ANTICIPATED POLICY RATE PATH IN POLICY SIMULATION: A CASE STUDY OF PAKISTAN

by

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CERTIFICATE

This is to certify that this thesis entitled: **"Anticipated Policy Rate Path in Policy Simulation: A Case Study of Pakistan"**. submitted by **Ms. Saima Sadiq** is accepted in its present form by the School of Economics, Pakistan Institute of Development Economics (PIDE), Islamabad as satisfying the requirements for partial fulfillment of the degree in Master of Philosophy in Economics and Finance.

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AUTHOR'S DECLARATION

I **Saima Sadiq** hereby state that my MPhil thesis titled "**Anticipated Policy Rate Path In Policy Simulation: A Case Study of Pakistan**" is my own work and has not been submitted previously by me for taking any degree from Pakistan Institute of Development Economics or anywhere else in the country/world.

At any time if my statement is found to be incorrect even after my Graduation the university has the right to withdraw my MPhil degree.

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Saima Sadiq

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ABSTRACT

Stabilizing output and inflation in Pakistan's economy is essential for sustainable economic growth, promoting macroeconomic stability, price stability and effective monetary policy. The study aims to estimate a macroeconomic model including the predetermined and the forward-looking variables and to conduct policy projections with restricted and unrestricted nominal and real policy rate for the Taylor rule and the optimal policy rule by keeping in view the dual mandate of minimizing the gap between the actual and the targeted inflation and the deviation between the actual and potential output. The study adopted the Rudebusch-Svensson and Linde model to estimate the macroeconomic model and for simulations to compare the monetary policy rules, the study opts the historical and stochastic simulation by adding demand and supply shocks in an economy. The time span of the study is from 1993Q1 to 2022Q4. The major findings of the study are that the Taylor rule is efficient in the historical and stochastic simulation of the backward-looking model, whereas the optimal policy rule is efficient in the historical and stochastic simulation of the forward-looking model. In the case of policy projection, the Taylor rule is efficient in minimizing the gap between the actual and the threshold level of inflation and the deviation between the actual and potential output in both the backward and forward-looking models. Based on the findings of this study, SBP should use rule-based policies rather than discretionary policies. This rule-based approach provides a systematic framework that helps central banks to make informed and data-driven decisions. Policymakers should consider adopting the Taylor rule as a guideline for formulating and implementing monetary policy and emphasizing forward-looking elements in decision-making to minimize the loss generated from the quadratic loss function.

Keywords: Optimal policy rule, Taylor rule, backward-looking and forward-looking variables

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LIST OF ABBREVIATIONS

AD	Aggregate Demand
ARDL	Auto Regressive Distributed Lag
BN filter	Beveridge-Nelson Filter
BP filter	Band-Pass Filter
CBE	Central Bank Expectation
CIR	Constant Interest Rate
CMR	Call Money Rate
CPI	Consumer Price Index
DSGE	Dynamic Stochastic General Equilibrium
ECB	European Central Bank
FOMC	Federal Open Market Committee
FPAS	Forecasting and Policy Analysis System
GDP	Gross Domestic Product
GMM	Generalized Method of Moments
HP filter	Hodrick-Prescott Filter
IFS	International Financial Statistics
INF	Inflation
LSM	Large-Scale Manufacturing
ME	Market Expectation
MP	Monetary Policy
MPC	Monetary Policy Committee

MSMEs	Micro, Small and Medium Enterprises
NFIS	National Financial Inclusion Strategies
OLS	Ordinary Lease Square
OMO	Open Market Operations
OPP	Optimal Policy Projections
POL	Pakistan Oilfields Limited
PSC	Private Sector Credit
SBP	State Bank of Pakistan
SVAR	Structural Vector Auto-regression
VAR	Vector Auto-regression
Y-GAP	Output Gap

CHAPTER 1
INTRODUCTION

Monetary policy is responsible to shape the interest rate in order to achieve certain macroeconomic objectives. The objective is to establish and maintain price stability, long-term economic growth, low unemployment rates, and financial stability. By maintaining sustainable output and to control inflation, the economy can experience various positive outcomes and help to avoid periods of excessive boom or recession.

The constancy of time is a significant distinction between policy rule and discretion. Policy rules are designed to offer a consistent and predictable framework for decision-making, whereas discretion permits policymakers to make decisions at their own based on the current situation. Discretionary policies are criticized for their inconsistency and short-term focus, which can result in unsatisfactory outcomes. The modern researchers emphasize on the importance of policy rules to improve policy effectiveness and create more stable and sustainable economic results, which have various advantages over discretion, for instance, (Meltzer, 2012), (Taylor, 1993)

The study aims to estimate a macroeconomic model and conduct policy projections considering projected policy rate paths, while comparing the performance of policy rules that are consistent with the monetary policy rule of inflation targeting. The study includes the optimal policy rule and the Taylor rule, to check which rule is efficient in minimizing the deviation between the actual and the targeted inflation and the deviation between the actual and the potential output gap. The central bank can attain agreement among the deviation of inflation and output stability by forecast targeting that minimizes the quadratic loss function. Forecast targeting, is a monetary policy approach that relies on forecasts of inflation and the real economy, in which the central bank aims to choose an interest rate path that ensures the forecasted inflation and resource utilization align with desired levels (Leeson et al., 2013).

To estimate the macroeconomic model, this study adopts the Rudebusch-Svensson and Linde models, which provide a framework to analyze and forecast the behavior of key economic variables. Historical and stochastic simulations, and projections are employed to compare the two monetary policy rules, for accessing the efficiency. In the historical simulation, demand and supply shocks are added to reflect real-world economic conditions. Meanwhile, in the stochastic simulation the series of thousands of demand and supply shocks are created and determine the average loss generated by the optimal policy rule and the Taylor rule.

1.1 Background

Monetary policy is just one part of overall economic policy, and its goals should align with the objective of improving economic well-being. While economic policy has various goals like employment, growth, stability, and fairness, monetary policy's impact is limited to a few specific areas. So, it's best to set realistic and achievable goals for monetary policy that support the broader economic objectives. There is a need to recognize the actual effects and limitations of monetary policy to determine its appropriate goals.

The transmission mechanism describes how monetary policy effects the economy through various channels. To implement monetary policy, central banks generally modify short-term interest rates. When the central bank reduces this rate, it affects the economy in several ways. Firstly, it can reduce real interest rates, which boosts economic activity by lowering borrowing costs. Secondly, it may cause the domestic currency to depreciate, making imports more expensive and increasing inflation. The proportion of imported items in the total price index determines the rate of inflation. Lower interest rates also promote consumption and investment, while a weaker currency might boost demand for exports and domestic items that compete with imports. This shows how monetary policy affects the economy.

Over the last few years, the main focus of central banks is price stability. In addition, there is a general agreement that monetary policy is more effective for achieving short-term stabilization. The monetary policy objective is to control an economy's interest rate and money supply to attain prices and financial stability. In Pakistan's economy, maintaining stable prices is the fundamental objective of monetary policy, however, without compromising the primary goal, the central bank assist with the financial stability. Monetary Policy Committee (MPC) manages the decision of monetary policy sentiments and involve in determining the policy interest rate, or target rate. SBP prepares forecasts of macroeconomic variables to support and enhance the forward-looking components of monetary policy formulation.

Projections for inflation, output gap, and other macroeconomic variables are important in the monetary policy decision-making process to keep the inflation at target level and output gap at the potential level. To attain a targeted inflation rate, setting a policy rate that minimizes loss function is important. However, the economy will encounter anticipated and unanticipated shocks while it takes the policy rate changes to completely affect inflation. During the process of monetary policy, the economy can face expected and unexpected changes that can affect inflation. It takes time for the policy rate changes to fully impact inflation. This means that even after the central bank adjusts interest rates, it may take some time for the effects on inflation to be seen. These delays occur because various factors can influence inflation, causing it to respond differently to policy actions.

Directly comparing inflation objective and results may not always offer a clear picture of monetary policy effectiveness. (L. Svensson, 2009) explores that the comparison of inflation outcome and the target is insufficient as Riks bank along with all other inflationary-targeting central banks use flexible inflation targeting instead of strict inflation targeting. The purpose of flexible targeting is to stabilize inflation at the desired level and the output gap at the potential level whereas rigid inflation targeting means stabilizing inflation without stabilizing the actual economy.

Recent macroeconomic policy has commonly focused on a central bank whose purpose is to minimize a quadratic loss function, with economic structure restricting a realistic balance of inflation and unemployment in the form of IS and a Phillips curve. It is important to keep inflation near to the central bank's target level. The central bank's weights for inflation and output are the important parameters in the loss function, their ratio reveals the CB implicit trade-off between output and inflation (Blanchflower et al., 2014).

The central bank of a country aims to achieve its monetary policy goals by controlling the supply of money and credit. However, in practice, it is challenging to attain the desired objectives, especially in struggling economies like Pakistan. Therefore, it becomes important to adopt the right policies at the right time with clear objectives in mind. When formulating monetary policy, a significant concern arises, whether the monetary authority follow a firm and systematic rule or have the freedom to take actions based on their judgment of the prevailing economic conditions This debate between rules and discretion is an important topic in economics. Advocates of rules argue that discretionary monetary policies can lead to problems of time inconsistency. This means that policymakers may be tempted to deviate from their initial plans in response to economic changes, which can result in suboptimal outcomes.

Considering the economy of Pakistan, flexible inflation targeting is efficient for achieving stability among the monetary policy (MP) objectives. The objectives of MP must be determined to choose the most efficient way to achieve them. Historically, it has been concluded that monetary policy should minimize the quadratic loss function i.e. losses generate due to the adjustments between inflation and output gap.

1.2 Problem Statement

In Pakistan, the country's monetary policy has involved a mix of rule-based and discretionary measures. The discussion over rules vs discretion in monetary policy has captured the interest of researchers. Various researchers proposed various monetary policy rules aiming the stabilize the objectives of monetary policy. The constant description of the monetary policy reaction function has been seriously challenged in the empirical literature (Malik & Ahmed, 2007) suggest a Taylor type monetary policy rule for Pakistan and in 2011, (A. M. Ahmed & Malik, 2011a) investigate the static and dynamic version of Taylor rule. (Saghir & Malik, 2017) investigate Taylor-type rules and McCallum rules (Hussain, M. 2005), (Aleem & Lahiani, 2011) and (Bunzel & Enders, 2010) have also worked on the estimation of different monetary policy rules. There are limited researches on projected policy rule to stabilize the trade-off between the inflation deviation from the targeted inflation and the output deviation from the potential output. So, by conducting a projection of monetary policy rule central bank can achieve the settlements between the deviation of inflation and output gap, in which the bank chooses the realistic combination of projected inflation and output gap that minimize the quadratic loss function and set policy rate accordingly (Debortoli et al., 2017). Therefore, it is important to examine the projected policy rate in determining efficient monetary policy rule that minimizing the expected quadratic loss function.

1.3 Research Problem

Based on the statement of the problem, this study narrowed my research problem into "*Anticipated Policy Rate Path in Policy Simulation: A Case Study of Pakistan*" and engaged my topic into the following research questions and objectives.

1.4 Research Questions

- Which monetary policy rule is efficient in the stabilization of inflation and output gap?
- What should be the simulated policy rule that minimizes the quadratic loss function?

1.5 Testable Hypothesis

The study includes the optimal policy rule and the Taylor rule. Based on these rules the testable hypothesis is shown in the table.

Table 1: Hypothesis Testing

Hypothesis Nature	Description
Null Hypothesis (H_o)	Taylor rule performs better than the optimal rule through projection.
Alternative Hypothesis (H_1)	Optimal policy rule performs better than the Taylor rule through projection.
Null Hypothesis (H_o)	Taylor rule performs better than the optimal rule by minimizing a quadratic loss function.
Alternative Hypothesis (H_2)	Optimal policy rule performs better than the Taylor rule by minimizing the quadratic loss function.

1.6 Research Objective

The fundamental research objectives of the study are

- To estimate the monetary policy reaction function and conduct policy projections considering different anticipated policy rate paths, while comparing the performance that are consistent with the monetary policy rule of inflation targeting.
- To simulate the policy rule by keeping in view the dual mandate of minimizing the gap between the actual and the threshold rate of inflation and the deviation between the actual and potential level of output.

1.7 Unit of Data Collection

Data collection is a method by which researchers collect related information to solve research problems and finally estimate the results. The choice of data collection depends on the type of research objectives. The UDC is from

- State Bank of Pakistan (SBP)
- International Financial Statistics (IFS)

1.8 Organization of the Study

The organization of the study is structured as follows; Chapter 2 comprises the literature review of the existing studies and the literature gap is also discussed in the chapter. The theoretical framework of empirical analysis and monetary policy framework of Pakistan has been discussed in chapter 3. Chapter 4 includes the research methodology, data collection, and discussion of the variables of the current study. Chapter 5 contains the results and discussion obtained from the estimation. The qualitative analysis is discussed in chapter 6. Finally, chapter 7 comprises of the summary and conclusion, and the policy recommendation.

1.9 Significance

Research on the debate of rule-based policies versus discretionary policies in the context of monetary policy is of predominant importance due to its direct implications on economic stability and growth. This debate seeks to determine whether central banks should adhere to predefined rules, or exercise discretion in responding to economic conditions. By delving into this debate, researchers can shed light on which approach is more effective in achieving the twin objectives of price stability and sustainable economic growth. Rule-based policies offer transparency and consistency, reducing uncertainty in financial markets and among the public. Conversely, discretionary policies provide flexibility to adapt to unique economic circumstances.

Furthermore, conducting projections on an interest rate estimate that, from the bank's perspective, projection aims to provide a balance between the objectives of monetary policy. A balanced policy rate projection contributes towards the predictability of monetary policy. This predictability enables market participants to respond to new information in a way that contributes towards the stabilization of the inflation level at the target and output gap at a potential level in a way when inflation is above the target level, the central bank increases the projected policy rate to make borrowing expensive that decreases the consumer spending to slow down the economy and when inflation is below the target level, the central bank decreases the projected policy rate to encourage borrowing that increases consumer spending to boost economic activity and hence contributes towards the predictability of monetary policy.

Another advantage of public policy rate projections is that they help central banks modify interest rate in the short and long term based on changing conditions of an economy. As discussed by (Bergo, 2006) minor differences emerged between Norges Bank's interest rate path and calculated forward rates until 2007. Market expectations fell after the report's publication, highlighting the divergence between the Bank's projected path and the market's anticipated path.

CHAPTER 2

LITERATURE REVIEW

The stabilization of macroeconomic variables and accurate projection of their future behavior is of paramount importance for policymakers, economists, and financial analysts alike. Achieving stability in key economic indicators such as inflation rates, GDP growth, unemployment levels, and interest rates is vital for the development of sustainable economic growth and avoiding recessions or periods of excessive volatility. Consequently, extensive research has been conducted in the field of macroeconomics to develop models and methodologies that can effectively stabilize these variables and provide reliable projections.

2.1 Macroeconomic Stability

Among the objectives of monetary policy, the study focuses only on two objectives as of (Taylor, 1993) rule: output and inflation which includes minimization of the loss function through projections. The term loss function incorporates the stabilization of the output gap and inflation targeting. Woodford (2001) navigates a intertemporal loss function that estimated the expected utility of the representative household corresponding to a particular policy in a quadratic approach whereas using the progressive open economy DSGE model (Benigno & Benigno, 2008) investigate GDP, output gap and inflation may be used to execute international monetary allocation through inflation targeting to minimize the quadratic loss function. Adolfson et al. (2008) study the adjustments among inflation targeting and the output gap in Ramses the forward-looking DSGE model. The study discovers that the variance between inflation stabilization and the output gap is substantially influenced by the potential output model used in the loss function. Severe volatility is examined, if potential production is considered as a smooth trend compared to the output level that would prevail if prices and wages were changeable, severe volatility is examined.

The central bank's goal is to support financial stability by minimizing the loss function. Akram & Eitrheim (2008) investigate whether financial stability is due to the stabilization of inflation and output or due to the stabilization of credit growth and asset pricing channel and find that the output stabilization enhances financial stability. In addition, the price of houses and equity enhance the stability of both inflation and output, contributing to a mixed impact on financial stability. In the decision-making of central banks judgments play an important role. The decision of Bayesian optimal monetary policy MP is obtained by minimizing the loss function. Gelain & Manganelli (2020) introduce the implicit choice of judgmental decision shown by Bayesian analysis of DSGE models, present an alternative approach to consolidate judgmental decisions, and determine the best monetary policy option compatible with a three-equation New Keynesian DSGE model, conclude that the central bank has a quadratic loss function and serves the dual purpose of stabilizing inflation and bringing the output gap to potential.

Compared to inflation targeting is a policy that aims to stabilize the level of inflation and stabilize the real economy. Walsh (2003) suggests that inflation targeting is dominated by speed limit targeting policies. On the other hand, price level and nominal income growth level were in contrast to policy regimes based on the difference in the gap. Prior studies claimed that when inflation is generally backward-looking, price level targeting has much worse results than inflation targeting, nominal income growth level, or speed limit level. In contrast, Ball (1999) presented the simple macroeconomic model which represents that the Taylor rule is efficient for determining the desirable monetary policy by minimization of a loss function. The model generates a set of results concerned with the Taylor rule, inflation targeting, and nominal interest rate targeting.

Chatelain and Ralf (2023) work on several findings on central bank funds rate rules that favor inflation. The study provides a procedure for simulating macroeconomic stability using funds rate regulation and examined the controllability of standard monetary policy transmission mechanism. Their study also establishes a Keynesian Taylor principle of reduced form funds rate rules that is conditional on the transmission mechanism parameters and for the estimation of inflation equation, provides econometric estimators that take into account the inverse causality of the negative response of funds rate rule.

A broad class of nonlinear sensible expectations models is examined, in which the goal of representatives is to maximize an objective function. Giannoni & Woodford (2010) demonstrate a target criterion that is dependable with the fundamental equations of the model, robust enough to imply a particular equilibrium, and committed to modifying the policy instrument to maximize the objective function. They also utilize a nonlinear DSGE model with an asymmetrical price setting. To organize monetary policy discussions and communicate monetary policy choices to the public, forecast targeting has gained popularity among central banks throughout the world.

Lin and Weise (2019) use a New Keynesian Model with robots and finds that if monetary policymakers aim to stabilize output and inflation, would need to focus on the stabilization of output rather than inflation. Furthermore, if policymakers have an employment stabilization goal in addition to the objective of output stabilization, then have to emphasize output stabilization due to the decline of the output-employment correlation.

Benchimol and Fourçans (2019) investigate the effects of different monetary policy rules on macroeconomic equilibrium. Using the (Smets & Wouters, 2007) DSGE model, they compare Taylor type rules versus nominal income rules. They test twelve monetary policy rules through Bayesian estimations over three different periods, capturing various economic environments. The findings indicate that NGDP level targeting strategies work better throughout the financial crisis and the Great Moderation, whereas a Taylor-type rule works best between 1955 and 1985. When it comes to minimizing the central bank's actual and predicted losses, NGDP in level regulations routinely beats other rules. To enhance interest rate choices, the study emphasizes the significance of regularly updating estimates and evaluating central bank losses using multiple empirical rules and models.

Monetary policy cannot parodist the best policy, but the optimal policy can still be adopted by focusing on an accurately defined inflation measure. Moving to the optimal policy shrinks the loss but cannot fully remove it, targeting the output gap almost replicates the optimal policy. (Rubbo, 2020) estimates the performance of the Taylor rule's two conventional objectives, output gap, and consumer inflation, and develops a general Phillips curve and a new inflation index, that that holds for any sector weighted-average inflation rate, as well as an equation for welfare as a function of the output gap and sectoral inflation rates, in the positive and normative effects of the New Keynesian model and that produces a Phillips curve but no endogenous cost-push shocks respectively and shows that targeting the new inflation index completely stabilizes the output gap because it overlooks the negative effect of sector-level inflation on allocative efficiency while stabilizing consumer prices results in an anticipated loss in comparison to the optimal policy.

(Purificato & Sodini, 2023) reveal that deviations from target macroeconomic variables in steady-state equilibrium result mainly from exogenous shocks or inter-country disparities and also introduce a temporal dimension that highlights the simple decision mechanism to equilibrium whereas (Rubbo, 2023) while revisited the New Keynesian model conclude that to stabilize output, policy must tolerate relative price distortions, implementable through a divine coincidence-targeted Taylor rule.

