A STUDY OF OPTIMAL SIZE OF GOVERNMENT: DIFFERENT APPROACHES

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LIST OF ABBREVIATIONS

PSE	Public Sector Efficiency
PSP	Public Sector Performance
EU	European Union
OECD	Organization for Economic Cooperation and Development
ASEAN	Association of South East Asian Nations
GDP	Gross Domestic Product
OLS	Ordinary Least Square
ADF	Augmented Dickey Fuller test
SBP	State Bank of Pakistan
EG	Engle Granger

ABSTRACT

The optimal size of government is an attempt at determining the extent of government role (without looking into its qualitative dimension) by applying optimization techniques on real-life data of public spending and taxation. The present study attempted to determine optimal size of government of Pakistan in balanced as well as unbalanced budget setting. Additionally, it explored the way budgetary deficit affects optimal size of government and deliberated upon as to which of the two approaches (i.e., balanced and unbalanced budget approach) is more suited to Pakistani context. The actual budgetary deficit data was used instead of arbitrarily assuming some figure. Moreover, for the purpose of estimation, the study employed co-integration technique instead of merely relying on OLS. Actually, the study used OLS on differenced variables and deployed co-integration to validate robustness of results. The results showed that in balanced budget setting the optimal level of taxation in Pakistan, while using OLS, is 18.87% of GDP. The Engel Granger two-step method not only confirmed the existence of long term relationship but also gave a largely consistent result i.e., 18.46% of GDP. The optimal level of spending in Pakistan, while using OLS, is 22.18% of GDP. Again, the Engel Granger two-step method not only confirmed the existence of long term relationship but also gave a largely consistent result i.e., 21.93% of GDP. In unbalanced budget setting, while using OLS, the optimal level of taxation was estimated to be hovering around 16 % depending on the level of fiscal deficit. It is, therefore, concluded that the unbalanced budget approach, allowing for fiscal deficit, is more suited to Pakistani context and calls for substantial improvement in tax effort.

Key Words: Optimal size of government, optimal level of taxation, optimal level of spending and optimal size in deficit economies.

CHAPTER I

INTRODUCTION

1.1 Background of the Study

Economic growth has been subject of extensive debate and intensive research in last few decades owing to potential inherent in it to transform lives across the globe. History bears witness to the fact that the countries with sustained economic growth have experienced remarkable improvement in their citizens' standards of living. In this backdrop, the government's role is increasingly being analyzed since its policies affect individuals' key decisions like saving and investment which in turn affect economic growth. Moreover, the government's revenue receipts determine the level of provisioning of public goods which are nonetheless an input in the production function of a private agent. A quantitative analysis of the government's role has been in the form of its size. Though the *size of government* appears to be an emerging topic in the domain of public finance, it is, actually, an attempt at determining the extent of government role (without looking into its qualitative dimension) by applying optimization techniques on real-life data of public spending and taxation. Three proxies have been commonly used for the size of government: taxes, public spending and employment level. A curve, popularly known as BARS curve (Barro, Armey, Rahn and Scully), is said to capture the relationship between size of Government and rate of economic growth (Forte & Magazzino 2010). Many papers have attempted to analyze the existence and shape of BARS curve using different econometric techniques depending on the type of data they had (i.e., time series, cross-sectional or panel).

The idea of optimum government has been beautifully summed up by one of its greatest proponents, Richard W. Rahn in a 2009 Washington Times article:

> *"Over the last few decades, many economists have done studies on the optimum size of government. A new study just completed shows that optimum size of government is less than 25 % of GDP. Optimum is defined as that point just before government becomes so large as to reduce the rate of economic growth and job creation. Governments are created to protect people and property. A government too small to establish the rule of law and protect people and their property from both foreign and domestic enemies is less than optimal".*

Thus, by optimal size we mean that the government is neither large enough to crowd out the private sector nor too small to protect its citizen and to perform basic functions.

Public spending (or taxation) is, in fact, a gauge to measure a state's capacity to deliver the goods, therefore, a key determinant of success of its development policy. A good spending effort enhances the state's capacity to finance public provisioning of essential goods and services and to extend social safety net for the disadvantaged sections of the society. It provides necessary wherewithal to a state for performance of its various functions whereas inadequate and insufficient resources leave the state crippled and handicapped and good governance remains a distant dream.

In this context, it is no surprise that an optimal level of spending and taxation can be worked out with the objective to enhance the economic growth of a country. In fact, it has already been worked out for many countries of the world though consensus regarding exact level remains elusive owing to varying methodologies and factors peculiar to the countries under study. The lack of consensus may be attributed to varying types of regimes, governments and differing views regarding functions of the government. Nonetheless, optimality debate at some point may lead to equity-efficiency debate. A government, actuated by equity concerns, may be so hyperactive that it finally ends up being inefficient owing to an atmosphere inimical to market forces. Conversely, a government, in a bid to be efficient, may end up leaving ordinary citizens to the mercy of market forces. The government in the former case will be oversized whereas in the latter case it will be undersized.

1.2 Statement of the Problem

Pakistan is a developing country with an estimated population of 196 million[1]. Around thirty nine percent of its population lives under multidimensional poverty[2]. Its GDP growth rate has also been far from impressive, averaging around 4.91 %[3]. No description of Pakistan's economic health would be complete without huge external debt it has accumulated over the years. Presently, the external debt is about $ 74 billion[4] and "the World Bank estimates the debt-to-GDP ratio to be at 67.4 %, considerably higher than 60 % limit". Therefore, in this context, the optimality debate becomes all the more relevant as the stakes are pretty high: the country is developing; a sizable chunk of population is living under poverty and the debt is gradually spiraling out of control. An optimal level of spending/taxation is required to be determined so that state does not stretch itself beyond limits and exhausts its resources. It must be borne in mind, however, that there are certain functions which are essential to the economic wellbeing of the people and thus crucial for economic

[1] Estimation of Population Welfare Department, Punjab (www.pwd.punjab.gov.pk)
[2] Multidimensional Poverty Index (MPI) 2015-16 was compiled by Ministry of Planning Development and Reform with technical support from UNDP Pakistan and the Oxford Poverty and Human Development Initiative (OPHI), University of Oxford.
[3] http://www.tradingeconomics.com/pakistan/gdp-growth
[4] State Bank of Pakistan (http://www.sbp.org.pk/ecodata/pakdebt.pdf)

growth of the country. For instance, national defense and maintenance of public order cannot be compromised at all. At the same too much activity on the part of the government may create inefficiencies or crowd out private sector. No doubt, there are certain institutions which are essential to the welfare of people at large but at the same time too much meddling into market affairs leads to inefficiency and is thus counterproductive. Therefore, the size of the government in Pakistan needs to be examined so that an appropriate level is determined which can strike a balance between important functions and efficiency market forces value so much.

1.3 Study Objectives

Optimal size of government has attracted considerable attention in recent years. Generally, two models, Barro (1990) and Scully (2006), have been the most frequently employed ones. Both have similar assumptions despite having differing empirics. The most fundamental assumption of both the models is a balanced budget which, by the way, is a rare phenomenon. In a balanced budget, spending equals tax receipts; thus, optimal level of taxation is also optimal level of spending. Most of the models are erected on these assumptions. However, Husnain et al (2015) extended the Scully model to an unbalanced budget setting and worked out optimal level of taxation. Though he calculated it, he did not analyze the impact of budgetary deficit (or surplus) on optimal level of taxation: how are they correlated? and how they impact each other in quantitative terms? These are important questions which need to be answered and will definitely shed further light on and give better understanding of optimal tax-to-GDP ratio. He overlooked this dimension because he arbitrarily assumed budgetary deficit to be around 1.5 % of GDP for all the countries under study. The present study plans to use Pakistan’s real budgetary deficit for the purpose so that not only more reliable and relevant results are obtained but also meaningful interpretations are made. Moreover, a comparison is required to be made of all these approaches to find out the most suitable one in Pakistani context.

1.4 Research Questions

The study will explore following questions:

1. What is the optimal level of taxation in Pakistan (i.e., tax receipts as a percentage of GDP)?
2. Does the budgetary deficit impact optimal level of taxation? How does the optimal level vary with deficit of the budget?
3. Of many approaches to calculate the optimal level of taxation, which one is the most suitable and relevant for Pakistan?

CHAPTER II

LITERATURE REVIEW

Section 2.1 deals with the theoretical underpinnings of the optimal size literature whereas Section 2.2 reviews empirical literature on the subject. In the former section, theoretical framework, as expounded by its greatest exponents, has been appraised. In the latter section, many studies conducted in different parts of the world, estimating optimal size of government, have been reviewed.

