series θ_1=0.8 and θ_2=0.8, without drift, a_1= b_1= 0 and θ_{21} = 0.8 after employing Hendry ARDL (1, 1) model, the power of current value of x is 86.8% which means only 8.6% power loss. The reason behind it is that we did not include linear trend in Hendry ARDL. If we include linear trend, it may provide more power. The figure of lag value of x is showing only 5.6% power which means 89.4% power loss. While the power of Pesaran ARDL model is 99.8% which means no power loss.

When the series have linear trend and tests have trend misspecification problem, then Pesaran ARDL model is performed well as compare to Hendry ARDL model in case of nonstationary time series. But both tests have size distortion problem in trend misspecification case. Same pattern has been found on other values of θ_{21} like, 0.6 and so on.

Though in some cases when the values of parameter*s* θ_1, θ_2, and θ_{21}, series are stationary the Pesaran ARDL model shows more power, yet we cannot consider it because as we have seen in size analysis the Pesaran ARDL model suffers badly in size problem and ARDL in all cases has less size problem. That is why, we cannot say that Pesaran ARDL model has more power.

Similarly results in tables 8.5 and 8.6 also show that two scenarios of lag dependent series with drift and with drift and trend depict the same fashion. So, the interpretations of these cases are approximately alike. The own lag values of y are highly significant in all cases, but one thing which is necessary to consider is that as we reduce the value of autoregressive terms, the significance of lag values also decreases in case of Hendry ARDL model and not in Pesaran ARDL model.

8.3.1 Robustness of Power to Misspecification

Table 8.7, shows the results of power of Pesaran ARDL model test under different specifications at different values of autoregressive parameter θ_2. At first we consider correct specification with drift and drift and trend cases. The first panel of table 8.7, describes the results of Pesaran ARDL model test when θ_2=0.8. The power in case of correct specification is 99.45% with drift. It shows 0% power loss when autoregressive parameter θ_2= 0.8 and x lag value coefficient θ_{21}= 0.8. The power under correct specification is 98.58% with drift and trend. It shows 0% power loss when lag value parameter of y is θ_2= 0.8 and x lag value coefficient is θ_{21}= 0.8. Table 8.8, describes the results of power of Hendry ARDL model under different specifications at different values of autoregressive parameter θ_2.

Table 8.7: Power Analysis of Hendry ARDL model by using Lag Dependent Series at Different Specifications

Pesaran ARDL model			
		$\theta_2 = 0.8$	
		Data Generating Process	
		Drift	***Drift and Trend***
Test Equation	***Drift***	99.45	99.99
	Drift and Trend	95.69	98.58
		$\theta_2 = 0.6$	
		Data Generating Process	
		Drift	***Drift and Trend***
Test Equation	***Drift***	99.52	99.26
	Drift and Trend	95.81	95.3
		$\theta_2 = 0.4$	
		Data Generating Process	
		Drift	***Drift and Trend***
Test Equation	***Drift***	99.7	90.9
	Drift and Trend	95.47	96.69
		$\theta_2 = 0.2$	
		Data Generating Process	
		Drift	***Drift and Trend***
Test Equation	***Drift***	99.85	81.08
	Drift and Trend	97.03	98.01

Table 8.8: Power Analysis of Hendry ARDL model by using Lag Dependent Series at Different Specifications

Hendry ARDL model			
		$\theta_2 = 0.8$	
		Data Generating Process	
		Drift	***Drift and Trend***
Test Equation	***Drift***	98.1	98.7
	Drift and Trend	99	99.9
		$\theta_2 = 0.6$	
		Data Generating Process	
		Drift	***Drift and Trend***
Test Equation	***Drift***	99.3	99
	Drift and Trend	99	99.9
		$\theta_2 = 0.4$	
		Data Generating Process	
		Drift	***Drift and Trend***
Test Equation	***Drift***	99.5	99
	Drift and Trend	99	100
		$\theta_2 = 0.2$	
		Data Generating Process	
		Drift	***Drift and Trend***
Test Equation	***Drift***	100	99
	Drift and Trend	99	100

Firstly, we consider correct specification with drift and drift and trend cases. The first panel of table 8.8, refers the results of Hendry ARDL model when θ_2=0.8. The power in case of correct specification is 98.1% with drift. It shows 0% power loss when θ_2= 0.8 and θ_{21}= 0.8. The power under correct specification is 99.9% with drift and trend. It shows 0% power loss at θ_2= 0.8 and θ_{21}= 0.8.

The second panel of table 8.7, illustrates the power results of Pesaran ARDL model test when θ_2=0.6. The power in case of correct specification is 99.52% with drift. It shows 0% power loss when θ_2= 0.6 andθ_{21}= 0.8. The power under correct specification is 95.3% with drift and trend, it shows 0% power loss when θ_2= 0.8 and θ_{21}= 0.6. The second panel of table 8.8 given above describes the power results of Hendry ARDL model when θ_2=0.6. The power in case of correct specification is 99.3% with drift, it shows 0% power loss when θ_2= 0.6 and θ_{21}= 0.8. The power under correct specification is 99.9% with drift and trend, it shows 0% power loss when θ_2= 0.8 andθ_{21}= 0.8.

Here we have seen both tests performed very well and they have no power loss at correct specification case. But the size analysis clears that in case of correct specification with only drift, the Pesaran ARDL model suffers in size distortion. Thus because of size problem we cannot say Pesaran ARDL model has 0% loss in this case. While Hendry ARDL model has no size problem in this case, so we can consider the power of Hendry ARDL model as real power. Nonetheless, in case of correct specification with drift and trend the Hendry ARDL model suffers in size problem but it is negligible because it is less than 10% nominal significance level. The order of statistics of power in case of correct specification is following:

$$Power\ (\text{Hendry}\ ARDL\ Model > Pesaran\ \ ARDL\ Model)$$

Similarly, when the values of autoregressive parameters are θ_2 = 0.4 and 0.2, same pattern has been found. It clearly indicates that in correct specification cases the Hendry ARDL model has good power while Pesaran ARDL model procedure is having huge power loss. We cannot compare the power of these tests and Hendry ARDL model because as we have seen in size analysis the cointegration procedure suffer in size distortion problem even in case of correct specification.

Secondly, we consider the case of under specification. Table 8.7, shows the results of power of Pesaran ARDL model test under different specifications at different values of autoregressive parameter θ_2. The first panel of table 8.7, describes the results of Pesaran ARDL model test when θ_2=0.8. The power in case of under specification is 99.9% with drift. It shows 0% power loss when θ_2= 0.8 andθ_{21}= 0.8. Table 6.31, shows the results of power of Hendry ARDL model under different specifications at different values of autoregressive parameter θ_2. The first panel of table 6.31 describes the results of Hendry ARDL model when θ_2=0.8. The power in case of under specification is 98.7% with drift. It shows 0% power loss when θ_2= 0.8 andθ_{21}= 0.8.

The second panel of table 6.29, describes the results of Engle and Granger cointegration test when θ_2=0.6. The power in case of under specification is 99.26%, it shows 0% power loss when θ_2= 0.6 andθ_{21}= 0.8. Table 8.8 shows the results of power of Hendry ARDL model under different specifications at different values of autoregressive parameter θ_2. The first panel of table 8.8, describes the results of Hendry ARDL model when θ_2=0.6. The power in case of under specification is 99.0% with drift. It shows 0% power loss when θ_2= 0.6 and θ_{21}= 0.8.

Thus, these results validate that Pesaran ARDL model test and Hendry ARDL model are having very good powers with 0% power loss. In case of under specification, Hendry ARDL model badly suffers in size distortion problem as we have seen in size analysis. So, we cannot say that ARDL has good power on the basis of these statistics that is in under specification according to our estimation Pesaran ARDL model works well because it has huge size distortion but less as compare Hendry ARDL model in this case. The order of power of ARDL in case of under specification is given in the following equation:

$$Power\ \ (Pesran\ ARDL\ Model > \text{Hendry}\ ARDL\ Model)$$

Similarly at all the values of autoregressive parameter θ_2 = 0.4 and 0.2 same pattern has been found.

At third we consider the case of over specification. Table 8.7, shows the results of power of Engle and Granger cointegration test over different specifications at different values of autoregressive parameter θ_2. The first panel of table 8.7, describes the results of Pesaran ARDL model test when θ_2=0.8. The power in case of over specification is 95.69%. It shows 0% power loss when θ_2=0.8 andθ_{21}= 0.8. Table 8.8, shows the results of power of Hendry ARDL model under different specifications at different values of autoregressive parameter θ_2. The first panel of table 8.8, describes the results of Hendry ARDL model when θ_2=0.8. The power in case of over specification is 98.1%. It shows 0% power loss when θ_2= 0.8 andθ_{21}= 0.8.

The second panel of table 8.7 describes the results of Pesaran ARDL model test when θ_2=0.6. The power in case of over specification is 95.81%. It shows 0% power loss when θ_2= 0.6 and θ_{21}= 0.8. The second panel of table 8.8, describes the results of Hendry ARDL model when θ_2=0.6. The power in case of under specification is 99.3% with drift. It shows 0% power loss when θ_2= 0.6 and θ_{21}= 0.8.

Thus, these results validate that the Hendry ARDL model has more power as compare to Pesaran ARDL model test. Also in this case Pesaran ARDL model and Hendry ARDL model have size distortions but the Pesaran ARDL model has more a compare to Hendry ARDL model. These size distortions are negligible because both are less than 10% nominal level of significance. In case of over specification Hendry ARDL model does not suffer in size distortion problem as we have seen in size analysis. So, we can say that ARDL has more power in case of over specification. The power of ARDL is given as follows;

$$Power \ \ (Hendry \ \ ARDL\ Model > Pesaran\ ARDL\ Model)$$

Similarly at all the values of autoregressive parameter θ_2 = 0.4 and 0.2 same pattern has been found. It clearly indicates that in over specification ARDL works good as compare to other techniques.

CHAPTER 9

TRACING DYNAMIC LINKAGES & SPILLOVER EFFECT BETWEEN PAKISTANI & LEADING FOREIGN STOCK MARKETS WITH ARDL & GARCH MODELS

9.1 Introduction

Modern econometric tools are used for investigating volatility co-movement between the financial markets. The global financial integration started in the mid-1980s, consequently risk and return Co-movements between the financial markets were observed at that time. The growing economic integration of intercontinental financial markets has attained significance since last three decades. The major factors behind this observed globalization are extensive growth of technology, easy capital flow and financial links between the economies. That is why, the analysis of the nature and level of linkages between different financial markets is significant for financial institutes, portfolio managers and market players. Engle et al. (1990) proposed the meteor shower hypothesis to trace out intra-market co-movements". The global financial crisis of 2008 was one of the worst financial crises of US history. It not only triggered imbalances in US economy but also impacted a major part of overall global economy. Most of the global financial crises initiated from US economy and due to the strong interdependence of US economy with other economies these crises impacted all integrated economies at some extent. The key reasons identified by the academic researchers behind this crisis were excessively relaxed monetary policy, regulatory failures in macro prudential and micro prudential levels, the accumulation of global balance of payment inequalities and flaws in the international financial planning (Kawai et al., 2012).

Owing to investment linkages with US economy the effect of financial crisis transmitted into Pakistan (Amjad & Din, 2010) and Dubai economy (Onour, 2010). Likewise, Dubai financial market had also impressively impacted Dubai's economic growth. The portfolio investment in Dubai financial market got reduced 24% in 2009. When financial crisis effects transmitted into Pakistan economy, the economy was facing some internal issues like political instability, bad governance, deficit in current account, rising unemployment, energy crisis and failure of macroeconomic policies. Pakistan and Dubai both countries have significant relationship in different sectors of economy. Dubai is one of the emerging markets of UAE. Over 1.2 million emigrants of Pakistan are providing their services in UAE. Their remittances significantly contributed to Pakistan's foreign reserves. UAE is the second prominent source of remittances from Pakistani emigrants. Pakistan expatriates provided $2.52 billion remittances in 2013-14 with 19.57% share in total remittances of. Similarly, UAE has major share in Pakistan exports and imports. In 2013-14 UAE had 8% share of total Pakistan's exports and 17% share in imports. At the occasion of UAE cityscape "7th annual property and real estate exhibition" 2008 in Dubai more than 100 Pakistani investors invested over $100 million for the booking of construction projects. There were a large number of Pakistani investors out of 40,000 visitors from all over the world who took part in this exhibition.

The objective of this study is to investigate the direct and indirect linkages between Pakistani and leading foreign stock markets in general and particularly during the global financial crisis of 2007-08. The leading stock markets having linkages with international financial system and US stock markets are selected from different regions of the world. In this study we analyze direct linkages between Pakistani and leading foreign stock markets by using whole data set. We also explore the indirect linkages between Pakistani and US stock markets through Dubai financial market by using whole and both subsets of data. All these findings help us in formulating more effective short run and long-run policies to tackle the effect of such global crises in favor of sustainable economic growth.

9.2 Literature Review

There exists strong integration of global economies through different financial or real links. Crisis in one part of the world is much likely to transmit to other parts. In 2007, when global markets experienced a huge wave of financial crisis due to United State sub-prime mortgage crisis. It not only impacted domestic economy of USA but also other economies of the world which are integrated directly or indirectly with US economy. Global financial crisis is one of the major factors which have shifted concentration on the dynamic linkages between the financial markets. Owing to the dynamic linkages, the information transmission is also existed between financial markets. The information transmission mechanisms were quantified through returns and volatilities (Padhi & Lagesh, 2012). Angkinand et al. (2009) investigated that how the financial crisis in US markets impacted 17 developed economies from 1973-2009, and they found the spillover effects from the US to other industrial countries were highest after collapse of U.S subprime mortgage market in the summer of 2007. Chelley (2005) explained the links of United State stock market with UK and European stock markets. The results showed strong bilateral relationship between US and UK stock markets while relationship of US market with other European economies was also found. The business cycle movements in the United Kingdom economy are more sensitive to disturbances in US relative to other European economies. Alsukker (2010) explored that US mortgage crisis 2008 affected Dubai financial market. Initially Dubai's economy tolerated the effect of global financial crisis but on 25 Nov, 2009

Dubai demanded suspension on debt repayment from world. Gomez and Ahmad (2009) examined that the neighbor country of Dubai, Abu Dhabi presented a loan of $10 billion for the management of its debt. At that time Dubai's amount of debt was roughly $59 billion and the overall global debt was 10 times more than that of Dubai's debt. Onour (2010) investigated that spillover effect of US Mortgage crisis 2008, badly affected oil producing countries including Dubai. The portfolio investment in Dubai financial market decreased up to 42%. When there was global financial crisis in 2008, Amjad and Din (2010) found that Pakistan economy was also facing an entire financial crisis at that time. Draz (2011) examined that Pakistan economy faced five financial crises (1958, 1974, 1979, 1997 and 2008) which badly affected Pakistan economy. They also explored that Pakistan economy was affected form global financial crisis 2008. There is empirical linkage between global financial crisis and Pakistani stock markets (Ali & Afzal, 2012; Zia-ur-Rehman et al., 2013; Attari & Safdar, 2013;Tahir et al., 2013). Mukherjee and Mishra (2008) explored that volatility spillover effect and linkages between India and other twelve emerging Asian and developed countries. Jeyanthi (2010), Sinha and Sinha (2010) investigated the relationships and volatility spillover effect among India, UK, Japan and USA stock markets and by incorporating the structural change. They concluded that there is volatility spillover from Japan and USA stock market's to Indian stock markets. Sok-Gee and Karim (2010) examined that there exists volatility spillover among five countries of ASEAN, Japan and USA. They found that USA stock market has more mean and volatility spillover effect on ASEAN stock markets as compare to Japan stock market. Alikhanov (2013) examined the volatility spillover effect between the eight European stock markets and oil price market and found a strong spillover effect from US to European stock and oil markets. Wongswa (2006) studied that there was strong indications of transmission of information from

US and Japan to Thailand and Korea. Due to the information transmission there was co-moment among the markets and also revealed the transmission from developed to emerging equity markets.

All the studies which we have reviewed conclude that the global financial crisis 2008 primarily, originated due to the creation and expansion of bubbles in housing and subprime markets of US. It triggered imbalances in US economy and also in those economies which were directly and indirectly integrated with US economy. Pakistan and Dubai economies were also affected by global financial crisis 2008 at some extent. At that time co-movements were also observed between the financial markets (particularly stock markets) due to direct and indirect linkages with global financial system. Studies related to Pakistan describe that the effect of global financial crisis 2008 transmitted into Pakistan economy through four main linkages, one of them is stock market. In this study we trace out direct and indirect linkages between Pakistani and leading foreign stock markets. These studies only investigated direct dynamic linkages between the financial markets (stock). There are no or very little traces of indirect information transmission. In this study, we have tried to find out direct linkages between the leading global stock markets and also indirect dynamic linkages between Pak-US stock markets through Dubai financial market.