2.2 Rules vs Discretion

In Pakistan, the country's monetary policy has involved a mix of rule-based and discretionary measures. The rule-based monetary policy follows predetermined frameworks or guidelines, providing transparency, accountability, and predictability. Discretionary policy, on the other hand, allows policymakers to make judgment-based decisions, offering flexibility and responsiveness to unique economic circumstances. Since the beginning of economics, economists have typically favored the application of monetary policy rule. In "The Wealth of Nations," Adam Smith emphasized the potential benefits of a well-regulated paper money system for encouraging economic stability and progress. However, there has been a recent trend away from rule-based approaches. From 2009 to 2013, following the global financial crisis, there was a period of deviation from rules, with central banks holding interest rates abnormally low for a prolonged period. There is a growing interest in returning to a more rules-based approach to monetary policy at the moment.

Various studies have suggested and proposed the superiority of rules over discretion using empirical models and historical analyses. (Meltzer, 2012) did a historical analysis and discovered that when monetary policy was more rule-based, the economy performed comparatively better. Suggesting that having clear standards and regulations for monetary policy implementation might lead to better economic results. (Shultz, 2014) explains the importance of having a strategy. It emphasizes the need of having a strategy-based monetary policy. Past policy decisions from many sectors show that having a plan gets you someplace, while not having a strategy is like being a strategist who adds up to nothing.

Inconsistency of discretionary policies has been demonstrated by (Kydland & Prescott, 1977) and (Barro & Gordon, 1983). The time-consistency distinction is a basic separation between policy rules and discretion. They argued discretionary policies of inconsistency and short-sightedness. Specifically, new research emphasizes on policy rules. The relevance of regulations is the primary reason for current scholars' interest in recent policy studies.

Taylor proposes monetary policy rule and claims that they are preferable to discretion. According to (Taylor, 1993), policy rules are preferable to discretion owing to time inconsistency. Rules should be flexible in response to changes in the money supply and monetary basis. Short-term interest rates may shift in reaction to changes in the price level and actual income. Taylor's simple rule provides that the federal funds rate rises if inflation exceeds a target or real GDP exceeds trend GDP.

Monetary policy is considered rule-based when it aligns with its stated commitments on how it will respond to observable economic changes in the future. (Dellas & Tavlas, 2022) investigate the historical evolution of the rules versus discretion debate in monetary policy, computing that the modern literature leans toward complex and activist rules, reflecting economists' confidence in their ability to understand and manage the economy. On the other side (Backus et al., 2022) debate on the long-term asset-pricing constraints in a macro term-structure model helps differentiate discretionary monetary policy from policy rules as the policy discretion significantly contributes to overall economic risk. When policymakers use discretion to ease monetary policy, it coincides with positive economic indicators but also signals negative long-term financial conditions.

2.3 Inflation Targeting and Macroeconomic Projections

In the monetary policy framework, inflation targeting is used by central banks to manage and control inflation within predetermined variables. It requires establishing a specific numerical inflation objective, often over the medium term, and implementing different policy instruments to attain it. Macroeconomic predictions are used by central banks to guide their decision-making process and decide the right policy actions. Macroeconomic projections anticipate significant economic indicators that include GDP growth, unemployment, and inflation based on available data and economic models. These predictions give critical insights into the economy's current and future status, allowing central banks to evaluate the effectiveness of their policy measures and make well-informed choices to keep price stability and foster viable economic growth.

Jain and S. Sutherland (2018) use panel data from twenty-three economies with the analysis that central bank predictions and forward guidance are relevant for private-sector differences about impending policy rate decisions, but less significant so for other private-sector macroeconomic projections. Projections of central bank target variables appear to have the largest impact as used by private-sector projections as monetary policy signals. Mokhtarzadeh & Petersen (2021) study the individual and aggregate forecast. Findings imply that all projections minimize the output gap but increase inflation disagreement. Central banks face two significant decisions when communicating and constructing their projects. They should first assume several assumptions, involving how people will think about the future and how the economy will develop. Second, central banks must decide which of their projections to communicate to the public that is simple to understand.

Hofmann and Xia (2022) Estimates that the policy rate projections announce by the key dimensions of central banks are credible and predictable but in limited ways. Market regulations to unexpected path changes reduce the credibility of central bank paths, with increasing projection horizon, predictability, and credibility declines. Furthermore, they discover that central bank policy rate projections are not unnecessary since, even after adjusting for the influence of the parallel central bank macro projections, they still have an impact on market expectations. The interest rate estimates are consistent with the macro projections, because of their empirical connection by a stabilizing Taylor rule.

Forecasts based on certain interest rate paths are usually criticized because many forward-looking models assume the presence of a unique equilibrium, central banks' projection. Galí (2011) presents three alternative methods for creating predictions, using two different versions of the new Keynesian model baseline framework. The three approaches CIR, ME, and CBE forecasts, all indicate the same path for the interest rate but provide different estimates for output and inflation. The effectiveness of forecasting depends on the interest rate path. In theory, there is no clear reason to prefer one method over the other to determine the interest rate path.

Real-world countries are currently pursuing explicit inflation targeting. Using a modest empirical model of the US economy. Rudebusch & Svensson (1999) investigate the effectiveness of policy rules that are in agreement with an inflation-targeting monetary policy regime. In this regard, policy rules have been partially prescriptive, entailing sorting and evaluating among numerous rules, and partly descriptive, directly related to what inflation-targeting central banks appear to be doing. Findings imply that some straightforward forward-looking rules are capable of doing fairly well from the latter viewpoint.

Ramses, the central bank of Sweden's open-economy medium-sized Dynamic Stochastic General Equilibrium (DSGE) model for projecting and policy analysis, demonstrates how to obtain the best possible policy projections. The conditional potential output, trend output, and unconditional potential output variations of the output gap were examined by (Adolfson et al., 2011) who also provided an overview of the applications of various output-gap concepts in loss functions. Their contribution includes providing a comprehensive review of the ideal policy forecasts in a linear quadratically estimated model with progressive variables.

The decisions that central banks make about their monetary policy are influenced by predictions of economic events. The advantages of forecasting are explained by (Amato & Laubach, 1999) in two ways. First, monetary policy influence's objective variables with advantageous lays. The second benefit is that by concentrating on an inflation projection, the central bank may accomplish several objectives while also increasing its public responsibility. A modest simulation model's results indicate that a five-quarter horizon produces very favorable results.

Orphanides and Wieland (2008) using projections of FOMC from the Humphrey-Hawkins reports, estimate the proposition in the context of FOMC policy settlements. Their findings show that rather than current economic development, the FOMC's decision may be primarily explained in terms of its predictions. Therefore, a rule based on forecasts better specifies FOMC in making decisions. federal funds rate's apparent variation from a Taylor-style rule based on outcomes may be interpreted as systematic reactions to data from FOMC predictions. (Epstein et al., 2022) presented a Quarterly Projection Model (QPM) that may be used for macroeconomic forecasting and policy analysis by extended New Keynesian model. This model incorporates policy tools like interest rates, credit growth targets, and exchange rate interventions to achieve price stability.

According to (L. E. O. Svensson, 2005a), forecast targeting creates the best policy projections of the target variables and the instrument rate using central-bank decisions, that minimize an intertemporal loss function, which significantly performs better than the monetary policy that adheres to the predetermined instrument rule. It is simple to develop optimum policy forecasts that match the best policy already committed from a timeless approach. The instrument rate prediction is more important than the actual instrument agreements. The existing instrument rate's reduced-form response function is generated by combining all of the variables to the

monetary-policy decision-making process, particularly the central bank's choices. It cannot be reduced to a straightforward response function like a Taylor rule. Svensson & Tetlow (2005) present a technique for optimal monetary policy while considering the policymakers' decisions. The method for optimal policy projections is demonstrated in a linear model that is Federal Reserve Board's US model. Optimal policy projections (OPPs) are projections of target variables, instruments, and other variables of interest that minimize that loss function for defined judgement terms given an intertemporal loss function that captures monetary policy objectives.
Alvarez & Sánchez (2019) illustrate the process used to create inflation forecasting by the staff of the Bank of Spain. Baseline forecasts are shown by fan charts that highlight the degree of uncertainty around them. Additionally, a qualitative risk analysis is included to highlight whether risks to the baseline are viewed as being on the upside, downside, or balanced. There are presented indicators of the possibility of deflationary occurrences and the monitoring of inflation objectives is taken into account. Issing (2004) addresses certain issues using projection for the European Central Bank, first at the internal level on both political and technical issues regarding monetary policy, then, at the external level that help to shape the policy judgments. Finally, the basic principles that have guided the policy from the start of the monetary authorities have not changed as a result of the decision to publish the projected inputs to decision-making.

Owusu (2020) estimated a forward-looking reaction function of the European and Swedish Central Bank. First, an estimated baseline model suggests that, in compared to the ECB, the Riksbank is a fighter when it comes to responding to expected inflation and output gaps. Second, the calculated baseline model roughly replicates both central banks' real interest rates, while with significant imperfections. Furthermore, the research builds on the baseline model by looking at new elements that properly explain ECB and Riksbank monetary policy. Furthermore, the Riksbank's monetary policy is influenced by the ECB's short-term interest and real exchange rates. The baseline model predicted an ECB inflation target of 2.18% and a Swedish target of 1.86%.

Svec & Tortorice (2022) govern the optimal interest rate policy in a New-Keynesian model, considering the effect on households' perceptions of central bank independence. The central bank may deviate from rational expectations to reinforce the perception of independence, especially in response to productivity shocks. Varying perceptions of independence over time lead to changing volatility in interest rate policy and macroeconomic outcomes.

In recent decades, central bank communication has emerged as a significant aspect of monetary policy frameworks. Due to the prolonged low-interest rate environment and the necessity for ongoing monetary support following the Great Financial Crisis, central banks have expanded their communication efforts in terms of both size and breadth. (Kryvtsov & Petersen, 2019) observe how central bank communication affects economic expectations. In controlled experiments, they find that different types of communication have varying effects. Announcing past interest rate changes has the largest impact, reducing forecast volatility and improving price stability. Forward-looking announcements have a lesser effect, especially if they lack clarity on timing. Their study emphasizes that communication is most effective when it is simple and relatable, benefiting less-accurate forecasters. The researchers suggest that improving the accessibility of central bank information to the general public can enhance communication effectiveness.

2.3 Literature Review Concerning Pakistan

The literature offers valuable insights into managing monetary stability and promoting sustainable economic growth in Pakistan. The existing literature explores the effectiveness of policy tools, the impact on inflation and output, exchange rate dynamics, and challenges specific to a developing economy. The studies also examine transmission mechanisms, interest rate adjustments, the role of financial markets, and the interplay between fiscal policy and the external sector. Ahmed et al. (2005) employ VAR to explore the monetary policy transmission mechanism in Pakistan and find that the interest rate channel, credit channel, and asset price channel are significant in Pakistan, whereas (Khan & Qayyum, 2007) finds that supply shocks and exchange rate channel are significantly associated with the demand side channel.

Waliullah (2010) study the long-term link among the money supply, price level, and GDP in Pakistan, highlighting the critical relevance of these variables for developing-country monetary policy design. The findings show a consistent long-run link between Pakistan's money supply (M1), GDP, and the Consumer Price Index (CPI), and the use of ARDL is emphasized for its advantages over traditional techniques in assessing causality and co-integration. Qayyum (2008) investigates Pakistan's Monetary Policy Framework and its efficiency in managing inflation. It concludes that when the monetary authority efficiently manages the money supply objective, the monetary authority is successful in managing inflation.

Jawaid et al. (2010) use yearly time series data from 1981 to 2009 to investigate the influence policies of fiscal and monetary on economic development in Pakistan. According to the co-integration study, both fiscal and monetary policies have a large and favorable impact on economic growth. However, the coefficient for monetary policy is found to be larger than that of fiscal policy. The study emphasizes that fiscal policy can be more effective in supporting economic growth by addressing issues such as corruption, resource leakages, and inefficient resource utilization. They recommend a combination and management of both monetary and fiscal policies for optimal outcomes.

Modern macroeconomics widely supports the idea that using policy norms or rules is more advantageous than discretionary decision-making when it comes to enhancing economic performance. This is particularly relevant for developing countries that lack sophisticated targeting regulations. Implementing basic instrument rules is seen as a feasible option in such cases. These rules serve as important prerequisites for improving economic outcomes in poor nations. Malik & Ahmed (2010) calculate the Taylor rule and assess its effectiveness as a monetary policy strategy to stimulate the economy. The findings revealed that the State Bank of Pakistan (SBP) has significantly deviated from the Taylor rule in its actual policy implementation. Conversely, counterfactual simulations demonstrated that adopting the Taylor rule as a monetary policy strategy could have led to improved macroeconomic performance, specifically in terms of inflation and output stability.

Whereas (Tariq and Kakakhel, 2018) examine the monetary policy of the State Bank of Pakistan operate both closed and open economy Taylor rule frameworks and found that the central bank's monetary policy was not limited to a closed economy Taylor rule. Instead, it followed an open economy Taylor rule, responding to both interest rate and real exchange rate movements. The study also highlighted the consistent nature of Pakistan's monetary policy across different exchange rate systems implemented over time. Tahir (2013) Using the backward-looking and forward-looking Taylor rules, compare the pre and post-reform eras to evaluate monetary policy conduct. By estimating the model through the Generalized Method of Moments (GMM) shows that no interest rate path was observed, which clarifies the monetary policy's inability to control inflation and stabilizes the output gap.

Ahmed and Malik (2011) estimate a monetary policy reaction function for Pakistan by estimating the data from 1992Q4 to 2010Q2. The findings reveal that the State Bank of Pakistan maintains a Taylor rule-compliant strategy that includes interest rate smoothing and currency rate control. This policy has remained consistent throughout most of the sample period, except for the last two years, where significant changes in parameters occurred due to a price hike and depreciation of the domestic currency. The study also finds non linearity in the reaction function, with a more aggressive response to inflation rates above 6.4 percent vs mild inflationary periods. Hussain et al. (2022) also investigates the monetary policy reaction function in Pakistan's economy utilizing data ranging from 2005Q1 to 2020Q3. Different rules, including static and dynamic ones, are estimated to understand the relationship between monetary policy variables. According to the findings, the State Bank of Pakistan does not consider the nominal exchange rate in its monetary policy. The paper also emphasizes the significance of the average lag of treasury bill, which has a substantial influence on assessing monetary policy in Pakistan.

Mushtaq et al. (2022) examine the non-linear Taylor rule of inflation-targeting in Pakistan using quarterly data from 2005 to 2019. The study finds that the response of the output gap and inflation to economic activity is weak, while the interest rate shows a positive and significant relation with the output gap. Additionally, the study reveals a negative relationship between inflation, interest rate, and exchange rate. When the exchange rate depreciation exceeds a certain threshold, inflation tends to rise while the interest rate decreases.

Tahir (2022) investigates Pakistan's monetary policy and emphasizes that monetary policy is more than just monetary operations or according to mechanical rules such as the New Keynesian Taylor Rule. The analysis discovers a considerable association between inflation and interest rates, but the influence of money supply is insignificant. In Pakistan, there is evidence of the output gap's effect, showing that the State Bank of Pakistan prefers a cyclical strategy over an aggressive policy stance. The study finds no indication of the active interest rate rule being used to alter future expectations.

2.4 Literature Gap

In the context of Pakistan, prior studies focused on estimating reaction functions and minimizing the actual loss function of monetary policy. The actual loss function represents the disparity between the targeted and actual levels of inflation, as well as the deviation along with the actual output and potential output, known as the output gap. The primary objective of this study is to provide projections for inflation, the output gap, and the policy rate, aiming to minimize the forward quadratic loss function associated with monetary policy. The central idea is to determine the projected policy rate rule that effectively stabilizes the monetary policy objective of targeting inflation and the output gap toward their potential levels. The loss function utilized in this study is defined as the squared deviation of inflation from the inflation target and the squared difference between actual output and potential output over a specific period. By analyzing and minimizing this loss function, policymakers can make informed decisions regarding the appropriate policy rate that promotes stability in both inflation and the output gap, supporting them with their respective potential levels.

2.5 Policy Review

During the meeting with the experts, they have suggested me to review the following policies.

2.5.1 National Financial Inclusion Strategy (NFIS)

NFI's strategy aims to promote financial inclusion by increasing access to formal financial services and promoting the usage of digital financial services in the country to bring people into the financial majority, with encouraging effects on economic growth and financial stability. The strategy was launched in 2015 and has set targets to achieve by 2020 by increasing the number of adult financial accounts from 10% to 50% and the number of women with formal accounts from 5% to 25% and also increasing the number of a transaction conducted through digital financial services from 1% to 10% of the total transaction. The strategy has four main objectives:

- To increase the availability and accessibility of financial services, including savings, credit, insurance, and payment services.
- To promote financial literacy and consumer protection to enable people to make informed decisions about financial products and services.

- To raise the use of digital financial services, including mobile banking and e-commerce, to improve financial inclusion and reduce the cost of financial transactions.
- To expand access to finance for Micro, Small, and Medium Enterprises (MSMEs), which are a vital source of employment and economic growth in Pakistan.

To achieve the above objectives, NFIS involves several measures:

- Expanding the reach of banks and other financial institutions to underserved areas.
- Promoting innovative financial products and services, such as branch less banking and mobile wallets.
- Financial education and awareness programs are offered to increase financial literacy and decision-making.
- Encouraging the development of microfinance institutions and other non-bank financial institutions to increase access to credit for low-income households and MSMEs.

2.5.2 Meeting of the Monetary Policy Committee (MPC) Held on April

The committee discussed a range of indicators, including a significant slowdown in economic activity, declining auto and petroleum sales, and a contraction in large-scale manufacturing. As a result of these factors, combined with monetary tightening and fiscal consolidation measures, projected growth for the year is expected to be significantly lower than the previous estimate of around 2 percent. Based on their discussions, the MPC decided to raise the policy rate by 100 basis points to 21 percent.

Economic Conditions Economic activity experienced a significant slowdown in the current fiscal year, reflected in declining auto sales, POL sales, domestic cement sales, and a contraction in Large-Scale Manufacturing (LSM). The agriculture sector was impacted by floods, resulting in lower cotton arrivals and reduced wheat production. Electricity generation and construction activities also declined.

Growth Outlook The current developments, combined with monetary tightening and fiscal consolidation measures, indicated that the projected growth for FY23 would be significantly lower than the earlier estimate of around 2 percent.

Global Scenario International financial conditions tightened further, leading to capital outflows from Emerging Market Economies. However, global economic prospects improved due to the recovery in emerging economies.

Balance of Payments The monthly current account deficit fell to its lowest level in the last 24 months, primarily driven by a sharp contraction in import volumes. Exports and remittances remained lower compared to the previous year, while investment inflows declined due to domestic uncertainty and weak growth prospects.

Fiscal Developments The fiscal deficit was restrained at 2.3 percent of GDP during Jul-Jan FY23, with a surplus in the primary balance. Tax revenue growth remained below target, while non-tax revenue increased due to the increase in Petroleum Development Tax.

Monetary Aggregates Broad Money (M2) growth accelerated due to increased government budgetary borrowings, while Private Sector Credit (PSC) witnessed deceleration due to high borrowing costs and regulatory measures.

Inflation Headline inflation surged to 35.4 percent in March 2023, mainly driven by energy price revisions, taxation measures, exchange rate depreciation, seasonal effects, and second-round effects. Core inflation also increased due to adjustments in administered energy prices and high inflation expectations.

Inflation Outlook The projection for FY23 remained unchanged, with an expected average national CPI inflation between 27-29 percent. The inflation projection for FY24 was revised downwards due to monetary tightening and the expected softening of international oil prices.

Financial Markets and Reserve Management Overnight repo rate averaged at the target of 20 percent, and market rates increased by around 200bps after the previous MPC meeting. Participation in primary auctions was focused on 3-month T-bills. Eurobond yields and CDS spreads remained elevated.

Model-Based Assessment Model-based assessments suggested a policy interest rate path to bring inflation within the target range of 5 to 7 percent in the medium term. The appropriateness of the current policy stance was discussed.

The MPC meeting held in January rise the policy rate by 100 basis points to 17 percent whereas the meeting held in April decided to rise the policy rate by 100bps to 21 percent, with seven out of nine members in favor of the increase. The increase in the policy rate indicates a tightening of monetary policy to address inflationary pressures and stabilize the economy. This decision suggests that the central bank aims to curb inflation and maintain macroeconomic stability. Overall, the meeting highlighted the significant slowdown in economic activity, challenges in the balance of payments, elevated inflation, and the need for monetary tightening to address these concerns.

