THE DYNAMIC ANALYSIS OF PUBLIC DEBT SUSTAINABILITY FOR PAKISTAN

Pakistan Institute of Development Economics

by

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PIDE2022FMPHILECO07

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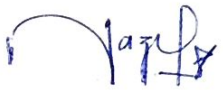

Date: October 25, 2024

Signature of the Student

Name of Student

Nazeef Ellahi

Dedicated

To my beloved Parents and Family

whose unwavering support, endless sacrifices, and boundless love have been my foundation and inspiration. Their guidance and encouragement have been the light that led me through every challenge, and I am forever grateful for their belief in me. This work is a tribute to their enduring strength and faith.

ACKNOWLEDGEMENT

All praise and gratitude are due to Almighty Allah, the Lord and Creator of the heavens and the earth, for granting me the strength and patience to complete this work. I extend my deepest respect to the Holy Prophet Hazrat Muhammad (PBUH), who illuminated our hearts with faith in Allah and embodied boundless compassion.

This journey would have not been possible without the profound support, insightful guidance, and enduring encouragement of a group of inspiring contributors to whom I owe my deepest gratitude. Firstly, I extend my heartfelt gratitude to my supervisor Dr. Haider Ali, whose expertise, insightful guidance, dedicated support, and sympathetic attitude have been pivotal in shaping this research. Dr. Haider's encouragement and constructive feedback throughout my MPhil journey have been invaluable and I am immensely grateful for his mentorship.

I would also like to acknowledge Dr. Hafsa Hina and Dr. Saud Ahmad, whose insights and guidance have been essential to my progress. Their contributions have enriched my understanding and significantly influenced the direction of this thesis. I am truly appreciative of their support and generosity in sharing their knowledge.

I am deeply grateful to my parents and family, whose unwavering love, sacrifices, and belief in me have been my constant source of strength. Their endless support and encouragement have been the foundation of my academic achievements, and I am forever indebted to them for their faith in me.

I also want to thank my dear friends, Aqsa, Iqra, Muneeb, Ajab, and Mazhar for their companionship and encouragement. Their understanding, patience, and sense of humor helped me navigate the challenging moments of this journey, and I am truly fortunate to have had them by my side. To everyone who has contributed to this work, both directly and indirectly, thank you for your support and encouragement.

Nazeef Ellahi

ABSTRACT

Most of the developed and emerging economies have the common goal of attaining long-term sustainable and higher economic growth. Domestic and external borrowings are the major sources of financing in these countries. The relationship between economic growth and public debt is not simple and unidirectional. Debt has both positive and negative implications, efficient and productive use of debt fosters economic development, but when used inefficiently and imprudently it impacts economic growth negatively. An increasing number of recent studies support the idea of a non-linear association between public debt (PD) and economic growth, with a threshold level at a certain point, above which excessive public debt adversely affects economic growth. This paper explores the nonlinear debt-growth nexus and estimates the threshold level of total public debt, domestic debt, and external debt for Pakistan and also analyzes the sustainability of public debt. The latest available time series data is utilized for the period 1973-2022. The Heteroskedasticity Consistent Lagrange Multiplier Test and Threshold Regression Model by Hansen (1996, 2000) are utilized for this purpose. The result reveals a non-linear relationship between public debt and economic growth, estimating the optimal public debt threshold for Pakistan to be around 58.5%. The threshold levels of domestic and external debt are 43.3% and 27.5% respectively. The estimated public debt threshold is in line with the 60% benchmark set by the FRDL Act, 2005. All the forms of debt affect economic growth adversely when the threshold levels are exceeded. Similarly, employing various empirical methods, including the stationarity test, co-integration approach, and fiscal reaction function approach, the study found public debt unsustainable in the country. The fiscal policies in the country are not responding efficiently to the increasing level of debt and the government expenditures are not according to its revenues. The government cannot meet its current financial obligation without additional financial assistance or facing financial distress. Using a debt-dynamic equation, this study simulates future debt-to-GDP ratio levels under various growth and fiscal deficit assumptions. The analysis indicates that a high GDP growth rate and lower fiscal deficits are crucial for reaching sustainable debt levels. To ensure sustainability, the study recommends maintaining a growth rate of 5-10% or higher, reducing the fiscal deficit to 1-2% along with efficient fiscal policies, cutting excessive expenditures, and increasing revenues. The implementation of these strategies can foster economic growth and ensure the sustainability of public debt in the country.

Keywords: Public debt, debt sustainability, economic growth, threshold level, fiscal deficit, threshold regression model, fiscal reaction function, FRDL act, co-integration

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List of Abbreviations

FRDL	Federal Responsibility and Debt Limitation
MAC DSA	Market Access Countries Debt Sustainability Analysis
LIC DSF	Low Income Countries Debt Sustainability Framework
IDA	International Development Association
PPG	Public and Publicly Guaranteed Debt
WDI	World Development Indicator
IFS	International Financial Statistics
PES	Pakistan Economic Survey
IMF	International Monetary Fund
GDP	Gross Domestic Product
SOE	State Owned Enterprises
PIBS	Pakistan Investment Bonds
PVBC	Present Value Budget Constraint
FRF	Fiscal Reaction Function
PRGT	Poverty Reduction and Growth Trust
NSS	National Saving Scheme
ADB	Asian Development Bank
TRM	Threshold Regression Model
PIDE	Pakistan Institute of Development Economics
PD	Public Debt
VECM	Vector Error Correction Model
LDCs	Least Developed Countries

Chapter

1 Introduction

"*The relationship between economic growth and public debt is not simple and unidirectional. The economic growth of a country and its ability to manage debt effectively are shaped by multiple factors. Furthermore, there is no universal public debt threshold that can specifically identify and differentiate between debt levels that are considered favorable (productive) and those deemed detrimental*" (IMF, 2020).

1.1 Background of the study

With the increasing level of public debt in many advanced and emerging economies, specifically after the global financial crisis in 2008 and the recent coronavirus pandemic in 2019, both policymakers and academics have raised questions about the optimal level of debt and the sustainability of public debt in the countries. Although a very high debt-to-GDP ratio is considered unsustainable for an economy, there is no rule of thumb that refers to a sustainable debt level[1]. The literature highlights economic growth, borrowing cost, primary balance, and responsible fiscal policy as key factors behind the sustainability of public debt (Abbas et al., 2019). Borrowing assists countries in promoting economic growth and resource management by providing financial assistance for development projects, fostering human capital and innovation, environmental protection, and smoothing consumption. However, excessive debt can become troublesome, leading to financial distress by reducing investment, lowering productivity, and slowing economic growth. Debt is a two-edged sword: it can be used to increase welfare and growth when managed and used efficiently and fairly, but debt also carries the risk of negative outcomes of financial and economic instability when used imprudently and inappropriately (Cecchetti et al., 2011). The literature is fairly divided on the debt growth nexus, with some studies debating the linear impact (either positive or negative) of public debt on economic growth (Akram, 2011; Woo and Kumar, 2015; Spilioti and Vamvoukas, 2015). In contrast, the others support a nonlinear relation between the two variables with a threshold effect at a certain level (Reinhart and Rogoff, 2010; Alsamara et al., 2024; Caner et al., 2010). This study aims to explore the non-linear relationship between public debt (PD) and economic growth, estimate the impact of debt on economic growth above and below the threshold level, and also assess the sustainability of PD for Pakistan.

Public debt sustainability is a crucial matter for the fiscal policy of a country since maintaining sustainable debt is essential for economic stability, fiscal discipline, and long-term growth. Debt sustainability pertains to a situation where a government meets its debt obligations without taking additional financial assistance and without being default (IMF, 2020). In other words, when the existing resources of a country become insufficient to meet its

[1] Japan's public debt to GDP exceeds 200 percent while Ukraine faces default with a lower obligation of 30 percent of GDP in 2015

finances, countries may incur debt from internal as well as external sources (i.e., by issuing securities, bonds, and bills or by securing loans from local or international lenders). However, if excessive debt turns into a debt distress situation (unsustainability), countries become unable to pay their debt obligations and debt restructuring becomes necessary or in extreme cases leads to default, which not only jeopardizes growth and investment but also leads to either higher borrowing costs or the loss of international financial market access.

The huge increase in PD after the Global Financial Crisis (GFC) and the recent coronavirus pandemic has prompted the discussion on the debt-growth nexus and debt sustainability. GFC was an era marked by a substantial rise in public debt and a simultaneous deterioration in economic growth rates across the globe and caused significant market instability and banking failures. The governments adopted unprecedented fiscal stimulus measures in an attempt to combat the economic crisis and stabilize the financial system, which resulted in an enormous accumulation of government debt in emerging and developed economies. The COVID-19 Pandemic has bought immense challenges to the economies globally, public debt level rises tremendously around the globe. The nation's debt-GDP ratios has risen to a large extend, and several nations faced financial defaults (i.e., Lebanon, Argentina). Many governments were forced to set aside emergency government funding to assist those affected by business closures. The Huge increase in government spending contributed to a notable increase in the national debt, which heightens the possibility of unsustainability and insolvency.

The debt-growth nexus and debt sustainability have long been an academic debate for economist and policymakers. The affiliation of lower economic growth and high debt level seems to be worrying for debt sustainability (Mehrotra and Sergeyev, 2021). Empirical literature in this context provides contradictory results. Most of the literature divulges that the relation between PD and economic growth is unidirectional (linear) and could be positive or negative, public debt helps to boost growth and has a positive effect on growth and total output (Elmendorf and Mankiw, 1999; Burhanudin et al., 2017), while some argue that the rise in PD will affect the economic growth negatively (Asteriou et al., 2021). The major question in this scenario is how public debt adversely affects the economic growth. Various factors contributed to this and the major channels through which government borrowing negatively affects economic growth are crowding out effect and debt overhang. In the case of the crowding out effect, the rise in spending by the public sector leads to a reduction in private investment. The government has to increase revenue for additional expenditures, either through taxing more or borrowing from domestic and international sources. Increased government borrowing raises the interest rates, increasing the cost of borrowing for the private sector, thus crowding out private investment and impacting economic growth negatively. In contrast, in the situation of debt overhang inefficiently managed debt leads to the utilization of borrowed fund for debt servicing instead of investment in developmental projects and productive purposes. The concept of debt overhang was prominently discussed by economist Paul Krugman in 1988. He emphasized that inefficiently managed borrowed funds can lead to a situation, where a significant portion of a country's resources is dedicated to debt servicing rather than productive investments.

The nonlinear association between PD and growth has recently drawn the attention of policymakers and academic debate. This non-linear hypothesis states that an increase in PD positively influences growth when the debt level is low and below a certain level (threshold), and turns negative when the debt accumulation rises above a particular threshold. Reinhart and Rogoff (2010) has proposed a standard for the debt-to-GDP ratios, that the debt threshold for the emerging and developed economies is 90%, beyond which PD as a percentage of GDP affects economic growth negatively. Nevertheless, this 90 percent threshold level has been challenged by several studies both in developed and developing economies, some suggesting ratios as low as 20 percent (Égert, 2015; Pegkas, 2019) and others as high as 200 percent (Butkus and Seputiene, 2018). Several studies suggested that the turning point is country specific and varies across countries due to variations in certain characteristics[2]. The debt-growth relationship varies across economies, depending on each country's macroeconomic indicators[3], institutions, and the policy implications (World Bank, 2017).

The estimation of a public debt threshold and the debt sustainability analysis (DSA) is significantly critical for a developing country like Pakistan as the domestic and external debt are increasing continuously and the growth rate is almost stagnant. DSA examines financing strategies to make sure they can be managed without requiring extraordinary measures like debt rescheduling, falling into default, or without running into problems with debt service. DSA gives prospective and probabilistic insights by connecting debt dynamics with macroeconomic policies, a dynamic instrument that illuminates risks, informs policy decisions, and ensures a country's economic future. As an early warning system, it finds deficiencies before crises arise, allowing for fast fiscal adjustments. It serves as the basis for fiscal policy, directing prudent choices that give priority to the sustainable allocation of resources. In addition to reducing risk, DSA promotes economic growth by guaranteeing reasonable debt limits and drawing in foreign capital through its function as a creditworthiness certification. Because DSA enables informed borrowing and helps creditors reduce risk, it benefits both borrowers and lenders. DSA has specific roles in a variety of circumstances, including industrialized, developed economies, extremely indebted countries, and underdeveloped nations.

In addition, the debt threshold provides policymakers with a crucial indication of a debt level above which the rise in public debt could potentially jeopardize economic stability. This information helps policymakers to examine the country's fiscal policies or take appropriate action. This could involve adopting austerity measures, improving revenue generating mechanisms, or exploring alternative sources for financing other than borrowing. Governments can use the threshold notion as a useful guide to help them manage their fiscal policies in a way that minimizes the risk of economic stability because of sovereign debts. Numerous studies have examined the debt-growth nexus in Pakistan, mostly focusing on the longterm linear association between the two variables. However, the non-linear perspective has been the subject of a few studies. This study will explore the nonlinear link between PD and economic growth,

[2]macroeconomic indicators, institutional framework, policy implementation

[3]Economic activity, exchange rate volatility, inflation, interest rate etc

and examine the threshold level of public, domestic, and external debt in Pakistan.

1.2 Global Debt Scenario

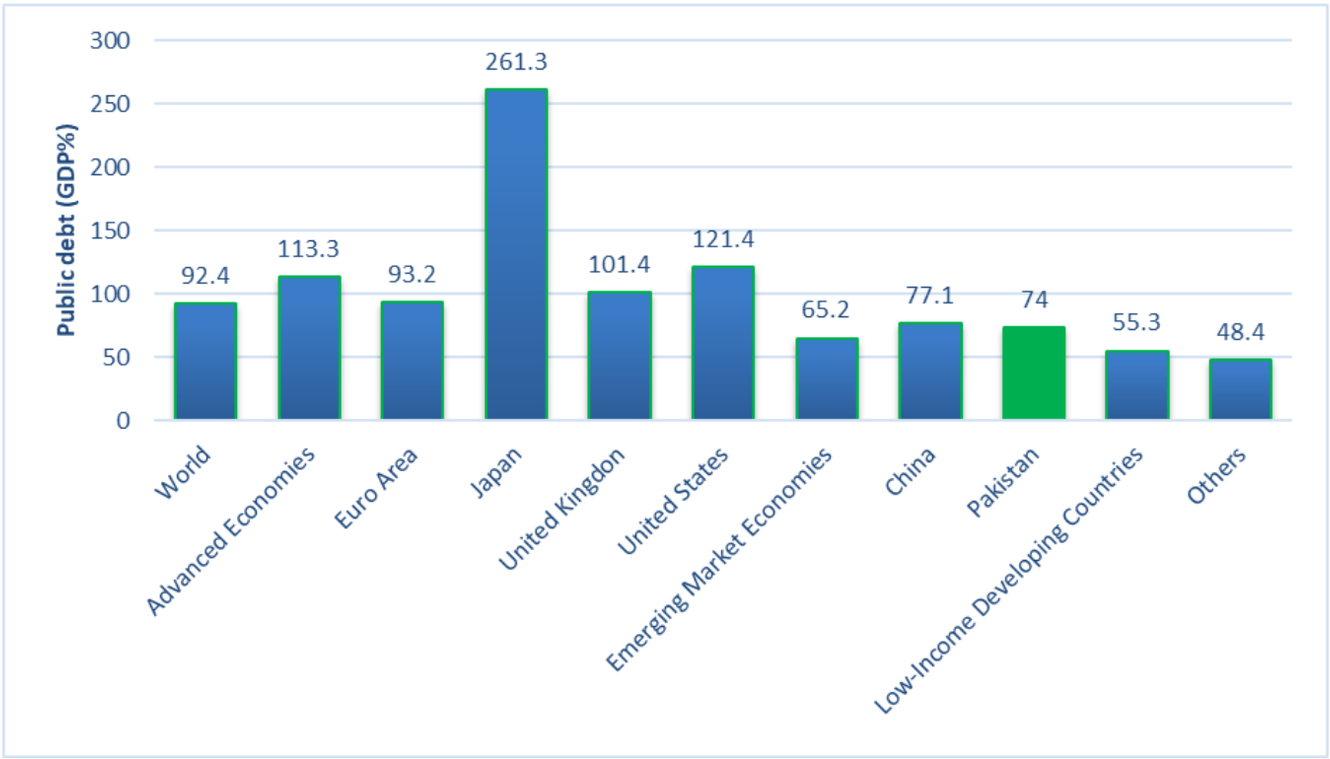
The recent International Debt Report[4] by the World Bank presents an alarming picture of the global debt situation, especially for developing countries. Developing nations paid the highest 443.5 billion dollars in 2022 to service their external and public and publicly guaranteed (PPG) debt, against the backdrop of the paramount increase in worldwide interest rates in 4 decades. According to the research, all developing nations have a 5% increase in debt service payments, including principal and interest, which took away vital funds from other key areas such as education, health, environment, agriculture, and development. Shockingly, the 75 economies which are eligible for World Bank's IDA[5] concessional financing, paid a record-breaking 88.9 billion dollars in debt servicing expenses in 2022, with interest payments more than tripling in the previous ten years. According to the research, the 24 poorest nations might see an increase in debt servicing costs of up to 39% in 2023 and 2024. Rising interest rates rendered debt more vulnerable, and in the last three years alone, there have been 18 sovereign defaults.

In the last decade, LICs[6] have taken on external debt at a higher rate than middle-income countries. Currently, almost 60% of LICs are either in or likely to experience financial distress (Minhaj et al., 2021). The report also underscores a structural imbalance between growth and debt accumulation, especially in LDcs, where debt has surged by 109% since 2012 while gross national income (GNI) has increased only by 33%. A substantial portion of the export profits of low-income nations are going toward interest payments, and the report points out that this scenario could get worse if interest rates continue to climb or if export revenues decline significantly. The report emphasizes the need for prompt actions and coordination between the debtor governments, official and private creditors, and multilateral financial institutions to tackle the mounting debt crisis. It recommends enhanced debt sustainability tools, greater transparency, sound policies, and expedited restructuring agreements to prevent a potential financial crisis.

Figure 1 presents the global public debt (percentage of GDP) for the world and the major economies, several nations exhibit high levels of debt; three prominent instances are Japan (261%), Greece (192.41%), and the United States (121.4%). The total world debt as a percentage of GDP is 92.4% and that of advanced and emerging economies are 113.5% and 65.2% respectively. Pakistan's debt level is hovering around 74% which is higher than the average debt level for most of the developing countries (65.2%). It's important to remember that a high debt-to-GDP ratio does not always indicate a forthcoming financial catastrophe because other important considerations when evaluating factors for debt sustainability include economic growth rates, debt-carrying capacity, and income generation capacity. According to the IMF LIcs should be more careful about the rise in public debt because they are more prone to external shocks due to higher external debt as compared to advanced countries.

[4]See: https://openknowledge.worldbank.org/handle/10986/40670
[5]International development association
[6]Less income countries

Figure 1: Central government debt of different economies.



Source: International Financial Statistics, 2022

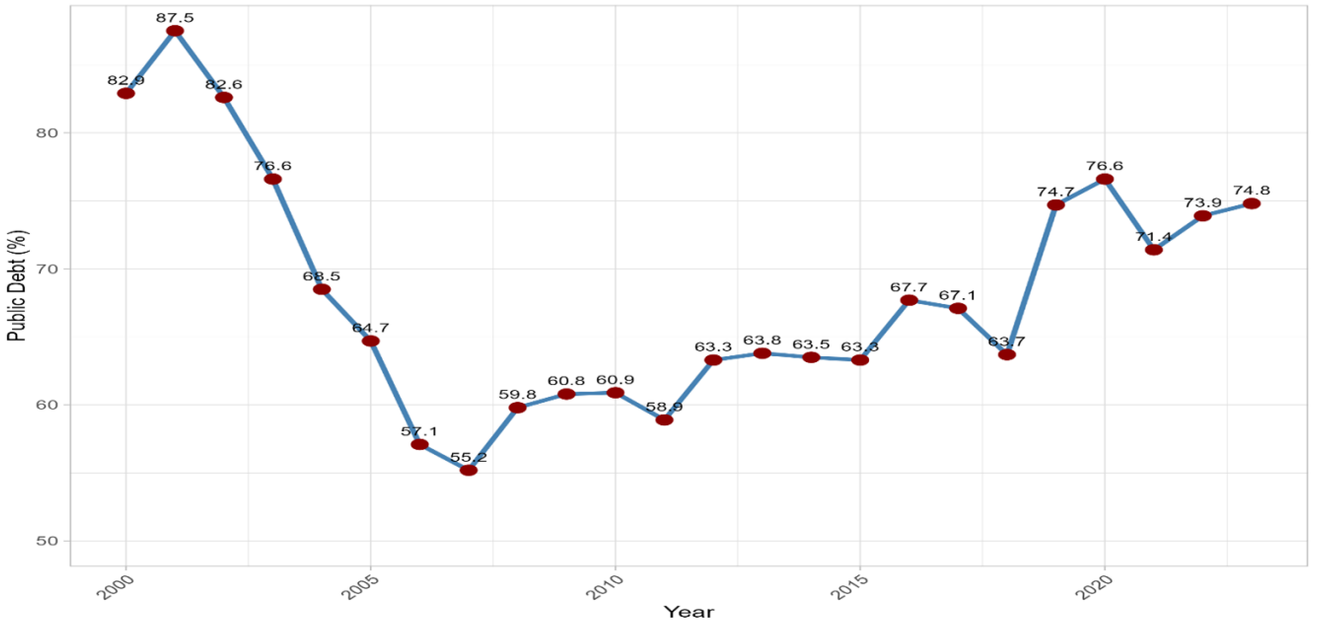
The global debt increased rapidly due to the COVID-19 epidemic, increasing to 97.1 trillion dollars in 2023, a 40% increase from 2019 (IMF, 2023). During the pandemic, governments took significant financial measures to stabilize the labor market and to halt insolvencies, resulting in a higher debt burden. With 33.2 trillion dollars in government debt, the United States alone accounts for more than a third of the world's total debt. Japan's aging population has resulted in one of the highest debt-to-GDP ratios, at 255%. North America, particularly the United States and Canada, has the highest levels of debt in the world, the emerging economies such as Egypt and Lebanon face high borrowing costs and default scenarios. Debt levels in Asia and the Pacific are comparable to those in North America, while 3.3% of the world's total is attributed to South America. Africa also confronts difficulties due to a large share of its public debt being held in foreign currencies and higher interest rates. IMF predicts that by 2028, the world's public debt will surpass 100% of GDP, hitting levels similar to those seen during the pandemic.