9.3 Econometric Methodology and Model Specification

To describe the variation of conditional variance with respect to time, Engle (1982) proposed Autoregressive conditional hetroscedastic (ARCH) model. Although ARCH model is a substantial contribution in econometric tools, it has some problems like long lag length and non-negativity restriction on parameters. Bollerslev (1986) introduced generalized autoregressive conditional hetroscedastic (GARCH) model, which improves the unique specification with the addition of lag value of conditional variance, which acts like smoothing term. GARCH model cannot analyze Asymmetric and leverage effect. Glosten, Jagannathan and Runkle (1993) proposed GJR-GARCH model. GJR-GARCH model is a significant extension of standard GARCH model. It contains asymmetric term in conditional variance equation. To avoid any non-convergence problem in this study, we employ appropriate univariate GARCH type model such as GARCH (p, q) and GJR-GARCH (p, q) to estimate volatility models and to explore mean and volatility spillover effect. Following the technique of Hamao et al. (1990), we explored spillover effect between Pakistani and foreign stock markets.

The financial series at level are trendy in nature. It is impossible to estimate a robust model if the series is trendy. To deal with trend we used the log difference return.

$$\mathrm{R_t} = \log_e(\mathrm{l_t}/\mathrm{l_{t-1}})$$

l_t= Financial time series at level i.e. stock indices and exchange rates at the end of time t.

l_{t-1}= First lag of financial time series.

9.3.1 ARCH (q) Model

Engle (1982) introduced the Autoregressive conditional hetroscedastic (ARCH) model. This model overcomes all short comings which existed in previous models. In this model Engle, introduced conditional mean and conditional variance equations. Empirically the conditional mean equation follows ARMA (p, q) process and the conditional variance depends upon the square of past values of error process ε_t.

The general description of ARCH model is:

Conditional mean equation

$$R_t = \alpha_0 + \beta X_t + \varepsilon_t \tag{9.1}$$

Where $\varepsilon_t = z_t \sigma_t, z_t \sim N(0,1)$

Conditional variance equation

$$\sigma_t^2 = \theta_0 + \sum_{i=1}^{q} \theta_i \varepsilon_{t-1}^2 \tag{9.2}$$

Where $\theta_0 > 0, \theta_i \geq 0$ i= 1,2,…….., q

In conditional mean equation R_t represents the return which is linear function of X_t. $where\ \beta$ shows the vector of parameters. Empirically βX_t illustrates ARMA (m, n) process with different specifications. In some cases it may be ARMA (0, 0). According to the "Efficient Market Hypothesis (EMH)" R_t represents mean reversion behavior and it is unpredictable. In conditional variance equation the restriction on coefficients is that they must be non-negative. σ_t^2 Represents conditional variance which depends upon lags of squared past value of ε_t process.

9.3.2 GARCH (p, q) Model

Linear ARCH (q) model has some problems. First, sometime we take long lag length 'q' due to this number of parameters are going to increase. As a result there is loss of degree of freedom. Second, there is non-negativity condition of parameters. Bollerslev (1986) proposed Generalized autoregressive conditional hetroscedastic (GARCH) model.

The general description of GARCH model is:

Conditional mean equation

$$R_t = \alpha_0 + \beta X_t + \varepsilon_t \tag{9.3}$$

Where $\varepsilon_t = z_t \sigma_t, z_t \sim N(0,1)$

Conditional variance equation

$$\sigma_t^2 = \theta_0 + \sum_{i=1}^{q} \theta_i \, \varepsilon_{t-1}^2 + \sum_{i=1}^{p} \varphi_j \, \sigma_{t-1}^2 \tag{9.4}$$

Where $\theta_0 > 0, \theta_i \geq 0, \varphi_j \geq 0$

In GARCH (p, q) model the conditional variance depends upon square of past values of process ε_t and lag of conditional varianceσ_{t-1}^2. The condition of non-negativity of parameter is also applied in this model.

9.3.3 Asymmetric GARCH Models

Simple GARCH type models deal with the symmetric effect of bad and good news on volatility. These models do not take into account the asymmetries which are associated with the distribution. In financial econometrics literature, Asymmetric GARCH type models consider the asymmetries of response to bad or good news. Asymmetric GARCH models account for leverage effect. The leverage effect indicates the negative correlation between the assets returns and the volatility of the assets return (Black 1976) which shows the magnitude of bad and good news are different.

9.3.3.1 GJR-GARCH (p, q) Model

Glosten, Jagannathan and Runkle introduced (GJR) model in 1993. GJR model is a significant extension of simple GARCH model. This model also captures the asymmetries in ARCH process. GJR model also account for the leverage effect in a financial series.

The general representation of the GJR model is:

Conditional mean equation

$$R_t = \alpha_0 + \beta X_t + \varepsilon_t \tag{9.5}$$

Where $\varepsilon_t = z_t \sigma_t, z_t \sim N(0,1)$

Conditional variance equation

$$\sigma_t^2 = \theta_0 + \sum_{i=1}^{q} \theta_i \varepsilon_{t-i}^2 + \sum_{i=1}^{q} \delta_i \varepsilon_{t-i}^2 G_t + \sum_{i=1}^{p} \varphi_j \sigma_{t-j}^2 \tag{9.6}$$

Where $\theta_0 > 0$, $\theta_i \geq 0$, $\varphi_i \geq 0$

$0 \leq \delta_i \geq 1$ Range of leverage effect parameter.

Gt = 1 when $\varepsilon_{t-1} < 0$ and Gt = 0 when $\varepsilon_{t-1} \geq 0$

Gt = 1 when $\varepsilon_{t-1} < 0$ illustrates bad news or the negative shock and Gt = 0 when $\varepsilon_{t-1} \geq 0$ indicates good news or positive shock. GJR model also shows that bad news has more impact ($\theta_i + \delta_i$). The good news has less impact (θ_i). If the δ_i> 0, it means that there is leverage effect and shows that response to shock is distinct. If the δ_i= 0, it means symmetric response to distinct shock (In other words both news have same impact). Condition (θ_i+φ_i+$\frac{\delta_i}{2}$ <1) shows the persistence of shock.

9.3.4 GARCH (p, q) Model (for Exploring Spillover Effect)

Conditional Mean equation

$$R_{t,k} = \alpha_0 + \beta X_t + \pi_1 R_{t,s} + \varepsilon_t \tag{9.7}$$

Where $\varepsilon_t = z_t \sigma_t, z_t \sim N(0,1)$

Conditional Variance equation

$$\sigma^2_{t,k} = \theta_0 + \sum_{i=1}^{q} \theta_i \, \varepsilon^2_{t-1} + \sum_{i=1}^{p} \varphi_j \, \sigma^2_{t-1} + \pi_2 R^2_{t,s} \tag{9.8}$$

$R_{t,k}$ Shows the return series of K market. $R_{t,s}$ describes the return series of S market which is used as a regressor in conditional mean equation of K markets return series. π_1, Represents the parameter of S market returns series. $\sigma^2_{t,k}$, Denotes the conditional variance of K market. $R^2_{t,s}$ Indicates the squared returns series of S markets which is used as a regressor in conditional variance equation of K markets return series. π_2, Demonstrates the parameter of squared return series of S market. We trace out the co-movements among these markets by following the technique of Hamao et al. (1990). According to Hamao et al. (1990), the residuals of one return series introduce as a regressor in conditional mean equation of other return series for mean spillover effect. For volatility spillover effect, the squared residuals of one return series is introduced as a regressor in conditional variance equation of other return series. Instead of using residuals and squared residuals we use return series and squared return series. According to "The Efficient Market Hypothesis (EMH) return are unpredictable and show mean reversion behavior". To check the mean spillover effect between two series, the return series of one market is introduced as regressor in other market return series. For volatility spillover effect the square of return series of one market is introduced as regressor in the conditional variance equation of other market.

9.4 Estimations and Analysis

In this section at first we employ the graphical and descriptive analysis on series for understanding about the characteristics of series. At second, we employed the GRACH models and ARDL models to measure the spillover effect between Pakistani and leading foreign markets.

9.4.1 Graphical Analysis

Figure 9.1, shows that in beginning all series have upward trend than sharp decline and then again there is an upward trend continuously. This sharp decline is due to global financial crises 2008. It also shows that series are overall upward trendy at level. In figure 9.1, the series are (KSE 100) from Pakistan Stock market, Standard and Poor (S&P 500), Dow Jones, Nasdaq 100 are from USA and Dubai financial market (DFMGI). Daily data are used for the period of 2005 to 2014. To check spillover effect we synchronized the data in term of dates.

Figure 9.1: Graphs of Series of Stock prices at level

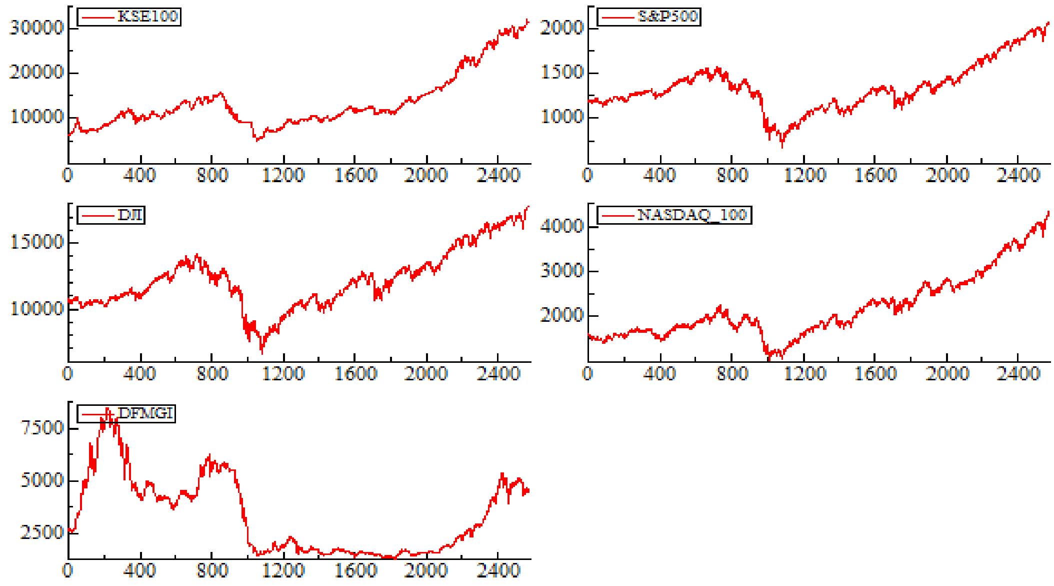
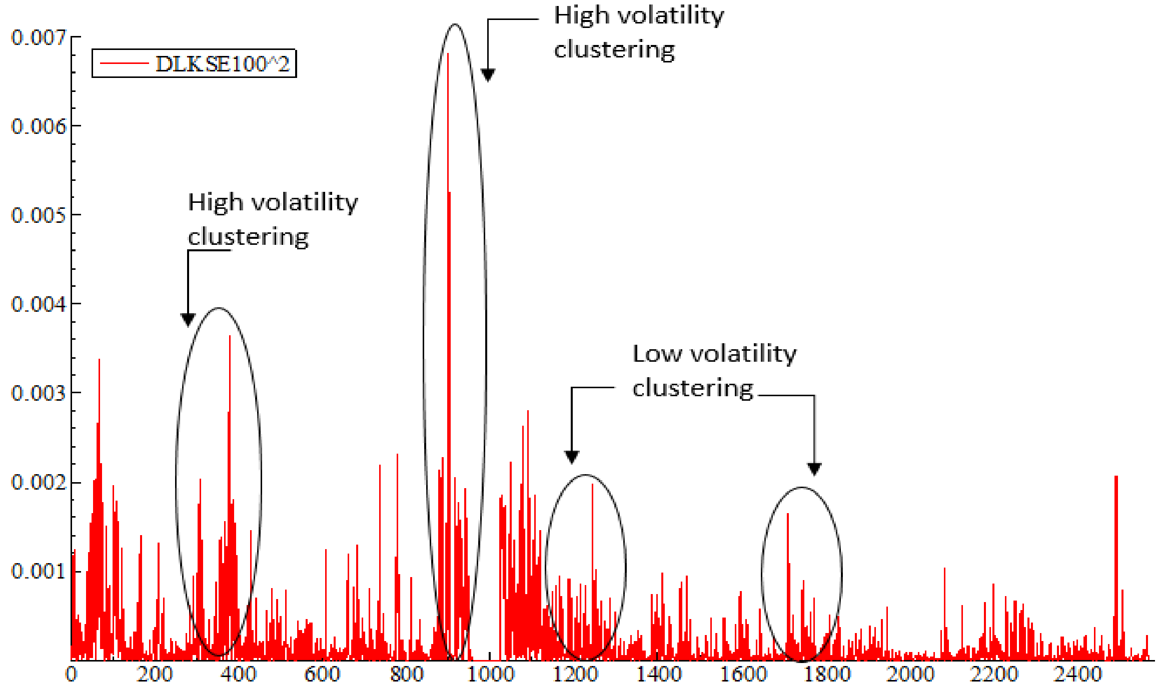


Figure 9.1 shows the series of raw data of stock market prices of KSE 100, S&P500, Dow Jones, Nasdaq 100, and DFMGI. All the series are upward trendy with some fluctuations. The downward break exist in all series, it is because of global financial crises.

Figure 9.2 given below represents return series of Karachi stock market indices. In financial econometrics, spread characterized as volatility. In return series spread does not remain constant, it is known as Hetroscedasticity. The circles in figure 9.2, are indicating the low and high volatility which denote the spread of problem of autocorrelation. If we combine all effects, then it indicates ARCH (Auto-Regressive Conditional Hetroscedasticity) effect. We can easily distinguish between low volatility clustering and high volatility clustering period. The greater depreciation from constant level (mean of return series) indicates high volatility clustering and less depreciation illustrates low volatility clustering. In the same way, we can plot and analyze return series of other stock markets.

Figure 9.2: Graph of Squared Return Series



The figure 9.3, illustrates the distribution of the return series. The distribution of return series is non-normal. In this graph green line shows the normal reference distribution of return series. The red line indicates the actual distribution of the return series. Histograms describes the outliers (extreme values) in return series. The distribution of return series has heavy tails and is leptokurtic. This all is due to different response of market players by having same information from the same market.

Figure 9.3: Graphs Distribution of the Return Series

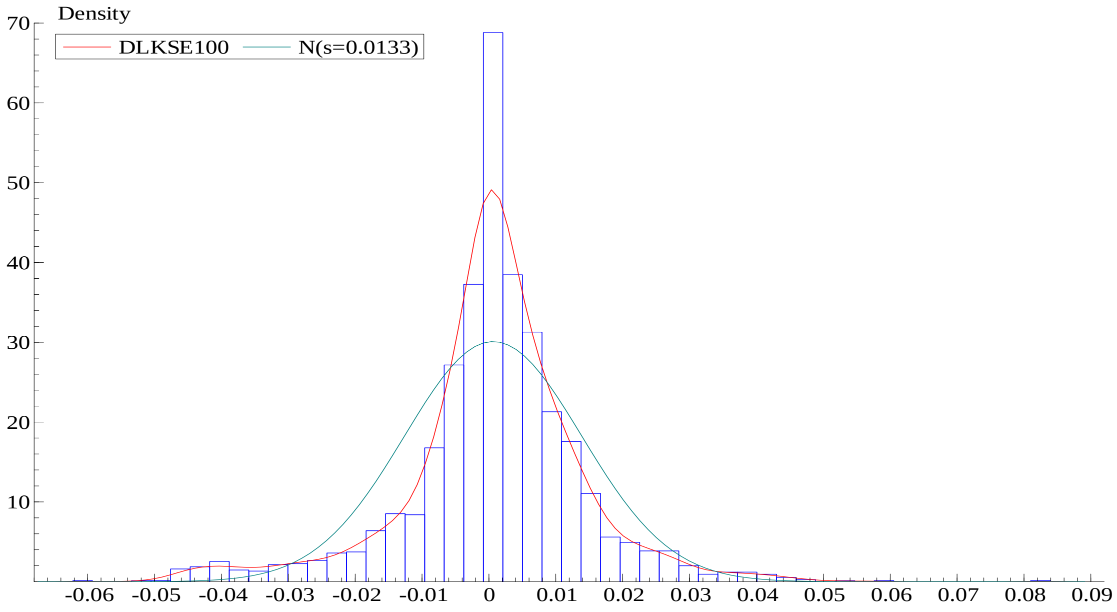


Figure 9.4, presents ACF (Auto-correlation function) and PACF (Partial Auto-correlation function) of return series. The green straight lines in this graph show 95 percent confidence interval, if there is any bar of ACF and PACF outside these lines it means that lag the values are significantly vary from zero. The ARMA (p, q) process species through the significant lags of ACF and PACF. The ACF species the MA (q) process, PACF species the AR (p) process. In this graph 1st, 2nd, 3rd, 4th, 10th, 17th and 18th lags of ACF are significant and 1st, 3rd, 4th, 10th, 11th, 12th, 17th and 18th lags of PACF are significant. These lags format ARMA (p, q) process in conditional mean equation. It means auto correlation and partial autocorrelation exist in the return series. We can also analyze cyclical behavior in return series through ACF and PACF graphs.

