

Agent Based Modelling for Monetary Policy Decisions



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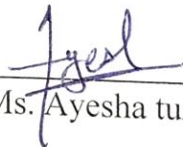
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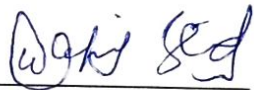
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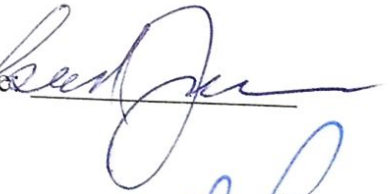
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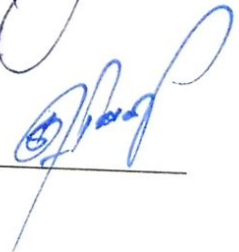
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Dedication

I dedicate this dissertation to Allah Rabb-ul-Izzat, who is also my creator, a steadfast supporter, and a source of inspiration, knowledge, and insight. Throughout this programme, He has been my source of strength, and I have only been able to fly on His wings. I also dedicate this work to my brother Abdur Rehman, who has supported me every step of the way and ensured that I put up all of my effort to complete what I have started. To my son Muhammad AbuBakr, who has been affected by this quest in every manner imaginable.

Acknowledgments

Gratitude is a debt that must be paid. In view of this fundamental truth, I've written a few words of gratitude to those who deserve it. First and foremost, I thank Allah Almighty for providing me with the strength and knowledge necessary to complete this dissertation. Knowledge is in His nature, and I am grateful to Him for sharing it with me.

Dr. Asad Zaman and Dr. Amena Urooj, my supervisors, were helpful in steering me in the proper way. I appreciate their faith in me and their careful and comprehensive scrutiny of my studies. My instructors' knowledge served as the initial stepping stone toward completing this course successfully. My faculty members' and colleagues' encouragement and collaboration provided me the confidence that I am capable of multitasking and will be able to complete the study effectively.

I owe a huge debt of gratitude to my brother, who has been my guiding light and source of strength for many years. All these years, it was his faith in me and his love for me that kept me going. I appreciate his patience with me throughout the process and for making my study endeavour a reality. I also want to appreciate my son, who just by being in my life gives me happiness, strength, warmth, and love. The greatest gift Allah Almighty has given me is you, Abu Bakr.

I am eternally grateful to all of my colleagues and friends. I pray that God, who will never be overcome by the necessity of giving, will give them with those who will aid them in whatever they are doing.

Abstract

The financial crisis of 2007/2008 sparked a wave of criticism of economic theory. These assaults are based on four basic criticisms: economists failed to predict the worst financial catastrophe since the Great Depression; authorities allowed bubbles to build unchecked; poor banking supervision; and macroeconomic policy models that were out of touch with reality. The DSGE models (dynamic, stochastic, and general equilibrium) and its two simplifying hypotheses: the representative agent and rationality, are the target in the case of macroeconomic models. Models of Dynamic Stochastic General Equilibrium (DSGE) are frequently employed as a tool for formulating policy. When these models failed to foresee the 2008 financial crisis and failed to address subsequent policy issues, they lost their prominence. Meanwhile, a DSGE model counterpart known as Agent-Based Modelling (ABM) emerged. In this study, an attempt is made to adapt a macroeconomic agent-based model of the Pakistani economy. For this, we employed an economic model for simulation and a surrogate modelling approach to investigate the parameter space of the macro agent-based model. Utilizing data from Pakistan, we fixed three model parameters before letting the remaining parameters undergo random exploration. Through an agent-based model with credit and capital, we attempted evaluate the monetary policies implemented by the monetary authority while taking into account the inter-bank market, the monetary authority combined with machine learning calibration techniques. We performed two monetary policy experiments in a calibrated agent-based model and investigated the impact on GDP, unemployment, and inflation. Our model suggests that agent-based macroeconomic models might be used to analyze monetary policy in Pakistan but there are computational limitations to addressed for better and improved simulation results. The comparison

of the effects of monetary policy on different macroeconomic variables can be done using the same paradigm.

Keywords: Financial Crisis, DSGE Models, Agent-based Modelling, Monetary Policy, Phillips Curve, Bibliometric Analysis, Surrogate Modelling

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CHAPTER 1

Introduction

Mainstream DSGE macroeconomic models failed to predict the global financial crisis and also failed to provide us with tools to address the Great Recession which followed. The basic assumptions of these models (rational expectations, representative agents, perfect competition etc) prevent the understanding of basic phenomenon underlying the crisis. On the other hand, agent based models are perfect substitute due to their extreme flexibility of set of assumptions regarding agent's behavior and interactions. In the sense that aggregate social system behaviour is the outcome of interaction and consideration at the level of individuals, monetary policy is a top-down tool whose impact is defined by bottom-up processes. A growing set of agent-based models Gatti et al. (2005); Raberto et al. (2008); Oeffner (2008); Mandel et al. (2010) used the Taylor rule to investigate the impact of monetary policy on the economy. These exercises in policy analysis are comparable to those done with DSGE models in this regard, but the complexity-rooted approach of ABM may offer fresh perspectives. Fewer central banks throughout the globe have begun implementing ABM, despite the fact that technology is ideally suited for offering insights for policy research. For Pakistan, there is no macro agent based model which makes simulations of situations more representative of real world. In order to use agent-based models (ABM) for safe policy exercise requires calibration and validation of these models to match the output of agent-based models with real world data.

1.1 Macroeconometric Models for Pakistan

A central bank, as a foresight-oriented policy agency, seeks to study trends in economic and financial variables and forecast their future course under many conceivable scenarios. The analytical framework needed to do so is provided by macroeconometric models. These models serve as a testing ground for scenario analysis and forecasting are made up of a collection of behavioural equations and definitional connections that replicate the dynamics of the real economy.

Some research groups and economists have made few attempts to construct big and medium macroeconometric models for Pakistan. A quantitative foundation for an experiment in economy-wide planning was intended by the PIDE Macroeconometric Model. Naqvi (1982); Naqvi et al. (1986). The production, spending, labour market, international trade, and fiscal and monetary sectors are among its four interconnected sub-models. The Integrated Social Policy Macroeconomic Model (ISPM), which was developed by Pasha et al. (1995), expressly acknowledged the connection between the growth of the macroeconomy and the social sector. It was regarded as good planning tool for the expansion of the social sector in order to tackle issues of income disparity, poverty, and the provision of social services.

Ten important macroeconomic variables were included in the model developed by Chishti et al. (1992), which was evaluated using vector autoregression on yearly data for Pakistan from 1960 to 1988. It empirically examined the magnitude of the expected and unexpected effects of monetary and fiscal policies, as well as shocks from external resources and remittances, on the economy. Naqvi (1982) can be regarded as the pioneering attempt within DSGE framework for Pakistan economy. A small number of studies have been done e.g. Haider & Khan (2008), Choudhri & Malik (2012), Ahmed et al. (2012), Haider et al. (2012), Choudhary et al. (2013), Ahmed & Pasha (2014) and Nawaz & Ahmed (2015). Some of them were created using a closed economy framework, while others expanded the concept to incorporate open economy characteristics.

1.2 Agent Based Models for Monetary Policy

Macroeconomic theories focus on collective behaviours that result from the micro level decisions made by specific economic players. The interactions between macroeconomic actors are ad hoc in that they merely serve as proxies for the results of individual decisions (Manfred (2003)). There have been macroeconomic agent-based models for 10 years or more. They have, for the most part, had remarkable success using generated data to replicate the empirical "stylized realities" of aggregate economic cycles and cross-sectional evidence (such as the distribution of company sizes)(Gatti & Desiderio (2015)). However, only a few studies have analyzed the effects of monetary policy, the processes of monetary transmission, and the efficacy of various monetary policy rules in these models. Using evolutionary algorithms, Gatti et al. (2005) demonstrate that an adaptable, discretionary Taylor rule outperforms a commitment method. Similar findings may be observed in Raberto et al. (2008), where an output gap-targeting policy rule outperforms a random monetary policy in both enhancing social welfare and containing inflation. When the economy is not mired in a liquidity trap, monetary policy can still be helpful, according to Oeffner (2008). A Taylor rule may raise macroeconomic volatility, according to Mandel et al. (2010). The EURACE simulator is then used to demonstrate how quantitative easing combined with an expansionary fiscal policy enhances macroeconomic performance overall but increases inflation and production volatilityCincotti et al. (2010). Although policies are accepted from the bottom up, they are executed from the top down. In order to determine which of the different approaches is most effective, this need a system knowledge at the micro level(Scharpf (1997)). Gualdi et al. (2017) examining the role and effectiveness of monetary policy of a 'Central Bank' that sets the interest rate such as to navigate the economy towards a given inflation and employment rate. The results show that Central Bank must steer in a slight opening: too little is not sufficient, too much leads to uncertainties and eagerly equivocating economies. Giri et al. (2019) discussed the execution of an Agent Based Model with a financial accelerator mechanism and the link between monetary policy and large-scale crisis events. The results show that unexpected and sharp increases of the policy rate can cause collapses. Secondly, after a crisis, recurring too soon and too fast to a normal monetary policy can create

a dual recession and the other finding show that the short-term interest rate fixed to the lower bound in the short run can effectively avoid more slowdown. Popoyan et al. (2017) develop an agent-based model to study the macroeconomic effect of another macro-prudential regulation and their probable connections with different monetary policy rules. The results show that a triple-mandate Taylor rule, intensive on production gap, inflation and credit growth. Provident regulation is the best policy mix to expand the stability of the banking sector and smooth output variations and the presence of an additional control does not always expand the performance of the macro-prudential regulation.

1.3 Research Gap

DSGE models are subject to various challenges, including issues with their internal coherence, their applicability to real-world data, their ability to replicate real-world phenomena, and the realism of their assumptions. In order for a policy model to be effective, it should be able to explain the reasons for the success or failure of policy interventions. While agent-based simulation models have the potential to be flexible and adaptable to a range of complex situations, they have not yet been widely used to guide policy decisions. Despite this, an increasing number of academics are utilizing AB models as a tool for assessing economic policy initiatives and making policy recommendations.

Currently, few countries analyze monetary policy using agent-based models. However, the Bank of England has recently collaborated with the University of Oxford and the Institute of New Economic Thinking to use ABM as a tool for policy regulations. In Pakistan, no attempts have been made to calibrate a macro agent-based model for policy experimentation. Our study aims to fill this gap by calibrating a pre-designed MABM to represent Pakistan's economy and investigate the effects of monetary policy on inflation and unemployment.

Assenza et al. (2015) presented an agent-based model with capital and credit markets, which was later extended by Silva et al. (2019) to include a monetary authority that follows the Taylor rule for monetary policy decisions. In our study, we use this extended macro agent-based model with a monetary authority to investigate the macroeconomic performance of Pakistan. The baseline model is calibrated using parameter values suggested by John Taylor, but it is un-

clear whether these values are applicable to Pakistan, as the State Bank of Pakistan does not formally accept any association to the rule.

To address this issue, we used modified the Taylor rule with lagged interest rates and examined its effects on the overall performance of the agent-based economy. We then calibrated agent-based model in such a way that it is adapted for Pakistan economy. For the purpose, we used the distribution of interest rate series of simulated data matches the distribution of real-world data. All parameter settings that provide positive calibrations were then empirically validated to minimize the gap between simulated and real-world data. The highlight of our study is the adaptation of an agent-based model to investigate the effects of monetary policy on the Pakistani economy, and to assess the suitability of the Taylor rule for this context. Through this adaptation of model, we contribute to the existing literature on agent-based modeling and provide valuable insights for policymakers.

1.3.1 Motivation of Study

Traditional economic tools may not adequately address certain aspects of the human experience, as identified by Tessaromatis (2018). One of these is irreducibility in computing, meaning that the behavior of a complex system cannot be fully reduced to a mathematical description that can accurately predict future outcomes. Another is emergence, which occurs when the collective behavior of individuals in a system produces phenomena that cannot be predicted based solely on the actions of individual agents. The third characteristic is non-ergodicity, which refers to the fact that the characteristics of a system may change over time, making it difficult to predict future outcomes based on past behavior. Finally, radical uncertainty refers to unexpected events or outcomes that cannot be fully accounted for in a probability distribution.

Given these complexities, agent-based simulations can offer a unique way to model and understand the economy, particularly in times of crisis. By allowing for flexible heuristics and interactive behaviors among agents, agent-based models can provide policymakers with more nuanced and realistic assessments of the economy, including the impact of central bank decisions on the broader economy. Furthermore, such models are crucial for developing and improving markets by identifying key features and dynamics that may not be captured by more

traditional economic tools.

1.3.2 Research Objectives

Summary of research goals;

- Investigate the advancements and potential applications of agent-based modeling in economics.
- Analyze the impact of monetary policy decisions on inflation and unemployment in the Pakistani economy using an agent-based model.
- Evaluate the effects of introducing a smoothing term in the monetary policy rule on the economy's response to policy changes.

1.4 Significance of Study

Agent-based modeling is a relatively new approach to modeling economic systems, but there is growing evidence to suggest that it is an effective and promising tool for understanding complex phenomena. However, despite the potential benefits of this approach, there are currently no macro-level agent-based models for policy analysis in the Pakistani economy or even in the South Asian region. This means that our research represents an important first step in applying this modeling approach to the Pakistani economy.

One of the strengths of agent-based models is that they allow for more direct comparisons between simulated and real-world data, going beyond just matching stylized facts between the two. By analyzing macro agent-based models calibrated on real-world settings, researchers can better understand how the economy has evolved over time and identify key factors that have influenced its behavior. This, in turn, can help policymakers make more accurate predictions about the effects of policy choices and develop more effective strategies for managing the economy.

By calibrating a macro agent-based model for Pakistan, we can simulate the effects of different monetary policy decisions on important macroeconomic variables such as inflation and unem-

ployment. The results of these simulations can be compared to actual data, which increases our confidence in the model's accuracy and usefulness. Policymakers can then use these simulations to better understand how different policy choices will affect the economy and make more informed decisions as a result.

Additionally, by introducing a macro agent-based model for policy actions in Pakistan, our research will open up avenues for further exploration and research in this domain within the country. This can lead to a better understanding of the economy and policy decisions, and ultimately result in more effective policymaking. It is our hope that this study will be a step towards establishing agent-based modeling as a useful tool for policymakers in Pakistan and the broader South Asian region.

1.4.1 Agent-Based Models: A Paradigm Shift in Economic Modeling

In today's economic modeling landscape, there is a growing recognition of the shortcomings of DSGE models (dynamic, stochastic, and general equilibrium). Their reliance on oversimplified assumptions, particularly the representation of agents by a single representative entity in a world characterized by heterogeneity, and the assumption of rational behavior, is a source of concern. These constraints, identified by the Sciences of Complexity and emphasized by behavioral and neuroeconomics, highlight the need for a paradigm shift.

Agent-based models emerge as a timely and necessary alternative to DSGE models, addressing their shortcomings. Fagiolo & Roventini (2012b) described ten key characteristics of agent-based models in Economics. These include a bottom-up perspective that derives aggregate properties from individual behavior, recognition of heterogeneity among agents, recognition of economic systems as evolving complex entities over time, incorporation of non-linear interactions and feedback loops, direct endogenous interactions among agents, acceptance of bounded rationality given the inherent complexity of systems, consideration of the adaptive nature of learning as agents respond to changing conditions, and consideration of the adaptive nature of learning as agents respond to changing conditions.

This paradigm shift toward agent-based models is critical because it more closely aligns with the principles of the Sciences of Complexity as well as the empirical evidence provided by

behavioral and neuroeconomics. Agent-based models provide a more robust framework for understanding and simulating real-world economic phenomena by embracing the multifaceted nature of agents and recognizing the dynamic, non-linear, and evolving aspects of economic systems.

1.4.2 Contribution of Study

Our study has made a significant and innovative contribution to the realm of agent-based modeling in the context of economics. We have undertaken two crucial advancements that significantly enhance the applicability and effectiveness of agent-based modeling in economic scenarios.

Firstly, we have successfully adapted a foundational simulation model that facilitates the thorough evaluation of diverse monetary policy scenarios through an agent-based modeling framework. This adaptation enables comprehensive comparisons between different monetary policy strategies, providing a more nuanced understanding of their potential outcomes. Secondly, our work introduces an innovative modification to the approach proposed by Lamperti et al. (2018). This modification integrates meta-modeling machine learning surrogates, an unexplored strategy within the domain of agent-based modeling in economics. This pioneering enhancement effectively facilitates parameter exploration, streamlining the intricate task of navigating the parameter space in agent-based models. By anchoring specific parameter values, we establish a targeted approach that yields more precise and insightful results. This novel technique holds the potential to reshape the comprehension of complex economic systems. Concurrently, as part of this process, a minor adjustment was implemented before initiating the random parameter exploration. We extracted adapted equations from a macro agent-based model and proceeded to estimate these equations using real-time data specific to Pakistan. During this procedure, we held constant the parameter values that exerted a direct influence on agents' behavior. Subsequently, akin to the approach outlined by Lamperti et al. (2018), we conducted analogous calibration procedures for the remaining variables. In essence, we embraced a concept akin to a combination lock, in which one digit remains fixed, while the other two undergo random exploration. This strategic maneuver facilitated the identification of a vector of parameters that

collectively serve as the key to unlocking the macro agent-based mechanism for Pakistan economy.

A central aspiration of our research is the creation of a macroeconomic model supported by micro-foundations, which offers an encompassing framework for policy analysis and broader macroeconomic inquiries. Significantly, our endeavors encompass the tailoring of agent-based modeling to align with the distinctive characteristics of the Pakistani economy and the unique research questions it poses. This facet is of paramount importance, as policy experimentation within any domain necessitates the development of an agent-based model finely attuned to the specific economic landscape. Consequently, our primary contribution lies in the calibration of a macro agent-based model meticulously customized to the nuances of the Pakistani context. This calibration empowers researchers and policymakers with localized insights, thereby fostering a deeper understanding of the intricate systems at play within the country.

1.5 Construction of Thesis

The following is a synopsis of the chapters to come.

Chapter 2: We adopted a computer-assisted scientific literature review technique. This bibliometric analysis, which includes research from 2000 to 2020, helps to emphasise a division that is still the focus of various ongoing inquiries. This is because, while academics in the field continue to describe ABM as a new and fascinating study topic, there is still a lack of a standard and clear grasp of the notion. Because it is still a work in progress, ABM research continues to flow, and researchers have been employing the technique to tackle policy concerns for the past two decades. In economic analysis, agent-based models have gained a lot of momentum. In addition, the number of papers published in Economics journals has increased. But it is still a long way from becoming a mainstream methodology, despite the fact that it is now being referred to and considered by an increasing number of economists – some of whom are well-known 'names' in traditional paradigms – and it is increasingly being used in a variety of topics and appearing in journals to which they had virtually no access until recently.

Chapter 3: FPAS refers to the central bank's approach to the economy. Rather of being a single model, it is a collection of them. The primary purposes of FPAS models (in general) are

to provide information for monetary policy decisions and to aid those decisions by structuring the analysis. In the 1960s, the Phillips curve was heralded as giving a model of inflation that had previously been omitted from traditional macroeconomic models. The Phillips curve, as changed by the natural-rate hypothesis into its expectations-augmented variant, remains the cornerstone in mainstream macroeconomic theory for connecting unemployment to inflation after four decades. Using quarterly data, we investigated the non-linearity of the Phillips Curve in the instance of Pakistan in this chapter. The policy implications of nonlinearity are also examined.

Chapter 4: Parameter estimation is the most important task in modelling complicated dynamical systems. This is an optimization task that entails a large number of evaluations of a computationally costly objective function. Surrogate-based optimization employs a computationally efficient predictive model to approximate the objective function's value. The chapter offers a description of the agent-based model we implemented, as well as the meta model that encapsulates the objective function, surrogate model, and replacement strategy model, as well as learning components.

Chapter 5: Provides a concise response to our research question(s) as well as a summary of the major findings. We reviewed the limits of our research in this chapter and made recommendations for further research.

CHAPTER 2

Bibliometric Analysis of Agent Based Modelling in Economics

Modelling is defined as a mathematical and statistical way of reproducing events and their possible consequences due to policy decisions (Shanahan et al., 2016). In econometric modelling, models formed on a theoretical framework are constructed using one or many exogenous variables, identifying quantitative relationships which generate various responses. Dynamic Stochastic General Equilibrium (DSGE) models have ruled as a tool for policy decisions. Then the 2008 financial crisis became the downfall of this modelling technique because of "no response" to policy problems afterwards. These models cannot predict a crisis or any non-linear event. Meanwhile, the Agent-Based Modelling (ABM) approach emerged as an alternative to DSGE models. ABM emergence property allows foreseeing complex behaviours. The Great Recession made policymakers see the "economy as [a] complex evolving system" consisting of heterogeneous agents and non-equilibrium state continuously change the economy's structure.

Bibliometric analysis is a method to explore the information-rich environment on research activities and findings extracted through data from research publications in academic journals and their citations. Bibliometric indicators help investigate the knowledge structure of a particular field and its scope in the future. In the framework of research developments, questions like "where are we now?" and "where will we be in the future?" are answered by this form of

Published article "Zehra, Ayesha, and Amena Urooj. 2022. A Bibliometric Analysis of the Developments and Research Frontiers of Agent-Based Modelling in Economics. *Economies* 10: 171. <https://doi.org/10.3390/economies10070171>" is an excerpt of the chapter.

analysis. There are two types of bibliometric analysis techniques: (1) performance analysis and (2) science mapping. In essence, performance analysis considers the contributions of research parts, whereas science mapping considers the relationships among them.

This chapter is dedicated to exploring the research development of agent-based modelling in economics using bibliometric analysis techniques. This aim is to use bibliometric analytic approaches to investigate the knowledge base of agent-based modelling in economics. We used both performance analysis metrics and scientific mapping methodologies to achieve the study's research aims. Also, ABM in economics is examined in terms of its conceptual and social structure.

2.1 What ABM Has and What DSGE Lacks

The primary instrument for generating policy judgments remains DSGE models. These models did not predict the financial crisis of 2008. These models' limitation in addressing many policy concerns is their data-driven approach and lack of macroeconomic data. These models' openness and transparency is a strength, but it also leaves them vulnerable to criticism. It is possible to draw attention to suspicious assumptions. Evidence of inconsistencies are readily apparent. It is possible to identify elements that are not included in the model (Christiano et al., 2018). DSGE modelling is a death tale that has been predicted. Joseph Stiglitz writes, "...much of the main parts of the DSGE model are defective—sufficiently seriously wrong that they do not give even a good starting point for creating a good macroeconomic model" (Stiglitz, 2018). Vines and Wills want DSGE models to be able to achieve what they desire, which is to allow modellers to get a quick look at crucial issues. Central banks routinely use estimated DSGE models for forecasting and quantitative policy analysis. Estimating these models and interpreting the results to formulate policy are both difficult tasks (Schorfheide, 2011). The issues and flaws of DSGE models are similar to those of generalised equilibrium (GE) models. Many researchers have discovered that the agents in DSGE models cannot be constrained so that their uniqueness and stability are preserved.

Assuming individual rationality does not automatically imply aggregate rationality, however. The representative agent assumption in these models is unreliable for policy analysis

because individual responses to shocks or parameter changes could not mirror aggregate responses. In DSGE models, solving systems of equations can lead to another difficulty of identification, resulting in skewed estimates of some structural parameters and raising doubts about statistical significance. This modelling technique cannot predict infrequent economic crises, which is not surprising given that fat tail densities are approximated distributions of macroeconomic time series (Fagiolo et al., 2008), and Gaussian distributed shocks are a typical assumption in DSGE models. The assumption that Representative Agents (RAs) are rational prevents these models from addressing distributional issues because it implies that one: agents are fully aware of the economy; two: agents are capable of understanding and solving any problem they encounter without making mistakes; and three: agents are aware that all others follow the same pattern. These issues demonstrate that DSGE models are ill-equipped to solve policy concerns and cannot forecast future crises. The DSGE approach is so enthralled by its internal logic that it confuses the model's precision with the real one. (Caballero, 2010).

ABM has evolved rapidly in economics over the previous two decades. Due to the following features of this modelling method;

- Bottom-up Perspective

A bottom-up approach is needed to address a decentralised economy in a way that is satisfying. In other words, aggregate features must result from a potentially uncontrolled micro dynamics occurring at the level of fundamental entities (agents). Contrast this with standard neoclassical models' top-down structure, where the bottom level often consists of a representative person and is restricted by stringent consistency criteria related to equilibrium and hyperrationality.

- Heterogeneity

Agents can vary from one another and are explicitly modelled. While in analytical models it might be beneficial to reduce the ways in which people can differ, with ABMs it is feasible to define various values of the parameters (such as preferences, endowments, location, social contacts, abilities, etc.) for different people owing to computational improvements. Typically, this is accomplished by selecting an appropriate distribution for each pertinent parameter, resulting in the addition of a small number of parameters (those

creating the distribution) to the model.

- A complex system approach that is always evolving

Agents exist in intricate systems that change throughout time. Therefore, rather than being a result of the modeler's consistency demands for rationality and equilibrium, aggregate features are believed to arise via recurrent interactions between basic entities.

- Non-linearity

In AB models, there are intrinsically non-linear interactions. Between micro and macro levels, non-linear feedback loops can exist.

