STOCK ANALYSIS AND EFFICIENT PORTFOLIO DIVERSIFICATION.

A CASE OF PAKISTAN EQUITY MARKET

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Author's Declaration

I Zahid Ali Shad hereby state that my MPhil Thesis titled "Stock Analysis and Efficient Portfolio Diversification, A Case of Pakistan Equity Market" is my own work and has not been submitted previously by me for taking any degree from Pakistan Institute of Development Economics or anywhere else in this country or world.

At any time if my statement is found incorrect even after my graduation the university has the right to withdraw my MPhil Degree.

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Dedication

I dedicate my MPhil thesis to my parents, who always wanted me to be best in every field, especially in my education career. I further dedicate my work to my all family members who always support me, especially my younger sister Faryal whose love and support give me strength in ups and down of life. May Allah gives us strength and courage and fulfill all our needs and prayers.

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Abstract

In today's global financial market, risk plays high fragile role in investment. Investors are looking to overcome this financial risk at any cost, with the global crisis arises day by day, investors have concern to diversify their portfolios effectively. The aim of this study is to construct an efficient portfolio of domestic level stocks using optimal hedge ratio which will minimize the overall variance of the portfolio. The study uses DBEKK model of MGARCH family for estimation of hedge ratio and uses minimum variance approach for the efficiency of portfolio selection. A sample of total 30 firms was used for portfolio construction which represents the six sectors of Pakistan stock exchange. Daily data from January 2014 to December 2019 were taken which make a total 1485 observations for each stock. The normality and stationarity of return series has been checked using Skewness, kurtosis and Jarque-bera test and KPSS test, while to know ARCH effect LM-ARCH test is used. Q-stat test is used to know serial correlation. After calculation of return of the actual series, variance and covariance of each return series has been calculated using DBEKK model, which were further used in calculation of portfolio return and portfolio variance. As we have used minimum variance approach in this study, so for minimum variance we have used reduction in variance formula for each 40 randomly constructed portfolios. Each portfolio is ranked on the basis of reduction in variance, the best portfolio is one which gives maximum reduction in portfolio variance. The finding of this study shows that it is possible to minimize variance or risk of a portfolio. The current method of estimation helps in reduction in variance of portfolios with three securities and have suggested best portfolio having minimum variance, and hence gives investors an opportunity to minimize their portfolio variance using current method of estimation and portfolio diversification.

Keywords: Portfolio Diversification, Multivariate GARCH, DBEKK Model, Minimum Variance Hedge Ratio, PSX.

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LIST OF ABBREVIATIONS

ARCH	Autoregressive Conditional Heteroscedasticity
CAPM	Capital Asset pricing Model
CPEC	China Pakistan Economic Corridor
DBEKK	Diagonal baba Engle Craft Kroner
KSE	Karachi Stock Exchange
MGARCH	Multivariate Generalized Autoregressive Conditional Heteroscedasticity
MPT	Modern Portfolio Theory
OLS	Ordinary least square
PSX	Pakistan Stock Exchange
PIDE	Pakistan Institute of Development Economics

CHAPTER 1

INTRODUCTION

1.1 Introduction

In today's financial world, Stock markets have a big chunk of overall financial investments. For any country stock market is an indicator of economic performance of that country. The performance of stock market depends upon the favorable conditions in the country for investors. The political stability, law and order situation, macroeconomics indicators like interest rate inflation, these factors play a role in the performance of stock markets. As in stock markets stock prices are highly volatile, and due to this high volatility investor can either make money or lose money depends upon the investment and volatility. To overcome the loses from investment an investors always tries to make a diversified portfolio, so that if one investment make lose the other investment can make gain and overcome overall lose. The risk and return of financial investment are further explained below.

1.2 Financial Risk and Return

Earnings received from invested capital from financial assets are known as returns. An investor invests their money today in order to gain a higher amount from the invested money in the future. Expecting more money in the future and investing for that in today is the assumption of utility maximization. There are two types of returns from financial assets, return in the form of capital gain and interest or dividend payments. The total return from financial asset return is the sum of return from capital gain and interest earned or dividend received for that period.

According to Gruber, Brown, and Goetzman (2009) the discounted value of all future cash flows is known as financial assets value at present.

While the expected value of the portfolio is determined by the mean value of the expected return from the proposed assets.

Risk is an undesirable outcome. According to the definition of risk, the deviation from the expected return is known as risk i.e. an investor is expecting a desired return but the actual return is not up to that expectations and less than that and is known as risk. Risk is of two types i.e Systematic risk or firm-specific risk which is also known as diversifiable risk because this type of risk can be diversified by making portfolios. While the other form of risk is systematic risk which is market-level risk and affect the whole markets and due to wide market factors, this type of risk is undiversifiable.

Risk is further divided into two types such as, Systematic risk which is also known as market risk, as this risk affects the whole market. Markowitz has termed the systematic risk as undiversifiable risk because no diversification or hedging can minimize the systematic risk. Systematic risk may occur due to natural disaster, political change, change in government policy, change in economic indicators etc. These changes affect the whole market and known as market risk or systematic risk.

While unsystematic risk refers to the risk associated with an individual firm or industry. This type of risk occur within a company or with in an industry. This risk is due to the internal failure of a company and this risk only affects a particular company or industry only. This type of risk can be minimized by diversification or hedging by using correct selection and allocation of assets. In this study, we have used portfolio diversification to hedge against unsystematic risk.

1.3 Diversification

Diversification is a choice for every investor to minimize the unsystematic risk associated with asset return. It all starts from the theory of Markowitz (1952) on portfolio diversification or also known as mean-variance theory. According to Markowitz (1952) an investor can minimize the amount of unsystematic risk by making a portfolio of distinct but negative correlated assets and hence earn more return with minimum risk bearing. The theory of Markowitz is also known as Modern portfolio theory. Grubel (1968) further extends the work of Modern Portfolio theory and takes it to international level. According To him, investors can diversify their portfolio to emerging markets as the emerging markets have a high growth rate and low correlation with developed markets. Malkeil and Mei (1999) explain in their book that in long Run also emerging Markets stocks will exceed those of developed markets.

Diversification has a lot of benefits in risk management, especially firm related risk or unsystematic risk can be fully minimized by using portfolio diversification. Markowitz (1952) if all security returns are statistically independent, then portfolio risk (variance) can be made infinitesimally small through sufficient diversification. But in real life, most of the securities are positively correlated, and if so it is hard to find out riskless portfolio.

A diversified portfolio is essential for risk-minimizing, but it is equally important to construct a portfolio which will be suitable for investors taste. A portfolio can be constructed by the preferences of investors, their willingness to take risk and the amount of risk which an investor can bear. There is a debate on the optimal level of diversification, as how many numbers of securities should be included to minimize risk. Secondly, the question arises is, which securities to be included in the portfolio.

The risk of portfolio depends upon the weightage of securities included, the variance of each security and the covariance among securities. Covariance plays an important role in portfolio selection i.e James and Mao (1970) if securities are positively and perfectly correlated than no diversification can make a diversification riskless. So for an efficient diversification the securities included in the portfolio must have low or negative correlation with each other.

1.4 The Pakistan Stock Exchange

The Pakistan stock exchange (PSX) formerly known as Karachi stock exchange, was established on 18th September 1947 and incorporated on 10th March 1949[1]. Before the incorporation of PSX on 11th January 2016, there were three stocks exchanges working with different management's i.e Karachi stock exchange, Lahore stock exchange, and Islamabad stock exchange. With the formation of the Pakistan Stock exchange, these three integrate their operation with one name that is Pakistan stock exchange. Started from six listed companies with a market capitalization of Rs 37 million, now PSX has 546 listed companies with capitalization of 7.692 trillion rupees[2]. These companies represent 35 sectors or group of industries. Currently, there are seven indices listed on PSX which are given below:

KSE 100 index, KSE 30 index, KSE All Shares, KMI 30 index, PSX KMI all Shares, BKTI (Tradable Bank index), OGTI (tradable Oil & Gas Index).The KSE 100 is considered to be the economic indicator for economic development in Pakistan.

In 2008 world crises the PSX which at that time was known as KSE or Karachi stock Exchange, also gone through crises and the indices touches the lowest point. The KSE market was suspended from august to mid of December, this was the longest period for KSE to suspend for

[1] https://www.psx.com.pk/psx/exchange/profile/about-us

[2] As on 31 December 2018.

such a long time. After 2013 PSX is performing well and shows no sign of any crash. PSX gives largest tax to government and employee thousands of workers. According to Bloomberg PSX is the best performing stock market in the world. PSX is not only giving local investors a chance but equal opportunity to international investors as well to diversify their stock portfolio.

A number of researches have done on PSX, widely in the field of optimal level of stock diversification and co-integration of PSX with other stocks exchange of the world. Current study has focus on different sectorial level stock diversification and its benefit.

1.5 Research Problem

Over time researchers have looked at different aspects of diversification. Like diversifying internationally to emerging markets, as emerging markets are having a different correlation with developed markets. A number of studies have been conducted on international diversification and researcher have find out that international or Regional diversification have better performance than with in market diversification. Solnik (2018), Cleary and Copp (1999) have worked on international diversification and find out significant benefits from international diversification.

But in the Modern world, markets over the world are integrating and the lower correlation is diminishing over time, and also the transaction cost and exchange rate fluctuation discourage international diversification. Investors are looking for alternatives, and focus on domestic level diversifications. Gupta and Basu (2011) have identified the diminishing benefit from international diversification and they have focused on domestic sector level diversification in Australian and Indian Market.

The current study have focused on Domestic sector level diversification and tried to find out the best possible benefit can get from local level diversification using different sectors level stocks.

1.6 Research Gap

Habibah, Sadhwani and Memon (2018), Zaidi (2017), Hussain, Hussain and Bhatti (2012), Joyo and Lefan (2019), Jebran (2014) All the above studies have focus on optimal portfolio size and integration of Pakistan stock market with other international equity markets. None of them have tried to find out any benefit we can take by investing in local level stocks. According to them, managers and investors can invest in international markets for diversifying their funds. The Current study has focus on Domestic level stock diversification rather than international level and will provide a better way of diversifying in domestic stocks.

Our study extends the existing literature of the multivariate GARCH model into the Pakistan market, which is a rarely investigated area of domestic sector diversification level. Besides suggesting that the domestic sector diversification effect exists in the Pakistan stock market, these results can also be extended to transitional markets in other developing economies. This study uses the DBEKK (Diagonal baba Engle Craft Kroner) MGARCH (Multivariate Generalized Autoregressive Conditional Heteroscedasticity) model to estimate time-varying volatility and co-volatility. These variances and co-variances are then used in the construction of portfolios. After completing portfolio construction, their performance is measured by minimum variance approach. Further the results will suggest the most efficient portfolio diversification.

Up to my knowledge, no one has tried to identify the benefits from domestic level diversification in the Pakistan market. This study have focused on the benefit of diversifying in

the local stock market especially sector level diversification using firm level stock prices other than international diversification. By the end of the study, we will be aware of the fact that, any benefit in sector-level diversification and which stocks makes a perfect portfolio.

1.7 Significance of the Study

This study have used time-varying model like MGARCH for estimation of variance and covariance of stocks. These estimations have been used for the construction of an efficient portfolio, and hence give investors an idea about the efficient ways of constructing an optimized portfolio diversification with minimum variance. The study has analyzed the stocks prices in kse-100 and give an actual portfolio, which can further be taken to real-time investing in that portfolio for an optimized return and minimum variance.

1.8 Objective of the Study

The objectives identified for the study are given as:

- To construct a portfolio that will give minimum variance.
- To get benefit from sector-wise portfolio diversification in the stock market.
- To give criteria for efficient portfolio selection.

1.9 Research Question

The questions which this study wants to find answers are:

- How portfolio variance can be reduced?
- Can we get benefit from diversifying in Domestic level? i.e diversifying stock using different sectors within a stock market.

- Which sectors stocks will give the perfect portfolio diversification?

1.10 Organization of the Study

The study is organized as follow,

Chapter 1 explains the introduction of the topic, Significant of the topic followed by research problem and research questions, introduction to PSX. Furthermore the next chapter is Chapter 2 which explains the Literature review with some theoretical background and empirical review followed by chapter 3 the methodology which briefly explains the data type, data measures, data collections, forecasting models and analyzing techniques. Chapter 4 explains the Results, their explanations, findings and last but no least chapter 5 explains the conclusion, future directions, recommendations and limitations of the study.

CHAPTER 2

LITERATURE REVIEW

2.1 Chapter Overview

After the work of Markowitz (1952) on portfolio diversification, a number of researchers have tried to understand the relationship between portfolio diversification, portfolio size, portfolio risk, and return. In this part of the paper, we have provided some theoretical background and a number of empirical reviews related to portfolio diversification.

2.2 Theoretical Review

Below are some theories related to diversification, risk and return. These theories will make base for empirical review and gives a better understanding of the topic.

2.2.1 Modern Portfolio Theory

Modern Portfolio Theory (MPT) by Markowitz (1952)[3], In his article Markowitz explained the risk and return of portfolio which is the basic finance theory also known as Markowitz's portfolio Selection theory, which gives a pathway for investors to diversify their portfolio and hence minimize risk. According to Markowitz's theory individual security having a high variance when combine with other securities having a negative correlation than the combined effect will produce a minimum variance level than individual. The MPT is based on mean-variance model, Markowitz derived a formula to calculate the variance of a portfolio and indicate the importance of diversification in portfolio in order to reduce total risk so for effective

[3] Harry Max Markowitz born on 1927, an American Economist, who received Nobel Prize in 1990 for his work for economics and finance.

diversification. The risk of portfolio is expressed as variance of asset return and return as a mean of expected return.

All investors have two types of risk, risk of individual asset or known as firm specific risk and systematic risk or also known as market risk. Although market risk effect all the market players and all securities ,while unsystematic risk or firm specific risk only effect a particular firm only .MPT explain that how an investor can minimize firm specific risk by diversifying his portfolio and including different securities which have negative or low correlation with each other.

The MPT aim is to reduce variance of return. The variance is considered as risk.MPT considers that investors are rational and markets are efficient. According to Gruber, Brown and Goetzman (2009) the covariance or co-volatility among two assets in a portfolio variance increases with the number of assets increase in the portfolio. The expected return of portfolio is the weighted average return of each asset in a portfolio. The portfolio variance depends upon the covariance and coefficient of correlation between two assets. Covariance or co-volatility is the measure of movement of mean of two assets with respect to each other over a period of time. The covariance may be of two type i.e positive covariance which means the mean returns of two assets are moving in same direction .While negative variance indicates that mean return of two assets are moving in opposite direction

Markowitz (1952) also introduce efficient frontier. According to him an efficient frontier is one which has the best possible expected return for a given level of risk. A given level of risk is considered to be as risk free rate .Which means that any return above that risk free rate for a given level of risk is considered as efficient frontier. An investor have the choice to select set of portfolio which are giving higher return for a given level of risk. The efficient frontier gives equilibrium point for risk and return, which point shows amount of risk an investor can bear

while having a certain amount of return. An investor can go beyond that equilibrium point by bearing high risk for high return.

This theory MPT was the base for portfolio diversification and the mathematical formulas gives information about risk of individual and portfolio, which helps while considering assets for diversification. This theory opens a debate that how to diversify a portfolio efficiently. This theory further explains the relationship between two asset classes included in the portfolio, their weights and the risk of each security which determine the overall risk of the portfolio. Current study also uses the MPT theory as a base for portfolio diversification.

2.2.2 The Capital Asset Pricing Model (CAPM)

Sharp (1964) [4]presented this model and is the extended work of Markowitz .This model gives relationship between an asset and its expected future return. According to the CAPM model, Asset return depends on risk-free rate and risk premium or beta of the securities.

The CAPM model assumes that, all investors want to maximize their expected utility for a single period of time. Second all investors have equal opportunity to land and borrow at risk free rate. Third all investors have same variance and covariance estimations of all assets. Fourth there are no cost and taxes included and fifth the quantities of all assets are fixed.

According to CAPM, as the un-systematic risk is diversifiable and hence does not required to be compensate while systematic risk is undiversifiable and hence required to be compensated which should be above risk free rate. As Risk-free rate is constant for all securities so it is beta or risk of individual asset which determine the differential in expected return for securities. This beta is also known as unsystematic risk, which is diversifiable according to Sharpe.