Figure 9.4: Graphs of ACF and PACF of Return Series

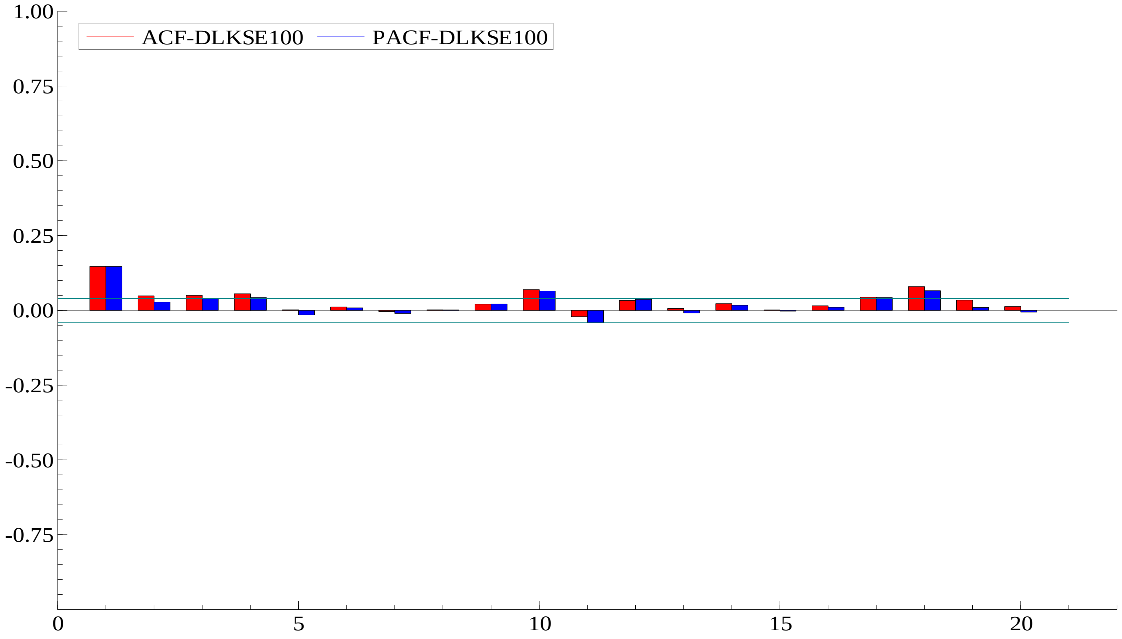
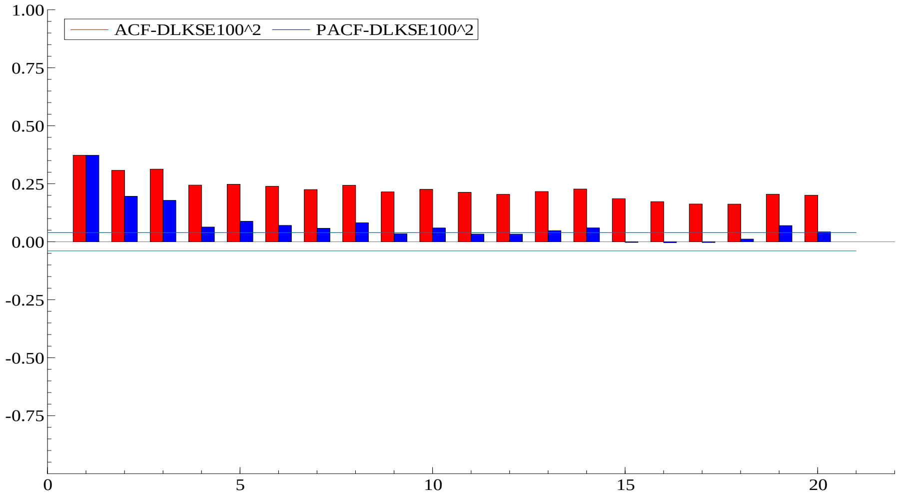


Figure 9.5, shows the graph of ACF and PACF of square return series. 1st to 20th lags of ACF significantly differ from zero and 1st……...8th, 10th, 13th, 14th, 19th and 20th lags of PACF are statistically significant. In the same manner square return series ACF and PACF may provide an indication about the critical lags in conditional variance equation structure of GARCH (p, q) model. It Means there is autocorrelation and partial autocorrelation in the square return series.

Figure 9.5: Graphs of ACF and PACF of Square of Return Series



9.5 Descriptive Statistics

The initial statistics of return series of stock markets indices are given in the table 9.1. They unveil some indications about the behavior of stock markets. The distributions of return are non-normal, heavy tails and leptokurtic. The mean of all return series are about zero which implies that return series show mean reversion behavior. Standard deviation of return series describe the dispersion from mean value which show that return series have greater standard deviation. It means more deviation from mean value. The skewness deals with the asymmetry of the distribution. The distributions of KSE 100, S&P 500, NASDAQ 100, DJI, and DFMGI return series are negatively skewed which means that the return of these stock markets are less than average return. The Jarque-Bera test with null hypothesis of normal distribution is employed. Jarque-Bera statistics of all return series are significant which shows the distribution of all return series are non-normal.

Table 9.1: Summary of Statistics of Stock returns

Series	Mean	Standard deviation	Skewness	Jarque Bera	Excess Kurtosis	Q-stat (5)	Q^2-stat (5)	ARCH 1-2	KPSS
KSE 100	0.0006	0.0132	-0.3854 (0.000)	1098.1 (0.000)	3.1075 (0.000)	76.120 (0.000)	1167.51 (0.000)	266.88 (0.000)	0.2073
S&P 500	0.0002	0.0127	-0.3409 (0.000)	14088 (0.000)	11.448 (0.000)	45.484 (0.000)	1131.31 (0.000)	266.72 (0.000)	0.1965
NASDAQ 100	0.0003	0.0136	-0.1587 (0.000)	7985.9 (0.000)	8.6282 (0.000)	24.928 (0.000)	765.777 (0.000)	156.96 (0.000)	0.2005
DJI	0.0001	0.0116	-0.0851 (0.077)	14168 (0.000)	11.499 (0.000)	45.037 (0.000)	1123.85 (0.000)	283.89 (0.000)	0.1548
DFMGI	0.0001	0.0183	-0.8778 (0.000)	13612 (0.000)	11.135 (0.000)	32.381 (0.000)	166.23 (0.000)	44.647 (0.000)	0.4874

Null Hypotheses (All Null Hypotheses are for n^{th} order)

KPSS H_0: Return series is level stationary, Asymptotic significant values 1% (0.739), 5% (0.463), 10% (0.347). Q-stat (return series) there is no serial autocorrelation. Q^2-stat (square return series) H0: there is no serial autocorrelation. Jarque-Bera H0: distribution of series is normal. LM-ARCH H_0: there is no ARCH effect. Use these Asymptotic Significance values of t-stat 1% (0.01), 5% (0.05), 10% (0.1) and compare these critical values with P-values (Probability values). P-values are in the parenthesis.

The Excess kurtosis of all returns series are significant which means that return series distributions are leptokurtic and it also indicates that probability of large values is more than normal return series. Q-stat of return series are significant, rejecting the null hypothesis of no autocorrelation return series. This shows that there is serial autocorrelation in return series. Q-stat of squared return series is significant, rejecting the null hypothesis of no autocorrelation in squared return series. This shows that there is serial autocorrelation in square return series. LM-ARCH test validates that there is ARCH effect in return series. KPSS is a unit root test with null hypothesis of stationary series. KPSS test results of all variables show that the estimated values lies in acceptance region [less than given three significance values 1% (0.739), 5% (0.463), 10% (0.347)] which shows the null hypothesis is accepted and return series are level stationary.

9.6 Tracing Spillover Effect

In section 9.6, we explored the direct and indirect linkages between Pakistani (KSE 100) and leading foreign stock markets (S&P 500, NASDAQ 100, DOWJONES, and DFMGI). We investigated the co-movements among these markets by using the technique of Hamao et al. (1990). We traced out the information transmission between individual markets to check the 'Meteor Shower' hypothesis of Engle. To explore mean spillover effect between two markets, the return series of one market is introduced as regressor in conditional mean equation of other market. If it is significant, it means there is mean spillover effect. For volatility spillover effect the square return series of one market is introduced as regressor in the conditional variance equation of other market. If it is significant, it means there is volatility spillover effect. If spillover is one sided, it is called unidirectional spillover. If it is two sided, it is called bidirectional spillover. In ARDL models, we introduced the series of one stock market into the equation of other stock market. If it is significant, it means there is a spillover effect from one series to other series. In ARDL models, we are using series without making them stationary or series at level because the experiments have proven in chapter 5, 6, and 7 that for ARDL modeling there is no need to make series stationary but in GARCH type modeling we used return series instead of series at level.

9.6.1 Tracing direct and indirect linkages between Pakistan, US and Dubai stock markets

Now we explore the spillover effect between Pakistani (KSE 100), USA (S&P 500, NASDAQ 100 and DJI) and Dubai financial market (DFMGI) by using daily data from 2005 to 2014.

Table 9.2.a Volatility Spillover Effect by using GARCH models (Bidirectional Analyses for Daily Data)

Parameters / Spillover direction	Mean spillover effect R_t (π_1)	Volatility Spillover effect R_t^2 (π_2)
KSE 100 to S&P 500	0.0169	0.0015
ARMA(1,1) GARCH (1,2)	(0.1293)	(0.4331)
S&P 500 to KSE 100	0.0067	-0.0010
ARMA(1,2) GJR (1,1)	(0.0000)	(0.0000)
KSE 100 to NASDAQ 100	0.0119	0.0045
ARMA(1,0) GARCH (1,1)	(0.4178)	(0.0973)
NASDAQ 100 to KSE 100	0.0116	-0.0009
ARMA(1,1) GJR (1,1)	(0.0000)	(0.0000)
KSE 100 to DJI	0.0096	0.0022
ARMA(0,0) GARCH (1,1)	(0.3784)	(0.1750)
DJI to KSE 100	-0.0019	-0.0024
ARMA(1,1) GJR (1,1)	(0.0000)	(0.0000)
KSE 100 to DFMGI	0.0650	0.0185
ARMA(1,1) GARCH (1,1)	(0.0443)	(0.0788)
DFMGI to KSE 100	0.0153	-0.0011
ARMA(1,0) GARCH (1,1)	(0.0000)	(0.0000)
S&P 500 to DFMGI	0.0820	0.0036
ARMA(1,1) GARCH (1,1)	(0.0028)	(0.5462)

Spillover direction \ Parameters	Mean spillover effect R_t (π_1)	Volatility Spillover effect R_t^2 (π_2)
DFMGI to S&P 500	0.0161	-0.0002
ARMA(1,1) GARCH (2,1)	(0.0328)	(0.1585)
NASDAQ 100 to DFMGI	0.0671	0.0022
ARMA(1,1) GARCH (1,1)	(0.0056)	(0.7160)
DFMGI to NASDAQ 100	0.0217	-0.0003
ARMA(0,0) GARCH (1,1)	(0.0257)	(0.6331)
DJI to DFMGI	0.0827	0.0046
ARMA(1,1) GARCH (1,1)	(0.0056)	(0.5194)
DFMGI to DJI	0.0159	-0.0001
ARMA(1,0) GARCH (2,1)	(0.0240)	(0.3720)

Null Hypotheses(All Null Hypotheses are for n^{th} order)

Mean spillover H0:π_1= 0 No mean spillover, volatility spillover H0: π_2= 0 No volatility spillover. P-values are in the parenthesis.

In table 9.2a, the parameter of return series π_1 and parameter of squared return series π_2 of S&P 500 are statistically significant in conditional mean and variance equations of KSE 100 but there is no reverse effect from KSE 100 to S&P 500. It shows there is unidirectional mean and volatility spillover effect from S&P 500 to KSE 100. Similarly, the parameter of return series π_1 and parameter of squared return series π_2 of NASADQ 100, DJI, and DFMGI in conditional mean and variance equations of KSE 100 are statistically significant. It means there is also mean and volatility spillover effect from NASADQ 100, DJI, and DFMGI to KSE 100. This clearly indicates that the disturbance in returns and volatility of return in NASADQ 100, DJI, and DFMGI affect the return and volatility of KSE 100 but there is no reverse effect from KSE 100 to these markets. The parameter of return series $\pi_1$of S&P 500, NASADQ 100, DJI are significant in conditional mean equations of DFMGI, which means there is mean spillover effect from S&P 500, NASADQ 100, and DJI to DFMGI. While the parameter of squared return series π_2 of S&P 500, NASADQ 100, and DJI in conditional variance equations of

DFMGI are statistically insignificant. It means there is volatility spillover effect from these markets to DFMGI. The parameter of return series $\pi_1$of DFMGI is significant in conditional mean equations of S&P 500, NASADQ 100, and DJI, which means there is mean spillover effect from DFMGI to S&P 500, NASADQ 100, and DJI. While the parameter of squared return series π_2 of DFMGI in conditional variance equations of S&P 500, NASADQ 100, DJI are statistically insignificant. It means there is volatility spillover effect from DFMGI to S&P 500, NASADQ 100, and DJI.

There are bidirectional mean and volatility spillover effects between KSE 100 and DFMGI. Because the parameter of return series π_1 and parameter of squared return series π_2 of DFMGI are statistically significant in conditional mean and variance equations of KSE 100 but there is also reverse mean and volatility spillover effect from KSE 100 to DFMGI. It shows there is bidirectional mean and volatility spillover effect between DFMGI and KSE 100.

This evidently shows that DFMGI, S&P 500, NASADQ 100, and DJI have direct mean and volatility effect on KSE 100 but also there is indirect effect from S&P 500, NASADQ 100, and DJI to KSE 100 through DFMGI. For the validations of results the residual analysis are employed the results are given in table (9.2.a)

Table 9.2.b Residual Analysis (of estimation in table 9.2.a, Daily data)

Table 9.2.b given in appendix shows that the validity of results is also approved by the residual analysis.

Table 9.2.b: Residual Analysis

Parameter Series	Jarque Bera	Q-Stat (5)	Q-Stat (10)	Q^2-Stat (5)	Q^2-Stat (10)	LM - ARCH (1-2)	LM-ARCH (1-5)
KSE 100 to	519.91	4.5809	7.5260	1.1042	9.8357	0.2195	0.2308
S&P 500	(0.0000)	(0.2051)	(0.4810)	(0.5757)	(0.1980)	(0.8029)	(0.9492)
S&P 500 to	4157.1	1.0655	9.6531	0.8109	2.9143	0.1368	0.1614
KSE 100	(0.0000)	(0.5869)	(0.2090)	(0.8468)	(0.9396)	(0.8721)	(0.9765)
KSE 100 to	211.08	2.7090	5.7941	5.9844	15.662	2.7596	1.1853
NASDAQ 100	(0.0000)	(0.6074)	(0.7603)	(0.1123)	(0.0474)	(0.0635)	(0.3138)
NASDAQ 100	5847.7	3.5112	16.882	0.8004	2.6895	0.1104	0.1585
to KSE 100	(0.0000)	(0.3193)	(0.0313)*	(0.8493)	(0.9523)	(0.8954)	(0.9775)
KSE 100 to	441.53	11.755	14.776	8.3740	18.127	3.8385	1.6138
DJI	(0.0000)	(0.038)*	(0.1404)	(0.0388)*	(0.0202)	(0.0216)	(0.1529)
DJI to	1198.1	4.8240	16.462	0.86921	2.1575	0.0613	0.1691
KSE 100	(0.0000)	(0.1851)	(0.036)*	(0.8328)	(0.9757)	(0.9405)	(0.9740)
KSE 100 to	13571	4.7337	10.608	3.0709	7.4353	0.1503	0.6138
DFMGI	(0.0000)	(0.1923)	(0.2248)	(0.3808)	(0.4904)	(0.8604)	(0.6893)
DFMGI to	2453.1	10.055	26.834	1.1986	4.4112	0.1127	0.2381
KSE 100	(0.0000)	(0.0395)*	(0.0148)*	(0.7533)	(0.8182)	(0.8934)	(0.9457)
S&P 500 to	13131	7.6998	13.595	3.1527	7.9966	0.1441	0.7641
DFMGI	(0.0000)	(0.0526)	(0.0929	(0.3686)	(0.4337)	(0.8658)	(0.6638)
DFMGI to	491.25	4.6889	6.6039	5.8411	12.523	2.5586	1.1151
S&P 500	(0.0000)	(0.1960)	(0.5799)	(0.0539)	(0.0845)	(0.0776)	(0.3500)
NASDAQ 100	13236	7.6817	13.448	3.1950	7.9185	0.1746	0.6385
to DFMGI	(0.0000)	(0.0530)	(0.0973)	(0.3625)	(0.4414)	(0.8398)	(0.6704)
DFMGI to	191.57	6.8647	9.7689	5.6420	14.328	2.6055	1.1185
NASDAQ 100	(0.0000)	(0.2308)	(0.4609)	(0.1303)	(0.0736)	(0.0741)	(0.3482)
DJI to	13059	7.5169	13.317	3.2428	8.1727	0.1524	0.6473
DFMGI	(0.0000)	(0.0571)	(0.1013)	(0.3556)	(0.4167)	(0.8586)	(0.6636)
DFMGI to	377.36	3.8701	6.8827	3.3924	9.9342	1.1531	0.6552
DJI	(0.0000)	(0.4238)	(0.6493)	(0.1833)	(0.1923)	(0.3158)	(0.6575)

Null Hypotheses(All Null Hypotheses are for n^{th} order)

Q-stat (return series) there is no serial autocorrelation. Q^2-stat (square return series) H0: there is no serial autocorrelation. Jarque-Bera H0: distribution of series is normal. LM-ARCH H_0: there is no ARCH effect. P-values are in the parenthesis.