1.3 Public Debt Scenario of Pakistan

Pakistan has been facing serious debt challenges for the past few decades. To finance its development projects and budget deficits, the country has frequently sought financial assistance and relied on domestic and external borrowing. The public debt trajectory of Pakistan has revealed a trend of debt accumulation, phases of debt reduction, and

phases of restructuring. The nation's debt burden has notably increased, triggering concerns about the possible threats to the country's long-term financial health and productivity. The current debt-to-GDP ratio of Pakistan has hovered around 75% for the past few years, which is above 60% as illustrated in Figure 2.

Figure 2: Public debt to GDP ratio of Pakistan

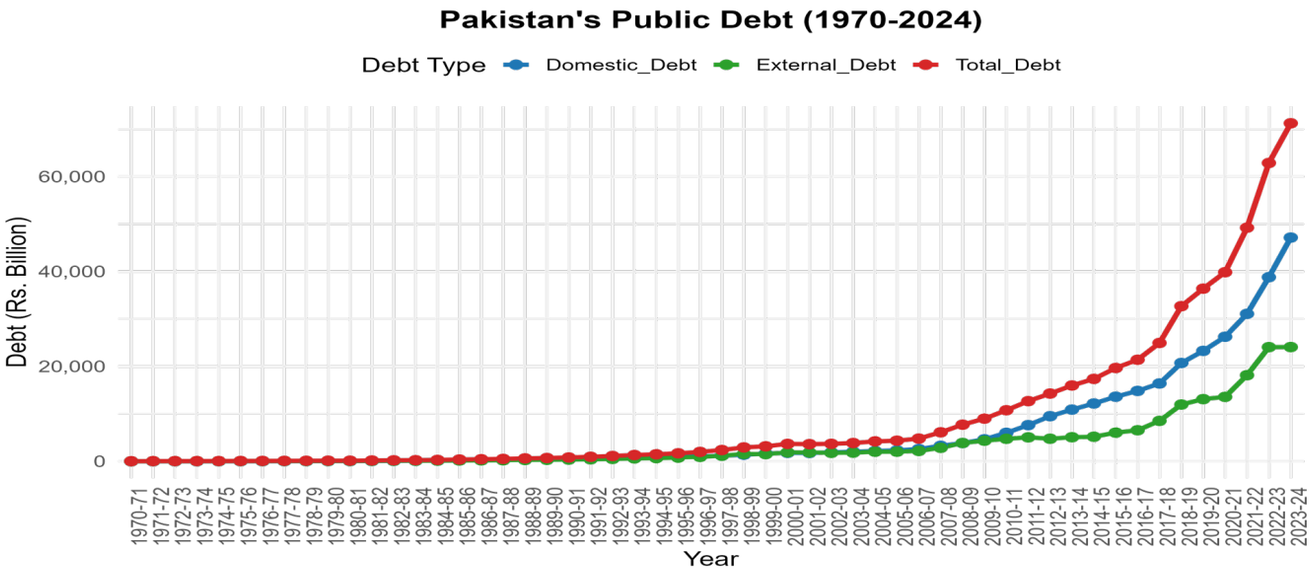


Source: Pakistan Economic Survey

However, this number is higher than the Fiscal Responsibility and Debt Limitation Act (FRDL)[7] 60% benchmark, which the government has not been able to meet over the previous years. The recent report[8] by the Ministry of Finance states that mark-up payments account for 95% of all federal taxes collected, leaving only 5% left for other services and provinces. Pakistan is one of those countries where the total domestic and external debt is continuously showing an upward trend. If the government does not act appropriately, more debt is expected to accumulate.
Figure 3 shows the trajectory of Pakistan's debt in three categories: domestic, external, and total public debt, from 2000 to 2023, measured in billions of Pakistani rupees (PKR). The graph shows an upward trend in all categories of debt. The domestic Debt and external debt both show an upward trend, with domestic debt surpassing external debt after 2010. However, public debt sharply escalates, particularly after 2010, indicating a swift accumulation of debt burden in recent years. In particular, the significant increase in public debt, particularly in the last decade, implies a more rapid accumulation of debt in recent years.

[7]FRDL act (2005) aims to efficiently manage PD in order to reduce the fiscal deficit and sustain a prudent debt to GDP ratio (60%), along with taking into consideration other related matters
[8]Pakistan Fiscal Operation-quarter ending December 2023

Figure 3: Trend in domestic, external and public debt of Pakistan (Billion PKR)



Source: Pakistan Economic Survey

The public debt of Pakistan increased in the 1980s and 1990s due to high real borrowing costs and low revenue. This situation was made worse in 1990 by high inflation-related real borrowing costs. But the country's growth was boosted between 2000 and 2007, which helped it handle its debt well. The public debt to GDP ratio significantly reduced from about 82% in year 2002 to 57% by fiscal year 2006, surpassing the reduction mandated by the FRDL Act, 2005. This was achieved by the effective use of funds to finance structural changes and infrastructure development. Successive high growth rates and debt reduction tactics were credited for this reduction in the public debt level, But between 2008 and 2014, several events—including a rise in the price of oil and other commodities, depreciation of the currency, higher security costs, revenue shortfalls, energy shortages, and natural disasters—caused the public debt to rise again. Increased interest payments, security costs, and budget deficits in the year 2012 to 2014 contributed to the greater debt levels, as did the need to borrow domestically because of the decline in foreign inflows. From the year 2015 to 2022, PD continuously increased due to continuous depreciation of exchange rate, natural calamities, and the COVID-19 pandemic.

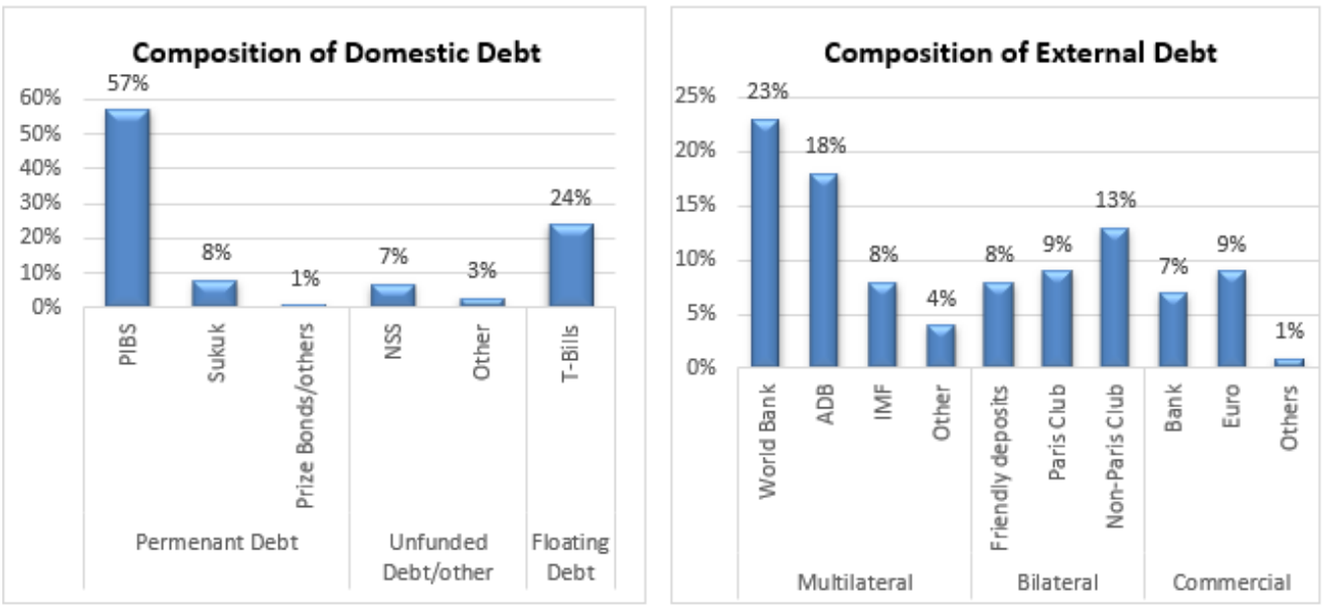
Examining Pakistan's national debt situation reveals a few important things. First, it becomes clear that the nation has been in a debt trap (domestic and internal) for a long time, mostly because the previous governments were unable to overcome the fiscal deficit. Secondly inefficient and unproductive use of loans, low revenue and lower tax base, inadequate policy response, mismanagement of domestic resources, higher cost of borrowing, exchange rate volatility, external shocks and economic challenges have further exacerbated the debt problems.

1.3.1 Components of Pakistan Public Debt

The overall public debt of Pakistan comprises of domestic and external debt. The total PD of Pakistan is about 67,525 billion PKR, where the portion of domestic debt is 43,432 and that of external borrowing is 24,093 billion PKR. The share of domestic debt as GDP percentage is 46.2% and that of external debt is 26.8%. Approximately 88 percent of the fiscal deficit was financed through domestic markets, while 12 percent came from external sources. Pakistan's domestic debt increased rapidly over the past few years, from 1,645 billion PKR in 2000 to 43,432 billion PKR in 2023. On the other hand, external debt also experienced an upward increase, starting at 1,527 billion PKR in 2000 and reaching 24,093 billion PKR in 2023. Both domestic and foreign debts have grown significantly over this time, with domestic debt eventually surpassing foreign debt. The nation's borrowing demands and financial difficulties throughout this period are indicated by the debt's rising tendency.

Figure 4 presents a graphical depiction of the composition of Pakistan's domestic and external debt, which demonstrates that to finance its current account and budget deficit Pakistan has relied on both domestic and external borrowing. Domestic debt is classified into permanent, floating, and unfunded debt to manage maturity and exchange rate risks. The largest portion of Pakistan's external debt comes from multilateral creditors, including the World Bank, IMF, and ADB, which indicates reliance on international financial institutions. Bilateral and commercial debts also play a substantial role in the debt composition of Pakistan.

Figure 4: Composition of Domestic and External Debt of Pakistan



Source: Ministry of Economic Affairs, State Bank of Pakistan and Debt management Office, Ministry of Finance

1.3.2 Domestic Debt

The major source through which the fiscal deficit is financed is the domestic debt. It comprises of three major categories, long-term and medium-term permanent debt, short-term floating debt, and unfunded debt. Treasury bills (T-bills) are short-term securities issued with maturities of 12 months or less. Pakistan Investment Bonds (PIBs) are long-term loans issued with maturities exceeding 12 months. The government also introduced a shariah complaint loan in 2009. The major portion of domestic debt is the permanent debt, which accounts for 67% of the total, with 57% PIBs, 8% Sukuk, and 1% other instruments. The Unfunded Debt, which includes NSS, makes up 10% of domestic borrowing, with 7% from NSS and 3% from other instruments. Floating Debt, primarily in T-Bills, constitutes 24% of the total domestic debt.

1.3.3 External Debt

Pakistan's external debt is sourced from different key sources:

1. The portion of loans from multilateral development partners (such as the IMF) is 53%, while the percentage from bilateral countries is 22%. These are mostly concessional (low interest rate and longer maturity period) in nature.

2. Eight percent comes from bilateral deposits (Saudi Arabia and China). These are one-year loans that are taken out for both budgetary support and balance of payments.

3. Approximately 7% of the external loan is provided by commercial banks. Most of these loans have market-based interest rates and are short- to medium-term, lasting one to three years.

1.4 Statement of the Problem (SOP)

Pakistan is facing serious debt challenges and economic growth in the country has remained stagnant in the past few years. The debt-to-GDP ratio is around 75% which has crossed the 60 percent benchmark set by the FRDL Act 2005, raising concerns about the sustainability of PD in the country, emphasizing the importance of assessing the debt growth nexus. This persistent increase in debt poses challenges for future generations and crowds out domestic and foreign investment in the economy. The empirical literature on the debt growth nexus shows mainly a linear association (Akram, 2011) (Woo and Kumar, 2015) (Spilioti and Vamvoukas, 2015), while also considering the nonlinear association between them (Reinhart and Rogoff, 2010). In Pakistan, most of the literature focuses on the long-term linear relation between economic growth and public debt, while only a few have discussed the nonlinear or heterogeneous impact of PD on growth. Moreover, there is also a need for an empirical study to evaluate the rationale for setting a 60 percent limit of debt to GDP ratio stipulated in the FRDL Act, 2005.

1.5 Research Problem

Given the background described above and recognizing that Pakistan has experienced a continuous rise in public debt for several decades, while economic growth has remained stagnant. This has led to higher concerns about the country's fiscal stability and long-term economic progress, with the threat of default looming in the past year. Given this backdrop, it is necessary to examine the relationship between PD and economic growth and to assess the sustainability of the country's public debt. Understanding these dynamics is important for formulating effective fiscal policies to prevent future crises and ensure long-term economic growth and stability.

1.6 Research Questions

1. What is the threshold level of public debt, domestic debt, and external debt for Pakistan?
2. What are the economic costs of exceeding the estimated debt thresholds for Pakistan?
3. To what extent does Pakistan's current debt profile and it's projected trajectory under different economic scenarios pose a risk to debt sustainability?

1.7 Research Objectives

The prime objectives of this research study are as follows;

1. To identify and estimate the existence of a prudent public debt, domestic debt and external debt thresholds for Pakistan.
2. To find out the impact of debt above and below the threshold levels on the economic growth of Pakistan.
3. To analyze the sustainability of public debt for Pakistan.
4. To provide valuable insight for the policymakers in developing debt management strategies that foster economic growth in the country.

1.8 Unit of Data Collection

The present study is based on time series annual data spanning a period of 50 years from 1973 to 2022. The data is secondary and obtained from different domestic and international sources including the International Monetary Fund (IMF), the World Development Indicator (WDI), the Pakistan Economic Survey (PES), and the State Bank of Pakistan (SBP).

1.9 Explanation of key terms

- **Public Debt**

  As per the definition of FRDL act 2005, PD is defined as the amount owed by the government (provincial and federal) serviced out of consolidated funds and debt owed to the IMF.

- **Default**

  A situation where a nation becomes unable to meet its financial obligations and the need to restructure or roll over of its debt becomes necessary. Defaults may hamper economic growth and investment, leading borrowing countries to lose market access and incur higher borrowing costs.

- **Debt Restructuring**

  It refers to the change in the maturity profile of debt and involves revising deadlines for principal and interest payments on an outstanding debt by mutual consent between the borrower and the creditor. In addition, the rescheduling agreement might include debt relief provisions designed to help the borrower get back on track financially and fulfill its commitments related to the rescheduled debt.

- **Debt Distress**

  A situation in which a country is experiencing difficulties in managing its debt obligations. Debt distress has serious outcomes such as lower growth, reduced government spending on essential services and investor confidence.

- **Fiscal adjustment**

  Fiscal adjustment refers to the measures implemented by a government to correct its fiscal imbalance, typically involving changes in government spending and taxation policies. The primary goal is to reduce budget deficits and manage public debt to ensure long-term fiscal sustainability

- **Crowding out effect**

  The crowding-out effect occurs when the rise in government borrowing drives up interest rates, which reduces or "crowds out" private investment as borrowing costs rise for businesses and individuals, thus reducing investment and hence economic growth.

- **Debt Overhang**

  Debt overhang arises when a sovereign total debt surpasses the present value of its future income, leading to a situation in which the major portion of a country's resources goes for debt repayment, instead of productive investment.

CHAPTER

2 Literature Review

2.1 Theoretical literature

The relationship between PD and economic growth has long been a crucial topic of economic debate, with different schools of thought—Classical, Keynesian, Neoclassical, and Endogenous Growth theories—offering differing views on how PD impacts economic growth. The classical school of thought considered public debt as a social burden, that adversely affects economic prosperity (Broner et al., 2014), they claimed that by reducing private investment and discouraging potential foreign investment, sovereign debt services overwhelm economic development (Krugman, 1988; Peter et al., 1965). On the contrary, according to the Keynesian school of thought, government borrowing is a viable strategy during economic recessions to foster economic recovery by stimulating demand (Elmendorf and Mankiw, 1999). They asserted that government borrowing crowds in domestic private investment and thus enhances economic activity, as long as the interest rate is lower than the economic growth rate (Blanchard and Perotti, 2002). They argued that during economic recessions, increased government borrowing can finance public spending and investment, which boosts aggregate demand, mitigates unemployment, and fosters economic growth by utilizing ideal resources. Monetarists, in contrast, express a different perspective and criticize government borrowing, arguing that political factors frequently influence fiscal decisions rather than economic needs. They suggest that higher PD can raise inflation expectations, which leads to higher long-term interest rates and a decline in the demand for money in the economy (Kwon et al., 2009). The position taken by the Neoclassical School emphasizes the possible impact that government debt may have on private sector activity and is therefore bad for investment and overall economic growth. Contributing to the debate, the Ricardians view the state's debt as a tax burden that will be postponed until future generations (Barro, 1974).

In contrast, modern economists argue that public debt, when used efficiently for productive purposes stimulates economic growth, but when used inefficiently and imprudently contributes to economic downturn (Checherita and Rother, 2012). They emphasize how crucial well-managed public debt is for promoting general economic development while acknowledging the possible disadvantages. Apart from these theoretical perspectives, a new debate, the non-linear or threshold effect theory supports the notion of a heterogeneous link between PD level and economic growth. This theory posits that PD positively influences growth at lower debt levels, while the effects turn negative once the PD crosses a particular limit (threshold level) (Reinhart and Rogoff, 2010; Spilioti and Vamvoukas, 2015; Law et al., 2021). Thus, the association between public debt and economic growth is not simple, with diverse opinions across various economic theories. These diverse perspectives emphasize the complex dynamics between debt and growth, recommending careful consideration when formulating and implementing fiscal policies.

2.2 Empirical literature

The empirical literature is fairly divided on the debt-growth nexus, conventionally presenting a linear association between PD and economic growth, which can be categorized as positive, negative, or insignificant. Most of the literature proposed that elevated debt levels are often linked with lower economic growth (Akram, 2011; Woo and Kumar, 2015), while others come across a positive link between the two variables (Bakar et al., 2008; Spilioti and Vamvoukas, 2015). On the contrary, a new debate on debt-growth nexus suggests a nonlinear or heterogeneous correlation between PD and economic growth. When the government debt-GDP ratio rises above a specific threshold, there is a substantial reduction in economic growth (Shah et al., 2024; Cecchetti et al., 2011; Caner et al., 2010; Reinhart and Rogoff, 2010). There is no accordant agreement on a single universal threshold level, beyond which beyond which debt effects growth rate adversely. Furthermore, this threshold level may differ significantly between developed and developing nations (Shah et al., 2024; Spilioti and Vamvoukas, 2015). This section reviews the existing literature on debt-growth nexus and the sustainability of PD.

2.2.1 Linear debt growth nexus

The linear association between PD and economic growth posits either a positive, negative, or insignificant relationship between the two variables. In the case of a positive relationship, the increase in government borrowing boosts economic growth by expanding output and aggregate demand (Yusuf and Mohd, 2021). PD can positively affect economic growth by offering productive investments in infrastructure, education, technological advancement, environmental protection, and healthcare, which improve long-term economic capacity and development (Bakar et al., 2008). In contrast, economic growth will decline with the rise in PD when the relation is negative. Public debt may negatively affect economic growth by crowding out private investment due to an increase in the interest rate and by shifting resources away from development projects toward debt servicing, leading to higher taxes or inflation, reducing economic activity (Akram, 2011; Ewaida, 2017; Panizza and Presbitero, 2014).

The primary channels through which public debt negatively affects economic growth are the crowding-out effect and debt overhang. Picarelli et al. (2019); Xu et al. (2021); Heimberger (2023) empirically identifies that higher debt levels are often associated with higher interest rates as governments compete for limited financial resources with the private sector, thereby crowding out private sector investment. Similarly, over the long term, elevated debt levels reallocate and divert resources from productive investments toward debt servicing, ultimately reducing economic growth, causing debt overhang (Krugman, 1988). The uncertainty surrounding debt sustainability can also weaken investor confidence, leading to a reduction in foreign and domestic investment (Cecchetti et al., 2011).