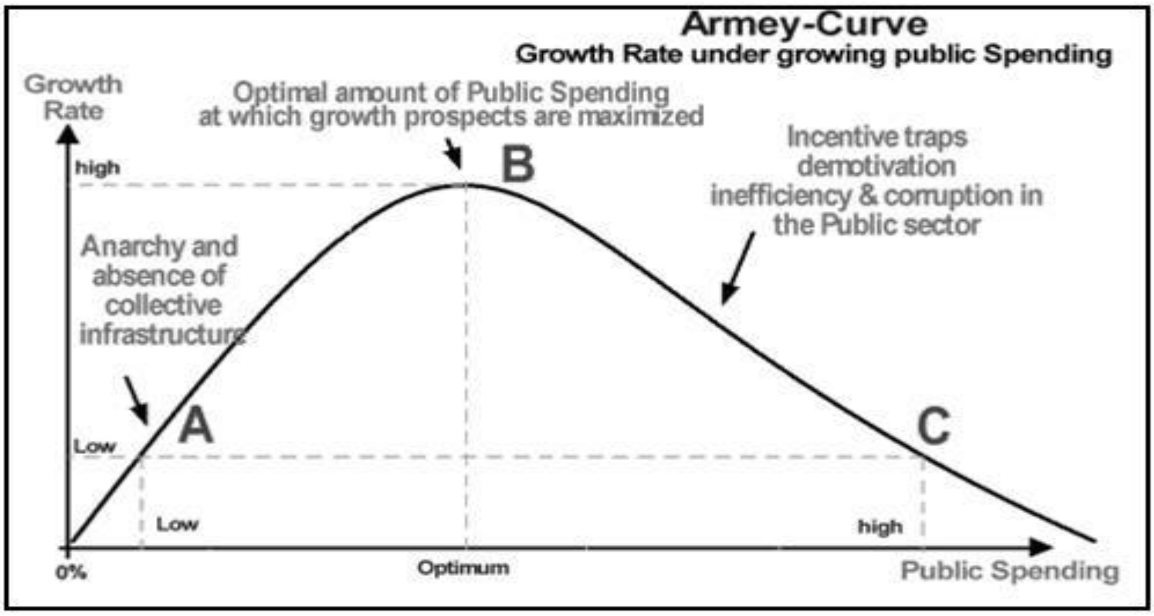
2.1 Theoretical Underpinnings

The BARS curve named after Barro, Armey, Rahn and Scully is said to capture the relationship between size of Government and rate of economic growth (Forte & Magazzino 2010). The pioneering studies were conducted by them. Their works are supposed to have lent theoretical framework to the optimal size literature.

Robert J. Barro (1990) introduced public sector into a constant returns to scale model of economic growth. He viewed the role of public sector as an input into private (Cobb Douglas) production (function) and mathematically demonstrated that the size of government impacts the economic growth rate in two ways: first, a higher spending inevitably leads to higher taxation which suppresses growth; secondly, a higher spending gives a fillip to economic growth by enhancing marginal productivity of capital. He believed that the second force is dominant as long as the government is small but the first one is dominant when the government is oversized. Finally, he concluded that in order to maximize growth, the government would choose the level of spending which equals marginal productivity of public expenditure to marginal productivity of private expenditure. The optimal level of government spending is achieved when marginal productivity equals unity.

Richard D. Armey (1995), an American economist-turned-politician, showed that as the size of government expenditure grows, so does the GDP but after a certain point, diminishing returns set in for economic growth. On this basis, he, as House majority leader in the US, called for smaller, smarter government. The basic idea, which subsequently gained wide currency, was that the relationship between size of government and Gross Domestic Product (GDP) is positive to a point beyond which it becomes otherwise.

Figure 2.1 Armey Curve: Growth Rate under growing public spending

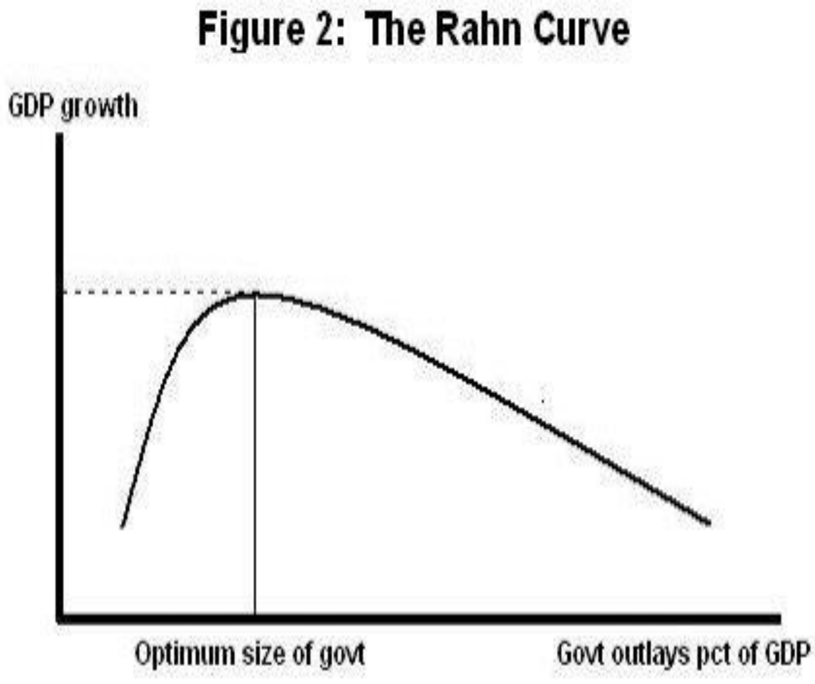


Source: Retrieved from http://www.americanthinker.com

The diagram gives a good idea of the curve. As is evident, there is growth optimizing level of spending before which anarchy and absence of collective infrastructure are characteristic features of the society.

Rahn *et al* (1996) concluded that the US public spending as a share of GDP is too big to maximize growth i.e., above and beyond optimal level. The high spending affects growth rate through two channels: first, the government uses resources less productively and efficiently as compared to the private sector and secondly, owing to high tax and regulatory burden, little resources are left with private sector for production.

Figure 2.2 The Rahn Curve

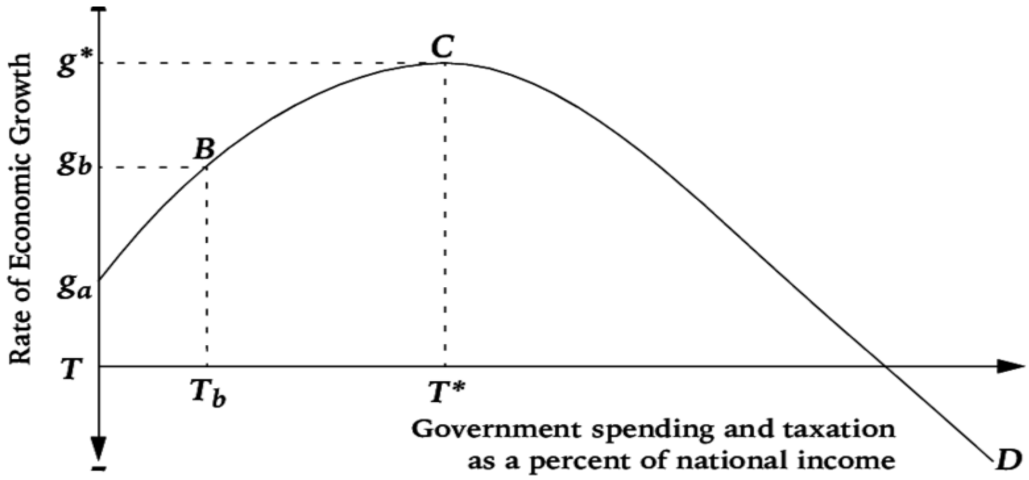


Source*:* Retrieved from economicshelp.org

The diagram shows that there is a growth-optimizing size of government. They were of the view that a little government does a lot of good whereas a lot of government does little good. As is shown in the figure, the Gross Domestic Product (GDP) growth rate reaches a climax with an increase in public spending until the growth rate starts falling. The climax of the curve signifies optimal size of government.