CHAPTER 3

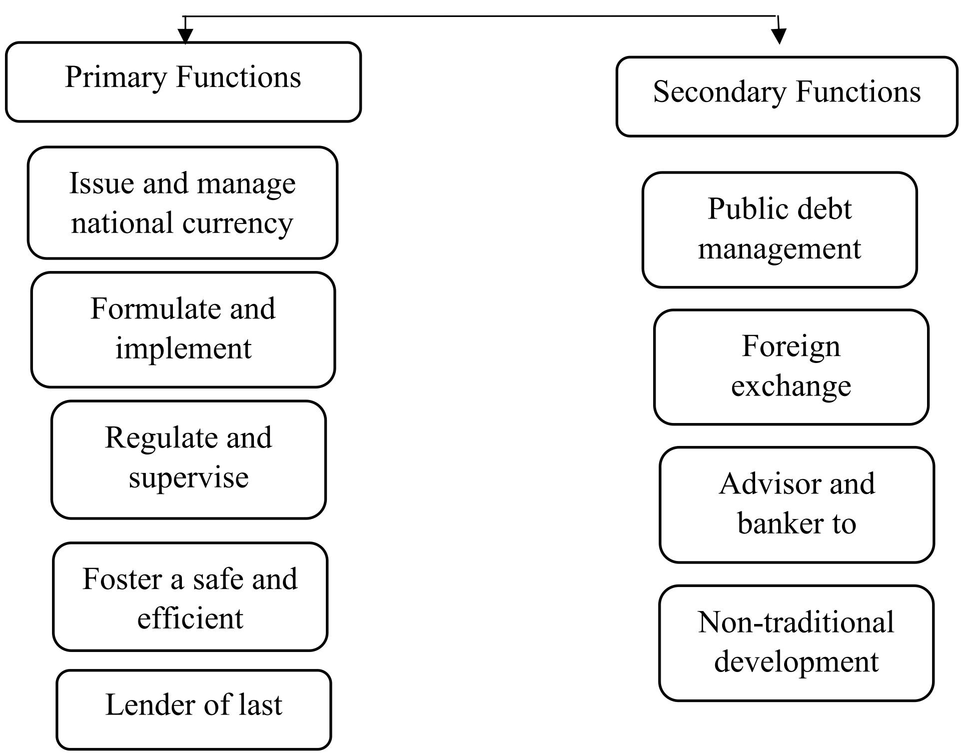
FORMULATION OF MONETARY POLICY

3.1 Background on Evaluation of Central Banks

Central banking originated in Europe. Some central banks, like the Riks bank of Sweden in 1668, the Bank of France in 1800, and The Netherland Bank in 1814, were initially established as government banks to bring order to currency issuance. Others, such as the Bank of England in 1694 and the First and Second Banks of the United States in 1791 and 1816 respectively, were created to fund government operations and finance wars. During the 20th century, most central banks were established for central banking functions, including regulating the banking system, controlling monetary policy, managing reserves, and promoting financial stability.

Function of Modern Central Bank

The primary and secondary function of a modern central bank is as follows



Source: Author Computation

Figure 1: Function of Modern Central Bank

Effective monetary policy management is important for preserving economic stability and guaranteeing the smooth operation of financial markets. Central banks around the world use a variety of policy instruments to affect interest rates and control the money supply in order to create price stability and foster long-term economic growth.

Monetary policy: Monetary policy is the use of policy instruments by central banks to influence interest rates and money supply in order to keep overall prices and financial markets stable.

Money supply management: Maintaining the worth of money and encouraging the population to keep domestic currency. The inflation (rise in prices) is when the same amount of money buys less quantity leading to the value of money drops and the deflation (decline in prices) is when the same amount of money buys more quantity leading to the rise in the value of money.

Objectives of monetary policy: the objective of MP is price stability that refers to regulating and maintaining low inflation.

Focus on price stability: To avoid costs of high inflation and this Low and stable inflation helps in achieving other objectives that facilitate investment decisions, maintain the value of the home currency, and protect the savings of the nationals.

Working of monetary policy: Monetary policy affects inflation and economic activity by altering aggregate demand through changes in interest rates or the money supply. This is known as the monetary policy transmission mechanism. Changes in policy rates have a direct influence on interbank and retail interest rates, as well as expectations. Retail interest rate changes, therefore, have an influence on consumer and company spending and investment, impacting aggregate demand. Finally, changes in aggregate demand have an impact on the total price level and inflation in the economy.

3.2 Monetary Policy Formulation in Pakistan

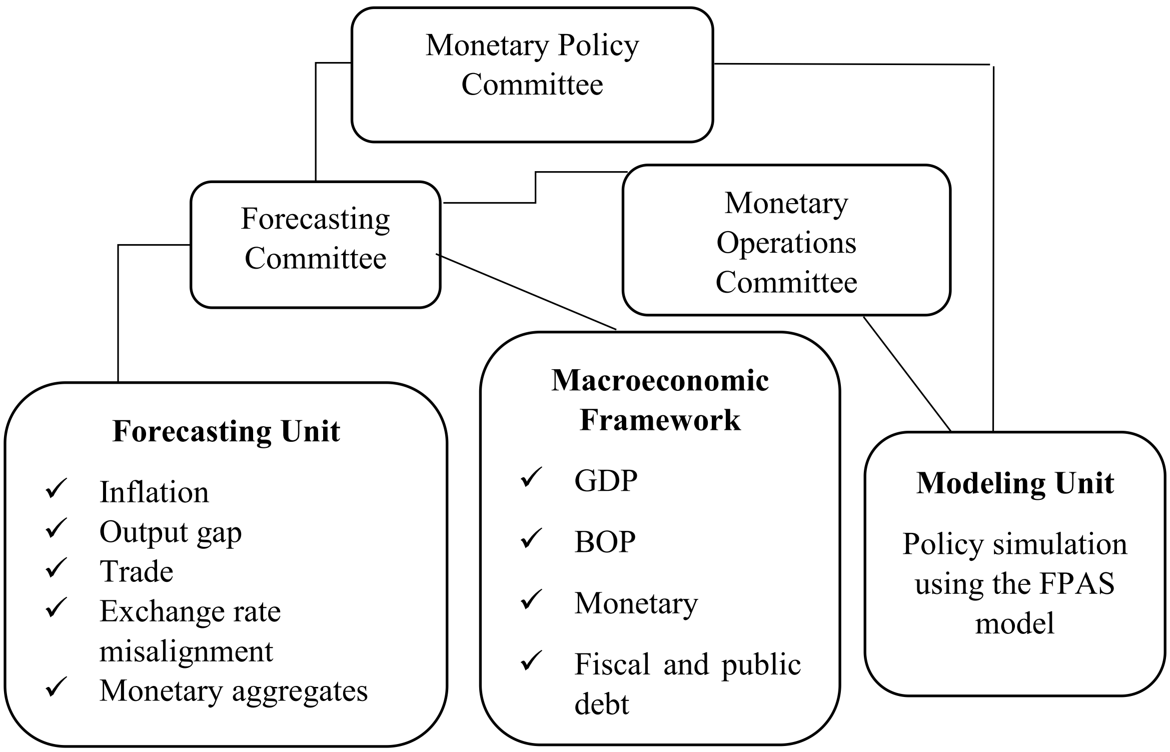
The SBP (Amendment) Act of 1956 defines the broad objectives of monetary policy in Pakistan, stating the need for the State Bank to regulate the monetary and credit system in order to promote economic growth, ensure monetary stability, and optimize the utilization of productive resources. These goals can be interpreted in a variety of ways. While most people think of monetary stability as ensuring price stability, it may also refer to the stability of the money supply or other monetary aggregates. Furthermore, efficient bank regulation and the preservation of the financial sector's soundness are critical for sustaining stable circumstances in the interbank market. The State Bank of Pakistan (SBP) focuses on establishing monetary stability by managing inflation and aligning it with government goals. Simultaneously, the SBP prioritizes financial stability by guaranteeing the smooth operation of the financial market and payments system, both of which are critical for a well-functioning economy.

3.2.1 Monetary Policy Decision-Making Process

i. Institutional Set-up

The Monetary Policy Committee (MPC) plays a significant role in the decision-making process of monetary policy. As per Section 9E of the SBP Act 1956, the MPC is responsible for formulating and recommending monetary policy actions, as well as approving and issuing monetary policy statements and other relevant measures. The MPC consists of ten members, including the chairman, three board members nominated by the board of the State Bank of Pakistan (SBP), three senior executives of the SBP, and three external economists appointed by the governor and federal government. The MPC held at least six times a year, in the second week of alternating months, to set the country's monetary policy position.

ii. Procedures



Soure: Fida, (2018)

Figure 2: Procedure of Monetary Policy Decision-Making Process

iii. Main Considerations of SBP

The primary and secondary considerations for the monetary policy decision-making process are

Primary Consideration

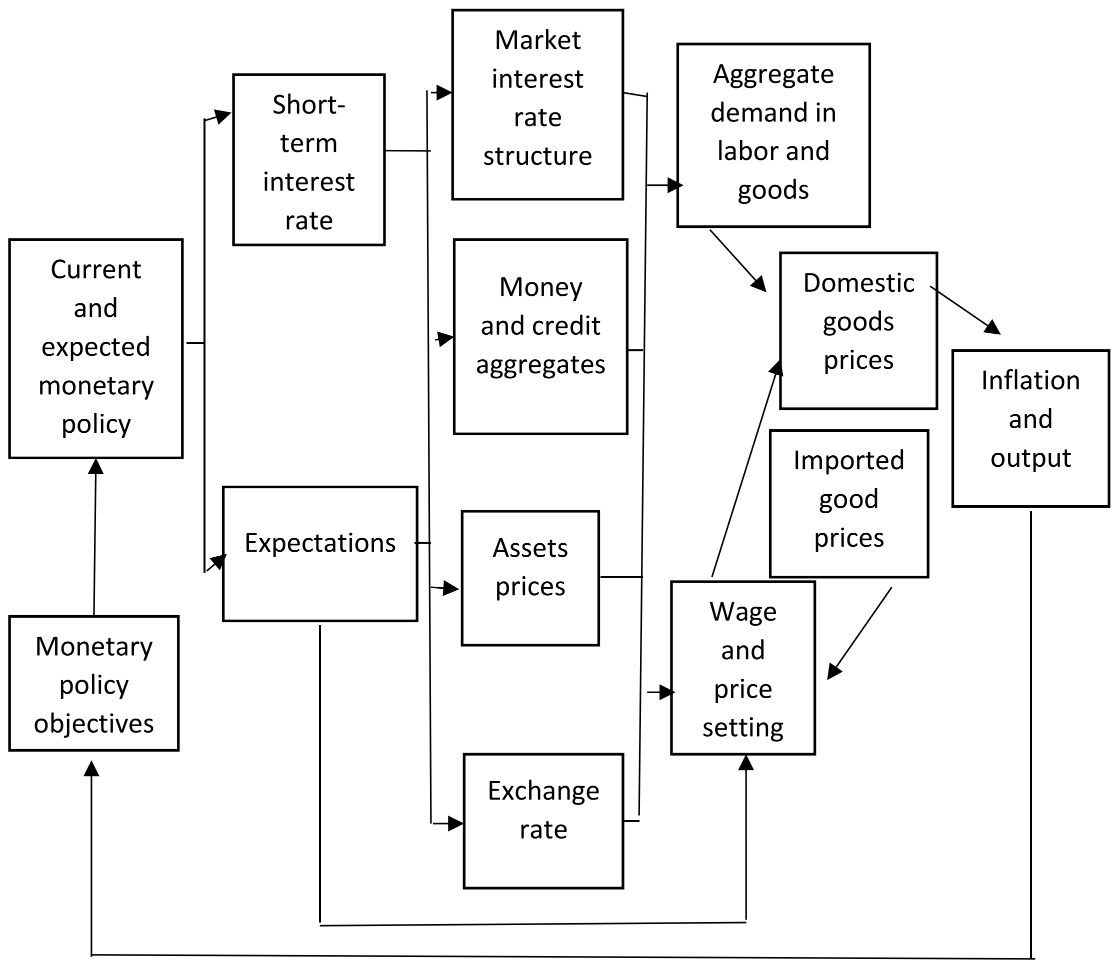
The Monetary Policy Committee (MPC) evaluates the near-term inflation trajectory and inflation expectations in relation to the objective. Price stability is indicated if inflation is expected to continue close to or below the target. In such cases, the MPC can implement accommodating monetary policies to boost economic development and increase investment and consumption, resulting in a good growth outlook.

Secondary Consideration

The Monetary Policy Committee (MPC) evaluates the external sector's sustainability, especially potential exchange rate pressures. Exchange rate fluctuations might have an impact on the inflation outlook. If there are significant pressures on the exchange rate, it can impact the cost of imported goods and potentially lead to inflationary pressures. As a result, the MPC considers these issues in order to guarantee the external sector's stability and to make informed monetary policy choices.

3.2.2 Implementation of Monetary Policy

The implementation of monetary policy involves setting targets and utilizing Open Market Operations to manage liquidity in the money market. The transmission of monetary policy occurs through various channels, including the interest rate, balance sheet, exchange rate, asset price, and expectation channel, as recognized by the State Bank of Pakistan.



Source: SBP

Figure 3: Transmission Mechanism of Monetary Policy

The interest rate channel is used in monetary policy to influence retail and lending interest rates through changes in policy rates, such as repo and KIBOR. This influences the real interest rate through money market operations, subsequently affecting economic activity with a time lag. The balance sheet channel operates by impacting the credit portfolios of financial intermediaries. When policy rates change, it affects the availability of loanable funds by altering the cash flow and net wealth of these intermediaries.

The exchange rate channel is another channel that links the domestic economy with the international economy. Changes in domestic interest rates can attract or deter foreign investors, impacting the exchange rate. The asset price channel operates through changes in the prices of real and financial assets. Lastly, the expectation channel plays a significant role, where the conduct of monetary policy influences the expectations of investors and the general public regarding future interest rates and inflation. This channel is crucial as market expectations can influence long-term interest rates.

3.3 Theoretical Framework for Empirical Analysis

Modern monetary macroeconomics is constructed on three equation New Keynesian model i.e. IS curve, the Phillips curve, and the equation of monetary policy rule that describes the relationship between fundamental economic variables and the objective of monetary policy. These three equations are interconnected and provide a framework for understanding how changes in monetary policy variables, such as interest rates or money supply, can impact inflation, unemployment, and economic output; the model also identifies the structural factors that influence the coefficient of a Taylor-type interest rate rule. Based on the preferred objective and the prevailing economic condition, these three equation model help policymakers to make decisions about setting monetary policy.

3.3.1 Aggregate Demand and Supply Curve

Monetary policy can affect the AD and AS curve through changes in interest rates, money supply, and other policy tools and trade-off among the variables of the Phillips curve specify that monetary policy can be used to influence the level of inflation and unemployment in the economy.

IS Curve and Phillips Curve

In IS curve equation current output is a function of lagged output and a function of real interest rate: aggregate demand equation with the goods market in equilibrium.

$$Y_t = \alpha Y_{t-1} + \beta\,(i_{t-j} - \pi_{t-j}) + \varepsilon_t \qquad (3.1)$$

where Y_t is the real GDP, π is the inflation and ί is the nominal interest rate, where ε_t is the demand shock in period t.

In the Phillips curve equation, current inflation is a function of lagged inflation and output gap-aggregate supply equation.

$$\pi = \alpha\,\pi_{t-1} + \beta y_t + \varepsilon_t \qquad (3.2)$$

where Y_t is the real GDP, π_{t-1} is the lagged inflation and ε_t is the supply shock in period t.

3.3.2 Equation of Monetary Policy Rule

The Monetary Policy Rule is an equation that describes how central banks set their policy interest rate based on their valuation of the current economic conditions and their objectives. The equation of monetary policy rule is used by the Central Banks to adjust interest rates to achieve their objectives that is the goals of price stability, maximum employment, and economic growth.

There are two sorts of monetary policy rules: instrument rules and targeting rules. The debate and discussion between these two emphases the difficulties such as simplicity, robustness, practicability, dependability, and technical feasibility, as well as the importance of policymakers' judgement in various policy regulations. The fundamental distinction between the two is described below.

3.3.2.1 Instrument Rule

The simple instrument rule is presented by (McCallum, 1988), (Taylor, 1993), and others. The monetary policy tool is specified by the instrument rules according to the economic situation i.e. the information that is available to the central bank. These regulations are easy to understand and requires limited information.

Svensson (2005) interprets the “instrument rules” as a specified function of both the predetermined and foreseeable variables. The rule is explicit instrument rule if the instrument is a direct function of a predetermined variable- the “explicit response function” and implicit instrument rule if the instrument is a direct function of a foreseeable variable- the “implicit response function”.

The equation for the rule can be written as

$$i_t = r + \pi_t + \beta_1 y_t + \beta_2(\pi_t - \pi^*) \quad (3.3)$$

where r is represented by the long-run real interest rate, y_t is the long-run output gap in period t, π_t represents the rate of current inflation, and π^* represents the rate of target inflation.

3.3.2.2 Targeting Rule

The targeting rule denote that the central bank is entrusted with minimizing a loss function that rises the variance between the targeting variable and its target level.

The word "targeting variable " or "having its target level for variable " has two interpretations in the literature. The first meaning is used to signify "setting a target for targeting variable" and "having a target" means using all the available information that is accessible to bring the target variable in line with the target or more accurately minimizing the quadratic loss function. According to the second interpretation the terms "targeting" and "targets" suggest a specific information limitation for the instrument rule, namely that the instrument must only rely on the gap among the target variable and the target level (lags of both this gap or lags of itself) (Rudebusch & Svensson, 1999b)

In 1990, New Zealand was the first to support inflation targeting (to keep inflation at a target level and to keep the output on track). Since many countries followed the inflation targeting regime including both developed economies (such as Australia, Sweden, Switzerland, and the United Kingdom) and emerging countries (such as Brazil, Chile, Columbia, Czech Republic, Iceland, Israel, Mexico, Peru, the Philippines, Poland, and South Africa). Switching to an inflation-targeting regime has also been discussed at the Federal Reserve in the US and the Bank of Japan. Inflation is one of the policy objectives of the ECB, which has a two-pillar framework along with monetary targets. Svensson (1997) exclaim this regime is an optimal monetary policy as the central bank’s loss function is specified and constrained by the economy’s transmission mechanism and the first-order conditions generated are implicit monetary policy response function. As Bernanke and Mishkin stated, the inflation forecast is the intermediate target.

In monetary policy, there are two types of targeting rules: the "general targeting rule" and the "specific targeting rule." A generic targeting rule identifies a functioning loss function and commits monetary policy to minimizing it. However, specific targeting rules, a need for establishing the tool is stated, such as an equalized marginal rate of transmission and substitution among the target variables. The State Bank of Pakistan has authority when it comes to defining operational targets. However, there is no quantitative constraint on government borrowing from the SBP, which changes the monetary policy approach. Due to this, SBP finds it difficult to target inflation, and credibility would be at risk if the SBP Act were not updated reasonably (Moinuddin, 2009).

The equation for simple Taylor rule

$$i_t = \alpha_0 + \alpha_\pi \pi_t + \alpha_y y_t + \varepsilon_t \tag{3.4}$$

where i_t represents the nominal interest rate, π_t is the inflation and y_t is the output gap whereas ε_t is the error term.

CHAPTER 4

RESEARCH METHODOLOGY

A quantitative analysis, where dynamic statistic general equilibrium DSGE model for backward-looking and forward-looking is used for the analysis of secondary data to find the projected policy rate path that minimizes the quadratic loss function, and for qualitative analysis interviews are conducted to discuss the topic and the results of the study from the personnel of State Bank of Pakistan or experts relevant to my area.

4.1 Method of Data Collection

The time-series quarterly data from 1993Q1 to 2022Q4 is used. The variables of the study of the policy rate, inflation, and output gap. For the given methodology, secondary data is gathered from the International Financial Statistics (IFS), and State Bank of Pakistan (SBP) for the variables of the policy rate, CPI used as a measure of inflation, and output gap as a measure of GDP. The output gap is estimated for simulation purposes.

4.2 Variables

The variables of the study are policy rate, inflation, and output gap.