Countries with higher PD are associated with lower economic growth, and those with higher growth rates have lower levels of debt (Ramos-Herrera and Sosvilla-Rivero, 2017). Égert (2015) highlighted that the relation between economic growth and public debt varies significantly across countries. Understanding the debt-growth nexus is essential for evaluating fiscal sustainability and its implications for long-term development. Calderón and Fuentes (2013) by a comprehensive analysis of panel data for many emerging economies from 1970 to 2010, proposed a significant negative impact of public debt on growth. The findings indicate that resilient institutions, practical and efficient domestic policies can overcome the adverse effects of PD on economic growth. A parallel reduction in PD and improvements in the policy frameworks could increase per capita growth rates by about 1.7% for the Caribbean and 2% for South-America, with conservative scenarios still yielding significant benefits of 1.5 and 0.85% points, respectively. The results draw attention to the importance of effective and efficient economic management to improve growth outcomes in the context of rising public debt, especially for developing economies.

Asteriou et al. (2021) investigated the debt-growth nexus in the short and long run for Asian economies from 1980 to 2012. Several econometric methods (mean and pooled mean group, dynamic fixed effect) were used for the analysis, and a negative correlation was found between government borrowing and economic growth in the short and long run. The countries selected in the study were suggested to prioritize only essential productive projects to avoid excessive government expenditure. Panizza and Presbitero (2013) explored the causal relationship between public debt and economic growth for a large sample of advanced economies. The finding highlights that while high levels of public debt do not necessarily show a direct significant effect on economic growth, they can lead to restrictive fiscal policies that hinder growth, especially during recessions, thus highlighting the need for future research to explore variations across countries and the mechanisms through which public debt impacts growth. Ale et al. (2023) analyzed the impact of external debt on economic growth among South Asian economies from 1980 to 2020. The results indicate a significant negative association between external debt and economic growth in both the short-run and long-run. The countries were suggested to promote investment and domestic saving to overcome the negative consequences and their dependence on external borrowing.

Akram (2011) strongly admits the existence of the "Debt overhang effect" in Pakistan as both GDP per capita and investment are negatively associated with public debt in the short as well as the long run. The ARDL co-integration technique was used to examine the impact of public debt on investment and economic growth. The study discourages over reliance on domestic and external borrowing and suggests the government to adopt alternate sources for financing. Debt is necessary to bridge financing gaps, its overuse, particularly external debt with restrictive conditions, hampers long-term economic stability, emphasizing the need for more prudent debt management (Rais and Anwar, 2012). The study explored the relationship between public debt and economic growth in Pakistan, highlighting the negative effects of excessive borrowing on GDP per capita. Using a simple OLS technique, the study reveals that both external and domestic debt constrain economic growth in the country, primarily due to poor fiscal management

and reliance on capital flows to cover persistent deficits. Similarly, Malik et al. (2010); Atique and Malik (2012) identify an inverse relation of domestic and external debt with growth from 1980-2010. The effect of external debt was more severe, as compared to domestic debt.

Yusuf and Mohd (2021) examined the long and short-term impacts of domestic and external debt on Nigeria's economic growth using the ARDL technique and revealed that external debt hinders long-term growth and enhances short-term growth, while domestic debt promotes economic growth in the long run. Debt service payments consistently reduce growth in both periods, whereas foreign reserves and domestic investment enhance growth. Additionally, high interest rates and foreign direct investment show varying effects depending on the time horizon. The study recommended that Nigeria should focus on domestic debt over external borrowing and ensure that borrowed funds are invested in productive sectors to sustain growth.

Similarly, several studies on selected developing economies revealed a statistically significant and positive association between GDP per capita and public debt. In India, Barik (2012) found that public debt has an indirect but positive effect on economic growth through investment channels. The study utilized annual time series data for the period 1982-2011, and the direct and indirect influence of public debt on economic growth was examined using the Solow Neoclassical Model. In the same way Owusu-Nantwi and Erickson (2016) examined the long-term casual association between public debt and growth in Ghana. The VECM and Johansen co-integration techniques were used for this purpose, the study confirms the existence of a positive link between the two variables and recommends the government acquire more debt for productive and developmental projects.

2.2.2 Non-Linear debt growth nexus

Reinhart and Rogoff (2010) were the pioneers in initiating the debate on the non-linear association between economic growth and public debt. Their study was based on the data from 3700 observations from 44 advanced and emerging economies, encompassing a wide range of political systems, institutions, monetary and exchange rate arrangements, and historical contexts. Using non-parametric approaches and histogram-based analysis their study suggested the existence of non-linear debt-growth nexus, with 90% PD as a percentage of GDP serving as the threshold level for both advanced and developing economies. If the ratio of debt to GDP is higher than 90%, PD will have a detrimental impact on economic growth. A new debate arose after the 90% threshold level of PD suggested by Reinhart and Rogoff (2010), where some researchers endorse their findings (Checherita and Rother, 2012; Baum et al., 2013), while others raised particular concerns (Yang and Su, 2018; Caner et al., 2010; Spilioti and Vamvoukas, 2015; Heimberger, 2023).

Law et al. (2021) provides new evidence on the existence of non-linearity between public debt and growth in 71 developing countries by employing a dynamic panel threshold technique from 1984 to 2015. Their results support the presence of a threshold at 51.65 percent debt to GDP ratio, which is significantly less than that found in earlier

studies (Reinhart and Rogoff, 2010). When public debt levels are high, debt has a statistically significant negative influence on economic growth; at lower levels, the effect is negligible. The results also show that stronger institutions generally offset the detrimental effects of public debt on economic growth. In twelve Euro economies Checherita and Rother (2012) investigated the association between per capita GDP growth and public debt for forty years using a fixed effect model, their findings were also similar to that of Reinhart and Rogoff (2010), having a threshold level of public debt between 90-100% of debt to GDP ratio. Alsamara et al. (2024) explored the heterogeneous link between public debt and economic growth in the MENA region from 1980 to 2021 using panel data employing the ARDL model and the threshold estimation technique. They found different turning points for oil and non-oil-producing countries, with debt threshold levels around 46-69% of GDP for oil-producing countries, and 71-84% of GDP for non-oil-producing countries.

Caner et al. (2010) tried to find out the tipping point of public debt and the impact of debt on growth above and below that point for a series of 99 emerging and developed economies. The threshold regression model by Hansen (2000) was employed from the year 1980 to 2008. The estimated threshold was 77% of the debt to GDP ratio for advanced economies, each additional point increase in debt to GDP ratio above this threshold reduces the growth rate by 0.017 percent, while for the emerging economies, the study estimated a threshold value of 64% as a percentage of GDP. Heimberger (2023) Using meta regression analysis tried to investigate the impact of public debt on growth from 47 primary studies. Their results could not confirm the existence of a uniform debt threshold, whereas the relation between the two variables was negative, admitting that the increase in public debt to GDP by 10% will lead to a decrease in annual growth rate by 0.14 percent at 95% confidence interval.

Shanmugam and Renjith (2023) examines the threshold level and assess the sustainability of PD in center and all states of India by employing statistical methods and threshold regression models, using data from 1990 to 2021. Based on the study they found the threshold level of 40% of the center government and 22% for all the states. Additionally, their findings imply that the national debt is not sustainable in the federal government as well as in every state. Their debt dynamics-based simulation exercises suggest that the nominal growth rate of the Indian economy should be 12 percent and that all states and the center should aim for a 2 percent fiscal deficit beginning in 2024. By doing this, the center will be able to reach the debt sustainability target by 2028 and all States by 2031. They suggest that the appropriate policy approach for all governments is to increase revenue while reducing public spending, such as ineffective subsidies. Further contributions to this literature include Baum et al. (2013), who used panel threshold models for 12 Euro area countries from 1990 to 2010. Their results revealed that while public debt has a short-term positive impact on growth, debt levels above 95% of GDP negatively affect growth.

Demonstrating the lack of consensus on debt growth nexus, where some studies identify a negative relationship, particularly in highly indebted countries like Nigeria and Sri Lanka, others find a positive or even non-linear relationship, dependent on the debt threshold based on Reinhart and Rogoff (2010) hypothesis, which sets a 90% debt-to-GDP

ratio as optimal threshold level, Rahman et al. (2019) provide a critical analysis of existing literature and found no universal consensus, as the relationship varied across countries, with some showing positive, negative, or non-linear effects, depending on factors like debt thresholds, economic conditions, and institutional quality. They argue that such a threshold is not universally applicable, as economic conditions, institutional quality, and macroeconomic stability vary across countries. The review highlights the need for country-specific fiscal policies rather than relying on generalized debt thresholds, emphasizing the importance of effective debt management to foster growth, especially in low- and middle-income economies.

Since most of the previous studies involve panel data analysis, recent studies suggested that the threshold level of public debt is country-specific and depends on different factors including institutional framework, quality of financial markets, economic conditions, and policy implications country (Rahman et al., 2019). Therefore, for the determination of debt threshold time series data is considered more appropriate. The recent studies mostly after 2015 have utilized time series data to find the specific threshold for different countries. Bhatta and Mishra (2020) estimated the threshold level of public debt for Nepal by introducing a quadratic bi-variate model in the ARDL technique. They found the growth maximizing public debt threshold around 33% for Nepal. Pegkas (2019) using the newly developed threshold regression model estimated the public debt threshold for Greece between 23.5% and 109.5%. Hsing (2020), to confirm whether the 90% debt threshold suggested by Reinhart and Rogoff (2010) for developing economies applicable to Italy or not utilizes an extended production function approach. Their results show that the threshold of government debt ratio for Italy is 105.5%, which is higher than the 90% threshold level suggested by (Hansen, 2000). Similarly, Obiero and Topuz (2023) by using a smooth transition regression model the threshold level of public debt for Kenya was found 33.29%.

The existing literature on the relationship between public debt and economic growth reflects varying results, with some studies suggesting a linear relationship, either negative, positive, or insignificant. On the contrary significant studies indicate a non-linear relationship, with a turning point at a certain point called the "threshold level" beyond which public debt begins to adversely affect economic growth. However, using panel data for such analysis may not be the appropriate solution, since different countries have different characteristics, such as economic conditions, institutional frameworks, political scenarios, and policies, Given these country-specific differences, employing time-series data for each nation, would provide a more accurate and tailored assessment of the prudent debt threshold. This approach allows for a deeper understanding of the dynamics at play and facilitates more effective policy recommendations for ensuring long-term debt sustainability.

2.3 Public Debt Sustainability

The concept of public debt sustainability has been extensively discussed in the economic literature, given the crucial repercussions that improper use of public borrowing can cause. Researchers have sought to find out how much public debt a country can bear without risking fiscal solvency. IMF (2020) defines PD sustainability as the ability of a country to meet its current and future debt obligations without additional financial assistance and any financial distress (default). The concept of debt sustainability is significantly different across advanced and low-income economies due to variations in their institutional framework, policies, and economic performance. On the other hand, most of the developing countries have structural economic challenges, such as lower tax bases, limited financial resources, and greater susceptibility to external shocks, making debt sustainability a more critical and fragile challenge.

Since countries have diverse and complicated economic systems, the concerns surrounding the sustainability of public debt have changed during the past few decades for each of those economies (Kaur et al., 2024). Some economies have sustainable debt strategies while others deal with imbalances in their fiscal and current account balance along with investment and saving disparities. The high debt to GDP ratio is a crucial challenge for many countries while others perform very well although their debt to GDP ratio exceeds 100% (Abbas et al., 2019). Generally, advanced economies have a higher potential to sustain their public debt due to efficient institutions, diversified industries, and intense financial markets, as seen in Japan, where the country's debt to GDP ratio exceeds 200% and was sustainable because of the country's ability to generate more revenue and borrow domestically at lower interest rates and its strong fiscal management. On the other hand, many developing and low-income countries encounter unsustainable debt, as evidenced by Sri Lanka and Greece, which have faced severe fiscal crises at much lower debt-to-GDP ratios compared to advanced economies, primarily due to limited access to financial markets, and higher borrowing costs.

A major portion of the literature focuses on fiscal sustainability, to asses whether their debt-to-GDP ratio remains stable over time (Bohn, 1995; De Mello, 2008; Ikikii, 2017). The traditional accounting approach considers a fiscal deficit sustainable if the debt-to-GDP ratio does not escalate, and the growth rate of debt will remain below the economic growth rate Bohn (1995), which also provides the basis for the DSA framework. This borrower-based approach, while foundational, has been criticized for its simplicity, as it fails to account for broader economic variables, such as the ability of countries to raise revenue and adjust to appropriation. Critics argue that this method neglects the significance of external shocks, a country's internal economic determinants, and political risks, which can affect the sustainability of debt significantly, particularly in middle and low-income countries.

The Present Value Constraint (PVC) approach shifts the focus toward a lender-based perspective, suggesting that a government's solvency depends on its future flow of resources relative to its present debt stock. The modern approach to debt sustainability involves statistical tests, starting with Hamilton and Flavin (1985) unit root test, which analyze the stationarity of public debt to identify potential bubbles, where the stationarity of debt series was

considered sustainable. Uctum et al. (2006) applied this test to G7 and selected Latin American and Asian countries, finding sustainability only in the G7.Trehan and Walsh (1991) explored the stationarity of quasi-differenced public debt and the cointegration of public debt and primary surpluses, concluding that such conditions indicate sustainability. Similarly, Bohn (1998, 1995), introduced the fiscal reaction function approaches to assess debt sustainability and to analyze whether a country's fiscal policies are responsive to the change in the stock of debt or not.Bohn (1998) approach was based on the response of the primary fiscal balance to changes in public debt resulting from economic shocks. He argued that a sustainable fiscal policy is one where the government systematically adjusts the primary balance in response to debt fluctuations, ensuring fiscal sustainability. The reasoning behind this is that if governments consistently fail to react to debt changes during shocks, it violates the no-Ponzi condition, causing public debt to potentially spiral out of control rather than converge to zero over time. Therefore, an increase in the debt-to-GDP ratio must be accompanied by a corresponding rise in the primary balance-to-GDP ratio to maintain long-term fiscal sustainability. This approach was dominantly used in the literature and is still in use.

Beqiraj et al. (2018) estimated debt sustainability by investigating the complex interplay between government debt accumulation and fiscal policy responses in 21 OECD countries from 1991 to 2005. Using second-generation panel co-integration tests to avoid cross-sectional dependency and to carry out further robust analysis, the authors tried to shed light on the two different government responses in terms of active; structural primary balance, and passive; cyclical primary balance, fiscal policies when debt to GDP increases. The results showed the inverse relationship between the structural primary balance and debt-to-GDP in the long-term depicting that governments of the OECD countries do not take any actions to reduce the debt-to-GDP ratio, hence the lack of debt sustainability. In the short-term, the government's response is recorded as asymmetrical which inclines the debt in economic expansion but fails to do so in economic recessions, hence the lack of debt sustainability in the overall period. Higher interest rates have made debt sustainability a key issue in macroeconomic policy when the interest on government debt is higher than the growth rate of the economy. The public debt servicing cost is impacted by secular stagnation, which is characterized by low production growth and low interest rates (Mehrotra and Sergeyev, 2021). where interest rate growth rate varies greatly, but r is often less than g. With slower trend growth enhancing debt sustainability and more output risk lowering interest rates but tightening the fiscal constraint, a continuous-time model with stochastic debt-to-GDP ratios demonstrates how these factors can reduce the debt-to-GDP ratio. The study recommends a budgetary ceiling of 150–220 percent of GDP for the United States.

To get access to the global capital markets, Ncube and Brixiová (2015) analyzed public debt management in African Countries right after the global financial crisis in 2009. Using a stabilizing primary balance approach which analyzes the variation in the debt-to-GDP ratio by breaking down variations in the public debt-to-GDP ratio over time with the help of real GDP growth, real interest rates, and primary fiscal balance to ensure debt sustainability in African nations from 2007 to 2012. The study found intricate details of how over the period African Nations saw a decline

in their debt-to-GDP ratios mainly caused by interest rate-growth differentials instead of strong fiscal policies. This study further reveals that the majority of the nations in the continent had their balances exceed the threshold required to maintain debt sustainability. Furthermore, the study suggested strengthening the fiscal policies is crucial to incline the growth rate of the country rather than relying on IRGD as it is prone to reductions in the future. The adversity caused by global financial crises put sustainable debt management on the spot as it affected both developed and developing nations as it is a crucial element of macroeconomic analysis in attaining economic prosperity.

After exploring the flaws in the traditional debt sustainability approaches, D'Erasmo et al. (2016) suggested three alternative approaches: empirical approach through linear fiscal reaction function to see the dynamics of primary balance and debt relationship, structural approach by using Neoclassical dynamic equilibrium model to estimate the impact of change in capital and labor taxes on debt sustainability, and a new conventional default risk model to stabilize the public debt. The study used historical data from the US from 1791-2014 for the FRFs and the cross-country data from 1951-2013. The empirical approach findings depicted a structural change in the fiscal sector post-financial crisis, primary balances saw a visible decline. The structural approach results show a different scenario for both the US and Europe. In the US capital taxes don't affect debt sustainability but the labor tax does incline with a slight increase in it. Whereas, in Europe, changes in both taxes don't ensure debt sustainability. The third approach shows that, unlike the other approaches, it allows governments to choose default even in the presence of fiscal solvency conditions to use default as a key element to reallocate the wealth among its nationals.

Ikikii (2017) critically examines Kenya's fiscal reaction function in response to public debt disturbances, revealing a counterintuitive negative relationship between rising debt and the primary surplus, which challenges conventional expectations of fiscal policy. The per unit increase in puclic debt to gdp ratio was linked with almos 15 percent reduction in the primary surplus. Building on Bohn's (1998) theoretical framework, the research employs robust econometric techniques, including Quantile Regression and ARDL models, to highlight the Kenyan government's difficulties in responding to short-term debt shocks, suggesting that severe fiscal adjustments may be necessary in the future. The findings underscore the need for policymakers to reevaluate fiscal strategies to enhance resilience and sustainability, particularly considering the dynamic economic environment and structural changes since 2014.

De Mello (2008) analyze Brazil's fiscal dynamics, showing that all levels of government adjust their primary budget surpluses in response to changes in public debt, specifically after the 1998 fiscal responsibility legislation. Similarly, Burger and Marinkov (2012) examine South Africa's fiscal policy and find that the government has historically responded to rising debt and it was confirmed through various econometric methods, including OLS, VAR, and GMM. Both studies highlight the importance of fiscal responsiveness to debt but also point out the challenges of maintaining sustainability. Moving forward, a deeper focus on the impact of external shocks, structural breaks, and non-linearities in fiscal behavior should be incorporated into debt sustainability analysis to ensure more comprehensive and adaptive fiscal strategies.

Campos and Cysne (2019) aim to evaluate the sustainability of Brazil's public debt through the estimation of time-varying fiscal reaction functions, utilizing monthly data from January 2003 to June 2016. The authors employ three distinct estimation methodologies: the Kalman filter, penalized spline smoothing, and time-varying cointegration to analyze the relationship between the primary deficit and the debt-to-GDP ratio. Their findings reveal a significant decline in the responsiveness of the primary deficit to changes in the debt-to-GDP ratio, indicating that Brazil's public debt trajectory became unsustainable in the latter years of the sample period. Consequently, the authors recommend a strong reversal in the fiscal reaction process, emphasizing the necessity of addressing underlying political and institutional challenges to enhance fiscal sustainability. This study contributes to the literature by highlighting the importance of time-varying models in understanding fiscal dynamics, while also suggesting avenues for future research to explore the broader implications of fiscal policy in emerging economies. Ogbeifun and Shobande (2020) investigated the relationship between public debt and primary balance in MIST countries (Mexico, Indonesia, South Korea, and Turkey) from 1990 to 2017, focusing on debt sustainability using fiscal reaction functions. It highlights the challenges posed by excessive reliance on public debt to finance developmental objectives, emphasizing the importance of effective debt management to avoid fiscal crises. The authors utilize fixed effect and feasible generalized least squares estimators to analyze government responses to debt accumulation, finding that primary balance improves by approximately 0.005 to 0.013 for every 1 percentage point rise in central government debt, indicating a positive fiscal reaction to rising debt levels. Furthermore, the study reveals that interest payments on debt significantly influence fiscal sustainability, while current account balances pose a potential threat to primary balance. The research advocates for policies that improve fiscal bases through reduced expenditures and reinforced tax revenues, contributing to the understanding of fiscal policy sustainability in emerging economies and suggesting that proper debt management is crucial for economic stability.

2.3.1 Public debt sustainability in Pakistan

The literature on debt sustainability in Pakistan reveals a complex situation of economic, political, and structural challenges. The sustainability of public debt is a crucial challenge for a country like Pakistan, as the country is dominated by a higher deficit and a huge debt burden (Chandia and Javid, 2013). Various studies have tried to assess the sustainability and project the country's debt trajectory using various methodologies.