[4] William Forsyth Sharpe, born on 1934, is an American Economist, Winner of Nobel Prize in 1990 for his extensive work in Economics Science.

$$ER_i = Rf + \beta_i (Rm - Rf) \quad (2.1)$$

Where**:**

ER_i expected return of investment

Rf =risk-free rate

β_i =beta of the investment

$(Rm - Rf)$ =market risk premium

The CAPM gives investors the opportunity to calculate the risk premium to be charged for any risky instrument included in their portfolio. The risk premium depends upon the risk of individual asset class or stock. CAPM model is important in this study, because it give investors the opportunity to calculate the premium amount to be charge for a systematic risk.

2.2.3 Utility theory

Utility theory[5] shows the preferences of individuals toward consuming a good and the level of utility gain from the consumption. In finance, utility theory is linked with risk and return, and how investors chose between risky security and high return security that will maximize the utility of investor. This selection of security depends upon the preferences of individuals, as risk lover investors prefer high return over low risk while risk-averse investors prefer lower risk over high return. These two types of investors can have the same level of utility with different combination level of risk and return and are irrespective of their attitude toward risk

Sharpe and Alaxender (1990) gives the idea that for some investors, an increase in wealth or return will give maximum satisfaction, while for some investors a decrease in risk or variance

[5] Utility theory definition source: Wikipedia.org

will give maximum utility. It's a vice versa either investors will be looking for higher return or either they will be looking for minimum risk for their investments. The preference of investor will decide, Risk averse investor will go for minimum risk, while risk taker investors will go for higher return without considering higher risk.

Utility theory is important in this study, as this study is about portfolio investments and investors have the choice to go for any investment, either risky investments or no or minimum risky investments depends upon of their preferences.

2.3 Empirical Review

The empirical investigation by different researchers using different data sets, a different approach and different market segments is being explained below. This will help further understanding the topic in detail.

2. 3.1 Optimal Portfolio Size

Even and Archer (1968) Examines the rate effect of increase in securities on the variation of returns. According to them, there exist a reliable and predictable relationship between portfolio size and variance. They used 470 stocks data from 1958 to 1967 from S&P Index and used OLS to regress the data set. Their result shows that there exist a predictable relationship between size of portfolio and portfolio performance. They concluded that an optimal number of portfolio can be constructed from the relationship between number of securities and variation of returns. If the number of securities in a portfolio increase, this will reduce the variance of portfolio, but the decrease in variance depend upon the correlation with the securities, negative correlated security is considered to be best for optimal portfolio .With the increase in the number

of securities, the variance can be reduced up to some level beyond which the variance of portfolio will not work .

Woerhaid and Persson (1993) the question which arise in diversification is what the number is of securities require for an optimal diversification. The use five different method to find out the best possible number of securities for a best portfolio diversification. After final conclusion they showed that a number of fifteen securities are enough for a best portfolio diversification. If diversification is less than that number that investor cannot get benefit from this portfolio diversification and if an investor diversify their portfolio with more that fifteen securities than he may lose extra money while over diversifying. So an optimal number of securities are very important while diversifying portfolios.

Cleary and Copp (1999) Studied Canadian stock return throughout 1985 to 1997. They took 222 stocks from Toronto stock exchange and find their mean and standard deviation monthly. The aim of their study was to find out optimal number of stocks that can best for portfolio. After analyzing the data and calculating optimal portfolio size. Their result indicates that a size of 30 to 50 stocks can capture diversification benefits and minimize risk to an optimal level. But these finding are for short-term only. The long-term optimal size may change due to change in risk and return of each stocks. However according to them a number of 10 securities can substantial enough for diversification. Which shows that optimal number of diversification can reduce variance of portfolio up to some limits beyond which reduction in variance is not possible.

Byrne and Stephen (2001) In their study to find the benefit of portfolio size, they uses number of assets and level of diversification on UK property assets .Their result shows that the size of the portfolio have negative related to unsystematic risk, but have a positive relation with systematic risk. According to them when diversification is done they were able to reduce unsystematic risk

only and not able to reduce market level risk or systematic risk. Because as systematic risk can affect all the assets in a portfolio and hence cannot reduce the overall risk of portfolio. This result shows that only unsystematic risk can be minimized by increasing the size of the portfolio up to optimal level. While systematic risk can be reduce by diversifying in international markets where markets are low integrated.

Keat and Kim (2011) Studies Malaysian stock exchange and tried to find out the effect of portfolio size and fund allocation method risk of portfolio. Risk in the portfolio decrease up to certain limit but after that at a decreasing rate. The number of optimal stocks for Malaysian stock are 11 are enough to minimize portfolio risk. The allocation of stock in portfolio when equal weights are assign to each asset perform poorly and when these weights are allocated based on correlation between assets this portfolio outperform all other portfolios. Hence suggesting for a minimum variance approach or an unequal weights to be assign to each assets in a portfolio so that maximum performance can gain from a portfolio.

Alexeev and Dungey (2014) in their study to address the number of stocks for an optimal portfolio, they find out that for the investors in the US stocks market a number of 8 to 10 stocks are sufficient enough to diversify away 90 percent of risk. They have used equally weighted assets in the portfolio. They took the data from S&P from 2001 to 2007 on daily basis with 5 minute duration between stock prices. According to them in the financial crises of 2007 to 2008 the magnitude of diversification was almost double of the size of the current optimal portfolio size. Which shows that the optimal size for portfolio depends the risk factors of the markets as well. Highly risky markets have high number of portfolio size for capturing risk.

Tapon and Vitali (2012) in their study of US, UK, Japan, Canada , Australia. They used daily data from year 1975 to 2011. According to them an increase in portfolio size will decrease the

risk of portfolio. The optimal portfolio size depends upon, the measure of risk, the risk of specific market, and the correlation between stocks over time .If securities in a portfolio have high correlation that the optimal portfolio size will increase and if they have negative correlation than few securities are enough for optimal portfolio size.

Stotz and Lu (2014) over the period of ten years, they have used 13000 stocks of Asia excluding Japan, and tried to find of optimal size of stocks portfolio. while adding equally weighted stocks in the portfolio of randomly selected stocks, the find out that ten stocks can minimize risk up to 64 percent while if ten more stocks are added in the portfolio, this only reduce the portfolio risk up to 74 percent. The fund managers if used the optimal size of ten stocks, they can get maximum utility out of it instant of using twenty stocks. This will reduce the diversification cost and help fund managers to actively change their portfolio.

Tsui, K.C and C.H (1983) in their study they used 40 stocks from Singapore stock exchange and tried to find out systematic risk as well as unsystematic risk of these stocks. The time span was from 1971 to 1983. They have used mean variance model and find out that a total twenty stocks can form an optimal portfolio and diversify a maximum amount of risk. The unsystematic risk can be reduced up to certain limit through optimal diversification, while the systematic level risk can be reduced by diversifying in different markets, which have low level of integration from local level stocks.

Statman (1987) in addition to portfolio size which was done by Even & Archer. Statman, used data from year 1979 to 1984 from S&P Index. While using randomly selected stocks he also allows borrowing and lending by firms. He used daily for analyzing risk and return and their result shows that stocks of 30 for borrowing investors and 40 stocks for lending investors can make a well-diversified optimal portfolio. He suggest that, the size of diversification should be

increased as long as the marginal benefit of diversification does not increase marginal cost of portfolio size. Furthermore if the size of portfolio increased without considering marginal increase in return or marginal decrease in risk than no portfolio will benefit the investor.

Goetzmann and Kumar (2008) shows US individual investors. Their preferences in investment, the level of diversification among young people, old age and their style of investments. According to them most of investors in the US under diversified their portfolio , the reason is due to over confident, biased investments, or superior information level. Other than that some investors do under diversification to save transaction cost and other relevant costs. This strategy of individual investors is very risky, as the amount any quality of information they are receiving may not be enough to rely on. The investors should go for optimal level for minimizing their portfolio risk and should rely on limited information and do not under diversify or over diversify their portfolios.

Bera and Perk (2008) they have used cross-entropy measure as an objective function for portfolio diversification instant of mean variance or minimum variance approach. They have used the mean variance covariance matrix and shrunk portfolio weights instant of using predetermine portfolio weights, like equal weights or minimum variance approach. The result of their study shows a high performance portfolio than mean variance approach and minimum variance approach.

There are various studied on optimal portfolio size in emerging markets. Gupta, Khoon and Shahnoon (2001) in their study of optimal portfolio size for the Malaysian stock market for the period 1988 to 1997. They used 213 stocks and calculate their mean-variance and found out that a well-diversified portfolio of stocks contains at least 27 randomly selected stocks. The further extend their study and find that 30 securities gave well diversification for borrower investors and

50 securities for lending investors. Meenakshi (2013) studied Indian stock market and try to find out the optimal size of portfolio. The data were taken from year 2001 to 2011. He applied OLS method and tried to find out relation between size and portfolio risk. According to him the risk of portfolio decrease with the increase in portfolio size due to the inverse relation between size of portfolio and portfolio risk. He further suggested that a number of 20 securities are enough for diversification. Beyond which if securities are added to a portfolio this will not reduce the risk of portfolio.

Patil (2007) studied Indian stock and tried to find out the optimal size of stocks to be included in portfolio so that risk is minimized at optimal level. According to him adding stocks in to portfolio defiantly reduce the risk of portfolio ,but this reduction in portfolio also have some cost , that is also known as diversification cost. So he want to find out what will be the optimal size of stocks for a best portfolio diversification. The result of his study shows that a size of ten to fifteen stocks are enough and can minimize the portfolio risk, beyond which if stocks are added to portfolio this will reduce the overall risk of portfolio but the marginal risk will no decrease.

Klein and Bawa(1977) this study finds the difficulties an investors face in optimal portfolio choice. According to the study, the limited information, the estimation risk, insufficient sample information and abdicate methodology adopted are the main problem an investors faces while calculating optimal level of diversification. If an investor find a proper way of calculating these errors, a perfect optimal portfolio can be formed. So for calculating the risk of a portfolio it is very important to calculate the mean variance of each return series properly and also use adequate method for calculating portfolio risk and return.

Tang (2004) portfolio diversification can eliminate the unsystematic risk and left behind only systematic risk. A portfolio size of ten to fifteen can eliminate most of the diversifiable risk and

leaving behind only systematic risk. This study focus on naïve strategy of equally weights while if 80 more stocks are added to this portfolio this can only eliminate extra 4 percent of risk. Which means that 100 stocks can only eliminate 99 percent of risk. So it is not viable to add more stocks than 20 in a diversified portfolio. He further suggest that the systematic risk can be minimize up to certain limit using low correlated assets.

Rowland (1999) this study focus on transaction cost, of domestic level portfolio diversification and international level portfolio diversification. According to the study the rate of diversification decreases with the increase in transaction cost both for domestic and international level portfolio diversification. Further the study suggest that with the increase in cost of diversification, active portfolio reallocation decreases, while passive portfolio allocation increases and investors are looking for capital gains. While the turnover rate of international portfolio is greater than domestic level portfolios. There is no especial reason for this turnover rate, it is the nature of international investors that determine this rate.

The above discussion shows that the optimal number of securities are not same for every market, each study and markets have different number of optimal size ranging from 10 to 50. Which means that size is not considered to be best variable for diversification, there should be other efficient way to diversify a portfolio. Santos (2015) studied Brazilian stock market and suggest that a better performance of portfolio can be obtained by fewer securities included in portfolio and by better allocation of assets in the portfolio. Further he also suggest that it is not optimal portfolios that matter but the correlation among securities and the better allocation of securities will outperformed in every aspect of portfolio diversification.

2.3. 2 Domestic Level Portfolio Diversification

After focusing on optimal size of portfolios. Researcher have also used domestic level stock portfolio diversification and tried to find out any benefit from diversification from local stock markets.

Cheng (2001) used US stock return data from year 1996 to 2001, he used different sectorial stock return and comapre their return,accourding to him tecnological sectors were in the rise ,while consumer products , energy,financial products were going down.while in long rurn this tecnological sector will burst and the consumer sector will pick up performance gradually.sectorial diversification is important for short term volaltility.A mix of different sectorial portfolio will benefit the investors in both short run and long run.

Gupta and Basu (2011) studied the different levels of the local market i.e. Australian and Indian markets. In their study, they use Indian and Australian equity sector indexes, while using time-varying asymmetric Dynamic conditional correlation (DCC) GARCH model for perfect estimation of correlation between stocks. They use data from 1997 to 2007 and find out that Indian markets and Australian markets are efficient. They uses weightage restrictions in portfolios and uses different weights to different assets and compare their results. Their result shows that few restricted portfolios show high performance than highly restricted portfolios.

Gupta and Li (2017) extend the work of (Gupta and Basu,2011) and extend it to three different markets. They also use the DCC GARCH model for estimation of correlation between sector indices of three different countries i.e. Australia, India, and China. Their results show a benefit from diversifying in sector-level stocks in all three markets. Which shows that other than

diversifying in international markets local stock diversification can also benefit the investors. According to them, political structure and market structure does not affect sector diversification.

Diversification is independent of political structure and market structure, because diversification can minimize unsystematic risk and political risk and market structure are systematic risk.

Ahmad, Ali, Ijaz and Ahmad (2018) while examining Colombo stock exchange for domestic level diversification benefit. They used data from year 2003 to 2016 and try to find out co-integration among different sectors, Multivariate co-integration and pair wise co-integration test were done to know to level of integration among sectors. Their result shows that no integration exist among sectors in Colombo stock exchange. Which means that investors have excellent opportunity to diversify their portfolio in local stocks and minimize their systematic risk rather going to international level stocks.

Krishanankutty and Tiwari (2011) Examines bombay srock exchange and try to find out that any benefits can have while sector portfolio diversification.The used weakly data from 1999 to 2011.While using cointegration test they find that no cointegration among sectorial indices in bombay stock exchange.Which again suggest that investors have an opportunity to diversify their portfolio in local level stocks.

2.3.3 International portfolio diversification

After a number of study have done on domestic level portfolio diversification,Reserchers have shift to internation level diversification and tried to find out any benefit can get from it.

Solnik (1974) examined stock returns of eight countries over the period of 5 years from 1966 to 1971. He used weakly data of more than 300 stocks for US, UK, France, Germany, Switzerland,

Belgium, Italy, and Netherland. While using a mean-variance model. The mean and variances of stocks were calculated for both domestic and international stocks and compared. He finds out that international diversification have low risk as compare to domestic portfolio diversification. According to Solnik (1974), a total of 20 securities of US Stocks can diversify risk up to certain limits beyond which cannot be minimized further. Further adding of securities will not decrease any risk and because all stock prices tend to move together. This shows that for international stock diversification the number of optimal stocks are less than domestic level diversification.

Meric and Meric (1989) their findings show that when portfolio is diversified across countries the reduction in portfolio risk is higher than when diversified in within industries in local market.Which shows that investors can get a minimum varinace of portfolio when they diversify their portfolio locally than internationally.The test for seasonality shows that the co-movement of international market are stable in sepember to may and are relatively unstabale from may to september.

Cleary and Copp (1999) According to them only 10 stocks can diversify away risk when diversified internationally as compared to local diversification which is 30 to 40 stock. According to them, it is due to the factor of the same co-movement of stock return in the local market, and when compared to international emerging markets which have low correlation. As correlation have important role in diversification of stocks. Which again shows the benefit of international diversification over domestic's level diversification.

Saritas and Egoren (2005) in their study they used the return of 15 international indices and the Istanbul stock exchange .The data covers from year 1998 to 2002. The findings of the study suggest that international indices doesn't shows superior performance than local stock exchange when formed portfolio diversification. This international diversification seems to be beneficial

but difficult to realize in practice. Hence according to them local level portfolio diversification is better than international level diversification.

Shawkey, Quenzil and Mikhel (1997) they studied the benefit of international diversification and shows that although international diversification have benefits but the correlation between international markets is unstable and hence make it difficult for investors to find an optimal portfolio diversification. Their finding suggest that investors should focus on optimal level diversification while investing in diversified portfolios. This will reduce the cost of extra investment in securities and increase the return of optimal diversification.