Table 2: List of Variables

Variable	Symbol	Measure	Source
Policy Rate	CMR	The policy rate is used as a measure of the call money rate (also known as the inter-bank offer rate)	IFS
Inflation	INF	Inflation as a measure of the CPI consumer price index.	IFS
GDP	GDP	The quarterly data of GDP from 1993 to 2017 is taken from the research of (Tahir et al., 2018). From 2018 to 2022 annual data is converted into quarterly data by using the technique of (Arby, 2008) and (Hanif et al., 2013). The data is later used for the calculation of the output gap.	SBP
Output gap	Y-Gap	The output gap is measured by taking the differential among logarithmic real and potential GDP, where potential GDP is estimated through the H-P filter and quadratic trend method.	

4.3 Discussion About the Variables

4.3.1 Policy Rate

The policy rate is the interest rate that the central bank charges for commercial banks for loans or deposits. The policy rate is the interest rate established by the State Bank of Pakistan to influence essential monetary indicators. The current study uses call money rate as in the literature researcher uses CMR for the estimation of reaction function and the Taylor rule. The call money rate, commonly known as the interbank offered rate, is a monetary policy instrument that is also equal to the federal funds rate in the United States. The CMR is used instead of the discount rate since a discount rate is just a policy tool for accomplishing aims that may also be achieve by other tools, such as open market operations and changes in the necessary reserve ratio. The majority of the empirical research supports this argument including Taylor (1993); Goodfriend (1993); Clarida et al. (1998; 2000); Malik (2007). The data on the call money rate has been extracted from IFS.

4.3.2 Inflation

Inflation is the rate at which the general level of prices for goods and services is rising. The inflation rate is measured as the consumer price index (CPI) growth rate. SBP uses inflation stability as the objective of monetary policy as documented by many researchers and SBP itself. Since the high-frequency inflation rate is not available so it is measured as a year-on-year increase in the CPI for Pakistan i.e.

$$\boldsymbol{\pi_t = \frac{CPI_t - CPI_{t-4}}{CPI_{t-4}} \times 100} \qquad (4.1)$$

4.3.3 Output Gap

The output gap is defined as the difference between the actual and the potential GDP and is used as a measure of the economic condition of an economy. The quarterly data on GDP is not readily available so the quarterly data of GDP from 1993 to 2017 is taken from the research of (Tahir et al., 2018). From 2018 to 2022 annual data is converted into quarterly data by using the technique of (Arby, 2008) and (Hanif et al., 2013). If the actual output is more than the potential output, it is characterized as a positive output gap then the economic boom often leads to a higher-than-average interest rate. The negative output gap occurs when the actual output is less than the potential output, reflecting a slowdown of an economy with low inflation and high unemployment. To minimize this gap central bank, adjust the policy rate accordingly.

In the literature, several studies have attempted to calculate the potential level of output and output gap through various estimation methods. Some used pure statistical approaches to estimate potential output, while others used structural or theoretical approaches. These various output gap estimating approaches result in changes in cyclical components such as amplitude, gap duration, range, and auto correlation. In the statistical approach, potential output is estimated by using economic theory, the most often used structural techniques are Structural Vector Auto Regressive (SVAR) and the production function, whereas the statistical approach estimates potential output using statistical processes. The linear trend technique, quadratic trend method, Hodrick-Prescott (HP) filter (Hodrick & Prescott, 1997), BN filter, Baxter and King filter, and Band-pass (BP) filter are some of the most well-known statistical procedures.

4.3.3.1 Hodrick-Prescott

The Hodrick-Prescott (HP) filter is a commonly filter for estimating potential output. It separates GDP into two parts: potential (Y^*) and cyclical component (C) (Hodrick & Prescott, 1997).

$$Y = Y^* + C \tag{4.2}$$

Where Y represents GDP, Y^* is the sum of the squares of its difference, as determined by minimizing the following loss function

$$minL = [\sum_{t=1}^{t} C_t^2 + \lambda \sum_{t=2}^{t} (\Delta Y_t^* - \Delta Y_{t-1}^*)^2] \tag{4.3}$$

$$= [\sum_{t=1}^{t} (Y_t - Y_t^*)^2 + \lambda \sum_{t=2}^{t} \{(Y_t^* - Y_{t-1}^*) - (Y_{t-1}^* - Y_{t-2}^*)\}^2] \tag{4.4}$$

λ is the smoothing parameter that is usually set as 1600 for quarterly data. The estimation results are given in the figure.

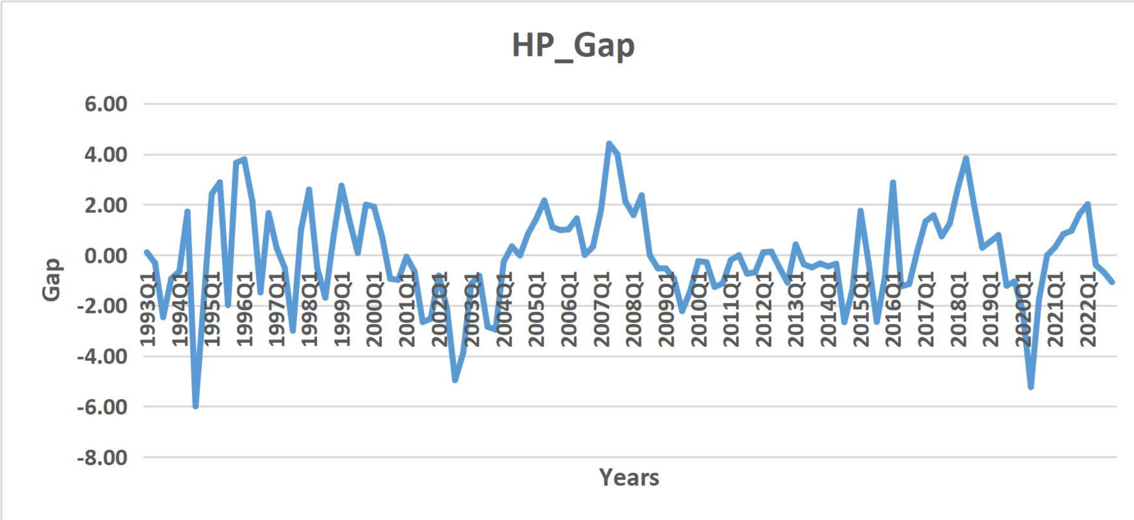


Figure 4: Output Gap through Hodrick-Prescott filter

4.3.3.2 Quadratic Trend

Another way for estimating potential output is the quadratic trend method. The advantage of this approach is that seasonal changes may be filtered using dummy variables. The cyclical component is again estimated as the difference between the actual values of the log of the output and the fitted values. In the literature for annual data, the following equation has been used (Satti & Malik, 2017).

The quadratic trend approach is another method for estimating potential output gap. This method has the benefit of allowing seasonal variations to be filtered using dummy variables. The cyclical component is evaluated once again as the difference along the actual and fitted values of the log of the output. The following equation has been used in the literature for yearly data (Satti & Malik, 2017).

$$y_t = \beta_0 + \beta_i t + \beta_2 t^2 + \varepsilon_t \quad (4.5)$$

For quarterly data, the equation for the quadratic trend approach is as follows

$$y_t = \alpha_0 + \alpha_1 t + \alpha_1 t^2 + \beta_2 D_2 + \beta_3 D_3 + \beta_4 D_4 + \varepsilon_t \quad (4.6)$$

Where t represents time and $D_2\ D_3\ and\ D_4$ are the dummy variables to adjust seasonality. The estimation results of the quadratic trend method are given in the figure below.

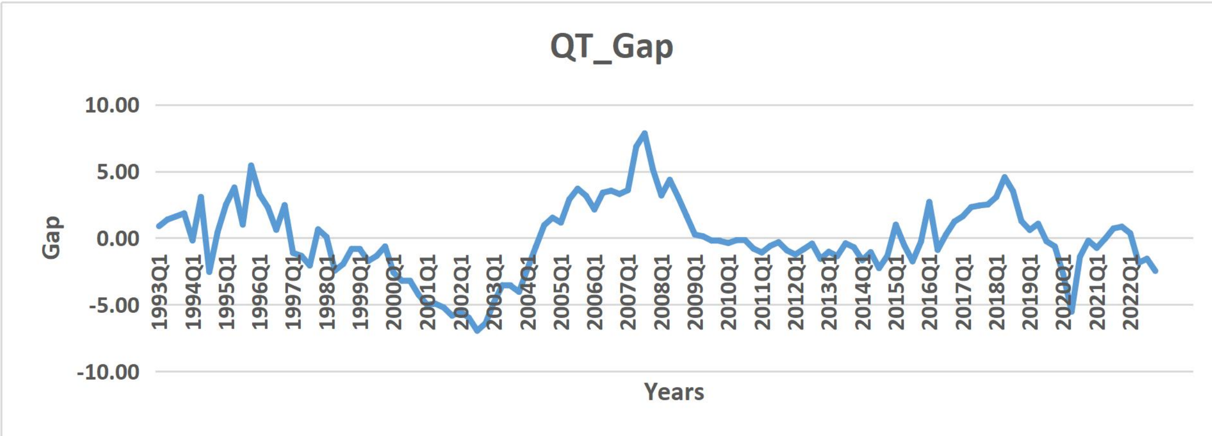


Figure 5: Output gap through Quadratic trend

The result shows that seasonality is well-adjusted by using the quadratic trend method.

Economic performance of Pakistan deteriorated during the 1990s, with annual GDP growth averaging 4.4 percent. Even though the different reforms were launched in the form of denationalization, which reduced the role of public foreign firms. However, the GDP growth rate remained low due to political uncertainty, frequent government changes, the removal of US help following the conclusion of the Afghan war, and sanctions following the nuclear test. According to the final evaluation, the economic slump began in 1992 and continued until 2002.

Following the recession of the 1990s, the recovery era began in 2002 and continued until 2007. According to the output gap calculation using the HP filter and quadratic trend approach, the economy began to improve in 2002 and peaked in 2007. The results are comparable to Pakistan's real economic situation because the average GDP growth rate stayed around 7% over this period. The recovery can be attributed to more liberal strategies for increasing Pakistan's share of global exports, privatization of the banking, telecommunications, oil, gas, and energy sectors, collaboration with coalition forces in the fight against terrorism, and increased remittances from abroad in the aftermath of the 9/11 event.

Pakistan's economic recovery, which began in 2002, did not endure long, and economic activity began to fail in 2008. The findings from the estimation of output gap show that the slowdown began in 2008 and continue through 2014. The causes of this recession include poor security, huge exogenous price shocks, and a worldwide financial downturn. The recovery era lasted from

2015 to 2018, and the economy again slowed down in 2019 due to economic reasons as it was a bad year for developing markets. Exorbitant imports and lower-than-expected inflows widened the account deficit, causing global monetary tightening, higher energy prices, and decreased investor confidence, likewise in 2020 the economy suffered from the Global Pandemic COVID-19 which is the main contribution to the negative economic growth rate.

4.4. Descriptive statistic

Table 3: Basic Statistics

		1993-1995	1996-2000	2001-2005	2006-2010	2011-2015	2016-2022
cmr	Mean	10.23	10.37	5.14	10.84	9.59	7.64
	Median	10.94	9.61	5.17	11.41	9.14	7.25
	S.D	1.93	2.87	2.67	1.82	1.99	2.10
inf	Mean	11.57	7.31	5.17	12.17	7.83	8.92
	Median	11.89	6.19	4.16	9.92	8.09	8.34
	S.D	1.58	3.28	2.83	5.53	3.48	5.78
y-gap	Mean	1.61	-0.91	-2.57	2.33	-0.92	0.38
	Median	1.50	-1.23	-3.82	2.61	-0.99	0.48
	S.D	2.03	1.99	3.57	2.52	0.67	2.16

The table shows some basic statistics (mean, median and standard deviation) over the period of 1993Q1-2022Q4. The results are summarized in table 3 and the actual behavior of variables by simple plot is given in appendix A. Regarding this descriptive analysis, there are some points to be discussed here. In the years from 1993-2000 interest rate indicating moderately tight monetary policy with the potential to control inflation and balance economic growth and with the implications for higher borrowing costs that could impact economic growth. 2001-2005 period for interest rate indicates moderate monetary policy with relatively lower borrowing costs to stimulate economic growth and investment. Again, in the years from 2006-2015 interest rate signals a tight monetary policy and in the period from 2016-2022 interest rate indicates neutral monetary policy maintaining stability without strong stimulus or restraint, which may promote economic equilibrium and manageable borrowing costs.

According to the inflation, the period 1993-1995, 2006-2010,2016-2022 signals significant inflationary pressures, likely requiring a tighter monetary policy to control rising prices and maintain economic stability, though this may impact consumer purchasing power and financial planning. Whereas in the period 1996-2000, 2011-2015 inflation suggest a need for attention from monetary policymakers, as it may indicate economic imbalances. Policy measures may be required to maintain price stability and economic well-being. In the years from 2001-2005 inflation implies a neutral monetary policy with a focus on sustaining growth and price stability.

According to the output gap, the basic statistic shows that in the period 1993-1995 output gap suggests an economy potentially operating above its potential, which might necessitate a tighter monetary policy to control inflation and maintain economic stability. While the periods 1996-2005 and 2011-2015 signals an economy operating below its potential, suggesting the need for a stimulative monetary policy to boost growth and reduce unemployment, but it might risk higher inflation. Again, in period 2006-2010 indicates an economy potentially operating above its potential, suggesting the need for a tighter monetary policy to control inflation and maintain economic stability. Furthermore, the period from 2016-2022 suggests the economy is operating close to its potential, allowing for a relatively neutral monetary policy to maintain stability without strong stimulus or restraint.

4.5 Constructing the Model

The study considers the DSGE models, for achieving the first objective the backward-looking model of (Rudebusch & Svensson, 1999a), the forward-looking model of (Lindé, 2005) model (a small empirical model) are estimated and for the second objective the historical and stochastic simulations are used. Where historical simulation is done by adding the supply and demand shocks of an economy and stochastic simulation is done by using the bootstrap technique.

4.5.1 DSGE Model

There has been phenomenal development in the formulation and estimate of dynamic stochastic general equilibrium (DSGE) models in recent years. This progress has garnered the increasing interest of central banks, emphasizing their huge potential in changing policy research and economic decision-making. DSGE models are strong analytical tools that provide a complete and coherent framework for economic policy discussions and analyses. At their core, DSGE models serve multiple critical purposes. Firstly, they excel in identifying the reasons for economic fluctuations, assisting economists and policymakers in delving deeply into the fundamental causes of economic ups and downs. These models provide insight into the causes of economic activity fluctuations by adding numerous economic shocks and disruptions.

DSGE models are also useful for projecting economic trends and evaluating the possible impact of policy changes. They let policymakers anticipate the consequences of their actions on key economic variables by simulating numerous policy scenarios, assisting in sensible and informed policy formation. These models also allow economists and policymakers to perform counterfactual experiments, allowing them to investigate "what if" possibilities. This skill is useful in determining how the economy might have performed under other conditions or policy options, offering significant insights into the decision-making process. Furthermore, DSGE models make drawing meaningful linkages between an economy's structural traits and observable, reduced-form parameters easier. This is a significant advantage, as it allows for a more robust connection between economic theory and empirical observations.

DSGE Model with a new Keynesian component for the Pakistani economy is used to explain aggregate economic phenomena such as growth, the business cycle, and fiscal and monetary policy impacts. The current study employs the small empirical backward-looking and forward-looking (Laseen & Svensson, 2009) approach to build the policy projection conditional on the policy rate path in a linearized Dynamic Stochastic General Equilibrium (DSGE) model.

4.5.2 The Rudebusch-Svensson (1999) Model: An Empirical Backward-Looking Model

The equations for this model are

$$\pi_{t+1} = \alpha_{\pi 1}\pi_t + \alpha_{\pi 2}\pi_{t-1} + \alpha_{\pi 3}\pi_{t-2} + \alpha_{\pi 4}\pi_{t-3} + \alpha_y y_t + \varepsilon_{\pi,t} \quad (4.7)$$

$$Y_{t+1} = \beta_{y1} y_t + \beta_{y2} y_{t-1} + \beta_r (i_{t-j} - \pi_{t-j}) + \varepsilon_{y,t} \quad (4.8)$$

The 4.7 equation is the aggregate supply equation or Phillips equation and the equation 4.8 is the aggregate demand equation where π_t represents inflation quarterly. i_t is the quarterly interest rate y_t is the relative gap between the actual GDP (y_t) and the potential (y^*) in percentage i.e. $\left(\frac{y_t - y^*}{y^*}\right)100$.

The loss function for the target variables are inflation, the output gap, and the policy rate is

$$L_t = \frac{1}{2}[\pi_t^2 + \alpha_y y_t^2] \quad (4.9)$$

The loss function is a combination of inflation, output gap, and interest rate where α_y and $\beta_{\Delta i}$ are the positive weights. π_t measured as the deviation from the inflation target which is equal to the stable level, y_t is the deviation from the potential output.

The reaction function for projections under the optimal policy and Taylor rule is,

$$i_t = \gamma_{\pi 1}\pi_t + \gamma_{\pi 2}\pi_{t-1} + \gamma_{\pi 3}\pi_{t-2} + \gamma_{y1}y_t + \gamma_{y2}y_{t-1} + \gamma_{i1}i_{t-1} + \gamma_{i2}i_{t-2} + \gamma_{i3}t_{t-3} \qquad (4.10)$$

$$i_t = \alpha_1\pi_t + \alpha_2 y_t \qquad (4.11)$$

where the policy rate counters the predetermined inflation and output gap with the standard coefficient of $\alpha_1$and α_2 respectively.

4.5.3 The Linde Model: Forward-Looking Model

The equations for the model of (Lindé, 2005) is

$$\pi_t = \alpha_f\pi_{t+1} + (1-\alpha_f)\pi_{t-1} + \alpha_y y_t + \varepsilon_{\pi,t} \qquad (4.12)$$

$$y_t = \beta_f y_{t+1} + (1-\beta_f)(\beta_{y1}y_{t-1} + \beta_{y2}y_{t-2} + \beta_{y3}y_{t-3} + \beta_{y4}y_{t-4} + \beta_r(i_t - \pi_{t+1}) + \varepsilon_{y,t} \qquad (4.13)$$

The equation 4.12 is the aggregate supply equation or Phillips equation and the 4.13 equation is the aggregate demand equation where π_t represents the inflation in period t. y_t represent the output gap in a current period. $\beta_r(i_{t-j} - \pi_{t-j})$ is the real interest rate, where $\varepsilon_{\pi,t}$, $\varepsilon_{y,t}$ is an i.i.d. shocks with zero mean.

The loss function for the target variables are inflation, the output gap, and the policy rate is

$$L_t = \frac{1}{2}[(\pi_t^2 + \alpha_y y_t^2] \qquad (4.14)$$

The loss function is a combination of inflation, output gap, and interest rate where α_y and $\beta_{\Delta i}$ are the positive weights.

The reaction function for projections under the optimal policy and Taylor rule,

$$i_t = \gamma_1\varepsilon_{\pi t} + \gamma_2\varepsilon_{yt} + \gamma_3\pi_{t-1} + \gamma_4 y_{t-1} + \gamma_5 i_{t-1} \qquad (4.15)$$

$$i_t = \beta_1\pi_t + \beta_2 y_t \qquad (4.16)$$

where the policy rate counters the predetermined inflation and output gap with the standard coefficient of $\beta_1$and β_2 respectively.

CHAPTER 5

RESULT AND DISCUSSION

5.1 Rudebusch-Svensson Model (Backward-Looking Model)

The model consists of the following equation

$$\pi_t = \alpha_{\pi 1}\pi_{t-1} + \alpha_{\pi 2}\pi_{t-2} + \alpha_{\pi 3}\pi_{t-3} + \alpha_{\pi 4}\pi_{t-4} + \alpha_y y_t + \varepsilon_{\pi,t+1} \quad (5.1)$$

$$Y_t = \beta_{y1} y_{t-1} + \beta_{y2} y_{t-2} + \beta_r\left(i_{t-j} - \pi_{t-j}\right) + \varepsilon_{y,t+1} \quad (5.2)$$

$$i_t = \gamma_{\pi 1}\pi_t + \gamma_{y1} y_t + \gamma_{i1} i_{t-1} + \gamma_{i2} i_{t-2} \quad (5.3)$$

The first equation is the Philips curve- the aggregate supply equation and the second is the aggregate demand equation. The final equation represents an interest rate rule that the central bank is expected to follow when determining the nominal interest rate. The model resembles the framework presented in the research conducted by (Rudebusch & Svensson, 1999a) for U.S. data. Quarterly data from 1993Q1 to 2022Q4 of Pakistan is used to analyze the link between the output gap, inflation, and interest rate within the transmission mechanism, and the model is estimated by using general regression i.e., through Ordinary Least Square (OLS). The estimated parameter results can be found in the table provided. The estimates are rounded to three decimal points.