According to the Pakistan Institute of Development Economics, lower economic growth and higher interest rates contribute to an increasing debt burden in Pakistan (Jalil, 2020). The study tried to answer the question of how much a threshold of economic solvency should be to acquire debt sustainability through three different scenarios: baseline, historic, and most extreme scenarios. The results depict that to tackle the severity of the debt burden, there is an urgency to adopt more investment-inducing, market-friendly, and transaction-friendly growth policies such as the Lucas-Romer policy of endogenous growth that prioritizes institutional reforms to incline the growth, eventually

resulting in lowered debt instead of focusing on the policies that lag in attaining higher return on assets, and result in severe conditions of debt increment.

The finance division also presents debt sustainability reports[9] every year to identify the major areas of concern in the field, using MAC DSA, to enhance debt transparency, fiscal sustainability, and efficiency in the management of public debt. According to the report for the year 2022-23, despite improvement in debt dynamics, the risk associated with public debt remain high over the medium term. The analysis suggested lowering borrowing costs and strengthening the nation's fiscal position and debt management to improve debt sustainability in the country. The Fiscal Risk Statement FY2023-24 for Pakistan provides a comprehensive overview of the potential risks and uncertainties facing the country's fiscal outlook. It identifies key areas of concern, including macroeconomic risks, public debt and guarantees, state-owned enterprises (SOEs), climate and natural disaster risks, and public-private partnership (PPP) risks. The document highlights the challenges posed by global economic conditions, fluctuations in commodity prices, and geopolitical tensions, which could significantly impact Pakistan's economic stability and growth prospects. Additionally, it emphasizes the importance of prudent debt management and fiscal discipline to mitigate these risks and ensure a more stable and sustainable fiscal outlook. The report acknowledges the collaborative efforts of various stakeholders in its preparation and provides recommendations for policymakers, investors, and other stakeholders to address these fiscal risks effectively. However, it also underscores the need for continued vigilance and proactive measures to navigate the complex and evolving fiscal landscape successfully.

Similarly, Sundus et al. (2022) highlights the need for stronger fiscal reforms to ensure the long-term sustainability of PD and economic stability. The research shows how Pakistan's economic growth is insufficient to cope with rising debt levels, trapping the nation in a downward spiral of borrowing and debt repayment. The study uses a dynamic debt model from the period 1975-2021 to examine the determinants and sustainability of public debt in Pakistan, focusing on fiscal deficits, exchange rates, and interest rates. The findings reveal persistent instability in borrowing positions, driven by fiscal indiscipline and adverse exchange rate movements, highlighting the macroeconomic vulnerabilities of Pakistan's external accounts.

Chandia and Javid (2013) applies a combination of fiscal reaction functions (FRF), debt dynamics, and vector autoregressive (VAR) models to evaluate the sustainability of Pakistan's debt. The methodology focuses on testing long-term relationships between the surplus-to-GDP ratio and the lag of the debt-to-GDP ratio. The study finds weak sustainability, where fiscal policies are marginally responsive to increasing debt. The FRF indicates a positive, but small, relationship between surplus and debt ratios, implying that past surpluses help maintain limited fiscal stability. The VAR analysis shows that government spending has a procyclical effect on the economy, negatively impacting consumption and output, consistent with Ricardian equivalence. Critically, while the paper demonstrates

[9]Debt Sustainability Report FY2022-23. Ministry of Finance, Pakistan. Available at: https://www.finance.gov.pk/publications/DSA_ReportFY2022_23.pdf

methodological thoroughness by incorporating cointegration analysis and impulse response functions, it offers a limited discussion of how external factors, political stability, and global economic changes may influence debt sustainability beyond the time period analyzed (1971–2008).

Similarly, Ejaz and Hyder (2019) used a probabilistic fan chart method to project the path of both external and total debt from FY2019 to FY2025. Their analysis reveals an alarming surge in total debt, with projections suggesting that Pakistan's debt-to-GDP ratio could reach 175% by 2025. The study provides a robust forecast of the debt trajectory, its reliance on probabilistic models minimizes the role of structural factors, such as governance failures, fiscal mismanagement, and corruption. By focusing primarily on macroeconomic indicators like GDP growth, interest rates, and exchange rates, the authors overlook critical institutional deficiencies that are instrumental in shaping Pakistan's debt dynamics. Moreover, while their call for fiscal austerity and improved domestic resource mobilization is valid, the practicality of such measures in a politically unstable and institutionally weak country like Pakistan is questionable. The paper assumes that policymakers can quickly pivot to fiscal responsibility without adequately accounting for the historical and political impediments to such reform. The overreliance on mathematical projections, even though insightful for short-term forecasting, failed to address the deeper systemic barriers that have historically undermined Pakistan's fiscal situation.

Mansoor et al. (2020) examines Pakistan's external debt management and finds that the primary balance has a weak relationship with the external debt-to-GDP ratio, indicating inadequate fiscal adjustments to manage the rising debt burden. They emphasize remittances and export earnings as potential mitigators, this view appears overly optimistic given Pakistan's structural challenges, such as a narrow industrial base, low productivity, and political instability. Similarly, Islam et al. (2023), using the Debt Sustainability Analysis (DSA) framework and Fiscal Reaction Function (FRF), highlights the inability of fiscal authorities to respond effectively to rising debt levels, predicting that external debt could stabilize by 2030 under optimistic growth assumptions of 10%.

2.4 Chapter Summary

A substantial number of literature explored the complex relationship between public debt and economic growth, with varying perspectives on its nature. Some studies argue that the relationship is linear, identifying positive, negative, or insignificant effects of public debt on growth (Spilioti and Vamvoukas, 2015; Heimberger, 2023; Akram, 2011). On the other hand, literature also highlights a non-linear relation (Caner et al., 2010; Reinhart and Rogoff, 2010; Shah et al., 2024), suggesting that public debt can stimulate growth up to a certain threshold, beyond which PD reduces economic growth. Furthermore, the suggestion of a single debt threshold for both developed and developing economies by Reinhart and Rogoff (2010) is also debatable, because due to differences in country-specific characteristics such as economic conditions, capacity to manage debt, institutional framework, and policy implementation processes, the threshold level might vary across countries (Égert, 2015; Rahman et al., 2019). Numerous studies have investigated the threshold level of public debt for different economies using both time series and panel data, identifying various thresholds, some closely aligned with Reinhart and Rogoff (2010), while others vary significantly. Despite such detailed works, the literature remains sparse, especially in the context of Pakistan. The studies have only focused on the linear relationship, ignoring the non-linear and threshold effect between public debt and economic growth. This leads us to the crucial gap in the literature. The understanding of the threshold level plays a vital role in efficient and effective policy formulation and implementation. Identifying the threshold level can benefit policymakers in managing public debt more effectively to overcome its adverse effects on economic growth, emphasizing the need for further empirical investigation into this nonlinear relationship in the context of Pakistan.

Debt sustainability and its challenges in the context of Pakistan have emerged in several themes. Different studies have established different arguments as there is a dire need to focus on fiscal reforms including austerity and efficient resource mobilization. Overall, the literature on debt sustainability suggests that Pakistan's debt trajectory is on an unsustainable path. There are plenty of studies that have estimated quantitative forecasts and macroeconomic variables. However, there is a huge lack of literature that focuses on underplaying the deeper structural and institutional issues that critically determine fiscal outcomes. Moreover, Pakistan's weak governance, lack of fiscal discipline, and inefficient tax collection system are consistently overlooked as the root causes of its fiscal imbalance according to the literature. These challenges are compounded by political instability, elite capture, and a history of ineffective fiscal reforms, which are often not sufficiently addressed in the literature.

This study addresses a critical gap in the literature by identifying the specific public debt threshold for Pakistan, utilizing a comprehensive time-series dataset, and incorporating key country-specific factors. By employing the latest methodologies, this research provides a more precise and contextually relevant assessment of Pakistan's prudent debt threshold, thereby contributing significantly to the understanding of debt sustainability. The findings offer valuable insights for policymakers, aiding in the formulation of more informed and effective economic strategies for Pakistan.

Table 1: Summary of Literature Review

Author	Time Period	Sample	Debt-Growth Nexus	Threshold
Reinhart and Rogoff (2010)	1946-2009	45 countries	Non-Linear	90%
Malik et al. (2010)	1980-2020	Pakistan	Inverse	-
Caner et al. (2010)	1980-2008	101 Economies	Non-Linear	64-77%
Akram (2011)	1972-2009	Pakistan	Inverse	-
Checherita and Rother (2012)	1970-2010	Euro Economies	Non-Linear	90-100%
Rais and Anwar (2012)	1976-2010	Pakistan	Inverse	-
Spilioti and Vamvoukas (2015)	1970-2010	Greece	Positive	-
Égert (2015)	1960-2010	49 countries	Non-Linear	20-60%
Jibran et al. (2016)	1972-2012	Pakistan	Inverse	-
Chen et al. (2017)	1991-2014	65 countries	Non-Linear	20-70%
Taher (2017)	1989-2014	Lebanon	Non-Linear	128.8%
Chudik et al. (2017)	1965-2010	40 countries	Inverse	-
Burhanudin et al. (2017)	1970-2015	Malaysia	Positive	-
Pegkas (2019)	1970-2017	Greece	Non-Linear	23.5%-109%
Hsing (2020)	1981-2019	Italy	Non-Linear	105%
Bhatta and Mishra (2020)	1976-2019	Nepal	Non-Linear	33%
Ud-Din et al. (2021)	1976-2022	Pakistan	Non-Linear	26.4%
Asteriou et al. (2021)	1980-2012	Asian Economies	Inverse	-
Law et al. (2021)	1984-2015	71 developing countries	Non-Linear	51.65%
Obiero and Topuz (2023)	1970-2018	Kenya	Non-Linear	33.29%
Shanmugam and Renjith (2023)	1990-2021	India	Non-Linear	40%
Ale et al. (2023)	1980-2022	South Asian Economies	Inverse	
Alsamara et al. (2024)	1980-2021	MENA region	Non-Linear	46-84%
Shah et al. (2024)	1990-2020	Developing countries	Non Linear	50% - 62.6%

CHAPTER

3 Data and Methodology

3.1 Research Strategy

To empirically examine the non-linear association between PD and economic growth, considering the prudent threshold effect, this study will imply the Threshold Regression Model (TRM) and LM test for threshold effect introduced by Hansen (2000, 1996). This method helps us to identify the non-linear debt-growth nexus and to investigate different correlations between debt and economic growth, both below and above the threshold level. Before applying the TRM model, the Heteroskedasticity consistent Lagrange multiplier test (LM) (Hansen, 1996) is used to examine whether there is a linear or nonlinear relation between PD and the real GDP growth rate. Different theoretical and econometrics methods are used to analyze the sustainability of PD in the country. This section provides the detail of the data, research methodology, and econometric models used in the study.

3.2 Conceptual and Theoretical Framework

3.2.1 Conceptual Framework

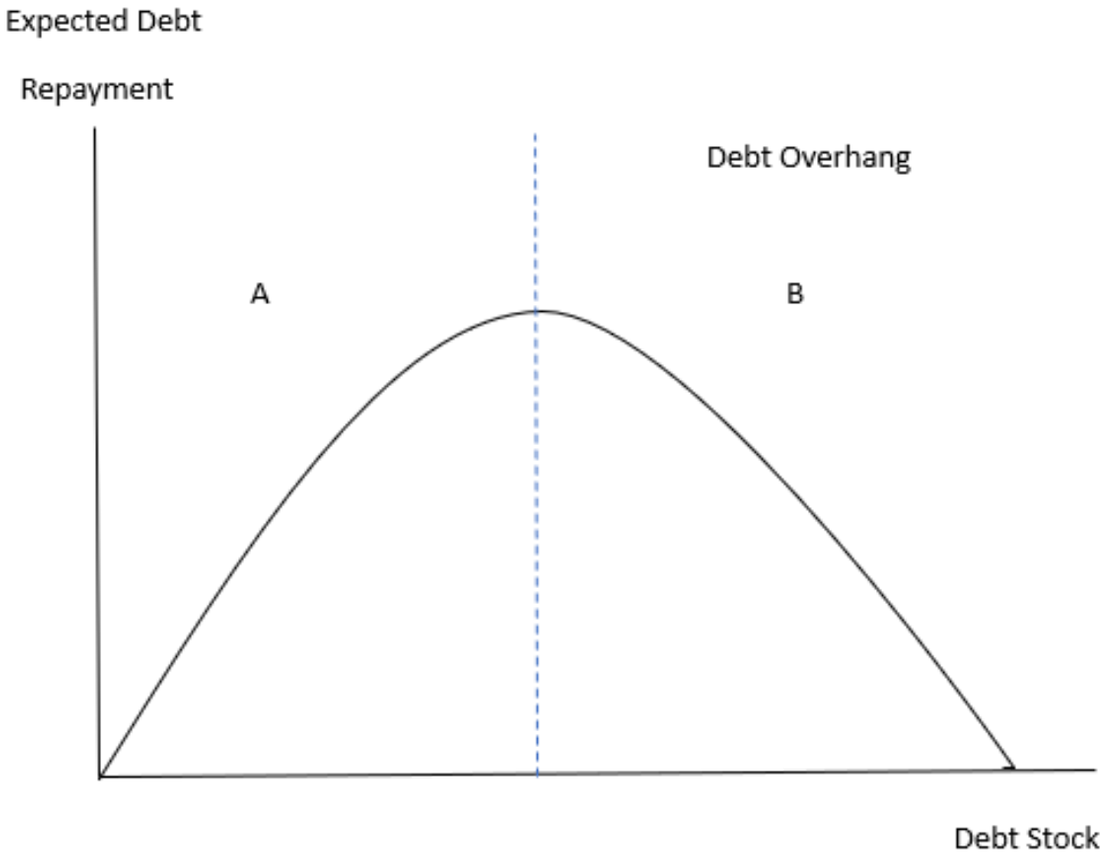
The capital stock in the less developed and emerging economies tends to be limited, due to lower productivity and poor investment, causing countries to consider alternative sources of financing. Faced with this economic obstacle, countries find themselves at a crossroads about whether to raise taxes or borrow more. The rise in taxes is more distortionary compared to borrowing, high taxes can impede private sector decisions and create market inefficiencies, which could affect overall economic growth (Barro, 1990), so most countries prefer to borrow from domestic and external sources. Debt, when used efficiently, positively impacts economic growth by providing funds for infrastructure development, technological advancement, and human capital, but in certain instances, it also causes debt overhang and a crowding effect, inversely affecting growth (Rosemary, 1993). According to recent studies, debt-growth nexus is nonlinear and growth decreases when the optimal debt threshold is exceeded (Reinhart and Rogoff, 2010). This hypothesis states that increases in government debt may have positive growth impacts when the debt level is low and turn negative when debt levels increase above a particular threshold.

3.2.2 Theoretical Framework: Debt Overhang Theory and Debt Laffer Curve

Economic theory proposes that a moderate level of borrowing in developing economies can potentially strengthen their economic growth. This argument stems from the observation that countries in their initial stages of development typically possess limited capital reserves, but they also have access to investment opportunities that have higher rates

of return as compared to more advanced economies. The key condition for this scenario to unfold positively is the efficient utilization of borrowed funds towards productive investments and on development projects, coupled with the absence of macroeconomic instability, policies distorting economic incentives, or significant adverse shocks. Under these circumstances, economic expansion is anticipated, enabling the timely repayment of debt. These predictions remain robust even in theories that acknowledge the realistic constraint that countries may not have unrestricted access to borrowing due to debt distress risks. The answer to the question of how elevated debt level leads to lessening economic growth can be best explained by "Debt Overhang Theory" presented by Krugman (1988), which suggests that if there's a possibility of future debt surpassing a country's repayment capacity, the anticipated costs of servicing debt will deter both domestic and foreign investment, ultimately hindering a country's economic progress.

Figure 5: Debt Laffer Curve



This concept is demonstrated by the debt "Laffer curve" which portrays a converse relationship between debt levels and the probability of debt repayment. The curve suggests that initially, as debt increases, so does the expected repayment. However, beyond a certain threshold, further increases in debt lead to diminishing expectations of repayment, resulting in a downward slope on the curve. Investors become cautious that heightened debt burdens will

necessitate increased future taxes or resources diverted towards debt servicing, disincentivizing current investment for future growth. The U-shaped pattern of the curve underscores the nonlinear connection between debt accrual and growth, highlighting the critical threshold where additional debt becomes counterproductive.

The debt overhang theory and debt Laffer curve illustrate the association between a nation's debt level and its ability to repay its financial obligation as demonstrated in Figure 5. Initially, as debt increases (from 0 to point A), so does its value, along with higher returns and increased likelihood of repayment (segment A). However, beyond a certain threshold (from A to B), labeled as segment B, excessive debt accumulation leads to greater debt stock and lower repayment probabilities, known as debt overhang. This overhang occurs when debt exceeds a country's repayment capacity. Essentially, the debt Laffer curve demonstrates that debt has a beneficial impact up to a point, but beyond that, it becomes detrimental to investment and economic growth. This situation arises due to investment disincentives caused by high debt levels. Consequently, the debt overhang poses challenges to economic development by hindering investment.

3.3 Data Collection

The present study employed annual time series data covering a 50-year period, from 1973 to 2022, to explore the non-linear association between PD and economic growth. The main variables are the real GDP growth rate (RGDP), the debt-to-GDP ratios, and several control variables (Z), collected from different local and international sources. To sustain a justifiable degree of freedom and avoid the issue of multicollinearity, only these variables were included in the model that is mostly used in growth studies. The prior research on the relationship between debt growth and the ideal debt threshold served as a guide for selecting the variables for this study. Caner et al. (2010); Cecchetti et al. (2011); Wright and Grenade (2014); Deng et al. (2023); Padda (2020) take into account several variables, including population growth, trade openness, labor input, domestic savings, capital stock, government consumption, gross capital formation and the exchange rate as the control variables.

The dependent variable in the threshold analysis is real GDP (RGDP) growth rate, while the primary explanatory variables are total public debt, domestic debt, and external debt, all expressed as a percentage of GDP. The other control variables include GDP deflator (INF) as a proxy for inflation, trade openness (TO), investment (GFCF), exchange rate (REER), and population growth (POP). Similarly, for the analysis of debt sustainability, the study utilizes variables such as the primary balance (PB) as the dependent variable, output gap[10], expenditure gap, inflation (INF), and government revenue (GR) and expenditure (GE). All the variables are expressed as a percentage of GDP except REER and population growth rate.

[10]The output gap is an economic measure that represents the difference between an economy's actual output and its potential output, where potential output is estimated by using HP filter

The description, sources, and definition of the variables used in the study are presented in Table 2.

Table 2: Variable description, definition/formula, and sources

Variable	Description	Definition/Formula	Sources of Data
Dependent Variable			
Economic Growth	RGDP	Real GDP growth rate	WDI[11]
Independent Variables			
Public Debt	PDGDP	Public Debt / GDP *100	IFS[12]
Investment	GFCF	GFCF (% of GDP)	WDI
Trade	TO	Exports + Imports/ GDP	WDI
Saving	Gross domestic saving	DS = S - (WR - NII)	WDI
Inflation	INF	Nominal GDP / Real GDP *100	PES[13]
Exchange Rate	REER	Real effective exchange rate	SBP[14]
Domestic Debt	DDGDP	Domestic Debt / GDP *100	PES
External Debt	EDGDP	External debt / GDP *100	PES
Primary Balance	PB	(Revenue – Primary Exp)/GDP	PES
Government Revenue	GR	GR / GDP * 100	IFS
Government Expenditure	GE	GR / GDP * 100	IFS
Expenditure gap	EGAP	G - G* / Yˆ*	WDI
Output gap	OGAP	Y - Yˆ*/ Y[15]	WDI

3.4 Model Specification and Estimation Technique

The threshold auto-regressive regime model developed by Hansen (2000, 1996) is used to ascertain the threshold level of public debt (domestic, external, and total public debt) and to inspect the impact of debt above and below the threshold level on economic growth. This estimation methodology is used due to its advantages over alternative techniques for estimating nonlinear functions. This method enables the identification of the threshold level, its significance, as well as the coefficients and their significance for different regimes simultaneously, all grounded in a robust theoretical framework. The model sorts the data set into different groups considering the value of an observed variable and determines whether it exceeds a specific threshold. Intrinsically, the model internally organizes the data into sets of observations that share the characteristic of exceeding or not exceeding a designated threshold. According to the definition of TRM, the regression coefficients within the model are characterized by variables that feature structural breaks are determined endogenously. Consider the given linear regression model.

$$Y_t = \alpha + \beta_1 X_t + \beta_2 Z_t + \epsilon_{1t} \tag{1}$$

Where Yt is the RGDP growth rate, Xt is the debt-to-GDP ratio, Zt is a set of control variables and Et is the error term. α, β_1 and β_2 are the coefficient parameters to be estimated.

Equation 1 represents a linear multivariate econometric model. However, as discussed in Chapter 1 (introduction) and Chapter 2 (literature review), the recent literature suggests that the connection between PD and economic growth is non-linear with a threshold effect at a certain turning point above and below which the relation between the two variables changes. In the econometric models with threshold effects, the estimation and inference process is a major challenge, because of the threshold parameter which is typically unknown. This leads to challenges in identifying the threshold and ensuring accurate estimation. Unlike conventional models where all the parameters are explicitly defined, nonlinear regression models involving threshold effects require the turning point[16] to be estimated from the data. Therefore, it is crucial to utilize a suitable and proper method for the estimation of the threshold effect. The methodology used in this study is briefly discussed in this section.