The table 9.2.b illustrate the post estimation results (Residual analysis). The Jarque-Bera test (Normality test) with null hypothesis the distribution of returns are normal which shows non normal residuals. The Q-stat are insignificant up to 10th lags which means that we should accept null hypothesis that there is no serial autocorrelation in the standardized residuals. The Q-stat on squared standardized residuals is insignificant up to 10th lags with null hypothesis that there is no serial autocorrelation in squared standardized residuals. LM-ARCH test is also insignificant up to 5th lags, accept null hypothesis, no ARCH effect remains in series residuals. These results show that there is no econometric problem left in residuals. It means the results of table 8.2b are valid. To find out the spillover effect from ARDL model, we put the return series of one market into the equation of other market. If the series coefficient is significant, it means there is spillover effect from one market to other. We employed F-test to test the joint significance of the lag values of independent variable. The results of ARDL model are following:

Table 9.3.a Volatility Spillover effect by using ARDL models (Bidirectional Analyses for daily data)

Spillover direction	F-stat	Spillover direction	F-stat
S&P 500 to KSE 100	7.7533 (0.0000)**	**KSE 100 to S&P 500**	2.1043 (0.0498)*
NASDAQ 100 to KSE 100	8.2386 (0.0000)*	**KSE 100 to NASDAQ 100**	1.0408 (0.3966)
DJI to KSE 100	8.5006 (0.0000)*	**KSE 100 to DJI**	2.7247 (0.0122)*
DFMGI to KSE 100	1.2125 (0.2966)	**KSE 100 to DFMGI**	1.7483 (0.1060)
S&P 500 to DFMGI	9.3317 (0.0000)*	**DFMGI to S&P 500**	1.8346 (0.0886)
NASDAQ 100 to DFMGI	8.0958 (0.0000)*	**DFMGI to NASDAQ 100**	0.93565 (0.4680)
DJI to DFMGI	7.0650 (0.0000)*	**DFMGI to DJI**	1.7298 (0.1101)

Table 9.3.a shows that the S&P 500, NASADQ 100, and DJI series coefficients are significant in the equation of KSE 100. It means there is spillover effect from S&P 500,

NASADQ 100, and DJI to KSE 100. But there is no spillover effect found from KSE 100 to S&P 500, NASADQ 100, and DJI because there coefficients are insignificant in the equation of KSE 100. It shows that there is unidirectional spillover effect from S&P 500, NASADQ 100, and DJI to KSE 100. The DFMGI series coefficients are significant in the equation of KSE 100 and KSE 100 series coefficients are significant in the equation of DFMGI. It means there is bidirectional spillover effect between DFMGI and KSE 100.

The results in table 9.3.a also show that the S&P 500, NASADQ 100, and DJI series coefficients are significant in the equation of DFMGI and DFMGI series coefficients are also significant in the equation of S&P 500, NASADQ 100, and DJI. It means there is bidirectional spillover effect between S&P 500, NASADQ 100, DJI and DFMGI. These results support the results of GARCH models because the directions of spillover remain same. This shows that DFMGI, S&P 500, NASADQ 100, and DJI have direct spillover effect on KSE 100 but also there is indirect effect from S&P 500, NASADQ 100, and DJI to KSE 100 through DFMGI. For the validation of ARDL results, we employed the residual analysis. The results are given following:

Table 9.3.b Residual Analysis of ARDL model with Daily data

Spillover direction	AR 1-7 test	ARCH 1-7 test	Hetero test
S&P 500 to KSE 100	3.6514 (0.0261)**	289.91 (0.0000)*	32.262 (0.0000)*
KSE 100 to S&P 500	0.92310 (0.3974)	58.190 (0.0000)*	12.279 (0.0000)*
NASDAQ 100 to KSE 100	0.11061 (0.8953)	54.757 (0.0000)*	11.154 (0.0000)*
KSE 100 to NASDAQ 100	0.90025 (0.4066)	77.090 (0.0000)*	5.8409 (0.0000)*
DJI to KSE 100	0.075671 (0.9271)	55.372 (0.0000)*	10.774 (0.0000)*
KSE 100 to DJI	1.7297 (0.1775)	69.070 (0.0000)*	12.863 (0.0000)*
DFMGI to KSE 100	0.29295 (0.7461)	62.399 (0.0000)*	12.641 (0.0000)*
KSE 100 to DFMGI	11.549 (0.0000)*	208.14 (0.0000)*	16.605 (0.0000)*
S&P 500 to DFMGI	13.320 (0.0000)*	194.00 (0.0000)*	16.519 (0.0000)*
DFMGI to S&P 500	0.32575 (0.7220)	55.367 (0.0000)*	14.025 (0.0000)*
NASDAQ 100 to DFMGI	0.24342 (0.7840)	76.028 (0.0000)*	7.7915 (0.0000)*
DFMGI to NASDAQ 100	1.6620 (0.3771)	51.3824 (0.0009)*	1.1127 (0.0024)*
DJI to DFMGI	14.024 (0.0000)*	199.10 (0.0000)*	16.741 (0.0000)*
DFMGI to DJI	0.73002 (0.4820)	65.852 (0.0000)*	14.952 (0.0000)*

LM-ARCH H0: there is no ARCH effect. P-values are in the parenthesis. AR H0: there is no autocorrelation in residuals. P-values are in the parenthesis. Hetero test H0: there is no Heteroscedasticity in residuals. P-values are in the parenthesis.

Table 9.3.b shows the residual analysis after employing ARDL model. The results indicate the ARCH effect and heteroscedasticity still prevail in residuals. It means that the results of ARDL model are not reliable. We are using series without making them stationary and data is in high frequency (daily data). It shows that ARDL model is unable to capture ARCH effect when we are working with daily data. So, we reduce the frequency of data and make it weekly by selecting closing price of last day of week. Again we measure the spillover effects through GARCH models and ARDL model.

Table 9.4.a Volatility Spillover Effect by using GARCH models (Bidirectional Analyses for Weekly Data Analysis)

Parameters / Spillover direction	Mean spillover effect R_t (π_1)	Volatility Spillover effect R_t^2 (π_2)
KSE 100 to S&P 500	0.0153	0.0019
ARMA(1,1) GARCH (1,2)	(0.1189)	(0.7802)
S&P 500 to KSE 100	0.0069	-0.0021
ARMA(1,2) GJR (1,1)	(0.0000)	(0.0000)
KSE 100 to NASDAQ 100	0.0129	0.0056
ARMA(1,0) GARCH (1,1)	(0.3131)	(0.1973)
NASDAQ 100 to KSE 100	0.0121	-0.0019
ARMA(1,1) GJR (1,1)	(0.0000)	(0.0000)
KSE 100 to DJI	0.0196	0.0026
ARMA(0,0) GARCH (1,1)	(0.2765)	(0.3754)
DJI to KSE 100	-0.0020	-0.0030
ARMA(1,1) GJR (1,1)	(0.0000)	(0.0000)
KSE 100 to DFMGI	0.0710	0.0201
ARMA(1,1) GARCH (1,1)	(0.0341)	(0.1879)
DFMGI to KSE 100	0.0243	-0.0015
ARMA(1,0) GARCH (1,1)	(0.0000)	(0.0000)
S&P 500 to DFMGI	0.0945	0.0041
ARMA(1,1) GARCH (1,1)	(0.0019)	(0.6891)
DFMGI to S&P 500	0.0253	-0.0005
ARMA(1,1) GARCH (2,1)	(0.0226)	(0.2891)
NASDAQ 100 to DFMGI	0.0743	0.0031
ARMA(1,1) GARCH (1,1)	(0.0024)	(0.8178)
DFMGI to NASDAQ 100	0.0221	-0.0005
ARMA(0,0) GARCH (1,1)	(0.0169)	(0.2901)
DJI to DFMGI	0.0791	0.0051
ARMA(1,1) GARCH (1,1)	(0.0048)	(0.5189)
DFMGI to DJI	0.0248	-0.0002
ARMA(1,0) GARCH (2,1)	(0.0161)	(0.1563)

Null Hypotheses(All Null Hypotheses are for nth order)

Mean spillover H0:π_1= 0 No mean spillover, volatility spillover H0: π_2= 0 No volatility spillover. P-values are in the parenthesis.

In table 9.4.a, the parameter of return series π_1 and parameter of squared return series π_2 of S&P 500 are statistically significant in conditional mean and variance equations of KSE 100 but there is no reverse effect from KSE 100 to S&P 500. It shows there is unidirectional mean and volatility spillover effect from S&P 500 to KSE 100. Similarly, the parameter of return series π_1 and parameter of squared return series π_2 of NASDAQ

100, DJI, and DFMGI in conditional mean and variance equations of KSE 100 are statistically significant. It means there is also mean and volatility spillover effect from NASADQ 100, DJI, and DFMGI to KSE 100. This clearly indicates that the disturbance in returns and volatility of return in NASADQ 100, DJI, and DFMGI affect the return and volatility of KSE 100 but there is no reverse effect from KSE 100 to these markets. The parameter of return series $\pi_1$of S&P 500, NASADQ 100, DJI are significant in conditional mean equations of DFMGI, which means there is mean spillover effect from S&P 500, NASADQ 100, and DJI to DFMGI. While the parameter of squared return series π_2 of S&P 500, NASADQ 100, and DJI in conditional variance equations of DFMGI are statistically insignificant. It means there is volatility spillover effect from these markets to DFMGI. The parameter of return series $\pi_1$of DFMGI is significant in conditional mean equations of S&P 500, NASADQ 100, DJI, which means there is mean spillover effect from DFMGI to S&P 500, NASADQ 100, and DJI While the parameter of squared return series π_2 of DFMGI in conditional variance equations of S&P 500, NASADQ 100, DJI are statistically insignificant. It means there is volatility spillover effect from DFMGI to S&P 500, NASADQ 100, and DJI.

There are bidirectional mean and volatility spillover effects between KSE 100 and DFMGI. Because the parameter of return series π_1 and parameter of squared return series π_2 of DFMGI are statistically significant in conditional mean and variance equations of KSE 100 but there is also reverse mean and volatility spillover effect from KSE 100 to DFMGI. It shows there is bidirectional mean and volatility spillover effect between DFMGI and KSE 100.

This evident shows that DFMGI, S&P 500, NASADQ 100, and DJI have direct mean and volatility effect on KSE 100 but also there is indirect effect from S&P 500, NASADQ 100, and DJI to KSE 100 through DFMGI. For the validations of results, the residual analysis are employed the results are given in table 9.4.b:

Table 9.4.b Residual Analysis of GARCH model with Weekly Data

Parameter / Series	Jarque Bera	Q-Stat (5)	Q-Stat (10)	Q^2-Stat (5)	Q^2-Stat (10)	LM - ARCH (1-2)	LM-ARCH (1-5)
KSE 100 to S&P 500	518.901 (0.0000)	5.3339 (0.2053)	6.5432 (0.3850)	4.6722 (0.1767)	6.7540 (0.1178)	0.2671 (0.6073)	0.1467 (0.1786)
S&P 500 to KSE 100	3363.1 (0.0000)	2.1315 (0.3878)	7.5021 (0.1094)	0.7779 (0.4478)	3.7813 (0.7706)	0.3287 (0.6893)	0.2875 (0.1767)
KSE 100 to NASDAQ 100	212.08 (0.0000)	2.5430 (0.5062)	5.6661 (0.5472)	4.9994 (0.4733)	13.606 (0.1334)	3.8721 (0.1765)	1.3765 (0.1152)
NASDAQ 100 to KSE 100	5478.6 (0.0000)	3.4302 (0.1165)	13.662 (0.1324)	0.2224 (0.4735)	1.7855 (0.6975)	0.65704 (0.2654)	0.1109 (0.6786)
KSE 100 to DJI	452.58 (0.0000)	12.645 (0.1488)	11.708 (0.2356)	6.3740 (0.1435)	15.786 (0.2442)	4.7765 (0.2016)	1.7338 (0.1121)
DJI to KSE 100	1178.2 (0.0000)	4.7840 (0.2665)	16.542 (0.3459)	0.8987 (0.6304)	3.2674 (0.7756)	0.1672 (0.6893)	0.2901 (0.2007)
KSE 100 to DFMGI	12067 (0.0000)	6.7557 (0.1542)	11.638 (0.1162)	3.0729 (0.8903)	8.4674 (0.5367)	0.4789 (0.4812)	0.6085 (0.7483)
DFMGI to KSE 100	2589.3 (0.0000)	10.035 (0.1465)	24.740 (0.1538)	2.2086 (0.7756)	5.3786 (0.4656)	0.3176 (0.6734)	0.6758 (0.7145)
S&P 500 to DFMGI	13451 (0.0000)	9.6668 (0.2676)	13.6705 (0.0889)	4.2527 (0.3636)	8.5786 (0.2787)	0.5254 (0.1356)	0.7987 (0.5667)
DFMGI to S&P 500	4712.2 (0.0000)	4.6559 (0.1561)	7.5549 (0.4760)	5.4311 (0.0983)	11.654 (0.1267)	2.6070 (0.1642)	1.5623 (0.1090)
NASDAQ 100 to DFMGI	12415 (0.0000)	4.6247 (0.1764)	13.408 (0.1353)	3.2050 (0.6167)	6.7878 (0.4631)	0.2463 (0.1076)	0.5780 (0.4464)
DFMGI to NASDAQ 100	146.54 (0.0000)	6.9207 (0.1472)	8.7869 (0.5766)	5.6020 (0.6501)	11.312 (0.1784)	1.7765 (0.1765)	1.0769 (0.4572)
DJI to DFMGI	14675 (0.0000)	7.5329 (0.2642)	13.3107 (0.6014)	3.2128 (0.5390)	9.1651 (0.2167)	0.2761 (0.7776)	0.6832 (0.3871)
DFMGI to DJI	357.06 (0.0000)	5.6601 (0.4535)	5.87927 (0.6945)	2.2342 (0.2800)	4.9670 (0.1467)	1.2782 (0.4876)	0.8971 (0.7871)

Null Hypotheses(All Null Hypotheses are for n^{th} order)

Q-stat (return series) there is no serial autocorrelation. Q^2-stat (square return series) H0: there is no serial autocorrelation. Jarque-Bera H0: distribution of series is normal. LM-ARCH H_0: there is no ARCH effect. P-values are in the parenthesis.

The table 9.4.b illustrate the post estimation results (Residual analysis). The Jarque-Bera test (Normality test) with null hypothesis the distribution of returns are normal which show non normal residuals. The Q-stat are insignificant up to 10th lags which means that we should accept null hypothesis that there is no serial autocorrelation in the standardized residuals. The Q-stat on squared standardized residuals is insignificant up to 10th lags with null hypothesis that there is no serial autocorrelation in squared standardized residuals. LM-ARCH test is also insignificant up to 5thlags, accept null hypothesis, no ARCH effect remains in series residuals. These results show that there is no econometric problem left in residuals. It means the results of table 9.4.b are valid. To find out the spillover effect from ARDL model, we put the return series of one market into the equation of other market. If the series coefficient is significant it means there is spillover effect from one market to other. We employed F-test to test the joint significance of the lag values of independent variable. The results of ARDL model are following:

Table 9.5.a Volatility Spillover effect by using ARDL models (Bidirectional Analyses for ARDL weekly data)

Spillover direction	F-stat	Spillover direction	F-stat
S&P 500 to KSE 100	6.7401 (0.0000)**	**KSE 100 to S&P 500**	5.1043 (0.0098)*
NASDAQ 100 to KSE 100	9.3381 (0.0000)*	**KSE 100 to NASDAQ 100**	2.1409 (0.3416)
DJI to KSE 100	8.5006 (0.0000)*	**KSE 100 to DJI**	3.1235 (0.0101)*
DFMGI to KSE 100	1.2308 (0.2966)	**KSE 100 to DFMGI**	1.9480 (0.2140)
S&P 500 to DFMGI	7.4518 (0.0000)*	**DFMGI to S&P 500**	1.8346 (0.0783)
NASDAQ 100 to DFMGI	8.1362 (0.0000)*	**DFMGI to NASDAQ 100**	1.23563 (0.5681)
DJI to DFMGI	8.0262 (0.0000)*	**DFMGI to DJI**	1.7298 (0.1704)

Table 9.5.a shows that the S&P 500, NASADQ 100, and DJI series coefficients are significant in the equation of KSE 100. It means there is spillover effect from S&P 500, NASADQ 100, and DJI to KSE 100. But there is no spillover effect found from KSE 100 to S&P 500, NASADQ 100, and DJI because there coefficients are insignificant in the equation of KSE 100. It shows that there is unidirectional spillover effect from S&P 500, NASADQ 100, and DJI to KSE 100. The DFMGI series coefficients are significant in the equation of KSE 100 and KSE 100 series coefficients are significant in the equation of DFMGI. It means there is bidirectional spillover effect between DFMGI and KSE 100.