- Endogeneity vs direct interactions

The direct interaction of agents. Through adaptive expectations, an agent's decisions today are directly influenced by the decisions other agents in the population have made in the past. Socioeconomic systems are by nature unstable. Economic systems are constantly being exposed to novelty, and new behavioural patterns are being created, both of which are powerful forces for learning and adaptation. Agents must therefore deal with "real (Knightian) uncertainty" (Knight 1921), which limits their ability to fully establish expectations about, say, technical results.

- Rationality with bounds

It is typically simpler to implement some type of optimum behaviour in models with general equilibrium solutions than it is to solve models where people make judgments based on acceptable rules of thumb or by observing what others have done. In ABMs, bounded rationality¹ comes into play because agents make decisions based on straightforward heuristics based on local information because they lack either perfect knowledge of how the system where they live functions or infinite computing power to process all the information that is available².

¹In contrast to the neoclassical theory of limited maximising, bounded rationality may be seen of as an alternative behavioural paradigm for economic actors. In fact, behaviour of agents often relies on heuristics, or relatively easy principles of decision-making, in order to make a satisfactory conclusion in a complicated environment with limited and partial information. Additionally, agents may pick up knowledge by their actions, interactions with other agents, and observations of their surroundings. When considering the economy as a whole, we must take into account that agents can interact with other agents directly when making decisions or discovering how the economy functions. At the level of the entire system, the interaction of diverse entities can result in complicated dynamics. The macro level can therefore be distinct from the basic aggregate of micro entities.

²The conventional technique confronts significant difficulties, even when only a little amount of information is taken into consideration. According to the mainstream approach, information is comprehensive and available to all

- Agents' ability to learn

Agents in AB models participate in open-ended exploration of settings that are constantly changing. This is a result of the formation of new behavioural patterns and the continual introduction of novelty, as well as the intricacy of the interactions between diverse agents.

- a market mechanism that is based on selection

Typically, agents go through a selection process. Consumers, for instance, choose the products and services offered by rival businesses. The selection criteria that are employed may also be intricate and multidimensional.

These models are computer simulations that use a top-down strategy to investigate developing dynamic patterns. Policies and the social behaviours that result from them act like a weather system constantly battered by storms and invasions. The ability to make large-scale modifications and crash systems are inherited. External disturbances throw the equilibrium condition off. ABM allows little effects like herding and fear driving bubbles and crashes to be amplified via feedback processes. Models are non-linear in mathematical terms, implying that the result may not be proportional to the cause. The capacity to represent emergent phenomena resulting from the interaction of each agent is a major advantage of this modelling technique. Emergent phenomena can have traits that are opposed to those of their constituents. Agent-based models are a natural way to describe a system of behavioural elements. ABM can explain that designing a virtual agent with a shopping basket is more natural than describing average effects using a synthetic basket density. The flexibility of these models allows for the addition of new agents and changes in behaviour, learning, evolution, and complexity by altering interaction rules.

2.2 Agent-Based Models and Macroeconomic Policies: A Review of Recent Findings

Recent studies have attempted to use the ABM approach to solve macroeconomic policy challenges. Napoletano et al. (2012) divided them into three categories of policy: independence of the central bank, bank regulation, and fiscal and monetary policies. With regard to fiscal policies as a useful instrument to combat economic downturns, the Great Recession has

agents. One of the fundamental tenets of the Walrasian tradition is that any sort of strategic action is disallowed and that the market, through the auctioneer, should be allowed to gather all available data. In reality, one may interpret the "revolution" in rational expectations as an effort to decentralise the auctioneer-led price-setting process.

reignited interest in them. By fusing Keynesian theories of demand creation and Schumpeterian ideas of technology-fueled economic growth (the K+S model), Dosi et al. (2010) explore the function of fiscal policy in a two sectors ABM. This model's study of the short- and long-term effects of fiscal policies is one of its innovative features. A basic duality about the efficacy of policies is included in DSGE models. More specifically, only supply-side policies may have real impacts over the long term; fiscal and monetary policies only have short-term effects on real variables. In an ABM developed by Russo et al. (2007) a population of heterogeneous, boundedly rational companies and consumers/workers engage in random matching protocols-based interactions. The model simulates firm growth-rate distributions, Okun and Phillips curves, as well as small- and large-scale regularities such fluctuating sustaining growth. On the policy front, they discover that if tax revenues are used to finance R&D investment, the average output growth rate is non-monotonically correlated with the tax rate assessed on corporate profits, whereas growth is adversely impacted if tax revenues are used to pay for unemployment benefits. In an ABM with the presence of families, businesses, banks, the government, and a central bank, Haber (2008) investigates the effects of various fiscal and monetary policies under various expectation creation mechanisms. The model is calibrated to produce "realistic" time series for GDP, unemployment, inflation rate and consumption. The approach then introduces positive fiscal (lower tax rate) and monetary shocks (higher money target). The paper's main finding is that both strategies boost GDP growth, lower inflation, and decrease unemployment. More complex agent expectations enhance the influence of monetary policy while decreasing the impact of fiscal policy. Dosi et al. (2011) expand the K+S model by include a bank that takes deposits from businesses and awards expensive loans to financially strapped businesses in a pecking order. The model is then used to evaluate the impact of monetary policy in various scenarios of income distribution. According to the results of the simulation, more income disparity increases production volatility, the unemployment rate, and the chance of major crises. The distribution of income affects how well monetary policy works: higher interest rates have no effect on the economy's dynamics until the level of economic inequality reaches an endogenous threshold, above which the average growth rate of the economy declines and the amplitude of fluctuations, the unemployment rate, and the probability of a crisis increase. Ashraf et al.

(2016) investigate the consequences of inflation on actual activities (1998). Boundedly rational agents trade a variety of items in the model, and specialist merchants mediate exchange. According to this model, high inflation rates cause both a decrease in mean GDP growth and an increase in GDP volatility. The cause is that greater price dispersion is likewise linked to higher mean inflation. In turn, greater price dispersion causes the demand that single traders confront to be more volatile, which has the effect of raising the rate at which traders file for bankruptcy. When inflation is strong, it has the effect of changing how the entire exchange system operates in the economy.

ABMs have also been used to research political economy problems pertaining to the institutional function of central banks and the public dissemination of monetary policy. By using an ABM in which heterogeneous nations select whether to implement central bank independence taking into consideration the behaviour of their neighbours, Rapaport et al. (2009) examine why throughout the 1990s many governments elected to cede authority to their central banks. The extent of the zone of influence of surrounding nations is positively correlated with the appearance and rate of adoption of central bank independence, according to simulation results carried out under a Monte Carlo exploration of the parameter space. Arifovic et al. (2010) use an ABM to investigate the time-inconsistency issue that central banks confront, where the interaction between a boundedly rational central bank and a population of heterogeneous agents determines the real inflation rate. The agents have two options: they may either accept the central bank's inflation rate announcement or use an adaptive learning strategy to predict inflation in the future. The central bank learns to maintain an equilibrium with a positive but varying proportion of "believers," and computer simulations of this model demonstrate that this result is Pareto superior to the equilibrium generated by traditional models.

2.3 Bibliometric Analysis: A Technique of Systematic Literature Review

One of the most important knowledge discovery methods is synthesising the results of earlier studies. The use of bibliometric analysis is growing in popularity (Zupic & Čater, 2015). In a qualitative study of published research papers, journals, and books, the bibliometric technique has been employed (Ellegaard & Wallin, 2015). It aids in the identification of frequently refer-

enced authors and institutions, related publications, and the keywords most commonly used in a given study filed (Daim et al., 2006). Furthermore, bibliometric analysis can be used to assess the publication's popularity among specialists and verify the author's reputation (R. Ball & Tunger, 2005). It also aids in literature review by leading the researcher to influential research works or publications, as well as objectively mapping the study field (Zupic & Čater, 2015). (Donthu et al., 2021) discussed in detail the methodology to conduct bibliometric analysis and concluded that bibliometric analysis can aid knowledge generation not just in business research but also in other sectors, thanks to a better comprehension of science. Bibliometric approaches are used for a variety of purposes, including performance analysis and science mapping (Cobo et al., 2011). Performance analysis is used to assess individual, institutional, and individual research and publishing performance. A generic approach of domain analysis and visualization is science mapping. A scientific discipline, a field of research, or topic areas related to specific research topics can all be included in the scope of a science mapping study. In other words, an area of scientific knowledge expressed through an aggregated collection of intellectual contributions from members of a scientific community or more clearly defined specialty is the unit of analysis in science mapping (C. Chen, 2017). It is worth mentioning that the advent of scientific databases like Scopus and Web of Science has made it relatively simple to acquire large volumes of bibliometric data, and bibliometric software like Gephi, Leximancer, and VOSviewer allows the analysis of such data in a very practical way. This has led to a recent increase in scholarly attention in bibliometric analysis. Citation analysis, co-citation analysis, bibliographic coupling, co-word analysis, and co-authorship analysis are some of the methodologies utilised in science mapping. Such methodologies are beneficial for illustrating the bibliometric and intellectual structure of a study field when combined with network analysis (Tunger & Eulerich, 2018; Baker et al., 2020). It is crucial to contrast bibliometric analysis now with other regularly employed review alternatives like meta-analysis and comprehensive literature reviews. In essence, systematic literature reviews, such as domain-, method-, and theory-based reviews, embody the acquisition, arrangement, and assessment of the existing literature using systematic procedures that are typically carried out manually. In contrast, meta-analysis estimates (1) "the overall strength and direction of effects or relationships," and (2) "the across-study variance in

Bibliometric analysis	
Main techniques	
<p>Performance analysis</p> <ul style="list-style-type: none"> Publication-related metrics <ul style="list-style-type: none"> Total publications (TP) Number of contributing authors (NCA) Sole-authored publications (SA) Co-authored publications (CA) Number of active years of publication (NAY) Productivity per active year of publication (PAY) Citation-related metrics <ul style="list-style-type: none"> Total citations (TC) Average citations (AC) Citation-and-publication-related metrics <ul style="list-style-type: none"> Collaboration index (CI) Collaboration coefficient (CC) Number of cited publications (NCP) Proportion of cited publications (PCP) Citations per cited publication (CCP) <i>h</i>-index (<i>h</i>) <i>g</i>-index (<i>g</i>) <i>i</i>-index (<i>i</i>-10, <i>i</i>-100, <i>i</i>-200) 	<p>Science mapping</p> <ul style="list-style-type: none"> Citation analysis <ul style="list-style-type: none"> Relationships among publications Most influential publications Co-citation analysis <ul style="list-style-type: none"> Relationships among cited publications Foundational themes Bibliographic coupling <ul style="list-style-type: none"> Relationships among citing publications Periodical or present themes Co-word analysis <ul style="list-style-type: none"> Existing or future relationships among topics Written content (words) Co-authorship analysis <ul style="list-style-type: none"> Social interactions or relationships among authors Authors and author affiliations (institutions, countries)
Enrichment techniques	
<p>Network analysis</p> <ul style="list-style-type: none"> Network metrics <ul style="list-style-type: none"> Degree of centrality Betweenness centrality Eigenvector centrality Closeness centrality PageRank Clustering <ul style="list-style-type: none"> Exploratory factor analysis Hierarchical clustering Island algorithm Louvain method Multidimensional scaling Simple centers algorithm Visualization <ul style="list-style-type: none"> Bibliometrix R Bibexcel Gephi Pajek UCINET VOSviewer SciMat Sci2 	

Figure 2.1: In the article "How to do a bibliometric analysis: An overview and guidance," Donthu et al. (2021) provided a procedure for carrying out bibliometric analyses.

the distribution of effect-size estimates and the factors that explain this variance”Donthu et al. (2021). Although bibliometric analysis is a useful technique for summarising and synthesising literature, it is vital to recognise that it has several drawbacks. First of all, because they are not developed solely for bibliometric analysis, bibliometric data of scientific databases like Scopus and Web of Science might have inaccuracies, which will inevitably damage any study that uses them. Scholars must thoroughly clean the bibliometric data they obtain, which includes deleting duplicates and incorrect entries, in order to reduce mistakes. Second, the bibliometric methodology’s very nature is a constraint. Given that bibliometric analysis is quantitative in nature and that the link between quantitative and qualitative outcomes is sometimes ambiguous, in particular, the qualitative statements of bibliometrics can be highly arbitrary (Wallin (2005)). In this sense, researchers should exercise extreme caution when making qualitative claims regarding bibliometric observations and, if necessary, support them with content analysis (Rudd & Whelan (2005)). Third, since bibliometric studies can only provide a short-term projection of the research field, academics should refrain from making overly optimistic claims about the research field and its long-term effects (Brzoza-Brzezina et al. (2021)). Despite these drawbacks, the bibliometric technique can enable academics to conduct ambitious retrospectives of business research by empowering them to get over their anxiety of working with huge bibliometric datasets. Indeed, bibliometric analysis can help create new knowledge in economics by enhancing our comprehension of science.

2.4 Study Design

The goal of this study was to fulfil two purposes. The first was to determine research trends over time, and the second was to investigate research content in order to assess the application of agent-based modelling techniques in various economic sectors. Hence we combined both bibliometric methodologies; performance analysis and science mapping to reach our objectives.

Research Objectives

The analysis is constructed in such a way as to achieve the following goals.

- Identifying the knowledge base of agent-based modelling and its intellectual structure

particularly in economics.

- Examine the research front/conceptual structure of agent-based modelling in context of monetary policy.
- Exploring the social network structure of agent-based modelling in economics.

Research Design

Many methods are described in section 2.3 of the bibliometric analysis. Citation and co-citation analysis (by author and journal), co-word analysis, and network analysis were among the bibliometric methodologies we considered.

Bibliometric Data Collection

The Scopus database offered a total of 1568 documents for examination. A sophisticated keyword selection was required for data extraction in order to provide a relevant set of data. The keywords chosen must match the following four criteria: high search volume, relevancy, high conversion value, and low competition. The search terms “agent-based modelling” AND (“DSGE” OR “monetary policy” OR “crisis” OR “central banks”) were used to extract bibliometric data.

Inclusion Criteria

After filtering the data based on the inclusion and exclusion criteria, a total of 1568 data were gathered. Three inclusion rules were followed: (1) Articles in which one of the keywords appears in the title, abstract, or keywords (2) The publication date ranges from 2000 to 2020. (3) Journal articles, conference papers, and book chapters. If they met all inclusion criteria, English language abstracts were included in the bibliometric review.

Exclusion Criteria

All documents with a core subject of agent-based modelling but not relevant to the field of economics were left out of the analysis.

Methodology and Software

We used the Scopus dataset in the analysis and the approaches listed above to answer our research questions. R was chosen as software for both visuals and quantitative analysis.

2.5 Knowledge Base of ABM: Results and Findings

2.5.1 Productivity Assessment

After the financial crisis of 2008, the agent-based model became a widely researched topic. The crisis was not predicted by DSGE models, and they also did not respond to policy questions. Since the most publications were in 2020, ABM has become a new topic in its development. ABM is also the ideal tool for experimenting with different policy scenarios during a pandemic (Figure 2.2). Countries such as the United States have played a critical role in the field's contin-

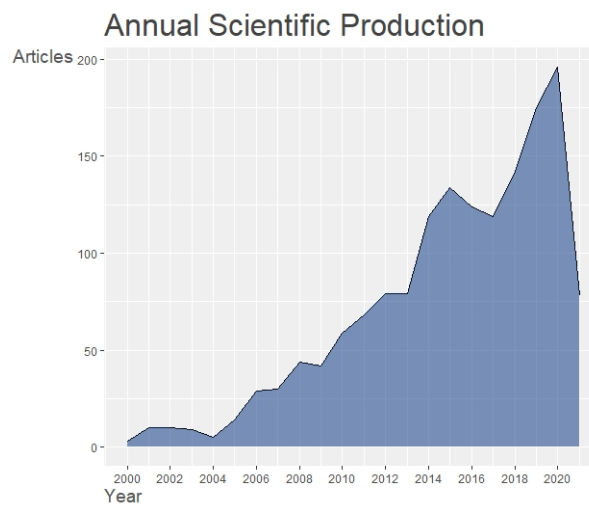


Figure 2.2: Annual scientific output on economics-related ABM: A Scopus database analysis (2000–2020).

uous progress. The authors are working with researchers from the same country as well as from other ones. The top nations and authors working on the issue of agent-based modelling in economics are shown in Figure 2.3. According to the findings, American researchers are putting a greater emphasis on this modelling technique and examining its possibilities for solving difficult challenges. Researchers prefer to collaborate with researchers from their own nation rather than researchers from other countries, according to stacked bar charts. Wilensky, being the most active contributor, also described how to use agent-based simulations to answer complex questions. His writings capture the thrill of re-creating social phenomena in computer simulations to better understand them (Wilensky & Rand, 2015) (See Figure 2.4). In Figure 2.5, we can see that in which year the authors were most productive and long lived. There are authors from economics who were working on agent-based modelling before the financial crisis took place.

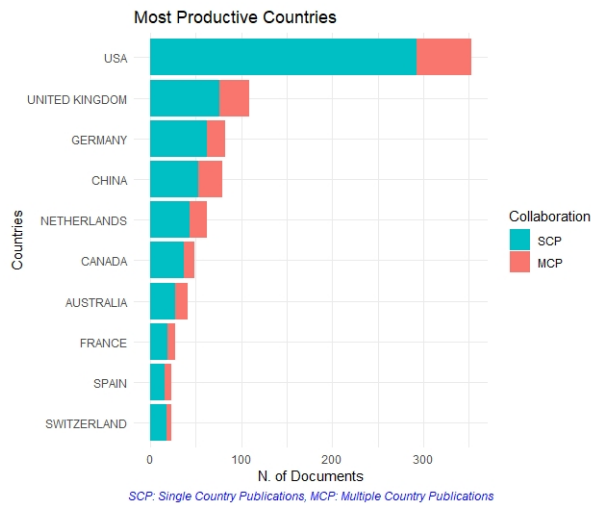


Figure 2.3: Ranking of countries based on scientific output on economics-related ABM (2000–2020).

But the focus of their research was not directed to answer the policy questions to overcome the aftershocks of crisis. Lotka’s Law is one of the most fundamental bibliometric rules, and it

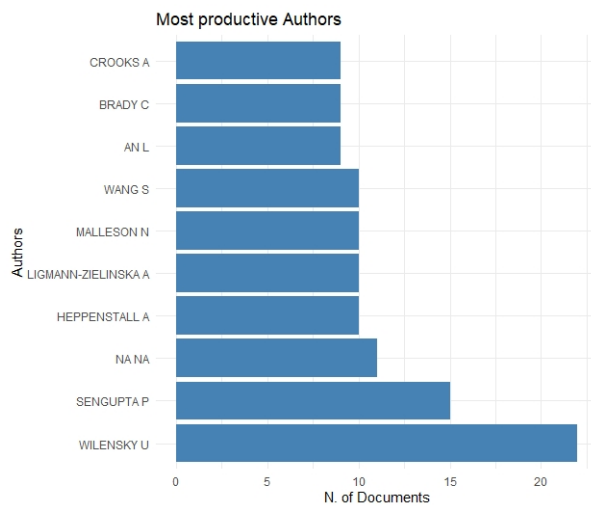


Figure 2.4: Ranking of authors based on scientific output on economics-related ABM (2000–2020).

deals with the frequency with which authors in a specific subject publish. The frequency of publishing by authors in a specific field is described by Lotka’s law, presented as follows:

$$f(x) = \frac{\beta}{x^\alpha}$$

where $f(x)$ is the frequency of authors having x publications, and x is the positive integer, representing number of publications. The parameter estimates of Lotka’s Law are $\beta = 2.82$ and

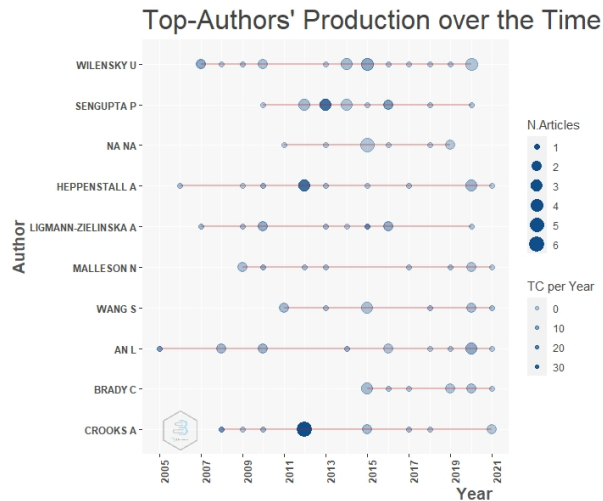


Figure 2.5: An overview of the author’s output over the years (2000–2020).

$\alpha = 0.97$. According to the findings, there are 2833 writers with a single ABM publication in economics. There are approximately 70 authors with at least five published works. Over 20 documents were released by only one contributor. Theoretical and observed frequency are depicted graphically in Figure 2.6. A journal’s impact on a specific study topic can be mea-

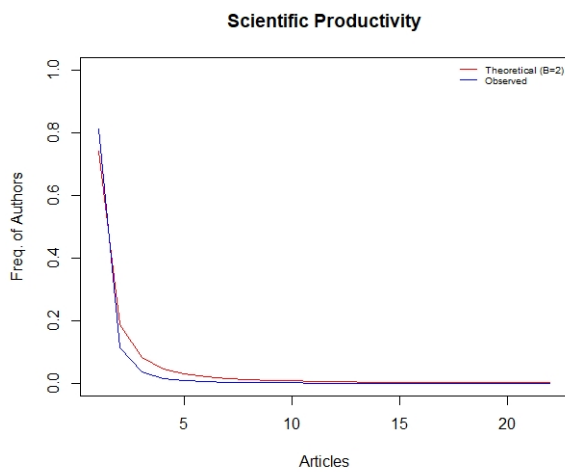


Figure 2.6: Lotka’s law of scientific productivity from 2000–2020 (authors publishing on economics-related ABM).

sured by its publications. It can be seen from Table 2.1 regarding ABM research in economics, Journal of Artificial Societies and Social Simulations (JASSS) is head and shoulders above in the league table. JASSS has the highest number of publications on the theme of agent-based modelling in Scopus index journals followed by Computer environment and urban system on the league table.

Table 2.1: Most Relevant Sources.

SR No	Sources	Publications
1	JASSS	210
2	COMPUTERS ENVIRONMENT AND UR- BAN SYSTEMS	42
3	SUSTAINABILITY (SWITZERLAND)	36
4	INTERNATIONAL JOURNAL OF GEO- GRAPHICAL INFORMATION SCIENCE	24
5	SOCIAL SCIENCE COMPUTER REVIEW	21
6	ENVIRONMENT AND PLANNING B: PLANNING AND DESIGN	20
7	TRANSPORTATION RESEARCH PART C: EMERGING TECHNOLOGIES	19
8	TRANSPORTATION RESEARCH PROCE- DIA	19
9	JOURNAL OF INDUSTRIAL ECOLOGY	15
10	ISPRS INTERNATIONAL JOURNAL OF GEO-INFORMATION	14

The sum of published documents on ABM in the top five journals is around 450 in ten years.

As outlined in Table 2.2, authors are ranked via the Dominance Factor (DF). (Kumar & Kumar, 2008) developed the formula as,

$$DF = \frac{\text{num of multi-authored publications of an author as first author (Nmf)}}{\text{total num of multi-authored publications (Nmt)}}$$

The value of dominance factor indicates collaboration in the field. A value less than 0.5, reflects a good sign for collaboration. Authors who have published nine or more publications on the theme of agent-based modelling are selected and their dominance factor is calculated by using the above formula. Sengupta and Wilensky are top authors with respect to publication number, i.e., 15 and 22, but they rank 6th and 10th, respectively. If the authors' dominance factor values are less than 0.5, this is a good sign of collaboration.

2.5.2 Importance Assessment

Table 3.1 is about author level metrics based on three indices. *H-index* measures the productivity as well as the impact of publication. The H-index is calculated as “author has H publications, and each publication has H or more citations”. Whereas *g-index* is one variant of H-index which gives credits for highly cited authors in the data set. (Hirsch, 2005), its inventor says: highly cited papers play a key role in the determination of H-index. The selected paper for the top h category are then dropped for the further determination of the H-index over time. This means that the H-index of the subsequent years are not influenced by the papers of the top category, even if the number of citations increases over time. Value of g-index is always either equal to or greater than H-index.

The m-index represents a modified version of the h-index, which showcases the h-index on a yearly basis from the year of the researcher's initial publication. While the h-index tends to naturally rise with the length of one's career, the m-index addresses this limitation, particularly when comparing researchers who have varying career durations within the same field. It's worth noting that the m-index presupposes continuous and uninterrupted research engagement starting from the year of the first publication. This index offers a nuanced perspective on a researcher's scholarly impact over time and provides a more equitable way of evaluating individuals with divergent career spans in academia. The M index in the table 3.1 shows the comparison of authors within the field of agent-based modelling but with very different career lengths. The h-index is constrained by the fact that it is time-based and field-specific, and it ignores highly cited works. In the early identification of young researchers, bibliometrics that account for time, such as the m-index, should be evaluated, ideally in conjunction with critical peer review.

Table 2.2: Dominance factor Ranking.

