NEXUS BETWEEN ENERGY SUBSIDIES AND FISCAL DEFICIT OF PAKISTAN

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CERTIFICATE

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To

My Parents, Siblings, and Family

"My Lord! Be merciful to them as they raised me when I was young."

(Surah Al-Isra' 17:24)

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ABSTRACT

In Pakistan, energy subsidy remains an important subject but at the cost of high fiscal imbalances. This study investigates the nexus between energy subsidies and fiscal deficit in Pakistan, addressing a critical policy concern amid ongoing macroeconomic instability. The primary objective is to assess how energy subsidies and the other macroeconomic variables influence the fiscal deficit, and to identify an optimal energy subsidy path that minimizes the fiscal imbalances. The study employs the Two-Stage Least Squares (2SLS) technique on simultaneous models for the time series data spanning 1981-2023. Building upon this structural model, the study applies optimal control theory, specifically the Linear Quadratic Regulator (LQR), to simulate fiscal policy scenarios over the period 2024-2028. The simulation results reveal that while higher subsidies initially help reduce public debt, and interest rates, however, sustained reduction in subsidies over time improve macroeconomic stability by lowering fiscal deficit, public debt, and inflation without severely compromising social support. The policy gap addressed by this research is the absence of an optimization framework for energy subsidy reform in Pakistan, a gap largely overlooked in existing literature. Findings also underscore the importance of targeted and phased subsidy rationalization coupled with macro-fiscal coordination. The study recommends integrating subsidy policies within a broader fiscal framework, institutionalizing data-driven planning, and adopting green technologies to ensure fiscal sustainability and inclusive economic growth.

Key Words: Energy Subsidy, Fiscal Deficit, Public Debt, Interest Rate, Inflation Rate, Linear Quadratic Regulator (LQR), Subsidy Rationalization.

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LIST OF ABBREVIATIONS

IEA	International Energy Statistics Agency
OECD	Organization for Economic Cooperation and Development
IMF	International Monetary Fund
TDS	Tariff Differential Subsidy
GoP	Government of Pakistan
PES	Pakistan Economic Survey
PSLM	Pakistan Social and Living Standards Measurement
HIES	Household Integrated Economic Survey
CGE	Computable General Equilibrium
LQR	Linear Quadratic Regulator
DARE	Discrete Algebraic Riccati Equation
FD	Fiscal Deficit
PD	Public Debt
Inf	Inflation Rate
IR	Interest Rate
ER	Exchange Rate
ES	Energy Subsidy
OGRA	Oil and Gas Regulatory Authority.
NEPRA	National Electric Power Regulatory Authority

CHAPTER 1

INTRODUCTION

1.1 Background

Pakistan is currently facing severe macroeconomic imbalances, with the fiscal deficit remaining one of the most persistent and critical challenges (PES, 2024). Fiscal sustainability has remained elusive, as the government struggles to manage rising expenditures in the face of weak revenue mobilization and increasing public debt. This fragile fiscal position has been further intensified by inflationary pressures, exchange rate instability, and external shocks (Hazmi et al., 2019). Further, Hussain et al. (2022) highlighted that the government has increasingly relied on subsidies, especially in the energy sector, as a policy tool to protect households from high fuel and electricity prices and also to support industrial competitiveness. Energy subsidies are defined by the International Energy Agency (IEA, 1999), and Organization for Economic Co-Operation and Development (OECD 1998) as government actions that lower the cost of energy production for producers or reduce prices paid by consumers. They can be explicit or implicit. While such subsidies can provide short-term relief, they often generate significant long-term fiscal, economic, and environmental costs.

Globally, energy subsidies remain substantial, with the IMF estimating them at USD 7 trillion in 2022, equivalent to 7.1% of global GDP. These subsidies have far reaching macroeconomic implications: they can distort market prices, misallocate resources, reduce incentives for investment in renewable energy, and place a heavy burden on public budgets. In developing economies, such as Pakistan, the fiscal cost is especially critical given the limited budgetary space and high debt servicing requirements.

The macroeconomic consequences extend beyond fiscal strain. Empirical studies highlight that persistent energy subsidies can reduce GDP growth by diverting resources away from productive investment as well as it has negative environmental consequences (Bauer, 2013). The deficit financing further exacerbates the inflationary pressure and hence the exchange rate imbalances occurs. Further, the subsidy also benefits the rich disproportionately; most of them are captured by the rich segment of society which create social imbalance as well, leaving the poor with small share of the relief while public resources are drained.

Pakistan's energy subsidies account for nearly 89% of total subsidy bill and 7.9% of current expenditures (GoP, 2023). Despite their large fiscal cost, evidence shows that only a fraction reaches low-income households (Khalid, 2022). This untargeted nature reduces the equity and efficiency of fiscal policy, while contributing to persistent fiscal deficits and limiting investment in growth-enhancing sectors such as infrastructure, education, and healthcare. Given Pakistan's limited fiscal space and high dependence on external financing, and the need for structural reforms, understanding the link between energy subsidies and fiscal deficit has become critical area for research and policy action.

1.2 Research Problem

Despite the substantial fiscal resources devoted as energy subsidies in Pakistan, the intended policy objectives of affordability and equitable access remain largely unmet. The current subsidy framework is untargeted, with the bulk of benefits accruing to wealthier households who consume more fuel and electricity. Tariff structures fail to differentiate adequately between income groups, and allocation mechanisms lack transparency and efficiency.

From a fiscal perspective, this misallocation is particularly costly. Energy subsidies have become the fourth-largest component of current expenditure after interest payments, defense, and grants. In FY2023, they represented nearly 88.7% of all subsidies, contributing significantly to a fiscal deficit of 6.53% of GDP which is well above the projected value 4.9%, while GDP growth fell to only 0.29%, far below the target of 5%.

This persistent fiscal gap reflects a deeper policy dilemma, how to balance short-term social relief with long-term fiscal sustainability. Continued reliance on untargeted subsidies exacerbates inefficiency, crowds out productive public investments, and constrains the government's ability to respond to economic shocks.

Further, the key problem in identifying all these issues is the absence of an evidence-based framework to determine the optimal level and structure of energy subsidies that can minimize the fiscal deficit without undermining social welfare or economic stability. This study addresses that gap by empirically quantifying the relationship between energy subsidies and fiscal deficit, and by applying an optimal control approach to identify a subsidy policy path that supports fiscal consolidation while maintaining essential welfare objectives.

1.3 Motivation

The motive behind this study lies in the urgent need to reconcile social welfare objectives with fiscal sustainability in Pakistan. The country's experience with large, untargeted energy subsidies has shown that they often fail to effectively target low-income households while placing substantial strain on public finances.

Moreover, existing literature on Pakistan has focused mainly on descriptive or partial-equilibrium analyses of subsidy impacts, with limited application of dynamic policy tools that could guide optimal subsidy levels over time. This study responds to this gap by using an integrated econometric and optimal control framework to quantify the relationship between energy subsidies and fiscal deficit, and to identify an optimal path that could reduce fiscal pressure without undermining economic stability.

1.4 Significance of the Study

In FY2023-FY2024, the fiscal deficit reached 6.53% of GDP, a level that is not only unsustainable but also carries serious macroeconomic consequences. This weakened budgetary position is largely driven by rising expenditures, particularly in the form of energy subsidies, which constitute a significant fiscal burden. In light of this, there is an urgent need for strategies that are not only theoretically sound but also practically implementable.

To address this challenge, the study proposes a control strategy by determining the optimal level of energy subsidies that can minimize the fiscal deficit without compromising the output growth. This approach positions subsidy reform as a tool for promoting economic expansion while ensuring fiscal sustainability. Furthermore, an in-depth understanding of the fiscal dynamics is essential for policymakers seeking to realign fiscal policy in a more sustainable direction.

The analysis is conducted in two stages. First, the relationship between energy subsidies, the fiscal deficit, and other key macroeconomic indicators[1] using a simultaneous equation model grounded in the Keynesian aggregate framework, a method well established in prior literature. The results from this macroeconomic modeling will then inform the second stage, the application of optimal control theory. This method is innovative in its nature as it is rarely applied in the context of energy subsidies. The purpose is to minimize the fiscal deficit while controlling the energy subsidies consequently this will provide the framework for policy makers regarding the fiscal policy.

Additionally, the study also evaluates the energy subsidy financing mechanism in Pakistan, shedding light on whether subsidies are governed by a structured disbursement framework or allocated arbitrarily. This dual focus on fiscal modeling and institutional analysis enhances the study's contribution to both academic research and policy development.

1.5 Research Objectives

The following are the research objectives of this study:

[1] Macroeconomic indicators will include government debt level, government expenditure, government revenues, inflation, interest rate, and exchange rate.

- To investigate the relationship between energy subsidies and other macroeconomic variables with the fiscal deficit of Pakistan.
- To optimize fiscal deficit while optimally controlling energy subsidies.
- To analyze the energy subsidy financing mechanism in Pakistan.

1.6 Research Question

These research questions are aligned with the objectives of the research which are given as:

- To what extent do changes in energy subsidy and macroeconomic variables interact with fiscal deficit in Pakistan's economy?
- How can adjustments in energy subsidies contribute to reducing Pakistan's fiscal deficit?
- How does Pakistan disburse energy subsidy within its fiscal framework?

1.7 Organization of the Study

This study is organized into different chapters. Chapter 1 includes introduction and research problems and also highlights the significance of the paper as well as objectives and research questions. The second chapter is about background and context of energy subsidies, chapter 3 is literature review about the implications of energy subsidies, and it also includes the literature gap. Chapter 4 incorporates theoretical framework and chapter 5 will be the methodology including the econometric models and its specifications. Further, chapter 6 includes the results and discussions while chapter 7 includes the qualitative analysis of the study. Lastly, the conclusion and recommendations are incorporated in chapter 8 of this study.

CHAPTER 02

BACKGROUND AND CONTEXT OF ENERGY SUBSIDIES

2.1 Overview of Energy Subsidies

Energy consumption is crucial in any economy and is heavily subsidized by the government. These energy subsidies benefit both consumers and producers which are defined as "an act by government that increases the price received by producers, declines the energy production cost or decreases the price that is given by consumers" IEA (1999), OECD (1998). But at the same time, it leads to negative impacts through various channels, including the aggregate output, the balance of payment issues, and public finances. Developing countries experience the negative effects of energy subsidies because of limitations to budget in the form of fiscal deficit hence it should consider fiscal constraints (Can, 2022).

2.2 Types of Energy Subsidies

Globally, the energy subsidies reached US$7 trillion, which is 7.1% of GDP during 2022 and this subsidy is both implicit and explicit and accounts for 82 and 18 percent (Black et al., 2023). The estimates of IMF staff regarding fuel subsidies expressed implicit subsidies as the major contributing factor to the fiscal burden rather than explicit subsidies. They are defining implicit subsidy as post-tax subsidy and explicit subsidy as pre-tax subsidy[2].

2.2.1 Explicit Energy Subsidy

The explicit subsidies are the amount of subsidies that are given to fuel products and the details of which are given in government documents explicitly as a part of subsidy provision. It is also known to pre-tax subsidy because no tax is included during the calculation process which is calculated with two approaches by IMF. The first one is price gap approach, and the second one is inventory method. In the price gap approach subsidies are simply the quantity of

[2] Methodology Fossil Fuel Subsidy Tracker (https://fossilfuelsubsidytracker.org/methodology/)

subsidized fuel consumed multiplied by the gap between the retail price and the supply cost. While, the inventory method focuses rather on identifying and quantifying the support measures to the energy production and consumption using the information from official government documents (Parry et al., 2014).

Example

Here is the example of pre-tax subsidies on fuels with price gap approach is given in order to understand how it is calculated.

If the subsidized price for gasoline is 20 cents per liter, the supply cost is 1 dollar per liter then the pre-tax price gap is equal to 0.8 (1-0.2) and if the quantity consumed is 500 million liters, then pre-tax subsidies are equal to = 0.8 * 500 = 400 million dollars. This indicates that the pre-tax subsidies are calculated as;

$(Pre - tax\ subsidies\ = (Pm - Ps)\ * Qs)$.

2.2.2 Implicit Energy Subsidy

IMF also estimated the implicit subsidies for different fuel products and for different regions, which is calculated as a post-tax subsidy. These implicit subsidies are not included in any government documents as it includes costs other than direct subsidies like the tax breaks, which is in case of developing countries are too high and actually represent the picture of the economy. Therefore, there is data limitation with implicit subsidy due to which the current analysis only focus on explicit energy subsidy.

The pre-tax subsidy exists when energy consumers pay a price below the supply cost of energy while post-tax subsidy is the sum of pre-tax subsidy with tax subsidy. The tax subsidy arises when energy taxes are too low. The purpose of these taxes are to enhance the revenue and to decline the negative environmental consequences because of excess fuel consumption due to subsidy (Parry et al., 2014b). The formula for the calculation of post-tax subsidy is;

$Post - tax\ subsidy\ = [(Supply\ cost\ of\ energy\ +\ Corrective\ tax)\ * (1\ +$

$VAT\ rate) - Retail\ price] * Quantity.$

Example

The international price of gasoline is USD 0.4 per liter, with transport and distribution margins of USD 0.2 per liter, the supply cost totals USD 0.6 per liter. Additionally, the value-added tax rate is 20% and corrective tax is USD 0.7 per liter, which makes the consumption of 200 million liters, then as per above formula the post-tax subsidy in this case will be 240 million USD.

2.3 Global Data of Energy Subsidy:

At the global level in 2022, explicit subsidies on natural gas are highest at 48%, then petroleum at 26%, electricity at 25%, and coal accounts for less than 1%. 97% of these explicit subsidies are for consumers and only 3% are for producers. In the case of total subsidies, including both implicit and explicit, the breakdown of fuel subsidies by their components is totally different in which coal accounts for 30%, natural gas 18%, petroleum 47%, and electricity 5% (Black et al., 2023).