The threshold autoregressive models developed by Tong (1978) was further enhanced by Hansen (2000, 1996), where Hansen (2000) introduced new tests for the threshold effect, estimation of the threshold parameters and to construct an asymptotic confidence interval for the parameters. The principle behind the Hansen (2000) estimation procedure is that, given an exogenous threshold variable, the sample is split into two regimes, above and below the threshold. This theory establishes the asymptotic distribution of the threshold parameters. By considering two regime structural equations in the threshold model:

$$Y_t = \alpha_1 + \beta_{11} X_t + \beta_{12} Z_t + \epsilon_{1t} \quad \text{if } X_t \leq \gamma \quad \text{Below the threshold level} \tag{2}$$

$$Y_t = \alpha_2 + \beta_{21} X_t + \beta_{22} Z_t + \epsilon_{2t} \quad \text{if } X_t > \gamma \quad \text{Above the threshold level} \tag{3}$$

Let Xt be the threshold variable used to divide the series into two regimes, Yt the dependent variable, and Zt the set of explanatory variables. Here, ϵ is the error term with white noise properties, and threshold value is represented by γ. If the threshold value γ is known, the model can be easily estimated using Ordinary Least Squares (OLS).

Initially, the threshold value is not known and must be calculated along with the other parameters. When the threshold variable is less than or equal to the threshold parameter, the model follows equation (2). Conversely, when the threshold variable exceeds the threshold parameter, the model follows equation (3).

The binary variable $d_t(\gamma) = \{X_t < \gamma\}$ is defined, where . is an indicator function that equals 1 when $X_t \leq \gamma$ holds and 0 otherwise. Setting $\dot{1}(\gamma) = \epsilon_t(\gamma)\epsilon_t(\gamma)$ equation (2) and equation (3) can be combined into one equation:

$$y_t = \alpha x_t + \beta z_t(\gamma) + \epsilon_t \tag{4}$$

[16] the point at which the relation between the two variables varies

where, the regression parameters$\alpha = \alpha_2, \beta = \alpha_1 - \alpha_2$ and α, γ, β need to be estimated. The RSS (residual sum square) from estimating these regression parameters is denoted as

$$S_1(\gamma) = \epsilon_t(\gamma)\epsilon_t(\gamma) \tag{5}$$

Hansen (2000) suggests estimating γ using the LS method, which minimizes the RSS based on the predicted threshold value. Consequently, the optimal threshold can be expressed as:

$$\gamma = \arg\min S_1(\gamma) \tag{6}$$

Given γ, the equation of regression becomes linear in β and δ, and the given OLS estimates of $\beta(\gamma)$ and $\delta(\gamma)$ are obtained by regressing the dependent variable to the regressors. In accordance with the procedure outlined above the linear equation in equation 1 can be written in the form of a non-linear threshold regression model as given below;

$$\begin{aligned} Y_t = {} & \beta_{0,1} \cdot 1(X_t \leq \lambda) + \beta_{0,2} \cdot 1(X_t > \lambda) + \beta_{1,1} X_t \cdot 1(X_t \leq \lambda) \\ & + \beta_{1,2} X_t \cdot 1(X_t > \lambda) + \beta_{2,1} Z_t \cdot 1(X_t \leq \lambda) + \beta_{2,2} Z_t \cdot 1(X_t > \lambda) + u_t \end{aligned} \tag{7}$$

In the above equation, 1 symbolizes the indicator function, which has the value 1 when the event inside happens, and 0 otherwise.

If we let $x_t(\gamma) = x_t I(\gamma)$, this will result in rewriting Equations 2 and 3 as in Equation 7. Yt indicates the long-runn RGDP growth rate and Xt corresponds to the public debt-to-GDP ratio. The control variables are represented by Zt. The unknown threshold value λ and the coefficients β_0 β_1 β_2 are estimated using the TRM developed by Hansen (2000). For the estimation of the above equation, the optimal threshold value is identified by minimizing the residual sum of squares (RSS), therefore the term threshold least square regression model is also used for this methodology in literature. The public debt-to-GDP ratio is used as a threshold variable. The initial step in determining the total public, external, and domestic debt threshold involves evaluating if the relationship between the two variables follows a linear or nonlinear pattern. In this context, conventional testing methods are not applicable, as the threshold parameter λ remains undefined under the null hypothesis (no threshold effect). Thus, Hansen (1996) recommends using a heteroscedasticity-consistent Lagrange Multiplier (LM) bootstrap method to derive the asymptotic critical and p-values. This approach involves using the standard F-statistic as a test with nearly optimal power against alternatives far from the null hypothesis Ho:

$$F = \frac{S_1 - S_2}{\hat{\sigma}^2} \tag{8}$$

Where S2 and S1 are the residual sum of squares. The bootstrap procedure achieves the first order distribution (asymptotic), which guarantees that the p-values produced are asymptotically valid. After estimating the threshold, the following step involves the evaluation of the statistical significance of the estimates. Hansen (2000) recommends employing the technique of bootstraping to simulate the empirical distribution of the following likelihood ratio test:

$$LR(\gamma) = \frac{S_1(\gamma) - S_1(\hat{\gamma})}{\hat{\sigma}^2} \tag{9}$$

where $S_1(\gamma)$ and $\hat{\gamma}$) are the residual sum of square under $H_0 : \gamma = \gamma_0$ and $H_:\gamma is not equal to \gamma_0$ respectively, and σ^2 is the residual variance. Threshold estimation methodology is utilized in this study due to its advantages over other methods for estimating nonlinear relationships. This method helps to determine the threshold levels, their statistical significances, and the coefficients, along with their significance, all derived at once from the data and grounded in robust theoretical foundations. Due to its robust threshold determination and higher applicability in nonlinear regressions, a vast number of recent studies have used this methodology to estimate the nonlinear relationship between different variables and for threshold determination (Buthelezi and Nyatanga, 2023; Nizam et al., 2020; Alfada, 2019; Munir et al., 2009; Law et al., 2013; Caner et al., 2010).

3.4.1 Threshold Regression Model for Public debt

The following econometric model is used to find the relation between PD and economic growth considering the prudent threshold effect following Caner et al. (2010); Munir et al. (2009); Akram (2011).

$$\text{RGDPt} = \beta_0 + \beta_1 \text{PDGDPt} + \beta_2 \text{GfCFt} + \beta_3 \text{TOt} + \beta_4 \text{INFt} + \beta_5 \text{POPt} + \beta_6 \text{REERt} + \text{Et} \tag{10}$$

Where RGDP is the Real GDP growth rate, DGDP is the Debt-to-GDP ratio, GFCF is the Gross fixed capital formation, TO is Trade Openness, INFis Inflation rate (GDP deflator), POP is the Population growth, REER is Exchange rate, The above equation can be written in nonlinear form as given below:

$$\begin{aligned} Y_t = {} & \beta_{0,1} \cdot 1(X_t \leq \lambda) + \beta_{0,2} \cdot 1(X_t > \lambda) + \beta_{1,1} X_t \cdot 1(X_t \leq \lambda) \\ & + \beta_{1,2} X_t \cdot 1(X_t > \lambda) + \beta_{2,1} Z_t \cdot 1(X_t \leq \lambda) + \beta_{2,2} Z_t \cdot 1(X_t > \lambda) + u_t \text{if } X_t \leq \gamma \end{aligned} \tag{11}$$

$$\begin{aligned} Y_t = {} & \beta_{0,1} \cdot 1(X_t \leq \lambda) + \beta_{0,2} \cdot 1(X_t > \lambda) + \beta_{1,1} X_t \cdot 1(X_t \leq \lambda) \\ & + \beta_{1,2} X_t \cdot 1(X_t > \lambda) + \beta_{2,1} Z_t \cdot 1(X_t \leq \lambda) + \beta_{2,2} Z_t \cdot 1(X_t > \lambda) + u_t \text{if } X_t > \gamma \end{aligned} \tag{12}$$

Where γ is the threshold level of PD, equation 11 represents the impact of PD below the threshold on economic growth and equation 12 represents the impact above the threshold.

3.4.2 Threshold Regression Model for domestic debt

The TRM for domestic debt is given below, where RGDP is the dependent variable, and the explanatory variables are domestic debt as a percentage of GDP (DDGDP), trade opennes (TO), inflation (INF) and Et is the error term.

$$\text{RGDPt} = \beta_0 + \beta_1 \text{DDGDPt} + \beta_2 \text{POPt} + \beta_3 \text{GFCFt} + \beta_4 \text{TOt} + \text{Et} \tag{13}$$

3.4.3 Threshold Regression Model for External Debt

The econometric model for external debt is given in equation 14, where the dependent variable is RGDP, EDGDP is the ratio of external debt to GDP, INF is inflation, TO is Trade Opennes and REER is the real effective exchange rate and Et is the error term.

$$\text{RGDPt} = \beta_0 + \beta_1 \text{EDGDPt} + \beta_2 \text{TO} + \beta_3 \text{INFt} + \beta_4 \text{REER} + \text{Et} \tag{14}$$

3.5 Test for threshold level

The Heteroscedasticity Consistent Lagrange multiplier test (LM) by Hansen (1996) is employed to test the nonlinear relationship between PD and economic growth. The LM test is mostly used to figure out the threshold effect in the relation betwixt two variables. The null hypothesis of LM test is of no threshold, while the alternative hypothesis is that there exists a threshold level below and above which the relation between the variables varies.

$$H0 = \beta_{0,1} = \beta_{0,2} = \beta_{1,1} = \beta_{1,2} = \beta_{2,1} = \beta_{2,2}$$

In the absence of a threshold, we accept Ho given in the equation above, and a simple, Ordinary Least Square model can be estimated and the relation will be considered linear. Equation (7) is estimated if a threshold effect exists, considering the unknown threshold value of λ. Since, they can replicate the asymptotic distribution, as shown by Hansen (1996), for this purpose, bootstrap p-values are used.

3.6 Debt Sustainability Analysis

3.6.1 Public Debt Sustainability: A Theoretical Aspect

The government expenditure usually exceeds its revenue as a response to the necessity of improving living standards, providing employment, investing in infrastructure, improving health and education services, and encouraging economic growth. To address this, governments rely on several sources such as increasing taxes, issuing currency or borrowing from external and domestic sources. While issuing currency would increase the supply of money which is expected to trigger inflation, it is an unfavorable option among economists, while taxes create distortions in the market, whereas in such a situation borrowing is considered a favorable alternative. Although historically seen as undesirable, modern economic thought considers government borrowing acceptable, provided it aligns with principles of sound fiscal management. In any given period t, the government must cover its expenditure through revenue generation and bond issuance. The government's budget constraint for the period t is represented by:

$$G_t + (1 + r_t)D_{t-1} = R_t + D_t \tag{15}$$

Where, the G_t is the government's primary expenditure[17], r_t interest rate in real term, R_t is the total tax revenue, D_t is the ratio of debt to GDP at period t, the previous debt stock is $D_t - 1$, $rD_t - 1$ is the interest on the previous debt. The primary balance is calculated as the difference between total government revenue and primary expenditures[18], (PBt = Gt - Rt) equation 15 can be written as:

$$D_t = (1 + r_t)D_{t-1} - PB_t \tag{16}$$

In terms of Nominal GDP (Yt), by assuming Yt increases at an annual rate of θ, the above equation can be converted in the form of ratios Yt as,

$$\frac{D_t}{Y_t} = \frac{D_{t-1}}{Y_{t-1}}\frac{Y_{t-1}}{Y_t}(1 + r_t) + \frac{P_t}{Y_t} \tag{17}$$

$$d_t = \left(\frac{1 + r_t}{1 + \theta_t}\right) d_{t-1} + p_t \tag{18}$$

If the growth rate is less than or equal to the interest rate, the necessary share of the primary surplus in GDP to maintain a constant debt-to-GDP ratio can be expressed as:

$$d_t = \frac{(1 - r_t)}{(1 + \theta_t)}d_{t-1} - pb_t \tag{19}$$

[17]The government total expenditure minus interest payments on public debt
[18]Primary balance = total revenue - primary expenditure

The equations show that at period t, the debt-GDP ratio depends on the interest payments on previous debts, the economy's growth rate, and PB (primary balance). The impact of interest on debt depends on the economy's growth rate. When the interest rate exceeds the growth rate (rt ¿ gt), the debt-to-GDP ratio will increase automatically

3.7 Econometric Approaches for Debt Sustainability

There are three main approaches to assess the sustainability of public debt, derived from the government budget constraint.

- Stationarity test Approach
- Co-integration Approach
- Fiscal reaction Function Approach

3.7.1 Stationarity tests

The first approach follows the methodology outlined by Hamilton and Flavin (1985), applying the stationarity (unit root) test to the series of public debt and primary balance (whether public debt is explosive or not), where the stationarity implies that the series will revert to a mean or a trend over time, suggesting that the debt will eventually stabilize relative to GDP, indicating sustainability. Whereas non stationary (presence of a unit root) suggests that the debt series follows a random walk and could grow without bound, which implies unsustainability. This approach was widely used in the literature to examine the mean reversal process of debt series to assess the sustainability of pd in different countries (Shanmugam and Renjith, 2023; Feve and Henin, 2000; Curtaşu, 2011)

3.7.2 Co-integration Tests

The second approach uses co-integration tests to identify a co-integrating relationship between revenue and expenditure Trehan and Walsh (1991); Stoian (2008). The co-integration regression is specified as:

$$r_t = \alpha + \beta z_t + \nu_t \tag{20}$$

where, κ_t and z_t are the government revenues and expenditures as a percentage of GDP. If both variables have the same order of integration$I(1)$, for the PVBC[19] to hold, it is a necessary and sufficient condition that κ_t and z_t be co-integrated. This implies that government revenues and expenditures are in accordance with each other which indicates a healthy budgetary policy.

Despite their broader applicability, stationarity and co-integration approaches were greatly criticized. Firstly, the unit

[19]Present Value Budget Constraint, the equation is derived in the appendix 1

root test is sensitive to structural breaks, which can lead to misleading results when such breaks occur (Uctum et al., 2006). Secondly, the rejection of unit roots in the series of debt-to-GDP ratios was considered a significant challenge. Some studies also found that the intertemporal budget constraint also holds when there exists no co-integration between two variables.

3.7.3 The Fiscal Reaction Function Approach (FRF)

The third approach to evaluate the sustainability of PD deals with how the government deals with the changes in its debt burden. It was developed to overcome the major criticisms on the previous approaches. The primary hypothesis when estimating a fiscal reaction function (FRF) is that the government amends the primary budget surplus in response to fluctuations in debt levels, aiming to maintain long-term debt sustainability. In line with the empirical literature (Bohn, 1998), the fiscal reaction function is typically derived from the government's inter-temporal budget constraint.

$$PB_t = a_0 + a_1 d_{t-1} + u_t \tag{21}$$

Bohn (1998) empirically estimated a FRF, where the primary surplus was regressed on the public lagged debt to GDP ratio while controlling for other factors that influence fiscal policy. Since the surplus in a given period depends on its previous value, the FRF equation is extended by incorporating the lagged primary surplus into the set of regressors. The fiscal reaction function can be represented as:

$$PB_t = a_0 + a_1 d_{t-1} + a_2 pb_{t-1} + a_2 C_t + u_t \tag{22}$$

If fiscal authorities respond systematically to rising debt by increasing the primary balance to maintain sustainable debt over time, the transversality condition of the budget constraint will be satisfied, and fiscal policy effectively prevents excessive debt accumulation Bohn (1995). PD will be considered sustainable if the primary balance remains stable and positively increases with the rise in the debt level. The present study is based on the fiscal reaction function given below, following Bohn (1998); Khalid et al. (2007); De Mello (2008); Potrafke and Reischmann (2015)

$$PB_t = a_0 + a_1 PB_{t-1} + a_2 PD_{t-1} + a_3 EXGAP_t + a_4 OGAP_t + a_5 INF_t + u_t \tag{23}$$

where: PB_t is the primary balance, PD_t is the ratio of public debt to GDP, INF_t is the inflation rate (CPI), EXGAP is the expenditure gap, $OGAP_t$ is the output gap and u_t is the error term. OGAP is the difference between actual and potential output, whereas EXGAP is the difference between actual and potential expenditure, which are estimated using the HP filter[20].The control variables output gap (reflecting the business cycle's effect on the budget, influenced

[20]Hodrick-Prescott filter

by the scale of automatic stabilizers) and inflation (capturing the impact of shocks on seigniorage revenues).

3.7.4 World Bank and IMF debt Sustainability Framework

The IMF has two frameworks for debt sustainability analysis, LIC DSF and MAC DSA. For market access economies, the framework was reviewed by the IMF Executive Board in 2021 and is currently in use after significant revisions and improvements. It applies to countries with access to the market, which includes countries that are not eligible for PRGT[21] fund facilities. The World Bank and the International Monetary Fund developed the LIC-DSF tool in 2005, and it was reviewed again in 2017 to assess the sustainability and vulnerabilities of low-income countries (LICs') external and public debt. This tool supports macroeconomic surveillance, policy decisions to avoid sovereign debt restructurings, and borrowing decisions for LICs that depend on concessional financing. It is particularly intended for these LICs. Examining all public and publicly guaranteed debt, both domestically and internationally, across a range of public sector organizations is part of the evaluation process. The LIC-DSF uses a standardized methodology that uses a forward-looking technique to categorize nations according to their ability to carry debt, their mechanical risk of external debt distress, and their overall risk of debt distress. Stress tests and debt burden indicators, such as solvency and liquidity metrics, are used to classify debt to evaluate its sustainability. The tool offers a comprehensive framework for assessing and managing the dynamics of sovereign debt by classifying nations as low, moderate, high, or in debt distress. uses MAC DSA as a critical tool to assess the viability of public debt in market access countries. The MAC DSA is essential to the IMF's primary lending and surveillance duties, since it helps determine the vulnerability of a country to sovereign stress and offers recommendations for mitigating it. The framework helps to determine whether more radical measures, such as sovereign debt restructuring or a mix of IMF finance and economic reforms, are required to achieve medium-term debt sustainability under programs supported by the IMF. The risk-based methodology of the MAC DSA takes into account extraordinary access to IMF resources, compares the public debt-to-GDP ratio against specified levels for advanced economies and developing markets, and calculates the public debt-to-GDP ratio as a baseline. In order to evaluate vulnerabilities, the framework takes into account a number of indicators, including debt composition, market indicators, and alternative scenarios. The analysis that results from this process helps decision-makers make well-informed choices about IMF conditionality and whether debt relief is necessary in debt restructuring operations. MAC DSA was reviewed by the IMF Executive Board in 2021 and is currently in use after significant revisions.

[21] Poverty Reduction and Growth Trust: serves as the main platform for financing low-income nations with concessional funding (zero interest rate)

3.8 Significance of the research

Empirical literature shows that the debt threshold level varies from one country to another depending on the country's institutional framework, debt management capacity, and macroeconomic indicators (WB IMF debt sustainability framework). To the best of our knowledge, very little research has been done to unravel the tipping point at which the rising debt level hurts Pakistan's economic growth. Therefore, it seems important to perform a threshold analysis to identify the debt threshold level and its possible impact on the pace of economic growth. The significance of this study lies in the following areas: The primary objective of this study is to determine the critical thresholds at which Pakistan's chances for economic progress are hindered by the accumulation of more debt. Since there isn't much research on this topic specifically in Pakistan, the expected findings of the study provide the policymakers with an extensive amount of insight. These understandings will assist in developing debt management plans that will be appropriate and productive while also considering Pakistan's particular economic conditions and institutional framework. The understanding of the threshold level is important for policymakers since it provides substantial directives regarding the optimal level of debt a country should manage. Adding another level of contribution to the assessment of its applicability in the nation's fiscal policy framework, this study will also attempt to determine whether the 60% debt-to-GDP ratio benchmark set by the Fiscal Responsibility and Debt Limitation (FRDL) Act, 2005 in Pakistan has been established based on empirical investigation or not. The study also explores the causal connection between public debt thresholds and the viability of a nation's public finances. Long-term economic stability depends critically on sustainable fiscal policy. By understanding the relationship between public debt levels and fiscal sustainability, policymakers can formulate strategies to maintain a balanced and sustainable fiscal stance. This involves ensuring that the accumulation of public debt remains within manageable limits, allowing the government to meet its financial obligations without causing a destabilizing impact on the economy. The study examines whether adhering to sustainable fiscal policies, informed by the identified public debt thresholds, can serve as a critical mechanism for achieving economic stability.

3.9 The Limitations of the study

1. This research study relies on secondary data obtained from various sources due to the non-availability of the data in a single source. Different sources may have varying methodologies, time frames, or definitions, leading to inconsistencies that can affect the accuracy and reliability of the findings.

2. The primary focus of this study is to explore the nonlinear impact of public debt on economic growth, rather than exploring the determinants of economic growth. Therefore, emphasis is placed on examining the debt-growth nexus, with less consideration given to other variables.