Gerald W Scully (1994) observed that the American productivity has declined substantially since 1950. He was not impressed by explanations like inadequate physical and human capital formation, excess of regulations, low research and development expenditures and the energy crisis. He maintained that there is strong evidence which points towards increasing size of government and corresponding rise in taxes. Beyond certain threshold, the government size becomes a huge drain on resources of private sector. Therefore, he developed an econometric model to work out optimal level of government. He concluded that in order to maximize growth, the taxation as a percentage of GNP should be between 21.5 and 22.9.

Figure 2.3 Scully Curve



Source: Retrieved from www.researchgate.net

The curve shows that there is g_a level of economic growth when there is zero level of spending and taxation (as a percentage of income). The low level of economic growth is because of inefficiencies prevailing in the system which may be attributed to the absence of government role: no public provisioning of public goods and services, no role of government in case of internal disturbance or external aggression and no dispute resolution mechanism in case of disagreements or conflicts. However, as we move along the curve from left to right the growth rate increases because taxation and spending improve overall economic efficiency of the system. The increase in efficiency is caused by positive externalities engendered by security (against internal disturbance or external aggression) dispute-resolution mechanism (in case of disagreements or conflicts) and public provisioning of goods and services. As the level of taxation increases, their negative effects start increasing: at lower level their advantages outweigh their disadvantages and at higher level their disadvantages outweigh their advantages. At T*, the growth rate reaches its optimal rate, g*. Between B and C, the rate of economic growth is more than corresponding increase in spending and taxation, as is evident from the curvature of the curve. At T*, marginal benefits of additional spending comes to a naught. Beyond T*, marginal benefits of additional spending become negative and Scully curve slopes down.

2.2 Empirical Literature

A delicate balance is required to be maintained while making public spending decisions. On one side, public spending impacts good governance positively by improving institutional arrangement for rule of law, transparency, accountability and participation; on the other side, there is a great like likelihood that by affecting individual saving and investment decisions, it may crowd out investment. The optimal size literature attempts to quantify that delicate balance.

Chao and Grubel (1998) used Professor Scully's theoretical model and econometric approach in order to estimate optimal level of government taxation and expenditure in Canada using data from 1929 to 1996. They assumed a balanced budget (with no surplus or deficit) meaning thereby that taxation will equal expenditure. They concluded that optimal level of taxation/ expenditure is about 34 % in Canada.

Vedder and Gallaway (1998) used the Armey curve to explain growth slowdown in the US and Europe where there had been "marked growth in the size of government relative to total output". On the other hand, growth rate in some Asian countries like Hong Kong, South Korea, China and India was impressive owing to private sector's faster growth than that of government. Actually, the former countries are located on the downward-sloping portion of the Armey curve whereas the latter countries are located on the upward-sloping portion of the curve. They estimated the Armey curve for the US using data from 1947 to 1997 and showed that the curve peaks when the government spending equals 17.45 % of GDP.

Scully, G. W. (2003) deployed an econometric model on the US data for tax rate, economic growth and income distribution from years 1960 to 1990 to work out growth-optimizing tax to GDP ratio. The results showed the optimal tax rate to be 19.3 % of GDP though actual rate was higher than it, resulting in slow growth in economy than would have been possible at the optimal rate. Unlike earlier studies on the subject, income distribution was also incorporated in the model. He also employed Gini coefficient (ranging from 0 to 1) as a measure of income inequality in his analysis and concluded that a 1% change in economic growth was linked with 0.00075 point change in Gini coefficient.

Afonso *et al* (2005) developed indicators to gauge Public Sector Performance (PSP) and Public Sector Efficiency (PSE) in twenty three industrialized nations. They employed different socio-economic indicators as proxies for PSP and public spending and its composition as proxies for resource utilization. They found out that small public sectors show better economic performance whereas larger ones display somewhat equal income distribution. Their analysis revealed that EU 15 countries are less efficient than the US and the average Organization for Economic Cooperation and Development (OECD) countries in this regard. They also pointed out that the EU countries were utilizing 27% more resources than the most efficient countries i.e. public sector performance level was same but resource utilization was higher for the EU countries.

Handler *et al* (2006) examined size, composition and structure of the government in a bid to explain differences in income level and economic growth across different countries. Besides giving an overview of studies linking public sector's structure and composition to its performance, they held (though subject to certain caveats) that a small government is more likely to be efficient. Of the two channels (i.e., direct and indirect) through which public sector influences economic performance, the paper mainly focused on the first channel. They also conducted production front analysis of different countries in the Europe and compared them to Japan and the US which in their opinion were working on the edges of production possibility frontier. Though they were a little hesitant in identifying a straightforward relationship between the government's size and structure and its economic performance, they observed, "it seems that small governments are on average more efficient than large governments".

Chobanov and Mladenova (2009) not only shed light on theoretical foundations of optimal size of government (as signified by inverted U curve) but using OECD countries' data also showed that the optimum size is no greater than 25% of GDP.

Grossman (1988) analyzed the relationship between government size and economic growth and unlike earlier studies introduced a non-linear relationship. He deployed a simultaneous equations model incorporating a non-linear relationship between the afore-mentioned variables. He estimated the model using the time series data of the US from 1929 to 1982. He estimated not only non-linear relationship but also linear relationship in order to demonstrate superiority of non-linear model. He concluded that beyond a certain point positive effects of government size are undone by the inefficiencies it causes.

Forte and Magazzino (2010) analyzed existence and shape of BARS curve for EU countries using panel type and time series econometric techniques on data from 1970 to 2009. BARS curve was generally found to exist. The panel data analysis for EU countries showed that BARS curve peaked at 37 % (expenditure share as a percentage of GDP) though actual average was higher i.e., 47 %. For 12 countries for whom sufficient time-series data was available BARS curve peak was found to exist at: Austria (38.21 %), Belgium (35.39 %), Denmark (38.63 %), Finland (40.38 %), France (39.49 %), Germany (41.99 %), Greece (39.33 %), Ireland (44.47 %), Italy (37.68 %), the Netherlands (35.52 %), Portugal (42.28 %) and the UK (43.50 %).

Afonso and Furceri (2010) analyzed the impact of level and volatility of public spending on growth for EU and OECD countries for the data from 1970 to 2004. It transpired that 1 % increase in public spending decreased output by 0.12 % and 0.13 % for OECD and EU countries, respectively. The volatility was also found adversely impacting growth in EU countries. Taxation and spending were further broken down into components for a threadbare analysis: it was observed that among total revenue, indirect taxes and social contributions (in terms of level and volatility) are the most detrimental to growth; on spending side, it was observed that public spending and subsidies impact growth negatively.

Karagianni & Pempetzoglou (2011) used non-linear Granger causality to work out causal relationship between national income and public spending in EU countries in the years following the Second World War. In this regard, they used different functional forms of Wagner's Law. Their results indicated non-linear causality between income and public spending. However, the pattern of causality between national income and public spending showed serious differences across countries. In the concluding remarks, they cautioned the policymakers that presence of nonlinearity cannot let them forecast exact size of a variable change caused by a shock. Therefore, they advised that nonlinear theoretical mechanisms and empirical regularities should be examined while analyzing and evaluating models of national income and public spending with combined dynamics.

Afonso & Jalles (2011) constructed a growth model showing that enhanced public spending adversely impacts private consumption and output per worker. Using a panel of more than one hundred countries and employing different proxies for government size and institutional quality, they observed that the government size has a negative effect on economic growth whereas institutional quality has a positive effect on it. They also illustrated a strong negative relation between government expenditures and economic growth beyond a certain level. Moreover, they found out that negative effect of government size on economic growth is more pronounced in countries characterized by weak legal system, and poor track record of liberties and political rights.

Facchini and Melki (2011) used time series data for the period from 1871 to 2008 to prove the existence of Armey curve for France. For inverted U-shaped curve between public spending and economic output, they used a quadratic model to work out the peak (i.e., growth optimizing spending level) which turned out to be 30 % as a share of GDP. They also added value to the theory of the curve by incorporating theories of government and state failures to account for a non-linear and inverted U-shaped relationship between public spending and economic performance. They theorized that the shape of Armey curve is attributable to its constituent curves: one standing for the costs of state failures and the other standing for benefits accruing from correction of market failures.

Altunc and Aydin (2013) tested validity of Armey curve for Turkey, Romania and Bulgaria from 1995 to 2011 using time series analysis. The results showed that Armey curve exists in three European countries and that optimal level of spending for Turkey, Romania and Bulgaria is 25.21 %, 20.44 % and 22.45 %, respectively. Surprisingly, in all the three countries, the government size was at least 10 % higher than the optimal size.