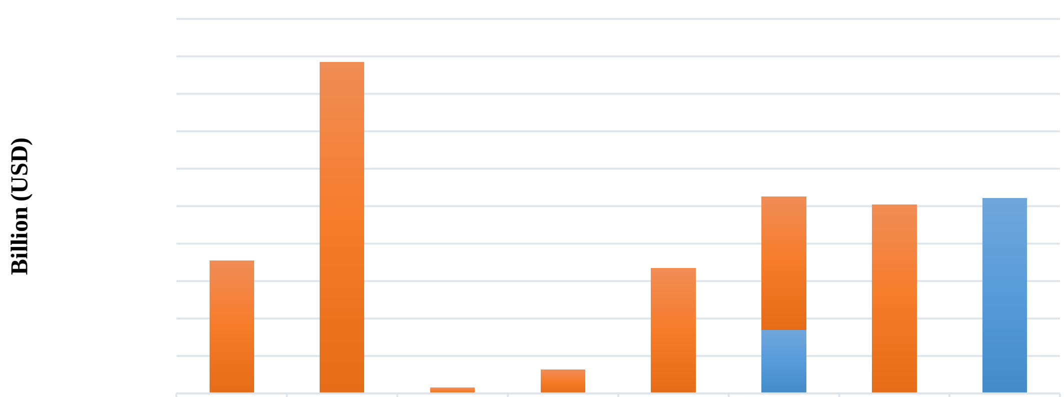
The IMF also estimates fuel subsidies both explicitly and implicitly for different regions and countries as well. It shows that explicit subsidies in 2022 are mostly given in the East Asia and Pacific (EAP) region accounts for 38%, it is then followed by the Middle Eastern and Northern African (MENA) region which accounts for 26%, and then Europe accounts for 16%. These regions are then followed by Latin America, South Asia and the Caribbean, ranging 5 to 12%, in comparison to North America and Sub-Saharan Africa (SSA) accounts for less than 3%.

This regional breakdown is different in the case of total subsidies (explicit plus implicit) in which EAP accounts for 48%, MENA, Europe, and North America have 11% of contribution each, and 5-9% in CIS, South Asia, and LAC, SSA accounts 2% of total subsidies which is smallest amongst all regions (Black et al., 2023).

2.4 Pakistan's Energy Subsidy Profile

Pakistan is one of the South Asian countries where fuel subsidies make up a significant portion of their budget. Both explicit and implicit subsidies are calculated by the IMF for Pakistan. Like Implicit subsidies are a major part of total subsidies globally, Pakistan has also implicit fuel subsidies as a major contribution to total subsidies. According to Guramani (2024) fuel subsidies in Pakistan is one of the major cause that significantly impact the fiscal deficit situation. These subsidies on different fuel products are estimated for 2023 by IMF as given in the following graph.

Figure 2.1 Subsidies on Fuel Products in Pakistan 2023



	Gasoline	Diesel	Kerosene	LPG	Oil - other	Natural gas	Coal	Electricity
Implicit	3.5	8.8	0.1	0.6	3.3	3.5	5.0	0.0
Explicit	0.0	0.0	0.0	0.0	0.0	1.7	0.0	5.2

Source:https://www.imf.org/en/Publications/WP/Issues/2023/08/22/IMF-Fossil-Fuel-Subsidies-Data-2023-Update-537281

This graph indicates that in Pakistan, implicit energy subsidies are major part of subsidies while explicit energy subsidies are only for natural gas and electricity. A major part of the implicit energy subsidy is diesel which is estimated at 8.8 billion USD while kerosene has the lowest implicit part making 0.1 billion USD. The explicit energy subsidies are only for natural gas and electricity. Electricity explicitly makes it 5.2 billion USD while natural gas makes it about 1.7 billion USD. We have data limitations regarding the implicit energy subsidies that’s why we will only use the explicit energy subsidy data in this study which is available in budget documents.

2.5 Distribution of Subsidy Benefits

Many households in Pakistan benefit from energy subsidies; however, the majority of these benefits are disproportionately captured by higher-income groups who consume more electricity.

For instance, data from the Pakistan Social and Living Standards Measurement (PSLM HIES) survey shows that the most subsidized group consists of households with monthly electricity consumption between 301 and 700 units. This group represents about 56% of electricity consumers, translating to approximately 16.8 million households.

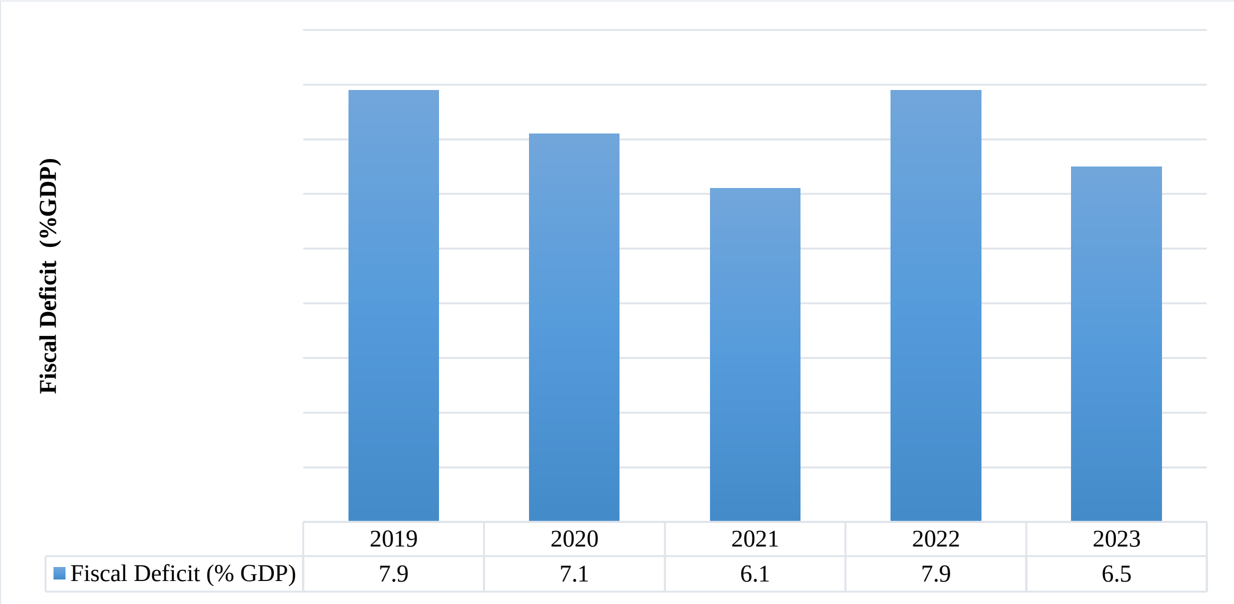
On an annual basis, the subsidy burden for the consumers falls in the range of 301 to 700 units of consumption, approximately making Rs. 454 billion, which on a monthly basis makes Rs. 25.8 billion if they use 500 units of electricity. This figure is striking when compared to budgetary allocations for social protection programs. In the 2021-22 federal budget, funding for social protection was cut by Rs. 255 billion, which is nearly half of what is spent on subsidies for just one consumer group (Younus, 2022).

This highlights a critical policy issue that many of these energy subsidies are not targeted effectively. As a result, wealthier households benefit the most, receiving a significant share of the total subsidies which includes the maximum subsidy available on electricity.

2.6 **Fiscal Implications of Energy Subsidies**

Over the past five years, Pakistan's fiscal deficit has persistently remained above 6% threshold, reflecting chronic imbalances between revenues and expenditures. The deficit reached its highest level in 2019 and again in 2022, peaking at a 7.9% of GDP. Despite policy targets for fiscal consolidation, the situation showed little improvement in FY2023, when the fiscal deficit stood at 6.53% of GDP which is equivalent to approximately Rs. 13 trillion that is significantly exceeding the projected target of 4.9%. The figure below illustrates the upward trend in fiscal deficit that remain high, highlighting the structural nature of fiscal imbalances.

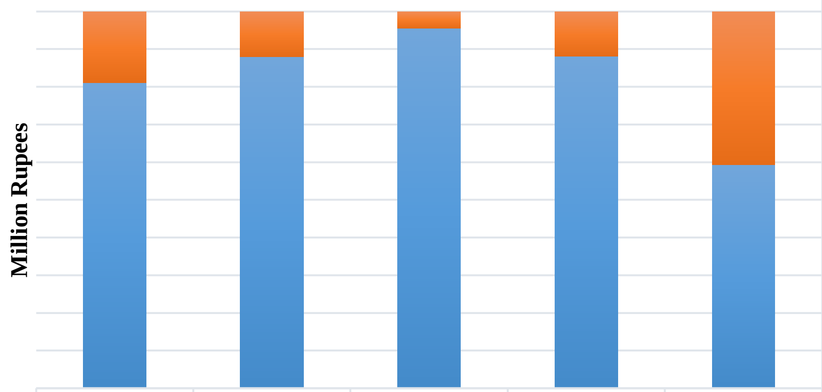
Figure 2.2 Fiscal Deficit of Pakistan (%GDP)



Source: Fiscal Development Pakistan Economic Survey 2022-2023.

A major contributor to this persistent deficit is the high level of subsidies, particularly those in the energy sector. In FY2022-23, energy subsidies accounted for 88.7% which is 972,000 million Rupees out of total 1,103,077 million Rupees, making them the fourth-largest component of current expenditure after interest payment, defense, and grants. While in FY23023-24, the energy subsidy declines to 59.2% making 634,600 million Rupees out of total 1,071,000 million Rupees. These subsidies not only represent a substantial fiscal outlay but also indicate a heavy reliance on budgetary resources to maintain artificially low energy prices.

Figure 2.3 Fiscal Burden of Budgetary Subsidies of Pakistan



	2019-20	2020-21	2021-22	2022-23	2023-24
Other Subsidies (million rupees)	66000	51650	67900	131077	436400
Energy Subsidy (million rupees)	283500	378350	1449000	972000	634600

Source: Budget in Brief, Ministry of Finance.

The above figure illustrates the fiscal burden of budgetary subsidies, including energy subsidies, showing how these allocations have mirrored the broader deficit trend, thereby placing continuous strain on Pakistan’s already limited fiscal space.

CHAPTER 3

LITERATURE REVIEW

3.1 Introduction

The energy subsidies have a long history, tracing back several decades. International organizations defined energy subsidies as a policy instrument used by any government to enhance the revenue to producers by reducing the cost of production to fuels that are a major portion of the production activity and also reducing the prices paid by consumers.

Hence, the provision of subsidies benefits both consumers and producers in the economy but has complex and multifaceted implications. These implications include the macroeconomic implications including growth, investment, exchange rate, and especially fiscal implications. Further, social implications are discussed in which income inequality, poverty, and welfare are prominent, and also the environmental implications are discussed here in which pollution damaging the environment and carbon dioxide emissions, etc. are discussed.

While there are short-term benefits, their long-term costs can be substantial, particularly in the above-mentioned sectors Vagliasindi, (2012). Here we will discuss mainly the macroeconomic implications and changes in macroeconomic conditions on the fiscal deficit situation of the country especially with the changes in energy subsidies.

3.2. Macroeconomic Implications of Energy Subsidy

Fuel Subsidies have its implications on the macroeconomic conditions of the countries.

Studies highlighted that fuel subsidies needed reforms because the intended results are not getting. IMF estimated energy subsidies globally for developed as well as developing countries and highlighted that these fuel subsidies have negative consequences on growth.

The negative impact of energy subsidies on growth surpasses their effects on fiscal balances and public debt, leading to diminished incentives for investors to allocate funds toward renewable energy sources. As a result, this diversion of investment hinders the potential for growth by reallocating resources away from other avenues of consumption that could stimulate economic development (Bauer, 2013).

It is also highlighted by Dartanto (2013) that there is the utmost need of a reduction in fuel subsidies due to the reason of severe budget deficit which is driven by a decrease in oil production, high prices of crude oil, and an enhancement in consumption.

Subsidy reforms were also taken by the Indian government which has positively contributed to economic growth, increasing the GDP growth to 7% but due to low demand in exports and below-normal rainfall it affects growth. Further, subsidizing diesel increases the subsidy burden and fiscal deficit, which can be financed by either printing money or by borrowing with the help of securities, both these raise inflation and lower GDP. For this alternative, if the government passes the rising international oil prices onto the economy, production costs will increase which will again result in higher inflation and reduced GDP (Ghosh, 2022).

The removal of fuel subsidies has implications on other macroeconomic and microeconomic variables as well which is highlighted by Ozili et al. (2023), who gave ideas on the implications of fuel subsidy reforms in Nigeria. The removal of fossil fuel subsidies has positive implications that lead the excess resources in terms of finance to divert into other sectors as well, it also incentivizes the oil refineries to produce more oil products and lessen the dependence of imported fuel while also generating employment opportunities. It reduces the budget deficit and generates surplus, also reduces the borrowing by government, mitigate corruption linked to the financing of fuel subsidies, enhance competition, revitalize the refineries domestically, and alleviate the exchange rate pressures.

The major implication of fuel subsidy is the fiscal burden which varies across different countries but is significant to developing countries. Fuel Subsidy has a positive and significant relation with fiscal deficit in India (Verma. et al, 2018). Yusoff (2017) also highlighted that fuel subsidy reform in Malaysia significantly reduced the government's fiscal deficit, with ratios consistently below the government's target of 4% of GDP. While overall fuel subsidy removal has positively impacted real GDP, certain sectors including energy itself faced with negative effects, further pressures on prices of energy and exchange rates also enhanced because of the removal of the subsidies on fuels.