CHAPTER

4 Result and Discussion

The primary aim of this study is to analyze the non-linear association between PD and economic growth and to estimate the external debt, domestic debt, and total public debt thresholds for Pakistan. The threshold regression model developed by (Hansen, 2000, 1996) is utilized for the estimation of thresholds and to determine the impacts of debt on economic growth below and above the threshold levels. Furthermore, the study also assesses the sustainability of PD using various theoretical and empirical methods (Stationary test approach, cointegration approach, and fiscal reaction function approach). The empirical findings of the study are discussed in this chapter.

4.1 Statistical Properties

The statistical properties of the variables used in the study are discussed in Table 3 for the period 1973-2022. The table reports the standard deviation, maximum, minimum, mean, and the number of observations for the variable. Most variables are measured as a percentage of GDP (public debt, domestic debt, trade openness, GFCF, external debt, inflation, and domestic saving), except RGDP, population growth, and real effective exchange rate.

The real GDP growth rate of Pakistan averaged 4.81 percent from the year 1973-2022 attaining the maximum value of 10.22 percent in the year 1980 and the minimum value of -1.27 in 2020. The real GDP growth has remained positive for most of the years except 2020 (-1.27). The standard deviation of RGDP is 2.15 and that of PD, domestic, and external debt are 11.84, 9.18, and 9.54. The total public debt and domestic and external debt-to-GDP ratios averaged 54.43, 32.30, and 34.84 respectively. This shows that Pakistan has accrued more domestic debt as compared to external debt during 1973-2022. The public debt to GDP reached the highest in the year 2020 (79.56), and the domestic and external debt reached the maximum value of 48.9 and 59.27 in the year 2020 and 1973 respectively.

Table 3: Statistical Summary

Variable	Mean	Median	Maximum	Minimum	Std	Observations
RGDP	4.81	4.58	10.22	-1.27	2.15	50
PDGDP	54.43	55.41	79.56	32.89	11.84	50
DDGDP	34.84	33.29	48.98	20.69	9.18	50
EDGDP	32.30	33.02	59.27	17.05	9.54	50
TO	31.21	32.34	38.50	21.46	4.19	50
GFCF	15.45	15.84	19.11	11.33	1.84	50
INF	10.32	9.02	68.01	0.92	9.48	50
MS	43.18	44.22	54.53	28.69	5.71	50
POP	2.66	2.86	4.42	1.20	0.79	50
REER	132.27	110.56	225.54	11.4	48.01	50

4.2 The Correlation Matrix

The correlation matrix is used to illustrate the relationship between variables using a correlation coefficient. The coefficient varies between 1 and -1 where 1 indicates a strong positive correlation between the variables, if it is 0 then the relationship is neutral, and -1 shows a strong negative correlation between the variables. The results of the correlation matrix are displayed in Table 4.

Table 4: Matrix of Correlation

	RGDP	PDGDP	DDGDP	EDGDP	INF	GFCF	POP	REER	TO
RGDP	1.000								
PDGDP	-0.463	1.000							
DDGDP	-0.256	0.707	1.000						
EDGDP	0.049	-0.270	-0.027	1.000					
INF	-0.136	0.064	-0.148	0.159	1.000				
GFCF	0.235	-0.366	0.042	0.213	-0.128	1.000			
POP	0.362	-0.586	-0.270	0.645	0.135	0.571	1.000		
REER	0.384	-0.671	-0.490	0.555	0.054	0.282	0.632	1.000	
TO	0.037	-0.310	0.183	0.322	-0.159	0.612	0.425	0.159	1.000

The findings demonstrate an inverse link between the growth rate of real GDP (RGDP) and public debt to GDP ratio (PDGDP), highlighting that the rise in PD reduces economic growth. Domestic debt (DDGDP) is positively linked to money supply (MS) and population growth (POP) but negatively correlates with RGDP and inflation (INF). External debt as a percentage of GDP (EDGDP) is positively associated with domestic savings(DS) and population growth but negatively associated with money supply (MS). Trade openness (TO) positively influences gross fixed capital formation (GFCF) but negatively relates to PDGDP. Lastly, POP is positively correlated with both PDGDP and EDGDP but negatively affects RGDP and MS, indicating that high population growth does not necessarily foster economic growth. RGDP shows a weak positive correlation with GFCF, suggesting slight investment influence. PDGDP positively correlates with DDGDP and POP, while negatively relating to EDGDP and trade openness.

The results show that for all the variables the correlation coefficient ranges between -0.128 to 0.6 which is reasonable to avoid multicollinearity in the base regression. The absence of multicollinearity improves the reliability of the regression coefficients, enabling clearer interpretations of each variable's influence. Consequently, the analysis can proceed without the risk of inflated standard errors or misleading statistical significance, ensuring robust findings.

4.3 Unit root test

The macroeconomic variables used in time series analysis are often non-stationary, which leads to spurious results in econometric analysis. Therefore, testing for unit roots is important to ensure reliable analysis. Stationarity is a property that indicates that the statistical characteristics, for instance, the mean and variance of a series, do not vary over time, indicating the long-term stability of the variables. Hence, the variables used in the regression analysis are tested for unit root at level and first difference. The test used in this study is the Augmented Dicky Fuller Test (ADF), with the null hypothesis of the unit root in the series and the alternate hypothesis of no unit root. The p-value and t-statistic guide in deciding whether to accept or reject the null hypothesis. When the p-value is less than 0.05 and the t-statistic falls below the critical values at the 1%, 5%, or 10% significance levels, we do not accept the null hypothesis.

Table 5: Results: ADF Unit Root Test

Variables	**Level**		**First Difference**		**Decision**
	Intercept	**Trend and Intercept**	**Intercept**	**Trend and Intercept**	
RGDP	-5.2873*** (0.0001)	-5.7718*** (0.0001)	- -	- -	I(0)
PDGDP	-0.60856 (0.8591)	-1.91616 (0.631)	-5.7462*** (0.0000)	-5.6976*** (0.0001)	I(1)
DDGDP	-1.0235 (0.7378)	-1.34421 (0.8647)	-6.4546*** (0.0000)	-6.3495*** (0.0000)	I(1)
EDGDP	-3.0693** (0.0356)	-3.38302 (0.0655)	- -	- -	I(0)
TOPEN	-2.3881 (0.1503)	-2.57018 (0.2951)	-6.1952*** (0.0000)	-6.0916*** (0.0000)	I(1)
GCF	-2.5796 (0.104)	-4.00073** (0.015)	- -	- -	I(0)
DS	-1.38074 (0.5841)	-1.50973 (0.8128)	-7.4363*** (0.0001)	-7.3732*** (0.0002)	I(1)
MS	-2.49749 (0.1223)	-3.3647 -	-6.6208*** (0.0000)	-6.4719*** (0.0002)	I(1)
POPG	-0.79336 (0.8117)	-4.12591** (0.011)	- -	- -	I(0)
PB	-2.839704* (0.2535)	-2.528723*** (0.0126)	- -	- -	I(0)
GE	-1.649318 (0.0606)	-2.085277 (0.5408)	-6.329306*** 0.0000	- -	I(1)
GE	-1.840978 (0.3569)	-2.222381 (0.4670)	-7.322516*** 0.0000	- -	I(1)

The results of the ADF test are presented in Table 5 which shows the unit root of the variables at the level and the first difference: with an intercept and with both trend and intercept. For real GDP (RGDP), the P value is less than 0.05 and the t stat is less than critical values (1%, 5%, or 10%) at the level, and with intercept, this implies that RGDP is stationary at the level. The findings furthermore reveal that Public Debt to GDP (PDGDP) and Domestic Debt to GDP (DDGDP) are not stationary at the level. However, the two variables become stationary after taking the first difference and, therefore, they become integrated of order one, I(0). Furthermore, External Debt to GDP is stationary at level, because there is a significant test statistic for the intercept, hence the debt ratio is of order I(0). Likewise, Gross Fixed Capital Formation (GFCF), population growth (POP), and primary balance (PB) are also stationary at the level. Trade openness (TO), Domestic saving (DS), and Money supply (MS) are non-stationary at the level but become stationary after taking the first difference.

The results of the unit root test reveal that the variables are stationary at different levels, I(0) and I(1), and according to economic theory, such analysis leads to spurious regression, when the proper methodology is not utilized. Therefore, all the variables that are integrated of order I(1) are made stationary I(0) by taking the first difference.

4.4 Determination of Public debt, Domestic debt and the External debt threshold

4.4.1 Test for threshold in Public debt

The Heteroskedasticity Consistent Lagrange (LM) test by Hansen (1996) is used to test the nonlinear association between public debt and the growth rate of real GDP. The null hypothesis of the LM test is of no threshold while the alternative hypothesis is that there exists a threshold below and above which the relation between the variables varies.

$$H0 = \beta_{0,1} = \beta_{0,2} = \beta_{1,1} = \beta_{1,2} = \beta_{2,1} = \beta_{2,2} \tag{24}$$

When there is no threshold effect, the above equation will not be rejected, allowing for the estimation of a linear regression model as presented in equation (1). On the other hand, in the presence of the threshold effect, equation (7) will be estimated. For this, bootstrap p-values are employed since they reproduce the asymptotic distribution, as demonstrated by Hansen (1996).

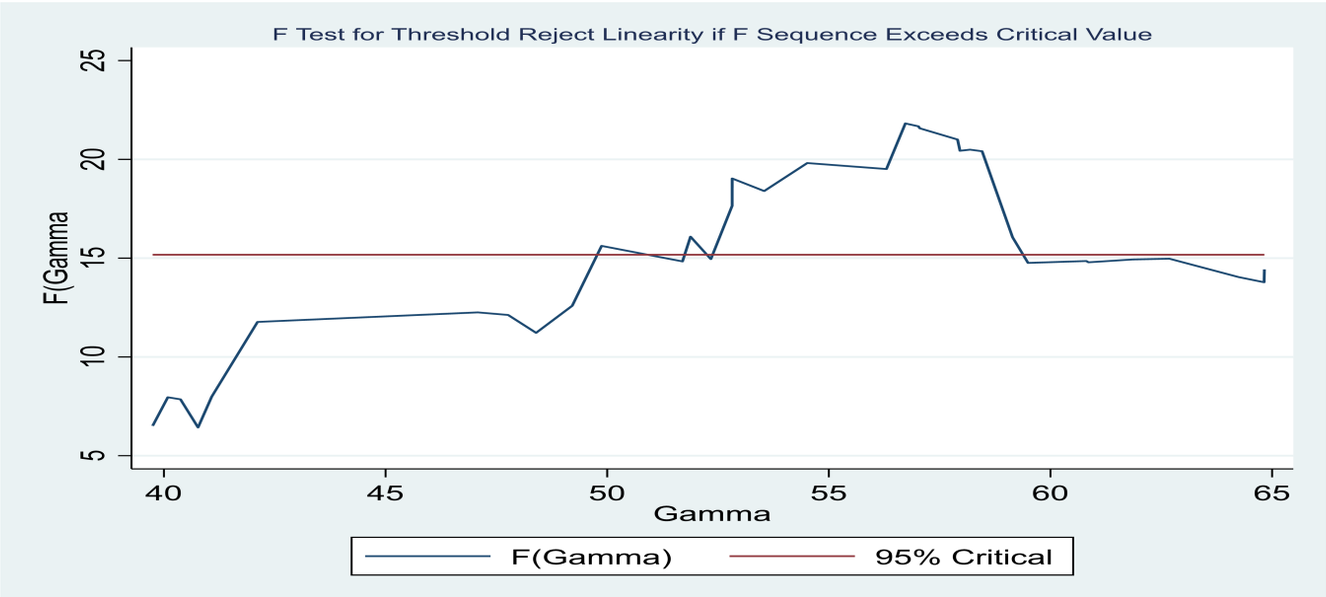
The results of the LM test for the effect of the threshold in public debt are presented in Table 6, which shows that by using 5000 bootstrap replications and the public debt-to-GDP ratio as threshold variable, the p-value is significant at 0.002, and the value of F statistics is 21.26, indicating that the relationship between PD and economic growth is not linear. Based on the P-value and F-statistics we do not accept the null hypothesis of no threshold effect, but rather accept the alternate hypothesis that there exists evidence of a threshold between PD and GDP growth rate. The study reveals a threshold effect in PD that is consistent with the findings of previous studies (Caner et al., 2010; Shanmugam and Renjith, 2023; Law et al., 2021; Obiero and Topuz, 2023).

Table 6: LM Test for Threshold Effect in Public Debt

Number of Bootstrap Replications	5000
Trimming Percentage	0.15
Threshold Estimate	56.7
Bootstrap P-Value	0.002
LM-test for no threshold	21.2576587
95% Confidence Interval	[57.9035561,64.8195303]

Similarly, Figure 6 displays the F test for the threshold effect in PD, where the red line denotes the critical values 95% and the blue line represents the F statistic (F (Gamma)) across various Gamma values (threshold). Evidence of nonlinearity is shown in the cases where the blue line crosses the red line, indicating that the linearity null hypothesis is rejected at the 5% significance level. More specifically, the crossing of the blue line implies the existence of large threshold effects, which suggests that the variable's relationship is probably nonlinear and that adding these threshold effects to the model will improve its fit compared to a simple linear one.

Figure 6: F test for threshold rejection if sequence exceeds critical values



4.4.2 Estimation of public debt threshold

The threshold regression model (Hansen, 2000) is used to find the threshold level of public debt and the impact of public debt on economic growth above and below the threshold level. The dependent variable is real GDP growth rate

(RGDP) while public debt-to-GDP ratio (PDGDP) is the threshold variable, and other explanatory variables include population growth (POP), inflation (INF), real effective exchange rate (REER) and gross fixed capital formation (GFCF). The model first estimates the linear relation between the dependent variable and the explanatory variables and then splits the sample into two regimes based on the threshold. The result of the linear regression model shows that PDGDP has a significant and negative impact on economic growth at a level of significance of 10% when the linear relationship between the variables is estimated using ordinary OLS. The control variables gross fixed capital formation, population rate, trade openness, and real effective exchange rate have insignificant but positive impacts, whereas Inflation shows a negative influence on the country's growth. The linear association is estimated only for comparison purposes.

The results of the threshold regression model reported in Table 7 indicate that the threshold level of public debt for Pakistan is 58.5%, which is closed to Caner et al. (2010); Shah et al. (2024); Law et al. (2021) threshold estimations for developing economies. The influence of public debt on economic growth varies when the PDGDP reaches 58.5 percent, which the model classified into two regimes; Regime 1, below the threshold level, and Regime 2, above the threshold level of 58.5%.

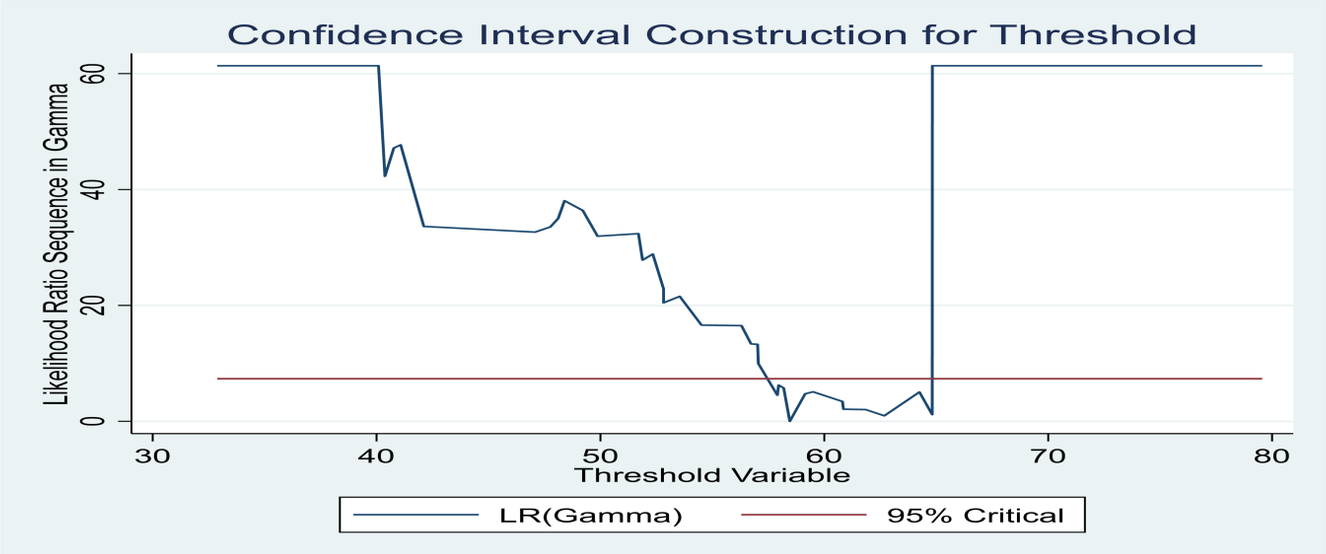
Table 7: Results: TRM for Public Debt
Dependent Variable: RGDP

Variables	**Regime 1: PDGDP** $\leq$ 58.5%			**Regime 2: PDGDP** $>$ **58.457%**		
	Estimate	**Standard Error**	**T-statistics**	**Estimate**	**Standard Error**	**T-statistics**
Intercept	11.8391	5.4652	2.1620	30.1610	8.7897	3.4266
PDGDP	-0.0323	0.0539	-0.5994	-0.2966	0.0809	-3.6692
GFCF	0.4911	0.1937	2.5370	-0.4669	0.3445	-1.3567
POP	1.3115	0.4452	2.9434	-1.0348	0.7918	-1.3072
INF	-0.2562	0.0568	-4.5058	0.1044	0.0290	3.5994
REER	0.0093	0.0091	1.0209	0.0196	0.0181	1.0809
D_TO	0.1460	0.1618	0.9026	0.4021	0.1341	2.9975
Observations	33			17		
Degrees of Freedom	26			10		
Sum of Sq. Errors	65.7805			32.4377		
Residual Variance	2.5300			3.2438		
R-squared	0.5299			0.5640		

Threshold Variable: Public Debt to GDP ratio

Table 7 also shows that in Regime 1, below the threshold level (58.5%), the PDGDP has no significant impact on economic growth, although the relation is negative. When PD rises above the threshold level (58.5% debt to GDP ratio) it affects economic growth negatively. Above the threshold level each additional percent increase in PD reduces the economic growth by 0.29 percent. The relationship is significant and quantitatively valuable. Determining the threshold level of public debt and its effects on economic growth both below and above is the main objective of this study. However, other control variables are also included in the model based on the existing literature and theories to enhance the robustness of the results. In Regime 1 below the threshold level of 58.5% inflation (INF) has a significant negative impact on the growth rate while GFCF has a positive relation with the GDP growth rate. Population growth and trade openness have an insignificant but positive impact on economic growth. The total number of observations in the regime is 33 and the R-square is 53%. In Regime 2 when the PDGDP exceeds 58.5%, each percentage increase in the debt-to-GDP ratio reduces economic growth by 0.29%. However, inflation has a significant negative impact on economic growth.

Figure 7: First Sample Split: Confidence interval constructed for threshold



Threshold Variable: Public Debt to GDP ratio

Figure 7 shows the normalized likelihood ratios as a function of the threshold in the debt-to-GDP ratio. The graph exhibits that the 95% confidence intervals of the threshold lie between 57.90 and 64.81. As discussed in Chapter 3, the LS (Least Square) estimate of the threshold is the value that minimizes the LRn(y) function, Which occurs at 58.5%. The asymptotic critical values at 95 percent, are represented by the dotted line. Where this line intersects with LRn(y), it defines a confidence interval of [57.9035561,64.8195303]. The estimated tipping point of PD (58.5%) falls precisely within the confidence intervals. These findings indicate that the threshold estimates are highly precise and also provide evidence of the presence of a single threshold.

4.5 Test for threshold in Domestic and External Debt

The LM test by Hansen (1996) is used to confirm the existence of a threshold effect in domestic and external debt. The null hypothesis of the LM test is that there exists no threshold effect and the relation is linear, while the alternative hypothesis is that there exists a threshold level, below and above which the relation between the variables varies. The results are illustrated in Table 8 for the domestic debt and Table 10 reports the results of the LM test for external debt.

Table 8: LM test for the threshold effect in domestic debt

Number of Bootstrap Replications	5000
Trimming Percentage	0.15
Threshold Estimate	43.39
Bootstrap P-Value	0.0434
LM-test for no threshold	12.45
Confidence Interval 95%	[25.6214263% , 44.77797%]

Table 8 reports the results of the LM test for the threshold effect in domestic debt as a percentage of GDP, the results confirm that using 5000 bootstrap replications and the domestic debt-to-GDP ratio as a threshold variable, the p-value (0.043) is significant at 5% (significance level), and the value of F statistics is 12.45, which implies that the relationship between domestic debt and economic growth is not linear. Based on the P-value and LM-test stat we accept the alternate hypothesis that there exists evidence of threshold effect in the relation between domestic debt and GDP growth rates. The F stat for threshold (f-gamma) crosses the 95% critical values, further confirming the results.