Santis, Bruno and Gerard (2012) used CAPM for world eight large equity markets. They used GARCH model for many assets at the same time, their findings shows that the equity market of US shows a steady return over many years as compare to other equity markets of the world. Other seven markets shows decline in return over a period of two decades. Which again shows the benefit of local level portfolio diversification rather going internationally.

Abru and Mendes (2009) in 2005 they use the behavior of investors, and their financial literacy on portfolio diversification. They have used survey technique on investors and find out the socio economic behavior of different investors. The result of the study shows that investor's education and level of financial knowledge have positive impact on portfolio diversification. The access of information the investors have also effect the number of assets included in the portfolio. A high level information access to the investor have high level of portfolio diversification. High knowledge of financial instruments will give the investors an edge over an illiterate one who only follows trends in the market.

Driessen and Leaven (2007) investigate the perspective of different international level diversification and its benefit. The study shows that diversification benefit depends upon country risk. If country risk is high than international diversification in that country will also be high. Thus suggesting to include a high risk country in portfolio while investing in international level stocks. Furthermore this risk of country is time varying as risk is a changing factor. With change in time the risk of a country also change and hence the portfolio should also be changed accordingly. There is a point that developed countries can diversify their portfolios in to less developed or under developed markets and can get benefit from low level of market integration.

Oloko (2018) they study investigates the portfolio diversification benefit of the US and UK stock investors in to Nigerian stocks market. This study used BEKK GARCH model for estimating conditional variance and covariance. The used these variance and covariance to calculate the optimal portfolio weights and optimal hedge ratios. Nigerian stocks market have one of the largest foreign investments in Africa. The evidence from the study shows that the US and UK investors have an opportunity to diversify their portfolio in Nigerian stocks market and can get benefit from it, as Nigerian stock market have low level of integration with US and UK stocks markets. The US and UK investors can minimize shocks by holding 10 percent US and 25 percent UK stocks and 16 percent Nigerian stocks in their portfolio.

Freanch and Poterba (1991) “the fortunes of different nation does not move together”. An investors have the opportunity in invest their fund in every part of the world as this world is a global market. While the benefit of international diversification has been identified long ago but still investors use local level stocks in their portfolios. This study have identified the preference of investors who prefers local level diversification over international level diversification. The answer is ,it is the choice of individual investors rather the institutional constrains.

Coeourdacier and Guibaud (2011) investigate bilateral cross border equity markets. They investigate whether domestic stock variance can properly be hedged using international level stocks are not. They used foreign stocks that have low correlation with domestic level stocks. While doing so their findings shows a better portfolio can be formed while including foreign stock in portfolios. Furthermore these stocks are not static as their correlation change with change in time.untill and unless international markets have low level of co-integration with local markets, it is viable to add international stocks in a local made portfolio.

Gilmore and McManus (2002) the paper examines both long and short term relationship between US stock market and three European markets. In short term low correlation was found between US stocks and other stocks, while Johnsen co-integration shows no long run correlation among them. Overall the result shows that US investors can get benefit from diversification into these three European stocks market. This shows the benefit of diversifying internationally. But for long run investors have to change their portfolio strategies accordingly. As the relation between US stock market and other international market is not clear.

Balli, Bashir and Louis (2013) in their study, they have used GCC (Golf Corporation Council) Sectorial returns. They investigate the sectorial return of different sectors, in across Country, with in country, their result shows that a mix of both across country and local level sectorial stocks, shows a higher return than a single local level or cross country level stocks portfolio diversification.

Abid, Lam, Mua and Kuan (2014) this paper has used portfolio optimization technique and stochastic dominance method to find out the performance of international diversification and domestic level diversification. The result of portfolio optimization shows that domestic level diversification have low risk as compared to international level diversification. Further the SD

model shows that there is no arbitrage opportunity exists between domestic and international level stocks. Although domestic level diversification dominate international level diversification but we cannot say that domestic level diversification can dominant international in all aspect.

The above discussion shows that although there is a benefit in international diversification, but still some study did not suggest an international diversification, as it is unstable and costly due to fluctuation in exchange rate and the integration among global financial institute discourage international level diversification.

2.3.4 Empirical evidence from Pakistan on Portfolio Diversification

Few studies have found from Pakistani Market, like Habibah, Sadhwani,and Memon (2018) have tried to identify the number of securities for an optimal portfolio size. The formed portfolio size of 40 securities randomly selected from Pakistan stock exchange, and find out that a portfolio size of 20 equity securities an minimize a significant amount of risk. They have taken monthly closing prices of stocks from 2009 to 2015, they use Markowitz's mean-variance model in their study. While randomly selecting stocks for optimal size they find a number of 20 stocks are enough for optimal diversification. Ahuja (2011) analyzed the same type of data for three year from 2007 to 2009 and find out that 10 stocks can diversify away a significant 52 % of portfolio risk.

Zaidi (2017) took data from 2003 to 2014 of 32 stocks from KSE 100 index. The data taken represent the major sectors shares which are traded in Karachi stock exchange. Monthly expected return, variance, and covariance have computed to know the efficient portfolio frontier. According to him when short selling is not allowed a portfolio of 9 stocks gives a return of 29.04 percent which is 1.614 percent greater than the return of KSE- 100 index. This study also uses

MPT mean-variance analysis while randomly selecting stocks. This study did not consider the risk factor of portfolio and only focus on return of portfolios.

Hussain, Hussain and Bhatti (2012) investigate the Pakistan equity market with the other seven equity markets like US, France, Germany, Japan, Hong Kong and Singapore. The purpose of this study was to know the integration of KSE with other countries stock exchange. Taking data from 1988 to 1993 they found out that the Pakistan equity market has low integration with other seven equity markets. Hence providing Pakistani market a viable market for international diversification and providing an opportunity for international portfolio managers to include KSE in their portfolio construction and also the institutional investors in Pakistan can get benefit from this low level of integration of KSE with rest of the international stock markets.

Muhmmad (2012) Investigate the linkage between KSE with other markets equity market and find out a strong positive correlation exists between KSE and US Market.While the UK ,China, India, and Germany have comparatively low level of correlation. Investors can get benefit from diversifying their portfolio in KSE due to their low integration with the KSE equity market. Which again shows an opportunity for international investors to use KSE stocks in their portfolios. Other than US market all other markets have low correlation with KSE and hence are good for portfolio diversification. The low integration of financial markets is very important when to diversify internationally, because if markets are fully integrated than it is no favorable to diversify in them.

Haroon, Hussain ,Waqar and Fraz (2010) have explored the relationship between the Asian equity market like (Karachi Stock Exchange, Dhaka Stock Exchange, Bombay Stock Exchange, and Colombo Stock Exchange).Using monthly data from the year 1999 to 2009 and using co-integration for long term relationship and VECM for short term relationship, they find out that no

relationship both in short-run and long run exist between these markets. Which again show that PSX is a best option for international investors and can benefit of international diversification and hence are suitable for international diversification.

Hassan, Salim and Abdullah (2008) examine long-run relationship between KSE and developed markets, like US, UK, Germany, Canada and Italy. Data covering from the year 2000 to 2006 using multivariate co-integration analysis. Their result shows no integration of KSE with the US ,UK, Germany, Canada, Italy, and Australia. Which again shows a positive sign for investors to keep PSX in their portfolio diversification. Investors can also diversify their fund internationally for short run as well.

Jebran (2014) in his study he use Pakistan stock market and some Asian stocks markets like India, Srilanka, China, Indonesia and Malaysia. The monthly closing prices of indices are taken from a period of year 2003 to 2013. They used johansen and juselius co-integration approach to check the relationship between stock markets. The result of the study shows, no relationship exist between Pakistan stock exchange with rest of Asian stock exchanges. Furthermore he concluded that the variance and change in Pakistan stock exchange is not due to change in other stock exchanges the changes is due to internal factors only.

Joyo and Lefan (2019) studies PSX and the co-movement among PSX and their trading partner like China, indonesia, malysia, UK and US.They used DCC-GARCH model to examine time varting corelation and volatility amomg PSX and other partner countries stock exchnage.The daily data from year 2005 to 2018 have taken for observation.Their results shows that before financial crisis in 2008 the PSX and patner countries have high integration while after 2008 the integration among PSX and partner countries decreases.and hence they suggest that an investor can use PSX and other trading partners as a best diversification portfolios.

Alvi, Chugtai and Ul Haq (2015) the objective of this study was to know the co-movement of Pakistan stock exchange KSE-100 with developed countries stock markets i.e Canada, UK, US, Australia, Germany, Japan and France. They have used the date span from year 2007 to 2014, and used multi-variant and bi-varieant johensen and jeselus co-integration.The study does not shows any co-movement of KSE with rest of stock markets and hence offers investors a great opportunity to invest their fund in domestic level stocks and also looking for international level stocks to add in their portfolio as well.

All the above studies have focus on optimal portfolio size and integration of Pakistan stock market with other international equity markets. None of them have tried to find out any benefit we can take by investing in local level stocks. According to them, managers and investors can invest in international markets for diversifying their funds. The Current study have focus on Domestic level stock diversification rather than international level and will provide a better way of diversifying in domestic stocks.

2.4 Multivariate GARCH model

The volatility and covolatility of financial series and their proper claculation is very important.For this purpose, Bollerslev (1986) was the first to suggest GARCH (Generalized Autoregressive Conditional Heteroscedasticity).GARCH model employs past variance and squared innovation to predict future variance. He used GARCH model as a time varying model for capturing volatility and co-volatility. According to him time series data requires model which is no static rather it is dynamic and require model like MGARCH, which is best in analyzing time series data, especially calculating variance or co-variance over time.

Tim, Robert and Jefry (1988) have used MGARCH model for estimation of covariance of assets return with market return. Where the returns of T- bills, stocks and bond and market return were taken and their covariance were calculated using time varying MGARCH model which is considered to be best model for time varying covariance according to them, GARCH model perfectly estimated the volatility and co-volatility among them which were helpful in hedging .

Baillie and Mayers (1991) used MGARCH for estimation of optimal hedge ratios for six commodities future. Chen et.al (2011) and Lee & lee (2012) also used MGARCH for optimal hedge ratio. In their study of effectiveness of different future contracts traded in Istanbul stock exchange have used static model like OLS and dynamic model like VECK-GARCH model .In their study they concluded that VECK Model of GARCH family out performed OLS in term of reduction in variance when used as an optimal hedge ratio estimator.

Gupta and Basu (2009) used DCC GARCH model to test efficient portfolio that will generate return above market level. Analyzing daily data of 10 Sectors of Indian market from year 1997 to 2007 they calculated correlation among these sectors using DCC GARCH model and give idea that correlation may change over time and with change in correlation portfolio performance may also change.

Gencer and Musoglu (2014) have used DEKK-GARCH model for analyzing transmission effect and spillover effect among gold prices, stock and bond prices. While their study of Turkish stock market with gold and bond prices ,they find out that Turkish stock prices have negative correlation with gold prices and hence are considered to be a unique investment and suitable for portfolio diversification. Rehman et,al.(2010),In their study they used KSE 100 index and compare it with other international indices. The data used from year 2005 to 2010 of PSX and international indices .While using EGARCH for calculating co-movement, they find that PSX

have negative movement of return with Global indices and hence they suggest that international investors can get benefit while investing in PSX.

Aun, Hassan and Malik (2007) in their study they have used daily return of US Stock indices from year 1992 to 2005.They have used multivariate GARCH model for estimating mean and conditional variances. According to them for portfolio allocation of stocks, it is very important to understand volatility transmission between stocks. They find a significant amount of shocks and volatility transmission among stocks and hence suggest for investors to hedge their portfolio in cross markets and other markets where there have low transmission between stocks.

So multivariate GARCH model is widely used for calculating variance and co-variance for stock returns and can calculate the time varying variances perfectly. We are using DBEKK model of MGARCH family in this study.

CHAPTER 3

DATA DESCRIPTION AND METHODOLOGY

3.1 Research Type

This study is empirical quantitative in nature. As the prices of stocks are historic and hence categorized as secondary in nature.

3.2 Sample Size

As the aim of this study is to construct efficient portfolio which will give minimum variance. For portfolio construction it is very important to choose securities wisely. For analyzing purpose this study uses stock prices listed in PSX, but there are 500+ companies listed on PSX, and we cannot take all of them, therefore six sectors from PSX have chosen as a sample for portfolio construction. Among these six sectors, each sector have given equal number of representation in the sample. A total 30 firms and their daily stock prices have used as a sample.

The six sectors used in this study are, 1 cement sector 2. Engineering sector 3. Textile sector 4. Pharmaceutical sector 5. Chemical sector 6. Transport sector

There are especial reasons for selecting these sectors for current study. As current government have given especial packages and compensation to construction sector and other than that CPEC have a big role in the demand of cement and engineering sector, which will benefit cement, engineering and transport sector in near future. Other than that in current years due to Covid-19 the only sectors which are in boom and are open are pharmaceutical and textile sector which have break all previous record of exports. Due to all these reasons, these sectors are included in

the sample and we assume that these sectors are the most prominent sectors to diversify in current scenarios.

3.3 Data Collection

The daily closing prices of 30 companies from six different sectors have collected from investing.com. and PSX website. As the study requires daily prices and synchronized data, it was very difficult to find complete set of data for synchronization. A total 60 firm were selected initially but after synchronizing we have only thirty firm left. The study uses daily prices from 1st Jan 2014 to 31 Dec 2019 which make 1485 synchronized observations for each firm. The daily return have been calculated from daily prices and further used for analyzing purpose.

$$R_t = \ln P_t / P_{t-1} \qquad (3.1)$$

Where in equation 3.1, R_t is return of stocks while Pt is current price and Pt-1 is previous price of stocks. Ln is for natural log of returns. The PSX have five working days starting from Monday to Friday.

3.4 Return of Portfolio

Earning received from invested amount is known as return. The above equation 1 shows the return of single stock. While the return of portfolio is the weighted average accumulated return from all assets in the portfolio. Following equation shows return from a portfolio.

$$R_p = W_1 * R_1 + W_2 * R_2 .....W_n * R_n \qquad (3.2)$$

Where,

Rp is the portfolio return,

w_1 is the weight of the first asset in the portfolio,

R_1 is the return on the first asset,

w_2 is the weight of the second asset, and

R_2 is the return on the second asset and so on.

While Wn and Rn is for 'n' number of terms.

3.5 Portfolio variance

By variance we mean, fluctuation of return series over time. Variance is a measure of risk. A portfolio variance shows overall fluctuations of a portfolio, it is the standard deviation of each security in the portfolio and correlation each pair of security in the portfolio.

$$\sigma_p^2 = \sum_{i=1}^{n}\sum_{j=1}^{n} wiwjCov(r_i, r_j)$$

$$\sigma_p = \sqrt{\sum_{i=1}^{n} w_i^2\sigma_i^2 + \sum_{i=1}^{n-1}(\sum_{j=i+1}^{n} 2w_i w_j \operatorname{cov} ij)\sigma_p} \qquad (3.3)$$

W_i = the portfolio weight of the first asset

W_j = the portfolio weight of the second asset

$Cov_{i,j}$ = the covariance between two assets.

The above equations shows the variance of a portfolio depend upon three basic things i.e weight of security in the portfolio ,variance of individual security and the covariance or co-volatility among securities that defines the variance of portfolio

According to MPT (modern portfolio theory),which is the basis for portfolio diversification ,a well-diversified portfolio is the one ,which have low correlated securities or negative correlate which will result in a lower portfolio variance. But in real life it is difficult to find such securities which are perfectly and negatively correlated.

3.6 Multivariate GARCH Model

Bollerslev (1986) was the first to suggest GARCH (Generalized Autoregressive Conditional Heteroscedasticity).GARCH model employs past variance and squared innovation to predict future variance. Volatility transmission and effect can be measured by using Multivariate GARCH. Correlations are critical when it comes to hedging strategies or constructing portfolios. A number of practitioners have used multivariate GARCH model for estimation of variance and correlation like, Bollerslev, Engle, and Wooldridge(1988), Bollerslev (1990), Kroner and Claessens (1991), Engle and Mezrich (1996).Other than that chang et.al(2010) , Lee and lee(2012) have used MGARCH for estimation of optimal hedge ratios .