Hok *et al* (2014) explored the inverted U shaped relationship between government spending and economic growth in eight ASEAN countries (i.e., Brunei, Cambodia, Indonesia, Malaysia, Philippines, Singapore, Thailand and Vietnam) for the data from 1995-2011. To be more precise, they had Armey Curve in mind. They employed unit root tests like Levin-Lin-Chu, Im-Pesara-Shin, Fisher ADF and Fisher PP during the course of estimation. They concluded that an inverted relationship exists between government spending and economic growth and that optimal level of government expenditure for the eight ASEAN countries is 28.5 percent.

Olaleye *et al* (2014) employed Nigeria's thirty years data from 1983 to 2012 to prove the existence of Armey curve. He also used a quadratic model to work out the optimal size of government which turned out to be 11 % of GDP. However, they subjected their results to caveat that owing to data and model limitations the optimal size may be understated and that the true optimal size may be actually higher.

Ahmad and Othman (2014) used Malaysian data from 1970 to 2012 to analyze relationship between public spending and rate of economic growth. They confirmed the existence of non-linear Armey curve. They also worked out optimal expenditure of government i.e., around 16.32 %. On its basis, they pointed out that level of government expenditure is lower than the optimal size and thus needs to be expanded in order to foster economic growth.

A few studies have been conducted in Pakistan on the subject. Husnain (2011) applied Scully model on Pakistani data for the period from 1975 to 2008 to determine optimal size of the government. Using time series data, he concluded that 21.48 % public expenditure share (as a percentage of GDP) is the optimal level. He further held that the existing government size (i.e., 22.7 % public expenditure) is a little higher than the optimal level and is required to be decreased in order to enhance efficiency. Therefore, he underlined the need for minor reduction in public expenditure.

Tabassum (2015) used Pakistani economy's data from 1976 to 2013 to verify inverted U shaped relationship between public spending and economic output. She not only bore out robustness of so called Armey curve but also worked out the optimal level of public spending which amounted to around 19.3 % of GDP whereas the actual size of government was 21.4 % of GDP. Therefore, she called for reduction in level of government expenditure in order to maximize growth.

Zareen & Qayyum (2014) examined in detail the relationship between public spending and other determinants of economic growth using a growth equation based on Barro model of endogenous growth. The data of Pakistan's economy from 1973 to 2012 was used and three dummies were introduced to capture 9/11, earthquake of 2005 and financial crisis. The estimation comprised two steps: first, Johansen Co-integration technique was used to estimate long-run economic growth equation; secondly, Error Correction Method was used to work out short-run growth elasticity of the each variable. They concluded that large government size negatively affects economic growth whereas accumulation in capital holdings increases economic growth in long run. They also found out that trade openness adversely affects economic growth in long run.

Husnain et al (2015) extended Scully model to unbalanced budget setting to work out optimal level of taxation for several South Asian countries, namely, Pakistan, India, Nepal and Sri Lanka. He underlined the need to increase level of taxation in view of his optimal tax rate.

There have been some studies on tax side, too. Engen & Skinner (1996) shed light on growth effects of taxation and tax reforms. Using multiple approaches, they estimated impact of a cut in marginal tax rate on long-term economic growth. They used US historical data, macro-level data of a large number of countries (i.e., top-down cross country regression) and micro-level studies (i.e., bottom up) to work out the quantum of the impact. The results indicated small effects of taxation on economic growth rate. However, the growth effects, though small, if sustained for long period of time, can be instrumental in improving the living standards, they concluded.

Padda & Akram (2009) concluded that tax revenue may be an effective tool to counter fiscal deficit which hampers economic growth and social welfare. Their empirical analysis yielded results in line with neo-classical growth models: changes in tax rates have permanent effects only on output whereas growth rates undergo temporary changes only. They also called for resorting to “optimal tax rate”, urging simultaneous use of fiscal instruments of tax and debt. In case of unexpected hike in expenditure, they suggested employing instrument of tax if hike is permanent and suggested issuing bonds if shock is transitory.

Pasha (2010) examined factors affecting Pakistan’s low tax-to-GDP ratio over a period of twenty years and underlined the need to undertake tax reforms. She also cited a study of Minh Lee which concluded that with present structure, Pakistan can raise its tax-to-GDP ratio by about 2.5 percent. She, however, concluded that success of tax reforms will depend on political will.

Chaudhry & Munir (2010) used time series analysis for period from 1973 to 2009 to analyze determinants of low tax-to-GDP ratio in Pakistan. They attributed it to “openness, broad money, external debt, foreign aid and political stability”. They suggested “boosting the openness, literacy level (and) political stability” in addition to “broadening the tax base”.

Fiscal solvency has also been discussed and probed into by numerous researchers. Smyth, D. J. and Hsing, Y. (1995) worked out a growth optimizing debt-to-GDP ratio using American data from year 1960 to 1991. They concluded that optimal debt is 48.9 % of the US GDP. They actually extended earlier studies of Barro (1979), Eisner (1992) and Joines (1991) etc in order to examine the impact of public debt on economic growth.

Chowdhury and Islam (2010) discussed that a debt-to-GDP ratio of 60% is deemed to be a ceiling for the developed world whereas 40 % debt-to-GDP ratio is considered to be a prudent limit for the developing and emerging economies. They also endorse Domar’s point of view who opined that the debt is not that much a problem as long as the growth rate of nominal GDP exceeds interest rate.

Fatima *et al* (2011) shed light on various consequences of budgetary deficit which directly or indirectly affect economic growth and investment. Using time series data of Pakistan stretching from 1980 to 2009, they concluded that the budgetary deficit negatively impacts economic growth.

Except for Husnain et al (2015), the entire optimal size literature appears to be standing on an unrealistic assumption of balanced budget. Though Husnain et al (2015) introduced budgetary deficit in the model, he arbitrarily assumed it to be 1.5 % of GDP. Therefore, a study based on realistic assumption is warranted. It would add to the existing literature in more than one ways:

- it would employ co-integration technique instead of merely relying on OLS.
- it would employ yearly data of budgetary deficit instead of arbitrarily assuming some figure and
- last but not the least it would also look into the way the budgetary deficit affects the optimal level of taxation.

The determination of appropriate level of taxation or tax-to-GDP ratio becomes all the more important when we make a comparative analysis of different countries. Not only there is a wide difference between the countries regarding their tax receipts, Pakistan fares badly on this count. In fact, Pakistan is one of the worst performers. A cursory glance also reveals that there is remarkable difference between the developed countries and the developing countries on account of tax-to-GDP ratio. The developed countries are excellent performers on this count whereas the performance of developing countries is dismal.

Table 2.1 Tax-to-GDP ratios of selected countries

Sr. No.	Country Name	Tax to GDP Ratio (%)
1	Austria	26.52745218
2	Belgium	26.21188526
3	United Kingdom	25.30991979
4	France	23.25453751
5	Australia	22.19711188
6	Turkey	21.39834766
7	Bhutan	14.37788345
8	Azerbaijan	13.49825501
9	Brazil	13.44169026
10	Indonesia	11.28530146
11	India	10.99857742
12	Sri Lanka	10.48662314
13	Bangladesh	8.962375782
14	Pakistan	10.00001000

Source: World Development Indicators (2013)

The table shows that the developed countries have a high tax-to-GDP ratio whereas the developing countries have low tax-to-GDP ratio. Low tax effort leads to under-provision of many public services and goods. On the other hand, a good tax effort provides the government with the necessary wherewithal to perform its functions. Australia, Austria, Belgium and France etc have high tax receipts and have equally immaculate public infrastructure and institutions whereas India, Sri Lanka and Bangladesh, like Pakistan, have poor tax effort and their public infrastructure and institutions are in tatters. Thus, one intuitively feels that the appropriate levels of taxation and spending are probably the most important public policy decisions which have lasting impact on growth trajectory of the country.

CHAPTER III

DATA AND METHODOLOGY

Section 3.1 explains the broader analytic framework whereas Section 3.2 deals with econometric methodology being employed. Section 3.3 sheds light on the source of data and the way the variables have been constructed in order to be deployed in the model.

3.1 Analytical Framework

As mentioned earlier, Scully model has been extensively used to estimate optimal size of government. It would not be, however, out of place to mention that the model assumes that the budget is balanced. Since most of the countries in the world do not fulfill this assumption, an extension of the model was devised to make it more relevant and useful. The present study plans to employ both the models to estimate the optimal size of government and to subsequently compare their results. Thus, not only Scully model (assuming balanced budget) will be estimated but also its variant allowing for budgetary deficit or surplus will be estimated.