Figure 8: F test for threshold rejection if sequence exceeds critical values (Domestic Debt)

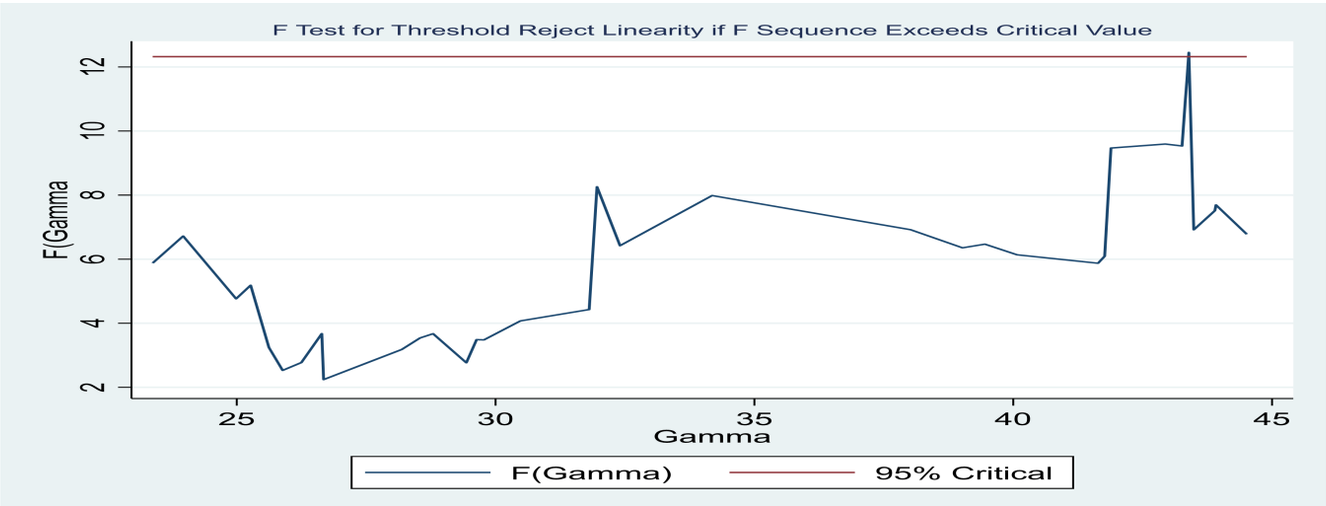


Table 9: Results: Threshold Regression for Domestic Debt
Dependent Variable: Real GDP Growth Rate

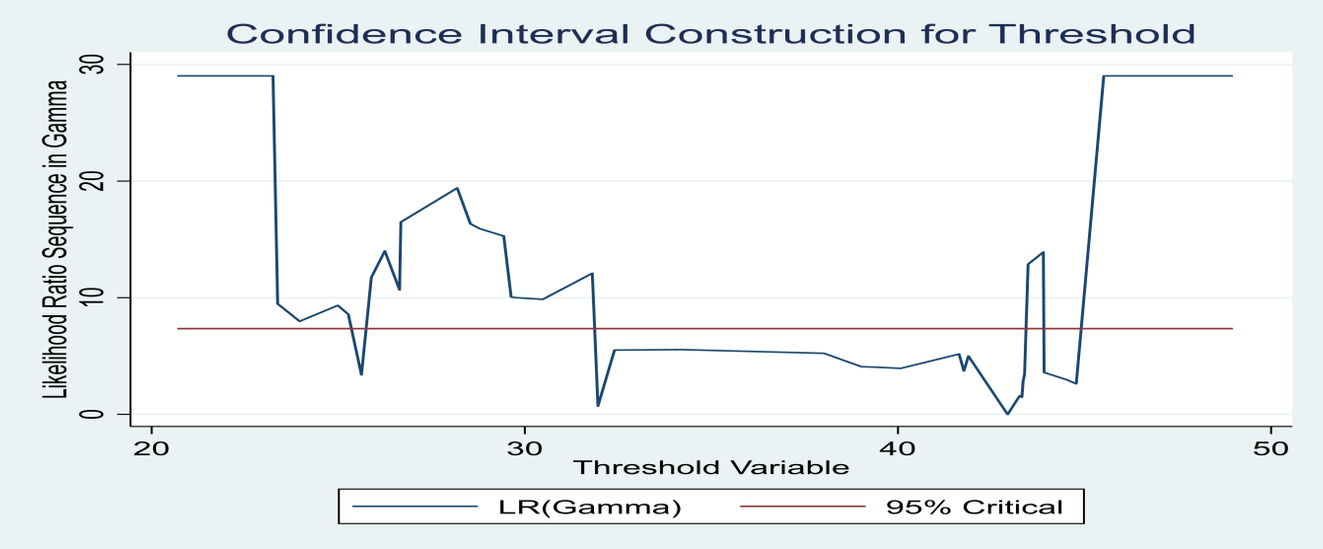
Independent Variables	**Estimate**	**Standard Error**	**T-statistic**
Global OLS Estimation, Without Threshold			
Intercept	1.1842	2.6462	0.4470
DDGDP	-0.1264	0.0668	-1.8924
POP	0.8408	0.4596	1.8284
GFCF	0.0919	0.2075	0.4423
TO	0.2188	0.0934	2.3412
Regime 1: DDGDP $\leq$ 43.3%			
Intercept	-0.1276	3.1330	-0.0407
DDGDP	-0.0776	0.0647	-1.1984
POP	0.6570	0.4365	1.5056
GFCF	0.2373	0.2497	0.9517
TO	0.1311	0.1039	1.2628
Regime 2: DDGDP $>$ 43.3%			
Intercept	1.6757	4.3583	0.3840
DDGDP	-0.4649	0.2413	-1.9266
POP	1.0219	1.1861	0.8613
GFCF	-0.0280	0.3207	-0.0872
D_TO	0.4700	0.1926	2.4411

Threshold Variable: Domestic Debt to GDP Ratio

The threshold regression estimates for domestic debt presented in Table 9, show that the threshold level of the domestic debt to GDP ratio is 43.3%. The results of the linear regression without threshold effect demonstrate that domestic debt (DDGDP) has a significant negative impact on the economic growth rate whereas population growth (POP) and trade openness (TO) have a positive and significant association with real GDP growth rate. In regime 1, when the domestic debt to GDP ratio is below the threshold level of 43.3%, it has no significant impact on economic growth. Similarly, all other variables do not exhibit any significant elation with the dependent variable RGDP. On the other hand when the domestic debt to GDP ratio exceeds the threshold level of 43.3% the relationship between DDGDP and RGDP becomes negative, where each percentage increase in domestic debt to GDP ratio reduces the

economic growth by 0.46 units. The 95% confidence interval (CI) for the threshold level lies between 26% and 44%. Figure 9 illustrates the normalized-likelihood ratios as the function of the threshold in domestic debt to GDP ratio. The 95% confidence interval of the threshold lies between 29 and 45. The graph shows that there are acceptable evidence of two regime orders.

Figure 9: Sample split confidence interval constructed for threshold in Domestic Debt



4.5.1 Threshold level for external debt

The results of the LM test for external debt show that using 5000 bootstrap replications and external-debt to GDP ratios (EDGDP) as the threshold variable the p-value (0.037) is significant at 5% and the F statistics is 12.42, indicating a nonlinear association of the real GDP growth rate with external debt, as outlined in Table 10.

Table 10: LM Test for Threshold Effect in External debt

Number of Bootstrap Replications	5000
Trimming Percentage	0.15
Threshold Estimate	27.348
Bootstrap P-Value	0.037
LM-test for no threshold	12.42
95% Confidence Interval	[27.589161,29.589161]

The result of the TRM for external debt reported in Table 11 demonstrates that the threshold regression analysis identifies a significant threshold at 27.5% of the EDGDP, where the relationship between the real GDP growth rate and external debt becomes negative when debt surpasses 27.5%. In regime 1, below the threshold level the

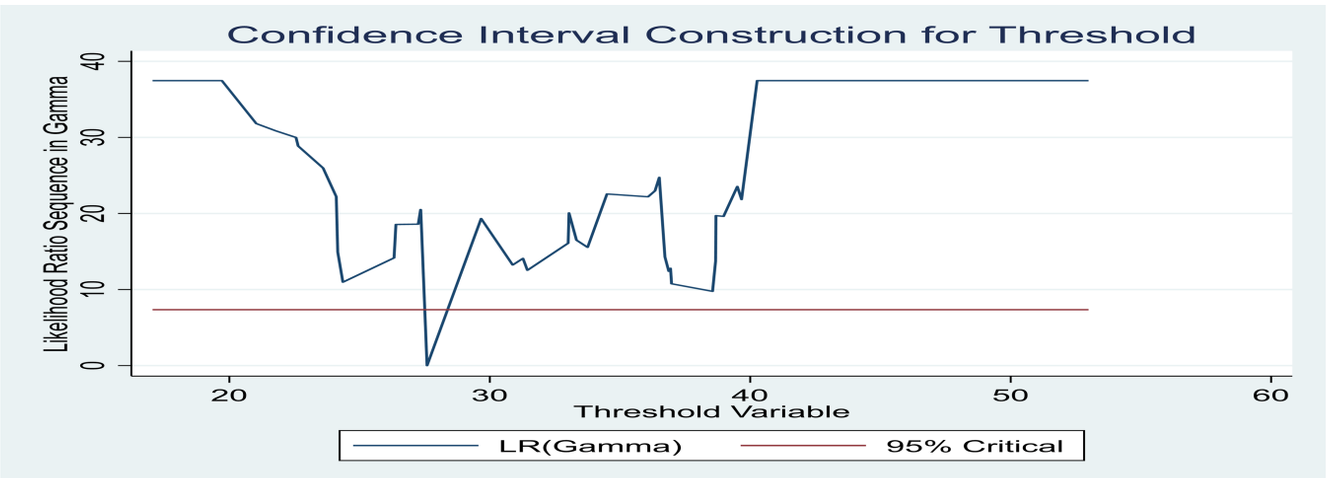
Table 11: Results: Threshold Regression for Domestic Debt
Dependent variable: Real GDP growth rate

Independent Variables	Estimate	Standard Error	T-statistic
Global OLS Estimation			
Intercept	3.9806	0.9650	4.130
EDGDP	-0.0659	0.0245	-2.692
D_TOPEN	0.1372	0.0973	1.411
INF	-0.0097	0.0336	-0.288
REER	0.0227	0.0054	4.243
Regime 1: EDGDP $\leq$ 27.5			
Intercept	6.6222	2.0967	3.161
EDGDP	-0.1666	0.1212	-1.376
D_TOPEN	0.4853	0.1402	3.460
INF	-0.2335	0.0543	-4.298
REER	0.0330	0.0059	5.574
Regime 2: EDGDP $>$ 27.5			
Intercept	7.9094	2.3005	3.437
EDGDP	-0.1407	0.0515	-2.725
D_TOPEN	0.0742	0.1154	0.643
INF	-0.0127	0.0209	-0.607
REER	0.0172	0.0057	3.019

Threshold Variable: External Debt to GDO ratio

results depict an insignificant relation between EDGDP and RGDP. But, when the threshold level is crossed the relation becomes negative and significant, where each percentage increase in external-debt as a percentage of GDP reduces the economic growth by 0.29%. Figure 10 displays the normalized likelihood ratios as a function external debt threshold, with a 95% confidence interval of [27.08, 29.58], providing evidence for a two-regime structure. The estimated threshold lies exactly between the confidence intervals, which indicates robustness of the results.

Figure 10: Sample split confidence interval constructed for threshold in External Debt



4.6 Empirical analysis of Debt Sustainability

4.6.1 Stationary Test Approach

The modern and simplest method to analyze PD sustainability is to check whether the debt-to-GDP series is stationary. If the debt to GDP series is non-stationary, the public debt will be considered unsustainable. The results of the Augmented Dickey-Fuller and Phillip Peron tests are reported in Table 12. The results report that the debt-to-GDP series has a unit root and is not stationary, indicating the unsustainable state of PD in the country during the period 1973-22. Similarly, the graph of the debt-GDP ratio is illustrated in Figure 11, which illustrates a continuous upward trend over time. The mean and median of the series are not constant, likely implying the presence of a unit root in the series, which indicates explosiveness and bubble term in PD.

Table 12: Results: ADF and PP Unit Root test (1973-2022)

Test	**ADF Test**	**PP Test**
Variable	PDGDP	PDGDP
Number of observations	49	49
Test Statistic	-0.609	-2.66
Critical Value (1%)	-3.587	-18.83
Critical Value (5%)	-2.933	-13.26
Critical Value (10%)	-2.601	-10.68
MacKinnon approximate p-value	0.8690	0.7877

Figure 11: Graph of public debt to GDP ratio (1973-2022)

4.6.2 Co-integration Test

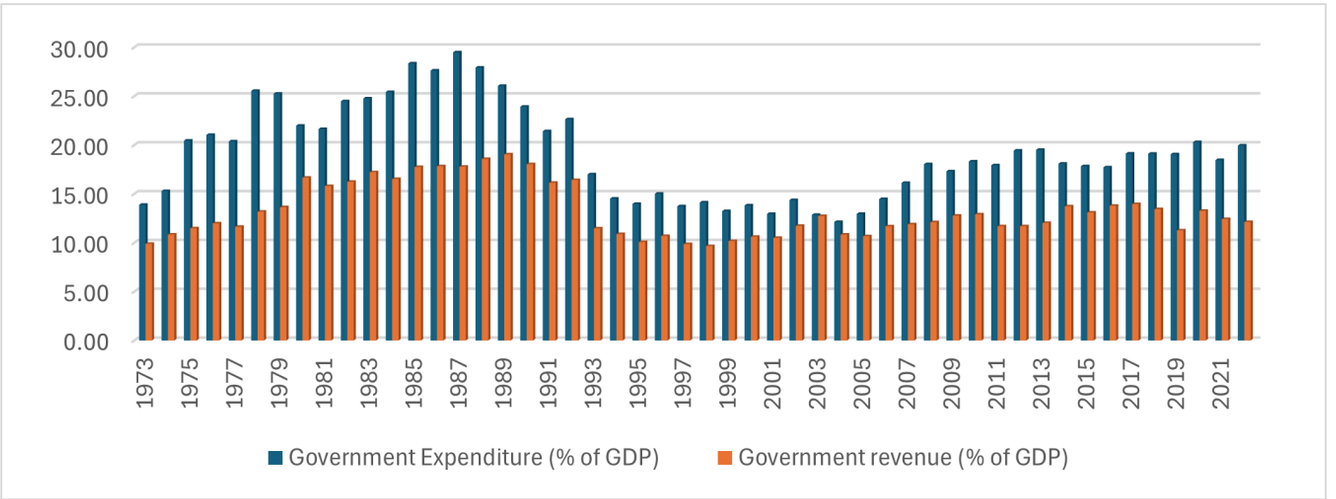
In order to check whether the government expenditures and its revenues are cointegrated or not, the Johansons cointegration test is used, as both the series are stationary at the first difference (integrated of order I(1)). Basically, it examines whether the two series move in a way that makes their link result in a stationary series. At a 5% significance level, the results reveal no cointegration between government revenues and expenditures(% of GDP). T trace statistics value 16.28565 is lower than the critical value (20.26184) for both at none and at most 1. The max Eigen statistic value is also lower than the critical value. Based on the results, it is concluded that government revenues and expenditures are not cointegrated. This suggests that there is an absence of a long-term relationship between the two series, indicating the unsustainability of public debt in the country. Government expenditure decisions are not made according to its revenues in Pakistan. This can also be observed in Figure 12, where the revenue-expenditure gap is quite large. The results of the Johansen co-integration test are provided in Table 13.

Table 13: Johansen Co-integration Rank Test

Unrestricted Cointegration

Hypoth. No of CE	Eigen-value	Trace-Stat	Critical-Value (0.05)	Probability (p-value)
None	0.244779	16.28565	20.26184	0.1615
At most 1	0.056859	2.809909	9.164546	0.6168
Hypoth. No of CE	**Eigenvalue**	**Max-Eigen Stat**	**Critical-Value (0.05)**	**Probability (p-value)**
None	0.244779	13.47574	15.89210	0.1155
At most 1	0.056859	2.809909	9.164546	0.6168

Figure 12: Governmet revenue and Expenditure (1973-2022)



4.6.3 Bohn's Fiscal Reaction Function Approach

The results of the Fiscal Reaction Function estimated using OLS are reported in Table 14. The results demonstrate that the primary balance (PB) has a positive and significant relation with its lagged value, while the lagged value of the public debt (PDGDP) shows an insignificant impact on the PB. The expenditure gap (EXGAP) and the inflation rate (INF) are statistically significant at level 1% and show negative signs, as expected, as government expenditure above its potential level reduces the primary balance. The output gap (OGAP) variable is statistically insignificant with a positive sign.

Table 14: FRF: OLS Results
Dependent Variable: Primary Balance

	Coefficient	**Stand. Error**	**t value**	**p value**	**[95% Confid. Interval]**	**Sign.**
LagPB	0.901	0.079	11.37	0.001	[0.741, 1.061]	***
LagPDGDP	0.061	0.048	1.28	0.209	[-0.036, 0.158]	
EXGAP	-0.410	0.151	-2.71	0.010	[-0.716, -0.105]	***
OGAP	0.046	0.117	0.39	0.695	[-0.190, 0.282]	
INF	-0.123	0.045	-2.71	0.010	[-0.215, -0.031]	***
Constant	0.757	0.494	1.53	0.132	[-0.239, 1.753]	
Mean dep. var			-2.056			
SD dep. var			2.454			
R^2			0.784			
No of obs			48			

Note: $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$,

The primary variable in the above table is the lag debt to GDP ratio. According to Bohn (1998), a positive and statistically significant association between the debt-to-GDP ratio and the PB is a key indicator of sustainable public debt. There is no significant relationship between PB and the lagged debt-to-GDP ratio (P-value = 0.209), this reflects the non-responsiveness of the government fiscal policies to the change in debt stock or the fiscal adjustments in response to rising debt are not strong enough to ensure long-term sustainability. Similar concerns are raised in the literature, where weak fiscal responses, especially in developing economies, often highlight limited institutional capacity, political constraints, or economic volatility. Additionally, the literature suggests that fiscal policy is counter-cyclic in a country when there is a positive relation between the PB and the output gap.

4.7 Analyzing time frame to achieve Threshold target: Simulation model

Hitherto, the current level of Pakistan's debt is unsustainable and it is considerably above the debt sustainability threshold of 58.5% as estimated and also has surpassed the 60% debt-to-GDP rule set by the FRDL Act, 2005. As discussed earlier, when PD, domestic debt, and external debt exceed their tipping points (threshold levels), they adversely impact the country's economic growth. There is a need to cut down the debt level (debt-to-GDP ratio) by about one-third a significant portion of about (14%). This section explores how Pakistan can attain a sustainable level of debt, and if so, when it would reach its threshold level. To achieve this objective, this study employs the following debt-dynamic-equation

$$d_t = f_t + d_{t-1}\left(\frac{1+r}{1+g}\right) \tag{25}$$

Where,

dt: is the Debt-GDP ratio

dt-1 : debt-to-GDP ratio of previous year

gt: Growth rate

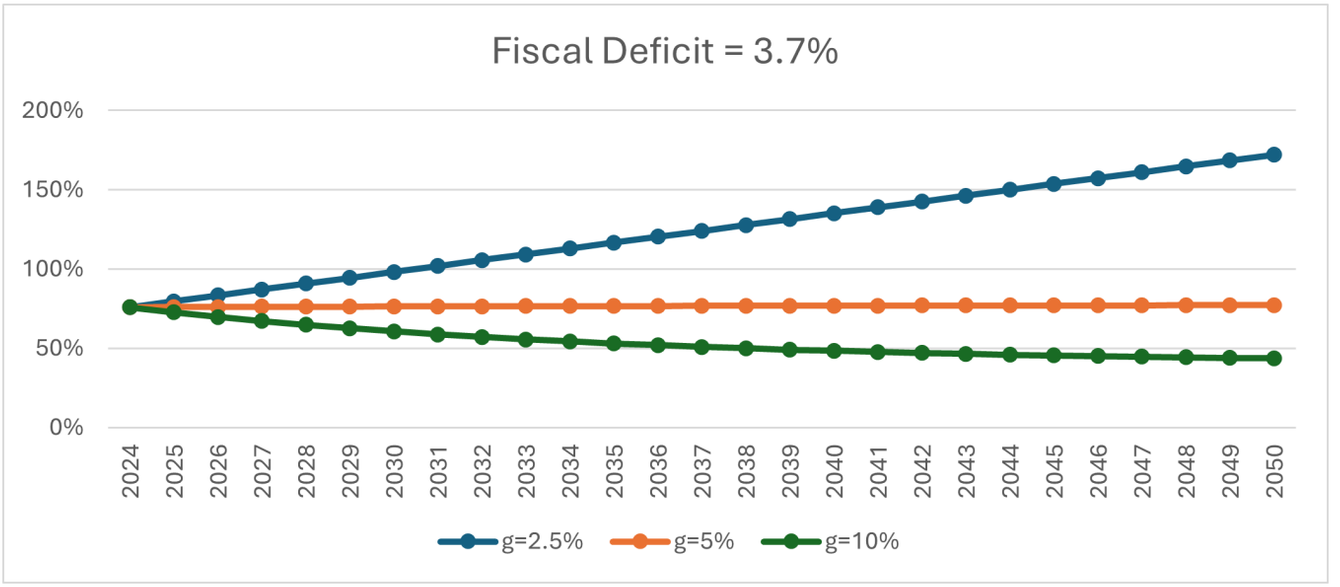
ft : Fiscal Deficit

The left side of the Equation shows the debt-to-GDP ratio for a particular year (i.e., at the time t) which depends on the economy's growth rate, fiscal deficit, and previous year's debt. Using this standard debt dynamic formula, this study simulates the debt-GDP ratio level for future periods (assuming, gt, ft, and dt-1). The objective is that, with various presumptions on these three elements, when and how Pakistan would reach the sustainable or threshold debt level? According to PES, the country's growth rate is about 2.38%, fiscal deficit is about 3.7% and debt to GDP ratio is around 74% for the fiscal year 2024. The rationale for the above statistics is provided hereunder: In Pakistan, the recent trend shows an average growth rate of around 5% (PES, 2024). The record growth rate is 10% in the history of Pakistan. With an inflation rate of 12%, the growth rate would be between 1 to 10%, given the current global and domestic scenario. A growth rate exceeding 10% is not possible. Similarly, based on recent fiscal compliance, the State Bank of Pakistan (SBP) norm of a 1-5% fiscal deficit is assumed. Therefore, various exercises are performed by assuming different growth rates (2.5%. 5%, 10%) and fiscal deficit values (2%, 3.7% and 5%), each of which is presented in detail below:

4.7.1 Exercise 1

In Exercise 1 (as depicted in Figure 13), 2.5%, 5%, and 10% growth rates are assumed from 2023-24 onward, with the current fiscal deficit (3.7%). Under these assumptions, Pakistan's debt-to-GDP ratio will decline only with 10% and 5% growth rates. It will reach the threshold of 60% by fiscal year (FY) 2028 with a 10% growth rate and by FY 2035 with a growth rate of 5%. however, It will increase continuously with a growth rate of less than 5%.