Further, the relationship of macroeconomic variables with fiscal sustainability other than energy subsidy is also studied for Indonesia. The result shows that in the long term, the subsidies and the Bank of Indonesia Interest Rate BIR positively affect fiscal deficit having a significant relationship. While the exchange rate negatively impacts fiscal deficit. In the short term, both subsidies and SBI have a positive impact on fiscal deficit, while the long-run analysis shows the balanced relationship among subsidies, exchange rate, and SBI with fiscal sustainability, this inflation, in the long run, does not maintain this balance (Hazmi et al., 2019).

Pakistan also faces the implications of energy subsidies specifically the macroeconomic implications in which fiscal sustainability is an important factor to consider. Khalid (2022) highlighted Pakistan as a case study in his paper to evaluate the ineffectiveness of untargeted subsidies on gasoline and diesel in Pakistan and its impact on cost push inflation in 2022. The results revealed that 89% of the subsidy domestically made about PKR 74.63 billion could be saved through targeted mechanisms that were equivalent to 9.2% of the 2021 development budget. The removal of these untargeted subsidies takes the consumption level of petroleum products to decline and decreases their share of total imports from 37% to 23%, hence a lot of fiscal saving occurs in this case. This paper is estimating the impact of reform in fuel subsidy to tackle the cost push inflation in Pakistan with the help of Household Integrated Economic Survey (HIES) 2018-19 micro data to know about the energy consumption of different quintiles.

3.3 Social and Environmental Implications of Energy Subsidies:

3.3.1 Social Implications of Energy Subsidies:

The Energy subsidies carry important social implications in any economy. It affects the equity, access, and welfare of the public. Although they can raise the affordability of energy for low-income households and also help in the alleviation of poverty related to energy, due to the untargeted subsidies, the benefits mainly go to wealthier consumers. Ahead of this, these energy subsidies can create a financial burden on the government budget which results in cuts in necessary services such as education and healthcare; consequently, the social inequalities are enhanced in this way.

Social inequality emerges when the purpose of welfare of energy subsidies for needy people is not met, and hence issues like income inequality and poverty arise. Many studies highlighted the welfare analysis of energy subsidies or fuel subsidy reforms in other countries as well as in Pakistan.

Umar (2013) highlighted about Nigeria that, when energy prices rose the 40% of the middle-income household welfare decreased as compared to the top and bottom 20% of the households. This is the case, because the middle-income group has more budget share on fuel. Although fuel subsidies appear to be expensive in shielding poor households, the welfare loss at the lower bottom might be higher due to subsidy cuts this is because of their smaller income base.

Similarly, the reform in energy subsidies in India indicated that price elasticity for the demand of fossils is low while income elasticities are high and a rise in price level by subsidy reform results in the erosion of income which will have harm on the welfare of consumers (Acharya, 2017).

According to Dartanto (2013), fuel subsidies in Indonesia are distributed in such a way that 30% of the richest proportion of the population gets almost 72% of energy subsidies while consuming 63.8% of total subsidies between 1998 and 2013. Further, the CGE simulation results highlighted that in Indonesia reducing the subsidies on fuels by 25% enhances poverty. Further, simulating the distributional effects of rising petroleum prices using the social accounting matrix (SAM) model indicated that the rise in petrol prices is regressive, disproportionately impacting poorer individuals more than wealthier ones. All ethnic groups face income losses, with Malay households typically the poorest on average, experiencing a greater reduction in real income as compared to households in China and India, which tend to be relatively wealthier (Saari et al., 2016). Similarly, Khalid (2022) highlighted that in Pakistan, it accounts that only 11% of total domestic fuel subsidies were directed to 40% of households, while approximately 55% of subsidies were allocated to 20% of rich people, this led to the result that in Pakistan subsidies to fuels are not targeted. Ahmad et al. (2020) in his paper indicated that the rise in the prices of energy impact the welfare of households significantly, while Ilyas et al. (2022) analyzed the impact of uniform and non-uniform increases in tariffs to different slabs of electricity consumption. It is highlighted that the benefits of targeting higher slabs improve the poor but a limited and major benefit goes to the rich. Further, the welfare of households through compensating variation estimated that welfare losses to households are significant in Pakistan when subsidies are removed which are higher for low-income groups, this leads to call for targeted subsidy programs to protect poor households (Mirza et al., 2022)

3.3.2 Environmental Implications of Energy Subsidies

Energy subsidy provision also leads to negative externalities in the form of environmental degradation through increased pollution, and Carbon dioxide emissions when used excessively. Many literatures highlighted the environmental impact of fuel subsidies.

According to Solarin (2020) if a 10% increase in fuel subsidies happens then 0.3% to 1.5% ecological footprints may rise which indicates the environmental degradation process. Similarly, Li et al. (2017) suggested that reducing fuel subsidies reduces the carbon emission ranges between 1.84% and 6.35% in different scenarios of reduction in petroleum, natural gas, and both petroleum and natural gas. But in mitigating climate change ideology here, the subsidy on gas is considered favorable for the industrial and commercial sectors. The reason for this is because of a slight reduction in carbon emission achieved the consequence of which is the a decline in GDP growth, and this is worthless.

Similarly, the onset of the impact of economic growth and fuel subsidies on the environment of Nigeria reveals that growth in terms of output per capita had a significant and positive relation with carbon emission, while fuel subsidies negatively impact the environment (Omitogun et al., 2021).

(Hassan et al., 2021) highlighted the environmental damages that are the consequences of using fossil fuels. The data were from 2005 to 2009 and when evaluated the impact of using these fuels on environmental damages on humans and on crops emerged as 42.68 to 13.7% for petroleum, 33.4 to 35.6% for natural gas, and 23.8 to 50.7% for natural gas respectively. Further the additional costs assciated to damage caused by the consumption of these fuels lead people to pay PKR 133 billion per year as taxes, health services, insurance, and for the declining living standards. Fossil fuels when consumed release Carbon dioxide in the air and damage the environment which alternatively has a detrimental impact on not only the health of humans but also on flora and fauna, the aquatic environment, the layer of ozone, crops, infrastructure, visibility, and many others including climate change.

3.4 Literature Gap:

Many studies have tried to evaluate the effect of reducing subsidies on fuel products and its implications globally and regionally, significant research gaps remain in the context of Pakistan. Existing studies largely focus on the welfare and distributional effects of subsidy reform impacts (Mirza et al., 2022; Ahmad et al., 2020; Hassan et al., 2021; Ilyas et al., 2022; Khalid, 2022; Samad et al., 2022) often highlighting the regressive nature of untargeted subsidies. Other strands of literature employ CGE, SAM, or Input-Output models to simulate subsidy reform impacts (Dartanto, 2013; Samad et al., 2022; Li et al., 2017), but these approaches primarily assess household welfare, consumption patterns, or environmental concequences rather than macro-fiscal sustainability.

In the case of Pakistan, very limited research has investigated the direct linkages between energy subsidies and fiscal deficit while simultaneously accounting for other key macroeconomic variables such as public debt, exchange rate, inflation, and interest rate. Furthermore, no study has attempted to optimize fiscal deficit through the lens of energy subsidy control, employing an optimal control framework to simulate alternative policy paths. This creates a methodological and policy gap in understanding how subsidies interact with broader macro-fiscal dynamics. Further, another unexplored dimension is the financing mechanism and distributional faireness of energy subsidies in Pakistan's fiscal framework.

Therefore, this study contributes by addressing three critical gaps:

Estimating the relationship between energy subsidies, fiscal deficit, and macroeconomic variables through a simultaneous model, applying the optimal control (DARE-LQR) framework to identify the subsidy path that minimizes fiscal deficit while maintaining macroeconomic stability, and incorporating qualitative insights to examine subsidy financing, distributional faireness, challenges and benefits.

Through this approach, the study not only extends the literature on energy subsidies and fiscal sustainability but also provides evidence-based recommendations for Pakistan's energy and fiscal policy reforms.

CHAPTER 4

THEORETICAL FRAMEWORK

The theoretical framework serves as the foundation for understanding the conceptual linkages between the key variables in this study, which are: Energy subsidies, fiscal deficit, public debt, exchange rate, inflation rate, and interest rate. It draws on established economic theories to explain the direction and nature of these relationships, guiding both the formulation of hypotheses and the interpretation of empirical results.

4.1 Keynesian Macroeconomic Theory

This study primarily aligns with the Keynesian school of thought, which emphasizes the role of government intervention in stabilizing the economy through fiscal policy. According to Keynes (1936), aggregate demand is the driving force behind economic activity, and fiscal instruments such as subsidies, taxation, and public spending plays a crucial role in influencing output and employment levels (Jahan et al., 2014).

Energy subsidies, as a component of government expenditure, can stimulate aggregate demand by reducing production costs for firms and lowering the energy prices for households. However, from a fiscal perspective, sustained subsidies can also widen the fiscal deficit if they are not matched by revenue generation (Allan, 2016). The Keynesian framework thus predicts a trade-off between the short-term growth stimulus of subsidies and their long-term fiscal sustainability.

4.2 Public Finance Theory and Fiscal Deficit

From the perspective of public finance theory, the fiscal deficit reflects the gap between government expenditure and revenue (Musgrave, 1959). Persistent deficits can lead to increased borrowing, higher public debt, and potential macroeconomic instability (Perotti et al., 1995). The theory highlights that subsidy expenditures, unless offset by efficiency gains or higher tax revenues, place upward pressure on the fiscal deficit.

In Pakistan, energy subsidies have historically been used to shield consumers from price volatility and to support industrial competitiveness (Helm, 2002). However, these subsidies often create budgetary strains, forcing the government to finance deficits through domestic or external borrowing.

4.3 Optimal Control Theory

Optimal control theory provides a mathematical basis for determining the most efficient allocation of fiscal instruments over the time to achieve macroeconomic stability (Kamien et al., 2012). In general, this framework is used to determine the best possible sequence of decisions or policies over time when a system's evolution is influenced by both current conditions and policy actions. It is particularly useful when there is a clearly defined policy objective, such as minimizing the fiscal deficit, stabilizing public debt, or controlling inflation, and when the system in question changes according to specific dynamic rules (Becerra, 2008). In this framework, the variables that describe the system at any given time are referred to as state variables which is influenced by a control variable that is available to policymakers. The policymaker's goal is expressed through an objective function, which quantitatively measures how close the system is to achieving its targets over a specified time horizon. The relationships between the current state, the control variables, and the future state are captured by dynamic constraint equations, which describe how the system evolves. By simultaneously considering the objective function and these constraints, optimal control theory identifies the policy path that best achieves the desired outcomes (Trentelman, 2013).

In economic policy applications, this approach is advantageous because it recognizes that decisions made today have long-term consequences and that future conditions influence present decisions. In the context of this study, optimal control theory is applied to design an energy subsidy path that minimizes deviations of fiscal deficit, public debt, and inflation from their target levels, while accounting for the dynamic interactions among key macroeconomic variables.

4.4 Exchange Rate and Inflation Dynamics

The Mundell-Fleming model and open-economy macroeconomics provide insight into how fiscal deficits and subsidies interact with exchange rates and inflation (Fendel, 2002). A higher fiscal deficit, if monetized or financed through external borrowings, can put pressure on the exchange rate, leading to currency depreciation. Currency depreciation, in turn raises the domestic cost of imported energy, potentially increasing the subsidy burden.

The inflationary pressures may also emerge when subsidies are reduced or removed, particularly if they had previously kept consumer prices artificially low (Coady et al., 2015). This creates a policy challenge of sequencing subsidy reforms in a way that minimizes macroeconomic shocks.

4.5 Energy Subsidy Theory

In literature energy subsidies can distort price signals, leading to the overconsumption of energy, reduced incentives for energy efficiency, and fiscal inefficiency (Coady et al., 2015). From welfare perspective, subsidies often disproportionately benefit higher-income households that consume more energy, undermining equity objectives (Mirza et al., 2022).

In Pakistan’s case, persistence of energy subsidies has contributed to circular debt in power sector, fiscal stress, and limited investment in renewable energy sources (World Bank, 2020). This reinforces the importance of aligning subsidy policy with both fiscal sustainability and energy transition goals.

CHAPTER 5

METHODOLOGY

5.1 Introduction

This chapter consists of three sections. In the ‘first section,’ we have explained the theoretical background of the models and their specifications. The second section explains about methodology, which is based on specifying a system of simultaneous equations consistent with Keynesian macroeconomic theory, following the approach of Syahrini et al. (2021). The estimation is conducted using the Two-Stage Least Squares (2SLS) method, which corrects for simultaneity bias and yields consistent parameter estimates in systems where endogenous relationships exist among variables. This section also discusses the application of Optimal Control Theory, in particular the Linear Quadratic Regulator (LQR), which is applied after estimation to determine the optimal trajectory for fiscal deficit reduction while respecting macroeconomic constraints. The third section describes the data sources, variable definitions, and construction procedures used in the analysis.

5.2 Model Specifications

The empirical framework of this study is built on a system of five simultaneous equations that shows the behavioral interaction among the fiscal deficit and macroeconomic variables such as energy subsidy, exchange rate, inflation, interest rate, and public debt. The aim of these models are to capture the complex interactions between these variables. Therefore, in this study, we have utilized the methodology adopted by (Syahrini et al., 2021). They used simultaneous models as defined by Keynes within macroeconomic theories. The rationale for using this framework is that there is simultaneity among the variables, along with these behavioral equations, there is an identity equation, that is the national income accounting identity of fiscal deficit as explained by Romer (2019).

The simultaneity amongst these variables are because of the explanatory variables. These explanatory variables used in the models are based on the macroeconomic theory of Keynes, fiscal dynamics, and empirical evidence from developing economies. The fiscal deficit is treated as a central transmission mechanism, influencing and being influenced by the macroeconomic variables, consistent with the "fiscal dominance" literature (Sargent et al., 1981) & (Catao et al., 2005). The functional form of each behavioral equation and its theoretical justification are presented below.