3.1.1 Balanced Budget

The growth maximizing or optimal size of government / taxation will be estimated based on model given by Scully (1994) and by Vedder and Gallaway (1998). The mathematical specification of the model is based on a non-linear Cobb Douglas production function which relates public sector as well as private sector to the output in following terms:

$$Y = \gamma \left(\frac{G}{Y}\right)^{\alpha} (1-\tau)^{\beta} \quad \text{- - - - - - - - - - (3.1}$$

Whereas

$$Y = \text{Total Output}$$

$$\alpha = \text{Relative Share of Government Sector in total output Y}$$

$$\beta = \text{Relative Share of Private Sector in total output Y}$$

$$\gamma = \text{Total Factor Productivity}$$

$$G = \text{Public Expenditures}$$

$$\tau = \text{Tax to GDP ratio}$$

Applying natural log, the above equation can be written as:

$$\ln Y = \ln \gamma + \alpha \ln \frac{G}{Y} + \beta \ln 1 - \tau \quad \text{- (3.2}$$

Differentiating it with respect to G will give a better idea of the impact of expenditures on growth:

$$\frac{\partial lnY}{\partial G} = [\ \frac{\partial lnY}{\partial \ln \frac{G}{Y}} \quad \frac{\partial \ln \frac{G}{Y}}{\partial \ln \frac{G}{Y}} \quad \frac{\partial ln \frac{G}{Y}}{\partial lnG}\ ] \quad \text{- - - - - - - -(3.3}$$

$$= \alpha \ \frac{Y}{G} \ \frac{1}{Y} \quad \text{- -(3.4}$$

$$= \alpha\, G^{-1} \quad \text{- -(3.5}$$

Differentiating again with respect to G:

$$\frac{\partial^2 ln\, y}{\partial G^2} = -\alpha\, G^{-2} \quad \text{- -(3.6}$$

Equations 3.6 can be explained intuitively. Since $-\alpha\, G^{-2} < 0$, the public expenditure impacts economic growth positively up to a point (captured as the maximum in the BARS curve) and then starts impacting adversely.

Since a balanced budget is assumed, expenditures equal tax receipts (i.e., G/Y = τ). Substituting it in equation 3.2 will yield:

$lnY = ln\gamma + \alpha ln\tau + \beta \ln(1-\tau)$ -(3.7

In order to work out optimal size of government or growth maximizing tax rate, let's apply first order conditions with respect to tax rate:

$$\frac{\partial lnY}{\partial \tau} = \frac{\alpha}{\tau} - \frac{\beta}{1-\tau} = 0$$

$$\frac{\alpha}{\tau} = \frac{\beta}{1-\tau}$$

$$\beta\tau = \alpha(1-\tau)$$

$$\beta\tau = \alpha - \alpha\tau$$

$$\beta\tau + \alpha\tau = \alpha$$

$$\tau \ \alpha + \beta \ = \alpha$$

Thus, growth maximizing tax rate in a balanced budget setting is:

$\tau = \frac{\alpha}{\alpha+\beta}$- (3.8

Equation 3.8 shows that optimal tax rate is the ratio of relative share of public sector in total output to the sum of relative share of public sector in total output and relative share of private sector in total output.

3.1.2 Unbalanced Budget

An extension of Scully model has been designed by Husnain et al (2015) to cater to the dynamics of an unbalanced budget. The extension makes it more relevant to the developing world which has a history of deficit budgets though even the developed world is not immune to it. The mathematical specification of the model is based on a non-linear Cobb Douglas production function which relates public sector as well as private sector to the output in following terms:

$$Y_t = A\,(\tau Y_{t-1})^{\alpha}\ (\theta Y_{t-1})^{\beta} (\ 1 - \tau - \theta\ \ Y_{t-1)}{}^{\gamma}$$ -(3.9

Whereas Y_t = Gross Domestic Product in period t.

Y_{t-1}= Gross Domestic Period in period t-1 (lag of GDP in period t)

A = Total Factor Productivity

α = Relative Share of Government taxes in total output Y

β = Relative Share of deficit in total output Y

γ = Relative Share of private sector in total output Y

τ = Tax to GDP ratio

θ = Deficit to GDP ratio

$1 - \tau - \theta$ = Share of private sector after deduction of tax and deficit

Mathematically, economic growth rate is defined as:

$$g = \frac{Y_t - Y_{t-1}}{Y_{t-1}}$$ -(3.10

Re-arranging equation 3.10 yields:

$$1 + g = \frac{Y_t}{Y_{t-1}}$$ - (3.11

Substituting equation 3.9 in equation 3.11 yields:

$$1+g=\frac{Y_t}{Y_{t-1}}=A\ \tau Y_{t-1}\ ^{\alpha}\ \ \theta Y_{t-1}\ ^{\beta}(\ 1-\tau-\theta\quad Y_{t-1})^{\gamma-1}$$

$$1+g=\frac{Y_t}{Y_{t-1}}=\ A\ (\tau)^{\alpha}(\theta)^{\beta}(1-\tau-\theta)^{\gamma}(Y_{t-1})^{\alpha+\beta+\gamma-1}$$

Assuming constant returns to scale, $\alpha+\beta+\gamma=1$

$$1+g=\frac{Y_t}{Y_{t-1}}=\ A\ (\tau)^{\alpha}(\theta)^{\beta}(1-\tau-\theta)^{\gamma}(Y_{t-1})^{1-1} \quad \text{- - - - - - - - - - - - - - - -(3.12}$$

$$1+g=\frac{Y_t}{Y_{t-1}}=\ A\ (\tau)^{\alpha}(\theta)^{\beta}(1-\tau-\theta)^{\gamma} \quad \text{- -(3.13}$$

Applying natural log on equation 3.13:

$$\ln\ \ 1+g\ \ =ln\ \frac{Y_t}{Y_{t-1}}\ \ =\ln A+\ \alpha\ ln\,\tau+\ \beta\ ln\,\theta+\gamma\ln\,(1-\tau-\theta) \quad \text{- - - - - -(3.14}$$

In order to find the optimal tax rate, let's apply first order condition with respect to tax rate:

$$\frac{\partial ln\ \ 1+g}{\partial\tau}=\frac{\alpha}{\tau}-\ \frac{\gamma}{1-\tau-\theta}=0$$

$$\frac{\alpha}{\tau}=\ \frac{\gamma}{1-\tau-\theta}$$

$$\gamma\tau=(\alpha-\ \alpha\tau-\alpha\theta)$$

$$\gamma\tau+\alpha\tau=\ \alpha-\alpha\theta$$

$$\tau\ \ \alpha+\gamma\ \ =\ \alpha\,(1-\theta)$$

Thus, the growth maximizing tax rate in an unbalanced budget will be:

$$\tau=\frac{\alpha\ \ 1-\theta}{(\alpha+\gamma)} \quad \text{- -(3.15}$$

Equation 3.15 can be written as:

$$\tau = \psi \quad 1 - \theta \text{ -(3.16}$$

Whereas

$$\psi = \frac{\alpha}{(\alpha+\gamma)}$$

A closer examination of equation 3.16 reveals that there are two determinants of optimal tax rate: ψ and θ. ψ is a reduced form parameter and is actually the ratio of relative share of government taxes in total output to the sum of relative share of government taxes in total output and relative share of private sector in total output. ψ is actually a ratio of regression coefficients of equation 3.14. The other determinant of the optimal tax rate is θ, the deficit to GDP ratio, or simply fiscal deficit. It is abundantly clear from the examination of equation 3.16 that as the deficit increases the optimal level decreases. It means that in a deficit economy the optimal tax rate is a little lower than that would have been in a balanced budget economy. The explanation is not far to seek.

$(1 - \tau - \theta)$ is the share of private sector after deduction of tax and deficit and any increase in fiscal deficit impacts it adversely. Since the private sector is the major driver of growth, decrease in its share will eventually impact the growth rate adversely.

3.2 Econometric Methodology

Therefore, in order to calculate optimal tax rate in balanced budget setting, we need to find out α and β which are regression coefficients of equation 3.7 and will be worked out by estimating it. However, in case of unbalanced budget we also have a third variable (i.e., budget deficit) in addition to regression coefficients α and γ of equation 3.14.