Table 4: Result of Rudebusch-Svensson Model (Backward-Looking Model)

Coefficient	IS Equation y_t	Phillips Curve π_t	Policy Rate rule i_t
π_t	-	-	0.099 (0.008)
π_{t-1}	-	1.366 (0.00)	-
π_{t-2}	-	-0.493 (0.00)	-
y_t	-	-	0.142 (0.015)
y_{t-1}	0.705 (0.00)	0.102 (0.087)	-
y_{t-2}	0.157 (0.09)	-	-
i_{t-1}	-	-	0.563 (0.00)
i_{t-2}	-	-	0.196 (0.024)
$i_{t-j} - \pi_{t-j}$	-0.039 (0.280)	-	-
S.E	1.487	1.721	1.674
D.W	2.010	2.037	2.071

Note: In the parenthesis are the probability values.

In the aggregate demand equation, the coefficient of 0.705 suggests that there is a positive relation between current output Y_t and output in the previous period (y_{t-1}). The coefficient is statistically significant at a probability level of 5%. The coefficient of 0.157 also shows a positive relationship and the coefficient is significant at a 10% level of significance. This means that, the current level of output is influenced by its previous values. A positive association shows that higher levels of output in the past are usually followed by higher levels of output in the present. Whereas the coefficient of the real interest rate shows a negative relation with the current output and the coefficient is insignificant. The significance of the real interest rate in the IS equation can vary depending on the specific context and time period. From the estimated coefficient of the real interest rate in the equation have shown to be statistically insignificant, indicating that other factors have a stronger influence on investment decisions and there is a need for rule-based policies for better outcomes. Factors such as alternative financing options, investor expectations, business confidence, and the macroeconomic environment can all affect the relation between the real interest rate and investment expenditure. Therefore, the relationship is not always straightforward or universally consistent across different economic situations. The relation of coefficients of the AD equation are consistent with the economic theory and the standard error is 1.487 which is not extremely huge.

The Phillips curve equation represents the relation between inflation π_t and output gap Y_t. 1.366 indicates that there is a positive relationship between current inflation and lagged inflation that is higher output is associated with higher inflation. The coefficient is statistically significant at a probability level of 5%. Likewise, -0.493 suggest a negative relation between current inflation π_t and the second lag of inflation (π_{t-2}). The coefficient for (π_{t-2}) is statistically significant at probability levels of 5%. This means that, the current inflation is only influence by the first lag. The coefficient of 0.102 suggests a positive relationship between the lagged output gap and the current inflation and is statistically significant at a probability level of 10%. The coefficients of the Philips curve equation are consistent with the economic theory and the standard error is 1.721 which is not extremely huge and the value of DW is 2.037 indicating that there is no autocorrelation.

Finally, the optimal policy rule equation represents a rule that guides the central bank's decision on setting the interest rate based on inflation, output, and the lagged interest rate. The coefficient for the current inflation rate in the Policy Rate rule equation is 0.099, suggesting a positive relationship between inflation and the policy rate. The standard error is 1.674 showing that the problem of autocorrelation is not there in the equation. The estimation results of the backward-looking model are consistent with the findings of (Malik & Ahmed, 2010)

5.1.2 Loss Function

A loss function is a function that quantifies inaccuracy by measuring the difference between expected and actual values. Its purpose is to apply optimization techniques to minimize this inaccuracy, hence enhancing a model's performance or accuracy. A macroeconomic model can be used to simulate data on the output gap and inflation rate. By applying equations 1 and 2, historical series of interest rates can be constructed using the optimal policy rule and Taylor rule.

The equation of the loss function can be written as

$$L_t = \frac{1}{2}[\alpha_\pi(\pi - \pi^*)^2 + \alpha_y(y - y^*)^2] \qquad (5.4)$$

Equation (5.4) explains a quadratic loss function, the formulas are squared to count positive and negative deviations equally.

5.1.3 Simulations

A backward-looking macroeconomic model of Pakistan is used for historical simulations. The residual terms obtained from the estimation of IS and Philips curve equation are used as the demand and supply shocks of an economy.

The equations for simulation with an optimal policy rule are given as

$$\pi_t = 1.366\pi_{t-1} - 0.493\pi_{t-2} + 0.102y_{t-1} + \varepsilon_{\pi,t}$$

$$Y_t = 0.705y_{t-1} + 0.157y_{t-2} - 0.039(i_{t-1} - \pi_{t-1}) + \varepsilon_{y,t}$$

$$i_t = 0.099\pi_t + 0.142y_t + 0.563i_{t-1} + 0.196i_{t-2}$$

And the equation for simulation with the Taylor rule is given as

$$i_t = 1.5\pi_t + 0.5y_t$$

Theoretically, the primary objective of SBP is to maintain price stability through controlling inflation to target level so higher weight is given to inflation and less weight to output gap. From the literature, if the State Bank of Pakistan strictly adheres to a specific rule, the parameter values in that rule should be 1.5 for the coefficient of inflation and 0.5 for the coefficient of output gap. These coefficient values are considered significant for maintaining stability within the economic system. If the SBP deviates from these values, it may lead to instability in the economy (Malik & Ahmed, 2010). (Laseen & Svensson, 2009) also uses the same weights i.e., 1.5 for inflation and 0.5 for output gap in Taylor rule for the construction of policy rate projections. In light of the ongoing debate on inflation within the monetary policy framework of Pakistan, the current study assigns a weight of 1.5 to inflation and 0.5 to the output gap in the Taylor rule equation, reflecting a higher priority given to price stability. However, in relative term the weight of inflation is 0.75 and 0.25 is for output gap which is equal to 1.

By adding the shocks in these equations, obtained a simulated series of interest rate, inflation, and the output gap. Variances of inflation and the output gap are then calculated to estimate the loss connected with the optimal policy rule and Taylor rule. Then equal weights are allocated to inflation and the output gap in the loss function. Simulation done through this process is known as historical simulations. The effectiveness of the optimal policy rule and the Taylor rule is also addressed in stochastic simulation, which generates 1000 series of demand and supply shocks and simulates output gap and inflation rate depending on each of these series. Bootstrap add-in is used for stochastic simulations in which the observed distribution of error terms is supposed to be the actual distribution and then the shocks are created from this observed distribution. The result of the historical simulation is in the table below.

Table 5: Loss in Actual and Historical Simulation Series

	Variance of y-gap	Variance of Inflation	Loss
Actual	7.794	23.023	30.817
Optimal Policy Rule	6.202	23.881	30.083
Taylor Rule	6.124	20.859	26.983

The results of table 5 obtained from the historical simulations show that the Taylor rule performed well in the macroeconomic model for Pakistan's economy as the loss for the Taylor rule is 26.983 which is less than the optimal policy rule and the actual loss which is 30.083 and 30.817 respectively. These findings indicate that in the framework of the historical simulation of the backward-looking model, the Taylor rule performed well than the optimal policy rule. This suggests that implementing the Taylor rule led to better results in terms of minimizing losses within the macroeconomic model.

Table 6: Loss in Stochastic Simulations

	Optimal Policy Rule	Taylor Rule
Average	35.773	29.585
S.D	9.5870	7.7409
Max	78.779	63.813
Min	16.025	13.430

The table 6 indicates that the same results are found from the stochastic simulation that the Taylor rule performed well in the macroeconomic model for the Pakistan economy as the average loss for the Taylor rule is 29.585 which is less than the 35.773 found in the optimal policy rule. Overall findings from the procedure of simulation are that the Taylor rule performs well in minimizing the quadratic loss function i.e. the stabilization of inflation deviation from its target level and output gap at its potential level. In both historical and stochastic simulations, the Taylor rule is an efficient strategy for stability and enhances overall economic performance.

5.2 Linde Model (Forward-Looking Model)

The forward-looking model consists of the following equations

$$\pi_t = \alpha_1 \pi_{t+1} + (1 - \alpha_1)\pi_{t-1} + \beta_0 y_t + \varepsilon_{\pi,t} \qquad (5.5)$$

$$y_t = \beta_o y_{t+1} + (1 - \beta_o) y_{t-1} - \beta_r (i_t - \pi_{t+1}) + \varepsilon_{y,t} \qquad (5.6)$$

$$i_t = \gamma_1 \varepsilon_{\pi t} + \gamma_2 \varepsilon_{yt} + \gamma_3 \pi_{t-1} + \gamma_4 y_{t-1} + \gamma_5 i_{t-1} \qquad (5.7)$$

The equation 5.5 corresponds to the Philips curve, which represents the relationship between inflation and unemployment in the aggregate supply equation. The 5.6 equation represents the aggregate demand equation. The equation 5.7 reflects an interest rate rule that the central bank is anticipated to adhere to when deciding the nominal interest rate. The model resembles the framework presented in the research conducted by (Lindé, 2005) using U.S. data.

The study uses quarterly data from 1993Q1 to 2022Q4 of Pakistan to estimate the forward-looking model and to analyze the link between the output gap, inflation, and interest rate within the transmission mechanism, and the model is performed by Generalized Method of Moment (GMM) using instrumental variables to compute a proxy for π_{t+1} and y_{t+1} . The estimated parameter results can be found in the table below.

Table 7: Results of Linde Model (Forward-Looking Model)

Coefficient	IS Equation y_t	Phillips Curve π_t	Policy Rate rule i_t
π_{t+1}	-	0.683 (0.00)	-
π_{t-1}	-	-	0.086 (0.07)
y_t	-	0.107 (0.06)	-
y_{t+1}	0.688 (0.00)	-	-
y_{t-1}	-	-	0.229 (0.04)
i_{t-1}	-	-	0.521 (0.00)
i_{t-2}	-	-	0.187 (0.03)
$i_{t-j} - \pi_{t-j}$	-0.039 (0.27)	-	-
e_pi	-	-	0.045 (0.42)
e_is	-	-	0.254 (0.00)
S.E	1.480	1.709	1.571
D.W	2.011	1.996	1.902

Note: In the parenthesis are the probability values

The estimation of the forward-looking variables of the macroeconomic model of Pakistan shows a positive relationship with the output gap and an inverse relationship with the real interest rate. The coefficient of 0.688 indicates that an increase in the future output gap leads to an increase in the dependent variable, suggesting a positive effect on investment or aggregate demand in the next period. The coefficient of y_{t+1} is statistically significant at a level of 5%. The real interest rate shows a negative relationship with the output gap in the aggregate demand equation and is statistically insignificant indicating there is a need for rule-based policies for better outcomes. The standard error of this estimated equation is 1.480 and the value of DW is 2.01 indicates that there is no autocorrelation.

In the equation of the Philips curve, the forward variable of inflation indicates a positive relationship, it is crucial to emphasize that the coefficient of expected inflation is significantly prominent in developing countries, which challenges the assumption that individuals in developing nations exhibit forward-looking behavior when making economic decisions. Whereas the current output gap shows a positive relation that means that an increase in one unit of current inflation increases the current output gap by 0.107 units. The results are consistent with the theory as the positive sign indicates that the economy is moving toward the boom. Both the coefficients ate statistically significant at levels 5% and 10% respectively. The standard error is not so high in the Philips curve equation i.e. 1.709 and the value of DW is 2 means there is no autocorrelation in the estimated equation.

The final equation of optimal policy rate shows a positive relationship with their independent variables, even though in the estimated equation the relation of the current interest rate with the demand and supply shock is positive. The coefficient of 0.086 suggests that lag inflation has a positive impact on the current policy rate, reflecting the determination of inflationary pressures, and is statistically significant at a p-value of 0.07 (at level 10%). The coefficient for the output gap (y_{t-1}) is 0.229, indicating that there is a positive association between the variables and significance at a probability value of 0.04. The coefficient for the 1st and 2nd lag of interest rate is 0.521 and 0.187 respectively, and the coefficients are statistically significant at the 5% level. The standard error is also not so high in the policy rate equation i.e. 1.571 indicating that there is no autocorrelation. The findings of the current study are consistent with the work of (Nawaz & Ahmed, 2015)

5.2.2 Loss Function

A loss function measures the degree of inaccuracy by quantifying the difference between expected and actual values. Its primary goal is to facilitate optimization techniques aimed at reducing this inaccuracy and thus improving the performance and accuracy of a given model. This technique may be used in the context of a macroeconomic model to simulate data on the output gap and inflation rate. Using equations of aggregate demand and supply, it is feasible to create historical interest rate sequences by adopting the optimal policy rule and the Taylor rule.

$$L_t = \frac{1}{2}[\alpha_\pi(\pi - \pi^*)^2 + \alpha_y(y - y^*)^2] \qquad (5.8)$$

Equation (5.8) explains a quadratic loss function

5.2.3 Simulations

The residual generated from estimating the IS and Phillips curve equations are used as demand and supply shocks for the economy in historical simulations of Pakistan's macroeconomic model. This method enables a forward-looking analysis that takes into account the interaction of various economic factors. By incorporating these shocks, the model aims to capture the unexplained movements and fluctuations in both aggregate demand and aggregate supply. The equations for simulation with an optimal policy rule are given as

$$\pi_t = 0.683 + (1 - 0.683)\pi_{t-1} + 0.107y_t + \varepsilon_{\pi,t}$$

$$y_t = 0.688y_{t+1} + (1 - 0.688)y_{t-1} - 0.039(i_t - \pi_t) + \varepsilon_{y,t}$$

$$i_t = 0.045\varepsilon_{\pi t} + 0.254\varepsilon_{yt} + 0.086\pi_{t-1} + 0.229y_{t-1} + 0.521i_{t-1} + 0.187i_{t-2}$$

And the equation for simulation with the Taylor rule is given as

$$i_t = 1.5\pi_t + 0.5y_t$$

Table 8: Loss in Actual and Historical Simulation Series

	Variance of y-gap	Variance of Inflation	Loss
Actual	7.794	23.023	30.817
Optimal Policy Rule	6.402	23.275	29.677
Taylor Rule	7.340	23.330	30.671

The historical simulations demonstrate that the optimal policy rule yielded favorable outcomes within the macroeconomic model for Pakistan's economy. The calculated loss for the Taylor rule is 30.671, which is slightly higher than both the optimal policy rule's loss of 29.677 and the actual loss of 30.817. These findings indicate that the optimal policy rule performed more efficiently in the historical simulation of the forward-looking model. This suggests that adopting the optimal policy rule would have led to better outcomes in terms of minimizing losses within the macroeconomic framework.

Table 9: Loss in Stochastic Simulations

	Optimal Policy Rule	Taylor Rule
Average	30.293	30.864
S.D	1.6872	1.7612
Max	36.531	37.654
Min	25.831	26.325

The stochastic simulation also yielded consistent results, demonstrating that the optimal policy rule was effective within the macroeconomic model for Pakistan's economy. In this case, the average loss for the optimal policy rule is 30.293, which was lower than the loss of 30.864 observed for the Taylor rule. The overall findings from the historical and stochastic simulation procedure indicate that the optimal policy rule performs well in minimizing the quadratic loss function. This implies that it successfully contributes to stabilizing the deviation of inflation from its target level and the output gap from its potential level. By considering relevant economic variables and optimizing policy decisions, the optimal policy rule proved to be more successful in achieving these stabilization objectives of the monetary policy compared to the Taylor rule.

The hypothesis testing in case of minimizing a quadratic loss function through simulation, the study accepts the null hypothesis and reject the alternative hypothesis when the predetermined variables for estimation of macroeconomic model is used whereas the study reject the null hypothesis and accept the alternative hypothesis when forward-looking variables are used.

5.3 Projections with Time-Varying Restriction on the Policy Rate

The projection for backward-looking and forward-looking models with restricted and unrestricted nominal and real policy rate with optimal policy and the Taylor rule is considered to check which rule is efficient in the backward and forward-looking model. Both rules provide a framework used in macroeconomics to investigate how central banks formulate monetary policy and alter policy rate in response to changes in inflation and output to effectively achieve their inflation target.

Projection of policy rate and the imposition of time-varying restrictions on the policy rate projection considers two types of restriction i.e. the nominal and real policy rate restriction. The nominal policy rate restriction projection follows a predetermined path and is assumed to be anticipated by both the central banks and the private sector of an economy. Whereas under the restriction on real policy rate projection is adjusted for expected inflation and is constrained to follow a predetermined path, real policy rate restrictions are also anticipated by the central bank and the private sector. To apply these restrictions, a stochastic deviation is introduced into the policy rule. The forecast of future deviations is chosen to meet the restrictions identified; the deviation follows a moving-average process. The central bank achieves equilibrium by announcing the policy rate path to the private sector, which integrates this forecast into their expectations. In the case of real policy rate restrictions, the central bank calculates the nominal policy rate projection consistent with the desired real policy rate path and announces it to the private sector.

Predetermine and forward-looking variables of inflation, output gap, and interest rate are used for projection in backward-looking and forward-looking models. Predetermine variables are the model's exogenous inputs that are considered to be known at the time of policy decision. These variables are a common approach in DSGE models, which are engaged in macroeconomic analysis and policy assessment. The model may represent the reality that agents must make decisions based on incomplete knowledge and that they may be vulnerable to exogenous shocks that are beyond their control by assuming that certain variables are predetermined. Whereas, the forward-looking variables are affected by the central bank's policy rate decision. On this basis, they reflect the economy's response to the policy rate, and the central bank must consider the responses of an economy while determining the policy rate.

The study constructed the projections by using the coefficients of backward and forward-looking model in MATLAB, the codes for constructing policy projections are attached in the appendix B.

5.3.1 Projections for Rudebusch-Svensson (Backward-Looking) Model

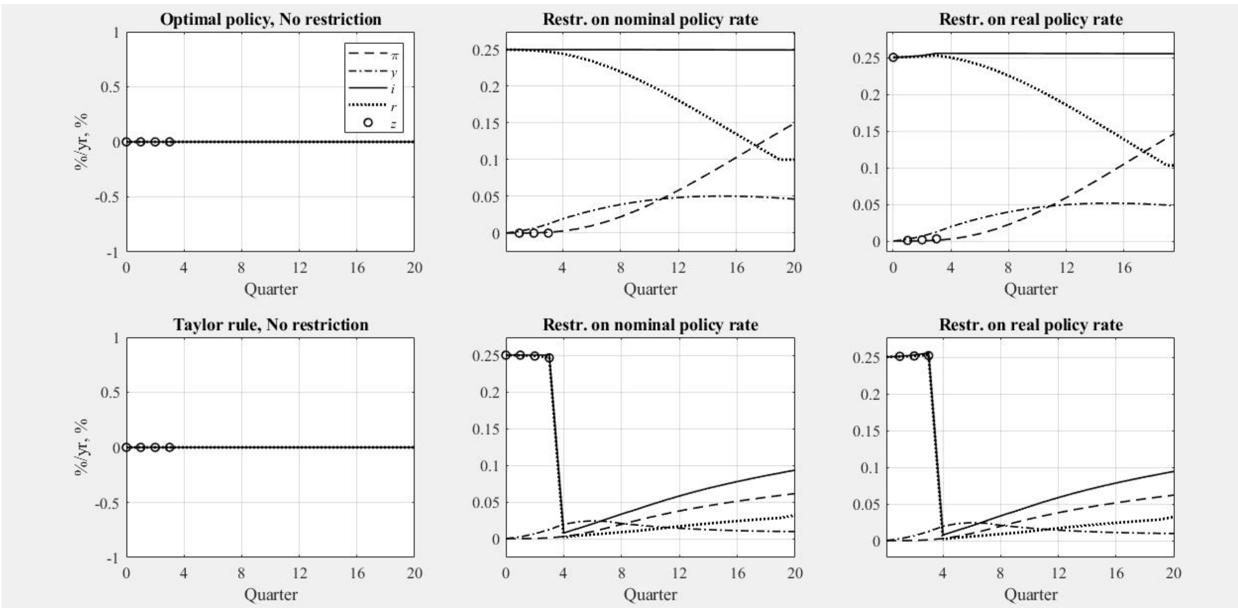


Figure 6: Projection for the Rudebusch-Svensson (Backward-Looking) Model for Optimal policy and the Taylor Rule.