5.2.1 Behavioral Equations

Behavioral equations capture the endogenous responses of variables in the system and are important in representing the dynamic interplay among economic variables in response to policy changes or external shocks. The behavior equations in macroeconomics, as per Keynes, are based on aggregate relationships. These behavioral models are specified as;

$$ES_t = \alpha_{11} + \alpha_{12}FD_t + \alpha_{13}ES_{t-1} + \mu_{1t} \dots\dots\dots\dots .5.1$$

$$PD_t = \alpha_{21} + \alpha_{22}FD_t + \alpha_{23}PD_{t-1} + \alpha_{24}ER_t + \mu_{2t} \dots\dots\dots\dots .5.2$$

$$ER_t = \alpha_{31} + \alpha_{32}FD_t + \alpha_{33}IR_t + ER_{t-1} + \mu_{3t} \dots\dots\dots\dots .5.3$$

$$IR_t = \alpha_{41} + \alpha_{42}FD_t + \alpha_{43}IR_{t-1} + \alpha_{44}ER_t + \alpha_{45}Inf_t + \mu_{4t} \dots\dots\dots\dots .5.4$$

$$Inf_t = \alpha_{51} + \alpha_{52}FD_t + \alpha_{53}IR_t + \alpha_{54}Inf_{t-1} + \mu_{5t} \dots\dots\dots\dots .5.5$$

Energy subsidies are modeled as a function of the fiscal deficit and their own past values. Government often expand subsidies during periods of fiscal imbalance, financing them through borrowing to pursue short-term political or social objectives (Clements et al., 2013) & (Monitor, 2019). The inclusion of lagged subsidy term reflects persistence, as subsidies, once introduced, tend to become entrenched in the policy framework due to political cycles and path dependence (Victor, 2009) & (Gupta et al., 2005).

Public debt dynamics are explained primarily by fiscal deficits, exchange rate, and its own lag.

Persistent fiscal deficits drive debt accumulation, a finding consistent with macroeconomic theory as defined by Keynes (Blanchard, 2013). While government spending, financed through borrowing, can boost aggregate demand but this borrowing further increases public debt. The exchange rate dynamics of debt are explained on the basis of Mundell-Fleming's open economy model. It argues that, whenever the deficit raises the debt, it crowds out investment undermining the investor's confidence and hence the currency depreciates it this way. Moreover, the autoregressive nature of the public debt is accounted for through the lagged debt term, reflecting the debt accumulation identity where the current debt stock builds on past obligations.

The exchange rate shows only a structural relation that links currency fluctuations to fiscal and monetary conditions as well as its own lagged trajectory. Large fiscal deficits tend to undermine investor confidence, increase reliance on imports, and exert depreciation pressures (Deravi et al., 1992). This was also suggested by the Keynesian theory that fiscal deficit can indirectly impact the exchange rate by affecting interest rates and investors' confidence. If a government's fiscal deficit leads to a higher debt burden, it may reduce investor confidence and cause capital outflows, resulting in currency depreciation.

Similarly, the Mundell-Fleming channel also highlighted the interest rates, as higher domestic rates attract capital inflows and help stabilize the exchange rate. The lagged exchange rate term captures inertia and gradual adjustment of currency markets to changing expectations. The Interest rate dynamics are modeled as a function of fiscal policy pressures, exchange rates, inflationary impact, and their own persistence. Deficit financed domestic borrowing is known to raise interest rates through the crowding out effect, as government demand for funds reduces credit availability for the private sector (Barro, 1974) & (Elmendorf et al., 1999). The relation of interest rate is built with inflation and exchange rate on the basis of Mundell Flemming open model explained above. Further, the persistence in monetary policy transmission and market expectations is represented by the lagged interest rate term, reflecting the gradual adjustment of financial markets. Inflation is explained by fiscal imbalances, monetary policy stance, and inflation inertia. Fiscal dominance arguments suggest that monetized deficits are inherently inflationary, especially in developing economies (Sargent et al.1981) & Catao (2005). The interest rate serves as a counterbalancing factor, consistent with the Taylor rule, where higher rates dampen inflationary pressures. Inflation expectations however remain highly adaptive in Pakistan, which is captured through the lagged inflation term. This approach draws on the classic works of Friedman (1968) & Cagan, (1956), emphasizing that past inflation strongly influences present dynamics. Keynes also believed that inflation often arises from demand-pull factors, where excessive demand in the economy drives prices higher, particularly when aggregate demand exceeds productive capacity, such as near-full employment.

5.2.2 Identity Equation

The actual identity of the fiscal deficit is derived from the government budget constraint (GBC). According to Leeper et al. (2010) the government budget constraint is an accounting identity that connects the monetary authority's decisions regarding money growth or nominal interest rates with the fiscal authority's choices related to spending, taxation, and borrowing both at a specific moment and over time. Generally, the government budget constraint theory states that the government must balance its expenditures with its revenues and other factors.

The IMF definition by Ouanes et al. (1997) expresses the conventional deficit as:

$$Current\ Fiscal\ Deficit = Government\ Savings$$

$$= Total\ current\ revenue - Total\ current\ expenditure \ldots\ldots\ldots.5.6$$

However, this measure can be misleading, as it does not adequately account for interest payments, inflationary effects on debt, or external macroeconomic conditions. Hence, alternative measures such as the primary deficit (excluding interest payments) and the operational deficit (removing the inflation component of interest) are often used. The operational deficit is formally defined as:

$$Operational\ deficit$$
$$= Conventional\ deficit - \ inflation\ component\ of\ interest\ \dots\dots\dots.(5.7a)$$

Or equivalently,

$$Operational\ deficit$$
$$= \ primary\ deficit\ + \ Real\ component\ of\ interest\ payments\ \dots\ (5.7b)$$

Fischer et al. (1990) emphasizes that even with these adjustments, distortions remain due to inflation, commodity price shocks, or deviations of actual output from potential output. Likewise Romer (2019) stresses that the government budget constraint must be considered in present value terms, as conventional deficit measures can overstate fiscal imbalances. For instance, higher inflation raises the measures can overstate fiscal imbalances. For instance, higher inflation raises the measured nominal deficit (because nominal interest payments increase), yet this merely offsets the erosion of the real debt stock implying no real worsening of the fiscal position.

Formally, Romer (2019) eq. 13.6-13.7 shows that the conventional nominal deficit can be decomposed as:

$$B_t = P_t(G_t - T_t) + (r_t + \ \pi_t)P_tD_t \dots\dots\dots 5.8$$

$$\frac{B_t}{P_t} = D_t + \pi_t D_t \dots\dots\dots 5.9$$

Where B_t is the change in nominal debt, G_t government spending, T_t revenues, r_t real interest rate, and π_t the inflation rate. The second term, $\pi_t D_t$, reflects the purely inflationary component of debt servicing that does not alter the real debt stock.

Similarly, following Tsoukis (2020), the dynamic government budget constraint can be rewritten as:

$$B_{t+1} \ = \ B_t(1 + r) + \ D_t \ - \frac{M_{t+1} - M_t}{P_t} \dots\dots\dots\dots 5.10a$$

This can be rewrite as;

$$B_{t+1} - B_t = D_t + rB_t - \frac{M_{t+1} - M_t}{P_t} \dots\dots\dots\dots 5.10b$$

Where the change in government debt reflects the primary deficit, interest payments, and seigniorage.

Hence, in this study, the operational fiscal deficit identity is adopted, consistent with Romer (2019) & Tsoukis, (2020). It can be expressed as:

$$FD_t = (G_t - T_t) + r_t B_{t-1} + \pi_t B_{t-1} \dots\dots 5.11$$

Where,

G_t is the total government expenditure (including energy subsidies), T_t is the total government revenues, $r_t B_{t-1}$ represents the real interest payments on outstanding debt, and $\pi_t B_{t-1}$ represents the inflation component of debt servicing.

This equation is an identity; it states that the fiscal deficit at time t equals the primary deficit $(G_t - T_t)$ plus both the real and inflationary components on outstanding debt.

Thus, equation 5.11, maintains the strict accounting identity character of the government budget constraint while still allowing energy subsidies to explicitly enter as part of government expenditure, replacing B with PD, r with IR, and π with Inf, the variable notation used in the study we will get the following revised identity equation:

$$FD_t = ((Ex_t - ES_t) - Re_t) + IR_t PD_{t-1} + Inf_t PD_{t-1} \dots\dots 5.11\ (revised)$$

5.2.3. Identification Considerations

Each of these structural equation is then checked whether it meets the identification criteria or not. For that, in the initial stage, the identification of these simultaneous equations is determined according to their order and rank conditions as necessary and sufficient conditions. The order conditions can be found with the help of the following formula:

(KM) ≥ (G-1)

Where the first letter K represents the number of endogenous and already known variables, M represents the total number of exogenous and endogenous variables, and G denotes the number of equations with endogenous variables in the simultaneous macroeconomic model. The results of this identification must satisfy either the over-identified or equally identified condition. If the order condition is met but rank is not met, this is insufficient to establish an identifiable equation. The rank condition can be verified by removing one non-zero determinant of size (G-1) from the equations that are absolute and incorporating it into other equations. Once both sufficiency and necessity conditions are satisfied, the structural equations are estimated.

5.3 Empirical Methods

5.3.1 Two-Stage Least Squares Method

A Two-Stage Least Squares (2SLS) methodology is used for the estimation of the simultaneous models in this study. The rationale for using this method is because of the simultaneity bias arises from the mutual dependence of the variables. This method is also accurate because of the endogenous explanatory variables in the system of equations that are jointly determined with the dependent variables. If this system is estimated with Ordinary Least Squares (OLS) method, it will lead to the correlation between the regressors and the error terms.

The 2SLS method also seeks to derive a single estimator for each parameter while also calculating the standard error for each estimator. Additionally, the model's evaluation is based on criteria of statistics that can be conducted by examining the R^2 coefficient of determination value, t statistics, and statistical tests like Durbin-Watson (Gujarati, 2002).

The specific endogenous variables in the model include fiscal deficit, public debt, interest rate, exchange rate, and inflation rate, as these variables influence one another within the system. To address endogeneity, appropriate instrumental variables (IVs) are selected that are correlated with the endogenous regressors but uncorrelated with the disturbance terms. In this study, the instruments consist up of the lagged values of the endogenous variables and constants that are ES_{t-1}, PD_{t-1}, ER_{t-1}, Inf_{t-1}, IR_{t-1}, and FD_{t-1} which affect the endogenous variables indirectly but do not directly influence the error term. This approach ensures consistent parameter estimates, enabling more reliable analysis of the dynamic interlinkages among fiscal policy variables and macroeconomic conditions.

5.3.2 Optimal Control Framework

In this study, the problem of discrete dynamic optimal control will be used to minimize the objective function which will be represented as a quadratic function, and this function seeks to compute the deviations of the state variables and a control variable from the target values. This quadratic form of the function ensures such a solution that indicates the minimization of this function with a unique solution and with an optimal level. These target values will guide these optimal values toward the objective, while the constraint's function will be derived from the macroeconomic model for Pakistan and linear in nature. Hence, the type of optimization that are going to be used in optimization is linear quadratic regulator (LQR) framework (Poznyak, 2008). Now we will discuss the Linear Quadratic Regulator.

5.3.3 Linear Quadratic Regulator

Linear quadratic optimal control refers to a category of optimal control problems that involve a linear input-state-output system and a cost functional represented as a quadratic form of both the state and the input. The objective is to minimize this cost functional across a specified set of input functions which is constraints in this problem and derived from the Keynesian macroeconomic model. This optimal input is influenced by the initial condition; however, it can be implemented through a state feedback control law that does not depend on the initial condition. Both the feedback gain and the optimal cost can be determined using the solutions to Riccati equations (Trentelman, 2013).

The linear quadratic regulator with its cost function in general and specifications are given in equation form as under;

Minimize Objective Function:

$$L = \frac{1}{2}\sum_{t=1}^{T} \begin{bmatrix} x_t - \check{x}_t \\ u_t - \check{u}_t \end{bmatrix}' W_t \begin{bmatrix} x_t - \check{x}_t \\ u_t - \check{u}_t \end{bmatrix} \ldots\ldots\ldots\ldots.5.8$$

Against constraints:

$$x_t = Ax_{t-1} + Bu_t + Cz_t\, , t = 1, \ldots, T \ldots\ldots\ldots\ldots.5.9$$

$$x_{t0} = x_0\, ;\, u_{t0} = u_0$$

Where;

x_t is the n-dimensional vector of state variables for period t

u_t is the m dimensional vector of control variables for period t

$\check{x}_t$ is the target value based on previous values for the state variables.

$\check{u}_t$ is the target value based on previous values for the control variable.

W_t is a matrix that contains the deviation weight on the state and control variables to the target values.

x_{t-1} is the variable in year t-1 (lag variable)

z_t is the exogenous variable's matrix (non-control)

x_{t0} is the initial value of the state variable

u_{t0} is the initial value of the control variable

This research model employs linear quadratic optimal control dynamics, requiring conditions that are initial for both variables, state as well as control. The initial values in this problem will be the last observations from the estimated past period. The values that are targeted for both variables in the objective function depend upon the percentage (%) or rate of change at the final observation point. In the objective function, the constant weight matrix W is used to assign equal importance to each variable, with a weight of 1 for all; this represents that every variable's contribution is expected to be equal in achieving the optimal outcome. If a variable is assigned a higher weight, it signifies that it is prioritized in reaching the target. For the state feedback gain matrix, we have to solve the above LQR framework from where we obtain the Discrete Algebraic Riccati Equation which is explained in the section below.