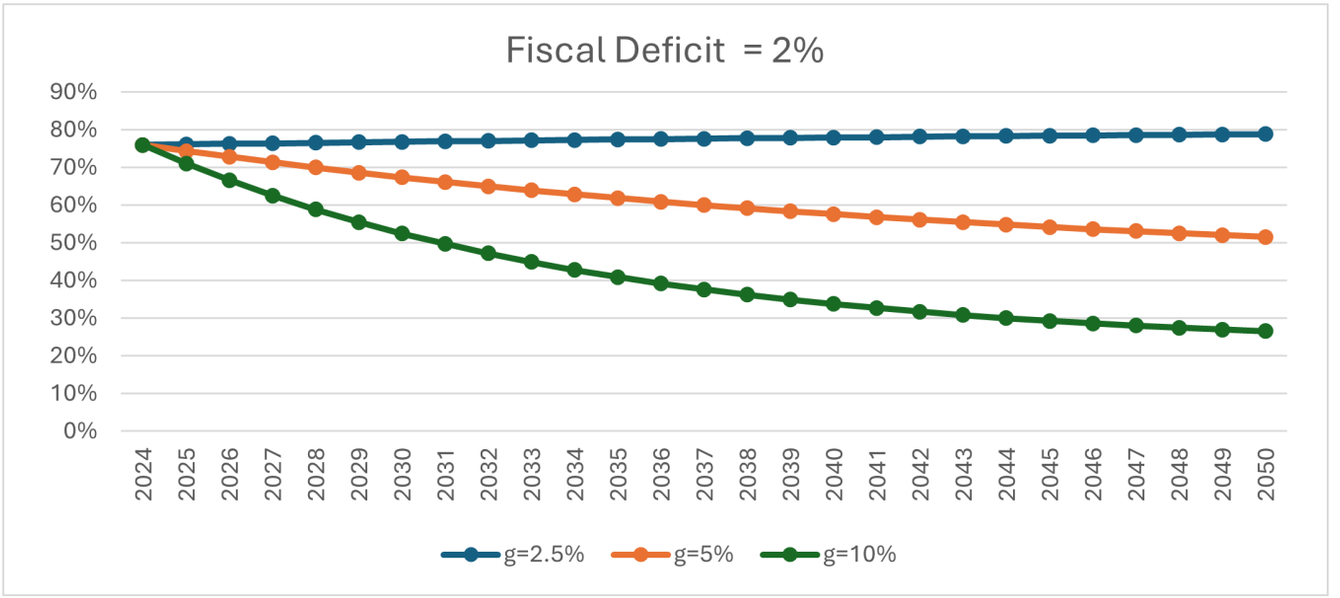
Figure 13: Scenario 1: When the Fiscal Deficit 3.7%



4.7.2 Exercise 2

In Exercise 2 as shown in Figure 14, growth rates of 2.5%, 5%, and 10% are assumed, along with a fiscal deficit of 2.0%. Pakistan's debt-to-GDP ratio is declining only with 10% and 5% growth rates. In the current situation of 2% FD the debt-GDP ratio will be brought to 60% by growth rate of 10% by FY 2027 and with a 5% growth rate by the FY 2037. The analysis also proposes that with 2% fiscal deficit and a growth rate below 5% it will not be viable to reduce debt.

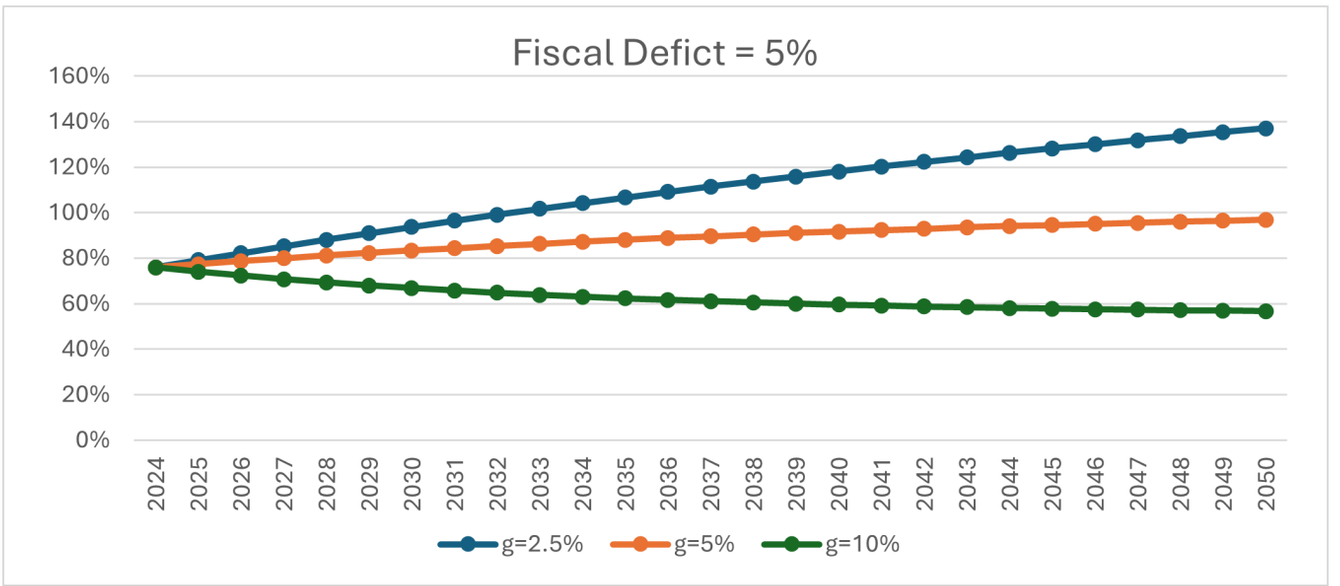
Figure 14: Scenario 2: When the Fiscal Deficit 2%



4.7.3 Exercise 3

Pakistan's economy is often on a roller coaster ride. Multiple domestic and international shocks impact the economy, particularly its revenue targets. In Exercise 3, a higher fiscal deficit of 5% is assumed. With this fiscal deficit, from 2023-24 onward, Pakistan's debt-to-GDP ratio will not decline by 5% and 2.5% growth rates. However, with a high growth rate of about 10% or above, it would attain the threshold level of 60% by FY 2039 and that of 58.5% by FY 2042. Therefore, by targeting a fiscal deficit of 2% to 3% and a GDP growth rate of around 5% or above, Pakistan would achieve its sustainable threshold debt level in the coming decades. The exercise 3 is graphically presented in Figure 15.

Figure 15: Scenario 4: When the Fiscal Deficit is 5%



CHAPTER

5 Conclusion and Recommendations

The primary objective of this study was to determine the threshold levels of public debt, domestic debt, and external debt, and to examine the effects of debt levels above and below these thresholds on Pakistan's economic growth. This study also assesses the sustainability of PD in the country. The threshold regression model developed by Hansen (1996, 2000) was employed for estimation purposes, on annual time series data for the period 1973-2022. Similarly, the sustainability of public debt is analyzed by using different empirical approaches (stationary test, co-integration test, and Bohn's Fiscal reaction Approach).

Firstly, the relationship between PD and economic growth was explored to examine whether the relation is linear or nonlinear, and the results of the LM test confirm the existence of non-linearity in all forms of debt (PD, domestic debt, external debt) with the economic growth in line with Reinhart and Rogoff (2010); Padda (2020); Caner et al. (2010). The threshold level of the public debt to GDP ratio was found to be 58.5%. The estimated threshold levels for the domestic and external debt-to-GDP ratios are 43.3% and 27.5%, respectively. The threshold level of external debt was close to the (Ud-Din et al., 2021) 26. 4% external debt threshold for Pakistan. The public debt as a percentage of GDP does not show any significant impact on economic growth below the threshold level (58.5%) but when the Public debt to GDP ratio exceeds 58.5%, it adversely affects the economic growth rate. Above the threshold level, each 1% increase in the PD to GDP ratio reduces the economic growth by 0.29%. The relationship is significant and quantitatively valuable. Similarly, domestic debt has no significant impact on economic growth below 43.3% of the domestic debt to GDP ratio, but above the threshold level of 43.3%, each percent increase in domestic debt to GDP ratio reduces the economic growth by 0.46%. The results are quite similar for external debt where below the threshold level of external debt to GDP ratio 27.5%, the relationship is insignificant but when the threshold level is exceeded, each percent increase in external debt reduces the RGDP by 0.14 units.

The current external, domestic, and total public debt to GDP ratios of Pakistan are 28.6%, 46.2% and 74.8% respectively[22] which are far above the estimated threshold levels (27%, 43% and 58.5%). The debt sustainability of Pakistan was examined by using different methods and the findings clearly indicate that the country faces substantial challenges in terms of managing its public debt. The ADF and PP unit root test indicates non-stationarity of PD during the period 1973 to 2022, suggesting unsustainable PD over the period. The Johansen co-integration test also discloses no long-term link between government revenues and its expenditures, further confirming the inefficiency of fiscal policies in managing public debt in the country, as revenues do not align with government expenditures. Additionally, the estimation of the fiscal reaction function by employing the Ordinary Least Square (OLS) shows

[22] as of Pakistan Economic Survey 2023-24

that the lagged debt-to-GDP ratio has no significant impact on the primary surplus, indicating that past debt levels do not effectively influence fiscal adjustments in the country. Collectively, these findings point to unsustainable debt in Pakistan, emphasizing the need for structural reforms.

The projections of Public debt based on a simulation model by assuming different growth rates (2.5%, 5%, 10%) and fiscal deficit (5%, 3.7%, 2%), imply that with the current fiscal deficit of 3.7%, the debt level can only be reduced by growth rates higher than 5% - 10% or above. Similarly, for the fiscal deficit of 5%, the projection results indicate that GDP growth rate of 10% or higher is required to shrink the debt level. Conversely, by assuming lower fiscal deficits of 1% and 2%, the projections indicate that reduction in the debt level is possible with growth rates below 5%. These findings highlight the significant impact of fiscal deficit and economic growth on the sustainability of public debt in Pakistan, suggesting that higher economic growth and lower fiscal deficit are the only viable strategies for debt sustainability.

According to our estimate, the threshold level of sustainable public debt is approximately 58.5%, while that of domestic debt is 43.3% and external debt is 27.5%. The current external, domestic, and total public debt to GDP ratios of Pakistan were 28.6%, 46.2% and 74.8% respectively, which are far above the estimated threshold levels. Debt reduces growth when exceeds the optimal threshold levels. Preserving a manageable debt level will stimulate economic growth and enable the nation to augment its revenue streams. The following policy recommendations can be suggested based on the findings of the study.

The government must take prompt actions to reduce the current debt levels (total pubic debt, domestic debt, and external debt) following the estimated debt thresholds, to ensure that debt remains at manageable levels, preventing it from hindering economic growth. The current debt levels are significantly above the estimated thresholds. The government should follow and implement a rule-based policy for debt management in order to maintain sustainable debt by setting targets for the debt-to-GDP ratios, regular debt sustainability analysis, annual borrowing ceilings, balanced debt maturity structures, transparency measures, contingent liabilities management, adherence to international standards, and structured debt restructuring and negotiation frameworks. The budget deficit in the country should be reduced either by increasing taxes or reducing government unnecessary expenditures. The government should utilize the borrowed funds only for productive purposes. To achieve debt sustainability, it is crucial to reduce the fiscal deficit. The optimal policy strategy involves cutting down excessive expenditures, including unproductive targeted subsidies. Since Pakistan's tax-to-GDP ratio is significantly lower than that of developed countries, there is substantial potential to increase revenues by raising tax bases across the board. In this regard, provinces must also play their role in reducing wasteful spending and increasing their revenues. The country should strive for a growth rate of 5-10% or above to ensure resilience in tax revenues and manage debt levels effectively. This requires a thorough analysis of the components of GDP and the formulation of both short-term and long-term growth strategies

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Chapter

6 Appendix A

In any given period t, the government must cover its expenditure through revenue generation and bond issuance. At period t, the government budget constraint is given by,

$$G_t + (1 + r_t)D_{t-1} = R_t + D_t \tag{3.8}$$

where, G_t is the government's primary expenditure[23], r_t interest rate in real term, R_t is the total tax revenue, D_t is the debt to GDP ratio at period t, $D_t - 1$ is the previous debt stock, $rD_t - 1$ is the interest on the previous debt. The difference between the government's total revenue and and primary expenditure is the primary balance[24], (PBt = Gt - Rt) equation 3.8 can be written as:

$$D_t = (1 + r_t)D_{t-1} - PB_t \tag{3.7.1}$$

In terms of Nominal GDP (Yt), by assuming Yt increases at an annual rate of θ, the above equation can be expressed in the form of ratios Yt as,

$$\frac{D_t}{Y_t} = \frac{D_{t-1}}{Y_{t-1}}\frac{Y_{t-1}}{Y_t}(1 + r_t) + \frac{P_t}{Y_t} \tag{ 3.7.2}$$

$$d_t = \left(\frac{1 + r_t}{1 + \theta_t}\right) d_{t-1} + p_t \tag{26}$$

and the growth rate of the economy is less than or equal to the interest rate, it follows that the share of the primary surplus in GDP necessary to keep the debt-to-GDP ratio constant can be expressed as:

$$d_t = \frac{(1 - r_t)}{(1 + \theta_t)}d_{t-1} - pb_t \tag{27}$$

The equations show that at period t, the amount of debt as a percentage of GDP depends on the interest payments on previous debts, the economy's growth rate, and the primary balance pb as a percentage of GDP. To fully grasp the mechanics of the government budget constraint, it is essential to recognize the possibility of indefinitely rolling over public debt. To illustrate this, we start by using equation (3) to understand how the current debt level connects to future levelsR˙t = 1 + r˙t$\frac{}{1+\pi_t and noting that d_{t+1}=R_{t+1}d_t+p_{t+1}}$

[23]The government total expenditure excluding interest payments on public debt
[24]Primary balance = total revenue - primary expenditure

, we can express the relationship as follows:

$$d_t = \frac{1}{R_{t+1}} d_{t+1} - \frac{1}{R_{t+1}} p_{t+1} \tag{28}$$

The period budget constraint establishes a connection between current debt and the present value of future debt along with the primary deficit. To accommodate the possibility of refinancing existing debt indefinitely, we need to solve equation (4) in a "forward" manner. This involves repeatedly substituting future debt values back into equation (4) as outlined below;

$$d_t = \lim_{n \to \infty} \frac{1}{R_{t+j}} \sum_{j=1}^{n} d_{t-j} - \sum_{k=1}^{\infty} \frac{1}{R_{t+k}} p_{t+k} \tag{29}$$

Considering the government's a priori infinite time horizon, its budget constraint, now expressed in a fully "intertemporal" manner, stipulates that any current debt level must be supported by the net present value of all future primary balances (the second term in (5)), adjusted for any remaining "terminal" value of the debt stock (the first term in (5)). Intuitively, it is evident that there cannot be a terminal debt stock that the government could conveniently liquidate at the end of time. No economic agent would willingly accept a government bond that could not be converted into funds for future expenditures. Furthermore, this argument can also be framed in terms of the impracticality of a Ponzi scheme, where a government continually finds willing agents to hold bonds solely to finance interest payments. Government solvency, therefore, necessitates that

$$\lim_{n \to \infty} \sum_{j=1}^{n} d_{t-j} = 0. \tag{30}$$

Under normal conditions regarding growth and interest rates, solvency mandates that public debt d_t should not exceed the present value of all future primary balances. In other words, primary deficits must eventually be completely counterbalanced by surpluses. Government solvency requires that

$$d_t = -\sum_{j=1}^{\infty} \frac{1}{R_{t+k}} p_{t+k} \tag{31}$$

This equation indicates that the current debt d_t is equal to the negative sum of the present values of future primary deficits, discounted by the appropriate interest rates. he challenge of evaluating government solvency is evident from equation (6), as it requires forecasting future fiscal policies (primary balances) over an infinite timeframe. This task is compounded by significant uncertainties regarding nominal economic growth, borrowing costs, and the primary balance itself. Consequently, assessing government solvency can be seen as a "known unknown" and a nearly impossible task (Wyplosz, 2011). Nevertheless, it is crucial for policymakers, taxpayers, and market participants to determine whether the relationship expressed in equation (6) holds, as unsustainable debt trajectories—like con-

tinually increasing debt ratios—can lead to default, restructuring, and hyperinflation, disproportionately affecting the poor who lack financial protections. Ultimately, recognizing the limitations of our understanding in this area necessitates a pragmatic approach, with various methods developed by practitioners and empirical economists to analyze public debt sustainability, emphasizing the need for specific patterns of primary balance behavior that ensure solvency, while raising the question of the necessary standards for sustainability.

Global OLS Estimation, Without Threshold for PD

Table 15: Global OLS Estimation, Without Threshold

Dependent Variable:	Real GDP Growth Rate		
Independent Variables	**Estimate**	**Standard Error**	**t-statistic**
Intercept	6.4039	4.0631	1.577
PDGDP	-0.0556	0.0342	-1.628
GFCF	0.0026	0.2000	0.013
POP	0.3794	0.4630	0.819
INF	-0.0130	0.0379	-0.342
REER	0.0039	0.0093	0.419
D TO	0.1298	0.1087	1.195
Observations			50
Degrees of Freedom			43
Sum of Squared Errors			165.8170
Residual Variance			3.8562
R-squared			0.2693
Heteroskedasticity Test (P-Value)			0.8522

6.1 Robustness Check

The results of the impact of external debt on economic growth by introducing a dummy for values above the threshold level of 27.5 equal to 1 and those below equal to zero

Table 16: OLS result for above the threshold

Variable	Coefficient	Std. Error	t-Statistic	Prob.
ED_GDP*EDABOVE	-0.0641	0.0350	-1.8332	0.0735
D_TO*EDABOVE	0.0700	0.1728	0.4048	0.6876
REER*EDABOVE	0.0214	0.0080	2.6754	0.0104
INF*EDABOVE	-0.0034	0.0414	-0.0815	0.9354
C	4.2592	0.4690	9.0825	0.0000
Model Summary				
R-squared	0.1856			
Adjusted R-squared	0.1115			
S.E. of regression	2.0260			
Sum squared resid	180.6050			
Log likelihood	-101.4880			
F-statistic	2.5066			
Prob(F-statistic)	0.0556			

Similarly, the impact of external debt on economic growth below the threshold level is also explored by introducing dummy values in the regression (1 for values below and zero for those above) the results are given below which show similar results as the threshold regression model used in the study.

Table 17: OLS result for below the threshold

Variable	Coefficient	Std. Error	t-Statistic	Prob.
ED_GDP*EDBELOW	-0.0446	0.1090	-0.4091	0.6844
D_TO*EDBELOW	0.3185	0.2306	1.3812	0.1742
REER*EDBELOW	0.0272	0.0200	1.3615	0.1803
INF*EDBELOW	-0.2564	0.1408	-1.8205	0.0755
C	4.8724	0.3793	12.8457	0.0000
Model Summary				
R-squared	0.1363			
Adjusted R-squared	0.0577			
S.E. of regression	2.0864			
Sum squared resid	191.5426			
Log likelihood	-102.9286			
F-statistic	1.7353			
Prob(F-statistic)	0.1593			