Rank	Author	Dominance Factor	Total Publications	Single Authored	Multi Authored	First Authored	Rank by Publications
1	ANL	0.67	9	0	9	6	6
2	MALLESONN	0.6	10	0	10	6	3
3	LIGMANN-ZIELINSKAA	0.57	10	3	7	4	3
4	CROOKS A	0.5	9	1	8	4	6
5	TANG W	0.5	9	1	8	4	6
6	SENGUPTA P	0.2	15	0	15	3	2
7	BRADY C	0.125	9	1	8	1	6
8	GILBERT N	0.11	9	0	9	1	6
9	HEPPENSTALL A	0.1	10	0	10	1	3
10	WILENSKY U	0.04	22	0	22	1	1

The m-index has the most potential for identifying early-stage high-potential researchers. An m-index of 1 is normal, 1-2 is above average, and >2 is exceptional, according to a suggested rule of thumb for interpreting the index (Ndwandwe et al., 2021).

Table 2.3: h-index top authors.

Sr No.	Authors	h-Index	g-Index	m-Index
1	AN L	6	7	0.352941176
2	BRADY C	3	6	0.428571429
3	CROOKS A	7	7	0.5
4	HEPPENSTALL A	5	8	0.3125
5	LIGMANN-ZIELINSKA A	7	9	0.466666667
6	MALLESON N	6	7	0.461538462
7	SENGUPTA P	7	12	0.7
8	WANG S	5	9	0.454545455
9	WILENSKY U	8	16	0.533333333

We attempted to plot average citations and average total citations (see Figure 2.7) over time in order to study the most cited literature on the application of ABM in economics. We can readily see that total citations were much higher for work published prior to 2008, with a downward trend after that. On the other side, due to the pandemic, average citations climbed in 2020.

2.5.3 Network Analysis

Figure 2.8 shows the social network maps of the co-occurrence matrix, collaboration matrix and coupling. The size of the nodes reflects the frequency of keywords in each cluster. A larger size suggests a stronger citation burst and suggests high significance of the subfield. The conceptual structure captured through keyword co-occurrence indicates diversity within

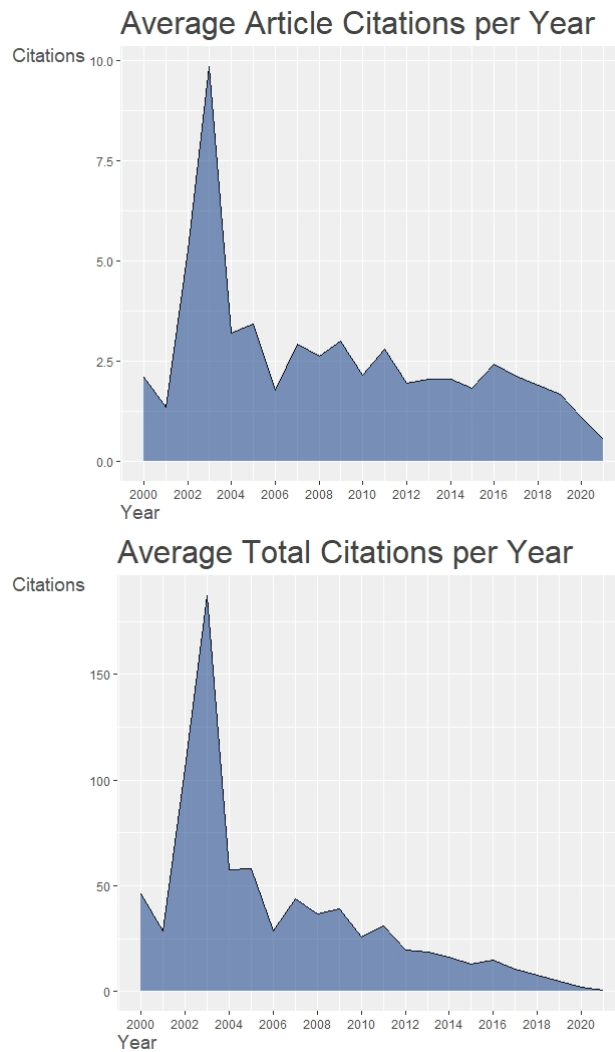


Figure 2.7: Citation analysis of published scientific documents.

the research sub-fields. The network indicates three clusters in Keyword Co-occurrence and Author's Coupling. Each cluster represents a theme/field of study in economics research using ABM. Three major themes emerge from the Keyword Co-occurrence network: human behavioral research, climate change and urban development, and the development of agent-based models that may be used to investigate various economic phenomena. The citing authors in the topic area that are mapped are known as author coupling, and these maps can be used to focus on research areas that are shared by many currently active authors. These are split into three groups in this instance. One of the primary cluster authors shares simulation platforms, while the other two work on computational simulations and numerical models together.

The expansion of international collaboration in ABM research was placed in a highly stratified way, resulting in a clear divide between the main contributing countries and many others

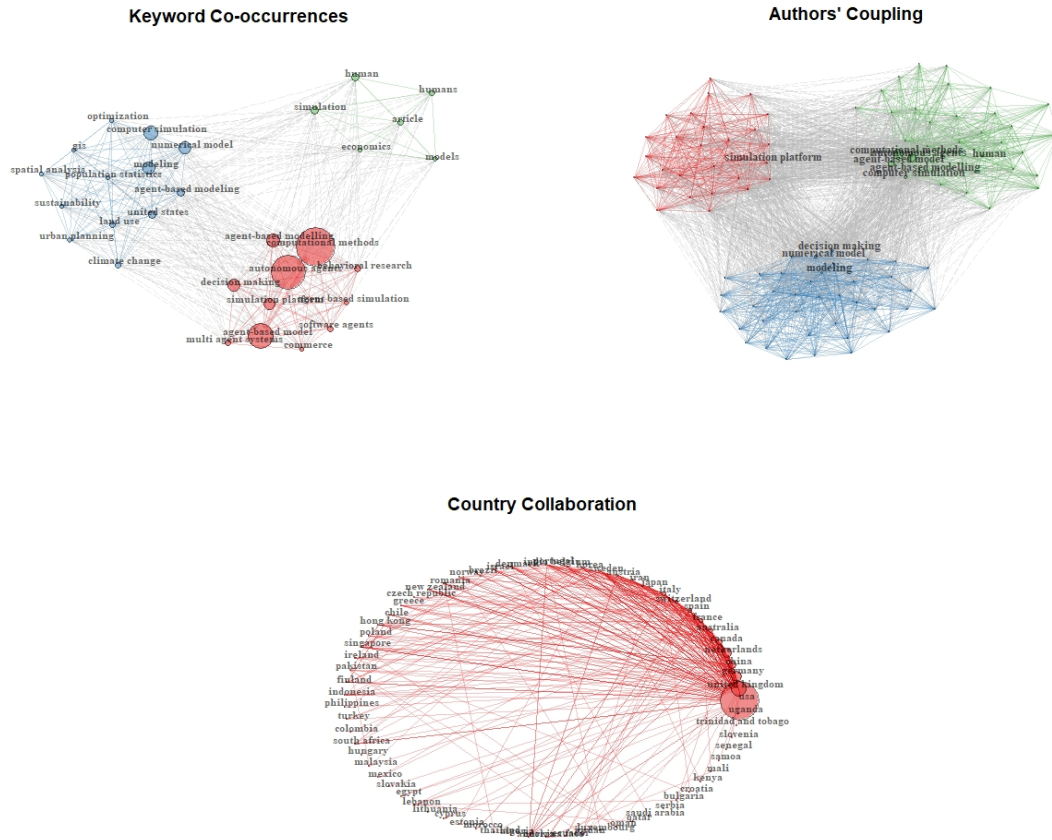


Figure 2.8: Network Analysis based on SCOPUS database.

who collaborated globally on a more occasional basis. The network is made up of a core that is dominated by research outputs from scholars in research-intensive countries, with numerous additional countries gravitating around that core—unsurprisingly, given the emphasis on English-language journals.

2.5.4 Research HotSpots

Conceptual structure of agent-based modelling in economics is shown in a thematic map (Figure 2.9). The map was constructed by using keywords with min word frequency of 250. Minimum cluster frequency per 1000 documents is 5 and number of labels to each cluster are 3. On the x-axis we have centrality, which measures degree of interaction of one network with other networks, while density on the y-axis is measure of internal strength of a network. Themes on the upper right quadrant, i.e., motor themes have well developed internal ties and

are important for the structure of the research field. Motor skills have strong centrality and high density. Niche themes are of marginal importance in the research field as they have well developed internal ties but unimportant external ties. Emerging or disappearing themes with low density and low centrality are weakly developed and marginal. Whereas basic themes are of high importance for the field of ABM in economics, but these are not well developed. We discovered that research on agent-based modelling in economics can be classified in two

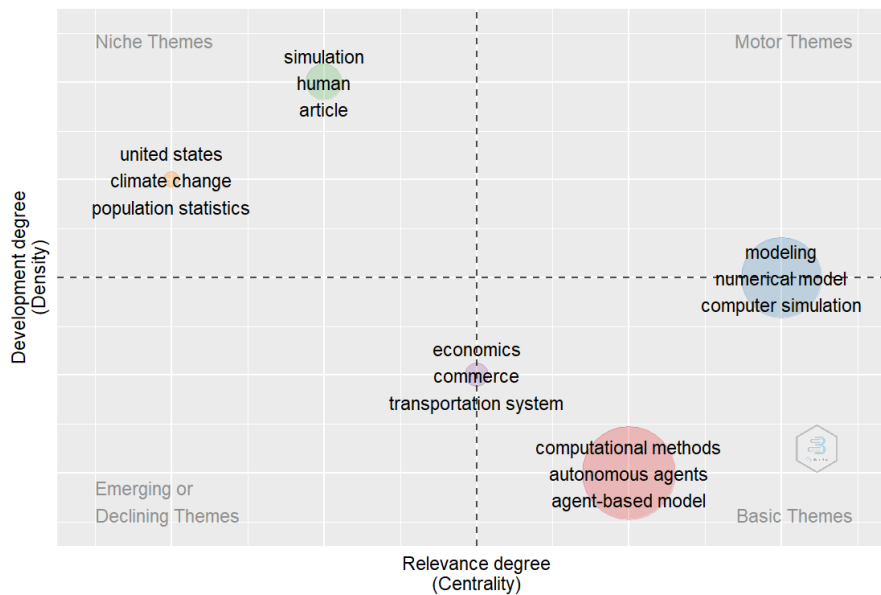


Figure 2.9: Thematic Map in Agent-based Modelling Research.

ways, the first being the development of an agent-based model, and the second being the use of a developed agent-based model to investigate policy possibilities, based on Keyword Co-occurrence analysis. The development of agent-based models employing economic theories, the application of diverse computational methods to calibrate these models, and the simulation of these models are all major topics in ABM currently. Although the largest bubble incorporates numerical models, computer simulations, and computational methods, these motor themes and basic themes both require a significant amount of research to get to a good measure of density and centrality.

2.6 Research Frontiers

There are several intriguing options for agent-based modelling research. ABM’s versatility in applying to new issues has always been one of its best attributes. While certain classes of

models have been established in fields like macroeconomics or financial markets, ABM has always been a transdisciplinary methodology that can be applied to problems involving a variety of rules, interactions, and behavioral phenomena (Steinbacher et al., 2021). ABMs can also be used to investigate issues that arise due to greater AI use, such as the societal impact of ranking algorithms and recommender systems and the potential reinforcement of social inequities and biases. ABMs can be used to build priors for machine learning algorithms in a semi-supervised manner in cases where the given data are noisy or biased, reducing errors and preventing the amplification of distortions. Artificial agents can also be incorporated into large-scale simulators once they have been built based on the behavior of human subjects (Dosi et al., 2020). Such synergy between ABM and experimental technique is still in its infancy, but it represents an exciting avenue for future research, in our opinion. Further research on the estimation of ABMs is also required, as little is known about the benefits and drawbacks of various techniques. The majority of current models allow for the formulation and stochastic approximation of a likelihood function. As models become more complicated, such approximations will become increasingly difficult. In such instances, Approximation through Bayesian Computation methods and GMM/SMM should be considered (ABC). This approach (Sisson et al., 2007; Toni et al., 2009) employs measurements (moments) of the data other than the likelihood to approximate the posterior distribution of the parameters using a rejection sampling or Markov Chain Monte Carlo technique. While this approach has gained much traction in ecological ABMs (Csilléry et al., 2010), economic applications are still a work in progress.

Recent research has concentrated on techniques for evaluating the fidelity of ABM outputs to reality (Marks (2013); Lamperti (2018); Barde (2017); Guerini & Moneta (2017)). However, both calibration/estimation and validation are hindered by substantial computation times, as indicated by Lux (2021). The most resource-intensive step in these processes is often the model simulation.

2.7 Conclusions

During the 2008 financial crisis, the discussion began when economists began to investigate the possibility of agent-based modelling techniques for answering policy problems and

performing “what-if” scenarios to aid policy decisions. This discussion about the future of agent-based modelling has yielded the desired result, with researchers currently working on developing agent-based models so that simulations based on simple rules can portray the complex economy. The European Central Bank is funding projects to construct agent-based economic models. The science of economics has long been in need of more robust methodologies that do not assume reasonable expectations and do not encourage optimism about the behaviours of agents. The economy appears to be a system, yet individual decisions cause the system’s complex nonlinearity. Similarly, agent-based models recognise individual interactions and adapt in response to them. The objective of the review analysis was to investigate the current developments in economics using this new modelling technique. According to the study’s findings, research on the topic of agent-based modelling in economics is growing at a quick pace. Researchers experiment and publish their findings in research papers, books, and conference papers. Researchers are increasingly collaborating in order to improve the quality of their publications. The United States of America (USA), the United Kingdom (UK), Germany, and China are the most productive countries. The most prominent research on this topic has been published in the United States of America (USA), and American-based journals have taken the lead in publishing research in this field. This might be due to the increased formation trend of academic journals in the USA. The most often used keywords by the researchers (e.g., decision making, sustainability, and commerce) indicate the hotspots in ABM research. The main purpose of adopting different bibliometric analysis methodologies was to uncover research trends and the substance of published work. The knowledge structure and research trends were discovered through co-occurrence analysis. According to the findings, several well-established economic themes benefit from ABM techniques, but many require them. Researchers are not introducing central banks in an agent-based economy and conducting more monetary policy experiments. However, monetary policy had a considerable impact during and after the Great Recession. The channels of collaboration among scholars worldwide were uncovered through social network analysis. In the field of agent-based modelling, researchers promote cross-national and intra-national collaboration, which fosters the creation of new ideas. Although there has been much research into using agent-based modelling to comprehend the complexities of economic prob-

lems, underdeveloped countries like Pakistan have been slow to adopt this modelling technique. Even after a decade of economic disaster, economists discover agent-based modelling. This policy decision modelling technique is not widely used in economics, especially in monetary policy issues. Agent-based models, in addition to existing ones, could be critical instruments for assessing economic policy.

CHAPTER 3

Inherited Flaws of DSGE Models

The inability of Dynamic Stochastic General Equilibrium (DSGE) models to account for nonlinearities has been subject to criticism in recent years. In response to this criticism, Brzoza-Brzezina et al. (2021) raise several important points that need to be addressed. Firstly, it is important to clarify whether DSGE models must be linearized and if all nonlinearities require specific treatment. Secondly, it is necessary to consider whether there are strategies available for dealing with nonlinearities.

Linearizing models can sometimes be advantageous, but it is important to investigate how much harm this approach can cause, given that it relies heavily on the type of nonlinearity being addressed. Even the simplest DSGE models may contain several nonlinear equations, such as the Euler equation which assumes a nonlinear relationship between consumption today and tomorrow. Linearization is only an approximation, and therefore it can produce variations from the true nonlinear relationship.

Another drawback of linearization is the existence of risk. Linear models assume no risk, but in reality, agents face risks and respond to them. This is also true for DSGE models when nonlinearities are considered. The impact of risk is significant when dealing with economic issues and neglecting it can lead to inaccurate results. The extent to which linearization is detrimental in this regard depends on the importance of risk given the specific economic issue being studied. For example, if the inquiry is about the consequences of a modest tightening of macroprudential policy, the effects of tighter loan-to-value ratios are likely to be prominent, and risk may be neglected. However, if the inquiry is about the impact of uncertainty shocks on savings and consumption, risk should not be disregarded.

While DSGE models are frequently linearized, this need not be a bad thing or lead to significant imprecisions. Economists have developed a variety of problem-solving techniques that can manage nonlinearities, and these techniques should be employed when nonlinearities are crucial.

3.1 Framework of Monetary Policy Decision Making in Pakistan

The monetary policy committee of the State Bank of Pakistan (SBP) meets every two months to review and communicate the stance of monetary policy in a statement. The Monetary Policy Department (MPD) of the SBP provides a comprehensive analysis of the local economy's present situation and the international financial and economic markets to the committee members. The MPD presents the information in the form of macroeconomic trends and changes and provides forecasts for essential factors such as inflation, exports, imports, currency rates, and money demand. These forecasts are combined into a macroeconomic framework based on financial programming to provide consistent estimates of the key macroeconomic variables for the current and next fiscal years, enabling the forward-looking element of monetary policy making to advance.

The SBP Research Department team uses a customized version of the Forecasting and Policy Analysis System (FPAS) to create model-based forecasts. The FPAS is a small to medium-sized Dynamic Stochastic General Equilibrium (DSGE) model that meets the requirements of general equilibrium and includes significant economic sectors. The model-based forecasts include key economic variables such as headline inflation, real interest rate gap, output gap, and real bilateral exchange rate gap. These forecasts are based on various policy scenarios and the projected interest rate path, incorporating the latest information available. The FPAS model also identifies relevant disturbances affecting these predictions based on its structural characteristics. Additionally, independent evidence such as inflation expectations and consumer confidence surveys accompany the forecasts and scenario assessments.

During the MPC meeting, Bank staff members provide the perspectives on macroeconomic circumstances and predictions and scenario assessments from the FPAS model. The Monetary Policy Committee uses this information to establish its judgments on monetary policy actions

after a protracted discussion and voting process. The SBP's senior management discusses monetary policy in the context of these predictions as well as macroeconomic developments and trends. Using the FPAS model, the Bank can provide a short- to medium-term outlook for the economy to aid in monetary policy decision-making. The FPAS model, along with other independent evidence, provides a detailed and accurate understanding of the country's economic performance, allowing the SBP to develop and implement monetary policies that are effective in promoting sustainable economic growth.

3.2 FPAS; A DSGE Modeling Approach

The Forecasting and Policy Analysis System (FPAS) is an essential tool used by the State Bank of Pakistan to approach the economy. It is a collection of models that provide information for decisions about monetary policy and support those decisions by organizing and systematizing the analysis. The risk assessment function of FPAS models comprises risk assessment to baseline prediction, alternative scenarios for certain anticipated shocks, and potential policy rule options.

The FPAS model provides model-based predictions for key variables, along with model-consistent confidence intervals, and evaluates uncertainty. The model is organized because each equation has an economic interpretation. The current FPAS model approved by the State Bank of Pakistan is a Dynamic Stochastic General Equilibrium (DSGE) model, which has come under heavy criticism for its inability to predict the economic crisis of 2008 and its lack of guidance for politicians in resolving the problem.

A new generation of DSGE models, known as large-scale New Keynesian models, emerged following the financial crisis. However, Blanchard (2016) explains that these models have three main faults. First, aggregate demand in the current FPAS model is computed using the consumption demand of a fully cognizant and endlessly lived representative agent. Second, the model is estimated as a system of equations rather than equation by equation, requiring the estimation of a large number of parameters. Calibration establishes a number of parameters a priori, while the other set of variables is estimated using the Bayesian technique, raising a dual problem. Finally, a standard DSGE study specifically distorts an existing core, making

it challenging for the layperson to comprehend how a particular distortion interacts with other distortions in the model.

At the Monetary Policy Committee meeting, Bank staff members provide the Bank’s perspectives on developing macroeconomic circumstances as well as predictions and scenario assessments from the FPAS model to help members establish their judgments on monetary policy actions. In summary, FPAS models are critical to monetary policy decisions, but their reliability and accuracy are subject to debate and scrutiny, particularly in the aftermath of economic crises.

3.3 Core Macro Model

The structure of dynamic stochastic general equilibrium models used for policy analysis is rather straightforward, with three interconnected blocks: a demand block, a supply block, and a monetary policy equation. Micro foundations are explicit assumptions regarding the behaviour of the primary economic actors in the economy—households, enterprises, and the government—that establish the equations that define these blocks. The ”general equilibrium” element of the models comes from the interaction of these agents in markets that clear every period. Because the FPAS model is a central bank research project, the advanced version of the FPAS models includes the foreign sector. State bank of Pakistan currently uses FPAS (Forecasting and Policy Analysis System) model for policy analysis which is customized DSGE model. The model comprises four equations which has economic interpretation and stochasticity is involved to quantify uncertainty in model’s forecast Ahmad et al. (2015).

1) Aggregate Demand

The equation for aggregate demand demonstrates that the output gap (\hat{y}_t) depends on the lagged output gap, the monetary conditions index (MCI), and the foreign demand gap (\hat{y}_t^*). The term ϵ_t^y represents the aggregate demand shock.

$$\hat{y}_t = \alpha_1 \hat{y}_{t-1} - \alpha_2 mci_{t-1} + \alpha_3 \hat{y}_t^* + \epsilon_t^y \quad (3.1)$$

$$mci_t = \alpha_4 (\hat{r}_t + cr_prem_t) + (1 - \alpha_4) (-\hat{z}_t) \quad (3.2)$$

$$cr_prem_t = \alpha_5 cr_prem_{t-1} + (1 - \alpha_5) (prem_t - cr_prem_{t-1}) + \epsilon_t^{cr_prem} \quad (3.3)$$

$$\ln z_t = \ln s_t + \ln p_t^* - \ln p_t. \quad (3.4)$$

The MCI (mci_t) is calculated as a weighted average of the real interest rate (\widehat{r}_t) and the real exchange rate in relation to their respective trajectories. The credit premium over the risk-free real rate is represented by cr_prem_t , and its shock is represented by $\epsilon_t^{cr_prem}$, which is independent and identically distributed. The real exchange rate gap-trend-deviation is represented by \widehat{z}_t , which is calculated as the logarithm of the nominal bilateral exchange rate (s_t) plus the logarithm of the foreign price level (p_t^*) minus the logarithm of the domestic price level (p_t).

2) Aggregate Supply

Core inflation (π_t^{core}) is modeled according to New Keynesian Phillips Curve with backward looking component and rmc_t represent real marginal cost and a function of domestic and foreign demand components.

$$\pi_t^{core} = \beta_1 \pi_{t-1}^{core} + (1 - \beta_1) E_t \pi_{t+1} + \beta_2 rmc_t^{core} + \epsilon_t^{\pi^{core}} \quad (3.5)$$

$$rmc_t^{core} = \beta_3 \widehat{y}_t + (1 - \beta_3) \widehat{z}_t \quad (3.6)$$

$E_t \pi_{t+1}$ is forward looking component of core inflation and $\epsilon_t^{\pi^{core}}$ is *i.i.d* shock to core inflation.

3) External Conditions

SBP impose augmented uncovered interest rate parity condition aligned to Pakistan condition as capital controls are there. UIP condition establish a link between domestic and foreign currency based on their interest rate differences and risk profile.

$$s_t = (1 - \gamma_1) E_t s_{t+1} + \gamma_1 \left(s_{t-1} + \frac{2}{4} (\overline{\pi}_t - \overline{\pi}_t^* + \delta \overline{z}_t) \right) + \left(\frac{i_t^* - i_t + prem_t}{4} \right) + \epsilon_t^s \quad (3.7)$$

$E_t s_{t+1}$ is forward looking part of exchange rate, $\overline{\pi}_t$ and $\overline{\pi}_t^*$ are inflation target differential between Pakistan and United States. The terms i_t , i_t^* , s_t and $prem_t$ denote quarterly domestic and foreign interest rates, Pakistan Rupee per dollar exchange rate and country specific Foreign Exchange (FX) risk premium respectively. ϵ_t^s is independently and

identically distributed.

4) Monetary Policy Rule

State bank of Pakistan target price stability with appropriate economic growth also keeping exchange rate in desired range.

$$i_t = a_1 \left[\frac{1}{1 - e_1} \Delta s_{t+1} - \frac{e_1}{1 - e_1} \Delta s_t + i_t^* + prem_t \right] + (1 - a_1) [b_1 i_{t-1} + (1 - b_1) \{i_t^n + b_2 (E_t \pi_{t+4} - \pi_t^T) + b_3 \hat{y}_t\}] + \epsilon_t^i \quad (3.8)$$

Monetary rule is the weighted average of uncovered interest rate parity and Taylor rule. ϵ_t^i is dependently and identically distributed.

3.3.1 DSGE Models and the New Keynesian Phillips Curve

The interplay between DSGE (Dynamic Stochastic General Equilibrium) models and the NKPC (New Keynesian Phillips Curve) is pivotal within the realm of macroeconomic modeling, particularly in the exploration of monetary policy and business cycles.