3.2.1 Augmented Dickey Fuller Test

Since time-series data is being used, its stationarity has been checked by employing ADF unit root test. Most of the time-series data exhibits a trend, rendering it non-stationary. Stationarity is an important concept related to time series processes and has been characterized by following features: series manifests mean-reverting behavior as it approaches a constant long run mean and has a variance which is finite and time-invariant. Regressions with non-stationary series are meaningless and are thus deemed spurious. The three forms of the ADF Test catering to three different options of deterministic part (none, constant and constant & trend) are as following:

$$\Delta y_t = \gamma y_{t-1} + \sum_{i=1}^{p} \beta_i \, \Delta y_{t-i} + u_t$$

$$\Delta y_t = \alpha_0 + \gamma y_{t-1} + \sum_{i=1}^{p} \beta_i \, \Delta y_{t-i} + u_t$$

$$\Delta y_t = \alpha_0 + a_2 t + \gamma y_{t-1} + \sum_{i=1}^{p} \beta_i \, \Delta y_{t-i} + u_t$$

3.2.2 Ordinary Least Square (OLS) Method of Estimation

The OLS method of estimation has been used for balanced as well as unbalanced budget setting. The regression coefficients have been used for calculating the optimal level of spending or taxation. For OLS, the data is required to be stationary. Therefore, data has first been stationarized at appropriate level as and where needed.

3.2.3 Engle Granger two-step Method

A co-integration technique has also been applied to find out long-run relationship between the variables. The Engle Granger two-step method has been used to confirm long-term relationship and to compare results with that of OLS. It has been used in

If two series y_t and X_t are not stationary but are co-integrated, their linear combination should be stationary. In short, if:

$$y_t = \beta X_t + u_t$$

The u_t must be stationary. Since u_t is unknown, first, above equation is estimated using OLS to get u_t. Then Engle Granger test/ stationarity test is applied on u_t to confirm its stationarity which eventually confirms co-integration. Finally, another regression is run on the first differenced variable from the estimated equation and the lagged residual u_{t-1}

$$\Delta y_t = \beta \Delta X_t + u_{t-1}$$

The regression coefficients are used to work out the optimal size.

3.3 Data

Table 3.1 Data Source

Notation	Variables	Source
Y	Gross Domestic Product (GDP)	State bank of Pakistan (SBP)
τ	Taxation as a percentage of Gross Domestic Product (GDP)	
S	Public Spending as a percentage of Gross Domestic Product (GDP)	
g	Gross Domestic Product (GDP) growth rate (annual %)	
θ	Fiscal Deficit as a percentage of Gross Domestic Product (GDP)	

The data of afore-mentioned variables for the years from 1976 to 2016 was obtained from *Handbook of Statistics on Pakistan Economy* by State Bank of Pakistan (SBP). Subsequently, different variables were constructed from these variables to suit the model specification. For instance, taxation, public spending and fiscal deficit were first taken as a percentage of GDP, then those were converted to decimal form in order to cater to the model. A few variables were also derived from these as shown in the following table. Last but not the least, before their final deployment in the equation, log transformation of all the variables was carried out.

Table 3.2 List of Constructed Variables

Constructed Variables	**Procedure**
$(1-\tau)$	By subtracting τ from unity in order to match the variables to the specifications of the model.
$(1-S)$	By subtracting S from unity in order to match the variable to the specifications of the model.
$(1-\tau-\theta)$	By subtracting τ and θ from unity in order to match the variable to the specification of the model.
$(1+g)$	By adding unity to g in order to match the variable to the specification of the model.

The variables mentioned in tables 3.1 and 3.2 have been used to estimate equations 3.7 and 3.14. The resultant coefficients have been used to calculate the optimal size.

CHAPTER IV

EMPIRICAL RESULTS AND DISCUSSION

Section 4.1 outlines results: it starts with ADF test and then describes results for balanced budget and unbalanced budget in Section 4.1.1 and 4.1.2, respectively. Section 4.1.1 not only gives results for OLS estimation and two-step Engle Co-integration Method but also sums up in tabulated form different diagnostic tests' results. Section 4.1.2 gives results for OLS estimation and sums up diagnostic tests' results. Section 4.2 generates discussion based on results and uses descriptive statistics for the purpose.

4.1 Results

The statistical analysis was preceded by a preliminary examination of the data. The purpose was to observe stationarity or otherwise of different variables so that appropriate method of estimation could be chosen in addition to subjecting the variables to appropriate stages of differencing. Thus, Augmented Dickey Fuller test was used to examine the above mentioned statistical properties. Except economic growth (g), none of the variables was found to be stationary at level. All other variables were stationarized at first difference. Results are reproduced as under:

Table 4.1 Augmented Dickey Fuller Test Results

ADF Unit Root Test Null Hypothesis: The series has a unit root							
Variables	Deterministic Part	Lag	Level		Deterministic Part	First Difference	
			ADF Calculated	P value		ADF Calculated	P value
LnY	C&T	0	-2.590741	0.2863	C	-6.126952	0.0000
LnT	C	0	-2.127606	0.2354	None	-8.559627	0.0000
Ln(1-T)	C	0	-2.132311	0.2336	None	-8.860240	0.0000
LnS	C&T	0	-2.181179	0.4866	None	-7.124612	0.0000
Ln(1-S)	C&T	0	-2.179031	0.4878	None	-7.245365	0.0000
Ln θ	C	0	-2.582494	0.1049	None	-8.031533	0.0000
$\text{Ln}(1-\tau-\theta)$	C&T	0	-2.268240	0.4408	None	-7.799657	0.0000
Ln(1+g)	C	0	-4.049180	0.0031	None	-10.19449	0.0000

Results show that the only variable that is stationary at level is growth which is dependent variable of equation 3.14. Since the equation caters to optimal level of taxation in unbalanced budget setting, OLS is deemed to be sufficient for estimating regression coefficients which are used to calculate optimal level of taxation.

It would not be out of place to mention that the coefficients (and thus explanation) of variables in difference form are not same as those in level form. Literature, however, tells us that for a deterministic equation, the coefficients do not change. Therefore, a similarity can be approximated as was suggested by Girouard and Christophe (2005) who found out that overall magnitude in two cases is similar[5].

4.1.1 Balanced Budget Setting (Taxation)

In balanced budget setting, taxation equals public spending. Therefore, within the balanced budget framework, two estimations were conducted: one for taxation yielding optimal level of taxation and the other for public spending giving optimal level of spending. The equation of the model which has been estimated is:

$$lnY = ln\gamma + \alpha ln\tau + \beta \ln(1-\tau)$$ - **-(3.7**

Ordinary Least Square (OLS) method of estimation is used. The variables were first stationarized at first difference. The estimation yielded following results:

Table 4.2 Ordinary Least Square Estimation (Taxation)

Dependent Variable is ΔlnY			
Variables	Coefficients	Standard Error	T-Values
C	0.135390	0.005600	24.17882
Δ Ln τ	-1.592211	0.589002	-2.703235
Δ Ln(1- τ)	-6.842595	3.254033	-2.102804
		R-Square =0.420722	DW =1.887546
Note: Δ denotes first difference operator.			

Different diagnostic tests were conducted to test problems like specification error, auto-correlation, heteroskedasticity, constancy of coefficients and normality of residuals. The results are reported as under:

[5] Khalid (2014) has cited the study in his Doctoral dissertation titled *Does Fiscal Policy Matter Evidence for Pakistan* at Pakistan Institute of Development Economics, Islamabad.

Table 4.3 Diagnostic Test Results

Diagnostic Test Results				
Test	Null Hypothesis	Test Statistics	P-value	Conclusion
Ramsey RESET F Test	Model is stable with no specification error.	1.22	0.23	Fail to reject null Hypothesis and thus conclude that model is stable with no specification errors.
Normality Test (Jarque Bera)	Residuals are normally distributed.	5.07	0.07*	Fail to reject null hypothesis and conclude that residuals are normally distributed.
Breusch-Godfrey LM F Statistics	No serial correlation in the residuals up to the second order.	0.53	0.76	Fail to reject null hypothesis and conclude that residuals are not correlated.
ARCH F Test	No autoregressive conditional heteroskedasticity up to the first order.	F-Statistics=0.69	Prob. F(1,38)= 0.41	Fail to reject null hypothesis and conclude that there is no ARCH effect.
		Obs*R-squared=0.72	Prob. Chi-Square(1)=0.40	
CUSUM of Squared Test		Stable		

*level of significance is 5 %.