Dr. Saud Ahmad in his PhD thesis “Hedging: an Islamic approach (2012)” have used three model for optimal hedge ratio calculation i.e. static hedge ration using OLS ,VAR (p,q) model and DBEKK(p,q,k) model. According to him among these three models DBEKK outperformed the others, when the optimal hedge ratios of DBEKK model are used, it increases the percentage reduction of variance in a portfolio. Current study also uses DBEKK model of MGARCH for estimation of variance covariance .As this model is perfect to estimate timely varying hedge ratios.

3.7 DBEKK Model (Diagonal baba Engle Craft Kroner)

The DBEKK model belongs to the family of MGARCH.). Engle and Kroner (1995) introduces BEKK model for the construction of conditional variance and covariance matrix.in order to reduce the number of parameters the DBEKK model is used.

The DBEKK (P,q,k) Model:

$$HH_t = CC^{'} + \sum_{i=1}^{q} . \sum_{k=1}^{k} . A_{ki}^{'} \varepsilon_{t-i} \varepsilon^{'}_{t-i} A_{ki} + \sum_{t=i}^{p} \sum_{k=i}^{k} G^{'_{ki}} H_{t-i} G_{ki}$$

Where,

$$H_t = \begin{bmatrix} h_{11,t} h_{12,t} h_{13,t} \\ h_{21,t} h_{22,t} h_{23,t} \\ h_{31,t} h_{32,t} h_{33,t} \end{bmatrix}, C = \begin{bmatrix} c_{11} 00 \\ c_{21} c_{22} 0 \\ c_{31} c_{32} c_{33} \end{bmatrix}$$

$$DiagA = [a_{11} a_{22} a_{33}] and$$
$$DiagG = [g_{11} g_{22} g_{33}]$$

Where h_{11} represent volatility while h_{12} represent co-volatility, and t indicates time while c is for constant.

3.8 Optimal Hedge Ratio as a Tool for Portfolio Construction

Optimal hedge ratio gives the perfect ratio of investment in any security in a given portfolio, which minimizes the portfolio variance. There are number of ways of estimating optimal hedge ratios.it depends upon the preference of investors and their objective functions (chen et.al, 2004).We will be using Minimum variance approach, as the main purpose of portfolio diversification is to minimize risk or variance. The objective function of minimum variance hedge ratio is to minimize variance of return of portfolio. For using minimum variance HR the expected return must be zero.

Ederington (1979) proposed minimum variance hedge ratio and reduction in variance of portfolio as a hedging effectiveness. If the reduction of portfolio variance is greater than unhedged portfolio variance then we can say that hedging works.

$$\Delta V = [V(UH) - V(H)] / V(UH) \qquad (3.4)$$

Where, ΔV is change in variance

VUH is variance of unhedged

VH is variance of hedged

For percentage change in variance the above formula can be written as,

% change in Variance= $[V(UH) - V(H)] / V(UH) * 100$

If the percentage reduction in variance is greater than 0 than we can say portfolio hedge worked and we can get benefit from hedging and if it is less than zero than hedging have no value.

The use of these different hedge ratios depend upon the investor preference and objective behind portfolio construction. Following are different hedge ratios, and we are using minimum variance hedge ratio among them.

Table 3.1 Different Hedge Ratios

Minimum variance hedge ratio
Mean variance hedge ratio
Maximum expected utility hedge ratio
Maximum MAG coefficient hedge ratio
Optimum mean MAG Hedge Ratio
Minimum GSV Hedge ratio
Maximum mean GSV hedge ratio
Sharpe hedge ratio

Source: Authors Computation

The minimum variance approach uses variances of individual and use them to calculated portfolio variance using different hedge ration. The variances of different portfolios are compared using percentage reduction in variance. The portfolio which gives minimum variance is considered to be best among others. Other than minimum variance hedge ratio there are other ratios as well, which have different objective functions. The use of these ratios depend upon the objective of the researcher .The main reason investors diversify their portfolio is to minimize risk rather than increase in return. As the objective of this study is to minimize the portfolio risk, so we are using minimum variance hedge ratio, to find our objectives.

3.9 Theoretical Estimation for Optimal Hedge Ratio

In this study we have used three return series, $r_1 r_2 r_3$ for portfolio construct and r_p is portfolio return having three securities. Here we are hedging r_1 using different combination of r_2 and r_3. Where $r_1 r_2 r_3$ are return of each series which can be calculate by using equation 3.1 as

$$R_t = \ln \frac{p_t}{p_{t-1}}$$

Let,

$$r_1 = a + b_{r2} + c_{r3}$$

$$r_p = r_1 - a - b_{r2} - c_{r3} \quad (3.5)$$

While portfolio variance Var (Rp)

$$Var(r_p) = Var(r_1 - a - b_{r2} - c_{r3}) \quad (3.6)$$

Var (r_1,r_2, r_3) are variances of each return series

And Var (r_p) is portfolio variance.

Where, b and c are optimal hedge ratios and calculated using following optimal hedge ratio formula 3.7

$$hedgeratio = Cov(r_1, r_2) / Var(r_2) \quad (3.7)$$

Equation 3.5 is used to find portfolio return while equation 3.6 is used to calculate portfolio variance, which are further used in minimum variance hedge ratios. While .7 is used for optimal hedge ratios.

CHAPTER 4

RESULTS AND INTERPRETATION

In this part of the chapter, we have interpreted and discussed our results and findings in detail. The methodology and techniques explained in previous chapter i.e in chapter 3, their results are shown and explained in this chapter.

4.1 Descriptive statistics

The below table 1 shows all the 30 return series and their descriptive states. All variables having 1484 number of observations, with minimum and maximum values followed by mean and standard deviations. The minimum value for each stock series is negative which show how risky stocks are. The mean return of AISHA, ABBT and PIBT are showing positive and high return other than that all other mean return are near to zero. The table below is self-explanatory which is clearly showing stats for each variable.

Table 1: Descriptive statistics of Return series

Sr. no	Return Series	Observations	Mean	min	S.D	max
1	DLMPLF	1484	-0.0001	-0.2159	0.0248	0.0651
2	DLCHRC	1484	0.0001	-0.1121	0.0238	0.0585
3	DLFAUC	1484	-3.8947	-0.0703	0.0212	0.0757
4	DLKOHC	1484	1.2831	-0.0687	0.0211	0.0601
5	DLLUKC	1484	0.0002	-0.0530	0.0196	0.0524
6	DLPION	1484	-0.0001	-0.0885	0.0243	0.0801
7	DLNISM	1484	-0.0001	-0.0638	0.0200	0.0571
8	DLNCHU	1484	-0.0001	-0.0659	0.0215	0.0545
9	DLKOHT	1484	0.0002	-0.0664	0.0240	0.0884
10	DLGULA	1484	0.0002	-0.183	0.0239	0.0897
11	DLDFSM	1484	0.0008	-0.2442	0.0509	0.2740
12	DLKOHI	1484	-0.0003	-0.1863	0.0463	0.2795
13	DLABBT	1484	9.0524	-6.9032	0.5661	6.9063
14	DLSEAR	1484	0.0010	-8.9037	0.5797	8.8867
15	DLHINL	1484	0.0012	-0.1861	0.0259	0.1529
16	DLGLAX	1484	-0.0004	-1.0543	0.0725	0.8111
17	DLIBLH	1484	0.0005	-0.6066	0.0527	0.6080
18	DLINTE	1484	0.0012	-0.0680	0.0323	0.6268
19	DLINTI	1484	-0.0009	-2.2905	0.0865	0.7537
20	DLCRST	1484	0.0001	-0.3058	0.0313	0.2388
21	DLDOST	1484	0.0004	-2.788	0.2243	2.773

22	DLAISHA	1484	8.7405	-0.4212	0.0427	0.4223
23	DLPIBT	1484	0.0025	-0.5598	0.0725	2.1617
24	DLPIAA	1484	-0.0001	-2.3246	0.1353	2.3094
25	DLPNSC	1484	0.0013	-3.1451	0.2645	3.1109
26	DLARCH	1484	0.0005	-2.6417	0.1502	2.6377
27	DLITHD	1484	-0.0001	-2.9939	0.2657	3.0395
28	DLICI	1484	0.0006	-3.429	0.3030	3.4538
29	DLDOL	1484	0.0008	-4.081	0.3361	4.1013
30	DLAGLI	1484	-0.0007	-0.5040	0.05248	0.44929

4.2 Checking Normality

The below table 2, shows normality results. Normality shows the normality test results of each return series. The skewness of the return series shows the measure of symmetry distribution. The value 0 for skewness means the series is normally distributed while negative and positive sign show the distribution pattern i.e. negatively skewed or positively skewed. So the table below shows that most of the return series are negatively skewed or having zero skewed.

While the Kurtosis is the measure of peakedness or flatness of the data set. The data set shows mix result for excess kurtosis, but most of them have a high peak as most of values are greater than 3 which is considered as high peak than normal distribution.

JB (Jarque bera) test hypothesis H0: Series is normally distributed. The normality of return series is checked by using JB (Jarque bera) test. except for three series , DLNCHU, DLKOHT, DLGULA all others are not normally distributed ,because the JB tests p vale for all rejects our

null hypothesis that was series is normally distributed and accept our alternate which is series are not normally distributed.

Table 2: Test for Normality

Sr. no	**Return Series**	**Skewness**	**Excess Kurtosis**	**Jarque-Bera**
1	DLMPLF	-0.690*	8.143*	3277.8*
2	DLCHRC	0.128*	0.518*	16.071*
3	DLFAUC	0.154**	0.789*	34.563*
4	DLKOHC	0.207*	0.637*	35.779*
5	DLLUKC	0.189**	0.920*	47.021*
6	DLPION	0.103***	0.367*	9.293*
7	DLNISM	0.045**	0.619*	18.826*
8	DLNCHU	0.007	0.261	3.772
9	DLKOHT	0.065	0.084	1.161
10	DLGULA	-0.093	2.445	288.98
11	DLDFSM	0.594*	4.827*	1187.3*
12	DLKOHI	0.681*	3.567*	701.16*
13	DLABBT	0.020	112.09*	6.036*
14	DLSEAR	0.074	116.54*	6.525*
15	DLHINL	-0.339*	6.5282*	2069.5*
16	DLGLAX	0.074	98.319*	4.644*
17	DLIBLH	-0.057	78.162*	2.935*
18	DLINTE	3.986*	63.748*	1.982*

19	DLINTI	-0.751*	90.497*	3.935*
20	DLCRST	0.056	19.949*	1.911*
21	DLDOST	-0.016	106.88*	5.487*
22	DLAISHA	0.484*	38.061*	6.963*
23	DLPIBT	-1.043*	48.872*	1.149*
24	DLPIAA	-0.058	97.467*	4.563*
25	DLPNSC	0.202*	110.87*	5.905*
26	DLARCH	-0.388*	107.81*	5.584*
27	DLITHD	0.146**	110.61*	5.877*
28	DLICI	0.091	111.63*	5.987*
29	DLDOL	0.128***	110.54*	5.870*
30	DLAGLI	0.514*	33.306*	5.334*

Significance at *1%,**5%, ***10%

4.3 Visualization of Actual Series

The graphical representation of the actual series are shown below, which show that series are not stationary.