DSGE models encompass a category of economic frameworks aimed at encapsulating the macroeconomic behaviors of an economy. By integrating equations that elucidate the decisions of diverse economic agents and their interactions in various markets, these models offer a dynamic portrayal of economic evolution over time. Their stochastic nature accommodates uncertainty through random shocks, and they encapsulate general equilibrium by accounting for interactions across multiple markets. DSGE models form a structured foundation for comprehending the intricate interplay between households, firms, and government entities within an economy.

In the same vein, the NKPC (New Keynesian Phillips Curve) occupies a specific equation within the broader canvas of DSGE models. Functioning as an offshoot of the Phillips Curve concept, which outlines an inverse correlation between unemployment and inflation, the NKPC takes the concept a step further. It incorporates insights from New Keynesian economics, highlighting the significance of price stickiness and nominal rigidities in steering economic fluctuations.

Within the NKPC, inflation's dynamics are shaped by a fusion of anticipated inflation, prevailing and preceding levels of economic activity (often gauged through output or unemployment

metrics), and supply shocks. The core tenet underlying the NKPC postulates that when the economy operates beneath its potential (characterized by elevated unemployment), firms encounter diminished pricing power due to feeble demand, thus leading to subdued inflation. Conversely, during phases of economic operation surpassing its potential (marked by lower unemployment), firms wield enhanced pricing leverage, potentially triggering inflationary pressures.

In synthesis, the synergy between DSGE models and the NKPC manifests through the incorporation of the latter as a constituent equation within the expansive DSGE modeling framework. By amalgamating the NKPC alongside other equations, DSGE models present a holistic depiction of how diverse economic variables interplay to mold an economy's overall behavior. This synergy proves vital in encapsulating the intricacies of inflation dynamics and its intricate relationship with tangible economic variables within the contours of the New Keynesian framework.

3.3.2 Price Setting Problems; Evolution of Phillips Curve

The Philips curve was originally introduced in the 1950s as a model for the United Kingdom, and it was used to study the relationship between inflation and the labor market. In 1958, A.W. Philips presented the concept of the Philips curve in his paper "The Relation Between Unemployment and the Rate of Change of Money Wage Rates in the United Kingdom, 1861–1957" published in the journal of *Economica*. The original results of the theory have since evolved due to increased computer power and larger datasets, revealing a weaker causation between inflation and employment in linear models, resulting in weaker results for the Philips curve (Granger & Jeon (2011)).

The Phillips curve has significant implications for demand management economic strategies as it is an interface of nominal and real variables (Motyovszki (2013)). The new Keynesian school of thought has taken a step toward effective demand-side policies that take the Phillips curve into account. However, in the 1970s, the concept of the Phillips curve changed as the economy faced a rise in both unemployment and inflation, leading to stagflation. As a result, economists argued that the Phillips curve could work in the short run, but not in the long run, as prices would increase along with unemployment, and there would be no trade-off between inflation

and unemployment (Humphrey (1985)). Hoover (2008) states that Philips Curve is the relationship between unemployment and inflation, lower the unemployment rate tighter the labor market. Philips curve is for the most developed economies. The history of the Philips curve in the American Perspective is discussed by Gordon (2008), and many empirical implications are on American data.

Recent research has also explored the effectiveness of the Phillips curve in the current economic environment. For example, some studies have suggested that the relationship between unemployment and inflation has weakened over time (Stock & Watson (2008)). In addition, there is evidence to suggest that the Phillips curve may have become flatter in recent years, meaning that a given level of unemployment is associated with a lower level of inflation than in the past (L. Ball & Mazumder (2019)).

Furthermore, recent research has shown that the Phillips curve may be asymmetric, meaning that the relationship between unemployment and inflation may differ depending on whether the economy is in a recession or expansion (Galí & Gambetti (2019)). These findings have implications for monetary policy, as central banks need to understand how changes in unemployment affect inflation, and how changes in inflation affect unemployment, in order to effectively manage the economy (Eser et al. (2020)).

3.3.3 New Keynesian Phillips Curve in Pakistan

The New Keynesian Phillips Curve (NKPC) has been estimated for various sectors of Pakistan in several studies. Hyder & Hall (2020) estimated the NKPC for the agriculture, manufacturing, and services sectors of Pakistan, finding that the manufacturing sector leads the services and agriculture sectors in terms of inflation dynamics. Another study estimated the NKPC for Pakistan from 1976 to 2006, showing that real marginal cost and the output gap are the major drivers of inflation (ul Haq Satti et al. (2007)). Mukhtar & Yousaf (2014) tested the standard and hybrid versions of the NKPC in Pakistan, revealing that expected inflation and the output gap are crucial for explaining inflation behavior in the country. Meanwhile, the hybrid NKPC suggests that price-setting behavior in Pakistan is relatively rigid. The study concludes that the NKPC model can serve as a benchmark for understanding inflation behavior in Pakistan.

Saeed & Riaz (2011) investigated whether the NKPC in Pakistan is forward- or backward-looking, finding that inflation is a constant fact for the country and that past inflation has explanatory power, indicating that inflation is both backward- and forward-looking. Khan (2021) tested the dynamic forces of inflation in Pakistan and found that the NKPC model accurately describes inflation dynamics in the country and is data-driven. Riffat et al. (2016) explored inflation dynamics and the NKPC in Pakistan, finding that external factors such as terms of trade and trade openness clearly affect inflation dynamics in the country.

Recently, ASJED & ALAM (2021) compared traditional and open economy Philips curves in the case of food, non-food, and core inflation using time-series data. The results show that domestic output gaps in the presence of global output gaps do not significantly affect core inflation in Pakistan, but the global economic crisis of 2008 has had a notable impact on core inflation in the country.

Gul et al. (2012) revisited the Phillips curve in Pakistan and examined whether inflation and unemployment are positively or negatively related. According to their findings, inflation and unemployment are negatively related in Pakistan from 1992 to 2010. As unemployment decreases, however, it does not necessarily mean that the labor force is fully employed.

Inflation refers to the general fall in a currency's buying power over time. Unemployment occurs when there is more demand for labor than there are job openings. The relationship between these two economic indicators has historically been adverse, as described by A.W. Phillips' 1958 Phillips curve theory. Governments and central banks usually rely on monetary and fiscal policy to prevent excessive economic stimulation or slowdown. However, many industrialized nations have abandoned the Phillips curve theory since the global economic crisis of the 1970s (Dellas & Tavlas (2010)).

Studies on the NKPC in Pakistan suggest that the model accurately describes inflation dynamics in the country and can serve as a benchmark for understanding inflation behavior. External factors such as terms of trade and trade openness clearly affect inflation dynamics in the country. Meanwhile, the negative relationship between inflation and unemployment in Pakistan has been confirmed by recent research.

3.3.4 Exploring Applicability of Phillips Curve in Pakistan

Visual inspection is a commonly employed technique in research to examine the relationship between two variables. SBP still views this PC concept as vital in Pakistan which means data should show an inverse relationship between unemployment and inflation. In other words, when the unemployment rate is low, the inflation rate should be high, and when the unemployment rate is high, the inflation rate should be low. In order to verify this assertion, We conducted a visual inspection by plotting Consumer Price Index (CPI) inflation and unemployment rate data spanning the years 1991 to 2021.

Figure 3.1 was used to visually inspect the data and interpret the relationship between inflation and unemployment over time. In the period from 1991 to 2000, there were fluctuations in inflation, but unemployment had minimal effect on inflation, indicating the absence of any relationship between the two variables. Similarly, from 2001 to 2010, although inflation fluctuated greatly, unemployment showed no significant reaction or response. In 2009, inflation reached its highest peak, but there was no corresponding peak in unemployment data, indicating a high level of invalidity regarding any inverse relationship in the graphical representation. Likewise, from 2011 to 2021, there was no evidence of an inverse relationship in the graph, indicating that no significant inverse relationship exists in Pakistan's inflation-unemployment data.

Previous investigations had also revealed a negligible or non-existent inverse relationship be-

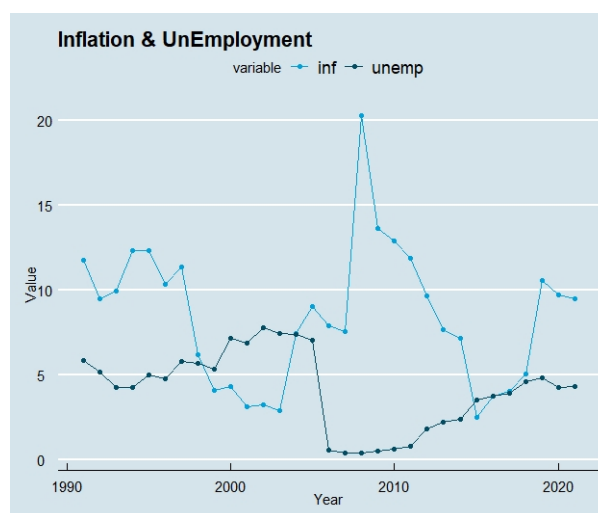


Figure 3.1: Unemployment and Inflation in Pakistan 1991-2021

tween the two variables. However, to validate these findings, we analyzed the Phillips curve in

Pakistan for the same time frame using CPI inflation and unemployment rate data, as shown in Figure 3.2. Upon reviewing the proposed shape of the Phillips Curve in literature, we concluded that the curve had a smooth concave shape. However, it should be noted that Figure 3.2 does not exhibit a regular or smooth shape.

Kendall rank test Abdi (2007) is a powerful tool for assessing the correlation between two

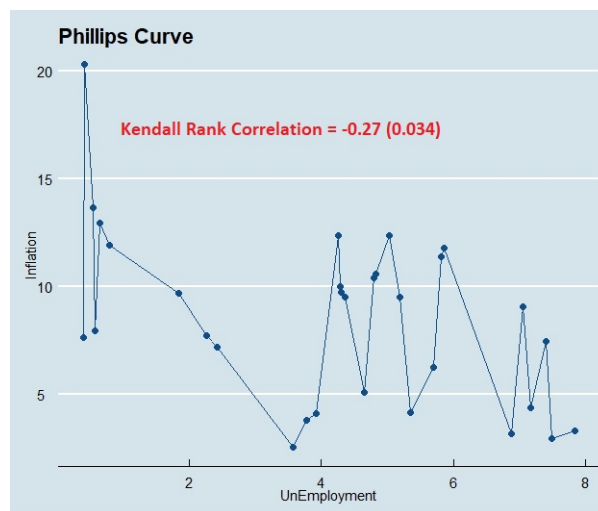


Figure 3.2: Relationship Inflation and Unemployment in Pakistan 1991-2021

variables when the data do not follow a normal distribution or when the relationship between the two variables is not linear. It provides a robust measure of association that is less sensitive to outliers and extreme values in the data. The null hypothesis of the Kendall rank test is that “there is no correlation between the two variables”. The alternative hypothesis is that there is a correlation between the two variables. The significance of the test is determined by comparing the calculated value of Kendall’s tau to a critical value from a table based on the sample size and significance level.

The correlation factor between inflation and unemployment in Pakistan was found to be -0.27, and the null hypothesis of no relationship was rejected at the 5% level of significance. This result confirms the existence of a very weak negative relationship between the two variables, which is almost zero. The Phillips Curve graph and correlation factor thus validate the presence of a weak relationship between inflation and unemployment in Pakistan. To further validate the behavior of the Phillips Curve in Pakistan over a divided time period, the data was divided into three parts (Figure 3.3). The first period covered the years 1991 to 2000, the second period

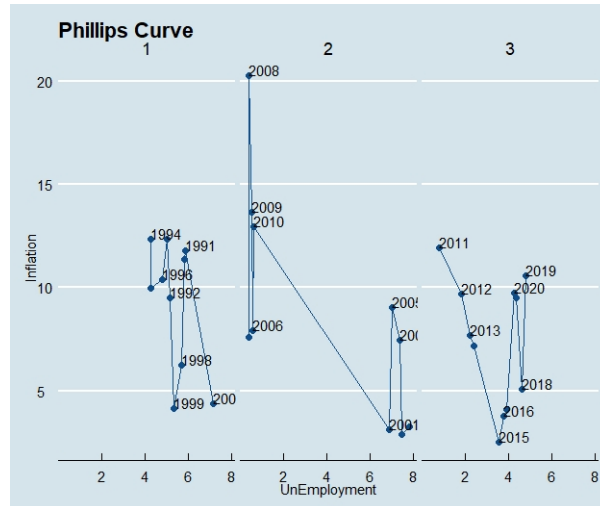


Figure 3.3: Inflation and Unemployment in Three Decades

covered the years 2001 to 2010, and the third period covered the years 2011 to 2021. Moreover, no clear pattern indicating the presence of an inverse relationship between inflation and unemployment was observed across the three decades.

Identifying Structural Change

According to Ploberger et al. (1989), a structural change test should be based on the cumulative sums of the common OLS residuals. Thus, the following criteria characterise the OLS-CUSUM type empirical fluctuation process:

$$W_n^0(t) = \frac{1}{\sigma \sqrt{n}} \sum_{i=1}^{\lfloor nt \rfloor} \hat{\mu}_i \quad (0 \leq t \leq 1). \quad (3.9)$$

The common Brownian bridge $W^0(t) = W(t) - tW(1)$ serves as the limiting process for $W_n^0(t)$. It begins in 0 at time $t = 0$ and ends in 0 at time $t = 1$. A single structural shift option would predict that the path will peak at around t_0 . The OLS-based CUSUM process may be shown to go beyond its limit, which provides evidence of a structural shift. Additionally, the procedure appears to point to just one substantial alteration (Figure ??). The OLS-based CUSUM process may be shown to go beyond its boundary, which provides evidence of a structural shift. Additionally, the procedure appears to show just one substantial out-of-bounds peak.

Using F test statistics is a somewhat different method of determining if the null hypothesis

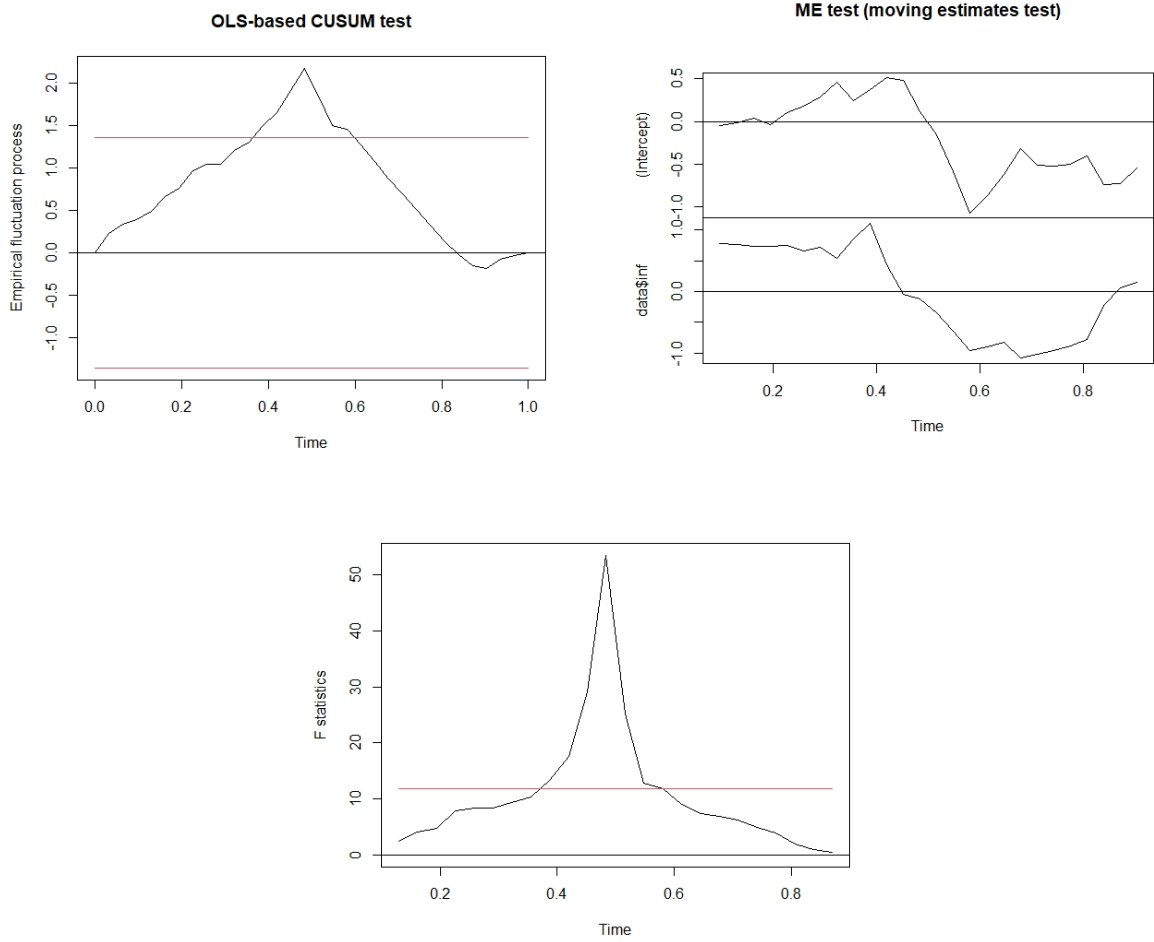


Figure 3.4: Testing for structural change in linear regression model

of "no structural change" is true. The alternative is stated in the F-tests, as opposed to generalised fluctuation tests, which are suited for a variety of patterns of structural changes. This is a significant distinction. As a result, the model (Eq. 3.10) may be used to formulate the alternative.

$$y_i = x_i^T \beta_i + \mu_i \quad (i = 1, \dots, n) \quad (3.10)$$

and,

$$\beta_i = \begin{cases} \beta_A & (1 \leq i \leq i_0) \\ \beta_B & (i_0 < i \leq n) \end{cases} \quad (3.11)$$

where some change in the interval $(k, n-k)$ is represented by i_0 . The first to propose such a test for structural change in the situation when the (possible) transition point i_0 is known, was Chow (1960). For the two sub samples determined by i_0 , he suggested fitting two distinct regressions

and rejecting whenever

$$F_{i_0} = \frac{\hat{\mu}^T \hat{\mu} - \hat{e}^T \hat{e}}{\hat{e}^T \hat{e} / (n - 2k)} \quad (3.12)$$

is too big, where $\hat{\mu}$ are the residuals from the restricted model, where the parameters are only fitted once for all observations, and $\hat{e} = (\hat{\mu}_A, \hat{\mu}_B)^T$ are the residuals from the complete model, where the coefficients in the sub samples are estimated individually. Test statistic Under the presumption of normality, F_{i_0} has an exact F distribution with k and $(n - 2k)$ degrees of freedom, while F_{i_0}/k has an asymptotic χ^2 distribution with k degrees of freedom. There are tests based on F statistics (Chow statistics) that are detailed in the following sections that do not need the definition of a particular change point. The "Chow test" drawback is that the transition point needs to be identified beforehand.

A structural change refers to a significant shift in the underlying economic conditions or policies that affect the relationship between inflation and unemployment. The presence of a structural change can have a significant impact on the Phillips Curve, altering its shape or position. For instance, a change in the monetary policy regime or fiscal policy can alter the relationship between inflation and unemployment, leading to a shift in the Phillips Curve. Similarly, changes in technology or the labor market can also affect the relationship between inflation and unemployment, leading to a change in the slope or curvature of the Phillips Curve.

The presence of a structural change can significantly affect the Phillips Curve, altering its shape, slope, or position. Therefore, it is important to take into account the possibility of structural changes when analyzing the relationship between inflation and unemployment using the Phillips curve.

3.4 Linear Modelling of New Keynesian Phillips Curve

Philips curve is main element of inflation dynamics and monetary policy. The NKPC describes how past inflation, future inflation, and current inflation derive demand. Nason & Smith (2008) identify the NKPC, weak identification of NKPC under GMM, and lack of higher-order dynamics in exogenous variables. Bårdsen et al. (2004) discussed the econometric evaluation of NKPC, the results show that economists should not accept NKPC too easily. Dees et al. (2009)

estimate the NKPC, and they estimated two issues weak instrument problems and steady states. The global vector autoregressive model steady states and it performs better for the steady states. Kleibergen & Mavroeidis (2009) used test procedures to revise the NKPC model and the results find out that U.S postwar data is stable with the inflation dynamics and mainly forward-looking, but it can be backward-looking as well. But recently Philips curve become flattered, and it is weak detection of the Philips curve.

The "new Keynesian Phillips curve," which is founded on microeconomic principles of the ideal conduct of economic actors, has emerged as a result of the nominal rigidity that characterised the 1980s and 1990s in combination with the rational expectations hypothesis. Typically, the linear new Keynesian Phillips curve is thought to take the following form:

$$\pi_t = \gamma_f E_t \pi_{t+1} + \theta(y_t - y^*)$$

The New Keynesian Phillips curve's primary critique pertains to its strictly forward-looking characteristics, which do not guarantee inflation persistence (Rudd & Whelan (2005)). In order to do this, it was suggested to incorporate some inertia into the inflation dynamic using a so-called "hybrid version" of the new Keynesian curve. Gali & Gertler (1999) make the assumption that a fixed fraction "w" of the enterprises that redefine their pricing employ a backward-looking price setting rule. As a result, this hybrid curve combines elements that are oriented both forward and backward. Hence,

$$\pi_t = \gamma_f E_t \pi_{t+1} + \gamma_b \pi_{t-1} + \theta(y_t - y^*)$$

This may be seen as a synthesis of the "old" and "new" Phillips curve. The capacity of the NKPC (including its hybrid form) to give a relevant empirical characterization of the inflation process has been questioned by recent research, in particular by Rudd & Whelan (2007). With an expectational component, excess demand pressure (output-gap), and "supply shock" variables, Gordon (2011) develops a new specification known as the "new Keynesian Triangle Phillips curve" and finds that it outperforms the NKPC. As previously mentioned inflation modelling at SBP is according to New Keynesian Phillips Curve with backward looking component and

rmc_t represent real marginal cost and a function of domestic and foreign demand components.

$$\pi_t^{core} = \beta_1 \pi_{t-1}^{core} + (1 - \beta_1) E_t \pi_{t+1} + \beta_2 rmc_t^{core} + \epsilon_t^{\pi^{core}} \quad (3.13)$$

$$rmc_t^{core} = \beta_3 \hat{y}_t + (1 - \beta_3) \hat{z}_t \quad (3.14)$$

3.5 Testing Non-Linearity of Phillips Curve

3.5.1 The Model

Empirical research on the nature of the short-term link between inflation and production have used a variety of approaches. While some authors are attempting to locate a non-linear Phillips curve in a certain functional form, others are attempting to ascertain if nominal demand shocks have unequal impacts on real activity. For the purpose of testing non-linearity of Phillips curve for Pakistan we applied the suggested Smooth Transition Regression Model (STR) of Granger et al. (1993), Terasvirta (2006). In comparison to rival structural break and nonlinear models, smooth transition regression (STR) models have a number of benefits. First, this model dispenses with the need for a previous non-linear functional form and enables the application of linearity tests as well as simulating the process of regime changeover. Theoretically, STR models are preferable to Markov regime switching and simple threshold models that force a rapid shift in the coefficients. Only when every agent acts simultaneously and in the same direction is it feasible for regimes to shift instantly. Third, because it provides for an economic intuition for the nonlinear dynamic defining why and when inflation sensitivity changes to output fluctuations, this model can offer a useful framework to explore potential nonlinearity in the interplay between the inflation rate and output. The STR model is expressed as follows:

$$\pi_t = \theta' Z_t + \psi' Z_t G(\gamma, c, s_t) + \epsilon_t, \quad t = 1, \dots, T \quad (3.15)$$

θ' and ψ^{prime} are parameters that need to be estimated for the linear and nonlinear parts of the model, respectively, where Z_t is the vector of explanatory variables. $\epsilon_t \rightarrow N(0, \sigma^2)$ The error term is expected to have an independent, identical distribution with a zero mean and constant variance. The transition function $G(\gamma, c, s_t)$ has a continuous range of 0 to 1 and is a

function of the transition variable s_t . The threshold's location parameter "c" shows where the change takes place, while the slope parameter γ gauges how smoothly one regime changes into another. Either an exponential or logistic transition function is possible. The logistic function (LSTR) is examined in this chapter as follows:

$$G(\gamma, c, s_t) = (1 + \exp\{-\gamma \sum_{j=1}^k (s_t - c_j)\})^{-1}, \text{ avec } \gamma > 0 \quad (3.16)$$

This transition function $G(\cdot)$ fluctuates depending on how far the transition variable deviates from the threshold (c). The transition function is equal to 0 and the linear part's coefficients describe the inflation-production tradeoff if this difference is negative ($s_t < c$). The transition function, on the other hand, tends to 1 when the difference is positive ($s_t > c$), and the parameters characterising this connection are equal to the sum of the linear and nonlinear coefficients ($\theta_i + \psi_i$). Between the two regimes, a continuum of factors will determine the Phillips curve's slope.