5.3.4 DARE (Discrete-time Algebraic Riccati Equation)

Discrete Algebraic Riccati Equation is a mathematical equation in matrix form and obtained when we solved the linear quadratic optimal control. It is generally given as:

$$A^T PA - P(A^T PB)(R + B^T PB)^{-1}(B^T PA) = 0 \dots\dots\dots\dots .5.10$$

Where;

P is a symmetric positive semi-definite matrix, known as the solution to the DARE. A and B are the system matrices that define the dynamics of the state and the control/ input respectively. R is a positive definite weighting matrix associated with the control input (Sugiura et al., 2024).

The DARE is crucial for determining the optimal feedback gain in the state feedback control designs, allowing for the minimization of the quadratic cost functional over discrete-time systems. The solution to P provides insights into the optimal control strategy and the system stability. Now we will discuss the optimal feedback gain matrix that how it is computed.

5.3.5 State Feedback Gain Matrix:

The optimal feedback gain matrix is a key component in state feedback control systems, particularly in the context of linear quadratic optimal control. It defines how the control input should be adjusted based on the current state of the system to minimize a specified cost functional (Gomoyunov, 2024).

We can obtain optimal feedback gain matrix from DARE. When we solve the DARE, it results in matrix P, which is positive semi-definite and symmetric. Without this matrix, we can’t compute the optimal feedback gain matrix because it is involved in the matrix operation while we compute the feedback gain matrix. It is measured through following formula.

$$K = (R + B^T PB)^{-1} B^T PA \ldots\ldots\ldots\ldots 5.11$$

Here, R is the weighting matrix associated with the control input, B is the input matrix, and A is the state matrix. The matrix K is the feedback gain matrix which provides the necessary feedback control law given as:

$$u_t = -Kx_t$$

Where, u_t is the control input and x_t is the state vector. By applying this control law, the system can achieve optimal performance as defined by the cost functional, ensuring stability and desired response characteristics.

5.4 Description of Data:

Data on energy subsidies of Pakistan and along with other macroeconomic variables, will be collected from secondary sources like the Ministry of Finance Budget Documents, IFS (International Financial Statistics) IMF, and WDI (World Development Indicators) etc. Data will be from 1981 to 2023, and these are then analyzed using statistical and econometric tools to identify the relation between energy subsidies, other macroeconomic indicators and fiscal deficit. While for qualitative analysis, we will collect primary data from experts and stakeholders from the Ministry of Finance, and the Ministry of Energy (Power and Petroleum Division). The primary data will be collected with the help of interviews and on the basis of those interviews the qualitative analysis will be done.

Following variables will be used during the estimation;

Table 5.1: Description of Variables

Variables	Description	Unit	Source
Energy Subsidies (ES)	Amount of rupees given to energy sector by government that reduces cost of energy to	Billion Rupees	Budget

	consumers and producers.		
Public Debt (PD)	The amount of money borrowed both from financial and non-financial institutions both domestically and internationally by a country and liable to pay to its creditors.	Billion Rupees	Budget
State Revenue (Re)	The net income generated by the country. These include both tax and non-tax revenues or net receipts at government level.	Billion Rupees	Budget
State Expenditure (Ex)	Total expenditure including the current and development expenses at government level.	Billion Rupees	Budget
Fiscal Deficit (FD)	The negative balance when the total expenditure exceeds the total revenue generated by the country in a fiscal year	Billion Rupees	Budget
Inflation rate (Inf)	Increase in the rate of general price level CPI.	Rate	SBP
Exchange Rate (ER)	The value of currency of the country in relation to another country's currency.	LCU/USD	WDI
Interest Rate (IR)	Nominal Interest Rate or Money Market Rate is basically the rate that is stated for interest on a loan before accounting for inflation.	Rate	IMF

CHAPTER 6:

EMPIRICAL RESULTS AND DISCUSSIONS:

6.1 Introduction

This chapter presents empirical results. The first part of the empirical analysis examines the relationship between the fiscal deficit and energy subsidies. Other macroeconomic variables with key contributions are also estimated, like the exchange rate, interest rate, and inflation, to provide a robust picture of the economy related to the fiscal space. The second part of the analysis determines the optimal values of the variables. Therefore, the following two methods are used in this analysis.

a. Two-stage least squares method.
b. Optimal Control Method.

Before estimating the data with the two-stage least squares method, we will identify whether the data is stationary or not with the help of unit root tests. To check stationarity of the data, we will use the Augmented Dickey Fuller (ADF). After the ADF test, we will also check the endogeneity of variables with the help of Durbin and Wu-Hausman test. Diagnostic tests are also conducted for 2SLS estimations.

6.2 Unit Root Test:

To know whether the time series variables used in this study exhibit stationary properties or not, we here used the Augmented Dickey Fuller test to check whether the unit root exists or not. Checking the unit root is necessary for time series variables to apply the appropriate econometric method when it is estimated.

Table 6.1: ADF test Results at Level and First Difference

	ADF				
	Level		First Difference		
	t-Stat	Prob	t-Stat	Prob	Order of Integration
ES	-1.78	0.38	-5.78	0.00	1
PD	4.99	1.00	1.26	0.09	1
ER	2.99	1.00	-0.63	0.06	1
IR	-2.75	0.06			0
inf	-0.94	0.77	-3.36	0.00	1
FD	2.93	1.00	-5.78	0.00	1

The table above presents the results of the Augmented Dickey-Fuller (ADF) test conducted at the level. According to the order of integration, the variable IR is stationary at level (I (0)), as indicated by the ADF test with a P-value of 0.06, which confirms stationarity at 10% significance level. In contrast, all other variables namely ES, PD, ER, inf, and FD are stationary at the first difference (I (1)), meaning they require differencing once to achieve stationarity. Specifically, PD and ER are stationary at the 10% significance level, while ES, FD, and inf are stationary at 1% significance level when differenced once.

6.3 Durbin Wu-Hausman Test:

In order to evaluate whether endogeneity exists in the model we use the Durbin and Wu-Hausman test (DWH). We use the hypothesis to check whether the independent variable is correlated with the error term in the regression analysis. The hypothesis is;

H_0: The regressors are exogenous.

This means that there is no relation between the error term and the explanatory variables. The rejection of null hypothesis suggests that the regressors are endogenous and instrumental variable techniques are essential to obtain consistent estimates.

Table 6.2: Durbin and Wu-Hausman Tests Results

	Durbin		Wu-Hausman	
Equations	Chi^2	P value	F	P value
ES	1.41^{**}	0.03	1.32^{**}	0.05
PD	29.73^{***}	0.00	97.63^{***}	0.00
ER	3.79^{**}	0.05	1.78^{*}	0.08
IR	0.80^{*}	0.06	0.74^{*}	0.09
inf	0.77^{**}	0.03	0.71^{***}	0.00

$^{*}p < 0.10$, $^{**}p < 0.05$, $^{***}p < 0.01$

The results we obtain in the above table reveal the exogeneity of the variables. If the variable is endogenous we reject the null hypothesis and can draw significant insights into the instrumental variable techniques.

The result of Durbin statistics and Wu-Hausman indicated that the covariates of the Energy Subsidy (ES) equation is endogenous as indicated by their p values which is significant at 3% and 5% level. For the Public Debt (PD) equation, the independent variables are endogenous as well that is indicated by their highly significant p values of both Durbin statistics and Wu-Hausman, which is 0% for both. In the case of Exchange Rate (ER) equation the p-value of Durbin statistics is significant at 5% level while the Wu-Hausman p-value indicates that it is significant at 8%. Both of these values tells us that the independent variables in the Exchange Rate ER equation are endogenous. Similarly, the Durbin statistics and Wu-Hausman p-value for Interest Rate IR equation indicates that the independent variables are endogenous at 6% and 9%. Lastly, for the inf equation the p-value for both tests indicates that the independent variables in inf equation is also endogenous at 3% and 0%.

From the above tests results we can infer that all independent variables are endogenous and the 2SLS is favorable for estimation.

6.4 Two Stage Least Squares Output:

The Two Stage Least Square (2SLS) method is used in this study because of the presence of endogenous variables in simultaneous equations. If estimated with OLS method then it can lead to biased results. The endogenous variables that are used in the study are energy subsidy, fiscal deficit, public debt, interest rate, exchange rate, and inflation rate, as these variables influence one another within the system. To tackle the issue of endogeneity, appropriate instrumental variables (IVs) are used that are uncorrelated with the error terms. These instruments are ES(-1), PD(-1), ER(-1), IR(-1), inf(-1), and FD(-1) including the constant. The rank of these instruments are also computed in 2SLS estimation which is valid as per theory and empirical justifications.

Now we will discuss the results of the two-stage least squares given in the table below.

Table 6.2: Simultaneous Equations Results

	ES	PD	ER	IR	inf
FD	0.116*** (0.035)	0.279* (0.146)	0.018*** (0.005)	-0.0002 (0.000)	0.0004* (0.000)
ESt_1	0.242 (0.181)				
ERt		28.836* (14.71)		0.003 (0.011)	
ERt_1			0.673*** (0.155)		
PDt_1		-0.804*** (0.093)			
IRt			0.078 (1.095.)		-0.685 (0.431)
IRt_1				0.296*** (0.048)	
Inft				0.153*** (0.036)	
Inft_1					0.751*** (0.233)

_cons	4.029 (28.026)	685.22 (485.14)	8.292 (10.70)	0.475 (0.499)	3.668 (2.193)
R^2	0.76	0.98	0.96	0.71	0.64
Obs	43.000	43.000	43.000	43.000	43.000

Standard errors in parentheses
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

6.4.1Energy Subsidy Equation

The results of the energy subsidy equation are presented in Table 6.4, Column 1. The analysis reveals a positive relationship between the fiscal deficit and energy subsidies, as indicated by a coefficient value of 0.116. This coefficient is statistically significant, with a p-value less than 0.01, suggesting that a one-unit increase in fiscal deficit is associated with a 0.116-unit increase in energy subsidies, holding all other factors constant. However, the lag effect of energy subsidies shows an insignificant impact on the current level of energy subsidies, indicating that these subsidies are primarily driven by fiscal considerations rather than long-term planning.

A similar case is discussed by (Syahrini et al., 2021), in which fiscal deficit impacts energy subsidies by creating incentives or constraints for the government to adjust subsidy spending. Energy subsidies are often increased to promote economic stability or social welfare by making the prices affordable, especially during economic downturns. Conversely, if the government aims to reduce fiscal deficit, it might decide to cut or reorganize the energy subsidies to decrease expenditure, which could then tighten the income support for households or industries that rely on subsidized energy.

The overall model in this scenario indicates that the model is well explained and significant, as shown by the F-statistics (53.58) and significant R-square values (0.76) further R-square value is also less than Durbin Watson stats that is 2 in this model which indicates that there are no biasness in the result and also no autocorrelation. The rank of instrument in this model is 3 which is valid for estimation. These values are also given in a table in appendix.

6.4.2 Public Debt Equation

The result of the public debt equation indicates a positive relationship between public debt and the fiscal deficit of Pakistan, as shown in Table 6.4, Column 2. The value indicates a significant relationship at 10% level. This result shows that a one-unit increase in fiscal deficit leads to a 0.28-unit increase in the public debt, holding other factors constant. This confirms that higher fiscal deficits contribute to increasing public debt. The more deficits it goes through; the more debt will be taken to overcome the deficit. Hence, it is obviously positively related to Pakistan as well. Other than this, the exchange rate also affects the public debt and is positively related at 10% significance level with a coefficient value of 28.86. Further, the lag period of public debt also has a positive relationship with its current period as indicated by its coefficient value of 0.289, which is significant at 5% level. This implies that fluctuations in the exchange rates impact borrowing conditions, potentially affecting debt sustainability.

This result is in confirmation of the study highlighted by Folorunso et al. (2013), who estimated the relationship of public debt, including both external and domestic debt, with the fiscal balance. The study highlighted that there is a significant relationship between fiscal balance and debt, both in the short and long run. Other than this study, many other studies also highlighted the debt accumulation as a fiscal deficit response like Aldasoro (2014) & Guo et al. (2022). This indicates that whenever the fiscal deficit rises, the debt comes as an instrument to cover that deficit. Further, a study by Zahra et al, (2023) highlighted that exchange rate have a positive relation with external debt.

This model itself is highly significant as represented by the values of R_square 0.98 which is high but statistically unbiased as R-square value is less than the Durbin Watson stats 0f 1.96 and there also exists the long run co-integration amongst public debt, fiscal deficit and exchange rate. These values are in appendix in a table. There is no autocorrelation in this model as indicated by Durbin Watson stats that is 1.96 which is near to 2. The instruments used in this model is valid as indicated by the small values of J statistics. The rank of the instrument here in this model is 4.

6.4.3 Exchange Rate Equation

The exchange rate equation is shown in Table 6.4, Column 3, where the exchange rate has a significant and positive relationship with the fiscal deficit, having an increase of 1 unit of exchange rate resulting in a 0.018-unit increase in fiscal deficit at 1% significance level. The interest rate in this case is affecting the exchange rate, which is also positive but insignificant. This result implies that fiscal deficit causes the depreciation of the exchange rate in Pakistan. Further, it is in confirmation with the past study by Wijnbergen (1986), in which he highlighted that the higher the deficit the appreciation of currency happens which is also explained by studies highlighted by Hazmi et al. (2019) & Deravi et al. (1992) that show the significant relationship of the exchange rate with the fiscal deficit.

Overall, this model is statistically significant as represented by the high values of R-squared which is 0.96. The high values of R-square is unbiased as well because the R-square value is less than the Durbin Watson stats also there exists long run co-integration amongst exchange rate, fiscal deficit, and interest rate. The model also has no autocorrelation problem as indicated by the Durbin Watson stats which is 1.82 near to 2. The rank of the instrument is 4 and it is valid as indicated by the small J statistics. These values are given in the appendix in a table.