The results in table 9.5.a also show that the S&P 500, NASADQ 100, and DJI series coefficients are significant in the equation of DFMGI and DFMGI series coefficients are also significant in the equation of S&P 500, NASADQ 100, and DJI. It means there is bidirectional spillover effect between S&P 500, NASADQ 100, and DJI and DFMGI. These results support the results of GARCH models because the directions of spillover remain same. This shows that DFMGI, S&P 500, NASADQ 100, and DJI have direct spillover effect on KSE 100 but also there is indirect effect from S&P 500, NASADQ 100, and DJI to KSE 100 through DFMGI. For the validation of ARDL results we employed the residual analysis. The results are given following:

Table 9.5.b Residual Analysis of ARDL model with Weekly Data

Spillover direction	AR 1-7 test	ARCH 1-7 test	Hetero test
S&P 500 to KSE 100	5.6758 (0.0261)**	469.61 (0.0000)*	31.412 (0.0000)*
KSE 100 to S&P 500	1.09315 (0.3974)	48.160 (0.0000)*	14.114 (0.0000)*
NASDAQ 100 to KSE 100	0.16067 (0.8953)	43.552 (0.0000)*	9.142 (0.0000)*
KSE 100 to NASDAQ 100	1.77024 (0.4066)	76.050 (0.0000)*	7.3601 (0.0000)*
DJI to KSE 100	0.178667 (0.9271)	65.573 (0.0000)*	12.750 (0.0000)*
KSE 100 to DJI	3.5691 (0.1775)	68.087 (0.0000)*	16.277 (0.0000)*
DFMGI to KSE 100	1.5694 (0.7461)	59.675 (0.0000)*	14.780 (0.0000)*
KSE 100 to DFMGI	12.156 (0.0000)*	187.169 (0.0000)*	19.616 (0.0000)*
S&P 500 to DFMGI	16.893 (0.0000)*	173.06 (0.0000)*	17.518 (0.0000)*
DFMGI to S&P 500	1.3772 (0.7220)	58.363 (0.0000)*	11.120 (0.0000)*
NASDAQ 100 to DFMGI	0.94649 (0.7840)	64.029 (0.0000)*	8.7516 (0.0000)*
DFMGI to NASDAQ 100	3.7290 (0.3771)	63.6893 (0.0009)*	2.3178 (0.0024)*
DJI to DFMGI	16.426 (0.0000)*	183.64 (0.0000)*	14.778 (0.0000)*
DFMGI to DJI	1.53078 (0.4820)	68.876 (0.0000)*	15.990 (0.0000)*

LM-ARCH H0: there is no ARCH effect. P-values are in the parenthesis.AR H0: there is no autocorrelation in residuals. P-values are in the parenthesis. Hetero test H0: there is no Heteroscedasticity in residuals. P-values are in the parenthesis.

Table 9.5.b shows the residual analysis after employing ARDL model. The results indicate the ARCH effect and heteroscedasticity still prevail in residuals. It means that the results of ARDL model are not reliable. We are using series without making them stationary and data is in high frequency (weekly data). It shows that ARDL model is unable to capture ARCH effect when we are working with weekly data. So, we reduce the frequency of data and make it monthly by selecting closing price of last day of week. Again we measure the spillover effects through GARCH models and ARDL model.

Table 9.6.a Volatility Spillover effect by using GARCH models (Bidirectional Analyses for monthly data)

Parameters / Spillover direction	Mean spillover effect R_t (π_1)	Volatility Spillover effect R_t^2 (π_2)
KSE 100 to S&P 500	0.1152	0.0325
ARMA(1,1) GARCH (1,2)	(0.2107)	(0.2111)
S&P 500 to KSE 100	0.0103	-0.0146
ARMA(1,2) GJR (1,1)	(0.0001)	(0.0021)
KSE 100 to NASDAQ 100	0.0148	0.0032
ARMA(1,1) GARCH (1,1)	(0.5167)	(0.0833)
NASDAQ 100 to KSE 100	0.01778	-0.1251
ARMA(1,1) GJR (1,1)	(0.0000)	(0.0002)
KSE 100 to DJI	0.0096	0.0022
ARMA(1,1) GARCH (1,1)	(0.3784)	(0.1750)
DJI to KSE 100	0.3532	0.5632
ARMA(1,1) GARCH (1,1)	(0.1171)	(0.0632)
KSE 100 to DFMGI	0.0711	0.0201
ARMA(1,1) GJR (1,1)	(0.0334)	(0.0657)
DFMGI to KSE 100	0.0167	-0.0014
ARMA(1,1) GARCH (1,1)	(0.0000)	(0.0000)
S&P 500 to DFMGI	0.0558	0.0041
ARMA(1,1) GARCH (1,2)	(0.0014)	(0.5462)
DFMGI to S&P 500	0.0231	-0.0028
ARMA(1,1) GARCH (2,1)	(0.0433)	(0.621)
NASDAQ 100 to DFMGI	0.0625	0.0164
ARMA(1,1) GARCH (1,1)	(0.0168)	(0.6183)
DFMGI to NASDAQ 100	0.0217	-0.0003
ARMA(1,1) GARCH (1,1)	(0.0364)	(0.8941)
DJI to DFMGI	0.0620	0.0032
ARMA(1,1) GARCH (1,1)	(0.0019)	(0.5194)
DFMGI to DJI	0.0314	-0.0031
ARMA(1,1) GARCH (1,1)	(0.0175)	(0.6421)

Null Hypotheses(All Null Hypotheses are for n^th order)

Mean spillover H0:π_1= 0 No mean spillover, volatility spillover H0: π_2= 0 No volatility spillover. P-values are in the parenthesis.

In table 9.6.a, the parameter of return series π_1 and parameter of squared return series π_2 of S&P 500 are statistically significant in conditional mean and variance equations of KSE 100 but there is no reverse effect from KSE 100 to S&P 500. It shows there is unidirectional mean and volatility spillover effect from S&P 500 to KSE 100. Similarly, the parameter of return series π_1 and parameter of squared return series π_2 of NASADQ 100, DJI, and DFMGI in conditional mean and variance equations of KSE 100 are statistically significant. It means there is also mean and volatility spillover effect from NASADQ 100, DJI, and DFMGI to KSE 100. This clearly indicates that the disturbance in returns and volatility of return in NASADQ 100, DJI, and DFMGI affect the return and volatility of KSE 100 but there is no reverse effect from KSE 100 to these markets.

The parameter of return series $\pi_1$of S&P 500, NASADQ 100, DJI are significant in conditional mean equations of DFMGI, which means there is mean spillover effect from S&P 500, NASADQ 100, and DJI to DFMGI. While the parameter of squared return series π_2 of S&P 500, NASADQ 100, and DJI in conditional variance equations of DFMGI are statistically insignificant. It means there is volatility spillover effect from these markets to DFMGI. The parameter of return series $\pi_1$of DFMGI is significant in conditional mean equations of S&P 500, NASADQ 100, and DJI, which means there is mean spillover effect from DFMGI to S&P 500, NASADQ 100, and DJI. While the parameter of squared return series π_2 of DFMGI in conditional variance equations of S&P 500, NASADQ 100, DJI are statistically insignificant. It means there is volatility spillover effect from DFMGI to S&P 500, NASADQ 100, and DJI.

There are bidirectional mean and volatility spillover effects between KSE 100 and DFMGI. Because the parameter of return series π_1 and parameter of squared return series π_2 of DFMGI are statistically significant in conditional mean and variance equations of KSE 100 but there is also reverse mean and volatility spillover effect from

KSE 100 to DFMGI. It shows there is bidirectional mean and volatility spillover effect between DFMGI and KSE 100. The evident shows that DFMGI, S&P 500, NASADQ 100, and DJI have direct mean and volatility effect on KSE 100 but also there is indirect effect from S&P 500, NASADQ 100, and DJI to KSE 100 through DFMGI. For the validations of results the residual analysis are employed the results are given in table 9.6.b:

Table 9.6.b Residual analysis of GARCH Model for Monthly Data

Parameter / Series	Jarque Bera	Q-Stat (5)	Q-Stat (10)	Q^2-Stat (5)	Q^2-Stat (10)	LM -ARCH (1-2)	LM-ARCH (1-5)
KSE 100 to S&P 500	745.80	6.8391	10.2670	3.35421	9.8357	0.2195	0.2308
	(0.0000)	(0.2051)	(0.4810)	(0.5757)	(0.1980)	(0.8029)	(0.9492)
S&P 500 to KSE 100	4254.3	1.0875	10.658	1.8104	3.9453	1.1662	0.1913
	(0.0000)	(0.4865)	(0.3097)	(0.9465)	(0.8306)	(0.3724)	(0.1745)
KSE 100 to NASDAQ 100	114.06	1.6091	7.7784	4.6781	12.751	3.5836	0.1556
	(0.0000)	(0.7073)	(0.3604)	(0.2023)	(0.1498)	(0.0859)	(0.2178)
NASDAQ 100 to KSE 100	6802.1	5.5109	18.451	0.7022	4.6876	0.3167	0.2505
	(0.0000)	(0.1114)	(0.7120)	(0.4462)	(0.0823)	(0.6907)	(0.7736)
KSE 100 to DJI	211.54	9.7567	11.241	6.3454	12.153	4.2378	1.3988
	(0.0000)	(0.1380)	(0.1137)	(0.3088)	(0.2321)	(0.1215)	(0.1322)
DJI to KSE 100	1293.3	6.5656	12.452	2.3714	5.1671	0.2614	0.2678
	(0.0000)	(0.1251)	(0.1163)	(0.1667)	(0.9654)	(0.9003)	(0.3700)
KSE 100 to DFMGI	237.81	5.7378	29.678	6.0897	6.4663	0.1605	0.3139
	(0.0000)	(0.3724)	(0.4543)	(0.4803)	(0.4608)	(0.6607)	(0.6890)
DFMGI to KSE 100	3454.4	13.090	24.784	2.1786	6.3189	0.2106	0.1378
	(0.0000)	(0.3952)	(0.3445)	(0.9833)	(0.4187)	(0.3532)	(0.2445)
S&P 500 to DFMGI	12167	8.8754	11.785	6.1922	5.9986	0.2982	0.7641
	(0.0000)	(0.0689)	(0.0734)	(0.1112)	(0.1207)	(0.1411)	(0.5981)
DFMGI to S&P 500	561.22	6.6763	8.6039	5.7610	11.740	3.0986	1.2354
	(0.0000)	(0.4567)	(0.7991)	(0.0765)	(0.0962)	(0.0800)	(0.8764)
NASDAQ 100 to DFMGI	11864	7.3812	14.788	4.1652	9.9189	0.1689	0.4398
	(0.0000)	(0.1535)	(0.0690)	(0.1656)	(0.3412)	(0.8189)	(0.4701)
DFMGI to NASDAQ 100	184.45	7.8781	8.9890	7.6765	11.748	3.5653	2.1565
	(0.0000)	(0.1306)	(0.5989)	(0.1672)	(0.0964)	(0.0956)	(0.3587)
DJI to DFMGI	10067	6.6789	12.578	4.8978	9.6577	0.3781	0.5876
	(0.0000)	(0.0635)	(0.5190)	(0.6751)	(0.5198)	(0.9576)	(0.3431)
DFMGI to DJI	467.76	7.7681	3.7687	3.7823	7.9564	3.7861	0.6981
	(0.0000)	(0.4238)	(0.4591)	(0.3431)	(0.1675)	(0.7818)	(0.5536)

Null Hypotheses(All Null Hypotheses are for n^{th} order)

Q-stat (return series) there is no serial autocorrelation. Q^2-stat (square return series) H0: there is no serial autocorrelation. Jarque-Bera H0: distribution of series is normal. LM-ARCH H_0: there is no ARCH effect. P-values are in the parenthesis.

The table 9.6.b illustrates the post estimation results (Residual analysis). The Jarque-Bera test (Normality test) with null hypothesis the distribution of returns are normal which show non normal residuals. The Q-stat are insignificant up to 10th lags which means that we should accept null hypothesis that there is no serial autocorrelation in the standardized residuals. The Q-stat on squared standardized residuals is insignificant up to 10th lags with null hypothesis that there is no serial autocorrelation in squared standardized residuals. LM-ARCH test is also insignificant up to 5thlags, accept null hypothesis, no ARCH effect remains in series residuals. These results show that there is no econometric problem left in residuals it means the results of table 8.6.b are valid. To find out the spillover effect from ARDL model we put the return series of one market into the equation of other market. If the series coefficient is significant it means there is spillover effect from one market to other. We employed F-test to test the joint significance of the lag values of independent variable. The results of ARDL model are following:

Table 9.7.a Volatility Spillover effect by using ARDL models (Bidirectional Analyses for ARDL Monthly Results)

Spillover direction	F-stat	Spillover direction	F-stat
S&P 500 to KSE 100	4.3644 (0.0149)**	**KSE 100 to S&P 500**	6.4908 (0.0022)*
NASDAQ 100 to KSE 100	3.2565 (0.0145)**	**KSE 100 to NASDAQ 100**	2.4784 (0.0683)
DJI to KSE 100	4.0464 (0.0043)*	**KSE 100 to DJI**	3.1857 (0.0966)***
DFMGI to KSE 100	3.4837 (0.0103)**	**KSE 100 to DFMGI**	2.2095 (0.0479)**
S&P 500 to DFMGI	6.0965 (0.0002)*	**DFMGI to S&P 500**	4.8537 (0.0012)*
NASDAQ 100 to DFMGI	6.7575 (0.0003)*	**DFMGI to NASDAQ 100**	3.8998 (0.0028)*
DJI to DFMGI	4.0773 (0.0020)*	**DFMGI to DJI**	3.7808 (0.0035)*

Table 9.7.a shows that the S&P 500, NASADQ 100, and DJI series coefficients are significant in the equation of KSE 100. It means there is spillover effect from S&P 500, NASADQ 100, and DJI to KSE 100. But there is no spillover effect found from KSE 100 to S&P 500, NASADQ 100, and DJI because there coefficients are insignificant in the equation of KSE 100. It shows that there is unidirectional spillover effect from S&P 500, NASADQ 100, and DJI to KSE 100. The DFMGI series coefficients are significant in the equation of KSE 100 and KSE 100 series coefficients are significant in the equation of DFMGI. It means there is bidirectional spillover effect between DFMGI and KSE 100.

The results in table 9.7.a also show that the S&P 500, NASADQ 100, and DJI series coefficients are significant in the equation of DFMGI and DFMGI series coefficients are also significant in the equation of S&P 500, NASADQ 100, and DJI. It means there is bidirectional spillover effect between S&P 500, NASADQ 100, DJI and DFMGI. These results support the results of GARCH models because the directions of spillover remain same. This shows that DFMGI, S&P 500, NASADQ 100, and DJI have direct spillover effect on KSE 100 but also there is indirect effect from S&P 500, NASADQ 100, and DJI to KSE 100 through DFMGI. For the validation of ARDL results we employed the residual analysis. The results are given following:

Table 9.7.b Residual Analysis of ARDL model with Monthly Data

Series	AR 1-7 test	ARCH 1-7 test	Hetero test
S&P 500 to KSE 100	1.8101 (0.0932)	1.3327 (0.2425)	1.8728 (0.0560)
KSE 100 to S&P 500	1.4507 (0.1933)	**3.7511 (0.0012)***	1.6972 (0.0909)
NASDAQ 100 to KSE 100	0.97862 (0.4511)	1.3996 (0.2136)	1.7213 (0.0626)
KSE 100 to NASDAQ 100	1.1241 (0.3546)	.87330 (0.5304)	1.4315 (0.1154)
DJI to KSE 100	1.4929 (0.1784)	1.3684 (0.2269)	1.5802 (0.0979)
KSE 100 to DJI	1.1241 (0.3546)	0.87330 (0.5304)	1.4315 (0.1154)
DFMGI to KSE 100	1.4929 (0.1784)	1.3684 (0.2269)	1.5802 (0.0979)
KSE 100 to DFMGI	**3.3040 (0.0033)***	1.4336 (0.2002)	**4.2217 (0.0001)***
S&P 500 to DFMGI	1.8983 (0.0776)	0.60082 (0.7539)	2.0017 (0.0749)
DFMGI to S&P 500	1.3842 (0.2203)	1.6889 (0.1201)	1.4790 (0.1328)
NASDAQ 100 to DFMGI	2.3331 (0.0501)	0.63288 (0.7277)	2.4133 (0.0859)
DFMGI to NASDAQ 100	1.2621 (0.2771)	1.3824 (0.2209)	1.1127 (0.3524)
DJI to DFMGI	2.1628 (0.0640)	1.0568 (0.3970)	1.9181 (0.0729)
DFMGI to DJI	0.38063 (0.9117)	1.3705 (0.2260)	0.78667 (0.7108)

LM-ARCH H0: there is no ARCH effect. P-values are in the parenthesis. AR H0: there is no autocorrelation in residuals. P-values are in the parenthesis. Hetero test H0: there is no Heteroscedasticity in residuals. P-values are in the parenthesis.