In general, there are three crucial steps that must be taken in order to specify the proper LSTR model to explain the nonlinear dynamics of inflation. The first step is to choose a suitable linear model that will be the foundation for the test of linearity versus the LSTR model. Then, in order to confirm the existence of non-linearity in the inflation response and identify the transition variable according to which the inflation modifies its response to the output gap, a linearity test must be used. After rejecting linearity, we move on to estimating the given nonlinear model. Verifying whether $y = 0$ in Equation is equivalent to testing linearity against the alternative of an LSTR (k) model. Due to the annoyance parameters θ' and c , the model cannot be recognised under the null hypothesis. To get around this issue, a Taylor series approximation around $y = 0$ is employed as a replacement, and the tests are based on this altered equation (Dijk et al. (2002); Teräsvirta et al. (1996a)).

$$\iota_t = \delta'_0 Z_t + \delta'_1 \hat{Z}_t s_t + \delta'_2 \hat{Z}_t s_t^2 + \delta'_3 \hat{Z}_t s_t^3 + \epsilon_t^* \quad (3.17)$$

where $\epsilon_t^* = \epsilon_t + \psi' Z_t R_3(\gamma, c, s_t)$ the remainder $R_3(\gamma, c, s_t)$ and $Z_t = (1, \hat{z}'_t)$ which is a vector of explanatory variables with constant and s_t as transition series. The null hypothesis of all

$\gamma'_i = 0$ against the alternative that at least one is non-zero is tested. Although it may be verified using an LM test with an asymptotic chi-squared distribution, it is advised to depend on the Fisher statistic with sample sizes due to its advantageous size characteristics. We used this test on one or more model variables. The transition variable chosen will be the one with the lowest p-value for the rejection of linearity if linearity is rejected for more than one variable.

3.5.2 Data Description

In order to test non-linearity of Phillips Curve, we examined quarterly data gathered for Pakistan from 2011:1 and 2020:3. The Hodrick-Prescott (HP) filter is used to compute potential output. The output gap is the percentage point difference between the industrial production (million rupees) and its potential value. Additionally, the difference between the exchange rate and prospective values proposed by the HP filter is used to determine the exchange rate gap. CPI inflation as a measure of inflation was considered. Prior to estimating the linear model, we

Table 3.1: Unit root test results

Variable	ADF Test(p-value)	KPSS Test (p-value)
Inflation	0.11 (0.99)	0.17 (0.02)
Output gap	1.25 (0.99)	0.16 (0.03)
Exchange rate gap	-3.37 (0.07)	0.13 (0.08)

used the traditional ADF and KPSS unit-root tests to see whether the relevant variables were stationary. Table 3.1 lists the outcomes of these tests.

3.6 Estimation and Empirical Findings

3.6.1 The Linear Phillips Curve: Specification and Estimation

In the sections that follow, we attempted to estimate the hybrid variants of the new Keynesian Phillips curve that have been suggested in the literature in order to see if a linear inflation-output connection can account for the dynamics of inflation in Pakistan. This Phillips curve's

estimation will enable us to decide on which to base the linearity test. The essential premise of the hybrid Phillips curve is that rational expectations are reasonable. Such expectations entail an orthogonality requirement for calculating the expectations' informative effectiveness. This logic makes the assumption that the forecast mistakes are independent of all accessible data. These variants of the Phillips curve are estimated by Galí & Gertler (1999) using a GMM with lagged instrumental variables. The labour share of income, the production gap, the difference in rates (long 10-month rate, short 3-month rate), wage inflation, and commodity inflation are four lags for inflation that are taken into account. Galí et al. (2005), in contrast, chose fewer instruments to reduce the estimate bias brought on by the amount of over-identifying limitations. We estimate hybrid NKPC using the extended method of moments, which substitutes the growth rate of inflation for the future predicted inflation rate. The fact that valid instruments could also be weak instruments is one aspect of reasonable expectations. We incorporate the lag of independent variables as a workable option. Therefore, the instruments employed are first lag of output gap, first lag of expected inflation, first lag of exchange rate gap, second lag of CPI inflation and discount rate. The J-test for over-identification developed by Hansen (1982), which assesses whether the instruments are orthogonal to the perturbation term, was used to confirm the validity of these instruments. The hybrid Phillips curve estimate highlights the importance

Table 3.2: Linear Phillips curve estimation

	π_{t-1}	π_{t+1}	$y_t - y^*$	$er_t - er^*$
Hybrid	0.102**	0.491***	-0.330*	0.12*
AIC	182.155		SBC	191.149
J-STAT (p-value)	0.967 (0.32)			

Null hypothesis is rejected at a significance level of **, 5%, and *, 10%. Akaike Information Criterion (AIC), Schwarz Information Criteria (SBC), and Hansen (1982) Overidentification Test (J-STAT) are all abbreviations for the same concept. All estimations use heteroscedasticity and autocorrelation-consistent standard errors.

of the preceding level of inflation. Future inflation predictions are given a lot of weight since the prospective component is still quite important. Although there is a statistically significant difference between the output gap coefficient and zero, it is negative. The negative value of the output gap coefficient raises questions about the reliability of this inflation-output relation. The

coefficient of exchange rate gap is also significant.

We can get the conclusion that the linear Phillips curve does not accurately depict the inflation process in Pakistan. The linear specification can be too limited since it doesn't account for any nonlinear effects brought on by the macroeconomic environment.

Given that both forward- and backward-looking behaviour has an impact on inflation in Pakistan, we cast doubt on the idea that the Phillips curve is linear and test for non-linearity of this connection using the hybrid model.

We performed diagnostic tests for the hybrid model residuals since the nonlinearity tests are sensitive to residual characteristics, particularly autocorrelation. Neither conditional heteroscedasticity nor autocorrelation are displayed in Table 3.3. Furthermore, residuals are properly distributed and do not exhibit excessive kurtosis or skewness.

Table 3.3: Residual tests and statistics

	ARCH/LM	LB	SK	J-B
Hybrid NKPC Residuals	3.979	12.212	-0.325	2.45
	(0.154)	(0.129)	(0.15)	(0.321)

3.6.2 Tests for Linearity and Estimating LSTR

As the model appears to be adequate, we now go forward to evaluate linearity based on the chosen hybrid curve in order to ascertain whether there is a non-linear connection and to identify the transition variable that regulates the regime change. According to Teräsvirta et al. (1996b) suggestion, the applied test presupposes linearity versus the LSTR alternative as the null hypothesis. It is carried out for different explanatory variables included in the hybrid model, such as the output gap and past and projected inflation. Table 3.4 presents the findings of the linearity tests. The table also lists the results of specification tests done by Teräsvirta (2006) to determine the ideal transition function for regime change.

The linearity null hypothesis is rejected exclusively for output gap, as is seen from Table 3.4. This result suggests that the difference of log value of industrial production from its potential

Table 3.4: Linearity Tests

Transition Variable	F-stat
Linearity Tests	
π_{t-1}	1.54 (0.39)
π_{t+1}	1.52 (0.58)
$y_t - y^*$	0.244 (0.06)
$er_t - er^*$	0.569 (0.03)
Transition Function Specification Tests	
H_02	2.549 (0.02)
H_03	0.371 (0.77)
H_04	0.415 (0.83)
H_05	0.987 (0.61)

Note: Fisher statistic-based tests with specified p-values in brackets.

Table 3.5: Logistic smooth transition regression (LSTR) estimation results.

	At Negative output gap	At Positive output gap	Threshold (c)
π_{t-1}	0.622**(0.127)	0.236**(0.093)	
π_{t+1}	-0.589**(0.14)	0.771**(0.105)	3.89**(0.06)
$y_t - y^*$	0.102*(0.05)	-0.025(0.03)	
$er_t - er^*$	0.651(0.04)	0.541(0.02)	
J-stats		4.81(0.62)	
ARCH/LM		0.29(0.96)	

Note: Standard errors are shown in parentheses; the null hypothesis is rejected at a significance level of **, 5%, and *, 10%. The term J-STAT stand for the Hansen (1982) overidentification test. All estimations use heteroscedasticity and autocorrelation-consistent standard errors.

controls the switching regime of inflation behaviour. Output gap is the transition variable that is being employed, which suggests that the link between the inflation rate and the exchange rate gap is dependent on the gap value(positive or negative). We used a logistic transition function with a single threshold estimate because the tests used to choose an acceptable transition function more strongly reject H_0 than other options. The nonlinear smooth transition model may therefore begin to be estimated (LSTR 1), it seems to reflect our data most accurately:

$$\pi_t = \theta_f \pi_{t+1} + \theta_g \pi_{t-1} + \theta_h (y_t - y^*) + \theta_i (er_t - er^*) + [\phi_f \pi_{t+1} + \phi_g \pi_{t-1} + \phi_h (y_t - y^*) + \phi_i (er_t - er^*)] \times (1 + \exp\{-\gamma \sigma_{(y_t - y^*)} ((y_t - y^*) - c)\})^{-1} \quad (3.18)$$

The two regimes in this model correspond to negative and positive output gap values. Critical point can be seen as the parameter "c" Table 3.5 presents the estimation outcomes of the LSTR model.

3.7 From Evidence to Policy

The relationship between inflation and the unemployment rate is a crucial factor to consider when creating monetary policy. Despite previous research suggesting that the Phillips curve holds for Pakistan, we aimed to examine the linearity assumption of the Phillips curve by test-

ing the linear form used in the State Bank of Pakistan's model. Our findings indicate that for Pakistan, the Phillips curve is non-linear. Therefore, while modeling prices using the Phillips curve may be appropriate for Pakistan, it should not assume linearity.

Dynamic stochastic general equilibrium (DSGE) models are widely used by policymakers to analyze the economy and make decisions about monetary policy. However, these models assume that the economy behaves in a linear and stable manner, which means that small changes in economic variables result in predictable and proportional changes in the overall economy. Many economic phenomena, however, do not follow a linear pattern and exhibit non-linearities, such as sudden shifts, threshold effects, or feedback loops. When these non-linearities are ignored, the resulting policy recommendations based on DSGE models may be inaccurate and ineffective. Therefore, it is crucial for researchers and policymakers to consider non-linearities in economic models and ensure that policy decisions are based on accurate and reliable information. For instance, if a DSGE model assumes a linear relationship between inflation and unemployment, policymakers may implement a policy that targets a certain level of inflation, expecting to see a corresponding decrease in unemployment. However, if the relationship between inflation and unemployment is non-linear, the policy may have unintended consequences and could even worsen the situation.

Ignoring non-linearities in DSGE models can lead to biased parameter estimates, incorrect impulse response functions, and inaccurate policy recommendations. The results of studies such as Hirose & Sunakawa (2015) serve as a warning to researchers and policymakers to avoid the common practice of estimating linearized DSGE models without considering non-linearity.

CHAPTER 4

Methodology

Our research required to choose a macro model which was able to reproduce the stylized facts (e.g. GDP, inflation rate etc) upon which the policy decisions are highly dependant. Our baseline model was presented by Assenza et al. (2015) and extended by Elder M. Silva introduced a monetary authority in the model. We used the extended model as our data generating process and match the properties of simulated data with real world data for the case of Pakistan. This required adapting the model for Pakistan economy, in the process we estimated the parameter values for three variables i.e human wealth, expected demand and average capital stock. Agent based models involves a large number of parameters due to their ability of reproducing many stylized facts, our model includes a total of 20 parameters out of which we fixed the values of memory parameter (human wealth and investment) and quantity adjustment parameter and will explore the values of other variables using surrogate modeling approach. The calibration methodology that we follow for Pakistan economy was presented by Lamperti et al. (2018). They tested their methodology on two agent based models. One Brock & Hommes (1998) and the other was "Island" growth model Fagiolo & Dosi (2003). We followed the same procedure to calibrate macro agent based model for Pakistan using data for GDP. Nevertheless, a minor adjustment was introduced before embarking on the random parameter exploration. We identified adaptive equations from macro agent-based model and estimated them using real-time data from Pakistan. In this process, we held constant the parameter values that influenced agents' behavior. Subsequently, akin to the approach outlined in Lamperti et al. (2018), we executed similar calibration steps for the remaining variables. Through parameter value exploration and minimizing the distance between output of ABM and real world data,

we explored the monetary policy effects on inflation and unemployment. Later on, modifying policy rule (by adding lagged interest rate) to observe the changes in the behavior of economy. The study mainly focused on agent based simulations and its ability to match reality.

4.1 Macroeconomic Agent Based Model

Assenza et al. (2015) designed a model for closed economy including firms (Capital and Consumption), households (workers as subagent of household), commercial bank and a central bank as agents. Firms have owners (capitalist), one for each firm. The workers supply labor to firms and households buy consumption goods and save in the form of deposits. Firms demand labor, produce and sell goods and demand loans from commercial bank. Commercial bank receives deposits from households and then extends the loans to the firms. Individual forecasting

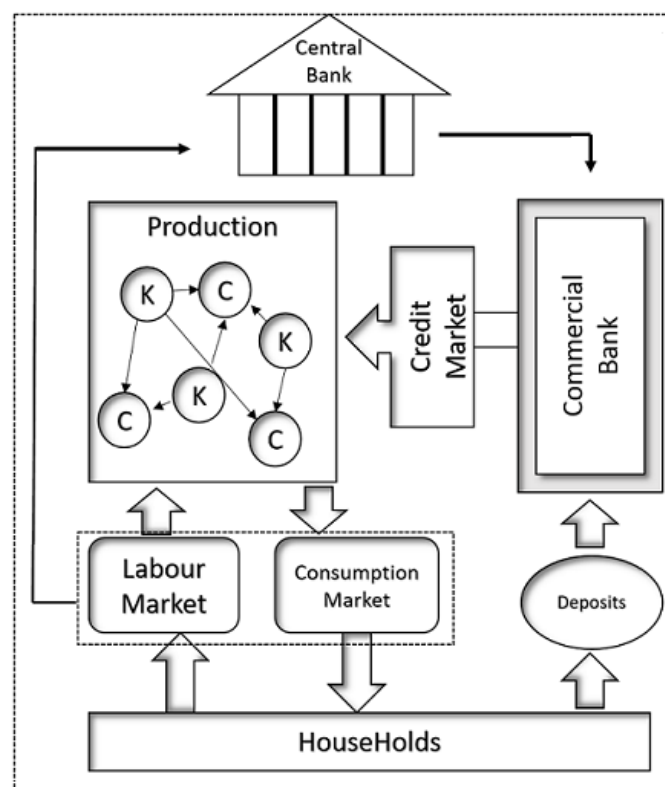


Figure 4.1: Model Mechanism

errors might cause some businesses to struggle financially, and the credit market will spread those errors to other businesses. Positive feedback made it possible for the model to replicate a succession of investments with sudden breaks. This happens as a result of the bank basing its

decision to grant credit on market risk. Therefore, even if a certain company may be in good financial standing, its access to credit may be limited as a result of the current circumstances. These conditions also have an impact on the emerging series of GDP and employment, which will likewise exhibit whiplash fluctuations. The households will choose how much to consume each period based on an adaption rule. The concept is that the household will assess its overall wealth at any given time, rather than just its current income, in order to choose how much to consume. The family will therefore update their perception of their own riches at each time (the amount deposited at the bank plus the revenue of the period). The enterprises will modify the amount produced and the prices they charge; by producing more or less, they will have an impact on the labour market (hiring or firing workers). The central bank strives to maintain a low unemployment rate while also attempting to keep prices stable. The households determine how much to consume of the items in each period based on the axiom that "proximate future is continuous of recent past." In order to make a profit, firms will endeavour to sell their goods, update their product pricing, and adjust the demand for their labour. The monetary authority use a simple Taylor rule to formulate the free interest rate. For this bank need to know the potential gross domestic product of this economy. As the labor productivity is known and constant in the in original model design, this job could be done $\bar{Y} = \alpha H$ where, \bar{Y} is potential GDP, α is productivity parameter and H is total number of workers. Since the number of workers is 3000 when model is initialized, the labor series employed will be determined using α , the productivity parameter¹. Inflation in the model depends upon current and previous price levels, whereas stochasticity is involved in price determination. The monetary authority apply the following Taylor rule:

$$r_t = \pi_t + r^* + \alpha_\pi(\pi_t - \pi^*) + \alpha_\gamma(Y_t - \bar{Y}), \quad (4.1)$$

where π , r and Y_t is rate of inflation, interest rate and current gross domestic product. r^* is natural rate of interest and π^* is inflation target. The information used by the monetary authority is also known by the other agents of this economy, except by the parameters of Taylor's rules.

¹We estimated the labour productivity by dividing Gross Domestic Product(GDP) with employment, taking annual data of variable "Gross Domestic Product(volume)" and "Employment" from International Financial Statistics(IFS) for the period 1985-2013. Average value of productivity in case of Pakistan is 0.11.

See Appendix A for complete ODD protocol of macro agent-based model.

4.2 Experimental Design and Empirical Settings

The main steps of our methodology to calibrate MABM (Macro Agent Based Model) are,

- 1) Initialize the model with specific initial conditions provided in Table. 4.1.
- 2) Adapting the behaviors of agents (consumers and firms) for Pakistan economy by fixing the parameter values estimated from real world data.
- 3) Through sampling, randomly drawing parameter values (See Table # 4.2 for theoretical support) in order to find positive calibrations. Positive calibration is a vector of parameters for which distribution of simulated interest rate series matches with real interest rate data series. Storing all such parameter vectors for case of Pakistan is part of the study.
- 4) Run the simulations to generate artificial time series of interest rates.
- 5) Set of all parameterizations which identify the same distribution of both samples (simulated and real world), we then identify a parameter vector which provide minimum distance between output of agent based model simulated data and real world interest rate data for Pakistan.

There are many different types of model parameters, however only few of them are utilised during calibration. Considering the subsequent four parameter types:

- 1) Parameters that are fixed to constants since we already know what their values are.
- 2) Elements that have been set to constants because they are essential to the canonical theory that the model's creator is attempting to illustrate
- 3) Variables whose values are speculative or uncertain.
- 4) The parameters we want the model to be insensitive. These are crucial but less frequent.

The third and fourth categories, and especially the third kind—tuning parameters with uncertain or unknown values—are primarily the focus of the calibration work. Unfortunately, there is a possibility that these parameters will be very nonlinear, have intricate relationships with other factors, and be stochastic. All of this could necessitate testing the model several times, but

ABM models can take a while to run. Automated calibration of agent-based models is therefore preferred D’Auria et al. (2020).

4.3 Initializing the Model

A medium-sized agent-based model was created, comprising of 200 consumption firms and 50 capital firms, each utilizing a position on the grid. The bank and central bank were assigned arbitrary positions that corresponded with one of the other firms, resulting in a total of 250 patches on the grid. A total of 250 capitalists who will be linked with one unique firm. The

Table 4.1: Initial Conditions to Setup and Run Agent Based Simulations.

Macro Agent Based Model with Central Bank		
Indicator	Brief Description	Initial value at $t = 0$
H	No. of Workers	3000
F_c	No. of Consumption firms	200
F_k	No. of capital goods firms	50
Z_e	No. of firms visited by unemployed worker	5
Z_c	No. of consumption firms visited by consumer	2
Z_k	No. of capital goods firms visited by C-firms	2
D_1^f	Initial liquidity of firms	10
k_1	Initial capital	10
Y_1^c	Initial production(C-firms)	5
Y_1^k	Initial production(Capital goods firms)	2
E_1^b	Initial equity of bank	3000
Y_1^h	Initial household personal asset	2

total of workers is 3,000. Since we know the total number of workers in our model, this made it

easier to keep track of employment in the economy. Table # 4.1 provides description of initial conditions to setup and run the simulation in agent based model. As described in section 2.3, model adaptation is on three variables i.e average capital stock, human wealth and expected demand. First we estimated the values of parameters (ξ, ν, ρ) using variables GDP per capita, capital stock and unemployment in Pakistan.

4.4 Surrogate Modeling in Agent-based Modelling: Literature Support

According to Windrum et al. (2007) and Fagiolo et al. (2019), the extreme flexibility of ABMs regarding, for example, Researchers have been able to examine the positive and normative repercussions of avoiding the usually oversimplifying assumptions present in most mainstream analytical models by using various forms of human behaviour, interaction patterns, and institutional setups. Macro and financial modelling have seen a trend in recent years toward richer, more complex models that target a greater number of stylized features and have a solid empirical foundation. The links between agent-based models and real-world data are a central topic in both theoretical analysis and methodological study. Numerous research have recently focused on the issue of estimating and calibrating ABMs. According to S.-H. Chen et al. (2012), ABMs must advance from stage I, which is the capacity to qualitatively evolve stylized facts, to stage II, where optimal parameter values are chosen using good econometric methodologies. When a model is sufficiently straightforward and behaves itself, one may derive a closed-form solution and calculate the distribution parameters for a particular model output (Alfarano et al. (2005), Boswijk et al. (2007)). More advanced methods are needed when complexity prohibits a closed form solution. Amilon (2008) uses the moments approach to estimate a financial market model with 15 parameters (but only 2 or 3 agents). The model is significantly impacted by the assumptions made about the noise term and stochastic components. To improve the similarity of exchange-rate models to real data, Gilli & Winker (2003) and Winker et al. (2007) propose a method and a set of statistics for constructing an objective function that can be estimated using indirect inference. Additionally, Franke & Westerhoff (2012) introduces a model competition for structural stochastic volatility models with limited parameters, while Franke (2009) refines this competition using the method of

simulated moments to estimate the parameters of an asset pricing model. Finally, Recchioni et al. (2015) evaluates the out-of-sample forecasting performance of the model they obtained through a simple gradient-based calibration approach.

Recently, parallel research has focused on developing techniques to assess how well ABM outputs can mimic reality (Marks (2013); Lamperti (2018); Barde (2017); Guerini & Moneta (2017)). A few of these contributions also offer unique metrics that may be used to create objective functions in place of longitudinal moments in an estimate context (such as the GSL-div presented in Lamperti (2018)). However, one common limitation of both these calibration/estimation and validation activities is the computation time, which is frequently extremely considerable. According to Lux (2021) the step for all of these strategies that takes the most computing is simulating the model.

Training algorithms for large macroeconomic agent-based models (ABMs) often requires lengthy Monte Carlo runs, which can take weeks to complete. Due to the considerable amount of time and resources required, most macro ABMs are not well-tested or calibrated, and previous studies have relied on relatively simple ABMs with few parameters, agents, and no stochastic draws. To achieve accurate estimation, conventional statistical approaches are utilized to reduce the number of parameters.

Theoretically, any estimator will converge to a smooth function's real value over a high-dimensional parameter space at a very slow rate (Weeks (1995); De Marchi & De Marchi (2005)). The assumptions of smoothness, linearity, and normalcy may not always hold true for ABMs despite the development of several techniques in the design of experiments literature to get around this issue (see the in-depth description in Lee et al. (2015)). Inadequately reflecting the complexity of micro-founded, multi-sector, and maybe multi-country phenomena, recent advances in agent-based macroeconomics have unfortunately resulted in the design of more complex models that call for vast sets of parameters.

New alternative methods must be developed to handle shorter computation times as well as the creation of appropriate calibration and validation procedures. By developing a computa-

tional technique that successfully trains a surrogate model to maximise specific calibration criteria or by reproducing statistical correlations between model-generated variables, our research shows how such challenges can be meaningfully related. Our method is somewhat similar to that of Dawid et al. (2014), who accelerated parameter exploration and revealed the dynamic effects of policies on the pertinent economic variables by using penalised splines techniques. On the other hand, our strategy focuses specifically on computing efficiency and is based on the two pillars of surrogate modelling and intelligent sampling.

The effectiveness of well-labeled samples can only be equaled by the most advanced surrogate modeling system. A set of parameters and the resulting output of an ABM using those parameters can be considered a labeled sample for ABMs. However, since these samples are only taken once, the process of sampling them is limited to a set number of samples, which can include various methods such as random or quasi-random sampling, Sobol sequence expansion, Latin-Hypercube sampling (Saltelli et al. (2010), and more advanced techniques. Additionally, due to the rarity of key ABM parameters and the high cost of evaluating them, it is important to carefully choose which parameter combinations to evaluate, taking advantage of the low cost of creating untested parameter combinations. This process of selecting the most informative subset of data over multiple sampling rounds is the foundation of active learning, which involves choosing parameter combinations from a large pool of untested options in order to optimize the generalization or learning performance of the surrogate model while staying within a predetermined evaluation budget (Settles (2009).

4.5 Structure of Surrogate

Surrogate models are built utilising a bottom-up, data-driven methodology. The technique often looks like the next search procedure.

- Initial sample selection (simulations to be run).
- Construction of surrogate model.
- Using machine learning algorithms, search surrogate model.
- Run the simulation, update it with the new location(s), and add it to the sample.
- Iterate above steps until the design is "good enough".

Following sections explains the design of surrogate model we will use to explore the parameter values for Pakistan using macro agent based model. Agent based models is a mapping $m : I \rightarrow O$ where I denotes the set of input parameters in Table # 4.2 and O is output of agent based model which in our case is simulated interest rate series. In real valued calibration setting of agent based model, function is defined as $\nu : O \rightarrow R$ where O is ABM output of interest rate and R denotes real values for the case of Pakistan. ν expresses the distance between the values and assigns a label to input vector $x \in I$ when the condition is met (Here the condition refers to the rule set to identify positive calibrations). We will train baseline model to efficiently approximate the value of $f(x) = \nu \circ m(x)^2$ using limited number of parameters (budget) to evaluate true behavior of ABM.

4.5.1 Training Surrogate

A trained surrogate provides an efficient way of exploring the behavior of agent based model over the entire parameter space. In order to train surrogate, three decisions which are aligned to basic machine learning methodology are important to make.