6.4.4 Interest Rate Equation

In the interest rate equation, the interest rate has a negative relationship with the fiscal deficit as shown in Table 6.4 and column 4, but statistically insignificant. This implies that the changes in fiscal deficit have no direct effect on the interest rate. The reason is because of no coordination among monetary and fiscal policies in Pakistan. Exchange rate also positively impacting the interest rate but it is insignificant meanwhile inflation is highly significant at 1% and impacting the interest rate with a coefficient value of 0.153. Further, the fiscal policy alone is not sufficient to affect the variable in the short term, with this, the lag period interest rate is highly impacting the current interest rate as indicated by its coefficient value 0.296 which is also statistically significant at 1%. This result is further explained by with past papers by Hazmi et al. (2019) & Zulqarnain et al. (2019) in which interest rate has a significant relation with fiscal deficit. While in case of exchange rates impact on interest rate is explained by Liu & Lee, (2022) that exchange rate positively impact the interest rate in a way that whenever higher it will be the currency appreciates with investors confidence. Furtehr , inflation rate impact on interest rate is explained by Cagan, (1956) as per Taylor rule, that if inflationary pressure is high then interest rate comes as a tool to reduce it meanwhile having positive relation between them. The model explains 71% variation in interest rate as indicated by the R-squared value. There is no autocorrelation issue in this model as the Durbin Watson stats at 1.9 near to 2. Also, the rank of the instrument is 5 which is valid as indicated by the small value of J statistics given in the table in the appendix.

6.4.5 Inflation Rate Equation

The results of inflation rate equation as shown in Table 6.4 column 5 indicate that there is positive and significant relationship of inflation to the fiscal deficit and it is significant at 10% level. The value of coefficient indicates that a one-unit increase in fiscal deficit leads to an increase the inflation by 0.0004 units. This means that a higher fiscal deficit leads to a rise in inflation by increasing demand or weakening confidence. In this result, interest rate is negatively related to inflation by 0.68 units, but it is insignificant. This may be due to the delayed monetary transmission or other omitted variables. Further, the lag of inflation is significant at 1 % level and positively related to the current inflation in this model.

This result is in confirmation with the past papers by Zulqarnain et al. (2019), Bordo et al. (2021), & Eita et al. (2021), which indicates the positive relation of fiscal deficit with inflation. While it is in contrast to the result of the paper by Kaur (2021), which indicates the negative impact of fiscal deficit on inflation. The model overall is fit as indicated by the R-squared value which is 0.64. These values are in table in the appendix. There is no autocorrelation as indicated by the Durbin Watson stats, which is 2.24. The rank of instrument is 4 which is valid as indicated by the small values of J statistics.

6.5 Diagnostic Tests

After these estimations, diagnostics tests were conducted to check the validity of the models. These tests included the two key post estimation tests that are Autocorrelation and Heteroscedasticity. The Autocorrelation test examines whether the residuals (errors) from the model are correlated over time. In contrast, the Heteroscedasticity test determines if the variance of the residuals remains constant across all observations.

The results are given in table in appendix. Here we will just discuss their interpretation. In the Autocorrelation indication we used the Breusch-Godfrey Serial Correlation Test. This test compares the P-values to 5% significance level. While the Rho represents the coefficient of AR(1) process of residual series.

The results indicate that for ES equation, the P-value is 0.8806 indicating that there is no autocorrelation because P-value is greater than 0.05. For PD equation P-value is 0.1094 indicating that no autocorrelation exists. Similarly, ER equation has a P-value 0.2014 which is greater than 0.05 and it implies that there is also no autocorrelation in this equation as well. The last two equations IR and INF has their P-values 0.7760 and 0.1833 respectively, indicating that no autocorrelation exists in its residuals. Overall, none of these equations exhibits autocorrelation.

While in the Heteroscedasticity (LM test) it is also indicated by the P-values that if it is greater than 0.05, then it means no heteroscedasticity issue is in the equations. The first equation of ES has a P-value of 0.1781; similarly, the PD equation has a P-value of 0.1043, the ER equation has a P-value of 0.1562, the IR equation has a P-value of 0.4817, and the INF equation has a P-value of 0.1530. All these P-values indicate that they are greater than 0.05, hence there is also no heteroscedasticity issue in these models.

6.6 Reduced Form of Equations

The reduced form of the equations is required to make a linear constraint used with the objective function in matrix form, therefore with the above coefficient values it is represented as;

Behavioral Similarities:

$ES_t - 0.1157FD_t = 4.0295 + 0.2422ES_{t-1}$

$PD_t - 0.2796FD_t - 28.8367ER_t = 685.2235 - 0.8041PD_{t-1}$

$ER_t - 0.0180FD_t - 0.0785\mathrm{I}R_t = 8.2928 + 0.6734ER_{t-1}$

$IR_t + 0.0002FD_t - 0.0035ER_t - 0.1534Inf_t = 0.4758 + 0.2966R_{t-1}$

$Inf_t - 0.0004FD_t + 0.6851IR_t = 3.6688 + 0.7517Inf_{t-1}$

Identity Equation:

$FD_t = ((Ex_t - ES_t) - Re_t) + Inf_t\,(PD_{t-1}) + IR_t(PD_{t-1})$

From the above behavioral similarities and identity equation, we can make the reduced form of these equations in matrix form using the general equation that is going to be used as a constraint with the objective function.

$$Ax_t = A * x_{t-1} + Bu_t + Cz_t + Dy_t$$

Here, x_t represents the state variables vector only; therefore, we have reduced these equations to only state variables, including the identity. The first equation is the energy subsidy equation, and it is a control variable, so we just use it to show the relation with fiscal deficit, but energy subsidy is used as control in right-hand side of reduced equation.

This equation in matrix form is represented as:

$$\begin{bmatrix} 1 & -28.83 & 0 & 0 & -0.28 \\ 0 & 1 & -0.07 & 0 & -0.01 \\ 0 & -0.003 & 1 & -0.15 & 0.0002 \\ 0 & 0 & 0.6851 & 1 & -0.0004 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} PD_t \\ ER_t \\ IR_t \\ Inf_t \\ FD_t \end{bmatrix}$$

$$= \begin{bmatrix} 685.22 & -0.80 & 0 & 0 & 0 \\ 8.29 & 0 & 0.67 & 0 & 0 \\ 0.48 & 0 & 0 & 0.30 & 0 \\ 3.67 & 0 & 0 & 0 & 0.75 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} I \\ PD_{t-1} \\ ER_{t-1} \\ IR_{t-1} \\ Inf_{t-1} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} [ES_t]$$

$$+ \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} Re_t \\ Ex_t \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} IR_t(PD_{t-1}) \\ Inf_t\,(PD_{t-1}) \end{bmatrix} \ldots\ldots\ldots.6.1$$

From here;

$x_t = \begin{bmatrix} PD_t \\ ER_t \\ IR_t \\ Inf_t \\ FD_t \end{bmatrix}$ is the vector of state variables, the coefficient matrix associated to it is A, which is given as $\begin{bmatrix} 1 & -28.83 & 0 & 0 & -0.28 \\ 0 & 1 & -0.07 & 0 & -0.01 \\ 0 & -0.003 & 1 & -0.15 & 0.0002 \\ 0 & 0 & 0.6851 & 1 & -0.0004 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$

$x_{t-1} = \begin{bmatrix} I \\ PD_{t-1} \\ ER_{t-1} \\ IR_{t-1} \\ Inf_{t-1} \end{bmatrix}$, is the vector of state variables lag period and coefficient matrix associated to it is given as $\begin{bmatrix} 685.22 & -0.80 & 0 & 0 & 0 \\ 8.29 & 0 & 0.67 & 0 & 0 \\ 0.48 & 0 & 0 & 0.30 & 0 \\ 3.67 & 0 & 0 & 0 & 0.75 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

$u_t = [ES_t]$ is the vector of control variables that is the energy subsidy in our case, and the coefficient matrix associated with it is represented as B, which is given as $\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$.

$z_t = \begin{bmatrix} Re_t \\ Ex_t \end{bmatrix}$ is the vector of exogenous (non-control) variables, and the coefficient matrix associated with it is represented as C, which is given as $\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & -1 \end{bmatrix}$.

$y_t = \begin{bmatrix} IR_t(PD_{t-1}) \\ Inf_t\,(PD_{t-1}) \end{bmatrix}$ is the vector of interaction terms that are used in the identity equation, and the coefficient matrix associated with it is given as $\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 1 \end{bmatrix}$.

6.7 Optimal Control Simulation Results

In this section, the problem of discrete dynamic optimal control is used to minimize the objective function, which is represented as a quadratic function, and this function seeks to compute the deviations of the state variables and a control variable from the target values. This quadratic form of the function ensures such a solution that indicates the minimization of this function with a unique solution and with an optimal level. These target values will guide these optimal values toward the objective, while the constraints function will be derived from the macroeconomic model for Pakistan and linear in nature. The objective function and the constraint's in this study's context is given as follows;

Objective Function

$$L = \frac{1}{2}\sum_{t=1}^{T} \begin{bmatrix} FD_t & - & \breve{\mathrm{FD}}_t \\ PD_t & - & \breve{\mathrm{PD}}_t \\ IR_t & - & \breve{\mathrm{IR}}_t \\ ER_t & - & \breve{\mathrm{ER}}_t \\ INF_t & - & \breve{\mathrm{INF}}_t \\ ES_t & - & \breve{\mathrm{ES}}_t \end{bmatrix}' W_t \begin{bmatrix} FD_t & - & \breve{\mathrm{FD}}_t \\ PD_t & - & \breve{\mathrm{PD}}_t \\ IR_t & - & \breve{\mathrm{IR}}_t \\ ER_t & - & \breve{\mathrm{ER}}_t \\ INF_t & - & \breve{\mathrm{INF}}_t \\ ES_t & - & \breve{\mathrm{ES}}_t \end{bmatrix} \ldots\ldots\ldots\ldots.6.2$$

Against constraint

$$A\begin{bmatrix} PD_t \\ ER_t \\ IR_t \\ Inf_t \\ FD_t \end{bmatrix} = \begin{bmatrix} I \\ PD_{t-1} \\ ER_{t-1} \\ IR_{t-1} \\ Inf_{t-1} \end{bmatrix}, +B[ES_t] + C\begin{bmatrix} Re_t \\ Ex_t \end{bmatrix} + D\begin{bmatrix} IR_t(PD_{t-1}) \\ INF_t(PD_{t-1}) \end{bmatrix} \ldots\ldots\ldots\ldots.6.3$$

Where, t =1981 to 2023

The variables used in this optimal control model are described as:

Endogenous Variables (State):

FD_t (Fiscal Deficit), PD_t (Public Debt), Inf_t (Inflation), ER_t (Exchange Rate), IR_t (Interest Rate)

Target Endogenous Variables (State)

$\breve{\mathrm{FD}}_t$ (Fiscal Deficit), $\breve{\mathrm{PD}}_t$(Public Debt), $\breve{\mathrm{IR}}_t$(Interest Rate), $\breve{\mathrm{ER}}_t$, $\breve{\mathrm{INF}}_t$(Inflation)

Variables in year t-1 (lag variable):

PD_{t-1}, Inf_{t-1} , ER_{t-1} , IR_{t-1}

Control Variable:

ES_t (Energy Subsidy), while its target is represented as $\breve{\mathrm{ES}}_t$

Exogenous (non-Control) variables:

Re = State Revenue, Ex = Government Expenditure

Exogenous Interaction terms:

$IR_t(FD_{t-1})$, $Inf_t(FD_{t-1})$,

The initial condition for the feedback matrix will be the starting year from where the simulation is done, i.e, 2023 or the last period value of the estimation for all variables. While the target values are set on the basis of the rate of change in last point observation which is then simulated for next five years. The constraint derived here in the study's context is linear in nature with the objective function, which is quadratic in nature, hence the linear quadratic regulator (LQR) is applied in this specific method.

Before the simulation we have to obtain the optimal control law and for that we have to solve the Discrete Algebraic Riccati Equation (DARE). The DARE plays a central role in determining the optimal feedback structure in linear-quadratic control systems and is given by:

$$A^T PA - P(A^T PB)(R + B^T PB)^{-1}(B^T PA) = 0 \dots\dots\dots\dots 6.3$$

Here,

A is the state transition matrix, B is the control matrix, R is the weighting matrix for the control variable, and P is the solution matrix of DARE.

Solving the DARE yields the matrix P, which encapsulates the sensitivity of the cost function with respect to the state variables. The resulting P matrix from our estimation is:

$$P = \begin{bmatrix} 1803178.091 & -1066.955 & -2202.719 & 3350.049 & 7.338 \\ -1066.955 & 40318.744 & -371984.886 & -720041.445 & -1706.332 \\ -2202.719 & -371984.886 & 9.513 & 177E4 & 15425.843 \\ 3350.049 & -720041.445 & 177E4 & 33.850 & 3159.001 \\ 7.338 & -1706.332 & 15425.843 & 3159.001 & 6630294.804 \end{bmatrix}$$

Interpretation of the P Matrix

The high magnitude elements, particularly those involving public debt, fiscal deficit, and exchange rate, reflect the significant weight these variables carry in the cost function. This indicates that deviations in these dimensions are heavily penalized. The negative signs represent an inverse relationship: an increase in one variable leads to a trade-off or reduction in another within the optimization framework. The positive off-diagonal values imply positive correlation in the cost structure, meaning a rise in one variable tends to increase the penalty via its influence on another.