Table 9.7.b shows the residual analysis after employing ARDL model. The results indicate there is no autocorrelation (AR) up to 7th lag, no ARCH effect up to 7th lag and no heteroscedasticity in residuals. It shows that the results of ARDL model are reliable. Only one time ARDL is unable to capture the ARCH effect and one time autocorrelation and heteroscedasticity. We regressed 14 regression two out of them are unable to capture the ARCH effect, it means on the basis of given sample in case of monthly data the accuracy of ARDL model is (100 - 2/14)*100 = 85.7%. We may not be able to generalize it on this small sample outputs but it is an effort to explore another way to

deal with financial series with ARCH effect. It shows that ARDL model is able to capture ARCH effect when we are working with monthly data.

9.7 Conclusion and Policy Implications

This study investigates the direct and indirect dynamic linkages between Pakistani and leading global stock markets. Daily data are used from 2005 to 2014. The appropriate univariate GARCH type models and ARDL models are employed to examine information transmission between stock markets and modeling volatility. The study examined the fluctuating nature and the magnitude of the spillover from US and Gulf equity markets to Pakistan stock market KSE 100. The unidirectional spillover effect is found from S&P 500, NASADQ 100, and DJI to KSE 100. The bidirectional spillover effect is found between DFMGI and KSE 100. While there is a bidirectional spillover effect amongst S&P 500, NASADQ 100, DJI, and DFMGI. This study concluded that there is direct and indirect spillover effect from leading foreign markets to Pakistan stock market.

One thing that is more important in the study is comparison of GARCH type models and ARDL model. The study concluded that the ARDL model is unable to capture ARCH effect when data are collected on daily and weekly basis. It only captures the ARCH when data are monthly or at less frequency. ARDL model performance on capturing the ARCH effect is 85.7% on the basis of given sample of monthly data. We may not be able to generalize these finding on this small sample outputs but it is an effort to explore another way to deal with financial series with ARCH effect.

We conclude that the investors are using these markets in their diversified portfolios. Despite the war and terror foreign investors are interested in Pakistani stock markets. Particularly the investment in energy sector is more attractive for foreign investors. The boom in KSE 100 is not a bubble created by local investors.

This study is an important tool for financial institutions, portfolio managers, market players and academician to diagnose the nature and level of linkages and information transmission between the financial markets. The financial managers get more understanding about the management of portfolio which is badly affected by the stock prices. The market players may use this information for portfolio diversification and hedging. The policy makers can minimize the effects of spread of stock prices. The stability of stock prices is very important for portfolio and foreign direct investments, which improves macroeconomic stability and positively affect the economic growth. Through these results the investors/market players of one market can guess the performance of other markets. This study also provides an alternative way to deal with ARCH effect.

CHAPTER 10

CONCLUSION AND FUTURE RESEARCH DIRECTIONS

The conventional econometrics literature considers nonstationarity only reasons of spurious regression since decades and propose unit root and cointegration procedures to handle this problem. As we discussed in chapter 2, the outputs of these procedures are not much reliable because of some specification decisions. These procedures are also unable to tackle the problem of spurious regression in stationary time series. We offer an alternative procedure for the treatment of this problem in stationary and nonstationary time series.

10.1 Conclusion

We concluded following results from this research that the unit root and commonly used cointegration procedures ordinarily provide misleading results. These procedures provide unreliable results due to some specification decisions. Under correct specification, they provide optimal size and power but in case of any misspecification they undergo in size distortion.

The specification decisions made on the basis of classical model selection techniques often provide spurious results in time series data. The reason behind it might be in case of nonstationarity these model specification techniques become worthless. The conventional econometric method OLS suffers a lot in size distortion problem, when the series are having unit root, and even in case of stationary time series. Whereas, the ARDL model has no size distortion problem in both (nonstationary and stationary) cases. However, in case of under specification: when data generating process contains linear trend but trend is not included in regression model then Both OLS and ARDL models suffer in size distortion problem. But even in case of under specification ARDL model significantly reduces the probability of spurious regression as compare to OLS.

The commonly used conventional cointegration procedures Engle and Granger, Johansen and Juselius, and Pesaran ARDL are having severe size distortion problem even in case of correct specifications, but ARDL does not show in size problem in case of correct specification. On the other hand, in case of over specification conventional cointegration procedures show size problem but ARDL has not size problem in case of over specification. Nonetheless, in case of under specification ARDL suffers in severe size distortion problem as compare to conventional cointegration test. It clarifies that the ARDL model is most robust model in case of correct and over specification but not in under specification.

The ARDL model provides better forecasting as compare to conventional cointegration procedures in case of real time series data. The ARDL model provides small deviation between actual and forecasted values. The experiments refer that ARDL model can be used as an alternative tool to tackle the problem of spurious regression in case of stationary and nonstationary time series.

10.2 Limitation of Study

There is some limitation of this study, first the data generating process which is being used in this study is only based on two variables. It means all the experiments are done on bivariate regression, but in case of multivariate the results might be varied at some extent. Second, this study only deals with autoregressive process but this study should be analyzed with autoregressive moving average method. Third, in this study the structural breaks are not incorporated in regression analysis.

10.3 Future Research Direction

The work of this study can be extended in future by over coming the limitations of this study. First, researcher can use multivariate data generating process for comparison of different econometric tools. Second, someone can check the size and power of these methods by including structural break in model.

REFERENCES

Agunloye, O. K., Shangodoyin, D. K., & Arnab, R. (2014). Lag Length Specification in Engle-Granger Cointegration Test: A Modified Koyck Mean Lag Approach Based on Partial Correlation. Statistics in Transition, *15(4),* 559-572.

Ahking, F. W. (2002). Model Mis-Specification and Johansen's Co-integration Analysis: An Application to the US Money Demand. *Journal of Macroeconomics, 24(1),* 51-66.

Aldrich, J. (1995). Correlations Genuine and Spurious in Pearson and Yule. Statistical Science, 364-376.

Ali, R. and M. Afzal (2012) Impact of Global Financial Crisis on Stock Markets: Evidence from Pakistan and India. *Journal of Business Management and Economics, 3 (7)*, 275-282.

Alikhanov, A. (2013). To What Extent are Stock Returns Driven by Mean and Volatility Spillover Effects? –Evidence from Eight European Stock Markets. Review of Economic Perspectives, *13(1),* 3-29.

Alsukker, A. S. (2010). Duabi Crisis. *Australian Catholic University*.

Amjad, R. and M. Din (2010). Economic and Social Impact of Global Financial Crisis: Implications for Macroeconomic and Development Policies in South Asia. *PIDE Monograph Series 1.*

Angkinand, A.P; J.R. Barth and H. Kim (2009). Spillover Effects from the U.S. Financial Crisis: Some Time-Series Evidence from National Stock Returns. *Edward Elgar Publishing, 1-36.*

Atiq-ur-Rehman, A. U. R., & Zaman, A. (2008). Model Specification, Observational Equivalence and Performance of Unit Root Tests. MPRA, 13489.

Attari, M. I. J and L. Safdar (2013). The Relationship between Macroeconomic Volatility and the Stock Market Volatility: Empirical Evidence from Pakistan. *Pakistan Journal of Commerce and Social Sciences, 7(*2), 309-320.

Black, F (1976) Studies of Stock Price Volatility Changes. *Journal of American Statistical Association,* 177-181.

Bollerslev, T (1986) Generalized Autoregressive Conditional Heteroskedasticity. *Journal of Econometrics, 31(1),* 307-327.

Carrasco Gutierrez, C. E., Souza, R. C., & Guillén, O. (2009). Selection of Optimal Lag Length in Cointegrated VAR Models with Weak Form of Common Cyclical Features. Fucape Business School, 16.

Chaouachi, K. (2013). False Positive Result in Study on Hookah Smoking and Cancer in Kashmir: Measuring Risk of Poor Hygiene is not the same as Measuring Risk of Inhaling Water Filtered Tobacco Smoke all over the World. *British journal of cancer, 108(6),* 1389.

Charemza, W. W., & Deadman, D. F. (1997). New Directions in Econometric Practice. *Books.*

Chelley-Steeley, P. L. (2005). Modeling Equity Market Integration using Smooth Transition Analysis: A study of Eastern European Stock Markets. *Journal of International Money and Finance, 24(5),* 818-831.

Choi, C. Y., Hu, L., & Ogaki, M. (2004). A Spurious Regression Approach to Estimating Structural Parameters. *Ohio State University Department of Economics Working Paper,* (04-01).

Davidson, J. E., Hendry, D. F., Srba, F., & Yeo, S. (1978). Econometric Modelling of the Aggregate Time-Series Relationship between Consumers' Expenditure and Income in the United Kingdom. *The Economic Journal,* 661-692.

DeJong, D. N., Nankervis, J. C., Savin, N. E., & Whiteman, C. H. (1992). The Power Problems of Unit Root Test in Time Series with Autoregressive Errors. *Journal of Econometrics, 53(1-3),* 323-343.

Dickey, D. A., & Fuller, W. A. (1979). Distribution of the Estimators for Autoregressive Time Series with a Unit Root. *Journal of the American statistical association, 74(366a),* 427-431.

Dickey, D. A., & Fuller, W. A. (1981). Likelihood Ratio Statistics for Autoregressive Time Series with a Unit Root. Econometrica: *Journal of the Econometric Society,* 1057-1072.

Draz, M. U (2011). Impact of Financial Crisis on Pakistan and China: A Comparative Study of Six Decades. *Journal of Global Business and Economics, Vol. 3,* No. 1, 174-186.

Engle, R. and Yoo Sam (1991). Forecasting and Testing in Co-integrated Systems, In Engle and Granger (eds.), Long Run Economic Relationships. Readings in Cointegration, *Oxford University Press, New York,* 237-67.

Engle, R. F. (1982) Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of United Kingdom Inflation. *Econometrica, Vol. 50,* No. 4, 987-1007.

Engle, R. F., & Granger, C. W. (1987). Co-integration and Error Correction: Representation, Estimation, and Testing. *Econometrica: Journal of the Econometric Society,* 251-276.

Engle, R. F; Takatoshi Ito and Wen-Ling Lin (1990) Meteor Showers or Heat Waves? Heteroskedastic Intra-Daily Volatility in the Foreign Exchange Market. *Econometrica, Vol. 58,* No. 3, 525-542.

Frey, B. S. (2002). Inspiring Economics: Human Motivation in Political Economy. *Edward Elgar Publishing.*

Glosten, L. R., Jagannathan, R., & Runkle, D. E. (1993). On the Relation between the Expected Value and the Volatility of the Nominal Excess Return on Stocks. *The Journal of Finance, Vol. 48,* No. 5, 1779-1801.

Gomez, S and H. Ahmad (2014) Volatility Transmission between Pals, Foes, Minions and Titans. IOSR. *Journal of Business and Management (IOSR-JBM), Vol. 16,* 122-128.

Granger IV, C. W., Hyung, N., & Jeon, Y. (2001). Spurious Regressions with Stationary Series. Applied Economics, 33(7), 899-904.

Granger, C. W., & Newbold, P. (1974). Spurious Regressions in Econometrics. *Journal of Econometrics*, *2*(2), 111-120.

Greene, W. H. (2003). Econometric Analysis. *Pearson Education* India.

Gaughan, P. A. (2009). Measuring business interruption losses and other commercial damages. *New York: John Wiley & Sons*.

Hamao, Y., Masulis, R. W., & Ng, V. (1990). Correlations in Price Changes and Volatility Across International Stock Markets. *Review of Financial Studies, Vol. 3,* No. 2, 281-307.

Hashimzade, N., & Thornton, M. A. (Eds.). (2013). Handbook of Research Methods and Applications in Empirical Macroeconomics. *Edward Elgar Publishing*.

Hassler, U. (2003). Nonsense Regressions due to Neglected Time-Varying means. *Statistical Papers, 44(2),* 169-182.

Hendry, D. F. (1980). Econometrics-alchemy or Science?. *Economica,* 387-406.

Hendry, D. F., & Richard, J. F. (1983). The Econometric Analysis of Economic Time Series. *International Statistical Review/Revue Internationale de Statistique,* 111-148.

Hendry, D. F., Pagan, A. R., & Sargan, J. D. (1984). Dynamic Specification. *Handbook of Econometrics, 2,* 1023-1100.

Höfer, Thomas; Hildegard Przyrembel; Silvia Verleger (2004). New Evidence for the Theory of the Stork. *Paediatric and Perinatal Epidemiology. 18 (1):* 18–22.

Harris, R. I. (1995). Using cointegration analysis in econometric modelling. *Harvester Wheatsheaf, Prentice Hall*.

Jeyanthi, B. J. Q. (2010). Interdependence And Volatility Spillovers Under Market Reforms: The Case Of National Stock Exchange. *International Business & Economics Research Journal, Vol. 9,* No. 9, 77-86.

Juselius, K. (1992). Testing Structural Hypotheses in a Multivariate Cointegration Analysis of the PPP and the UIP for UK. *Journal of Econometrics, 53(1-3),* 211-244.

Leybourne, S. J., & Newbold, P. (2003). Spurious Rejections by Cointegration Tests Induced by Structural Breaks. *Applied Economics, 35(9),* 1117-1121.

Mukherjee, K.N. and R. K. Mishra (2008). Sto ck Market Integration and Volatility Spillover: India and its Major Asian Counterparts. *MPRA Paper No. 12788*.

Nelson, C. R., & Plosser, C. R. (1982). Trends and Random Walks in Macroeconmic Time Series: some Evidence and Implications. *Journal of Monetary Economics*, *10*(2), 139-162.

Onour, I. A. (2010). The Global Financial Crisis and Equity Markets in Middle East Oil Exporting Countries. *MPRA Paper No. 23332, 1-19.*

Padhi, P. and M. A. Lagesh (2012). Volatility Spillover and Time-Varing Correlation Among the Indian, Asian and US Stock Markets . *Journal of Quantitative Economics, Vol. 10,* No. 2, 78-90.

Perron, P. (1990). Testing for a Unit Root in a Time Series with a Changing Mean. *Journal of Business & Economic Statistics, 8(2),* 153-162.

Pesaran, M. H. (1997). The Role of Economic Theory in Modelling the Long Run. *The Economic Journal, 107(440),* 178-191.

Pesaran, M. H., & Smith, R. (1995). Estimating Long-Run Relationships from Dynamic Heterogeneous Panels. *Journal of Econometrics, 68(1*), 79-113.

Pesaran, M. H., Shin, Y., & Smith, R. J. (1996). Testing for the' Existence of a Long-Run Relationship' (No. 9622). *Faculty of Economics, University of Cambridge.*

Phillips, P. C. (1986). Understanding Spurious Regressions in Econometrics. *Journal of Econometrics, 33(3),* 311-340.

Phillips, P. C., & Perron, P. (1988). Testing for a Unit Root in Time Series Regression. *Biometrika,* 335-346.

Plosser, C. I., & Schwert, G. W. (1978). Money, Income, and Sunspots: Measuring Economic Relationships and the Effects of Differencing. *Journal of Monetary Economics, 4(4),* 637-660.

Rehman, A. U., & Malik, M. I. (2014). The Modified R a Robust Measure of Association for Time Series. *Electronic Journal of Applied Statistical Analysis, 7(1),* 1-13.

Sapsford, Roger; Jupp, Victor, eds. (2006). Data Collection and Analysis. Sage. ISBN 0-7619-4362-5.

Schwert, G. W. (2002). Tests for unit roots: A Monte Carlo Investigation. *Journal of Business & Economic Statistics, 20(1),* 5-17.

Simon, H. A. (1954). Spurious Correlation: a Causal Interpretation. *Journal of the American Statistical Association, 49(267),* 467-479.

Sinha, P. and G. Sinha (2010). Volatility Spillover in India, USA and Japan Investigation of Recession Effects. *MPRA Paper No. 21873.*

Sok-Gee, C and M. Z. A. Karim (2010). Volatility Spillovers of the Major Stock Markets in ASEAN-5 with the U.S. and Japanese Stock Markets. *International Research Journal of Finance and Economics, Vol. 44,* 156-168.

Su, J. J. (2008). A note on Spurious Regressions between Stationary series. *Applied Economics Letters, 15(15),* 1225-1230.

Sun, Y. (2004). A Convergent t-Statistic in Spurious Regressions. *Econometric Theory, 20(05),* 943-962.

Song, H., & Witt, S. F. (2012). Tourism demand modelling and forecasting. *Routledge.*

Tahir, S. H; H. M. Sabir; Y. Ali; S. J. Ali and A. Ismail (2013). Interdependence of South Asian & Developed Stock Markets and Their Impact on KSE (Pakistan). *Asian Economic and Financial Review, Vol. 3,* No. 1, 16-27.

Thomas, R. L. (1997). Modern econometrics: an introduction. Addison-Wesley *Longman.*

Ventosa-Santaulària, D. (2009). Spurious regression. *Journal of Probability and Statistics*, 2009.

Wongswa, J (2006) Transmission of Information across International Equity Markets. *Review of Financial Studies, Vol. 19,* No. 4, 1157-1189.

Yule, G. U. (1926). Why do we sometimes get Nonsense-Correlations between Time-Series?--a study in Sampling and the Nature of Time-Series. *Journal of the Royal Statistical Society*, *89*(1), 1-63.