Step 1: Machine Learning Algorithm

The choice of machine learning algorithm is an important decision to make. *Extreme gradient boost tree* (XGBoost) T. Chen & Guestrin (2016) seems a better option because it avoids smoothness assumption and the ability of this algorithm learning an ensemble Breiman (2001a) of classification and regression trees (CART) Breiman et al. (1984), these properties makes it an appropriate choice to train model. CART trees treated as function, the gradient resulting from the ensemble of CART trees can be minimized. Given weight to each parameter vector and boosting them in the direction of gradient so that total loss is minimum. Boosting brings out significantly the importance of learning non-linearity (agent based models are unique in modeling non-linear systems) in the sample. The learning mechanism of the algorithm can be described as: In each round the trees will learn over the boosted parameter vector, with

²In mathematics, function composition is an operation that takes two functions f and g and produces a function h such that $h(x) = g(f(x))$. In our composition operation, the function ν is applied to the result of applying the function m to x .

Table 4.2: Parameters of Assenza Model with Monetary Authority.

Macro Agent Based Model with Central Bank		
Parameters	Brief Description	Theoretical Support
ξ	Memory Parameter	(0, 1)
τ	Dividend payout ratio	(0, 1)
r	Initial risk free interest rate	(0, 1)
ρ	Quantity adjustment parameter	(0, 1)
η	Price adjustment parameter	U(0, 0.1)
μ	Bank gross markup	(1, ∞)
α	Productivity of labor	(0, 1)
κ	Productivity of capital	(0, 1)
γ	Probability of investment	(0, 1)
ζ	Bank's loss parameter	(0, 1)
θ	Installment on debt	(0, 1)
δ	depreciation of capital	(0, 1)
ν	Investment memory parameter	(0, 1)
$\bar{\omega}$	Desire capacity utilization rate(long run)	(0, 1)
ω	wage	(0, ∞)
r^*	Natural interest rate	(0, 1)
α_π	Taylor rule parameter of inflation	(0, 1)
α_γ	Taylor rule parameter of productivity	(0, 1)

increased penalty according to boosted weights. *XGBoost trees* algorithm construct those CART trees which are specialized to deal with subset of sample which were not easy to learn

up-till current round. In simpler words the procedure considers it as ensemble of "weak" approximations that together builds strong approximations Freund (1995); Freund et al. (1996); T. Chen & Guestrin (2016).

Step 2: Sampling Procedure

Selection of an appropriate procedure for sampling can be employed to select or generate samples from parameter space is important to train surrogates. Sampling very carefully selects the parameterizations of agent based model for which the evaluation of should be based on the performance of the surrogate model. In our research we will draw the parameter samples through *quasi random Sobol*³ (Morokoff & Caflisch (1994)) sampling over the parameter space. Once the training set increases then samples will be drawn through "total variations" analysis (Saltelli et al. (2010)).

Step 3: Performance Criterion

In real valued calibration setting of agent based model our aim is to minimize the *mean squared error (MSE)*,

$$MSE = \frac{\sum_{i=1}^N (\hat{y}_i - y_i)^2}{N} \quad (4.2)$$

where \hat{y}_i are interest rate predictions of surrogate over N number of evaluations. This approach is in line with approach presented by Recchioni et al. (2015).

4.5.2 Training Procedure

Machine learning algorithm will be initialized with pre-evaluated fixed subset of parameter combinations that are drawn with sampling technique discussed in step 2 of previous section.

³The Sobol sequence is a base-2 digital sequence that fills space in a properly uniform manner. While quasi random sampling will enable the algorithm to disperse the sampling points in partially uniform manner.

These initial training sets will be evaluated through ABM in order to record *labels*⁴. These *labels* will then be used to train our first surrogate model. After training the first surrogate model, combinations of parameters will be drawn from the entire parameter space and evaluated through the ABM. These parameter combinations will be recorded as 'labels' to train the subsequent surrogate model. A random subset of points, denoted as x_i , will be selected from the surrogate model's predicted positive calibrations, and their true labels, denoted as y_i , will be evaluated using the agent-based model. This process will be repeated, and new points will be added to the training set for the subsequent surrogate model in each round of training. This iterative process will continue until the desired training budget is achieved.

Criteria for identification of Positive Calibrations

A positive calibration by definition is a parameter vector $x \in I$ whose label is contained in set C s.t $C = \{x : \nu(x) \leq \alpha\}$. The condition for positive calibration in our research will be employed on Kolmogorov-Smirnov two sample test. It is non-parametric test with null hypothesis that "two samples are drawn from the same distribution". Test do not specify the common distribution beforehand and test statistics can be calculated as:

$$D_{RW,S} = \sup_r |F_{RW}(r) - F_S(r)| \quad (4.3)$$

where r is the interest rate series, F_{RW} and F_S are empirical distribution function of real world (RW) and simulated (S) samples. In real valued calibration setting when $P(D > D_{RW,S})$, at 5% level of significance, will be considered as an expression of models fit to real data.

4.6 Estimating Adaptive Equations for Pakistan Data

In our preliminary data analysis we are interested to find the values of memory parameter of human wealth(ξ), memory parameter of investment(ν) and quantity adjustment parameter(ρ) for the case of Pakistan. Through estimating the adaptive equations we get values of memory parameters (human wealth and investment) and quantity adjustment parameters. These

⁴In machine learning, *label* is the output we get after training the model.

parameter values remained fixed in our study. The main data sources are World Development Indicator(WDI) and International Financial Statistics(IFS).

4.6.1 Memory Parameter of Human Wealth

Adaptive equation of Human wealth suggest that it is weighted average of current and past income,

$$\bar{Y}_{c,t} = \xi Y_{c,t-1} + (1 - \xi) Y_{c,t}. \quad (4.4)$$

In order to estimate the parameter value for the case of Pakistan, we considered macroeconomic indicator of wealth and income as GDP per capita (PPP) and GNI per capita respectively. The data was collected from WDI on annual frequency for the period starting from 1990 to 2019. Parameter (ξ) turned out to be significantly positive when we regress wealth on lagged value of income. Our estimated equation becomes,

$$\widehat{Y}_{c,t} = 0.86 Y_{c,t-1} + (1 - 0.86) Y_{c,t} \quad (4.5)$$

with adjusted R squared equals 0.75 and DW stats ⁵ is 2.3 with low p-value.

4.6.2 Estimating Quantity Adjustment Parameter

Adaptive equation involving quantity adjustment parameter ρ suggest that it is the weight assigned to difference of actual and expected demand. Demand indicator of an economy is unemployment rate. We collected data of unemployment rate from International Financial Statistics (IFS) from 1985 to 2019. To get values of expected demand, we first estimated an ARIMA model of unemployment rate in order to have a series of expected demand. Following are the ACF, PACF and Fitted ARIMA with drift.

ACF and PACF plots assists in determining the appropriate order of the AR (autoregressive) and MA (moving average) components for the ARIMA model. The significant lags in the plots help determine the p, d, and q values for the ARIMA(p, d, q) model, where p represents the

⁵The Durbin Watson (DW) statistic is a test for autocorrelation in the residuals from a statistical regression analysis.

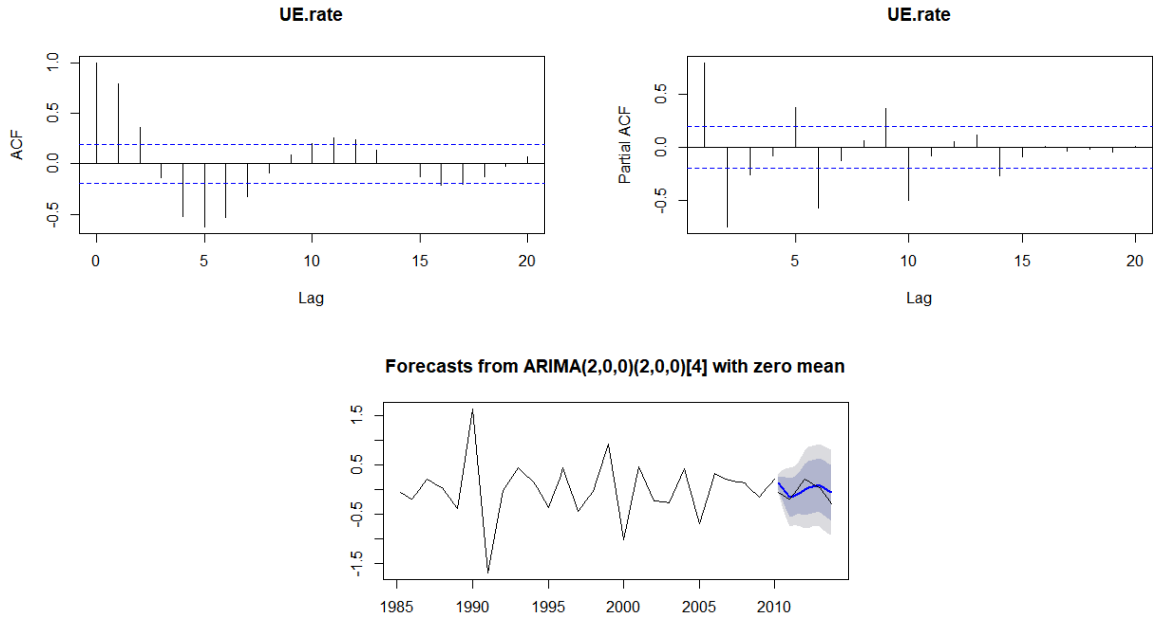


Figure 4.2: Forecasting Unemployment rate using ARIMA Model

order of the AR component, d represents the order of differencing, and q represents the order of the MA component. The inferred order of the ARIMA model indicates an autoregressive order of 2, no differencing, and a moving average order of 2. The model also incorporates seasonality with a period of 4. The inclusion of the seasonal component helps capture and account for the recurring patterns that occur at quarterly frequency in the data.

Once we obtained a series of expected demand we then estimated the following equation,

$$Y_{i,t}^e = Y_{i,t-1}^e + \rho(Y_{i,t-1}^d - Y_{i,t-1}^e). \quad (4.6)$$

ρ is significant with value 0.36 (p value=0.00). Adjusted R squared is 88 percent and DW stats is 3.01.

4.6.3 Estimating Memory Parameter of Investment

In order to estimate the following adaptive equation,

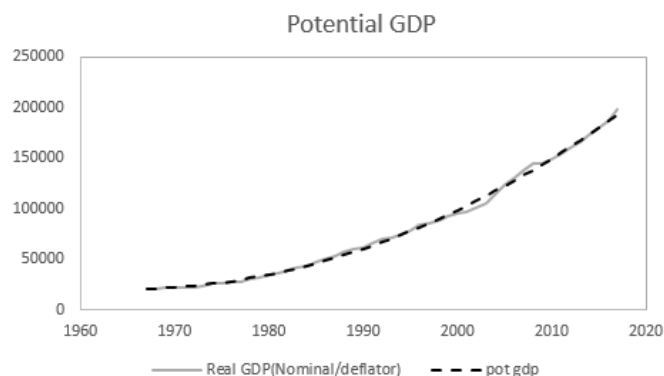
$$\bar{K}_{i,t-1} = \nu \bar{K}_{i,t-2} + (1 - \nu) \omega_{i,t-1} K_{i,t-1}, \quad (4.7)$$

we first constructed a series of capacity Utilization rate(ω). Rate is defined as,

$$\text{Capacity Utilization Rate} = \frac{\text{Real Output}}{\text{Potential Output}} \times 100.$$

Real GDP is ratio of nominal GDP and GDP deflator and demonstrate a quadratic trend.

We have collected data of macroeconomic indicators 1) Gross Capital formation (% of GDP),



2) Nominal GDP and 3) GDP deflator for over lapping time period and calculated ω . using the data for the period of 1969-2019. Estimated adaptive equation for the case of Pakistan is,

$$\widehat{K}_{i,t-1} = 0.83\overline{K}_{i,t-2} + (1 - 0.83)\omega_{i,t-1}K_{i,t-1} \quad (4.8)$$

with Adjusted R squared equals 68 percent but DW stats close to zero indicate autocorrelation in residuals.

4.7 Estimating Taylor Rule for Pakistan

In order to carry out the estimation procedure of monetary policy rules(Taylor rule without smoothing term and Taylor rule with smoothing term), we used OLS methodology by using the quarterly data for the period 1990Q1 to 2019Q4. Table describes static and dynamic version of Taylor rule fits in case of Pakistan. Money Market rate as policy instrument, CPI inflation and real gap(difference of actual and potential GDP where potential GDP is calculated by fitting quadratic trendline.) are key variables used to estimate the rules. The residual series of estimated rules were found stationary. Low DW stats in static Taylor rule indicates the

other objectives of SBP, which was obvious in the second specification when we introduced a smoothing term.

	Taylor Rule without Smoothing term	Taylor Rule with smoothing term
Constant	7.94(0.00)	2.00(0.001)
Inflation rate	0.387(0.066)	0.26(0.059)
Output gap	0.058(0.007)	0.023(0.044)
Lagged Interest rate		0.707(0.000)
Adj. R square	0.18	0.63
DW stats	0.59	2.34
F-stats	13.83(0.000)	64.85(0.000)

The table of parameters(Table. 4.3) may now be reconsidered where we have fixed parameters and parameters that need to be investigated using surrogate approach once we have calculated the behavioral parameters and parameters of the Taylor rule.

4.8 Software Procedure Extension

The utilization of both *Python* and *NetLogo* is integral to transforming the agent-based model into a functional software program. While the foundational Macro agent-based model was developed within the *NetLogo* environment, the efficient algorithms available in *Python* were harnessed to systematically explore the parameter space. To address priorities such as speed, scalability, and user-friendliness, the *NL4Py* library was introduced. Designed for seamless orchestration of *NetLogo* simulations, *NL4Py* empowers even domain experts with limited coding background to efficiently parallelize *NetLogo* simulations. This integration is fortified by the extensive suite of open-source Python packages for machine learning and analytics Gunaratne & Garibay (2018), amplifying the capabilities of the overall approach. Complex adaptive systems, as represented through *NetLogo*, inherently yield a multitude of potential outcomes even within seemingly basic models. By leveraging *NL4Py*, the process

Table 4.3: Parameters of the Assenza model with monetary authority after the use of data to adjust the parameters of behavioural equations for Pakistan economy

Macro Agent Based Model with Central Bank		
Parameters	Brief Description	Theoretical Support
ξ	Memory Parameter	0.86
τ	Dividend payout ratio	(0, 1)
r	Initial risk free interest rate	(0, 1)
ρ	Quantity adjustment parameter	0.36
η	Price adjustment parameter	U(0, 0.1)
μ	Bank gross markup	(1, ∞)
α	Productivity of labor	(0, 1)
κ	Productivity of capital	(0, 1)
γ	Probability of investment	(0, 1)
ζ	Bank's loss parameter	(0, 1)
θ	Installment on debt	(0, 1)
δ	depreciation of capital	(0, 1)
ν	Investment memory parameter	0.83
$\bar{\omega}$	Desire capacity utilization rate(long run)	(0, 1)
ω	wage	(0, ∞)
r^*	Natural interest rate	(0, 1)
α_π	Taylor rule parameter of inflation	(0, 1)
α_γ	Taylor rule parameter of productivity	(0, 1)

of parallelizing experiments in Python was streamlined. This optimization was crucial for leveraging high-performance computing infrastructure and conducting the substantial volume of simulations required within a reasonable time frame. A significant advantage lies in Python's repository of robust machine learning and analytical tools, which are freely accessible to researchers and practitioners alike.

Pseudo Code

Set

- Agent Based Model $ABM \in \mathbb{R}^J$
- Sampling distribution $\nu \in \mathfrak{X}^J$
- Calibration function $C(\cdot)$
- **Fixing parameters of adaptive equations**
- Learning algorithm A , with parameters Θ
- Evaluation budget B
- Initial training set size $N \ll B$
- $X^{training} \in \mathbb{R}^{N \times J}$
- Calibration labels $Y^{training} \in \mathbb{R}^N$ real-valued outcome case (at least 1 positive calibration)
- Hyper-parameter optimization algorithm (HPO)

Initialize

- Per-round sampling size $S \ll B$
- Per-round out-of-sample size $K \gg B$

while $|Y| < B$, **repeat**

- 1) $\Theta = HPO(A(\Theta, X^{training}, Y^{training}))$
- 2) Draw out of sample points $X^{OOS} \in \mathbb{R}^{K \times J} \sim \nu$
- 3) Select $X^{sample} \in \mathbb{R}^{S \times J}$ from X^{OOS}
- 4) Evaluate $X^{training} = X^{training} \cup X^{sample}$
- 5) Evaluate $Y^{sample} = \{C(ABM(X_i^{sample}))\}_{i=1 \dots S}$
- 6) Evaluate $Y^{training} = Y^{training} \cup Y^{sample}$

end while

To comprehend intricate adaptive systems, external control of agent-based models remains pivotal. These inquiries frequently necessitate a substantial number of resource-intensive sim-

ulation runs. While the preferred choice for many agent-based modelers is *NetLogo*, it lacks direct *Python* API accessibility. In response to these requirements, *NL4Py* emerges as a *Python* library, designed with a focus on speed, scalability, and user-friendliness, enabling the concurrent execution of *NetLogo* simulations. *NL4Py* empowers domain scientists, even those with limited coding experience, to effectively parallelize *NetLogo* simulations utilizing a wide array of open-source *Python* machine learning and analytics packages. Our proposed methodology optimizes parameter space exploration by efficiently navigating primarily along stiff directions. We substantiate the effectiveness of our approach by applying it to a medium-sized agent-based model, where it successfully captures the entire spectrum of potential unemployment rate dynamics. This innovative strategy, when employed with agent-based models, holds the promise of refining parameter sensitivity studies and cultivating a more comprehensive and robust understanding of their underlying properties.

4.9 Considerations for ABM policy experiments

ABMs may be set up to be a very effective tool for addressing policy queries in more practical, adaptable, and modular frameworks. ABMs do, in fact, offer a number of benefits over neoclassical tools like the DSGE model, which we categorise into two divisions in the sections that follow: theory and empirics Fagiolo & Roventini (2012a). Alternative world generators, or speculative DGPs that resemble the unknown one, are what ABMs are. In contrast to neoclassical models, the structure of ABMs makes it easier to connect them to the data. There are two methods to go about this. The first step is to verify the inputs of ABMs, which entails fine-tuning modelling assumptions about specific behaviours and interactions to make them more comparable to the ones that have been observed. By, for example, limiting the space of parameters, individual behaviours and interactions, and beginning circumstances to those that let the model to replicate the stylized facts of interest, one may verify the model on the output side. This enables a level of realism that is far higher than that displayed by, for example, DSGE models Farmer & Foley (2009). Additionally, agent-based models can target a rich ensemble of stylized facts at many levels of aggregation due to the theoretical flexibility (i.e. micro vs. macro regularities). In further detail, macroeconomic agent-based models can often sim-

ulate macroeconomic stylized facts including endogenous growth and economic fluctuations, the onset of financial crises and severe downturns, relative volatilities, and co-movements of macro aggregates at business cycle frequencies. The same model may also jointly account for stylized microeconomic facts such as wage and income inequality, firm productivity dynamics, company investment patterns, firm size and growth rate distributions, and firm productivity dynamics and dynamics. This is a significant benefit of ABMs over DSGE models cannot reproduce any micro empirical regularities by construction given the representative-agent assumption and are often created to explain just one or two single macro stylized facts (see the debate in Aoki (2006) for additional information).

Consider the ABM descriptive analysis process shown in figure 4.3. Keep in mind that mi-

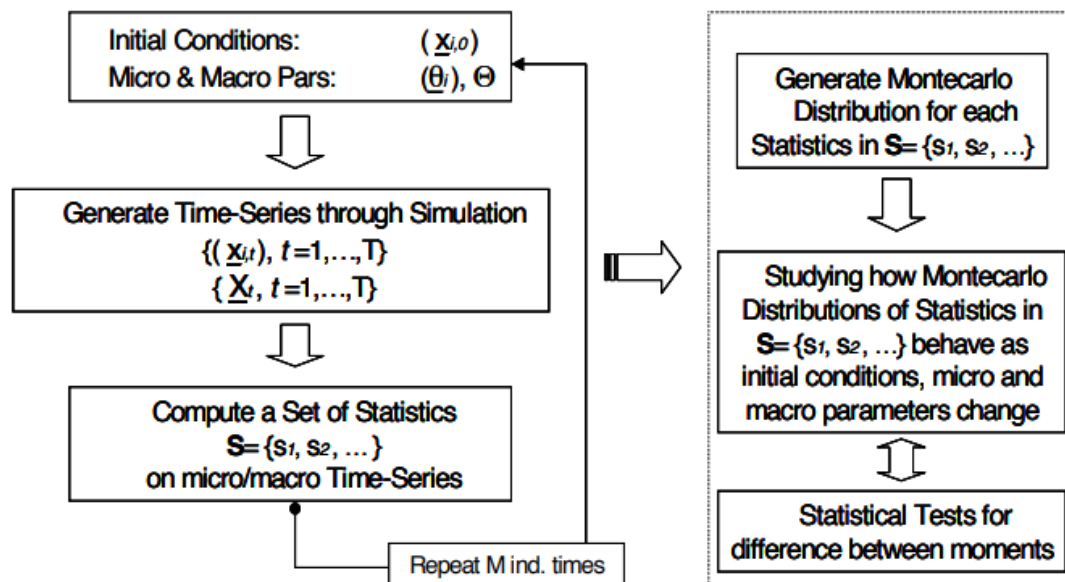


Figure 4.3: A representative process for analysing an agent-based model's results.

Source: Dosi & Roventini (2017)

cro and macro parameters can be created to imitate real-world important policy factors like tax rates, subsidies, interest rates, money, etc. as well as other important behavioural measurements affecting individual incentives in growth, innovation, other technologically related policies. Additionally, starting circumstances could serve as initial endowments and hence characterise various distributional settings. Additionally, alternate institutional, market, or industrial configurations may be simply represented by the interaction and behavioural norms used by economic players. Since all of these components are freely interchangeable, it is possible to investigate a

vast array of alternative policy experiments and rules, the results of which can be evaluated in both qualitative and quantitative ways (for example, by performing conventional statistical tests on the distributions of the statistics in S). All of this could be accomplished while maintaining the model's capacity to replicate both microeconomic empirical regularities (such as firm size distributions, firm productivity dynamics, firm investment patterns, etc.) and existing macroeconomic stylized facts (e.g., persistence of output growth-rate fluctuations, relative standard deviations, cross-correlations, etc.).

Results and Discussion

5.1 Parameter Importance

Using the XGBoost technique, we can undertake parameter sensitivity analysis for free as part of our surrogate modelling process. In particular, the machine learning technique offers a simple method for evaluating the surrogate's explained variance in accordance with the percentage of instances a parameter was "split-on" in an ensemble (Archer & Kimes (2008); Breiman (2001b); Silva et al. (2021)). Since each tree is constructed according to an optimal splitting of the potential values for a certain parameter vector and it gradually focuses on difficult-to-predict samples, splits indicate the relative relevance of parameters in defining the output criteria of the ABM. Quantitative evaluation of the surrogate model's sensitivity to the user-defined conditions given by the parameter is thus provided by the relative number of splits over a particular parameter. This also makes it possible to prioritise parameters according to how crucial they are in creating model behaviour that complies with the user-specified requirements. The results of this approach should be understood as a rank-based statistic because it is non-parametric. Particularly, only the particular instantiation of the ensemble is characterised by the relative significance values related to the number of splits. The generated counts shed light on each parameter's relative performance. The number of splits for each parameter would vary depending on the number of trees. By virtue of the law of large numbers, as the number of trees gets closer to infinity, the number of splits will eventually reach the real ratio of splits per parameter (See Figure 5.1). Comprehending parameter importance in Agent-Based Models is pivotal for refining model reliability, precision, and applicability in decision-making. By identifying influential

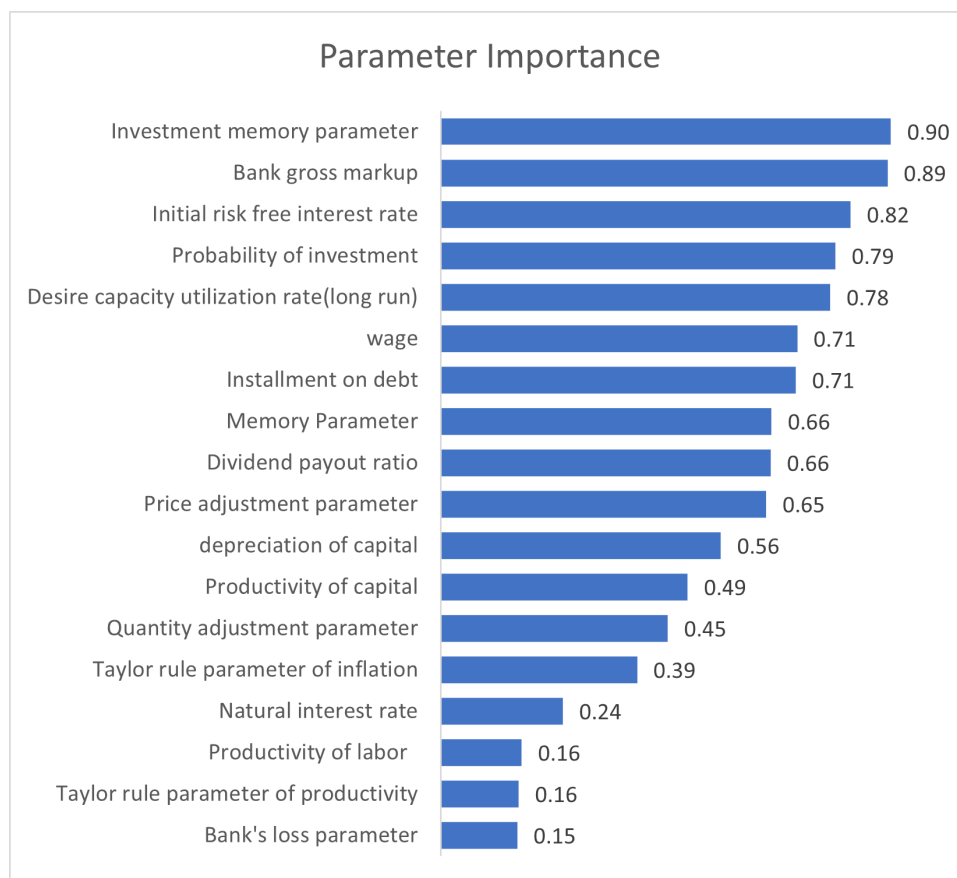


Figure 5.1: Importance of each parameter (feature) in shaping behaviour of the Macro Agent Based Model.

parameters, researchers can enhance model calibration, policy formulation, and predictive capabilities, further enhancing the utility of ABMs in tackling multifaceted real-world situations. The horizontal stretch of each bar in figure 5.1 indicates the magnitude of impact a parameter holds: longer bars signify parameters with more significant effects, while shorter ones denote those of lesser importance. This ranking allows us to readily identify the pivotal drivers behind the model's behavior. The ranking also aids in pinpointing which parameters merit closer scrutiny for accurate calibration, efficient optimization, or effective decision-making. By highlighting the relative contributions of individual parameters, these plots empower researchers to focus attention, and extract policy insights with a clearer understanding of the underlying system's sensitivities and intricacies. It's important to note that the results presented in figure 5.1 are contingent on the chosen analysis methods and assumptions.