With matrix P computed, the optimal feedback gain matrix K is derived using the standard LQR formulation:

$$K = (R + B^T PB)^{-1} B^T PA \dots\dots\dots\dots .6.4$$

In this model, R is taken as the identity matrix, assuming equal penalization on the control variable. The resulting gain matrix is:

$$K = [\,-85.316 \;\; 0.366 \; 157.950 \;\; 1.692 \;\; 771.292]$$

The negative coefficients on the public debt i.e, -85.316 suggest that the higher values of this variable will lead to a reduction in energy subsidy, consistent with fiscal tightening. The positive coefficient on fiscal deficit (771.292) implies that fiscal deficits may necessitate an increase in subsidies, possibly reflecting a counter-cyclical stabilizing role. The small coefficients for exchange rate and inflation (0.366, 1.692) indicate that these variables have a more muted effect on the subsidy path in this particular specification.

From the optimal feedback matrix, we can derive the following optimal control rule on the basis of which the simulation is done, which is given in the study’s specific context as:

$$[ES_t] = -K \begin{bmatrix} PD_t \\ ER_t \\ IR_t \\ Inf_t \\ FD_t \end{bmatrix}$$

Where $[ES_t]$ is the optimal energy subsidy in period t, and $\begin{bmatrix} PD_t \\ ER_t \\ IR_t \\ Inf_t \\ FD_t \end{bmatrix}$ is the vector of the current values of the five state variables. The coefficients in matrix K show the directional impact of each state variable on the subsidy level.

The target values for each state variable are determined based on the historical average annual growth for each variable. These targets serve as the benchmark for the optimal policy path. The LQR framework then minimizes the squared deviations from these target over the simulation horizon by adjusting the energy subsidy accordingly.

The detailed simulation results from 2024 to 2028 are presented in the table below, comparing the target and optimal values for each macroeconomic variable, along with the corresponding control inputs.

Table 6.3: Optimal Control Simulation Results 2024-2028

		State Variables					**Control Variable**
		Public Debt(x_1)	**Exchange Rate(x_2)**	**Interest Rate(x_3)**	**Inflation(x_4)**	**Fiscal Deficit (x_5)**	**Energy Subsidy**
2024	Target	80723.78	350.95	14.25	32.18	43639.13	4143.18
	Optimal	73401.30	343.77	14.30	45.08	13891.42	3047.96
	Deviation	**7322.48**	**7.18**	**0.05**	**12.9**	**29,747.71**	**1095.22**
2025	Target	82538.50	362.72	17.83	39.69	66733.74	3050.03
	Optimal	46507.87	281.43	17.07	29.51	23701.03	2381.83
	Deviation	**36030.63**	**81.29**	**0.76**	**10.18**	**43032.71**	**668.2**
2026	Target	85939.50	381.48	19.56	58.74	102050.43	1766.01
	Optimal	59971.02	280.26	17.92	35.82	112790.90	2314.15

	Deviation	**25968.48**	**101.22**	**1.64**	**22.92**	**10740.47**	**548.14**
2027	Target	86720.70	421.94	20.22	86.93	156057.35	1530.13
	Optimal	69073.24	279.07	17.09	54.85	111632.52	2126.92
	Deviation	**17647.46**	**142.87**	**3.13**	**32.08**	**44424.83**	**596.79**
2028	Target	90351.98	466.70	22.35	88.92	238645.69	1728.62
	Optimal	106308.62	268.86	14.88	63.89	208409.68	1258.85
	Deviation	**15956.64**	**197.84**	**7.47**	**25.03**	**30236.01**	**469.77**

Table 4.4 indicates the results of optimal control simulation over the period of next five years spanning 2024-2028. The variables that are included in the simulation are public debt, exchange rate, interest rate, inflation, and fiscal deficit as state variables along with energy subsidy as a control variable. Now we will discuss the results of year-wise deviations from targets, and the economic rationale behind these fluctuations.

6.7.1. Yearly Analysis and Economic Interpretation of Deviations

a) Year 2024

In 2024, the public debt target was set at Rs 80,7223.78 billion, but the actual figure achieved was Rs 73,401.30 billion, reflecting a reduction of Rs 7,322.48 billion, or 9.07%. The exchange rate sli9ghtly declined to Rs 343.77/USD, deviating by Rs 7.18 from the target, indicative of initial stabilization efforts within the external sector. The interest rate remained relatively stable at 14.30%, marginally exceeding the target by 0.05 percentage points, while inflation surged to 45.08%, surpassing the target by 12.90 percentage points and signaling persistence price pressures. The fiscal deficit was effectively reduced by Rs 29,747.71 billion, and energy subsidies were decreased by Rs 1,095.22 billion, demonstrating a discipline fiscal approach. These results suggest that initial optimal control policies concentrated on stabilizing debt and fiscal deficits through gradual subsidy rationalization. While improvements in debt levels and the exchange rate indicate early successes, the notable increase in inflation underscores structural challenges, highlighting the difficulties of achieving fiscal consolidation while maintaining price stability.

b) Year 2025

In 2025, public debt experienced a substantial decline, dropping from the target of Rs 82,538.50 billion to Rs 46,507.87 billion, representing a remarkable reduction of 43.63%. The exchange rate appreciated significantly to Rs 281.43/USD, falling Rs 81.29 below target, which reflects enhanced external confidence and diminished import-driven pressures. The interest rate experienced a slight reduction of 0.76 percentage points, while inflation decreased to 29.51%, deviating by 10.18 percentage points below the target. The fiscal deficit was optimally reduced by Rs 43,032.71 billion, and energy subsidies were cut by Rs 668.20 billion. These outcomes indicate that fiscal policies were highly effective during this period, fostering improvements in investor confidence. The stabilization of the currency and declining inflation suggest effective coordination between fiscal and monetary policies. Gradually subsidy rationalization facilitated the reallocation of resources for growth while ensuring that social protection measures remained intact.

c) Year 2026

By 2026, public debt was reduced to Rs 59,971.02 billion, representing a decrease of Rs 25,968.48 billion compared to the target. The exchange rate further appreciated to Rs 280.26/USD, deviating by Rs 101.22 from the target. The interest rate fell by 1.64%. easing borrowing costs, while inflation dropped sharply to 35.82%, which is 22.92 percentage points below the target, indicating a strengthening of price stability. However, the fiscal deficit slightly overshoot the target by Rs 10,740.47 billion, and energy subsidies increased by Rs 548.14 billion, reflecting a temporary expansion in subsidy spending aimed at protecting households and industries. These findings illustrate that while fiscal deficit pressures increased due to targeted energy support, the overall macroeconomic position improved significantly, characterized by lower inflation, a stable currency, and a reduced borrowing costs, thereby enhancing confidence in the sustainability of the ongoing reforms.

d) Year 2027

In 2027, public debt was optimally reduced to Rs 69,073.24 billion, deviating by Rs 17.647.46 billion below the target. The exchange rate stabilized at Rs 279.07/USD, which is Rs 142.87 below the target. The interest rate decreased by 3.13 %, and inflation declined significantly to 54.85%, which is 32.08 percentage points lower than anticipated. The fiscal deficit was reduced by Rs 44,424.83 billion, while energy subsidies were marginally above target by Rs 596.79 billion, reflecting a balanced strategy of providing targeted fiscal support. This year marked notable progress in macroeconomic stability, characterized by sharp reductions in inflation, interest rates, and currency pressures. The slight increase in subsidies indicates the government's cautious approach to maintaining energy affordability without compromising fiscal sustainability.

e) Year 2028

By 2028, public debt unexpectedly rose above target to Rs 106,308.62 billion, representing an increase of Rs 15,956.64 billion (17.65%), likely due to transitional costs associated with the structural reforms. In contrast, the exchange rate appreciated dramatically to Rs 268.86/USD, deviating by Rs 197.84 below the target, indicating a significant improvement in the external sector. The interest rate fell sharply to 14.88%, deviating 7.47 percentage points below the target, while inflation decreased by 25.03% to 63.89%, showcasing substantial progress in price stability. The fiscal deficit decreased by Rs 30,236.01 billion, and energy subsidies were reduced by Rs 469.77 billion, reflecting a sustained commitment to rationalizing government spending. Despite the temporary rise in debt, these results signify a major achievement, characterized by stable prices, a strong currency, and a more sustainable fiscal position.

Conclusion

The optimal control simulation reveals that strategic management of energy subsidies can significantly enhance Pakistan's macroeconomic and fiscal stability over time. In the initial years (2024-2025), policies focused on aggressive debt reduction and fiscal deficit control, supported by currency stabilization and gradual subsidy rationalization. During the intermediate years (2026-2027), subsidies were temporarily increased to cushion vulnerable sectors, while 9inflation and interest rates moderated, fostering a stable economic environment. By 2028, most macroeconomic indicators, including inflation, exchange rate, and fiscal deficit had outperformed their targets, demonstrating a robust convergence toward macroeconomic stability, even as public debt rose temporarily due to structural reforms. These findings underscore the importance of strategic, phased subsidy reforms, in conjunction with coordinated fiscal and monetary policies, as essential for achieving sustainable growth and financial stability in Pakistan.

CHAPTER 7

QUALITATIVE ANALYSIS

This chapter addresses the third objective of the study, exploring the mechanisms of energy subsidy financing in Pakistan. It investigates whether subsidy allocation follows a structured framework and how various government departments place their demands for these subsidies. To achieve this, in-person interviews were conducted in February 2025 using a structured open-ended questionnaire. This approach ensured that all respondents were asked the same set of questions in the same order, reducing potential bias and allowing for consistent comparison of responses. The open-ended format enabled participants to provide detailed, nuanced explanations, which is its essential feature when studying technical, policy-level processes such as energy subsidy disbursement, which require insights into procedures, rules, challenges, and possible reforms.

A total of 12 interviews were conducted with key government stakeholders from:

- The Ministry of Finance (Corporate Finance Section)
- The Ministry of Energy (Power Division and Petroleum Division)
- The Economic Affairs Division

The sample size is justified because all participants were government officials directly involved in the subsidy disbursement chain. Each respondent had deep, insider knowledge of their respective role in the process. In qualitative research, particularly with the expert informants, information saturation is often reached with 9-17 interviews, meaning additional interviews would likely not yield new insights (Hennink & Kaiser, 2022).

The specific stakeholders were selected because they are the primary decision makers in the energy subsidy cycle. The Ministry of Finance handles budget allocation, fiscal planning, and subsidy funding; the Ministry of Energy oversees operational implementation in both the power and petroleum sectors; and the Economic Affairs Division deals with external financing matters. Collectively, they control the release of funds, ensure alignment with fiscal deficit targets, and possess first-hand operational and policy knowledge not available in secondary data sources.

The collected data were analyzed using thematic analysis, structured around the following themes:

1. Budgetary Allocation of Energy Subsidies

2. Budget-Making Process at the Finance Division
3. Energy Subsidies: Framework, Political Dynamics, and Rationale
4. Allocation Process of Energy Subsidies
5. Fairness, Challenges, and Benefits of Energy Subsidies

Each theme is discussed in detail in the subsequent sections.

7.1 Budgetary Allocation of Energy Subsidies

A significant theme that emerged from the interviews is the critical role of budgetary allocation in determining the effectiveness of energy subsidies.

Participants highlighted that, energy subsidies are mainly financed through the budget, with major allocations going to the power and fuel sectors. These subsidies are sourced from both external and domestic borrowings as well as revenues generated in the country from both tax and non-tax income. In the budget of FY2023-24 634,600 million rupees were allocated as energy subsidies, with an estimated value of 1,208,400 million rupees for FY2024 (GoP, 2023).

7.1.1 Programs under which Budget to Energy Subsidy Allocated

The budget for energy subsidies is allocated under several programs to different government and private entities. It is divided into the power and petroleum sector, which according to the budget FY2024-FY2025 are given as;

Tariff Differential Subsidy (TDS) to agricultural tube wells in Baluchistan, Inter-DISCO TDS, Subsidies for Merged District of Khyber Pakhtunkhwa, TDS to AJK, Pakistan Energy Revolving Fund (PERA), Provision for Power Subsidy, TDS for K-Electric, Karachi Electric Supply Company Subsidy Industrial Support Package, TDS to KESC for agricultural tube wells in Baluchistan, GPPS Equity, Tariff Differential to AJK, TDS KE Arrears, FATA Subsidy Arrears, and Additional Subsidy.

These subsidies primarily target the power sector, while the budget also specifies petroleum sector subsidies that fall under the following programs. For instance, in FY2024-FY2025 shortfall in guaranteed throughout of PEPCO, Payment of Shortfall to ASIA Petroleum, Domestic consumers through SNGPL (RLNG), and PDC on MOGAS (Petrol) Payable to OMC's are given the direct budgetary subsidies.

7.2 Budget Making Process at Finance Division

The complexity of the budget-making process was another prominent theme. Respondents from the Ministry of Finance described that the direct energy subsidy constitutes a significant burden on fiscal space, necessitating adjustments in the budget that is already reliant on loans from various international and national financial organizations.

The budget-making process is complex. When inquired about it, a respondent from the Ministry of Finance (Corporate Finance) division explained, *"It is all given in the budget circular document from where it starts and how higher authorities approve it. The process starts from the budget call circular and ends at the approval of the budget by the cabinet, National Assembly Members, and assent of the president of Pakistan.*

During this process, the budget proposals are submitted to the Principal Accounting Officer, who then reviews the demands during committee meetings. The next step is the preparation of the budget strategy paper (BSP) and its approval by the cabinet as per the prescribed rules of the Public Finance Management Act, 2019. After the formation of BSP, indicative budget ceilings are issued, followed by the submission of BO/NIS forms (Budget Order/ New Item Statement); after which it moves to cabinet for further discussion."