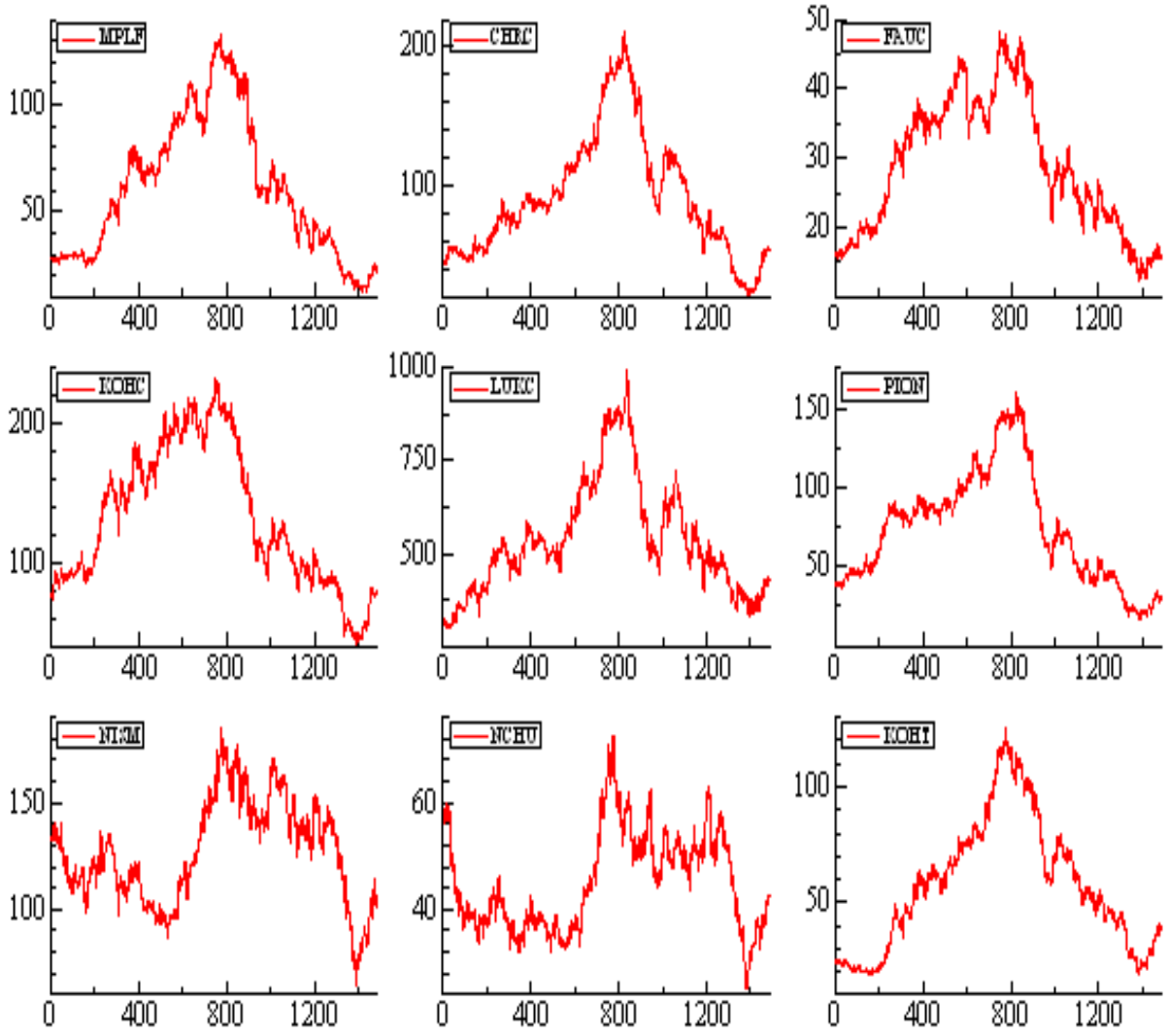

Figure 4.1 : Visualization of actual series 1-9

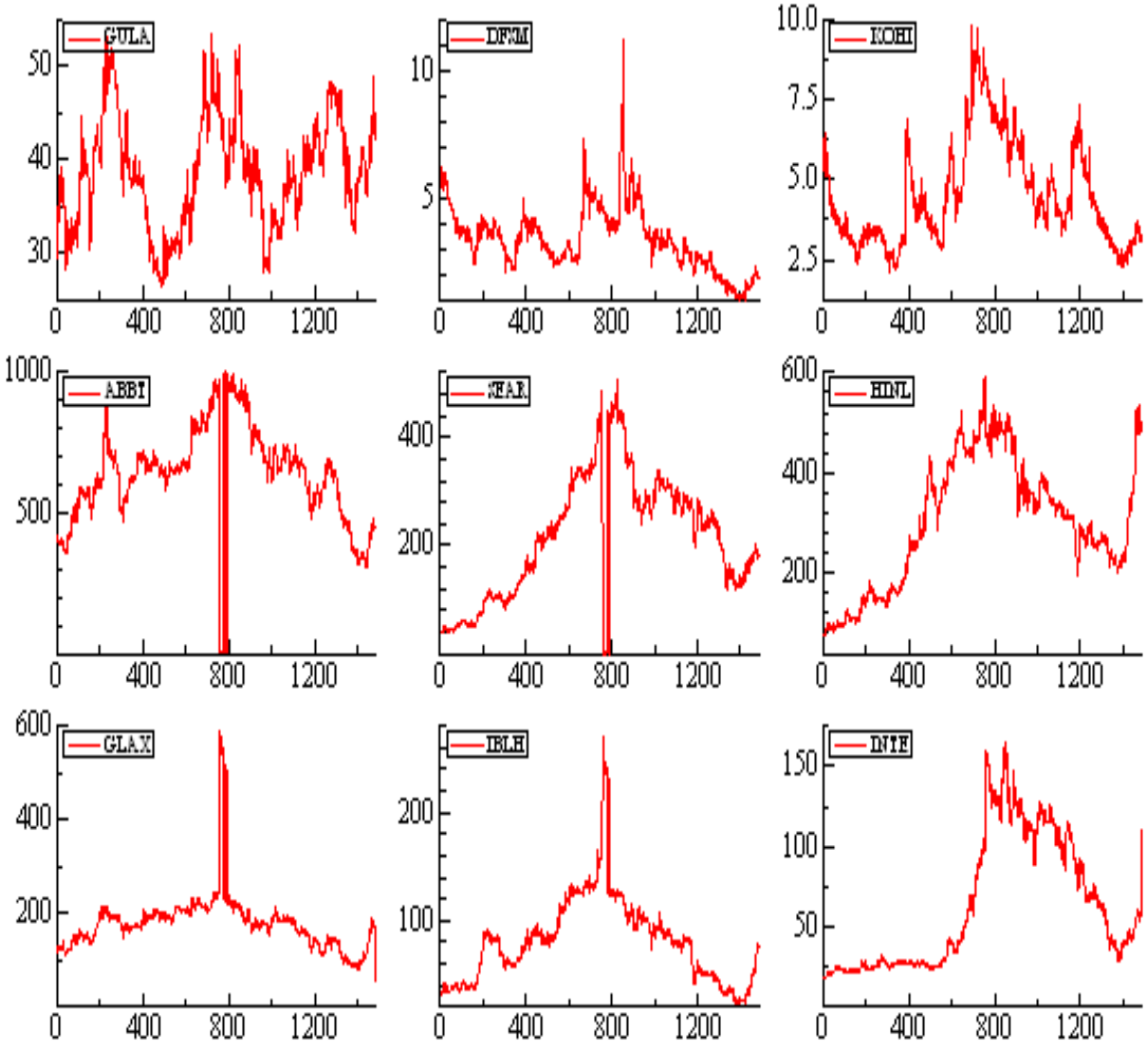

Figure 4.2 :visualization of actual series 10-18

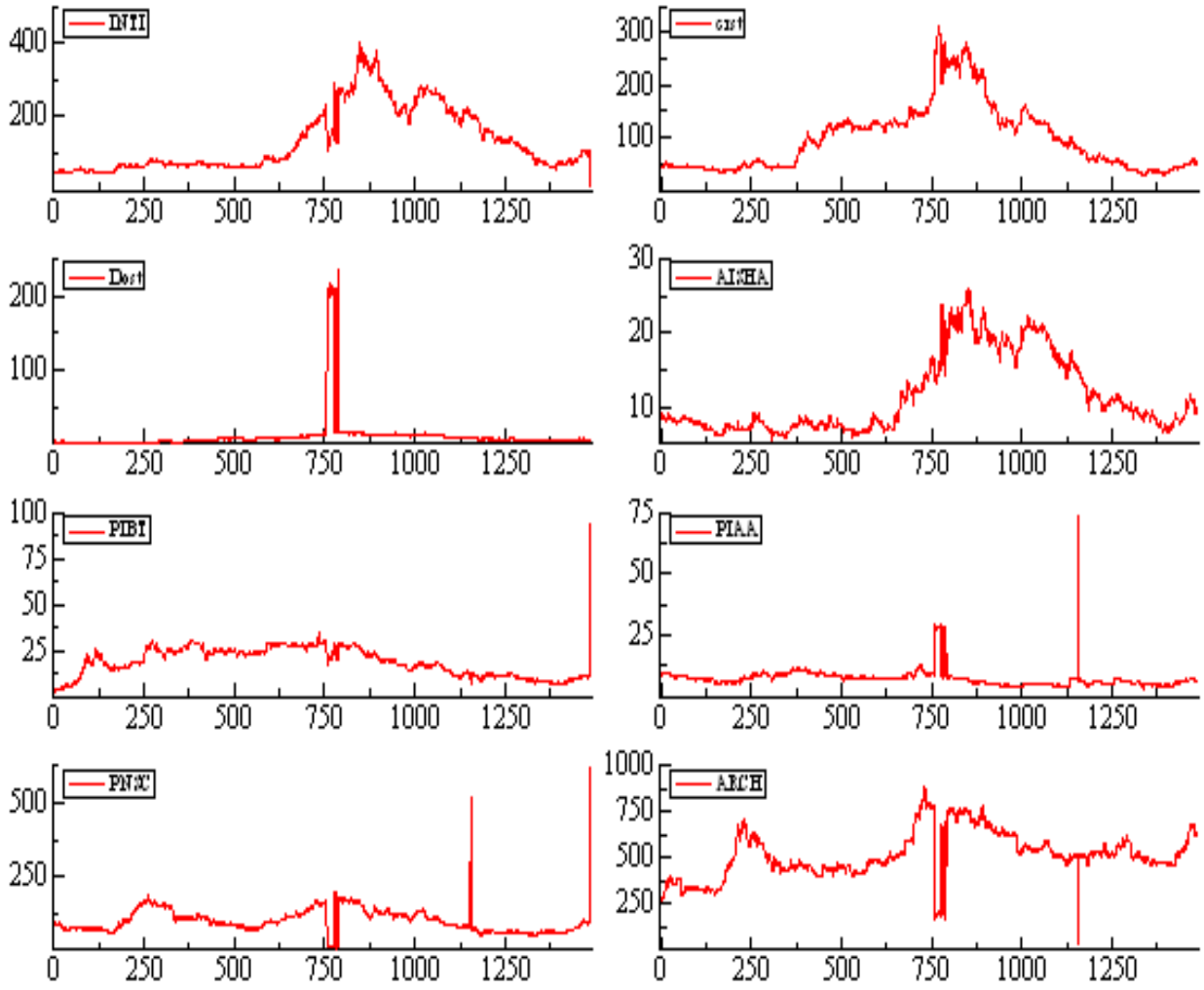

Figure 4.3 :Visualization of actual series 19-26

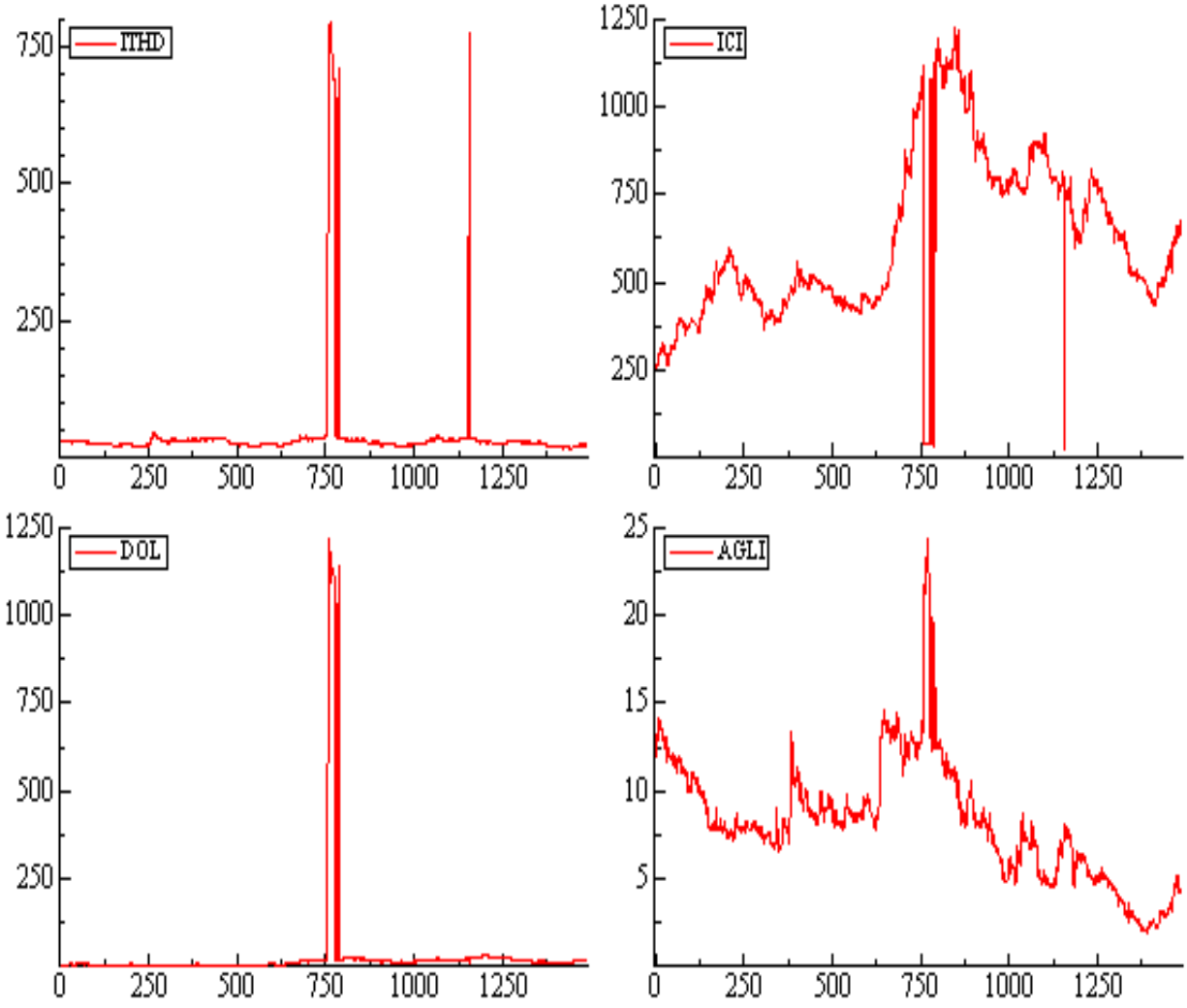

Figure 4.4: Visualization of actual series 27-30

The above graph for actual series shows a trendy behavior with changing mean over time. Which means that the actual series are non-stationary and for proper analysis first we have to make these series stationary by taking log return of actual series.

4.4 Checking Stationarity of Return Series

We have seen the actual series through graph that the series are not stationary. We can clearly see that the mean is changing over time. This will create a problem in our analysis, to make the series stationary we have taken the log return of each series by computing log return formula.

i.e. $R_t = \ln \frac{p_t}{pt-1}$

The following table shows kpss statistics results for stationarity of return series.

Table 3: KPSS Test Result

Sr.no	Variables	KPSS Statistics
1	DLMPLF	0.727
2	DLCHRC	0.521
3	DLFAUC	0.486
4	DLKOHC	0.613
5	DLLUKC	0.227
6	DLPION	0.854
7	DLNISM	0.072
8	DLNCHU	0.159
9	DLKOHT	0.491
10	DLGULA	0.052
11	DLDFSM	0.059
12	DLKOHI	0.081
13	DLABBT	0.004

14	DLSEAR	0.004
15	DLHINL	0.468
16	DLGLAX	0.041
17	DLIBLH	0.155
18	DLINTE	0.137
19	DLINTI	0.045
20	DLCRST	0.367
21	DLDOST	0.015
22	DLAISHA	0.075
23	DLPIBT	0.264
24	DLPIAA	0.012
25	DLPNSC	0.005
26	DLARCH	0.015
27	DLITHD	0.005
28	DLICI	0.004
29	DLDOL	0.007
30	DLAGLI	0.033

1%(0.739) 5% (0.463) 10%(0.347)

The kpss statistics result for all return series that after taking log return of actual series now the return series become stationary. The statistics result of kpss is lover that critical values that are (0.7390, (.465) and (.347). Hence accepting null hypothesis which is H0: series are stationary I=0

As the series now become stationary, and are ready for further analysis.

4.5 Visualization of Return Series

After taking log return of the series, it can be seen through graph that mean is not changing over time. Which indicate that the return series are stationary now. The return series of series is presented in below in visualization form. The trendy nature of series has now gone and the return series are now ready for further analysis.