5.2 Robustness Analysis

Now, to evaluate the robustness of our training method in relation to several surrogate models. More particular, we contrast the more straightforward and popular Logit surrogate with the XGBoost surrogate utilised in the earlier research. Our comparative exercise is carried out using a macro agent-based model that is fully stochastized and has an additional Monte Carlo (MC) on the seed parameter that determines the stochastic terms of the model.

We employed a more relaxed requirement, wherein we focused on maintaining a positive average growth rate (AGR) of over 0.5 percent, while concentrating on a binary outcome setting that resulted in lower outcomes. This allowed for a comparison with the outcomes of a previous exercise in Fagiolo & Dosi (2003). To obtain an average GDP growth rate as our output variable, we allocated a budget of 500 evaluations of the actual macro ABM and conducted a Monte Carlo experiment with a size of 100 for each parameter combination. This exercise is more comprehensive than the previous sections, as we created a surrogate model that established the correlation between the parameters and the Monte Carlo average of their ABM evaluations. However, it should be noted that this process takes considerably longer to converge to the required statistic than the parameter combination in the actual ABM. We used the XGBoost and the quicker, less accurate Logit as surrogates to show the effectiveness of our active learning

methodology. The former, which was used in the studies performed in the preceding sections, offers more precision in return for more expensive processing expenditures. This latter model is a typical statistical one that is frequently used in this kind of regression analysis. The primary objective is to enhance the accuracy of the resultant models, i.e., to increase the number of actual positive evaluations that are accurately predicted prior to being reviewed. Therefore, while training the surrogate model, the F1-score was set as criteria to evaluate the performance of these surrogate models. This is a crucial issue since applying the suggested strategy in practice prevents us from evaluating every point in our sample space. Only positive predictions are given labels in real-world evaluation, and the performance that results can only be assessed in terms of true positives and false positives, with a bias for the former.

Starting with a random sample of 1000000 points from the parameter space, the experiment begins. The first surrogate is provided with 35 labelled parameters chosen at random from the 1000000 points, in accordance with the total-variation sampling approach in Saltelli et al. (2010)), given the set budget of 500 evaluations of the genuine ABM for both the XGBoost and Logit. A surrogate will then be fitted to the labelled parameters across a number of cycles in order to forecast a labelling over the 1000000 points. The suggested technique will then use the projected labels to choose points that will be added to the collection of labelled points at the end of each round. The process is repeated until the budget for genuine assessments has been achieved, at which point a new surrogate is learnt. The suggested method yields accuracy values for Logit and XGBoost of 95.89 percent and 93.28 percent, respectively, that are similar. Since the precision of the two surrogates barely differed from one another, it is possible that our training technique still yields satisfactory results even when the common Logit statistical model is used. However, when the Platt scaling method is used to adjust the XGBoost projected probability, its accuracy increases to 99.35 percent. Additionally, scaled XGBoost performs far better than Logit in terms of true vs. false positives. one could like the quicker Logit surrogate when erroneous positives are affordable due to its lower computational costs and lack of requirement for hyperparameter tweaking when utilising the more accurate XGBoost surrogate. Nevertheless, both the Logit and XGBoost examples benefit from suggested surrogate modelling method(See Table 5.1).

Table 5.1: Performance of surrogate modelling utilising the learning process

Surrogate Algorithm	True Negative	False Positives	False Negatives	True Positives	Precisions
logit	62	22	61	355	94.17%
XGBoost	178	17	0	305	94.72%
XGboost (scaled)	193	2	0	305	99.35%

Only True and False Positives are provided when positive anticipated calibrations are examined.

5.3 Policy Experiments

We proceeded with the requirement to endogenize the base interest rate, which is exogenous in the original model, in order to take monetary policy experiments into consideration in our investigation (Suggested by Silva et al. (2021)). First, let's assume that the central bank uses Taylor's rule (Taylor (1993)) to determine the interest rate. The base interest rate influences the interbank interest rate, which the banks take into account when extending loans to businesses (as the MABM is designed). According to Taylor's rule, the central bank raises interest rates during periods of rising inflation but lowers them during periods of widening output gaps. In order to see the effects of inflation and unemployment, we adjusted the rule by include a lag of the dependent variable.

5.3.1 Taylor Rule without smoothing term (Baseline)

The parameter settings for doing experiment simulations are shown in Table 5.2. When compared to a normal distribution, the distribution seems to be asymmetric, and the peaks exhibit a high kurtosis. Histogram showing the unemployment rate throughout experiments using traditional monetary strategies. We assume an experiment to have 20,000 total periods (10 runs with 2500 periods each). The histogram for overall unemployment utilises bins of 50 (Figure 5.2). By adjusting the number of repetitions and the length of the time series, we aimed to strike a balance between capturing meaningful trends and insights within the constraints of available

Table 5.2: Policy experiment 1; Explored parameter values using machine learning methodology(See Chapter 4)

Macro Agent Based Model with Central Bank		
Parameters	Brief Description	Theoretical Support
ξ	Memory Parameter	0.86
τ	Dividend payout ratio	0.20
r	Initial risk free interest rate	0.01
ρ	Quantity adjustment parameter	0.36
η	Price adjustment parameter	0.08
μ	Bank gross markup	1.20
α	Productivity of labor	0.50
κ	Productivity of capital	0.33
γ	Probability of investment	0.25
ζ	Bank's loss parameter	0.002
θ	Installment on debt	0.025
δ	depreciation of capital	0.02
ν	Investment memory parameter	0.83
$\bar{\omega}$	Desire capacity utilization rate(long run)	0.85
ω	wage	1.00
r^*	Natural interest rate	0.01
α_π	Taylor rule parameter of inflation	0.387
α_γ	Taylor rule parameter of productivity	0.058

resources. This strategy enables us to manage the computational demands of each run while still obtaining valuable insights from the simulation results. While a larger number of repetitions could potentially enhance statistical significance, the practicalities of computational efficiency guided our decision-making process. It's an approach often taken to ensure that our analysis remains feasible and produces relevant findings given the available computing environment.

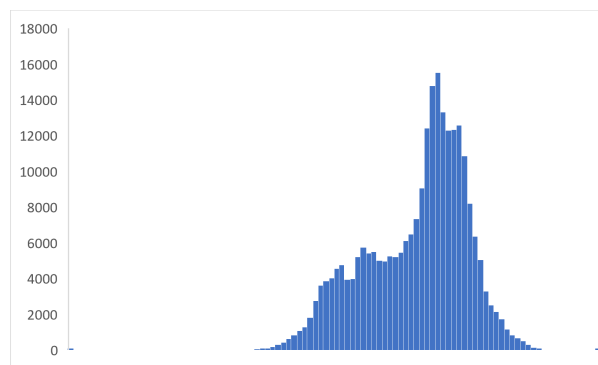


Figure 5.2: Histogram of Unemployment rate across monetary policy experiment (When Central bank uses Taylor rule without smoothing term.)

5.3.2 Taylor Rule with smoothing term

The parameter settings for doing experiment simulations are shown in Table 5.3. A change in monetary policy does not significantly affect unemployment. Histogram showing the unemployment rate throughout experiments using traditional monetary strategies. We assume an experiment to have 20,000 total periods (10 runs with 2500 periods each¹). The histogram for overall unemployment utilises bins of 50. Figure 5.3). Agent-Based Modeling (ABM) heavily relies on visualization techniques for output analysis. However, in our specific case, we encountered a situation where the discernible changes resulting from the simulation were nearly impossible to identify through conventional visualization methods, such as plotting distributions or graphs. This predicament led to the outcome where figure 5.2 and figure 5.3 appeared identical due to the lack of observable differences. The key findings are shown in Tables 5.4.

¹By adjusting the number of repetitions and the length of the time series, we aimed to strike a balance between capturing meaningful trends and insights within the constraints of available resources. This strategy enables us to manage the computational demands of each run while still obtaining valuable insights from the simulation results. While a larger number of repetitions could potentially enhance statistical significance, the practicalities of computational efficiency guided our decision-making process. It's an approach often taken to ensure that our analysis remains feasible and produces relevant findings given the available computing environment.

Table 5.3: Policy experiment 2; Explored parameter values using machine learning methodology(See Chapter 4)

Macro Agent Based Model with Central Bank		
Parameters	Brief Description	Theoretical Support
ξ	Memory Parameter	0.86
τ	Dividend payout ratio	0.20
r	Initial risk free interest rate	0.01
ρ	Quantity adjustment parameter	0.36
η	Price adjustment parameter	0.008
μ	Bank gross markup	1.20
α	Productivity of labor	0.50
κ	Productivity of capital	0.33
γ	Probability of investment	0.25
ζ	Bank's loss parameter	0.002
θ	Installment on debt	0.025
δ	depreciation of capital	0.02
ν	Investment memory parameter	0.83
$\bar{\omega}$	Desire capacity utilization rate(long run)	0.85
ω	wage	1.00
r^*	Natural interest rate	0.01
α_π	Taylor rule parameter of inflation	0.26
α_γ	Taylor rule parameter of productivity	0.023
α_L	Taylor rule coefficient of lagged interest rate	0.707

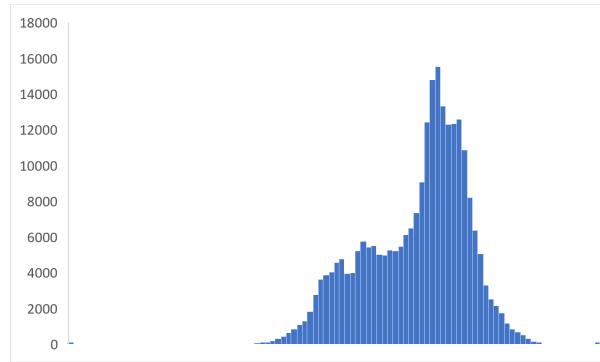


Figure 5.3: Histogram of Unemployment rate across monetary policy experiment (When Central bank uses Taylor rule without smoothing term.)

The numbers reflect the proportions between the average values of each variable calculated in a policy scenario and those attained in the benchmark scenario. Additionally, we give the p-value for the t-test, whose null hypothesis states that the ratio between the two scenarios is equal to 1. Failure to do so would mean that the two scenarios there compared result in the same results for the study variable. A ratio that is much greater (lower) than one would suggest that the first scenario performs better (is better) than the second in the pairwise comparison if the variable has a positive economic connotation, such as GDP growth. The inverse relationship between an economic variable and macroeconomic performance, such as unemployment, is true. The results of our simulation exercises indicate that, under the assumption that historical accounting standards are in place, the application of the Taylor Rule with a smoothing term results in some improvement compared to the baseline scenario. In particular, when a smoothing term is in place, the average growth rates of GDP, consumption, and investment are higher. This is because the central in the model permits the banks to maintain lower levels of the bad-debt-to-equity ratios, upholding thus their function as fund producers through their lending activities. Overall, this promotes expansion of the real sector of the economy. A higher inflation rate, which is 10% higher than in the baseline scenario while the unemployment rate stays the same, is the price of such a strategy. The findings in Table 5.5 indicate that the Taylor rule with the smoothing term compared to Taylor Rule without a smoothing component, is more efficient while maintaining a positive average growth rate (AGR) of over 0.5 percent. While the unemployment rate continues to be close to 10%, a policy rule with a smoothing period can increase GDP by about 1.4 percent. The choice of a monetary rule with a smoothing term is based on

Table 5.4: Comparing effects of monetary policy rules on aggregate variables

Variable	GDP	Unemployment	Inflation
Taylor Rule (with smoothing term vs without smoothing term)			
ratio	0.999	2.328	0.976
p-value	0.998	0.008	0.336

the specific characteristics of the Pakistani economy, which include high levels of volatility and uncertainty. This approach provides stability and predictability in a context where there is significant variability in economic conditions. By incorporating a memory component in the

Table 5.5: Adapted agent-based model policy scenarios for Pakistan

	No policy	Taylor Rule	Taylor rule with smoothing term
GDP	-2.1%	-2.4%	-1.4%
Inflation	0.2%	0.0%	0.5%
Unemployment	10.2%	10.6%	9.1%

Average value of 20 runs, each run ending after 5000 ticks.

decision-making process of agents, our model captures the notion that economic agents often consider past experiences and information to shape their expectations and behaviors. In the context of monetary policy, the inclusion of a smoothing term allows for a gradual adjustment in interest rates, which helps to mitigate potential disruptions and provide stability to the economy.

5.4 Implications of Taylor Rule Variations and Relevance to Monetary Policy

This section aims to discuss the implications of Taylor rule variations and their relevance to the broader discourse on monetary policy, despite the simplified representation of the State Bank’s policy framework. While Taylor rule testing is commonly conducted in Dynamic Stochastic General Equilibrium (DSGE) models, the distinct assumptions and characteristics

of Agent-Based Models (ABMs) offer unique insights. DSGE models provide a structured framework for analyzing macroeconomic phenomena, including monetary policy. However, the introduction of agent heterogeneity, bounded rationality, and interaction dynamics in ABMs allows for the exploration of emergent behaviors and non-linear effects. This sheds light on how policy rules impact diverse agents within complex economic systems. Consequently, ABMs enhance our understanding by incorporating real-world complexities that might not be fully captured by traditional DSGE models, thereby enhancing our ability to comprehensively assess the implications of Taylor rules.

The ABM experiment assumes that the full complexity of the State Bank's monetary policy framework is not replicated, allowing for a focused analysis of the impact of Taylor rule variations on agent decision-making and macroeconomic outcomes. This simplification enables insights into the general effects of different rule scenarios, rather than attempting to replicate the State Bank's specific policies.

The primary objective of the experiment is to examine how agents' behaviors and macroeconomic variables respond to different Taylor rule variants. Although the explicit consideration of the State Bank's monetary policy is absent, the experiment provides valuable insights into the implications of alternative rule scenarios. By isolating the impact of the Taylor rule variations, this research contributes to the broader understanding of monetary policy rules and their effects.

Despite the simplified representation of the State Bank's policy framework, the findings of this research hold relevance to the broader discourse on monetary policy. By studying the effects of different Taylor rule scenarios, the experiment provides insights into the relationship between interest rate rules and macroeconomic outcomes. These implications can inform policymakers and contribute to the ongoing discussions surrounding monetary policy design and implementation.

It is important to acknowledge the limitations of the experiment. Future extensions of this research could incorporate a more detailed representation of the State Bank's policy framework, enabling a comprehensive analysis that aligns more closely with actual monetary policy practices. This would provide a more nuanced understanding of the State Bank's policy

implications within an agent-based model framework.

5.5 Conclusion

Agent-based modeling can be particularly useful for studying complex economic systems that are characterized by multiple interdependent agents, such as financial markets, supply chains, or social networks. Such systems are often difficult to model using traditional econometric approaches, which rely on simplifying assumptions and ignore many of the complexities and nonlinearities that arise in real-world settings. In contrast, agent-based models allow researchers to capture these complexities by modeling the behavior of individual agents and their interactions with each other.

One of the key advantages of agent-based modeling is its ability to generate a wide range of possible outcomes and scenarios, including extreme events that are difficult to predict using traditional models. This is because the intrinsic randomness and frictions in agent-based models can lead to nonlinear dynamics and emergent phenomena that are not easily captured by linear models. By exploring different scenarios and policy responses, policymakers can better understand the potential consequences of different decisions and develop more robust strategies.

In the case of Pakistan, the model was adjusted at the aggregate level, using means and standard deviations of GDP growth, unemployment rate, and inflation rate. The limitation of policy space meant that different monetary policy rules could only produce limited effects on their own. However, despite these limitations, the customized model showed promise for policy analysis and scenario development, and the analysis can be expanded in many different directions. Incorporating a smoothing term into the Taylor rule can enhance the performance of agent-based models in the context of the Pakistani economy. This can help to mitigate the impact of short-term fluctuations and enable the central bank to focus on longer-term trends, resulting in more stable policy outcomes and reduced volatility in key economic variables such as inflation and output. Although the effects of the smoothing term may not be immediately visible in the short run, the long-term benefits can be significant in improving the overall performance of the model when applied to Pakistan's unique economic conditions. Therefore, policymakers and

researchers should consider the inclusion of a smoothing term in the Taylor rule when utilizing agent-based models to examine policy in Pakistan, as it can lead to more reliable and robust results that are specific to Pakistan's economic context.

Our investigation further highlights certain limitations associated with the utilization of the machine learning surrogate approach proposed by Lamperti et al. (2018) within the realm of agent-based modeling. Our experimental results bring to light the possibility that the surrogate approach may not offer the same level of efficiency as posited by the authors, potentially necessitating considerable computational resources to achieve accurate predictions. Although Lamperti et al. (2018) methodology introduces an innovative solution to mitigate the computational complexities inherent in Agent-Based Model (ABM) evaluations, its effectiveness encounters constraints when confronted with more intricate scenarios. While the methodology convincingly demonstrates its prowess in trials involving relatively modest ABMs, each characterized by a maximum of 10 parameters, its efficiency gradually wanes as it grapples with larger and more complex models. Particularly noteworthy is its less potent computational cost reduction performance when applied to a sizable ABM encompassing 22 parameters. This observed decline in computational efficiency presents a potential impediment to its applicability in situations necessitating comprehensive parameterization. Our findings also illuminate the necessity for a judicious consideration of the constraints surrounding the construction of the surrogate. It emerges that constructing the surrogate with an excessively extensive range of parameter combinations and an inadequately sized training sample could be unfeasible. Consequently, researchers venturing into the domain of machine learning surrogates must vigilantly deliberate upon the associated computational demands and limitations intrinsic to this approach. Adequate computational resources and substantial training data are essential prerequisites to ensuring its successful implementation.

In conclusion, agent-based modeling is a powerful tool for analyzing complex economic systems, but it requires careful consideration of model assumptions and parameterization. Our study demonstrates the potential of agent-based modeling for policy analysis in the context of Pakistan, but also highlights the limitations of using machine learning surrogates in this approach. Policymakers and researchers should be aware of the computational costs and limita-

tions of this method and ensure that they have adequate resources and training data. Additionally, our analysis suggests that incorporating a smoothing term into the Taylor rule can improve the accuracy and reliability of agent-based models in Pakistan's unique economic context, leading to more stable policy outcomes and reduced volatility in key economic variables. Overall, our study contributes to the ongoing discussion on the use of agent-based modeling and machine learning in economics and underscores the need for further research to refine and improve these approaches.

While the State Bank of Pakistan plays a critical role in the formulation and implementation of monetary policy, my research focuses specifically on the impact of monetary policy rules in an agent based economy which is adapted for Pakistan. As such, my analysis is focused on the implications of monetary policy rules for key macroeconomic variables such as inflation, output, and employment, rather than on the specific policies of the State Bank itself. While the State Bank's policy decisions may have an impact on these variables, my research seeks to identify the most effective policy rules for achieving the desired outcomes, regardless of the specific policy decisions made by the State Bank.

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ODD: Macroeconomy from the bottom up: Studying the implications of a monetary authority's behavior in a macroeconomic agent-based model with a central bank.

ODD Template

The model description follows the ODD (Overview, Design concepts, Details) protocol for describing individual- and agent-based models (Grimm et al. 2010).

Introduction

Since the last financial-economic crises the economist has a new challenge: generate tool robust enough to resist time of crises. In difficult times the conventional tool of analysis had demonstrated limited support to the policymakers. Hommes (2015) had shown us that the atomistic optimizing agents underlying existing models do not capture the behaviors during a crisis period. Their point is, we need a better way to deal with heterogeneity among agents. This brings us an alternative motivation for economic choices based on behavioral economics draws on psychology to explain decisions made in a time of crisis circumstances, which need to be modeled into the agent-based framework.

The model we work on here could find as origins from Brock and Hommes (1997), Brock and Hommes (1998), going thru Gatti et al. (2011), and it is close to Assenza et al. (2015). In Assenza et al. (2015) you can find the core of our model, in its works they generating a macroeconomic agent based model with credit and capital. That means one virtual economy which has a consumption market, has a job market and also has a capital goods market. This model is capable of reproducing stylized facts, such as the series of GDP, unemployment, and series of prices. The interesting thing is, this model had created endogenously crises; which here will mean a sequence of time when GDP shrinks abruptly and induce a high level of unemployment. But, so far, the model has no government and no monetary authority. We include the figure of the monetary authority.

Purpose

We inserted a monetary authority into a Macroeconomic Agent-Based Model with Capital and Credit and verify the interaction between the central bank and the rest of the economy, thus offer a tool for policymaker simulate monetary policies.

Entities, state variables, and scales

- *Agents*. We use agents to represent the minimal unit of a behavior of the members at this economy; they will represent the participants of the productive, consumption, and the financial sector of an economy.
- *Spatial units*. The patches of the grid will be occupied by only one firm per patch. There will be as many patches as a number of firms.
- *Environment*. The households can transit among the patches freely. The position of the firms is constant during all the experiment; they do not change their address. All the patches have the same characteristics. Each period will represent a quarter, the simulations may be running for an arbitrary number of periods.
- *Collectives*. We will divide our agents into three types: (i) the first one, will be the firms, responsible for the production in this economy; (ii) the householders, among them there will be the workers and the owners of the firms; (iii) the financial sector, we will stylize the financial sector in a way which will a commercial bank, and a monetary authority – which will manage the basic interest rate of our economy.

AGENTS

Production sector

The firms will be divided into two groups: (i) producers of consumption goods; and (ii) the producers of capital goods. The first one is absorbed by the households, and the second one is buying for the firms which produce consumer goods.

Financial sector

One commercial bank will receive the deposit of the households and firms. The bank will not charge for this service, but he will charge the firms that need for credit. The bank receives the deposit of the agents into this economy and supplies the credit market. There is a minimal rate (risk-free interest rate); this rate is decided by a central bank. The central bank does not make any contact with the productive sector or the households; it only controls the minimal interest rate.

Households

The households of this economy are divided into two groups: the workers; and the capitalists. The workers sell their workforce in the job market to the firms, receive wages, and use the wages to buy goods into the consumption market. The capitalist is the owners of the firms and the bank. Each firm has only one owner. Each capitalist has the same share of the bank.

Information

The available information is limited; any agent has access to see complete information. The firm which produces consumption goods will know the level of price practiced in their sector and the quantities demanded from their clients. The firms which produce capital goods will know the level price of their sector and also the quantities demanded from their clients. These will be the information available during the decision process of the firms.

The households can visit a restricted number of firms per period. They will have as information the prices of the firms visited, and the quantities ready to sale by these firms. At the moment to decide how much and from whom to buy this will be the set of information used by the households.

The bank will need to decide whether to offer or not a loan and what rate to use in each transaction. For that, the bank will measure the financial health of the firm, this means, find out how much leveraged the firm has. The bank will know how much money the firm has deposited with it, and the bank will know how much credit the firm had taken before. As all this information is available for each firm individually; the bank can construct a risk for the entire market.

The central bank has a concern about production and about the prices movement – inflation. It will decide what to do with the free interest rate based on his observation of these two variables.

Process overview and scheduling

Time will be a discrete variable where each period will represent a quarter. In each period the firm will decide how much to produce and what price to use. The households will decide how much to consume, if do not consume all the income within the period they will save their remaining money – they made a deposit in the bank.