The optimal level was calculated in following manner:

$$\tau = \frac{\alpha}{\alpha+\beta}$$

$$\alpha = -1.592211$$

$$\beta = -6.842595$$

$$\text{Optimal rate} = \frac{\alpha}{\alpha+\beta}*100 = 18.87\ \%$$

However, these results are to be taken with a pinch of salt as the data was not originally stationary and was made stationary at first difference. The use of difference entails certain consequences. For instance, the process of differencing does not yield a unique long term solution. The urge to have models based on short run as well as long run properties without being bogged down by the stationarity issues underlined the need of a model that deals with the variables measured in their levels and offers short-run as well as long-run properties. That's where co-integration comes into play.

The Engle Granger 2-step method has, therefore, been deployed to determine the existence of co-integrating relationship and to work out optimal size of government. First, OLS estimation has been used to estimate residuals and then Engel Granger test has been applied to determine stationarity of the series on the residuals. The stationarity is actually indicative of co-integrating relationship. Secondly, another regression is run on the first differenced variables from the estimated equation and the lagged residual.

First, we estimated residual/error by running following regression:

Table 4.4 Engle Granger Step-1

Dependent Variable is lnY			
Variables	Coefficients	Standard Error	T-Values
C	6.108185	2.810157	2.173610
Ln τ	-2.109481	1.008694	-2.091299
Ln(1- τ)	-10.51157	5.512159	-1.906979
@Trend	0.135788	0.000821	165.4647
		R-Square=0.999194	DW= 0.599040

α = -2.109481

β = -10.51157

Optimal rate = a/(a+b)*100= 16.71 %

Engel Granger test/stationarity test was applied on the residuals of the above model:

Table 4.5 Engle Granger Test

Engel Granger Test $\Delta u_t = \varrho u_{t-1} + \Sigma_{i=1}^{\rho} \delta_i \Delta u_{t-i} + \varepsilon_t$		
No	*t-Statistic*	*Prob.**
Augmented Dickey-Fuller test statistic	-4.349143	0.0001

Since the critical values of EG exceed -3, another regression has been run on the first differenced variables from the earlier regression and lagged residual was introduced as an independent variable. This step completes the dynamic error correction model.

Table 4.6 Engle Granger Step-2

Dependent Variable is ΔlnY			
Variables	Coefficients	Standard Error	T-Values
C	0.135780	0.005299	25.62488
ΔLn τ	-1.663638	0.557936	-2.981772
ΔLn(1- τ)	-7.347506	3.085405	-2.381375
EC(-1)	-0.270593	0.116862	-2.315483
		R-squared=0.495810	DW=1.708645

Different diagnostic tests were conducted to test problems like specification error, auto-correlation, heteroskedasticity, constancy of coefficients and normality of residuals. The results are reported as under:

Table 4.7 Diagnostic Test Results

Diagnostic Test Results				
Test	Null Hypothesis	Test Statistics	P-value	Conclusion
Ramsey RESET F Test	Model is stable with no specification error.	1.65	0.10	Fail to reject null Hypothesis and thus conclude that model is stable with no specification errors.
Normality Test (Jarque Bera)	Residuals are normally distributed.	1.37	0.51	Fail to reject null hypothesis and conclude that residuals are normally distributed.
Breusch-Godfrey LM F Statistics	No serial correlation in the residuals up to the second order.	1.06	0.58	Fail to reject null hypothesis and conclude that residuals are not correlated.
ARCH F Test	No autoregressive conditional heteroskedasticity up to the first order.	F-Statistics=0.19 Obs*R-squared=0.21	Prob. F(1,37)=0.66 Prob. Chi-Square(1) =0.65	Fail to reject null hypothesis and conclude that there is no ARCH effect.
CUSUM of Squared Test		Stable		

The optimal size was thus calculated as under:

α = -1.663638

β = -7.347506

Optimal rate = a/(a+b)*100= 18.46%

A comparison of optimal level of taxation calculated through different methods of estimation reveals that by and large the results of OLS estimation and Engle Granger two-step method are similar. So not only a long-term relationship has been tested and estimated but also the possibility of spurious regression has been ruled out. In both cases, the optimal level of taxation is above 18 % and far above the current tax-to-GDP ratio of Pakistan, underlining the need to improve tax effort.

4.1.2 Balanced Budget Setting (Spending)

Under balanced budget setting, taxation equals spending. But actually, this assumption does not hold as taxation and public spending differ. In the preceding section, the equation was estimated with taxation as a variable but in the following section public spending would substitute taxation. As shown in table 1, all variables have been stationarized at first difference. Therefore, first data was stationarized at first difference and subsequently OLS estimation was applied and finally different diagnostic tests were conducted to test problems like specification error, autocorrelation, heteroskedasticity and normality of residuals etc. The results of estimation were as under:

Table 4.8 Ordinary Least Square Estimation (Public Spending)

Table 7 Dependent Variable is ΔlnY			
Variables	Coefficients	Standard Error	T-Values
C	0.132073	0.007487	17.63939
ΔLn S	1.714061	0.808586	2.119825
ΔLn(1-S)	6.010606	2.970569	2.023385
		R-Square =0.11	DW =1.68

Different diagnostic tests were conducted to test problems like specification error, auto-correlation, heteroskedasticity, constancy of coefficients and normality of residuals. The results are reported as under:

Table 4.9 Diagnostic Test Results

Diagnostic Test Results				
Test	Null Hypothesis	Test Statistics	P-value	Conclusion
Ramsey RESET F Test	Model is stable with no specification error.	0.08	0.934	Fail to reject null Hypothesis and thus conclude that model is stable with no specification errors.
Normality Test (Jarque Bera)	Residuals are normally distributed.	2.53	0.28	Fail to reject null hypothesis and conclude that residuals are normally distributed.
Breusch-Godfrey LM F Statistics	No serial correlation in the residuals up to the second order.	2.13	0.34	Fail to reject null hypothesis and conclude that residuals are not correlated.
ARCH F Test	No autoregressive conditional heteroskedasticity up to the first order.	F-Statistics=0.67 Obs*R-squared=0.69	Prob. F(1,38)= 0.42 Prob. Chi-Square(1)=0.40	Fail to reject null hypothesis and conclude that there is no ARCH effect.
CUSUM of Squared Test		Stable		

The optimal level of spending was thus calculated as under:

$\alpha = 1.714061$ $\beta = 6.010606$

$$\text{Optimal rate} = \alpha /(\alpha + \beta)*100 = 22.18\%$$

However, these results are to be taken with a pinch of salt as the data was not originally stationary and was made stationary at first difference. The use of difference entails certain consequences. For instance, the process of differencing does not yield a unique long term solution. The urge to have models based on short run as well as long run properties without being bogged down by the stationarity issues underlined the need of a model that deals with the variables measured in their levels and offers short-run as well as long-run properties. That's where co-integration comes into play.

The Engle Granger 2-step method has, therefore, been deployed to determine the existence of co-integrating relationship and to work out optimal size of government.

First, OLS estimation has been used to estimate residuals and then Engel Granger test has been applied to determine stationarity of the series on the residuals. The stationarity is actually indicative of co-integrating relationship. Secondly, another regression is run on the first differenced variables from the estimated equation and the lagged residual.

First, we estimated residual/error by running following regression:

Table 4.10 Engle Granger Step-1

Dependent Variable is lnY			
Variables	Coefficients	Standard Error	T-Values
C	17.17999	1.633925	1.633925
Ln S	2.231526	0.673925	3.311239
Ln(1-S)	7.967152	2.443358	3.260738
@Trend	0.137442	0.000862	159.5269
		R-squared= 0.999234	DW= 0.848723

α = 2.231526 β = 7.967152

Optimal level of spending = α /(α + β)*100=21.88 %

Engel Granger test was applied on the residuals of the above model:

Table 4.11 Engle Granger Test

Engel Granger Test $\Delta u_t = \varrho u_{t-1} + \Sigma_{i=1}^{\rho} \delta_i \Delta u_{t-i} + \varepsilon_t$		
No	*t-Statistic*	*Prob.**
Augmented Dickey-Fuller test statistic	-3.607614	0.0006

Since the critical value of EG exceeds -3, another regression has been run on the first differenced variables from the earlier regression and lagged residual was introduced as an independent variable. This step completes the dynamic error correction model.