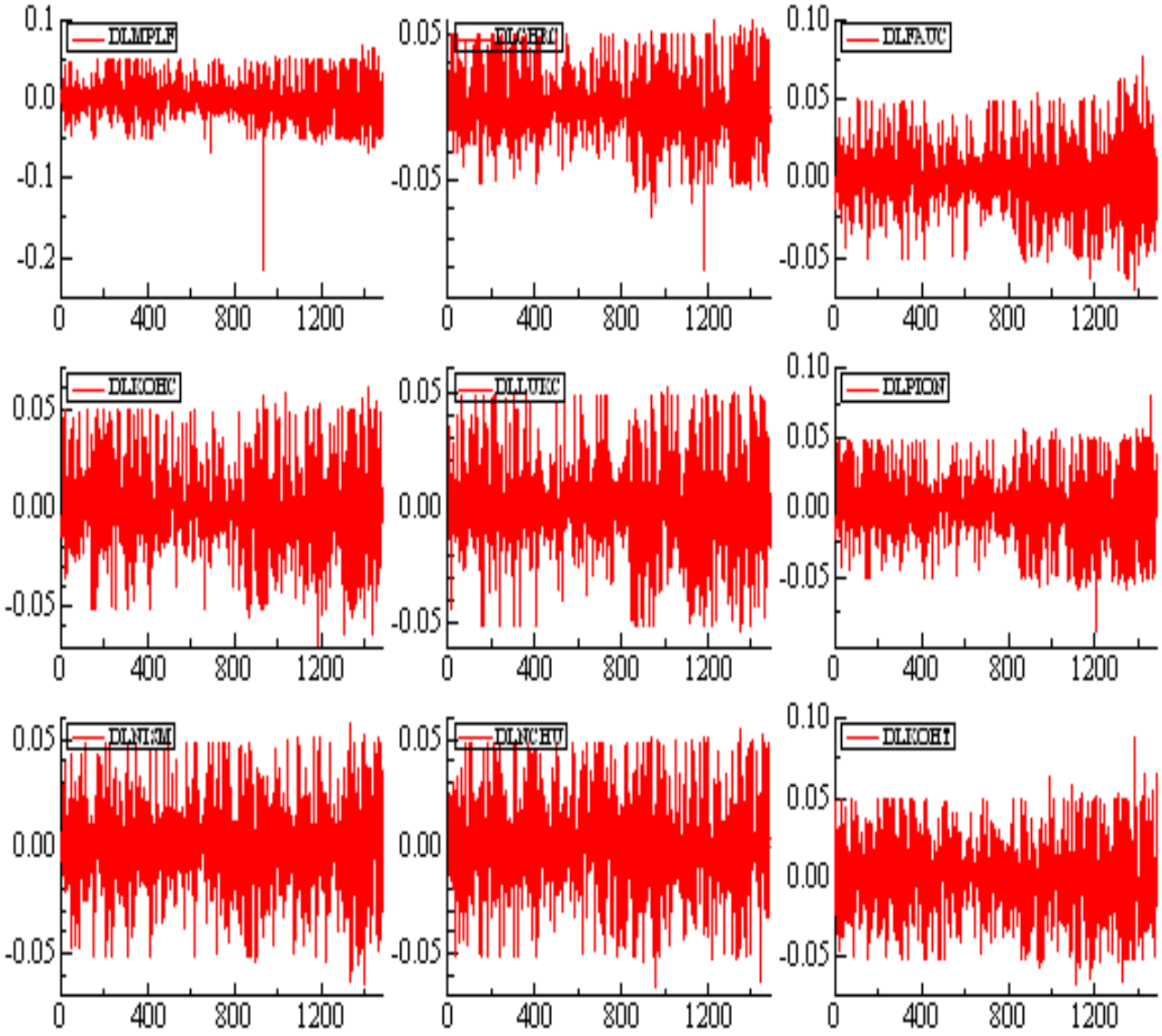

Figure 4.5 :Visualization of retun series 1-9

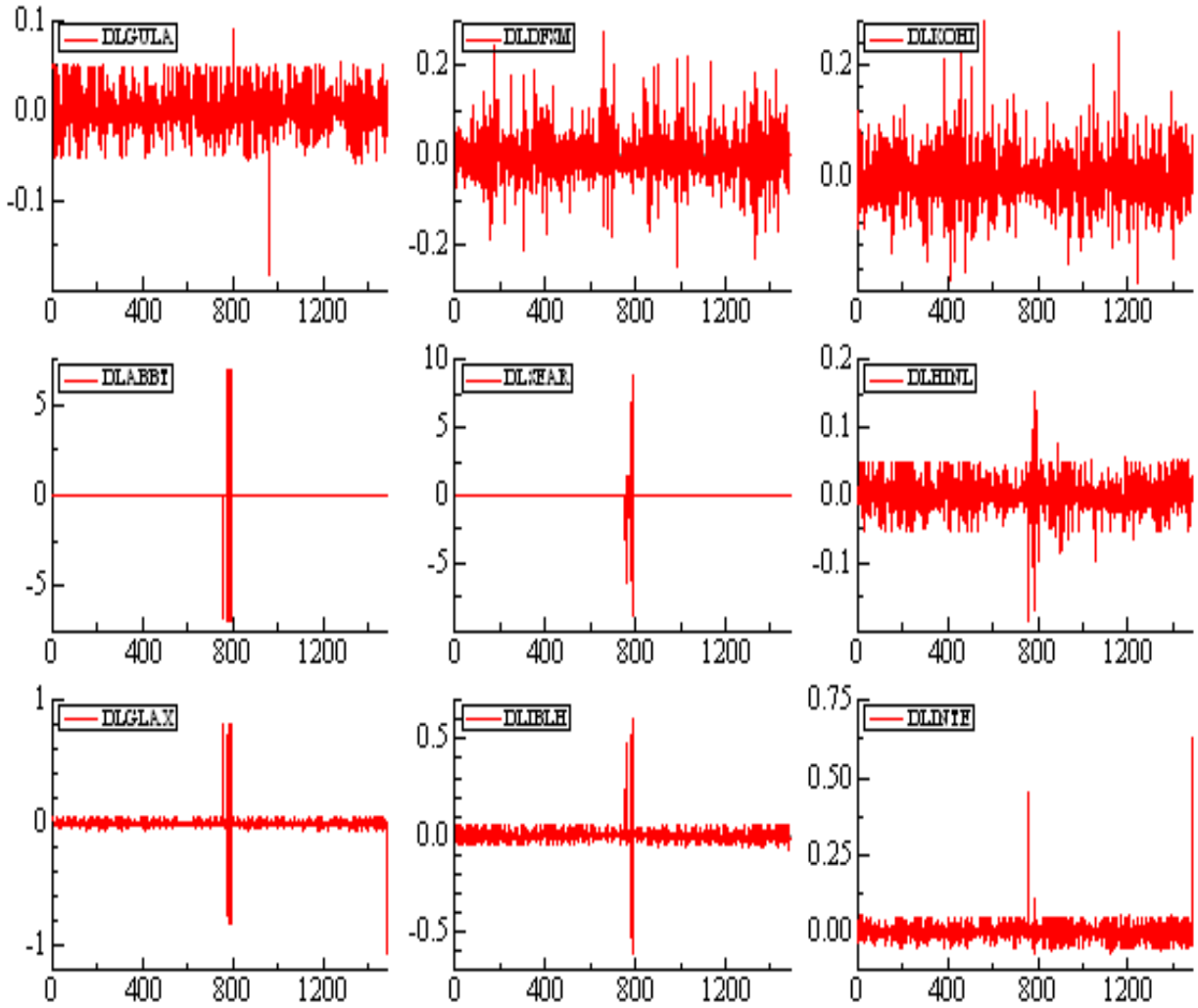

Figure 4.6 :Visualization of retun series 10-18

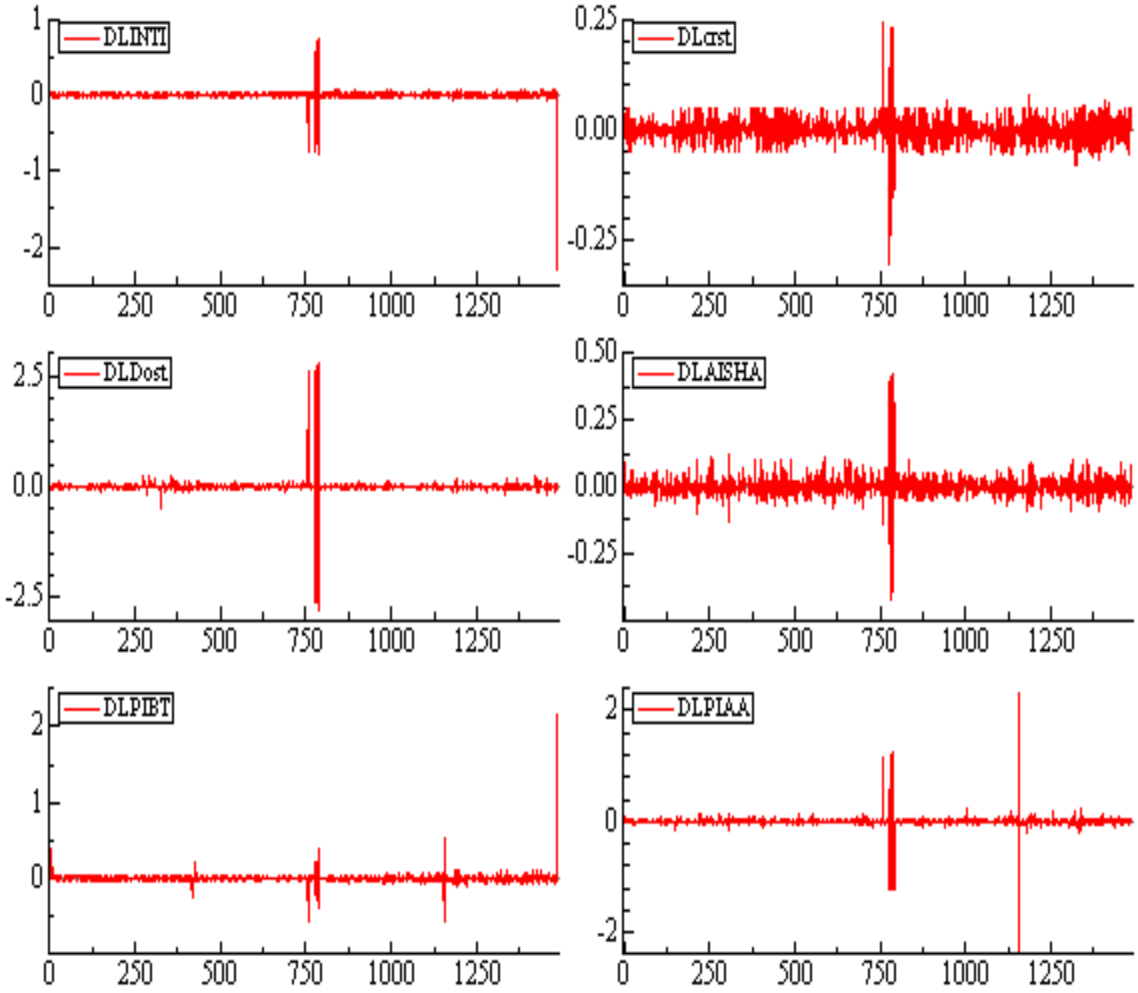

Figure 4.7 :Visualization of retun series 19-24

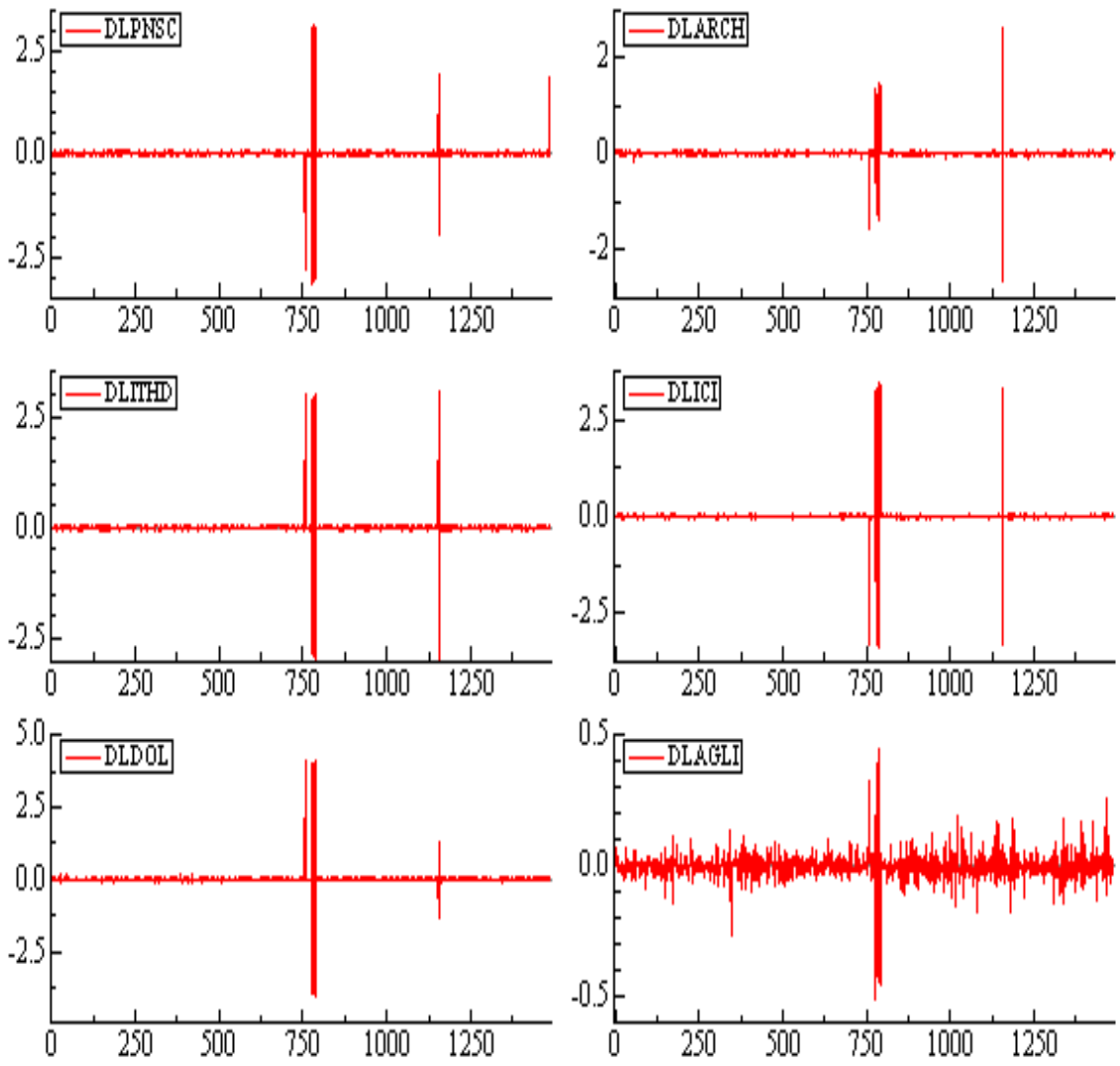

Figure 4.8: visualization of retun series 25-30

All the above graph represent the return series by taking log return, it is clearly seen that the series have become stationary now.

4.6 Serial Correlation Test

Auto correlation or serial correlation occurs when the value of variable depends upon its own previous value. This generally happens in time series data. When the variable depends upon its own lag than this effect will go on in leading values. The serial correlation test using Q-stat both on raw data and squared data of return series is shown below.

The hypothesis for serial correlation is H0 = no serial correlation exist

The p value in the following table for both raw and squared dated for each return series reject the null hypothesis and accept alternate which means that series have serial correlation both in raw data and squared data. Except DLNCHU and DLKOHT, their raw data does not shows serial correlation but their squared data shows that they have serial correlation. Over all we can say that serial correlation exist in all return series.

Table 4: Q_ Stat test result

Sr. No	**Return Series**	**Q-stat on Raw Data**	**Q-stat on Squared Data**
1	DLMPLF	Q(5) = 38.137 [0.0000]** Q(10) = 44.077 [0.0000]**	Q(5) = 12.944 [0.0239]* Q(10) = 21.842 [0.0159]*
2	DLCHRC	Q(5) = 22.065 [0.0005]** Q(10) = 25.629 [0.0042]**	Q(5) = 175.154 [0.0000]** Q(10) = 224.542 [0.0000]**
3	DLFAUC	Q(5) = 17.050 [0.0044]** Q(10) = 25.636	Q(5) = 184.799 [0.0000]** Q(10) = 238.816

		[0.0042]**	[0.0000]**
4	DLKOHC	Q(5) = 32.279 [0.0000]** Q(10) = 44.722 [0.0000]**	Q(5) = 118.272 [0.0000]** Q(10) = 164.278 [0.0000]**
5	DLLUKC	Q(5) = 54.926 [0.0000]** Q(10) = 59.013 [0.0000]**	Q(5) = 98.767 [0.0000]** Q(10) = 121.656 [0.0000]**
6	DLPION	Q(5) = 11.460 [0.0429]* Q(10) = 20.365 [0.0259]*	Q(5) = 219.827 [0.0000]** Q(10) = 362.877 [0.0000]**
7	DLNISM	Q(5) = 18.349 [0.0025]** Q(10) = 25.484 [0.0044]**	Q(5) = 60.350 [0.0000]** Q(10) = 91.567 [0.0000]**
8	DLNCHU	Q(5) = 11.719 [0.0388]* Q(10) = 14.116 [0.1677]	Q(5) = 46.466 [0.0000]** Q(10) = 51.306 [0.0000]**
9	DLKOHT	Q(5) = 10.046 [0.0739] Q(10) = 15.609 [0.1113]	Q(5) = 138.923 [0.0000]** Q(10) = 178.062 [0.0000]**
10	DLGULA	Q(5) = 42.965 [0.0000]** Q(10) = 51.1180 [0.0000]**	Q(5) = 17.264 [0.0040]** Q(10) = 20.258 [0.0269]*
11	DLDFSM	Q(5) = 10.348 [0.0659] Q(10) = 11.3651 [0.3297]	Q(5) = 35.978 [0.0000]** Q(10) = 44.877 [0.0000]**
12	DLKOHI	Q(5) = 17.928	Q(5) = 54.669

		[0.0030]** Q(10) = 19.582 [0.0334]*	[0.0000]** Q(10) = 57.247 [0.0000]**
13	DLABBT	Q(5) = 939.228 [0.0000]** Q(10) = 1292.57 [0.0000]**	Q(5) = 919.667 [0.0000]** Q(10) = 1252.48 [0.00000]**
14	DLSEAR	Q(5) = 854.647 [0.0000]** Q(10) = 1112.16 [0.0000]**	Q(5) = 896.369 [0.0000]** Q(10) = 1125.21 [0.0000]**
15	DLHINL	Q(5) = 27.566 [0.0000]** Q(10) = 40.486 [0.0000]**	Q(5) = 499.431 [0.0000]** Q(10) = 554.539 [0.0000]**
16	DLGLAX	Q(5) = 819.001 [0.0000]** Q(10) = 1066.87 [0.0000]**	Q(5) = 993.617 [0.0000]** Q(10) = 1260.62 [0.0000]**
17	DLIBLH	Q(5) = 745.137 [0.0000]** Q(10) = 976.763 [0.0000]**	Q(5) = 1222.50 [0.0000]** Q(10) = 1490.89 [0.0000]**
18	DLINTE	Q(5) = 6.951 [0.2242] Q(10) = 9.457 [0.4892]	Q(5) = 0.056 [0.9999] Q(10) = 0.103 [1.0000]
19	DLINTI	Q(5) = 681.669 [0.0000]** Q(10) = 947.191 [0.0000]**	Q(5) = 875.508 [0.0000]** Q(10) = 1184.97 [0.0000]**
20	DLCRST	Q(5) = 93.019 [0.0000]** Q(10) = 151.232 [0.0000]**	Q(5) = 329.780 [0.0000]** Q(10) = 525.304 [0.0000]**

21	DLDOST	Q(5) = 938.278 [0.0000]** Q(10) = 1258.25 [0.0000]**	Q(5) = 990.509 [0.0000]** Q(10) = 1295.51 [0.0000]**
22	DLAISHA	Q(5) = 270.381 [0.0000]** Q(10) = 424.671 [0.0000]**	Q(5) = 911.748 [0.0000]** Q(10) = 1389.78 [0.0000]**
23	DLPIBT	Q(5) = 143.693 [0.0000]** Q(10) = 187.826 [0.0000]**	Q(5) = 267.709 [0.0000]** Q(10) = 284.496 [0.0000]**
24	DLPIAA	Q(5) = 832.977 [0.0000]** Q(10) = 1134.32 [0.0000]**	Q(5) = 972.002 [0.0000]** Q(10) = 1294.72 [0.0000]**
25	DLPNSC	Q(5) = 928.289 [0.0000]** Q(10) = 1292.10 [0.0000]**	Q(5) = 936.768 [0.0000]** Q(10) = 1301.06 [0.0000]**
26	DLARCH	Q(5) = 892.138 [0.0000]** Q(10) = 1182.50 [0.0000]**	Q(5) = 890.993 [0.0000]** Q(10) = 1137.20 [0.0000]**
27	DLITHD	Q(5) = 937.047 [0.0000]** Q(10) = 1265.63 [0.0000]**	(5) = 943.121 [0.0000]** Q(10) = 1250.62 [0.0000]**
28	DLICI	Q(5) = 946.441 [0.0000]** Q(10) = 1290.50 [0.0000]**	Q(5) = 968.258 [0.0000]** Q(10) = 1290.87 [0.0000]**
29	DLDOL	Q(5) = 926.081 [0.0000]** Q(10) = 1263.59	Q(5) = 920.470 [0.0000]** Q(10) = 1236.78

		[0.0000]**	[0.0000]**
30	DLAGLI	Q(5) = 285.655 [0.0000000]** Q(10) = 383.234 [0.0000]**	Q(5) = 821.860 [0.0000]** Q(10) = 1091.90 [0.0000]**

Significant at *1%, **5%, ***10%

The above shows the result of Q-state up to 10 lag and their result shows that all series have serial correlation up to 10 lag. If the series have serial correlation up to 10 lag than no need to look for up to 20 and 50 lags, 10 lags are enough and their results shows serial correlation exist.