Zia-ur-Rehman, M; Z. A. Shah and R. Mushtaq (2011). Sources of Return and Volatility Spillover for Pakistan: An Analysis of Exogenous Factors by using EGARCH Model. *International Conference on Business and Economics Research, Vol. 16,* 151-157.

APPENDIX A

Lemma 1. Let's suppose $\{Y_t\}_1^\infty$ *and* $\{X_t\}_1^\infty$ *are first order autoregressive and generated by equation (3.12). If the error terms sequences* $\{u_{yt}\}_1^\infty$ *and* $\{u_{xt}\}_1^\infty$ *and independent and if* $\{u_{yt},\ u_{xt}\}_1^\infty$ *sustains Assumption 1 conditions then as* $T \uparrow \infty$,

(a)

$$T^{-3/2}\sum_{1}^{T} Y_t \Rightarrow \sigma_{u_y}\int_0^1 W(t)\,dt$$

$$T^{-3/2}\sum_{1}^{T} X_t \Rightarrow \sigma_{u_x}\int_0^1 N(t)\,dt$$

(b)

$$T^{-2}\sum_{1}^{T} Y_t^2 \Rightarrow \sigma_{u_y}^2\int_0^1 W(t)^2\,dt$$

$$T^{-2}\sum_{1}^{T} X_t^2 \Rightarrow \sigma_{u_x}^2\int_0^1 N(t)^2\,dt$$

(c)

$$T^{-2}\sum_{1}^{T} (Y_t - \overline{Y})^2 \Rightarrow \sigma_{u_y}^2[\int_0^1 W(t)^2\,dt - \{\int_0^1 W(t)\,dt\}^2]$$

$$T^{-2}\sum_{1}^{T} (X_t - \overline{X})^2 \Rightarrow \sigma_{u_x}^2[\int_0^1 N(t)^2\,dt - \{\int_0^1 N(t)\,dt\}^2]$$

(d)

$$T^{-2}\sum_{1}^{T} Y_t X_t \Rightarrow \sigma_{u_y}\sigma_{u_x}\int_0^1 N(t)W(t)\,dt$$

(e)

$$T^{-2}\sum_{r}^{T}(Y_t - Y_{t-r}) \Rightarrow (r/2)\{\sigma_{u_y}^2 W(1)^2 + \Omega_{u0_y}\} + \sum_{j=1}^{r}(r-j)\Omega_{u_{yj}}$$

$$T^{-2}\sum_{r}^{T}(X_t - X_{t-r}) \Rightarrow (r/2)\{\sigma_{u_x}^2 W(1)^2 + \Omega_{u0_x}\} + \sum_{j=1}^{r}(r-j)\Omega_{u_{xj}}$$

(f)

$$T^{-1}\sum_{r}^{T}(Y_t - Y_{t-r}) + T^{-1}\sum_{r}^{T}(X_t - X_{t-r}) \Rightarrow r\sigma_{u_y}\sigma_{u_x}W(1)N(1)$$

where N(t) and W(t) are defined as independent Wiener processes on the C[0, 1] and $\Omega_{u_{xj}}$ and $\Omega_{u_{yj}}$ are define as following:

$$\Omega_{u_{yj}} = \lim_{T\to\infty} T^{-1}\sum_{j=1}^{T} E\left(u_{yt}, u_{yt-1}\right) \qquad j = 0, 1 \qquad (3.26)$$

$$\Omega_{u_{xj}} = \lim_{T\to\infty} T^{-1}\sum_{j=1}^{T} E\left(u_{xt}, u_{xt-1}\right) \qquad j = 0, 1 \qquad (3.27)$$

Furthermore, (a) to (f) all condition holds regardless of initial conditions consigned to Y_0 and X_0. In lemma 1 C[0, 1] defines the real valued continuous functions space on interval [0, 1]. The Wiener processes W(t) and N(t) in lemma are stochastically independent and their path lie between this interval [0, 1]. The results of (a) to (f) of lemma 1 establish that properly standardized moments of sample of sequences $\{u_{yt}\}_1^\infty$ and $\{u_{xt}\}_1^\infty$ converge weakly to suitably defined functional of Wiener processes N(t) and W(t). Every functional of them has a defined non degenerate distribution. The arrow sign "⇒" in lemma is representing the weak convergence of related probability measures. So, in case (a) we infer that $T^{-3/2}\sum_1^T X_t$ converges in the distribution to distribution of functional $\sigma_{u_x}\int_0^1 N(t)\,dt$ of Wiener process N(t) on C[0, 1]. We also infer that normal limiting distribution of $T^{-3/2}\sum_1^T X_t$ with mean zero and variance is given below:

$$\sigma_{u_x}^2 E\left\{\int_0^1\int_0^1 N(t)N(s)dtds\right\} = 2\sigma_{u_x}^2\int_0^1\int_0^r E\{N(r)N(s)\}dsdr \quad (3.28)$$

$$\sigma_{u_x}^2 E\left\{\int_0^1\int_0^1 N(t)N(s)dtds\right\} = 2\sigma_{u_x}^2\int_0^1\int_0^r sdsdr \quad (3.29)$$

$$\sigma_{u_x}^2 E\left\{\int_0^1\int_0^1 N(t)N(s)dtds\right\} = \sigma_{u_x}^2/3 \quad (3.30)$$

For further understanding of these results see Philips (1986, appendix). Now it is easy to derive the theorem 1 by considering derived results of lemma 1.

Theorem 1. Let's assume the regression equation (3.12)is estimated by employing linear regression model and it sustains all condition of lemma 1, then as T approaches to infinity

(a)

$$\hat{\beta} \Rightarrow \frac{\sigma_{u_y}^2\left\{\int_0^1 N(t)W(t)dt - \int_0^1 N(t)dt\int_0^1 W(t)dt\right\}}{\sigma_{u_y}^2\left\{\int_0^1 W(t)^2\,dt - (\int_0^1 W(t)dt)^2\right\}},$$

$$\hat{\beta} = \left(\frac{\sigma_{u_y}^2}{\sigma_{u_x}^2}\right)\xi;$$

(b)

$$T^{-2}\hat{\beta}_0 \Rightarrow \sigma_{u_x}\left(\int_0^1 N(t)dt - \xi\int_0^1 W(t)dt\right);$$

(c)

$$T^{-1/2}t_{\hat{\beta}_1} = \left(\frac{U}{v^{\frac{1}{2}}}\right)$$

where

$$U = \int_0^1 N\,(t)W(t)dt - \int_0^1 N\,(t)dt \int_0^1 W\,(t)dt,$$

$$v = \left\{\int_0^1 N\,(t)^2\,dt - (\int_0^1 N\,(t)dt)^2\right\}\left\{\int_0^1 W\,(t)^2\,dt - (\int_0^1 W\,(t))^2 dt\right\}$$

$$- \left\{(\int_0^1 N\,(t)W(t)dt - \int_0^1 N\,(t)dt \int_0^1 W\,(t)dt)^2\right\};$$

(d)

$$\mathrm{T}^{-1/2}\mathrm{t}_{\hat{\beta}_0} \Rightarrow \frac{\left\{\int_0^1 \mathrm{N}\,(\mathrm{t})\mathrm{dt} - \xi \int_0^1 \mathrm{W}(\mathrm{t})\mathrm{dt}\right\} \text{ x } \left\{\int_0^1 \mathrm{W}(\mathrm{t})^2\mathrm{dt} - (\int_0^1 \mathrm{W}(\mathrm{t})\mathrm{dt})^2\right\}}{[\mathrm{v}\int_0^1 \mathrm{W}(\mathrm{t})^2\mathrm{dt}]^{\frac{1}{2}}};$$

(e)

$$R^2 \Rightarrow \frac{\xi^2\left\{\int_0^1 W(t)^2 dt - (\int_0^1 W(t)dt)^2\right\}}{\int_0^1 N\,(t)^2 dt - (\int_0^1 N\,(t)dt)^2};$$

(f)

$DW\vec{P}0,$

$$TDW \Rightarrow \left\{\left(\frac{\Omega_{u0_y}}{\sigma^2_{u_y}}\right) + \xi^2\left(\frac{\Omega_{u0_x}}{\sigma^2_{u_x}}\right)\right\}\left[\int_0^1 N(t)^2 dt - (\int_0^1 N(t)dt)^2\right.$$

$$\left. - \xi^2\left\{\int_0^1 W(t)^2 dt - (\int_0^1 W(t)dt)^2\right\}\right];$$

(g)

$s \geq 1$ fixed for all,

$$T(r_s - 1) \Rightarrow -\frac{A_s}{B} \; and \; the \; r_s = 1 + O_P(T^{-1});$$

where

$$A_s = \left(\frac{s}{2}\right)\left[\{N(1) - \xi W(1)\} - \left\{\int_0^1 N(t)dt - \xi \int_0^1 W(t)dt\right\}\right]^2$$

$$+ \left(\frac{s}{2}\right)\left\{\int_0^1 N(t)dt - \xi \int_0^1 W(t)dt\right\}^2$$

$$+ \left\{s\left(\frac{\Omega_{u0_y}}{2\sigma_{u_y}^2}\right) + \sum_{j=1}^{s}(s-j)\left(\frac{\Omega_{u_y}}{\sigma_{u_y}^2}\right)\right\}$$

$$+ \xi^2\left\{s\left(\frac{\Omega_{u0_x}}{2\sigma_{u_x}^2}\right) + \sum_{j=1}^{2}(s-j)\left(\frac{\Omega_{u_x}}{\sigma_{u_x}^2}\right)\right\},$$

$$B = \int_0^1 N(t)^2 dt - \left(\int_0^1 N(t)dt\right)^2 - \xi^2\left\{\int_0^1 W(t)^2 dt - \left(\int_0^1 W(t)dt\right)^2\right\};$$

(h)

$$T^{-1}Q_k = \sum_{s=1}^{k} r_s \vec{P}\, k;$$

where

where N(t) and W(t) are the independent Wiener processes on the C[0, 1]. Theorem 1 explains the Monte Carlo simulation results stated by Granger and Newbold. In theorem 1 the (c) shows the conventional t-ratio for slope coefficient $t_{\widehat{\beta}_1}$and (d) shows the t-ratio for intercept $t_{\widehat{\beta}_0}$. These t-ratios are used to check the statistical significance of regression parameters, but in this regression analysis case they do not follow limiting distribution. In fact, $t_{\widehat{\beta}_1}$ and $t_{\widehat{\beta}_0}$ distributions diverge as T increases $C_{0.}$ So, these significance tests are not having asymptotically corrected critical values. The rejection of null hypothesis should be expected when we use conventional asymptotic critical value (such as 1.96) as we increase the sample size in context of weak stationary time series. Thus huge rejection of null hypothesis in Granger and Newbold experiment (T=50), therefore it arises as no surprise. Definitely, it is anticipated by correct asymptotic theory. Granger and Newbold in their experiments also suggest to use 11.2 as a new critical value instead of conventional critical value 1.96 at 5% level of significance but this suggestion does not have any foundation in asymptotic theory according to our results.

APPENDIX B

	OLS	ARDL (1, 1)			ARDL (2, 2)			
	xt	*xt-1*	*xt*	*F-stat*	*xt-2*	*xt-1*	*xt*	*F-stat*
				ALB				
Percentile 25	8.396	-0.724	-0.181	1.896	-0.125	-0.468	-0.388	1.564
Percentile 50	10.426	0.011	0.313	2.950	0.318	-0.124	0.444	2.312
Percentile 75	12.875	0.687	1.364	4.169	0.831	0.406	1.135	2.793
Percentile 5	6.171	-1.810	-1.175	0.772	-0.787	-1.636	-0.827	1.264
Percentile 95	14.847	1.507	2.560	5.627	1.461	0.719	2.051	3.510
Positive significant	30	0	5	14	0	0	2	6
Negative significant	0	1	0	0	0	1	0	0
Total significant	30	1	5	14	0	1	2	6
Percentages	100	3.333	16.667	46.667	0	3.333	6.667	20
				AUS				
Percentile 25	16.810	-1.881	0.548	1.194	-1.274	-0.551	0.619	0.042
Percentile 50	25.525	-1.105	1.474	2.588	-0.485	-0.314	1.392	0.064
Percentile 75	39.582	0.076	2.181	4.105	0.001	0.137	1.918	0.105
Percentile 5	11.622	-4.481	-0.443	0.211	-2.320	-1.211	-0.282	0.020
Percentile 95	57.888	0.840	3.613	15.682	1.063	1.342	2.828	0.210
Positive significant	30	0	10	13	0	0	6	0
Negative significant	0	6	1	0	2	0	0	0
Total significant	30	6	11	13	2	0	6	0
Percentages	100	20	36.667	43.333	6.667	0	20	0
				AUT				
Percentile 25	17.870	-4.059	1.006	1.442	-0.233	-2.593	1.385	0.913
Percentile 50	24.822	-2.224	2.793	5.166	0.294	-1.401	2.716	2.864
Percentile 75	31.650	-0.638	4.534	11.668	0.812	-0.478	4.392	8.225
Percentile 5	10.718	-5.058	-0.329	0.455	-0.771	-4.611	-0.095	0.207
Percentile 95	41.686	0.737	8.051	38.383	1.988	0.329	7.845	24.909
Positive significant	30	0	19	18	2	0	19	15
Negative significant	0	16	0	0	0	11	0	0
Total significant	30	16	19	18	2	11	19	15
Percentages	100	53.333	63.333	60	6.667	36.667	63.333	50
				BHR				
Percentile 25	15.164	-0.568	-0.113	2.281	0.250	-1.181	-0.215	1.306
Percentile 50	18.598	-0.085	0.900	4.026	0.570	-0.669	0.810	1.920
Percentile 75	31.326	0.637	2.277	6.252	1.056	0.027	2.190	2.892
Percentile 5	10.899	-1.787	-1.031	0.487	-0.382	-1.883	-0.477	0.401
Percentile 95	65.590	1.190	3.917	14.122	2.545	0.684	3.399	6.521
Positive significant	30	0	9	18	3	0	9	7
Negative significant	0	1	0	0	0	1	0	0
Total significant	30	1	9	18	3	1	9	7
Percentages	100	3.333	30	60	10	3.333	30	23.333

					BHS			
Percentile 25	12.391	-2.437	0.600	0.850	0.132	-1.618	1.075	1.378
Percentile 50	15.610	-1.886	1.968	2.001	0.469	-1.276	1.923	2.186
Percentile 75	19.372	-0.333	3.090	4.885	0.994	-0.637	2.538	3.783
Percentile 5	7.849	-3.690	-1.026	0.163	-0.855	-2.402	-0.496	0.687
Percentile 95	29.251	1.131	4.014	9.688	1.691	0.413	4.626	8.969
Positive significant	30	0	14	10	0	0	13	10
Negative significant	0	11	0	0	1	3	0	0
Total significant	30	11	14	10	1	3	13	10
Percentages	100	36.667	46.667	33.333	3.333	10	43.333	33.333
					BHA			
Percentile 25	14.872	-1.643	1.206	1.766	0.366	-2.657	0.967	1.201
Percentile 50	17.167	-1.140	1.746	2.987	1.079	-1.629	1.963	3.049
Percentile 75	28.970	-0.611	2.320	4.139	2.217	-0.502	2.659	4.634
Percentile 5	11.670	-2.170	-0.114	0.523	-0.343	-3.374	-0.072	0.322
Percentile 95	47.314	0.738	3.015	6.896	3.157	0.262	3.765	6.523
Positive significant	30	0	14	12	10	0	15	16
Negative significant	0	3	0	0	0	12	0	0
Total significant	30	3	14	12	10	12	15	16
Percentages	100	10	46.667	40	33.333	40	50	53.333
					BHA			
Percentile 25	14.872	-2.654	1.610	2.440	-0.023	-1.746	1.191	2.863
Percentile 50	17.167	-1.784	2.401	4.177	0.407	-1.330	2.126	4.176
Percentile 75	28.970	-1.073	3.428	7.948	1.005	-0.542	2.575	4.838
Percentile 5	11.670	-3.307	0.046	0.440	-0.879	-2.376	0.189	0.745
Percentile 95	47.314	0.690	4.430	14.468	1.792	0.580	3.693	8.770
Positive significant	30	0	18	17	1	0	18	20
Negative significant	0	12	0	0	0	4	0	0
Total significant	30	12	18	17	1	4	18	20
Percentages	100	40	60	56.667	3.333	13.333	60	66.667
					BRN			
Percentile 25	12.628	-0.760	0.789	5.533	0.443	-1.347	-0.194	0.532
Percentile 50	14.361	-0.253	1.382	11.673	0.730	-0.976	0.871	0.894
Percentile 75	16.770	0.365	2.228	17.201	1.053	-0.476	1.718	1.992
Percentile 5	8.998	-1.501	-0.304	0.490	-0.260	-1.767	-0.880	0.257
Percentile 95	20.028	1.586	3.373	30.018	1.894	0.507	2.340	3.604
Positive significant	30	1	9	24	1	0	4	3
Negative significant	0	0	0	0	0	0	0	0
Total significant	30	1	9	24	1	0	4	3
Percentages	100	3.333	30	80	3.333	0	13.333	10
					BWA			
Percentile 25	15.805	-2.252	1.246	1.311	-0.099	-2.218	1.186	1.218
Percentile 50	22.309	-1.463	2.042	2.584	0.485	-1.532	2.250	2.026
Percentile 75	32.581	-0.649	3.273	6.445	1.120	-0.722	3.332	4.935
Percentile 5	11.493	-4.355	0.201	0.488	-0.740	-3.133	0.246	0.396
Percentile 95	48.080	0.653	4.395	10.600	2.180	0.418	4.204	7.371