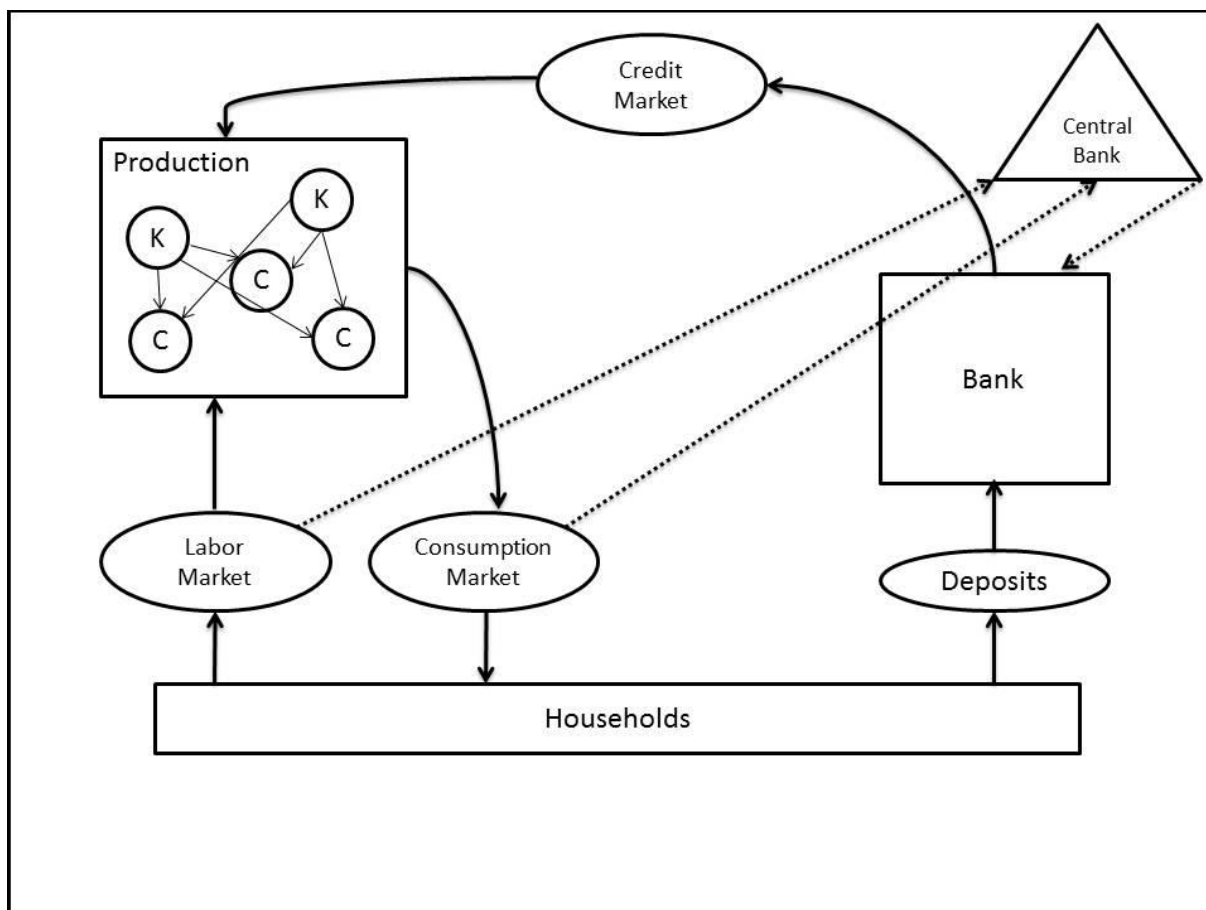


Figure 1: Agents and markets; production sector with firms that produces capital goods (K) and consumptions goods (C)

Job market

The unemployed workers will visit a restricted number of firms trying to find a job position. The wages are homogenous, so the worker will accept the first job offer they receive. The productivity is homogenous among the workers, and through the time, then the firms will contract the first worker to apply for their vacancy position.

Consumption market

The households have a certain amount of money that need to consume for each period, they will visit a restricted number of firms and try to buy their goods at the firm with the lower price. In that case, the firm with the better price will not have the enough quantity for the household, so the households will buy in the next firm. When there are not enough goods in all firms visited the household will save money.

Capital market

The consumption firms will need to combine labor and capital in a way to produce goods. The capital and the labor are perfect complements (a Leontief production function). The consumption firm will visit the capital producer's firms and try to buy for the lower price, similar to the behavior of the households in the consumption market.

Credit market

Sometimes the firms will need to access the credit market. Doing so, the firms will ask the bank for a loan. The bank will measure if it has enough available money for that firm which is asking for a loan, and the bank will also decide which interest rate he will apply for that loan. For the formation of the interest rate, the bank will use this set of information together with the free interest rate – which is determined by the central bank.

Design concepts

Basic principles. The macroeconomy we simulated here emerge as a result from the behavior of individuals which are trying to maintain their consumption level through the time. That comes from the permanent income life circle hypothesis, which means, the individual will consume not only in a base in their income of the moment, but will also consider their expectation of the entire life wealth. The firm will have some monopolistic power; so, some firms may have extraordinary profit performances, at the same time some other firms may be in trouble times. The limitation of information is a key fact that induces these situations described before. As the economy will not be run at full capacity, sometimes in a period of crises, we have space for some adjustments. The introduction of a monetary authority into the model comes in this direction.

Emergency. The individual errors at the forecast formation may drag some firms at difficult financial times, it will be transmitted to the others firms by the credit market. As a result of a positive feedback, we will be able to see a series of investment with abrupt interruptions. This occurs because the bank uses the risk of the market to decide whether or not conceive credit. So, one particular firm could be in a good financial health situation, but see their credit go restricted because the environment is not in a good time. The series of GDP and employment which emerge are also affected by these circumstances and will also show abrupt movements.

Adaptation. The households will use an adaptation rule to decide how much to consume each period. The idea is the household will measure the wealth of their entire life, not only the revenue of the period, at moment to decide how much to consume. So, in each period the household will reformulate their knowledge of their own fortune (the amount deposited at the bank plus the revenue of the period). The firms will adapt their quantity produced and their price practiced; increasing or decreasing their production they will affect the job market (hiring of firing workers).

Objectives. The central bank tries to keep the unemployment level low, but at the same time, it would like to avoid big changes at the level price. The households will consume the goods using a hole of thumbs to decide how much to consume in each period. The firms will try to have profit, for that they try to sell their production and update their prices goods, as well adjust their labor's demands.

Learning. Not applied. The rules will be the same for all the experiment.

Prediction. The households, when behaving as consumers, think the proximate future is a continuous of the recent past, their rule of thumb for consuming will not change over time. That means, if he has a job at this time, for example, it will behave as it also has a job in the next period – what may not be true.

Sensing. The households can see the prices and the quantity available at the firms they visit. The consumer goods' production firms can see the price and the quantity available at the firms they visit trying to buy capital goods.

Interaction. The households interact with firms by opportunism, each period they can visit and buy from different firms, there is no such thing as fidelity's behaviors. The firms will interact with each other in an indirect way into their own sector. They will compete by price and by quantity. Since the households have no fidelity, firms have a difficult job trying to formulate their forecast; the competition will happen using the information of the previous period. The bank has a direct interaction with the firms, receiving and conceiving financial services.

Stochasticity. When the firms need to chance their prices, up or down, we used a stochastic way to do so. The firm will raffle the number from a uniform distribution every time it needs to create a new price and then charge the new price with a small variation using this mechanism.

Collectives. For the point view of the bank, there are two groups. The consumer goods' firm production and capital goods' firm. The bank will use these two collectives to generate a risk market, which increases with the number of firms in financial difficulty into each sector.

Observation. The series of GDP, price, and unemployment rate can be exported and analyzed. The credit market behavior can also be exported, via the quantities of loans conceive and denied, the connection with the GDP is a key to understanding the crises. These data may be collected by the end of the simulations

Initialization

We start with 200 consumptions firms and 50 capital firms, each firm uses a position of the grid. The bank and the central bank will use arbitrary positions, their address will coincide with one of the others firms. So, the grid has 250 patches. Will exist 250 capitalists, and they will be linked with one unique firm each one. The total of workers is 3,000.

The initial conditions and the parameters are disposals at Table 1, these parameters were settled using real data from the FRED's database from Federal Reserve among the years of 1955 to 2013. Taylor's parameters are described as in Taylor (1993). Details can be found at the Assenza at al. (2015).

Table 1: Parameters and initial conditions.

Parameters	Description	Value
T	Number of periods	3000
H	Number of workers	3000
F_c	Number of consumptions firms	200
F_k	Number of capital goods firms	50
Z_e	Number of firms visited by unemployed workers	5
Z_c	Number of consumptions firms visited by a consumer	2
Z_k	Number of capital goods firms visited by a C-Firm	2
ε	Memory parameter, human wealth	0.96
τ	Dividend payout ratio	0.20
χ	Fraction of wealth devoted to consumption	0.05
r	Initial risk free interest rate	0.01
ρ	Quantity adjust parameter	0.90
η	Price adjust parameter (random variable)	U(0,0.1)
μ	Bank gross mark-up	1.20
α	Productivity of labor	0.50
κ	Productivity of capital	1/3
γ	Probability of investing	0.25
ζ	Bank's loss parameter	0.002
θ	Installment on debt	0.05
δ	Depreciation of capital	0.02
ν	Memory parameter (investment)	0.50
$\bar{\omega}$	Desire capacity utilization rate	0.85
w	Wage	1.00
D_1^f	Initial liquidity of all firms	10
K_1	Initial capital	10
Y_1^c	Initial production (consumptions firms)	5
Y_1^k	Initial production (capital goods firms)	2

E_1^b	Initial equity of the bank	3000
E_1^h	Initial households' personal assets	2
r^*	Natural interest rate	0.01
π^*	Desire inflation for the monetary authority	0.01
α_π	Taylor' rule parameter for inflation	0.50
α_y	Taylor' rule parameter for product	0.50

Submodels

At the consumption market the workers and the capitalists will behavior at the same way. Their sources of income will be:

$$Y_{c,t} = \begin{cases} w, & \text{if he is a worker with job contract,} \\ \tau\pi_{f,t-1}, & \text{if he is capitalist receiving dividends.} \end{cases} \quad (1)$$

$Y_{c,t}$ is the actual income, $Y_{c,t} \in (0, \infty) \subset \mathbb{R}$;

w is the wage, $w \in (0, \infty) \subset \mathbb{R}$;

τ is the dividend ratio, $\tau \in (0, 1) \subset \mathbb{R}$;

π is the profit of the period, $\pi \subset \mathbb{R}$.

The households have a limited rationality, they will use a rule of thumb to decide how much to consume. First, he estimates their own lifetime wealth, $\bar{Y}_{c,t}$, as a proxy of their own future income, that means, he expected in the future their wealth will be similar then nowadays. For that, they will use this adaptive rule:

$$\bar{Y}_{c,t} = \varepsilon\bar{Y}_{c,t-1} + (1 - \varepsilon)Y_{c,t}, \quad (2)$$

$\bar{Y}_{c,t}$ is the lifetime wealth estimative, $\bar{Y}_{c,t} \in (0, \infty) \subset \mathbb{R}$;

ε is the memory parameter, $\varepsilon \in (0, 1) \subset \mathbb{R}$.

If the household has no income in determined period he still consumes. In this situation he will decrease his savings. Also, will occur decrease in the agent's savings account when the consumptions of the period are bigger than the income:

$$D_{c,t} = D_{c,t-1} + Y_{c,t} - C_{c,t}, \quad (3)$$

$D_{c,t}$ is the savings, $D_{c,t} \in (0, \infty) \subset \mathbb{R}$;

$C_{c,t}$ is the total consumption of the period, $C_{c,t} \in (0, \infty) \subset \mathbb{R}$.

Each household will visit a number Z_c of firms by period. They will try to consume the goods for the lower price. The firms each produce consumptions goods will need labor and capital, at the beginning of period the firm will check their own position $(P_{i,t}, Y_{i,t})$, it is their set of prices and quantities practiced at previous period. The $P_{i,t}$ represent the last price used. The firm also know their real quantity sold $Q_{i,t}$. As the sales just occur after the firm take their production levels choice, it may happen a queue of clients unsatisfied or an undesirable storage of goods at the end of the period. The goods of the consumption market will be perishables ones, so the stock that did not had sold will go to waste, i.e., they will not be available to be sale in the next period. So, the firm will try to find the correct level of production. For that the firm will use two pieces of information: the current level of price practiced in their sector; and their own forecast error, it is, the difference between their expectation of sales and the real sales.

$$\Delta_{(i,t)} = Q_{i,t}^e - Q_{i,t}^d, \quad (4)$$

$\Delta_{i,t}$ is the forecast error, $\Delta_{i,t} \subset \mathbb{Z}$;

$Q_{i,t}^e$ is the actual production, $Q_{i,t}^e \subset \mathbb{N}$;

$Q_{i,t}^d$ is the quantity demanded, $Q_{i,t}^d \subset \mathbb{N}$.

$$P_t^c = \frac{1}{Q} \sum_{i=1}^N Q_{i,t}^d P_{i,t}^c, \quad (5)$$

P_t^c is the level price in the consumption sector, $P_t^c \in (0, \infty) \subset \mathbb{R}$;

$P_{i,t}^c$ is the price in i^{th} firme, $P_{i,t}^c \in (0, \infty) \subset \mathbb{R}$;

$Q_{i,t}^d$ is the level quantity sold by i^{th} firme, $Q_{i,t}^d \subset \mathbb{N}$;

Q is the total quantity sold, $Q \subset \mathbb{N}$.

$$Q_{i,t+1}^e = Q_{i,t}^d - \rho \Delta_{i,t} \text{ if } \{ \Delta_{i,t} \leq 0 \text{ and } P_{i,t}^c \geq P_t^c \} \text{ or } \{ \Delta_{i,t} > 0 \text{ and } P_{i,t}^c < P_t^c \}, \quad (6)$$

ρ is the quantity adjust parameter, $\rho \in (0,1) \subset \mathbb{R}$.

$$P_{i,t}^c = P_{i,t}^c (1 + \eta_{i,t+1}) \text{ if } \{ \Delta_{i,t} \leq 0 \text{ and } P_{i,t}^c < P_t^c \} \text{ and } \{ \Delta_{i,t} > 0 \text{ and } P_{i,t}^c \geq P_t^c \}, \quad (7)$$

η is a price adjust paramer, $\eta \in U(0,0.1) \subset \mathbb{R}$.

The firm will produce using labor and capital available:

$$\hat{Y}_{i,t} = \min\{\alpha N_{i,t}, \kappa K_{i,t}\}, \quad (8)$$

$\hat{Y}_{i,t}$ is the actual production of the i^{th} firme;

$K_{i,t}$ is the capital in use by the i^{th} firme, $K_{i,t} \in (0, \infty) \subset \mathbb{R}$;

$N_{i,t}$ is the number used by the i^{th} firme, $N_{i,t} \subset \mathbb{N}$;

α is the productivity of labor, $\alpha \in (0,1) \subset \mathbb{R}$;

κ is the productivity of capital, $\kappa \in (0,1) \subset \mathbb{R}$.

The capital which is in use by the firm will suffer depreciation; the firm will need to invest, by new capital goods, in a way to keep their level of production.

$$K_{i,t+1} = (1 - \delta \omega_{i,t}) K_{i,t} + I_{i,t}, \quad (9)$$

δ is the depreciation of capital, $\delta \in (0,1) \subset \mathbb{R}$;

$w_{i,t}$ is the capacity utilization by the i^{th} firme, $w_{i,t} \in (0,1) \subset \mathbb{R}$;

$I_{i,t}$ is the investment made by i^{th} firm, $I_{i,t} \in (0, \infty) \subset \mathbb{R}$.

There are fluctuations into the firm's demand; the firm will not be in a situation that he is using their full capacity. That means the firms will be looking for the long run at the moment to target their desire amount of capital, doing so:

$$\bar{K}_{i,t-1} = v\bar{K}_{i,t-2} + (1-v)\omega_{i,t-1}K_{i,t-1}, \quad (10)$$

$\bar{K}_{i,t}$ is the long run desire capital by the i^{th} firm, $\bar{K}_{i,t} \in (0, \infty) \subset \mathbb{R}$;

v is the memory of investment parameter, $v \in (0,1) \subset \mathbb{R}$.

When the firm knows their long run desire capital the firm will be able to calculate their level of investment:

$$I_{i,t}^r = \frac{\delta}{\gamma} \bar{K}_{i,t-1}, \quad (11)$$

γ is the probability to invest, $\gamma \in (0,1) \subset \mathbb{R}$.

Just readjusting, we can rewrite the law of capital as:

$$K_{i,t+1} = \left(\frac{1}{\bar{\omega}} + \frac{\delta}{\gamma}\right) \bar{K}_{i,t-1} - \delta\omega_{i,t}K_{i,t}, \quad (12)$$

$\bar{\omega}$ is the long run desire capacity utilization, $\bar{\omega} \in (0,1) \subset \mathbb{R}$.

Then the quantity of labor can be finding. The firm does not know a priori if she will have all that labor, the firm will open some vacancies at the job market and hope to same free worker to apply.

$$K_{i,t+1}^* = \omega_{i,t+1}^* K_{i,t+1} \quad (13)$$

$\omega_{i,t+1}^*$ is the desired capacity to be use by the i^{th} firm,

$$N_{i,t+1}^* = \min \left\{ K_{i,t+1}^* \frac{\kappa}{\gamma}, K_{i,t+1} \frac{\kappa}{\gamma} \right\}. \quad (14)$$

The Equation (14) shows us the quantity of labor will be need. When the capital available is restrict, $K_{i,t+1} \frac{\kappa}{\gamma}$, the quantity of labor will depend of that. If there is abundant capital the firm will use only a portion of that, then the quantity of labor is relative to that.

So far, we are talking about the decision process of the firms which produces goods for the consumption market. The firm that produces capital goods has a different job. They will not use the combination of labor and capital; they will only use labor into their production process. Also, as their product is capital goods they will be able to storage their unsold production. Whit that in mind we may construct their rules for decision of prices and quantities:

$$Q_{j,t+1}^k = (Q_{j,t}^k + \Delta_{j,t}^k)(1 - \delta^k) \quad (15)$$

$\Delta_{j,t}^k$ is the variation of stock of the j^{th} firm, $\Delta_{j,t}^k \subset \mathbb{R}$;

δ^k is the depreciation of capital in stock, $\delta^k \in (0,1) \subset \mathbb{R}$.

Their rule for price formation is similar to the firms into the consumption market:

$$P_{j,t}^K = P_{i,t}^K(1 + \eta_{j,t+1}) \text{ if } \{\Delta_{j,t}^k \leq 0 \text{ and } P_{j,t}^K < P_t^k\} \text{ and } \{\Delta_{j,t}^k > 0 \text{ and } P_{j,t}^K \geq P_t^k\} \quad (16)$$

An adaptive rule, considering the forecast error and the current stock, will guide their productions decisions:

$$Q_{j,t}^* = Q_{j,t}^K - \rho \Delta_{j,t}^k \text{ if } \{\Delta_{j,t}^k \leq 0 \text{ and } P_{j,t}^K \geq P_t^k\} \text{ or } \{\Delta_{j,t}^k > 0 \text{ and } P_{j,t}^K < P_t^k\}, \quad (17)$$

$$Q_{j,t}^K = \alpha N_{j,t}. \quad (18)$$

As we can see in Equation (18) the production into the capital goods sector will depends only by the level of labor. The labor productivity is the same in both sector and time independent. To need for new workers, or necessity to dismiss some, the firm look at their desire production, $Q_{j,t+1}^*$, and for the productivity of labor, α :

$$N_{j,t+1}^* = \frac{Q_{j,t+1}^*}{\alpha}. \quad (19)$$

Will be times when the expectation will not be realize, the firms may see difficult to honor their bills (pay the wages and for investments). Their assets will be deposit at the bank, and will be the bank that may help the firms at the time of short money. We can define the need for financial help as the gap between the expenses of the period and the revenue. The Equation (20) resume that for a firm that product consumption goods. The firms which produce capital goods will differ because they don't need to invest, so we can resume that in Equation (21).

$$F_{i,t}^C = \max\{wN_{i,t} + P_{t-1}^K I_{i,t} - D_{i,t-1}, 0\}, \quad (20)$$

$F_{i,t}$ is the financial gap of the i^{th} firm, $F_{i,t} \subset \mathbb{R}$;

$D_{i,t}$ is the assets of the i^{th} firm, $D_{i,t} \subset \mathbb{R}$.

$$F_{j,t}^K = \max\{\omega N_{j,t} - D_{j,t-1}, 0\}, \quad (21)$$

Once the bank has this information he will move on in their job to measure the risk of the firm. The next step is the look at the leverage ratio of the firm that will expose the dimension of the financial problem of firm. When the firm is in perfect financial the leverage ratio tends to zero, that means, the firm are working only with their own money. Otherwise, when the firm has little assets their leverage ratio tends to one.

$$\lambda_{f,t} = \frac{L_{f,t-1} + F_{f,t}}{E_{f,t-1} + L_{f,t-1} + F_{f,t}}, \quad (22)$$

λ is the leverage ratio, $\lambda \in (0,1) \subset \mathbb{R}$;

L is the accumulated debt, $L \in (0, \infty) \subset \mathbb{R}$;

E is the equity or assets, $E \in (0, \infty) \subset \mathbb{R}$.

In an ideal situation, when the firm has no doubt about the return a loan, the bank would target this minimal gross rate:

$$R = \left(1 + \frac{r}{\theta}\right), \quad (23)$$

R is the gross rate, $R \in (0,1) \subset \mathbb{R}$;

r is the free interest rate, $r \in (0,1) \subset \mathbb{R}$;

θ is the installment on debt, $\theta \in (0,1) \subset \mathbb{R}$.

Likely the bank will find the firms with no ideal situation, then the bank will need to evaluate the risk of the loan do not return. He will calculate the risk for each firm individually. For that the bank will use the leverage ratio of the market to construct a logistic regression ($\phi_{f,t}$) and the leverage ratio of the firm to measure the time life expectance of a firm, $T_{f,t}$. When the firm has low leverage ratio their time life tend to infinity and their situation tends to ideal situation describe above.

$$T_{f,t} = \frac{1}{\phi_{f,t}\lambda_{f,t}}, \quad (24)$$

$\phi_{f,t}$ is a logist regression for mesuare market risk , $\phi_{f,t} \subset \mathbb{R}^+$;

T is the expected survival time, $T \subset \mathbb{R}^+$.

The gross rate applied to a particular firm will then depend on that set of information: the time life expectance of the firm, which also depends of the market risk, the free interest rate practiced at the moment by the monetary authority, and the installment on debt:

$$R_{f,t} = (\theta + r_{f,t}) \frac{1 - (1 - \theta)^{T_{f,t} + 1}}{\theta}, \quad (25)$$

If we reorganize the equation above, and resume $\Xi = 1 - (1 - \theta)^{T_{f,t} + 1} / \theta$, we have:

$$r_{f,t} = \mu \left(\frac{1 + \frac{r}{\theta}}{\Xi} - \theta \right), \quad (26)$$

μ is the bank's markup, $\mu \in (1, \infty) \subset \mathbb{R}$.

After all, the bank will apply a markup and find their interest rate applicable for the firm in Equation (26). The bank needs now decide how much credit will be available for the firm. He already known the current stock of loan ($L_{f,t-1}$) and also know their new need for loans. The bank has a limit of acceptable loss, ζE_t^b , and he will only conceive new credit when this loss limit is not exceeded:

$$\phi_f(\Delta L_{f,t} + L_{f,t-1}) \leq \zeta E_t^b, \quad (27)$$

ζ is the bank loss parameter, $\zeta \in (0,1) \subset \mathbb{R}$.

The monetary authority will use a simple Taylor rule to formulate the free interest rate. For this he will need to know the potential gross domestic product of this economy. As the labor productivity is known and constant this job could be done:

$$\bar{Y} = \alpha H, \quad (28)$$

\bar{Y} is the potential GDP;

α is the productivity of labor, $\alpha \in (0,1) \subset \mathbb{R}$;

H is the total of workers, $H \in \mathbb{N}$.

The current inflation level is described as the difference between the current and the previous level of price:

$$\pi_t = \frac{P_t - P_{t-1}}{P_{t-1}}, \quad (29)$$

π_t is the actual inflation.

Then the monetary authority has the tools to apply the following Taylor rule:

$$r_t = \pi_t + r^* + \alpha_\pi (\pi_t - \pi^*) + \alpha_Y (Y_t - \bar{Y}), \quad (30)$$

r_t is the current free interest rate;

r^* is the natural interest rate, $r^* \in (0,1) \subset \mathbb{R}$;

α_π is a parameter, $\alpha_\pi \in (0,1) \subset \mathbb{R}$;

α_Y is a parameter, $\alpha_Y \in (0,1) \subset \mathbb{R}$;

Y_t is the current gross domestic product.

The information use by the monetary authority is also known by the other agents of this economy, except by the parameters of Taylor's rules. This is our first attempt to include a monetary policy in the model. The desired inflation is taken as fixed at the first moment, we may ease this assumption in the future. Also the parameter of Taylor rule – each one could indicate if the monetary authority is more inclined to regulate inflation or support production – could verify other sets of value to them.

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