4.7 ARCH Effect

The time series data often poses to ARCH (Autoregressive conditional Heteroscedasticity) effect, which, means that the squared residuals of time series exhibit autocorrelation. When time series data have ARCH effect the volatility transmission cannot be calculate properly. It is essential to use other test for calculating volatility when series have ARCH effect.

LM ARCH test is used and the results is shown in below table. All the 30 return series shows an ARCH effect, the hypothesis for LM ARCH test is :

H0 : Series have no ARCH effect

H1: Series have ARCH effect

All the following return series shows significance, LM ARCH test, which means we have to reject our null and accept alternative hypothesis. That is all the series have ARCH effect and we

cannot use ARCH based models and we can have to go for Multi variant GARCH models for estimation of variance and covariance.

Table 5: LM ARCH Test

Sr. no	Return Series	ARCH 1-2 test	ARCH 1-5 test	ARCH 1-10 test
1	DLMPLF	F(2,1148) = 4.6686 [0.0096]**	F(5,1142) = 2.2693 [0.0457]*	F(10,1132)= 1.6983 [0.0762]
2	DLCHRC	F(2,1148) = 53.056 [0.0000]**	F(5,1142) = 23.371 [0.0000]**	F(10,1132)= 12.318 [0.0000]**
3	DLFAUC	F(2,1148) = 44.543 [0.0000]**	F(5,1142) = 25.190 [0.0000]**	F(10,1132)= 13.391 [0.0000]**
4	DLKOHC	F(2,1148) = 29.881 [0.0000]**	F(5,1142) = 16.116 [0.0000]**	F(10,1132)= 9.0458 [0.0000]**
5	DLLUKC	F(2,1148) = 31.471 [0.0000]**	F(5,1142) = 16.001 [0.0000]**	F(10,1132)= 8.5255 [0.0000]**
6	DLPION	F(2,1148) = 45.008 [0.0000]**	F(5,1142) = 27.631 [0.0000]**	F(10,1132)= 16.080 [0.0000]**
7	DLNISM	F(2,1148) = 19.470 [0.0000]**	F(5,1142) = 9.5666 [0.0000]**	F(10,1132)= 5.9114 [0.0000]**
8	DLNCHU	F(2,1148) = 17.894 [0.0000]**	F(5,1142) = 7.8132 [0.0000]**	F(10,1132)= 4.1612 [0.0000]**
9	DLKOHT	F(2,1148) = 37.592 [0.0000]**	F(5,1142) = 19.584 [0.0000]**	F(10,1132)= 10.610 [0.0000]**
10	DLGULA	F(2,1148) = 3.7426 [0.0240]*	F(5,1142) = 2.7257 [0.0186]*	F(10,1132)= 1.3765 [0.1857]
11	DLDFSM	F(2,1148) = 13.349 [0.0000]**	F(5,1142) = 6.3208 [0.0000]**	F(10,1132)= 3.8158 [0.0000]**
12	DLKOHI	F(2,1148) = 19.494 [0.0000]**	F(5,1142) = 8.6756 [0.0000]**	F(10,1132)= 4.6457 [0.0000]**
13	DLABBT	F(2,1148) = 323.32	F(5,1142) = 146.90	F(10,1132)= 82.796

		[0.0000]**	[0.0000]**	[0.0000]**
14	DLSEAR	F(2,1148) = 347.09 [0.0000]**	F(5,1142) = 158.03 [0.0000]**	F(10,1132)= 96.217 [0.0000]**
15	DLHINL	F(2,1148) = 179.95 [0.0000]**	F(5,1142) = 75.470 [0.0000]**	F(10,1132)= 39.701 [0.0000]**
16	DLGLAX	F(2,1148) = 365.29 [0.0000]**	F(5,1142) = 156.31 [0.0000]**	F(10,1132)= 87.172 [0.0000]**
17	DLIBLH	F(2,1148) = 528.82 [0.0000]**	F(5,1142) = 223.11 [0.0000]**	F(10,1132)= 121.52 [0.0000]**
18	DLINTE	F(2,1148) = 0.0138 [0.9863]	F(5,1142) = 0.0112 [1.0000]	F(10,1132)= 0.0101 [1.0000]
19	DLINTI	F(2,1148) = 346.75 [0.0000]**	F(5,1142) = 155.69 [0.0000]**	F(10,1132)= 85.461 [0.0000]**
20	DLCRST	F(2,1148) = 58.946 [0.0000]**	F(5,1142) = 41.335 [0.0000]**	F(10,1132)= 26.508 [0.0000]**
21	DLDOST	F(2,1148) = 362.11 [0.0000]**	F(5,1142) = 159.05 [0.0000]**	F(10,1132)= 89.500 [0.0000]**
22	DLAISHA	F(2,1148) = 351.45 [0.0000]**	F(2,1148) = 351.45 [0.0000]**	F(10,1132)= 117.43 [0.0000]**
23	DLPIBT	F(2,1148) = 84.083 [0.0000]**	F(5,1142) = 43.516 [0.0000]**	F(10,1132)= 22.361 [0.0000]**
24	DLPIAA	F(2,1148) = 341.02 [0.0000]**	F(5,1142) = 152.16 [0.0000]**	F(10,1132)= 85.758 [0.0000]**
25	DLPNSC	F(2,1148) = 331.13 [0.0000]**	F(5,1142) = 154.40 [0.0000]**	F(10,1132)= 86.772 [0.0000]**
26	DLARCH	F(2,1148) = 309.61 [0.0000]**	F(5,1142) = 134.82 [0.0000]**	F(10,1132)= 75.033 [0.0000]**
27	DLITHD	F(2,1148) = 336.92 [0.0000]**	F(5,1142) = 148.62 [0.0000]**	F(10,1132)= 83.004 [0.0000]**
28	DLICI	F(2,1148) = 355.53 [0.0000]**	F(5,1142) = 159.06 [0.0000]**	F(10,1132)= 88.929 [0.0000]**

29	DLDOL	F(2,1148) = 323.66 [0.0000]**	F(5,1142) = 144.56 [0.0000]**	F(10,1132)= 80.764 [0.0000]**
30	DLAGLI	F(2,1148) = 267.49 [0.0000]**	F(5,1142) = 123.86 [0.0000]**	(10,1132)= 75.277 [0.0000]**

Significance level at * 1%, **5%, *** 10%

4.8 DBEKK-GARCH Model for Optimal Hedge Ratio

Optimal hedge ratio is critical for efficient portfolio construction. Calculating variance, co-variance properly for estimation of optimal hedge ratios is very important. Optimal hedge ratio can be calculated by static models like OLS and dynamic time varying model like GARCH models.

As the return series possess ARCH effect, which is why we cannot use ARCH based modeling for estimating volatility. We have used DBEKK-GARCH model for estimation of variance and co-variance estimations and the reason for using this model is explained in previous chapter of methodology section.

Table 6 : Portfolio with hedge Ratios and Percentage Change in Variance

Sr.no	Constructed portfolios	b*	c*	% Change in Variance
1	MPLF/NISM,ABBOT	-1.7605	-3.2308	-.001
2	MPLF/NCHU,SEAR	-0.1501	-0.1930	21.05
3	MPLF/KOHT,GALX	0.4809	0.4561	**76.05**
4	MPLF/BANU,INTE	0.3307	0.4283	41.05
5	MPLF/INTI,PIBT	0.3225	0.2511	-4.43
6	MPLF/PNSC,ARCH	-0.1289	2.8401	26.05
7	MPLF/HINL,ITHD	0.2336	0.2505	38.05
8	MPLF/KOHI,HINL	0.0964	0.2114	24.05
9	MPLF/KOHI,IBLH	0.0651	0.2947	38.05
10	MPLF/AISHA,PIAA	0.2938	0.3262	58.05
11	MPLF/ AISHA,PIAA	0.2938	0.3262	58.05
12	MPLF/INTE,ITHD	0.4217	0.2785	-88.21
13	MPLF/NISM,ICI	0.5815	0.2900	48.05
14	MPLF/KOHT,INTI	0.4683	0.3474	32.05
15	MPLF/GULA,DOL	-1.0412	2.0945	-80.57
16	MPLF/NISM,AGLI	0.5773	0.1749	46.05
17	MPLF/ABBT,ARCH	2.1301	2.8801	34.05
18	MPLF/INTE,ITHD	0.4217	0.2785	-88.21
19	MPLF/SEAR,PNSC	0.4273	0.3042	-99.74
20	MPLF/NISM,ITHD	0.5830	0.3923	31.05
21	MPLF/SEAR,LICI	0.4134	0.2868	**74.05**

22	MPLF/CHRC,FAUC	1.03264	1.2163	-8.05
23	MPLF/NCHU,HINL	4.5401	2.240	52.05
24	MPLF/HINL,PIAA	0.2160	0.2828	41.05
25	MPLF/INTI ,ITHD	0.3557	0.2870	-98.75
26	MPLF/KOCH,LUKH	0.8218	0.9798	5.50
27	MPLF/AISHA,ARCH	0.2881	0.2881	60.05
28	MPLF/PIBT,ICI	2.8401	2.7401	-97.63
29	MPLF/PIAA,DOL	0.3310	0.3094	47.00
30	MPLF/PNSC,AGLI	0.2874	0.1639	-96.87
31	MPLF/DFSM,CRST	0.14842	0.3118	36.05
32	MPLF/DOST,DOL	0.2461	0.2951	40.05
33	MPLF/DFSM,DOST	0.1255	0.2036	12.05
34	MPLF/DOST,AGLI	0.2472	0.1901	30.05
35	MPLF/KOHI,DOST	0.0944	0.2172	15.05
36	MPLF/AISHA,AGLI	0.2798	0.1782	**64.05**
37	MPLF/KOHT,DOL	0.4656	0.2670	55.05
38	MPLF/LUKC,PION	1.0897	1.0235	4.05
39	MPLF/GULA,DFSM	0.3174	0.1582	29.50
40	MPLF/IBLH,ICI	2.8901	2.6701	54.05

Source: Author's own Calculation

Where b* and c* are optimal hedge ratios.

The above table shows a portfolio of three different securities while taking one security as fixed underlining asset so that at the end we can compare the performance of different asset with the fixed underlining asset. From portfolio 1 to portfolio 40, there has been use different randomly

combination of stocks, which gives different portfolios return and portfolio variance .The percentage reduction in variance of portfolios have been calculated and shown in front of each portfolio.

$$\Delta V = [V(UH) - V(H)] / V(UH) \qquad (4.1)$$

Where, ΔV is change in variance

VUH is variance of unhedged

VH is variance of hedged

The above equation is used to calculated change in variance of hedged portfolio and when multiplied with 100, gives percentage change in variance.

Some portfolios gives negative reduction in variance while some gives a positive portfolio variance reduction. The best portfolio is the one which percentage risk reduction is greater than zero. In the above portfolios, the one which gives greater reduction in variance is chosen as a best portfolio. The variation in percentage reduction in variance is due to correlation among securities return, highly correlated firm will gives maximum portfolio variance and negatively or low correlated firm will give minimum variance.

After looking into all 40 portfolios, 11 out of them gives a negative reduction in portfolio variance. Which means that these 11 portfolios did not have any benefit in investing into it rather these portfolios not only increase the risk but an investors may bear a huge lose. Other than these 11 portfolio rest of them gives a positive reduction in variance of portfolio. Which means that an investor can consider these other for their portfolio construction. But if we chose the top three best portfolios which gives minimum variance and overall high percentage in risk reduction we will choose the following as the best portfolios,

Portfolio # 1 $MLPF$ / $KOHT, GALX$ with 76.05% reduction in variance

Portfolio # 2 $MLPF$ / $SEAR, LICI$ with 74.05% reduction in variance

Portfolio # 3 $MLPF$ / $AISHA, AGLI$ with 64.05 % reduction in variance

We have seen that most of the portfolios give a positive percentage reduction in variance. The percentage reduction in variance is greater than 0 and a few of them have percentage reduction near to 100% which is desirable for an investor. The reason of this reduction in variance is due to the co-volatility of stocks returns. When two stocks returns move in same direction, this will defiantly benefit when the stocks perform well, the problem occurs when there is risk involve and the variance also move in same direction. This will increase the variance of portfolio too because the two stocks included in the portfolio are moving in same direction, if one stock have lose the other also bear lose and hence a loss for portfolio too. Therefore it is very important for a well-diversified portfolio to include stocks that are low correlated and there co-volatility move in different direction. The main reason the above portfolio have minimum variance or we can say have maximum reduction in variance is due to the movement of covariance.

CHAPTER 5

CONCLUSION AND RECOMMENDATION

5.1 Conclusion of the Study

As the purpose of this study was to construct an efficient portfolio of stocks .By efficient we means a portfolio which uses minimum resources or we can say which have low cost and that portfolio will give high return. Previously a number of studies have done on stocks analysis and stock portfolio diversification. Many researchers have tried to find out the optimal number of stocks or securities to be included in a portfolio to make it optimal. Their number of optimal portfolio size is greater than 10 securities as discussed in empirical review portion of literature. The current study only uses three securities and try to find best possible portfolio out of it. As the best portfolio is one which uses a minimum number of securities and that will give minimum portfolio variance.