Positive significant	30	0	16	12	3	0	19	12
Negative significant	0	10	0	0	0	8	0	0
Total significant	30	10	16	12	3	8	19	12
Percentages	100	33.333	53.333	40	10	26.667	63.333	40
	CAN							
Percentile 25	18.501	-2.961	1.630	1.605	0.370	-3.458	1.363	1.598
Percentile 50	26.644	-2.329	2.566	4.048	0.847	-2.295	2.973	4.040
Percentile 75	32.815	-0.733	3.852	7.511	1.966	-0.845	3.932	7.056
Percentile 5	11.754	-5.048	-0.028	0.175	-1.147	-4.599	0.256	0.739
Percentile 95	42.234	0.559	4.897	14.125	2.764	0.037	4.947	10.474
Positive significant	30	0	18	16	7	0	19	17
Negative significant	0	18	0	0	0	17	0	0
Total significant	30	18	18	16	7	17	19	17
Percentages	100	60	60	53.333	23.333	56.667	63.333	56.667
	COG							
Percentile 25	10.009	0.968	-1.470	0.703	0.228	-0.539	-0.933	0.826
Percentile 50	11.552	1.338	-1.136	1.585	1.160	-0.043	-0.427	1.807
Percentile 75	15.836	1.850	-0.504	3.929	1.839	0.513	-0.104	2.927
Percentile 5	8.452	0.006	-2.878	0.145	-0.882	-1.943	-1.966	0.524
Percentile 95	19.444	3.744	0.485	8.940	2.848	1.550	0.969	6.204
Positive significant	30	6	0	8	5	1	0	7
Negative significant	0	0	6	0	0	1	1	0
Total significant	30	6	6	8	5	2	1	7
Percentages	100	20	20	26.667	16.667	6.667	3.333	23.333
	CPV							
Percentile 25	15.952	-2.308	1.635	4.653	-0.007	-2.152	1.478	2.154
Percentile 50	19.772	-1.678	2.626	7.088	0.424	-1.287	2.658	4.862
Percentile 75	30.061	-0.733	3.347	12.887	0.900	-0.619	3.551	7.531
Percentile 5	10.429	-3.122	-0.297	0.925	-0.766	-3.120	0.137	0.654
Percentile 95	40.789	1.520	5.069	20.723	1.373	1.087	5.312	11.163
Positive significant	30	1	21	23	0	0	20	20
Negative significant	0	9	1	0	0	8	0	0
Total significant	30	10	22	23	0	8	20	20
Percentages	100	33.333	73.333	76.667	0	26.667	66.667	66.667
	CRI							
Percentile 25	16.990	-2.364	1.778	2.886	0.694	-2.433	1.278	1.848
Percentile 50	20.390	-1.554	2.791	5.371	0.993	-1.455	2.436	2.893
Percentile 75	36.624	-1.015	3.241	7.720	1.843	-1.231	3.128	4.425
Percentile 5	11.430	-2.890	-0.600	1.135	-0.385	-3.474	-0.447	0.703
Percentile 95	73.699	1.375	4.422	12.170	2.580	0.313	4.042	7.460
Positive significant	30	1	21	22	7	1	18	15
Negative significant	0	14	1	0	0	12	0	0
Total significant	30	15	22	22	7	13	18	15
Percentages	100	50	73.333	73.333	23.333	43.333	60	50

				DEU				
Percentile 25	14.031	-3.387	1.273	1.076	-0.048	-2.968	1.281	0.842
Percentile 50	17.749	-1.722	2.015	2.523	0.485	-1.969	2.231	2.952
Percentile 75	23.109	-0.555	3.766	7.162	1.661	-0.539	3.817	5.339
Percentile 5	9.577	-5.580	-0.498	0.375	-0.375	-5.081	-0.257	0.383
Percentile 95	33.929	0.741	6.776	24.347	3.488	0.308	8.207	24.112
Positive significant	30	0	15	13	6	0	16	15
Negative significant	0	14	0	0	0	14	0	0
Total significant	30	14	15	13	6	14	16	15
Percentages	100	46.667	50	43.333	20	46.667	53.333	50
				DMA				
Percentile 25	14.959	-0.602	0.222	1.466	-0.687	-0.558	-0.016	1.098
Percentile 50	18.880	-0.033	1.040	3.108	-0.011	-0.222	1.059	1.917
Percentile 75	24.497	0.300	1.657	3.988	0.561	0.275	1.640	2.514
Percentile 5	9.883	-1.933	-0.568	0.151	-1.264	-0.794	-0.704	0.441
Percentile 95	29.761	1.434	2.618	6.270	1.136	1.067	2.016	4.009
Positive significant	30	1	4	13	0	0	3	5
Negative significant	0	2	0	0	0	0	0	0
Total significant	30	3	4	13	0	0	3	5
Percentages	100	10	13.333	43.333	0	0	10	16.667
				DNK				
Percentile 25	13.098	-3.912	0.633	1.271	-0.045	-3.031	0.886	0.873
Percentile 50	16.929	-2.903	2.505	4.574	0.391	-2.024	2.514	3.199
Percentile 75	23.994	-0.964	3.957	8.809	1.509	-0.546	3.711	5.541
Percentile 5	8.479	-5.623	-0.359	0.303	-0.458	-5.847	-0.181	0.121
Percentile 95	32.845	0.599	5.541	16.909	3.572	0.288	6.145	17.071
Positive significant	30	0	18	18	5	0	19	16
Negative significant	0	17	0	0	0	14	0	0
Total significant	30	17	18	18	5	14	19	16
Percentages	100	56.667	60	60	16.667	46.667	63.333	53.333
				FIN				
Percentile 25	15.573	-3.675	1.694	2.117	0.013	-3.319	1.685	2.330
Percentile 50	18.707	-2.722	3.216	5.646	0.979	-2.357	3.818	5.080
Percentile 75	28.095	-1.329	4.592	15.914	1.639	-0.828	4.744	10.669
Percentile 5	9.454	-6.067	-0.377	0.125	-0.535	-6.787	0.139	0.229
Percentile 95	32.912	0.581	7.025	27.008	3.796	0.136	7.787	26.272
Positive significant	30	0	22	20	6	0	20	20
Negative significant	0	16	0	0	0	16	0	0
Total significant	30	16	22	20	6	16	20	20
Percentages	100	53.333	73.333	66.667	20	53.333	66.667	66.667
				FJI				
Percentile 25	14.931	-0.447	0.462	1.075	0.208	-1.312	0.581	0.683
Percentile 50	19.209	-0.013	0.759	1.670	1.007	-0.767	0.939	1.590
Percentile 75	22.579	0.434	1.200	3.644	1.454	-0.448	1.481	3.229
Percentile 5	9.724	-1.430	-0.145	0.317	-0.387	-2.126	-0.092	0.176
Percentile 95	29.765	0.873	2.048	4.706	2.589	0.620	2.463	5.204

Positive significant	30	0	3	8	4	0	5	9
Negative significant	0	0	0	0	0	4	0	0
Total significant	30	0	3	8	4	4	5	9
Percentages	100	0	10	26.667	13.333	13.333	16.667	30
	FRA							
Percentile 25	17.139	-3.665	0.893	1.544	-0.052	-2.722	1.358	1.003
Percentile 50	23.643	-2.475	2.663	4.392	0.689	-1.983	2.564	2.879
Percentile 75	29.082	-0.720	3.948	10.197	1.306	-0.722	4.145	7.243
Percentile 5	9.994	-7.303	-0.823	0.293	-0.424	-5.766	-0.355	0.252
Percentile 95	42.656	0.549	8.870	44.177	3.237	0.020	8.769	30.125
Positive significant	30	0	19	17	5	0	19	13
Negative significant	0	17	0	0	0	14	0	0
Total significant	30	17	19	17	5	14	19	13
Percentages	100	56.667	63.333	56.667	16.667	46.667	63.333	43.333
	GAB							
Percentile 25	10.046	-0.199	-0.150	0.563	0.314	-1.251	0.138	1.008
Percentile 50	12.305	0.131	0.162	1.171	1.023	-0.946	0.569	1.762
Percentile 75	14.034	0.548	0.512	2.396	1.514	-0.121	0.873	2.267
Percentile 5	8.067	-0.754	-0.479	0.046	-0.888	-1.682	-0.561	0.760
Percentile 95	15.853	1.247	1.472	3.551	2.496	1.447	1.375	3.042
Positive significant	30	0	0	2	3	0	0	2
Negative significant	0	0	0	0	0	0	0	0
Total significant	30	0	0	2	3	0	0	2
Percentages	100	0	0	6.667	10	0	0	6.667
	GNB							
Percentile 25	7.791	-0.136	-0.487	1.458	-0.125	-0.450	-0.375	1.140
Percentile 50	9.236	0.506	-0.218	2.036	0.269	-0.068	0.068	1.503
Percentile 75	10.535	0.972	0.633	3.020	0.736	0.333	0.898	2.194
Percentile 5	6.943	-1.758	-0.873	1.003	-1.448	-1.430	-0.819	0.751
Percentile 95	12.457	1.262	2.183	4.994	1.304	1.035	2.076	4.248
Positive significant	30	0	2	4	0	1	2	3
Negative significant	0	2	0	0	1	0	0	0
Total significant	30	2	2	4	1	1	2	3
Percentages	100	6.667	6.667	13.333	3.333	3.333	6.667	10
	GRD							
Percentile 25	14.856	-1.721	0.684	0.969	-1.011	-0.707	0.504	0.907
Percentile 50	20.103	-0.989	1.712	2.709	-0.581	-0.363	1.776	2.013
Percentile 75	25.729	0.195	2.549	4.788	0.208	0.108	2.146	3.534
Percentile 5	8.826	-2.752	-1.026	0.197	-1.678	-1.160	-0.677	0.260
Percentile 95	34.477	1.239	3.389	8.781	0.818	1.434	2.797	6.288
Positive significant	30	0	12	10	0	0	12	10
Negative significant	0	6	0	0	0	0	0	0
Total significant	30	6	12	10	0	0	12	10
Percentages	100	20	40	33.333	0	0	40	33.333

	GUY							
Percentile 25	9.656	-0.208	-0.184	4.350	0.170	-0.554	-0.036	3.654
Percentile 50	12.371	0.263	0.636	6.640	0.847	-0.137	0.830	5.582
Percentile 75	15.055	0.909	1.142	8.441	1.404	0.143	1.301	7.340
Percentile 5	7.707	-0.655	-1.230	1.010	-0.929	-1.264	-1.152	1.300
Percentile 95	17.222	1.949	1.886	12.131	1.924	1.227	1.910	10.745
Positive significant	30	2	2	24	2	1	2	24
Negative significant	0	0	1	0	0	1	1	0
Total significant	30	2	3	24	2	2	3	24
Percentages	100	6.667	10	80	6.667	6.667	10	80
	HKG							
Percentile 25	15.329	-2.459	1.112	1.282	0.403	-3.183	1.125	1.231
Percentile 50	21.496	-1.703	2.326	3.020	1.252	-2.721	2.933	3.849
Percentile 75	31.931	-0.434	2.973	5.437	2.381	-1.020	3.626	5.307
Percentile 5	11.977	-3.511	-0.162	0.517	-0.669	-3.972	-0.204	0.454
Percentile 95	42.996	0.477	4.341	9.649	3.340	0.347	4.330	8.288
Positive significant	30	0	17	12	11	0	19	18
Negative significant	0	10	0	0	0	16	0	0
Total significant	30	10	17	12	11	16	19	18
Percentages	100	33.333	56.667	40	36.667	53.333	63.333	60
	HND							
Percentile 25	15.886	-2.291	1.078	1.991	-0.060	-1.944	0.994	1.927
Percentile 50	19.554	-1.696	2.493	4.644	0.432	-1.354	2.481	3.522
Percentile 75	37.537	-0.510	3.309	6.270	1.088	-0.732	3.451	4.666
Percentile 5	11.692	-2.908	-0.146	0.170	-1.478	-2.629	0.109	0.333
Percentile 95	67.890	0.396	4.714	11.906	1.644	0.833	4.389	9.119
Positive significant	30	0	17	19	1	0	16	18
Negative significant	0	11	0	0	1	6	0	0
Total significant	30	11	17	19	2	6	16	18
Percentages	100	36.667	56.667	63.333	6.667	20	53.333	60
	IRL							
Percentile 25	16.916	-3.236	0.545	1.758	0.332	-3.381	1.176	1.991
Percentile 50	21.414	-2.008	2.691	5.040	1.459	-2.626	2.497	3.968
Percentile 75	27.687	-0.316	3.879	11.236	2.223	-0.757	3.674	8.828
Percentile 5	10.558	-4.318	-0.168	0.148	-0.371	-4.440	0.146	0.485
Percentile 95	34.194	0.987	5.662	20.874	3.161	0.150	4.877	10.333
Positive significant	30	0	19	18	10	0	18	17
Negative significant	0	15	0	0	0	16	0	0
Total significant	30	15	19	18	10	16	18	17
Percentages	100	50	63.333	60	33.333	53.333	60	56.667
	IRQ							
Percentile 25	10.596	-0.454	0.014	3.000	-0.055	-0.622	0.004	1.706
Percentile 50	12.504	0.275	0.439	3.649	0.284	-0.153	0.542	2.093
Percentile 75	15.807	0.642	1.168	6.010	0.620	0.522	1.217	3.062
Percentile 5	7.864	-1.288	-0.566	1.955	-0.532	-1.245	-0.536	0.997
Percentile 95	17.337	1.477	1.928	7.496	1.130	0.921	2.052	4.241

Positive significant	30	0	2	17	0	0	2	8
Negative significant	0	0	0	0	0	0	0	0
Total significant	30	0	2	17	0	0	2	8
Percentages	100	0	6.667	56.667	0	0	6.667	26.667
				ISL				
Percentile 25	15.218	-2.033	1.439	2.393	-0.608	-1.118	1.576	2.450
Percentile 50	22.446	-1.383	2.056	4.611	-0.174	-0.726	2.268	4.247
Percentile 75	26.878	-0.579	2.800	6.498	0.045	0.010	2.732	5.797
Percentile 5	10.372	-2.721	-0.606	0.749	-1.924	-1.733	-0.100	0.632
Percentile 95	32.525	1.360	4.256	14.273	0.764	1.343	3.800	8.005
Positive significant	30	0	15	17	0	0	18	17
Negative significant	0	8	1	0	2	1	0	0
Total significant	30	8	16	17	2	1	18	17
Percentages	100	26.667	53.333	56.667	6.667	3.333	60	56.667
				ISR				
Percentile 25	16.627	-2.038	0.751	1.359	0.117	-2.900	0.904	1.246
Percentile 50	23.356	-1.563	1.945	2.330	0.927	-1.583	2.207	2.703
Percentile 75	36.942	-0.083	2.628	3.774	1.853	-0.376	2.935	3.663
Percentile 5	11.188	-2.848	-0.444	0.309	-1.039	-3.424	-0.059	0.359
Percentile 95	54.479	0.531	3.698	7.627	2.503	0.430	4.105	6.452
Positive significant	30	0	15	11	7	0	18	11
Negative significant	0	8	0	0	0	12	0	0
Total significant	30	8	15	11	7	12	18	11
Percentages	100	26.667	50	36.667	23.333	40	60	36.667
				ITA				
Percentile 25	8.800	-4.508	0.540	3.082	-0.242	-3.214	0.898	1.186
Percentile 50	11.029	-3.003	2.587	7.349	0.460	-2.153	2.431	3.528
Percentile 75	14.458	-1.015	3.859	13.297	1.391	-0.747	3.873	7.678
Percentile 5	6.016	-8.592	-0.437	1.206	-1.060	-5.878	-0.167	0.522
Percentile 95	19.717	-0.277	7.323	37.304	2.315	-0.121	8.879	30.980
Positive significant	30	0	18	21	3	0	18	17
Negative significant	0	18	0	0	0	15	0	0
Total significant	30	18	18	21	3	15	18	17
Percentages	100	60	60	70	10	50	60	56.667
				SLV				
Percentile 25	15.863	-2.128	1.455	3.662	0.527	-2.936	0.995	1.016
Percentile 50	21.691	-1.308	2.396	6.120	1.274	-1.818	2.250	2.884
Percentile 75	25.586	-0.841	3.085	9.890	1.827	-0.769	3.234	4.895
Percentile 5	9.642	-3.500	-0.852	1.282	-0.509	-4.322	-0.468	0.396
Percentile 95	34.104	2.290	4.497	23.788	3.106	0.760	4.529	8.620
Positive significant	30	3	17	24	6	0	17	14
Negative significant	0	8	1	0	0	12	0	0
Total significant	30	11	18	24	6	12	17	14
Percentages	100	36.667	60	80	20	40	56.667	46.667