Figure 6 shows the projection for the Rudebusch-Svensson model with unrestricted and restricted nominal and real policy rates for optimal policy and the Taylor rule. The top row shows the projection with restricted and unrestricted model nominal and real policy rates for optimal policy. The bottom row shows the projections with unrestricted and restricted model nominal and real policy rates for the Taylor rule.

The projections begin in 0 quarter from the steady state, supposing all the predetermined variables are set to zero. The projections in the left column of panels are provided without any restriction on the nominal or real policy rate path. This indicates that for the first four quarters (quarters 0-3), the optimal instrument rule and the Taylor rule predict zero projected deviations. The economy remains in a steady state over this time, with inflation (shown by a dashed curve), output gap (represented by a dashed-dotted curve), nominal policy rate (represented by a solid curve), and real policy rate (represented by a dotted curve) all remaining at zero.

The projection in the middle column of the figure shows nominal policy rate is restricted to 25 basis points both for optimal policy and the Taylor rule. Quarter 1 shows the projection of optimal policy with no restriction. The upward shift of the nominal policy rate path in the first 3 quarters is projected that inflation is reduced and also reduce the output gap. The shift of the real policy rate path is also exactly the same as for the nominal policy rate. So, the real and nominal policy rates have the same effect on inflation and output gap. Quarter 4 shows no restriction for the Taylor rule. The upward shift of the nominal policy rate path in the last 3 quarters is projected that inflation is reduced and it also reduces the output gap. The shift of the real policy rate path is also exactly the same as for the nominal policy rate. However, the optimal policy rate path shows more variation than the Taylor rule. The optimal policy path has more variation to bring the negative inflation and output gap back to zero so Taylor's rule is more appropriate and efficient than the optimal policy. The next step is to restrict the real policy rate equal to one this is shown in the right column of the figure. This restriction again gives similar results. Again, with this restriction, the Taylor rule is more efficient than the optimal policy.

Since there is no forward-looking variable in the Rudebusch-Svensson model there would be no difference between these projections with the change in restriction on the policy rate path of both real and nominal policy rates.

5.3.2 Projections for Linde (Forward-Looking) Model

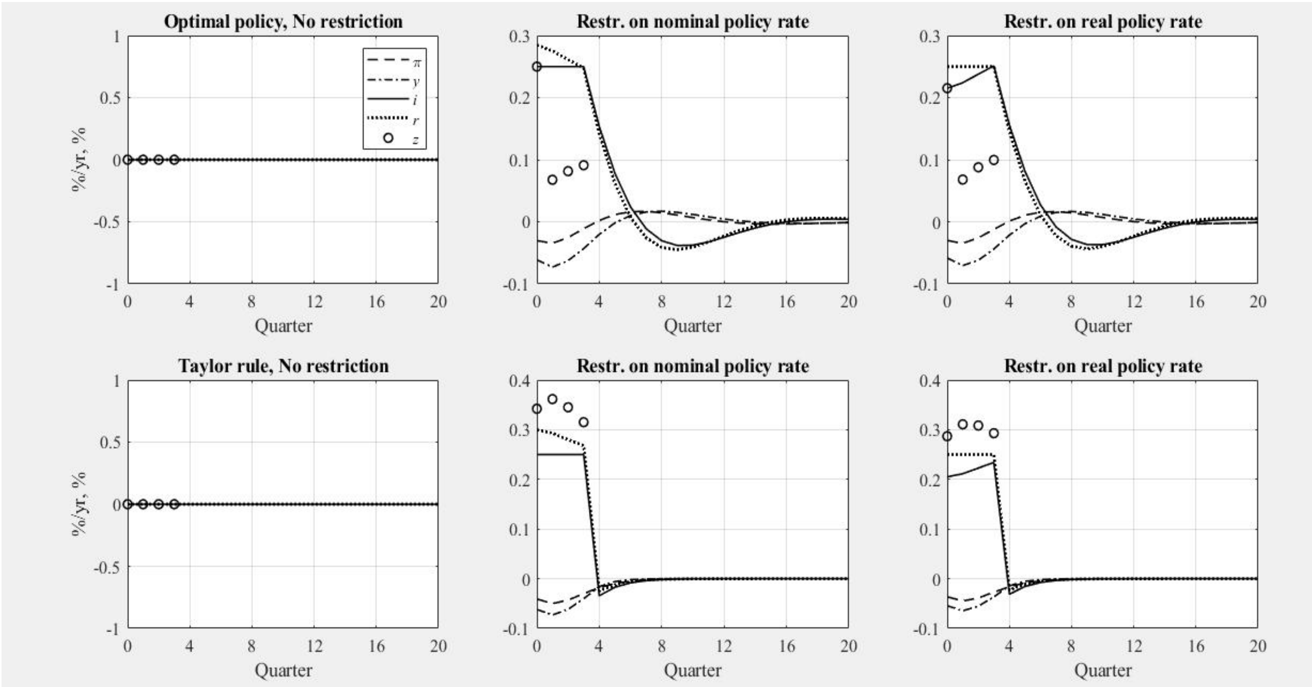


Figure 7: Projection for the Linde (Forward-Looking) Model for Optimal Policy and the Taylor Rule

Figure 7 shows the projection for the Linde model with unrestricted and restricted nominal and real policy rates for optimal policy and the Taylor rule. The top row shows the projection with unrestricted and restricted model nominal and real policy rates for optimal policy. The bottom row shows the projections with unrestricted and restricted model nominal and real policy rates for the Taylor rule.

The projections in the left column of panels are provided without any restriction on the nominal or real policy rate path. The economy remains in a steady state over this time, with inflation, output gap, nominal policy rate, and real policy rate all remaining at zero. The projection in the middle column of the figure shows nominal policy rate is restricted to 25 basis points both for optimal policy and the Taylor rule. Quarter 1 shows the projection of optimal policy with no restriction. The upward shift of the nominal policy rate path in the first 3 quarters is projected that inflation is reduced and also reduce the output gap. The shift of the real policy rate path is less than as for the nominal policy rate. Quarter 4 shows no restriction for the Taylor rule. The upward shift of the nominal policy rate path in the last 3 quarters is projected that inflation is reduced and it also reduces the output gap. The shift of the real policy rate path is also exactly the same as for the nominal policy rate. Again, here the real policy rate shifts less than the nominal policy rate. However, the optimal policy rate path shows more variation than the Taylor rule. The optimal policy path has more variation to bring the negative inflation and output gap back to zero so Taylor's rule is more appropriate and efficient than the optimal policy. In the Linde model, the magnitude of projection is higher than in the Rudebusch-Svensson. The next step is to restrict the real policy rate equal to one this is shown in the right column of the figure. This restriction again gives similar results. Again, with this restriction, the Taylor rule is more efficient than the optimal policy.

In backward-looking model and the forward-looking model, the Taylor rule is efficient and easily minimizes the deviation between the inflation from its target level and the output gap from its potential level. According to the testable hypothesis the study accepts the null hypothesis and reject the alternative hypothesis in both backward- and forward-looking model where the predetermine and forward-looking variables are used for the estimation of macroeconomic model of Pakistan.

CHAPTER 6

QUALITATIVE RESEARCH

This chapter is based on the qualitative section. For this, a questionnaire is prepared and an interview is conducted with monetary policy experts of the National Institute of Banking and Finance (NIBAF), Ministry of Finance, and a telephone interview is conducted with monetary policy experts of State Bank of Pakistan (SBP). The questionnaire is designed in such a way first to discuss the topic in general and then the findings of the study are also discussed to check the reliability of research.

6.1 Objective of Monetary Policy

The primary objective of monetary policy is price stability. The State Bank of Pakistan is responsible for regulating the monetary policy of Pakistan. I asked the respondents of the State Bank through telephonic interviews about the objective of monetary policy. One respondent of State Bank answered that price stability is the primary objective of the State Bank of Pakistan whereas financial stability and support for economic growth are the second and third objectives. The objectives of monetary policy are also interlinked as when there is an increase in employment it leads to an increase in the economic growth of an economy. Just like that exchange rate stability and interest rate smoothing are linked and price stability is also linked with exchange rate stability. However, all the respondents address the objectives of monetary policy including economic growth, price stability, financial markets, exchange rate stability, employment and interest rate smoothing.

6.2 Significance of Projection

The projections play a vibrant role in economic decision-making and policy formulation. Here are some reasons for the importance of projections addressed by respondents. One respondent answered that projections provide policymakers with essential guidance for setting the instruments of monetary, fiscal and other policy instruments to achieve macroeconomic objectives such as price stability, full utilization of economic resources and generating sources for employment. Similarly, one respondent answered that projections serve as a basis for developing robust economic plans and strategies. In the context of business and investors, projections reduce uncertainty and boost transparency providing confidence among businesses, investors and financial markets. However, the common response given by all the respondents is that they considered projections to serve as a critical tool for decision-making, policy formulation, risk assessment, and economic planning. Reliable projections contribute to stable economic growth, attract investments, and promote a favorable business environment.

6.3 Essential Factors for Projection

There are many factors that are essential while doing projection. Respondents highlight the factors that should be considered when determining the projected policy rate path such as the macroeconomic indicators that should consider the recent trend and forecast inflation, growth, employment and the output gap. However, one respondent explains in detail that during projections, the policy rate should align with the monetary policy objective and also consider the credit availability, liquidity condition and market expectations. The other factors highlighted by the respondents are the quality of data as projections heavily rely on accurate and reliable data. High-quality data that is comprehensive, timely, and consistent allows for robust analysis and modeling.

6.4 Recent Monetary Policies of Pakistan

The specific monetary policy actions and measures employed by central banks, including the State Bank of Pakistan, depending on the economic conditions, objectives, and challenges faced by the country at any given time. These policies are typically determined by a monetary policy committee or board, which assesses economic data, conducts analysis and makes decisions in line with the central bank's mandate. While discussion all the respondents are asked about the recent monetary policies of Pakistan issued by the State Bank. The policies highlighted by the respondents are the National Financial Inclusion Strategy, Financial Stability Report, State of Economy, Minutes of Monetary Policy Committee etc. However, they give more emphasis on the National Financial Inclusion Policy and Minutes of the Monetary Policy Committee. These two policies are also reviewed in Chapter 2 of this study.

6.5 Weight in Taylor Rule

Some studies use equal weight to the Taylor rule (e.g. inflation and output). In this study, we give higher weight to inflation and lesser weight to output. To address this and to get the opinion of monetary policy experts on this we asked them "In the Taylor rule why do researchers give higher weight to inflation and less weight to the output gap?" In response to this question, some experts answered that there should be equal weight assigned to both inflation and output. However, one respondent from SBP answered that in today's scenario of Pakistan, it is preferable to give higher weight to inflation and low weight to output. This is also collinear with this study. The reason they highlight to give higher weight to inflation is that Pakistan is suffering from an issue of high inflation. The current debate in documents of monetary policy issued by the State Bank is about inflation. So, it will be more appropriate to give high weight to inflation and less weight to output while calculating the Taylor rule.

6.6 Taylor Rule for Policy Formulation

The next question asked to the respondent is about the use of the Taylor rule while making policy. In response to this, they highlighted the process for policy formulation followed by SBP. In the formulation of monetary policy, the State Bank of Pakistan (SBP) follows a process where central bankers analyze the economic situation and provide their analysis based on their subjective judgments. This analysis is then presented to the Monetary Policy Committee (MPC) of Pakistan. The MPC, consisting of ten members including the chairman, board members, senior executives of the SBP, and external economists, is responsible for making decisions on the monetary policy regime. The functions and powers of the MPC are defined in Section 9E of the SBP Act 1956. The committee formulates and recommends monetary policy actions, approves and issues monetary policy statements, and implements other relevant measures. The decision-making process involves a collaborative effort between the Central Bank officials and external experts who contribute their expertise to ensure a well-informed and balanced monetary policy decision.

6.7 Relationship of Inflation and Output Gap

As the main objective of the interviews is to discuss the result of this study with the policy experts. In order to achieve this objective, I also discussed the findings with the respondents, and they debated the link between inflation and the output gap. Some respondents agree that the relation between inflation and the output gap is negative. Similarly, some respondents, citing economic theory, agree that there is a positive relation between inflation and the output gap. However, several respondents stated that the link between inflation and the output gap might be both positive and negative. They highlighted that it depends on the economic condition and some external factors of the specific country.

6.8 Conclusion

The main conclusion drawn after all the discussion is first to address the objective of monetary policy. The main objectives of monetary policy are economic growth, price stability, financial markets, exchange rate stability, employment and interest rate smoothing. The respondents also highlighted the importance of projection. As the projections play a vibrant role in economic decision-making and policy formulation. The respondent suggests that in the case of Pakistan, more weight should be assigned to inflation in the Taylor rule. They also give a positive response on the current finding of the study.

CHAPTER 7

SUMMARY AND CONCLUSION

It has been discussed in the literature that rules are preferable over discretion. Rule-based monetary policy can perform better than discretionary policy. Empirical research in the Pakistan context additionally indicates that rule-based monetary policy may enhance Pakistan's macroeconomic performance, and several Pakistani economists propose monetary policy rules to the State Bank of Pakistan to implement monetary policy. In the study both backward and forward-looking model has been estimated, where the backward-looking macroeconomic model is estimated through OLS, and the forward-looking macroeconomic model is estimated through GMM using the instrumental variable for forward-looking variables.

Broadly the study mainly comprises of two objectives for price stability and sustainable economic growth through flexible inflation targeting i.e., minimizing the quadratic loss function over the period 1993Q1 to 2022Q3. The first objective is to estimate a macroeconomic model including the predetermined and the forward-looking variables and to conduct policy projections with restricted and unrestricted nominal and real policy rate for the Taylor rule and the optimal policy rule. For the estimation of predetermine and forward-looking variables, the study uses the Rudebusch-Svensson model (backward-looking model) and Linde model (forward-looking model). For the projections, the study utilized the coefficients derived from the estimation of the backward and forward-looking macroeconomic models. These coefficients were applied to forecast the behavior of the variables included in the models, allowing for the projection of future outcomes based on the estimated relationships observed in the historical data. After analyzing two different rules the findings include that the Taylor rule is efficient in both backward and forward-looking models as there is less deviation to bring the negative inflation and output gap back to zero and in order to minimizes the deviation between the inflation from its target level and the output gap from its potential level. The equilibrium depends on the policy rule for defined restrictions on the policy rate path.

The second objective is to simulate the loss function by comparing the performance of the optimal policy rule and the Taylor rule by keeping in view the dual mandate of minimizing the gap between the actual and the targeted inflation and the deviation between the actual and potential output. For this purpose, the study conducts historical and stochastic simulations to check which rule is efficient in minimizing the deviation between the two. Historical simulation is done by adding the supply and demand shocks to the economy and stochastic simulation by using the bootstrap technique which generates 1000 series of demand and supply shocks and simulates output gap and inflation rate depending on each of these series. On the basis of simulations, it is found that the Taylor rule performs well in the historical and stochastic simulations of the backward-looking model. On the other hand, the optimal policy rule demonstrates efficiency in minimizing quadratic loss function in both historical and stochastic simulations of the forward-looking model. These results provide valuable understandings for policymakers in formulating effective monetary policy strategies to ensure long-term price stability and economic growth.

7.1 Policy Recommendation

Based on the findings of this study, the following policy recommendations can be made to promote price stability and sustainable economic growth through the stabilization of inflation and output. SBP should use rule-based policies rather than discretionary policies. This rule-based approach provides a systematic framework that helps central banks to make informed and data-driven decisions. Policymakers should consider adopting the Taylor rule as a guideline for formulating and implementing monetary policy, as the Taylor rule has demonstrated efficiency in minimizing deviations and bringing inflation and output gaps back to their targets. Emphasizing forward-looking elements in decision-making, such as inflation forecasts and economic expectations, is crucial to minimize the loss generated from the expected quadratic loss function. It allows policymakers to act swiftly and decisively to counter potential economic challenges. By implementing these policy recommendations, policymakers can create a conducive environment for price stability and sustainable economic growth in Pakistan.

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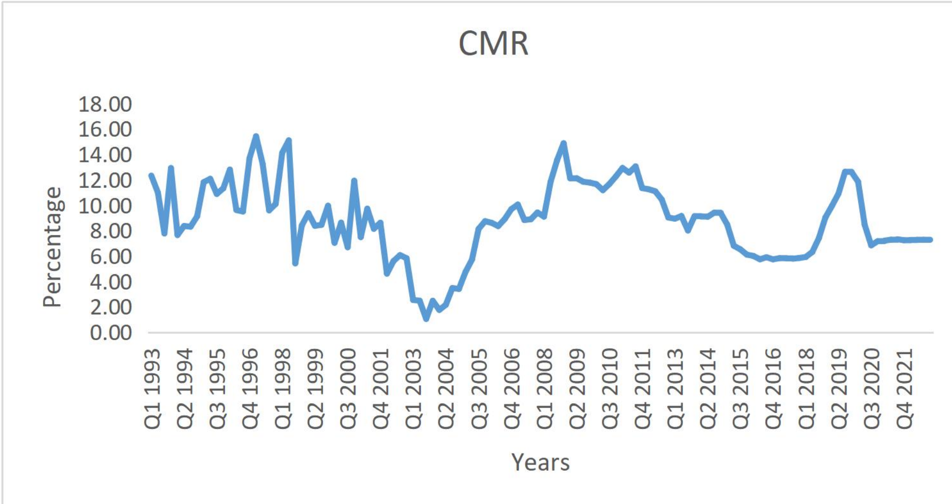
APPENDIX A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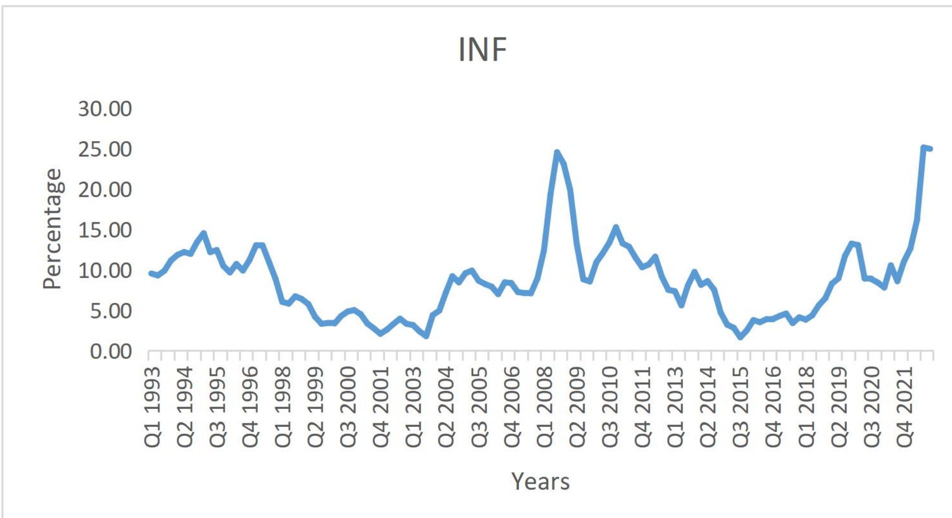
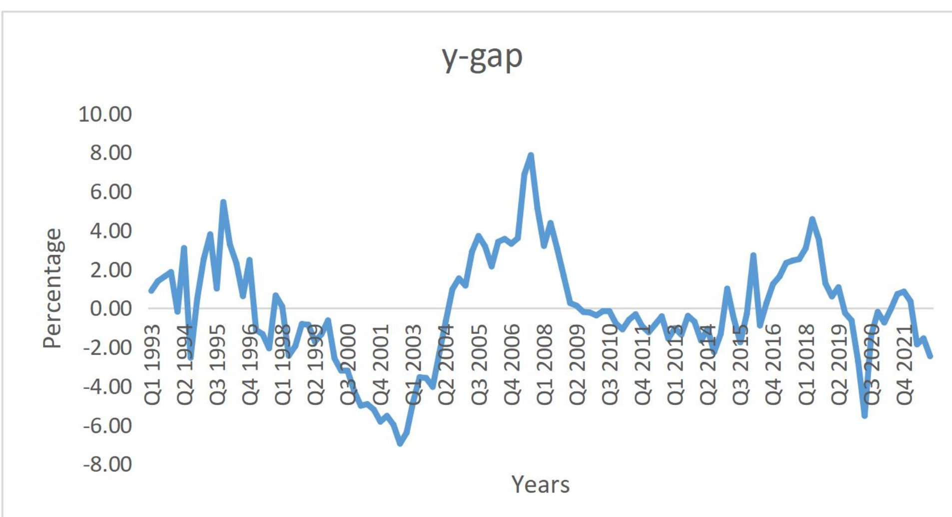
10.00
8.00
6.00
4.00
2.00
0.00
Percentage
Q1 1993
Q2 1994
Q3 1995
Q4 1996
Q1 1998
Q2 1999
Q3 2000
Q4 2001
Q1 2003
Q2 2004
Q3 2005
Q4 2006
Q1 2008
Q2 2009
Q3 2010
Q4 2011
Q1 2013
Q2 2014
Q3 2015
Q4 2016
Q1 2018
Q2 2019
Q3 2020
Q4 2021
Years