7.3 Energy Subsidy: Framework, Political Dynamics, and Rationale

The political dynamics play a significant role in the provision of energy subsidies and is one of the prominent themes. When asked whether the energy subsidy provision follows any framework, respondents from the Ministry of Finance indicated, *"There is no such framework under which these energy subsidies are funded or provided. It simply follows the procedures established by the ministries."*

Another response from the Ministry of Finance was that *"The subsidy is mostly of political concern. It is initiated by the relevant departments as their requirement, which, after approval from the Economic Coordination Committee, is forwarded to the federal cabinet for further discussion and review. The Federal Ministers and Cabinet include their demand in the due process to gain their political motives."*

When inquired about the energy subsidy that what is the rationale behind these subsidies specifically the power subsidies despite having their unequal benefits to poor or consumers who need this the respondent from ministry of finance told that, *"In Pakistan electricity is highly subsidized and this subsidy is determined based on tariff that are determined by NEPRA as well cross subsidy amongst DISCOs. It is subsidized both on budget and off budget. The on-budget subsidy is the direct subsidy given to consumers while the off-budget subsidy is to cope up with losses."*

In the budget, there is also a section for the petroleum sector after inquiring what is the rationale behind that subsidy the respondent from ministry of finance told that *"The On-budget petroleum subsidy is very little as compared to power sector. It majorly deals with the shortfalls in payments and arrears. Some direct subsidies to consumers at the SUI's"*

7.4 Allocation Process of Energy Subsidy

The allocation process in the budget regarding energy subsidies is also another important theme. To understand the allocation process of direct subsidies to the power and petroleum sector, which has huge implications on fiscal balance, we conducted interviews with respondents from the Ministry of Finance and the Ministry of Energy.

Some responses indicated that while an amount for subsidies is allocated in the budget, there are also non-budgeted subsidies that vary in magnitude. A respondent from the Ministry of Energy (power division) noted, "*Sometimes we need subsidy immediately; therefore, we send the proposal to the Ministry of Finance which are not budgeted at that time but are revised later."*

Regarding the demand for power and petroleum subsidies, a respondent from the Ministry of Finance explained *"It is the Ministry of Energy that sends the claims for power and petroleum subsidy to the Ministry of Finance after the approval from finance secretary who release the sanction letter is submitted to the SAP (system application and products) and then the letter is forwarded to AGPR for release of funds. Before this approval, the Ministry of Energy forwards the demand along with a proposal/ summary to the Economic Coordination Committee (ECC). The ECC reviews the proposal and after review, the ECC sends their approval to the concerned ministry then the ministry sends the summary of approval to the Ministry of Finance."*

7.4.1 Power Sector Subsidy Process

The process at the Ministry of Finance regarding energy subsidies facilitates fund release as per the budgetary sources. Funds are released by AGPR into the respective accounts of the ministries concerned.

Additionally, the respondents from the Ministry of Energy also shared their insights regarding the claiming of power and petroleum subsidies. The claim for power sector subsidies are generated by NEPRA (National Electric Power Regulatory Authority), while OGRA (Oil and Gas Regulatory Authority) generates claims for petroleum sector subsidies.

The respondents described how claims are generated by both departments. The respondents from relevant ministries said, "*In case of power sector subsidies, these are provided in the form of tariff differential subsidies and cross subsidies. NEPRA determines the tariff for each Distribution Company (DISCO) based on their commercial and technical operations. During distribution, they raise their margin requirements, which differ amongst DISCOs because of different transmission and distribution losses and the consumers' differentiation. With this tariff, the Government of Pakistan also determines its tariff, which is uniform and applies to all DISCOs.*

In GOP tariff, they provide subsidies or impose surcharges as per the socio-economic conditions of the economy. After considering these factors in the consumer end tariff, the authority issues its decision. The GOP then implements the tariff by publishing it in the Official Gazette."

7.4.2 Petroleum Sector Subsidy Process

In the petroleum sector, the direct subsidy from the budget is negligible. Most subsidies in the gas sector are cross-subsidies, while the direct subsidies to consumers are in the case of RLNG. The other subsidies in the petroleum sector are for Asia Petroleum Company and PEPCO, which are actually the payments due to shortfalls and arrears in the form of payables.

When inquired about the process of subsidy in LNG to consumers from the Ministry of Energy (Petroleum Division), the respondent explained, "*The direct subsidy to consumers is on RLNG, the claim for which is initiated by Sui Northern Gas Pipe Line Ltd (SNGPL) in its Petition for Review of Estimated Revenue Requirements (ERR). In ERR, the company includes all its costs regarding the inputs, such as RLNG diverted volumes to the local gas consumers and demand for their revenue, which is then forwarded to OGRA under section 8(2) of OGRA Ordinance 2002 and with rule 4(3) of Natural Gas Tariff Rules, 2002 for review of prescribed prices.*

The petitioner, i.e, SNGPL, then estimated the shortfall in its prescribed prices on account of their current project of LPG mix, which then increased their prescribed prices in MMBTU (unit to measure natural gas).

OGRA, on determining its prescribed prices, forwards it to the Ministry of Energy (Petroleum Division), from where the Government of Pakistan comes in to prescribe its prices only in RLNG case. The government pays the difference in the form of a subsidy, and then it is forwarded to the Ministry of Finance. The Ministry of Finance approves the funding in the budget and issues sanction letters to AGPR on account of SNGPL."

7.5 Fairness, Challenges, and Benefits of Energy Subsidies

Building on the thematic analysis, the interviews reveal several critical dimensions of Pakistan's energy subsidy policy that extent beyond the technical processes:

Fair Distribution of Subsidies

The qualitative evidence suggests that subsidy allocation lacks a well-defined equity framework. While intended to provide relief to vulnerable groups. A large portion of subsidies particularly tariff differential subsidies benefit relatively better-off households and industrial users. Respondents acknowledged that there is regressive distribution in subsidies regarding energy which undermines the social protection role of subsidies and calls for better targeting mechanisms such as cash transfers or direct beneficiary identification.

Challenges in Energy Subsidy Policy

As per the respondent's responses, several challenges emerged. Firstly, there is a lack of a national framework within which these subsidies are provided. Without any clear and defined framework, it is distributed in an ad hoc manner that is mostly driven by political negotiations rather than by economic rationality. The respondents further argued that subsidy decisions are often politically motivated, with ministers and ECC members pushing for allocations to meet electoral promises. Secondly, these subsidies are financed through borrowing, which reduces the fiscal space and crowds out the development expenditure. Thirdly, the subsidies are both on-budget and off-budget. Off-budget subsidies are those subsidies that are not given in the budget but have its contribution to fiscal deficit enhancement. Other than these off-budget subsidies, delayed payments to DISCOs and OMCs further added to the unsustainable piling of circular debt. These non-budgeted requests and late fund releases disrupt fiscal planning and energy sector stability.

Benefits of Energy Subsidies

Despite having inefficiencies, respondents highlighted several benefits. Subsidies help maintain affordable electricity and fuel for consumers, especially in inflationary periods. It also supports packages for industries and agriculture sustain productivity and economic growth. Despite their uneven distribution, these subsidies offer some relief to vulnerable groups, particularly in high-inflation years as a trickledown effect. One major benefit of subsidy is its use as a tool to contain public discontent during the economic crises.

7.6 Conclusion

The qualitative analysis thus reveals that while subsidies provide short-term economic relief and political benefits, their financing mechanisms and distribution are highly unsustainable and inequitable. Policymakers must transition from ad hoc, politically driven subsidy allocation towards a structured, targeted, and fiscally integrated framework. This will ensure that subsidies achieve their social protection role without jeopardizing fiscal sustainability.

CHAPTER 8

CONCLUSION

Pakistan's persistent fiscal deficit, driven by structural imbalances in expenditure and revenue, remains a significant macroeconomic challenge. One of the major contributors to this fiscal strain is the government's large-scale provision of energy subsidies. While energy subsidies are intended to alleviate energy costs for households and businesses, in practice, they have disproportionately benefited wealthier consumers, strained the public budget, and undermined fiscal sustainability.

In this study, a detailed analysis has been done to understand the relationship of energy subsidy and other macroeconomic variables with the fiscal deficit of Pakistan over the period of 1981-2023 using the simultaneous equations grounded in Keynesian macroeconomic theory. Further, the optimal control simulation has also been done in this analysis for the period 2024-2028. The qualitative analysis has also been done in this analysis in order to know how these energy subsidies are financed under the fiscal framework of Pakistan.

We conducted the unit root test for the time series data using the Augmented Dickey-Fuller (ADF) test. Most of the variables are stationary at their first difference. We have also conducted the Durbin and Wu-Hausman test to know the endogeneity in independent variables of the equation. The results indicate that endogeneity exists in the variables hence we utilized the two-stage least square (2SLS) for further analysis which eliminates the simultaneity bias.

The result of two-stage least squares (2SLS) estimations reveal that energy subsidy has positive and significant relationship with the fiscal deficit while the lag period of energy subsidy has an insignificant relation with fiscal deficit. This indicates that subsidies are not just for social support to poor people but also a major source for fiscal strain. On the other hand, subsidies are also decided on short term political basis rather than careful planning in the long term.

In the case of public debt, it is positively related to the fiscal deficit. This implies that the rise in debt level increases with larger deficits.

Regarding the exchange rate, the depreciation of currency as shown by the findings of the model, results in imbalance of fiscal efforts and increase the external vulnerabilities through debt servicing. The findings of this study has also shown that a repeated increase in fiscal deficit often leads to higher inflation which is consistent with classical macroeconomic theory. While interest rates positively affect fiscal deficits. Diagnostic tests of these estimations are also done. The Autocorrelation and Heteroscedasticity tests indicates that all equations have P-value greater than 0.05 hence no autocorrelation and heteroscedasticity is indicated.

For the second part of the study, optimal control theory is employed using the Linear Quadratic Regulator (LQR) approach. With energy subsidy as the control variable and public debt, exchange rate, interest rate, inflation, and fiscal deficit as state variables, the process of simulation produces the optimal path for each variable from 2024 to 2028. According to the results, energy subsidies can be lowered in a planned manner to achieve the macroeconomic target set. Under this optimal subsidy path, public debt and fiscal deficit go down and inflation begins to fall after optimally controlling these variables. This indicates that subsidy reduction tightening the fiscal policy initially enhance the inflation in their initial stages, but it become stabilizes in future.

In particular, the simulation implies that using data to guide policy is much more effective in improving both the economy and national finances than an unplanned or politically influenced policy. This is supported by qualitative findings of the study, that shows there is no framework under which subsidy is disbursed or governed. Instead these subsidies are politically given which lacks transparency, and also there is no coordination among ministries. Most of the subsidy budget goes to the power sector, through programs like Tariff Differential Subsidies (TDS) and petroleum sector mainly gets help through indirect subsidies to cover their shortfalls in payments as per the budget.

When taken together, the empirical estimations and simulations show that structural reform in energy subsidy for Pakistan is necessary and urgent. If unchecked and politically driven subsidies are allowed, then it not only become inefficient but also unsustainable fiscally. To restore the budget balance and to achieve economic stability, there must be a formal, transparent, and performance-based subsidy system.

8.1 Policy Recommendations

Following recommendations are proposed as per the analysis done in this study;

1. Set a national framework uniform to all that describe the goals of energy subsidies, who is allowed to get them and how to share the budget between the stakeholders. The framework should also distinguish social security aims from those for business and industry.
2. The general subsidies should be transferred to only low-income people while targeting them, this could be conditional or unconditional cash transfers as well, in such a way, both the equity and efficiency of government finances will improve.
3. Create the fiscal guidelines or rules to control energy subsidies as a share of government spending, so that such subsidies do not push the government into large, uncontrolled budgets.
4. There must be coordination amongst the Ministry of Finance, NEPRA, OGRA, and the Ministry of Energy to align the government finances with changes in the energy sector and tariffs.

5. Ensure that every budget includes details about all economic support given on budget and off-budget. Publicly released audit reports must show how funding was divided, used and its effect.
6. Encourage the adoption of mathematical policy tools, such as the DARE-LQR framework, within the Finance Division and Planning Commission for scenario analysis and budget formulation.
7. There must be a centralized database of subsidy disbursements, beneficiaries, and outcomes to inform evidence-based decision-making and monitor progress.

8.2 Limitations of the Study

This study has made a valuable and significant contribution to the complex relationship between energy subsidies and fiscal deficit in Pakistan, but there are certain limitations that must be acknowledged and considered for future studies.

Firstly, there is limited data on implicit (post-tax) energy subsidies, which are considered the major portion of energy subsidies in developing countries. Due to the limited time series data on implicit energy subsidies, we only focus on explicit (pre-tax) energy subsidies in our analysis that are documented in government budgets.

Secondly, in qualitative analysis limited number of interviews were conducted due to time constraints. While these interviews provided great insight, it would have been more helpful to have a broader representation from all regions and administrative tiers.

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APPENDIX

Summary of 2SLS Models

Model / Instrument Spec.	R^2	F-stat	DW Stat	J-Stat	Inst. Rank
FD(-1), ES(-1)	0.7686	53.5857	2.0070	0.0000	3
FD(-1), PD(-1), ER(-1)	0.9897	1194.53	1.9641	0.0000	4
FD(-1), ER(-1), IR(-1)	0.9609	321.76	1.8216	2.45E-41	4
FD(-1), ER(-1), Inf(-1), IR(-1)	0.7113	19.91	1.9199	0.0000	5
FD(-1), INF(-1), IR(-1)	0.6487	9.4283	2.2484	2.93E-41	4

Diagnostic Tests

Autocorrelation (Breusch-Godfrey Serial Correlation LM Test)

Equations	LM Statistic	Rho	P value > Chi^2
ES	0.0225	-0.0954	0.8806
PD	6.7541	0.4757	0.1094
ER	1.4313	0.8006	0.2014
IR	0.0809	0.0618	0.7760
INF	1.7705	-0.2714	0.1833

Heteroscedasticity

Equations	LM Statistic	P value > Chi^2
ES	0.2799	0.1781
PD	1.3147	0.1043
ER	2.1354	0.1562
IR	3.6773	0.4817
INF	5.2707	0.1530