Other than optimal size of stock portfolio ,different researcher have used different securities other than stocks return for portfolio diversification, like many researcher have used Gold, Oil prices ,Bonds and Forex ,which is quite expensive and risky investment. This study only uses stock return for portfolio construction and successes in doing so. Present study has used 30 stocks from 6 different sectors listed in Pakistan stock exchange. After doing some basis analysis and calculating variances and covariance of stock return using DBEKK GARCH model, which is considered to be best model for time varying variances. We have constructed 40 portfolio of different combination using three securities. Return and variance of each portfolio was constructed and then change in variance of each portfolio have been calculated. Minimum variance approach have used for performance valuation of portfolios, because the reason an

investor do diversification is to minimize the overall risk. After successfully analyzing, the study come up with best three portfolios which gives minimum variances are given below:

Portfolio # 1: MPLF/ KOHT, GALX

Portfolio # 2: MPLF/ SEAR, LICI

Portfolio # 3: MPLF/ AISHA,AGLI

In the above three portfolios Meaple leaf cement is taken as underlining asset and two other stocks is used to hedge against variance. In first portfolio kohinor textile mills and Glaxo smith cline were used as hedging instrument and they perform well in reduction of variance. While in second portfolio Again Meaple leaf cement is taken as underlying asset and searl laboratories Pakistan and ICI Pakistan ltd is used as a hedging instrument to minimize portfolio variance and they also perform well in minimizing portfolio variance. Last but not least the third portfolio where Meaple leaf cement is an underlining asset and Aisha steel mills and Agri Tec limited is used to hedge against portfolio variance. We have seen that as Meaple leaf cement is common in all three portfolios, The others stocks which make the best portfolios are, searl laboratories and Glaxo smith kline and Aisha steel mills engeneering and steel sector sectors while Nishat chunian and kohanor textile mills belongs to taxtile sectors while ICI ltd and AGLI belongs to Chemical sectors. The reason these stocks well performed in the portfolio is due to the low correlation or low level of integration with Meaple leaf cement all other stocks have high level of integration with underlining asset.

Bottom line of the discussion is that ,sector vise we cannot say that ,we can use a particular sector to hedge against cement sector because, if a stocks from a sector make a good portfolio, while other from the same sector did not make a best portfolio in term of reduction in variance.

So we cannot rely on sectors only. Investors have to go for firm level analysis while making portfolios.

The three portfolios that gives the minimum variance and considered to be the best portfolio among all 40 portfolios. The finding of this study shows that an efficient portfolio can be constructed using stocks from different sectors, but it is not considered to be best for all time, means that the portfolios should change with time because the dynamics of co-volatility changes with passage of time.

5.2 Recommendation for Investors

Stock investors rather they are individual or institutional investors, they have the fear of change in stock prices. Investors are always worried about variance and hence tried their best to minimize that variance in stock prices. The best way to minimize those variances is to make diversified portfolio, but diversification is not that easy, diversification is an art. If diversification is not done properly not only investors may lose returns but the loss may be big enough that he may not come to that level again. So this study gives investors a proper path way to invest in stock portfolio and how an investor can reduces the overall risk of portfolio.

An investor while investing in portfolio of different stocks, he should always do various analysis of variance and co-variance, portfolio return and portfolio variance and compare each portfolio with the best. But as every investor have their own preferences and limitation of resources he can have any combination of securities in a portfolio which gives return or variance above a certain limit taken by the investor. Investors can add different sectors and different stocks and calculate their portfolio variances using current method and chose among them accordingly to their preferences.

In the above suggested three portfolios, an investor can use them and compare their return with the market return, and if their return are higher than market return than these portfolios are the best option for them to invest. Other than that this study is especially useful for those investors who cannot add optimal number of stocks in their portfolio which is for PSX between 20 to 40 stocks, this study gives them idea to invest in small number of stocks i.e 3 stocks and reduce their portfolio variance.

5.3 Recommendation for Future Researches

As this study have used daily data from 1st January 2014 to 31st December 2019, while taking 30 stocks form six different sectors, other researcher can extend the number of stocks and can add more sectors in the sample ,the number of period and see the benefit from it. Other than that, this same analysis can be perform after few years and can compare and see that the top best portfolios are still the best portfolios, are the hypothesis is true that with change in time portfolio performance also changes?

Other than that the same Analysis can be repeat by using different objective function like mean variance approach or shape ratio as a performance measure of portfolios.

5.4 Limitation of the Study

Historical data for stock prices of firm listed in PSX were not readily available, this limits the amount of data to be obtained. Also there was time constraint to perform the study.

The study does not address the issue that how much money an investors required for a portfolio and also doesn’t take consideration of transaction cost and other relevant costs while

diversification.so investors should think about these cost as well while investing. Other than that this study does not include all stocks listed in KSE -100 only thirty stocks from six sectors have been included in the sample.

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Annexure A

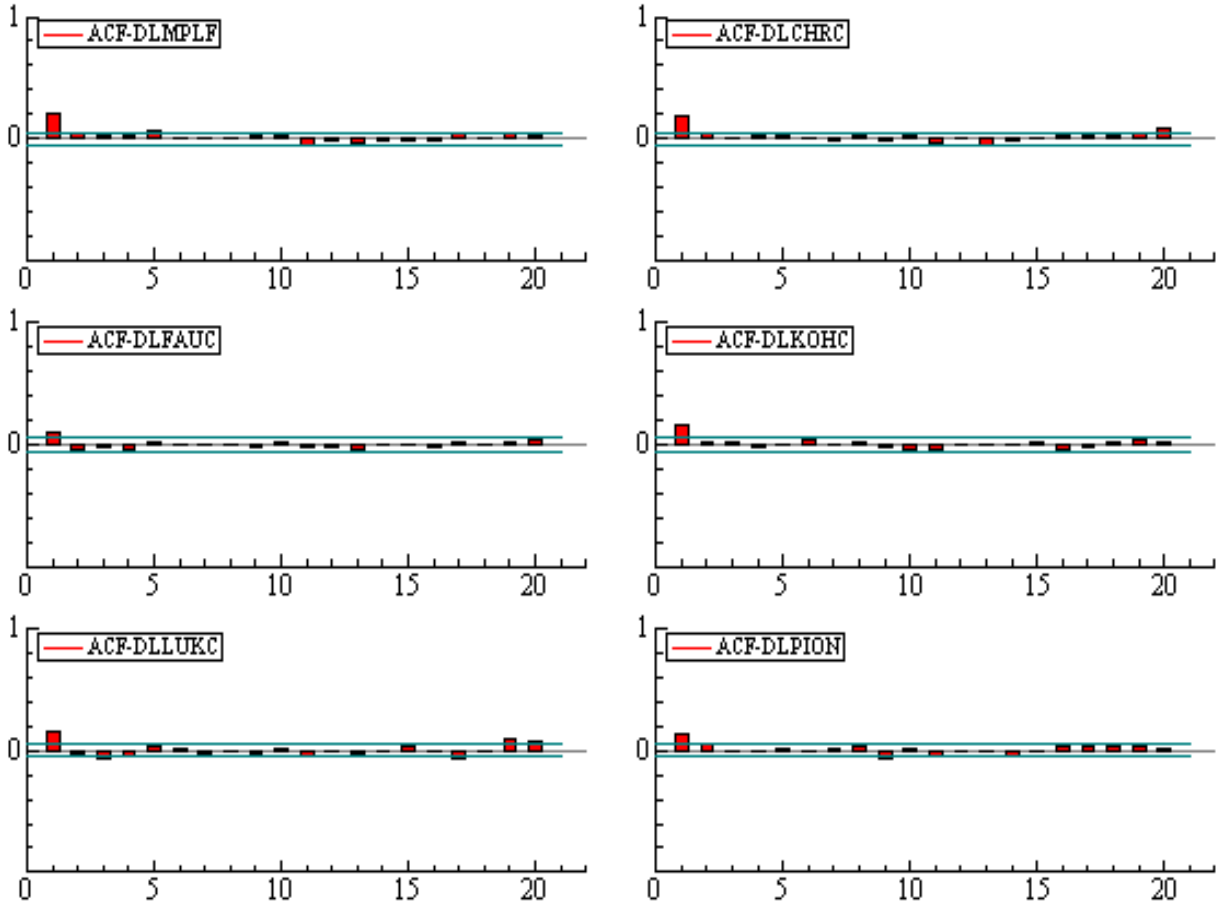
List of companies used in the sample and their symbols.

Sr. no	Symbols	Company Names
1	DLMPLF	Maple Leaf Cement
2	DLCHRC	Cherat Cement Company
3	DLFAUC	Foji Cement Company
4	DLKOHC	Kohat Cement Company
5	DLLUKC	Lucky Cement Company
6	DLPION	Pioneer Cement Company
7	DLNISM	Nishat Mills Ltd
8	DLNCHU	Nishat Chunian Ltd
9	DLKOHT	Kohinor Textile Mill
10	DLGULA	Gul Ahmad Textile Mill
11	DLDFSM	Dewan Farooq Mill
12	DLKOHI	Kohinoor Industries Ltd
13	DLABBT	Abbott Laboratories Pakistan
14	DLSEAR	Searl Laboratories Pakistan
15	DLHINL	Highnoon Labourtries Pakistan
16	DLGLAX	Glaxosmith Kline Pakistan
17	DLIBLH	IBL Health Care
18	DLINTE	International Steels Ltd
19	DLINTI	International Industries Ltd
20	DLCRST	Crescent Steel Production
21	DLDOST	Dost Steel Ltd
22	DLAISHA	Aisha Steel Mills
23	DLPIBT	Pakistan International Bulk Terminal
24	DLPIAA	Pakistan International Airline
25	DLPNSC	Pakistan National Shipping Corporation

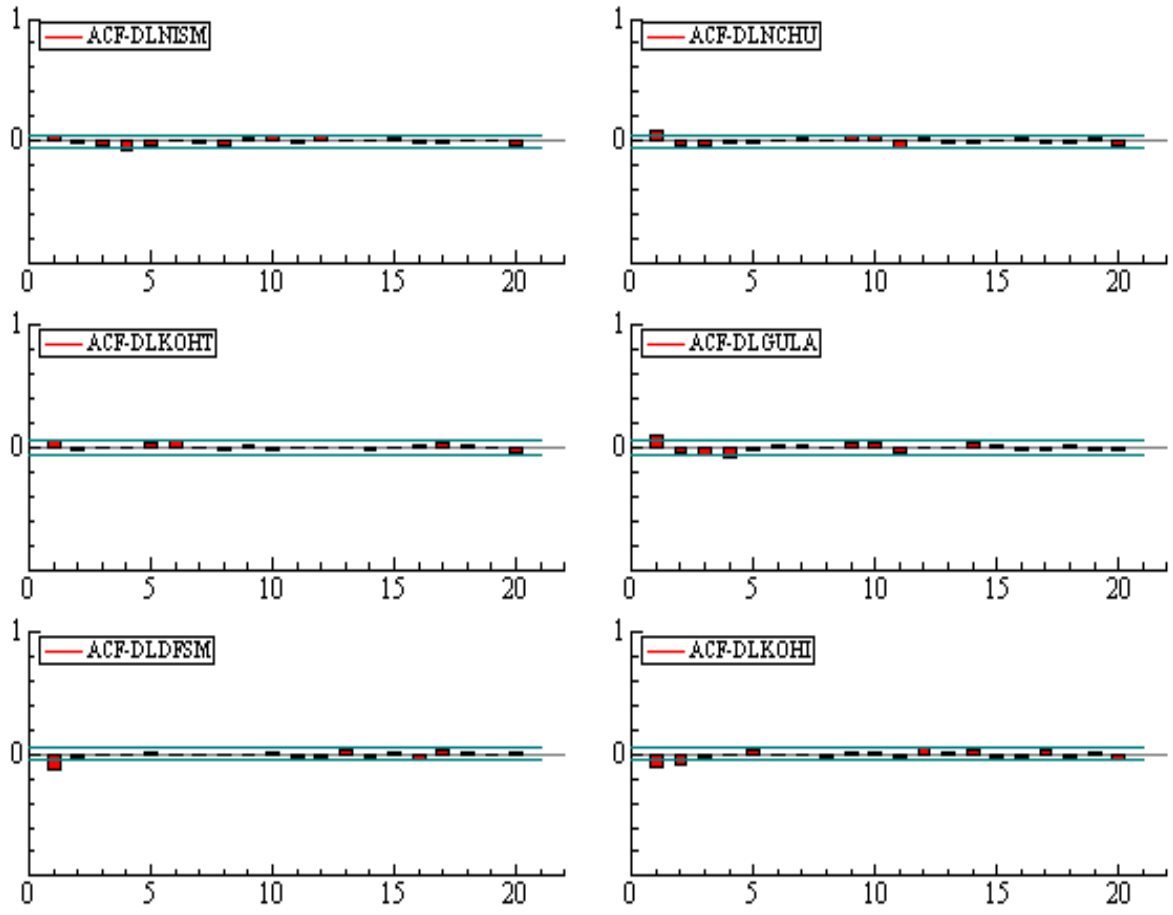
26	DLARCH	Archroma Pakistan Ltd
27	DLITHD	Ithedad Chemicals
28	DLICI	ICI Pakistan Ltd
29	DLDOL	Descon Oxycam Ltd
30	DLAGLI	Agritec Ltd

Annexure B

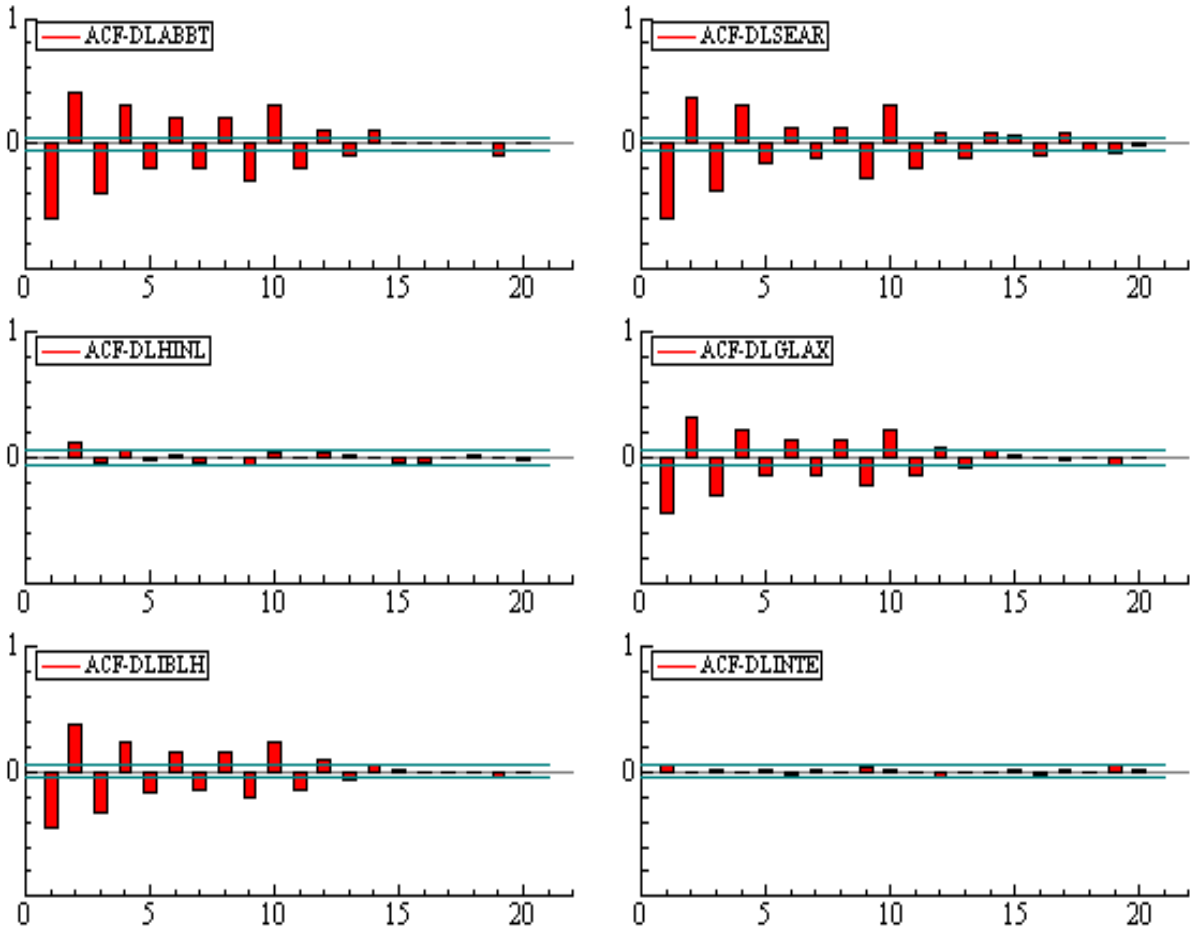
ACf (Auto correlation function) of each return series.



ACf (Auto correlation function) of each return series.



ACf (Auto correlation function) of each return series.



ACf (Auto correlation function) of each return series.