INF
30.00
25.00
20.00
15.00
10.00
5.00
0.00
Percentage
Q1 1993
Q2 1994
Q3 1995
Q4 1996
Q1 1998
Q2 1999
Q3 2000
Q4 2001
Q1 2003
Q2 2004
Q3 2005
Q4 2006
Q1 2008
Q2 2009
Q3 2010
Q4 2011
Q1 2013
Q2 2014
Q3 2015
Q4 2016
Q1 2018
Q2 2019
Q3 2020
Q4 2021
Years


y-gap
10.00
8.00
6.00
4.00
2.00
0.00
-2.00
-4.00
-6.00
-8.00
Percentage
Q1 1993
Q2 1994
Q3 1995
Q4 1996
Q1 1998
Q2 1999
Q3 2000
Q4 2001
Q1 2003
Q2 2004
Q3 2005
Q4 2006
Q1 2008
Q2 2009
Q3 2010
Q4 2011
Q1 2013
Q2 2014
Q3 2015
Q4 2016
Q1 2018
Q2 2019
Q3 2020
Q4 2021
Years

APPENDIX B

```
% Plot parameters
T = 20      ;  % Horizon for z_tau, zbar_tau and plots
BlackWhitePlot = 1; %if 1, black and white lines in plots
FixScale = 0; % if = 1, apply ymin, ymax
ymin = -2;
%ymin = -0.3;
ymax = 2.5;
%ymax = 0.3;
%ymax = -ymin;
GridOn = 1; % 1 = grid in plots
BoxOff = 0; % 0 = box off in plots
SupTitle = 0; % 1 = suptitle in figure
tauv = [0:1:T]'; %time index for plots from period 0 to T
% Choose a model RS and Lindé model described in Appendix to "Monetary Policy
% with Judgement: Forecast Targeting" ForcTargAppendix.pdf
Model = 1;  % 0 = RS model
            % 1 = Lindé model
% Set initial predetermined variables
% X_0 = [epspi_0;epsy_0;pi_-1;y_-1;i_-1]
% epspi_0 is a shock in the Phillips curve, epsi_y0 a shock in the
% aggregate demand curve (zpi_0 and zy_0 in Svensson (2005)
if (Model~=0) & (Model~=1)
   warning('Incorrect choice of Model - You have to choose either 0 (RS) or 1 (Lindé)')
   return
end
if Model == 1
   X0  = [ 0;0;0;0;0];
   disp('AntIRPathsRSL, Linde model, Optimal policy and Taylor rule');
elseif Model == 0
```

```
  X0  = [ 0   ; 0  ; 0    ; 0   ; 0 ; 0 ; 0 ; 0 ; 0];
  disp('AntIRPathsRSL, Rudebusch-Svensson Model, Optimal policy and Taylor rule');
end
% Set time-varying restriction on levels nominal and real instrument-rates levels
tau1 = 0; % tau1>=0 is first period for binding restriction on i_t+tau|t or r_t+tau|t
ibar = [0.25;0.25;0.25;0.25];
rbar = ibar;                              % levels.
% For simplicity, restrict lenght of ibar and rbar to be the same.
% Stop if not fulfilled.
if size(ibar,1)~=size(rbar,1)
  display('Warning: ibar and rbar have different lenghts!')
  return
end
Ti = size(ibar,1)-1;
%tau2 = tau1+size(ibar,1)-1;
tau2 = tau1+Ti;
% The Ti+1-vector of corresponding (additional) intercepts in the instrument rule,
% z_t+tau|t for tau = tau1,...,tau2, will be determined endogenously to make the
% instrument rate satisfy i_t+tau|t = ibar_t+tau|t or r_t+tau+t
% for tau = tau1,...,tau2.
% If tau1>0, the intercepts z_t+tau|t for tau = 0,1,...,tau1-1 are set to
% zero
% Set z0 directly (option)
 Setz = 0; % 1 = set z0 directlyf
 %z0 = [1;1;1;1;1;1;1;1;1];
 z0 = [0;0;0;0;0;1];
 %z0 = ibar;
 if Setz;
  Ti = size(z0,1)-1;
  %tau2 = tau1+size(z0,1)-1;
```

```
    tau2 = tau1+Ti;
  end
% z-vector for no restriction
z0NR = zeros(Ti+1,1);
z0NR = [zeros(tau1,1);z0NR];
% % Set z0 directly
% Setz = 0; % Set z0 directly
% z0d = [1;1;1;1;1;1;1;1;1;1];
% z0d = ibar;
% if Setz;
%     Ti = size(z0d,1)-1;
%     %tau2 = tau1+size(ibar,1)-1;
%     tau2 = tau1+Ti;
% end
% Set coefficients for simple instrument rule (implicit Taylor rule)
% i_t = f_xpi*pi_t + f_xy*y_t
f_xpi = 1.5;
f_xy = 0.5;
% Loss function parameters, for optimal policy
del = 1;
lamy = 0.5;
lami = 0;   % weight on i_t^2
lamDi = 0.3; % weight on (i_t-i_t-1^2
% Target variables
% Y_t = [ pi_t; y_t; i_t; i_t - i_t-1]
%
% Period loss function is
% L_tau = (1/2)*( pi_tau^2 + lamy*y_tau^2 + lami*i_tau^2 + lamDi*(i_tau-i_tau-1)^2 )
%
% Intertemporal loss function
```

```
% Sum{ E_0 del^tau * L_tau, tau = 0:Inf } for 0<del<=1
% Predetermined variables
% X_t = [epspi_t;epsy_t;pi_t-1;y_t-1;i_t-1]
% epspi_t and epsy_t are here zero-mean i.i.d. shocks to inflation and the
% output gap, respectively (in judgment terms zpi_t and zy_t in Svensson
% (2005)
%
% Forward-looking variables
% x_t = [pi_t;y_t]
%
% Instrument i_t
% Model
% [X_t+1;H*x_t+1|t] = A*[X_t;x_t] + B*i_t + [eps_t+1;0]
% Y_t = D*[X_t;x_t;i_t]
% L_t = (1/2)* Y_t'*W*Y_t = (1/2)* [X_t' x_t' i_t]*Wtil*[X_t;x_t;i_t]
% WW = D'*W*D
% W = Diag(Lamv)
Lamv = [ 1; lamy; lami; lamDi];
% Instrument rule, time-varying intercepts
% G_x*x_t+1|t + G_i*i_t+1|t = f_X*X_t + f_x*x_t + f_i*i_t + z_t
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% LINDÉ MODEL
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
if Model == 1 % Lindé
   % Matrices
   % Dimensions
   nX=5; nx=2; ni=1; nY=4;
   n=nX+nx;
```

```
A = zeros(n,n);  B = zeros(n,ni); H = zeros(nx,nx);
D = zeros(nY,n+ni);
Sigepsepsm = zeros(nX,nX); %Covariance of iid shocks to X_t+1
% Substitute parameters from Linde (2002)
omef=0.683;
%    if SimpleNKmodel, omef=1; end; %simple NK model
gam=0.107;
betf=0.688;
%    if SimpleNKmodel, betf=1; end; %simple NK model
betr=0.039;
A(3,6)=1;
A(4,7)=1;
A(6,1)=-1;A(6,3)=-(1-omef); A(6,6)=1; A(6,7)=-gam;
A(7,2)=-1;A(7,4)=-(1-betf); A(7,7)=1;
%display(A);
A_z = zeros(tau1+1+Ti,tau1+1+Ti);
A_z(1:tau1+Ti,2:tau1+Ti+1) = eye(tau1+Ti);
B(5,1)=1; B(7,1)=betr;
%display(B);
A11 = A(1:nX,1:nX);       A12 = A(1:nX,nX+1:nX+nx);       B1 = B(1:nX,:);
A21 = A(nX+1:nX+nx,1:nX); A22 = A(nX+1:nX+nx,nX+1:nX+nx); B2 = B(nX+1:nX+nx,:);
H(1,1)=omef;
H(2,1)=betr; H(2,2)=betf;
%     if SimpleNKmodel, H(1,1)= del*H(1,1); %standard simple NK model
%        cc = SimpleNK(gam,lamy,del); %root for simple NK model
%     end;
%display(H);
D = zeros(nY,nX+nx+ni);
D(1,6)=1;
D(2,7)=1;
```

```
D(3,8)=1;
    D(4,5)=-1; D(4,8)=1;
  %display(D);
  W = diag(Lamv);
  WW = D'*W*D;
  %Sigepsepsm(1,1)=sig2epspi; Sigepsepsm(2,2)=sig2epsy;
  %Sigepsepsm(1,2)=sigepspiy; Sigepsepsm(2,1)=Sigepsepsm(1,2);
  % Instrument rule (implicit Taylor rule)
  G_x=zeros(ni,nx); G_i=zeros(ni,ni);
  f_X=zeros(ni,nX); f_x=[f_xpi f_xy]; f_i=-1;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% RUDEBUSCH-SVENSSON MODEL
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
elseif Model == 0  % RS
  nX = 9; nx = 0; ni= 1; nY=4;
  A = zeros(nX,nX);  B = zeros(nX,ni); D = zeros(nY,nX+ni);
  Sigepsepsm = zeros(nX,nX); %Covariance of iid shocks to X_tau+1
  %   Substitute parameters
  [alfv,betv,sig2epspi,sig2epsy,sigepspiy] = RSParms; % Get parameters
  A(1,1:5)=alfv';
  A(2,1)=1; A(3,2)=1; A(4,3)=1;
  A(5,1:4)=betv(3)*ones(1,4)/4; A(5,5:6)=betv(1:2)';
  A(5,7:9)=-betv(3)*ones(1,3)/4;
  A(6,5)=1;
  A(8,7)=1;
  A(9,8)=1;
  B(5,1)=-betv(3)/4; B(7,1)=1;
  D = zeros(nY,nX+ni);
```

```
    D(1,1)=1;
    D(2,5)=1;
    D(3,10)=1;
    D(4,7)=-1; D(4,10)=1;
    H = zeros(1,1);
    W = diag(Lamv);
    WW = D'*W*D;
    % Instrument rule (implicit Taylor rule)
    G_x=zeros(ni,nx); G_i=zeros(ni,ni);
    f_X=zeros(ni,nX); f_i=-1;
    f_X([2 6])= [f_xpi f_xy];
    A11 = A(1:nX,1:nX);
    A12 = [];%A(1:nX,nX+1:nX+nx);
    B1 = B(1:nX,:);
    A21 = [];%A(nX+1:nX+nx,1:nX);
    A22 = [];%A(nX+1:nX+nx,nX+1:nX+nx);
    B2 = [];%B(nX+1:nX+nx,:);
end
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Solutions with time-varying intercepts in policy rule
% Create Htil and Atil for the different policy rules with time-
% varying intercepts
% Cases
% a: Taylor rule, restrictions on nominal and real instrument rate
% b: Taylor rule, no restriction
% c: Optimal policy, restriction on nominal and real instrument rate
% d: Optimal policy, no restriction
% a:
% Htila and Atila is for the instrument-rule
```

```
% G_x*x_t+1|t + G_i*i_t+1|t = f_X*X_t + f_x*x_t + f_i*i_t + z_t
% with time-varying intercepts z_t+tau|t.
% The predetermined variables are [X_t;z^t] (nX+tau1+1+Ti).
% The non-predetermined variables are xtil_t = [pi_t;y_t;i_t] (nx+ni).
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
if Model == 1 % Lindé
   Htila = [H   zeros(nx,ni);
      G_x G_i         ];
   Atila = [A11     zeros(nX,tau1+1+Ti) A12          B1 ;
      zeros(tau1+Ti,nX+1)  eye(tau1+Ti) zeros(tau1+Ti,nx+ni) ;
      zeros(1,nX+tau1+1+Ti+nx+ni)                   ;
      A21      zeros(nx,tau1+1+Ti) A22          B2 ;
      f_X      1 zeros(ni,tau1+Ti) f_x          f_i];
   m1a = nX+tau1+1+Ti; % # of predetermined variables, inluding zsupt
   cutoff = 1;
   m1a = nX+tau1+1+Ti; % # of predetermined variables, inluding zsupt
   HHtila = [eye(nX+tau1+1+Ti)         zeros(nX+tau1+1+Ti,nx+ni);
      zeros(nx+ni,nX+tau1+1+Ti) Htila               ];
%     [Fa,Ma,HHa,Ja,Ka,Na,Pa,geva] = DiffEqu3a(HHtila,Atila,m1a,cutoff,0);
   [Fa,Ma,HHa,Ja,Ka,Na,Pa,geva] = DiffEqu4(HHtila,Atila,m1a,cutoff,0);
   % b:
   % Htilb and Atilb is for the standard instrument rule,
   % G_x*x_t+1|t + G_i*i_t+1|t = f_X*X_t + f_x*x_t + f_i*i_t + z_t,
   % w/o additional intercepts
   Atilb = [A11  A12 B1 ;
      A21  A22 B2 ;
      f_X  f_x f_i];
   m1b = nX; % # of predetermined variables, inluding zsupt
   HHtilb = [eye(nX)         zeros(nX,nx+ni);
```

```
    zeros(nx+ni,nX) Htila         ];
%    [Fb,Mb,HHb,Jb,Kb,Nb,Pb,gevb] = DiffEqu3a(HHtilb,Atilb,m1b,cutoff,0);
  [Fb,Mb,HHb,Jb,Kb,Nb,Pb,gevb] = DiffEqu4(HHtilb,Atilb,m1b,cutoff,0);
   % c:
  % Htilc and Atilc is for the optimal instrument rule,
  % 0 = F_iX*X_t + F_iXi*Xi_t-1 - i_t + z_t, with intercepts z_t+tau|t, where
  % Xi_t = M_XiX*X_t + M_XiXi*Xi_t-1
  % Solve for the optimal policy to get F_iX, F_iXi, M_XiX, M_XiXi
  % Create HtilOpt and AtilOpt, for optimal policy
  %[HtilOpt,AtilOpt] = CreateCCMM( A,B,H,WW,del );
  % Use new function CreateHtilAtil
  [HtilOpt,AtilOpt] = CreateHtilAtil( A,B,H,WW,del );
  m1 = nX+nx;    % # of predetermined variables, including Xi_t-1
  m2 = nx+ni+nX; % # of nonpredetermined variables, including xi_t
  m = m1+m2;
  cutoff = 1;
  %[F,M,HH,J,K,N,P,gev] = DiffEqu2( HtilOpt,AtilOpt,m1,cutoff );
%    [F,M,HH,J,K,N,P,gev] = DiffEqu3a( HtilOpt,AtilOpt,m1,cutoff,0 );
  [F,M,HH,J,K,N,P,gev] = DiffEqu4( HtilOpt,AtilOpt,m1,cutoff,0 );
  % Now solve for the model with time-varying intercepts in the policy rule
  % The predetermined variables are [X_t;Xi_t-1;z^t] (nX+nx+tau1+1+Ti)
  % The non-predetermined variables are xtil_t = [pi_t;y_t;i_t] (nx+ni)
  Htilc = [H   zeros(nx,ni);
     G_x G_i        ];
  A11 = A(1:nX,1:nX);      A12 = A(1:nX,nX+1:nX+nx);      B1 = B(1:nX,:);
  A21 = A(nX+1:nX+nx,1:nX); A22 = A(nX+1:nX+nx,nX+1:nX+nx); B2 = B(nX+1:nX+nx,:);
  M_XiX = M(nX+1:nX+nx,1:nX); M_XiXi = M(nX+1:nX+nx,nX+1:nX+nx);
  F_iX = F(nx+1:nx+ni,1:nX); F_iXi = F(nx+1:nx+ni,nX+1:nX+nx);
  Atilc = [A11     zeros(nX,nx+tau1+1+Ti) A12          B1 ;
     M_XiX    M_XiXi    zeros(nx,tau1+1+Ti+nx+ni)    ;
```

```
      zeros(tau1+Ti,nX+nx+1)  eye(tau1+Ti) zeros(tau1+Ti,nx+ni) ;
      zeros(1,nX+nx+tau1+1+Ti+nx+ni)                  ;
      A21      zeros(nx,nx+tau1+1+Ti) A22          B2 ;
      F_iX  F_iXi    1 zeros(ni,tau1+Ti+nx)        -1];
   m1c = nX+nx+tau1+1+Ti; % # of predetermined variables, inluding zsupt
   HHtilc = [eye(nX+nx+tau1+1+Ti)         zeros(nX+nx+tau1+1+Ti,nx+ni);
      zeros(nx+ni,nX+nx+tau1+1+Ti) Htilc              ];
%     [Fc,Mc,HHc,Jc,Kc,Nc,Pc,gevc] = DiffEqu3a(HHtilc,Atilc,m1c,cutoff,0);
   [Fc,Mc,HHc,Jc,Kc,Nc,Pc,gevc] = DiffEqu4(HHtilc,Atilc,m1c,cutoff,0);
   % Check solution
   Testm = F(1:nx+ni,:)-Fc(:,1:nX+nx);
   disp(['Norm of F(1:nx+ni,:)-Fc(:,1:nX+nx): ' num2str(norm(Testm)) ' (should be zero)'])
end
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%
if Model == 0 % RS
   % a:
   % Htila and Atila is for the instrument-rule
   % G_x*x_t+1|t + G_i*i_t+1|t = f_X*X_t + f_x*x_t + f_i*i_t + z_t
   % with time-varying intercepts z_t+tau|t.
      % The predetermined variables are [X_t;z^t] (nX+tau1+1+Ti).
   % The non-predetermined variables are xtil_t = [pi_t;y_t;i_t] (nx+ni).
   % Htila = [H   zeros(nx,ni);
   %          G_x G_i         ];
   Htila = zeros(1,1);
   H=Htila;
   A11 = A(1:nX,1:nX);
   A12 = [];%A(1:nX,nX+1:nX+nx);
   B1 = B(1:nX,:);
   A21 = [];%A(nX+1:nX+nx,1:nX);
```

```
A22 = [];%A(nX+1:nX+nx,nX+1:nX+nx);
B2 = [];%B(nX+1:nX+nx,:);
% Atila = [A11     zeros(nX,tau1+1+Ti) A12          B1 ;
%          zeros(tau1+Ti,nX+1)  eye(tau1+Ti) zeros(tau1+Ti,nx+ni) ;
%          zeros(1,nX+tau1+1+Ti+nx+ni)                  ;
%          A21      zeros(nx,tau1+1+Ti) A22          B2 ;
%          f_X      1 zeros(ni,tau1+Ti) f_x          f_i];
Atila = [A11     zeros(nX,tau1+1+Ti)     B1 ;
    zeros(tau1+Ti,nX+1)  eye(tau1+Ti) zeros(tau1+Ti,nx+ni) ;
    zeros(1,nX+tau1+1+Ti+nx+ni)                  ;
    f_X      1 zeros(ni,tau1+Ti) f_i];
m1a = nX+tau1+1+Ti; % # of predetermined variables, inluding zsupt
cutoff = 1;
m1a = nX+tau1+1+Ti; % # of predetermined variables, inluding zsupt
% HHtila = [eye(nX+tau1+1+Ti)         zeros(nX+tau1+1+Ti,nx+ni);
%           zeros(nx+ni,nX+tau1+1+Ti) Htila              ];
HHtila = [eye(nX+tau1+1+Ti)         zeros(nX+tau1+1+Ti,nx+ni);
    zeros(nx+ni,nX+tau1+1+Ti) Htila              ];
%     [Fa,Ma,HHa,Ja,Ka,Na,Pa,geva] = DiffEqu3a(HHtila,Atila,m1a,cutoff,0);
[Fa,Ma,HHa,Ja,Ka,Na,Pa,geva] = DiffEqu4(HHtila,Atila,m1a,cutoff,0);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% b:
% Htilb and Atilb is for the standard instrument rule,
% G_x*x_t+1|t + G_i*i_t+1|t = f_X*X_t + f_x*x_t + f_i*i_t + z_t,
% w/o additional intercepts
% Atilb = [A11  A12 B1 ;
%          A21  A22 B2 ;
%          f_X  f_x f_i];
Atilb = [A11  B1 ;
```

```
    f_X  f_i];
  m1b = nX; % # of predetermined variables, inluding zsupt
  HHtilb = [eye(nX)         zeros(nX,nx+ni);
    zeros(nx+ni,nX) Htila         ];
%     [Fb,Mb,HHb,Jb,Kb,Nb,Pb,gevb] = DiffEqu3a(HHtilb,Atilb,m1b,cutoff,0);
  [Fb,Mb,HHb,Jb,Kb,Nb,Pb,gevb] = DiffEqu4(HHtilb,Atilb,m1b,cutoff,0);
  %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
  % c:
  % Htilc and Atilc is for the optimal instrument rule,
  % 0 = F_iX*X_t + F_iXi*Xi_t-1 - i_t + z_t, with intercepts z_t+tau|t, where
  % Xi_t = M_XiX*X_t + M_XiXi*Xi_t-1
  % Solve for the optimal policy to get F_iX, F_iXi, M_XiX, M_XiXi
  % Create HtilOpt and AtilOpt, for optimal policy
  %[HtilOpt,AtilOpt] = CreateCCMM( A,B,H,WW,del );
  % Use new function CreateHtilAtil
  [HtilOpt,AtilOpt] = CreateHtilAtil( A,B,[],WW,del );
  m1 = nX+nx;    % # of predetermined variables, including Xi_t-1
  m2 = nx;%+ni+nX; % # of nonpredetermined variables, including xi_t
  m = m1+m2;
  cutoff = 1;
  %[F,M,HH,J,K,N,P,gev] = DiffEqu2( HtilOpt,AtilOpt,m1,cutoff );
  %[F,M,HH,J,K,N,P,gev] = DiffEqu3a( HtilOpt,AtilOpt,m1,cutoff,0 );
  % Solution for backward model
  [F,M,HH,J,K,N] = ComAlgB3( A,B,WW,del,1 );
  % Now solve for the model with time-varying intercepts in the policy rule
  % The predetermined variables are [X_t;Xi_t-1;z^t] (nX+nx+tau1+1+Ti)
  % The non-predetermined variables are xtil_t = [pi_t;y_t;i_t] (nx+ni)
  %   Htilc = [H   zeros(nx,ni);
  %           G_x G_i        ];
```

```
   Htilc = [H];
    A11 = A(1:nX,1:nX);
   A12 = [];%A(1:nX,nX+1:nX+nx);
   B1 = B(1:nX,:);
   A21 = [];%A(nX+1:nX+nx,1:nX);
   A22 = [];%A(nX+1:nX+nx,nX+1:nX+nx);
   B2 = [];%B(nX+1:nX+nx,:);
   M_XiX = [];%M(nX+1:nX+nx,1:nX);
   M_XiXi = [];%M(nX+1:nX+nx,nX+1:nX+nx);
   F_iX = F(nx+1:nx+ni,1:nX);
   F_iXi = [];%F(nx+1:nx+ni,nX+1:nX+nx);
   %   Atilc = [A11      zeros(nX,nx+tau1+1+Ti) A12          B1 ;
   %           M_XiX    M_XiXi    zeros(nx,tau1+1+Ti+nx+ni)    ;
   %           zeros(tau1+Ti,nX+nx+1)  eye(tau1+Ti) zeros(tau1+Ti,nx+ni) ;
   %           zeros(1,nX+nx+tau1+1+Ti+nx+ni)                  ;
   %           A21      zeros(nx,nx+tau1+1+Ti) A22          B2 ;
   %           F_iX  F_iXi    1 zeros(ni,tau1+Ti+nx)       -1];
   Atilc = [A11      zeros(nX,nx+tau1+1+Ti) B1 ;
      zeros(tau1+Ti,nX+nx+1)  eye(tau1+Ti) zeros(tau1+Ti,nx+ni) ;
      zeros(1,nX+nx+tau1+1+Ti+nx+ni)                  ;
      F_iX    1 zeros(ni,tau1+Ti+nx)        -1];
   m1c = nX+nx+tau1+1+Ti; % # of predetermined variables, inluding zsupt
   HHtilc = [eye(nX+nx+tau1+1+Ti)         zeros(nX+nx+tau1+1+Ti,nx+ni);
      zeros(nx+ni,nX+nx+tau1+1+Ti) Htilc               ];
%    [Fc,Mc,HHc,Jc,Kc,Nc,Pc,gevc] = DiffEqu3a(HHtilc,Atilc,m1c,cutoff,0);
   [Fc,Mc,HHc,Jc,Kc,Nc,Pc,gevc] = DiffEqu4(HHtilc,Atilc,m1c,cutoff,0);
   % Check solution
   Testm = F(1:nx+ni,:)-Fc(:,1:nX+nx);
   disp(['Norm of F(1:nx+ni,:)-Fc(:,1:nX+nx): ' num2str(norm(Testm)) ' (should be zero)'])
end
```

```
if Setz
   z0ia = [zeros(tau1,1);z0];
   z0ra = z0ia;
else
   % Find time-varying intercepts z0i and z0r in instrument rule to satisfy
   % restrictions on nominal and real instrument rate, respectively
   % a:
   % Write restriction on nominal instrument-rate path as
   %   z0ia = [zeros(tau1,1);z0i];
   %   ibar = Ria * [X0;z0ia] = Ria_X*X0 + Ria_z*z0i
   %   z0i = inv(Ria_z) * (ibar - Ria_X*X0)
   %   Ria = Fa_i * [Ma^tau1;Ma^(tau1+1);...;Ma^(tau1+Ti)]
   Fa_i = Fa(nx+ni,:);
   Ria = Fa_i*Ma^tau1;
   for j=1:Ti
      Ria = [Ria;Fa_i*Ma^(tau1+j)];
   end
   Ria_X = Ria(:,1:nX);
   Ria_z = Ria(:,nX+tau1+1:nX+tau1+1+Ti); % Select columns corresponding to z0i,
                                          % not z0ia.
   z0i=inv(Ria_z)*(ibar-Ria_X*X0);
   z0ia = [zeros(tau1,1);z0i];
   %Write restriction on real instrument-rate path as
   %   z0ra = [zeros(tau1,1);z0r];
   %   rbar = Rra * [X0;z0r] = Rra_X*X0 + Rra_z*z0r
   %   z0r = inv(Rra_z) * (rbar - Rra_X*X0)
   %   Rra = (Fa_i-Fa_pi*Ma) * [Ma^tau1;Ma^(tau1+1);...;Ma^(tau1+Ti)]
   %   since r_t = i_t - pi_t+1|t
   Fa_i = Fa(nx+ni,:);
   if Model==1 % Lindé
```

```
        Fa_pi = Fa(1,:);
    end
    if Model == 0 % RS
        Fa_pi = Ma(1,:);
    end
    Rra = (Fa_i-Fa_pi*Ma)*Ma^tau1;
    for j=1:Ti
        Rra = [Rra;(Fa_i-Fa_pi*Ma)*Ma^(tau1+j)];
    end
    Rra_X = Rra(:,1:nX);
    Rra_z = Rra(:,nX+tau1+1:nX+tau1+1+Ti); % Select columns corresponding to z0i,
    % not z0ia.
    z0r=inv(Rra_z)*(rbar-Rra_X*X0);
    z0ra = [zeros(tau1,1);z0r];
end
% Find projections of pi,y,i,r for restriction on i_t and restriction on r_t
% and without restrictions
% a:
% Y_t+tau|t = D_til*Ma^tau * [X0;z0]
% Dtila = D*[eye(nX) zeros(tau1+1+Ti);
%           Fa                ];
Dtila = D*[eye(nX) zeros(nX,tau1+1+Ti);
    Fa                ];
% Restriction on i_t
Ymai = Dtila*[X0;z0ia];
for jj=1:T
    Ymai = [Ymai (Dtila*Ma^jj * [X0;z0ia])];
end
pivai = Ymai(1,:)';
pileadvai = [pivai(2:end);pivai(end)];
```

```
rvai = Ymai(3,:)'-pileadvai;
z0vi=zeros(1+T,1);
z0vi(1:tau1+Ti+1)=z0ia;
% Restriction on r_t
Ymar = Dtila*[X0;z0ra];
for jj=1:T
   Ymar = [Ymar (Dtila*Ma^jj * [X0;z0ra])];
end
pivar = Ymar(1,:)';
pileadvar = [pivar(2:end);pivar(end)];
rvar = Ymar(3,:)'-pileadvar;
z0vr=zeros(1+T,1);
z0vr(1:tau1+Ti+1)=z0ra;
% b:
% Without restrictions
Dtilb = D*[eye(nX);
   Fb    ];
Ymb = Dtilb*X0;
for jj=1:T
   Ymb = [Ymb (Dtilb*Mb^jj * X0)];
end
pivb = Ymb(1,:)';
pileadvb = [pivb(2:end);pivb(end)];
rvb = Ymb(3,:)'-pileadvb;
z0vNR=zeros(1+T,1);
z0vNR(1:tau1+Ti+1)=z0NR;
% c:
% For the optimal policy rule
%   Xtil0 = [X0;Xi_-1];
Xtil0 = [X0;zeros(nx,1)];
```

```
if Setz
   z0ic = [zeros(tau1,1);z0];
   z0rc = z0ic;
else
   % Write restriction on nominal instrument-rate path as
   %   z0ic = [zeros(tau1,1);z0i];
   %   ibar = Ric * [Xtil0;z0ic] = Ric_Xtil*Xtil0 + Ric_z*z0i
   %   z0i = inv(Ric_z) * (ibar - Ric_Xtil*Xtil0)
   %   Ric = Fc_i * [Mc^tau1;Mc^(tau1+1);...;Mc^(tau1+Ti)]
   Fc_i = Fc(nx+ni,:);
   %  Ric = Fc_i*eye(nX+nx+tau1+1+Ti);
   Ric = Fc_i*Mc^tau1;
   for j=1:Ti
      Ric = [Ric;Fc_i*Mc^(tau1+j)];
   end
   Ric_Xtil = Ric(:,1:nX+nx);
   Ric_z = Ric(:,nX+nx+tau1+1:nX+nx+tau1+1+Ti); % Select columns corresponding
   % to z0i, not z0ic.
   z0i=inv(Ric_z)*(ibar-Ric_Xtil*Xtil0);
   z0ic = [zeros(tau1,1);z0i];
   %Write restriction on real instrument-rate path as
   %   z0rc = [zeros(tau1,1);z0r];
   %   rbar = Rrc * [Xtil0;z0rc] = Rrc_Xtil*Xtil0 + Rrc_z*z0r
   %   z0rc = inv(Rrc_z) * (rbar - Rrc_Xtil*Xtil0)
   %   Rrc = (Fc_i-Fc_pi*Mc) * [Mc^tau1;Mc^(tau1+1);...;Mc^(tau1+Ti)]
   %   since r_t = i_t - pi_t+1|t
   Fc_i = Fc(nx+ni,:);
   if Model == 1 % Lindé
      Fc_pi = Fc(1,:);
   end
```

```
    if Model == 0 % RS
        Fc_pi = Mc(1,:);
    end
    Rrc = (Fc_i-Fc_pi*Mc)*Mc^tau1;
    for j=1:Ti
        Rrc = [Rrc;(Fc_i-Fc_pi*Mc)*Mc^(tau1+j)];
    end
    Rrc_Xtil = Rrc(:,1:nX+nx);
    Rrc_z = Rrc(:,nX+nx+tau1+1:nX+nx+tau1+1+Ti); % Select columns corresponding
    % to z0r, not z0rc.
    z0r=inv(Rrc_z)*(rbar-Rrc_Xtil*Xtil0);
    z0rc = [zeros(tau1,1);z0r];
end
% Find forecasts of pi,y,i,r for restriction on i_t and restriction on r_t
% and without restrictions
% Y_t+tau|t = D_tilc*Mc^tau * [Xtil0;z0]
% Dtilc = D*[eye(nX) zeros(nX+nx,tau1+1+Ti);
%           Fc                      ];
Dtilc = D*[eye(nX) zeros(nX,nx+tau1+1+Ti);
    Fc                      ];
% Restriction on i_t
Ymci = Dtilc*[Xtil0;z0ic];
for jj=1:T
    Ymci = [Ymci (Dtilc*Mc^jj * [Xtil0;z0ic])];
end
pivci = Ymci(1,:)';
pileadvci = [pivci(2:end);pivci(end)];
rvci = Ymci(3,:)'-pileadvci;
z0vic=zeros(1+T,1);
z0vic(1:tau1+1+Ti)=z0ic;
```

```
% Restriction on r_t
Ymcr = Dtilc*[Xtil0;z0rc];
for jj=1:T
    Ymcr = [Ymcr (Dtilc*Mc^jj * [Xtil0;z0rc])];
end
pivcr = Ymcr(1,:)';
pileadvcr = [pivcr(2:end);pivcr(end)];
rvcr = Ymcr(3,:)'-pileadvcr;
z0vrc=zeros(1+T,1);
z0vrc(1:tau1+Ti+1)=z0rc;
% d:
% Optimal policy w/o restrictions
Dtild = D*[eye(nX) zeros(nX,nx);
    F(1:nx+ni,:)        ];
Ymd = Dtild*Xtil0;
for jj=1:T
    Ymd = [Ymd (Dtild*M^jj * Xtil0)];
end
pivd = Ymd(1,:)';
pileadvd = [pivd(2:end);pivd(end)];
rvd = Ymd(3,:)'-pileadvd;
%-----------------------------------------------------------------------
% Plots
% Default font size and font
set(0,'DefaultAxesFontSize',12,'DefaultAxesFontName','Times');
set(0,'DefaultAxesLineStyleOrder',{'--','-.','-',':','o'}); % line styles for all plots
% Black and white plot defaults for printing
%whitebg('w') %create a figure with a white color scheme; needed?
if BlackWhitePlot;
    set(0,'DefaultAxesLineStyleOrder',{'--','-.','-',':','o'}); % line styles for all plots
```

```
    set(0,'DefaultAxesColorOrder',[0 0 0]); % set default line color to black
    set(0,'DefaultLineLineWidth',1.2) end;
% Default font size
% set(0,'DefaultAxesFontSize',14,'DefaultAxesFontName','Times');
% Remove defaults during session:
%set(0,'DefaultAxesLineStyleOrder','remove');
%set(0,'DefaultAxesColorOrder','remove');
%Plot projections for anticipated instrument-rate paths
figure;
subplot(2,3,2);
%subplot(1,1,1);
Plotmci = [Ymci(1,:)',Ymci(2,:)',Ymci(3,:)',rvci,z0vic];
%plot(tauv,Plotmci(:,1:4),tauv(tau1+1:tau1+1+Ti),Plotmci(tau1+1:tau1+1+Ti,5));
plot(tauv,Plotmci(:,1:3));
hold on
plot(tauv,rvci,':','LineWidth',2)
plot(tauv(tau1+1:tau1+1+Ti),Plotmci(tau1+1:tau1+1+Ti,5),'o');
hold off
title('Restr. on nominal policy rate');
xlabel('Quarter');
%ylabel('%/yr,');
set(gca,'XTick',[0 4 8 12 16 20 24 28 32])
if FixScale; axis([0 T ymin ymax]); end;
% legend('\pi','\ity','i','r','z','x','Location','Best');
%legend('\pi','\ity','i','r','z','x','Location','NorthEast');
if GridOn; grid on; end;
if BoxOff; box off; end;
subplot(2,3,1);
Plotmd = [Ymd(1,:)',Ymd(2,:)',Ymd(3,:)',rvd,z0vNR]; %,zmpi(:,1)];
plot(tauv,Plotmd(:,1:3))
```

```
hold on
plot(tauv,rvd,':','LineWidth',2)
plot(tauv(tau1+1:tau1+1+Ti),Plotmd(tau1+1:tau1+1+Ti,5),'o');
hold off
title('Optimal policy, No restriction');
xlabel('Quarter');
ylabel('%/yr, %');
set(gca,'XTick',[0 4 8 12 16 20 24 28 32])
if FixScale; axis([0 T ymin ymax]); end;
%legend('\pi','\ity','i','r','{z}_{\rm\pi}');
legend('\pi','\ity','\iti','\itr','\itz','Location','NorthEast');
if GridOn; grid on; end;
if BoxOff; box off; end;
subplot(2,3,3);
%subplot(1,1,1);
Plotmcr = [Ymcr(1,:)',Ymcr(2,:)',Ymcr(3,:)',rvcr,z0vrc];
%plot(tauv,Plotmcr(:,1:4),tauv(tau1+1:tau1+1+Ti),Plotmcr(tau1+1:tau1+1+Ti,5));
plot(tauv,Plotmcr(:,1:3))
hold on
plot(tauv,rvcr,':','LineWidth',2)
plot(tauv(tau1+1:tau1+1+Ti),Plotmcr(tau1+1:tau1+1+Ti,5),'o');
hold off
title('Restr. on real policy rate');
xlabel('Quarter');
%ylabel('%');
set(gca,'XTick',[0 4 8 12 16 20 24 28 32])
if FixScale; axis([0 T ymin ymax]); end;
%legend('\pi','\ity','i','r','z');
%legend('\pi','\ity','i','r','z','Location','NorthEast');
if GridOn; grid on; end;
```

```
if BoxOff; box off; end;
% suptitle1('Optimal policy');
% print -depsc AntIRPaths1.eps
% figure(2);
subplot(2,3,5);
%subplot(1,1,1);
Plotmai = [Ymai(1,:)',Ymai(2,:)',Ymai(3,:)',rvai,z0vi];
plot(tauv,Plotmai(:,1:3))
hold on
plot(tauv,rvai,':','LineWidth',2)
plot(tauv(tau1+1:tau1+1+Ti),Plotmai(tau1+1:tau1+1+Ti,5),'o');
hold off
title('Restr. on nominal policy rate');
xlabel('Quarter');
%ylabel('%');
set(gca,'XTick',[0 4 8 12 16 20 24 28 32])
if FixScale; axis([0 T ymin ymax]); end;
%legend('\pi','\ity','i','r','{z}_{\rm\pi}');
%legend('\pi','\ity','i','r','z');
if GridOn; grid on; end;
if BoxOff; box off; end;
subplot(2,3,4);
Plotmb = [Ymb(1,:)',Ymb(2,:)',Ymb(3,:)',rvb,z0vNR]; %,zmpi(:,1)];
plot(tauv,Plotmb(:,1:3))
hold on
plot(tauv,rvb,':','LineWidth',2)
plot(tauv(tau1+1:tau1+1+Ti),Plotmb(tau1+1:tau1+1+Ti,5),'o');
hold off
title('Taylor rule, No restriction');
xlabel('Quarter');
```

```
ylabel('%/yr, %');
set(gca,'XTick',[0 4 8 12 16 20 24 28 32])
if FixScale; axis([0 T ymin ymax]); end;
%legend('\pi','\ity','i','r','{z}_{\rm\pi}');
%legend('\pi','\ity','i','r');
if GridOn; grid on; end;
if BoxOff; box off; end;
subplot(2,3,6);
%subplot(1,1,1);
Plotmar = [Ymar(1,:)',Ymar(2,:)',Ymar(3,:)',rvar,z0vr];
plot(tauv,Plotmar(:,1:3))
hold on
plot(tauv,rvar,':','LineWidth',2)
plot(tauv(tau1+1:tau1+1+Ti),Plotmar(tau1+1:tau1+1+Ti,5),'o');
hold off
title('Restr. on real policy rate');
xlabel('Quarter');
%ylabel('%');
set(gca,'XTick',[0 4 8 12 16 20 24 28 32])
if FixScale; axis([0 T ymin ymax]); end;
%legend('\pi','\ity','i','r','{z}_{\rm\pi}');
%legend('\pi','\ity','i','r','z');
%legend('\pi','\ity','i','r','z','Location','SouthOutside','Orientation','Horizontal');
if GridOn; grid on; end;
if BoxOff; box off; end;
if SupTitle;
   %suptitle1('Optimal policy (1st row) and Taylor rule (2nd row)');
end;
```