Table 4.12 Engle Granger Step-2

Dependent Variable is ΔlnY			
Variables	Coefficients	Standard Error	T-Values
C	0.136494	0.006384	21.37985
ΔLnS	1.941876	0.685207	2.833999
ΔLn(1-S)	6.912153	2.521289	2.741516
EC(-1)	-0.405993	0.156262	-2.598154
		R-squared= 0.273891	DW= 1.499929

Different diagnostic tests were conducted to test problems like specification error, auto-correlation, heteroskedasticity, constancy of coefficients and normality of residuals. The results are reported as under:

Table 4.13 Diagnostic Test Results

Diagnostic Test Results				
Test	Null Hypothesis	Test Statistics	P-value	Conclusion
Ramsey RESET F Test	Model is stable with no specification error.	0.20	0.84	Fail to reject null Hypothesis and thus conclude that model is stable with no specification errors.
Normality Test (Jarque Bera)	Residuals are normally distributed.	0.33	0.84	Fail to reject null hypothesis and conclude that residuals are normally distributed.
Breusch-Godfrey LM F Statistics	No serial correlation in the residuals up to the second order.	3.03	0.0615	Fail to reject null hypothesis and conclude that residuals are not correlated.
ARCH F Test	No autoregressive conditional heteroskedasticity up to the first order.	F-Statistics=0.012	Prob. F(1,37)= 0.91	Fail to reject null hypothesis and conclude that there is no ARCH effect.
		Obs*R-squared=0.012	Prob. Chi-Square(1)=0.90	
CUSUM of Squared Test		Stable		

The optimal level of spending was thus calculated as under:

α = 1.941876 β = 6.912153

Optimal rate = α /(α + β)*100= 21.93%

A comparison of optimal level of public spending calculated through different methods of estimation reveals that by and large the results of OLS estimation and Engle Granger two-step method are similar. So not only a long-term relationship has been tested and estimated but also the possibility of spurious regression has been ruled out. In both cases, the optimal level of spending is around 22 % and above the current level of spending, underlining the need to improve it so that badly needed investments in infrastructure may be made and growth optimizing route may be taken.

4.1.3 Unbalanced Budget Setting

The model equation which has been estimated is:

$$\ln\ 1+g\ = ln\ \frac{Y_t}{Y_{t-1}}\ = \ln A + \ \alpha\ ln\,\tau + \ \beta\ ln\,\theta + \gamma \ln\,(1-\tau-\theta)\ \text{- - - - -}\ \textbf{(3.14}$$

Whereas the optimal size is: $\tau = \frac{\alpha\ 1-\theta}{(\alpha+\gamma)}$

The optimal size can be written as : $\tau = \psi\ \ 1-\theta$. First, ψ was calculated and then annual fiscal deficit was added to capture change caused by it.

Table 4.14 Ordinary Least Square Estimation (Taxation) in unbalanced budget

Dependent Variable is ln(1+g)			
Variables	Coefficients	Standard Error	T-Values
C	0.047119	0.003173	14.85117
Δ Ln τ	0.057775	0.096271	0.600133
ΔLnθ	0.004823	0.037231	0.129536
ΔLn(1- τ- θ)	0.289669	0.524217	0.552574
		R-Square = 0.042070	DW =1.172051
Note: Δ denotes first difference operator.			

Different diagnostic tests were conducted to test problems like specification error, auto-correlation, heteroskedasticity, constancy of coefficients and normality of residuals. The results are reported as under:

Table 4.15 Diagnostic Test Results

Diagnostic Test Results				
Test	Null Hypothesis	Test Statistics	P-value	Conclusion
Ramsey RESET F Test	Model is stable with no specification error.	0.33	0.74	Fail to reject null Hypothesis and thus conclude that model is stable with no specification errors.
Normality Test (Jarque Bera)	Residuals are normally distributed.	1.01	0.60	Fail to reject null hypothesis and conclude that residuals are normally distributed.
Breusch-Godfrey LM F Statistics	No serial correlation in the residuals up to the second order.	9.36	0.052	Fail to reject null hypothesis and conclude that residuals are not correlated.
ARCH F Test	No autoregressive conditional heteroskedasticity up to the first order.	F-Statistics=0.50 Obs*R-squared=0.52	Prob. F(1,38)= 0.49 Prob. Chi-Square(1)=0.47	Fail to reject null hypothesis and conclude that there is no ARCH effect.
CUSUM of Squared Test		Stable		

The optimal level of taxation in unbalanced budget setting was thus calculated as under:

$\alpha = 0.057775$ $\gamma = 0.289669$

$$\psi = \frac{\alpha}{(\alpha+\gamma)}*100 = 16.27\%$$

The optimal level is 16.27 $1-\theta$ % whereas θ is the fiscal deficit, meaning thereby, the optimal level varies over the time with variations in fiscal deficit.

4.2 Discussion

The results obtained so far can be summed as under:

Table 4.16 Summary of Results

No	Approach	Variable	Technique	Result
1	Balanced Budget	Taxation	OLS	18.87%
			Engel Granger	18.46%
2	Balanced budget	Public Spending	OLS	22.18%
			Engel Granger	21.93%
3	Unbalanced Budget	Taxation	OLS	16.27 $(1-\theta)$ %

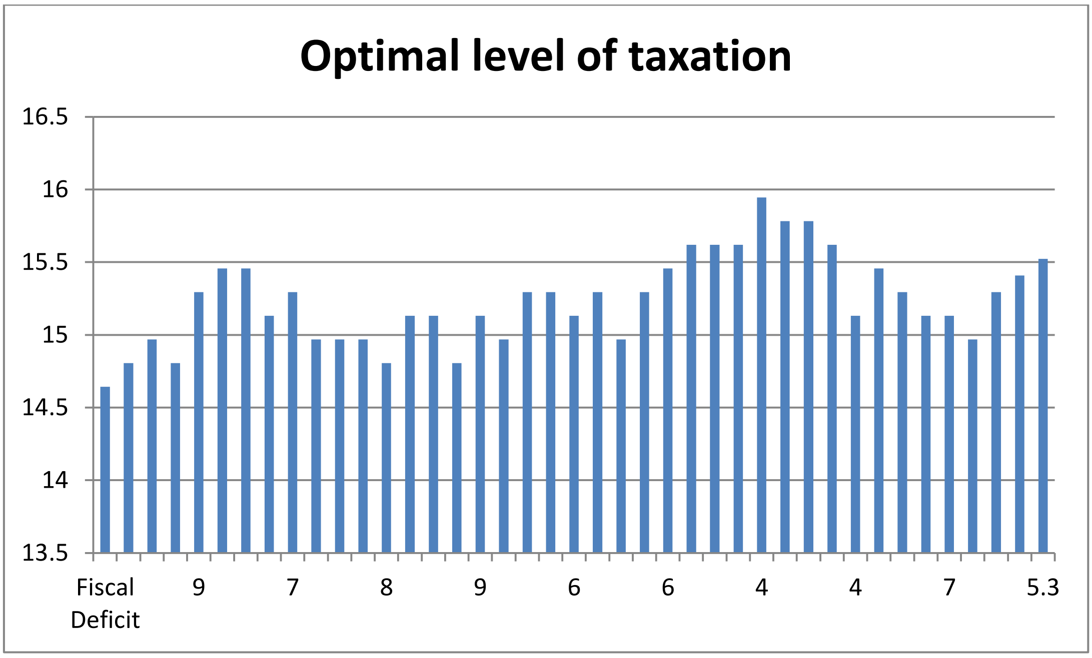
Of all the results, the OLS results obtained in unbalanced budget setting are the most reliable results owing to multiple reasons. First and foremost, balanced budget is more like an ideal and is hard to find in real life; especially, in third world countries which have a long history of running fiscal deficits for an extended period of time. So much so, even the developed countries do not have balanced budgets on a regular basis. The US, for instance, is running huge budgetary deficits for long. So are many other developed countries of the developed world. Therefore, results obtained from balanced budget setting do not have much practical utility. On the other hand, unbalanced budget setting has wider application and acceptability and results can have wider relevance, application and acceptability.

Secondly, in balanced budget setting the data was not stationary at level and was thus stationarized at first difference whereas in unbalanced budget setting the data was stationary at level and was thus not subjected to manipulation. In short, the latter case is better than the former because of its reliance on data in pure form.

Last but not the least, the unbalanced approach places due importance on fiscal deficit which is a key determinant of public spending and tax effort. It would not be out of place to draw a comparison between a household and the government on this point. A household while making domestic budget gives due consideration to gap between income and proposed expenditures. It not only actuates an individual to cut his expenditures but also to augment his income through different possible sources.

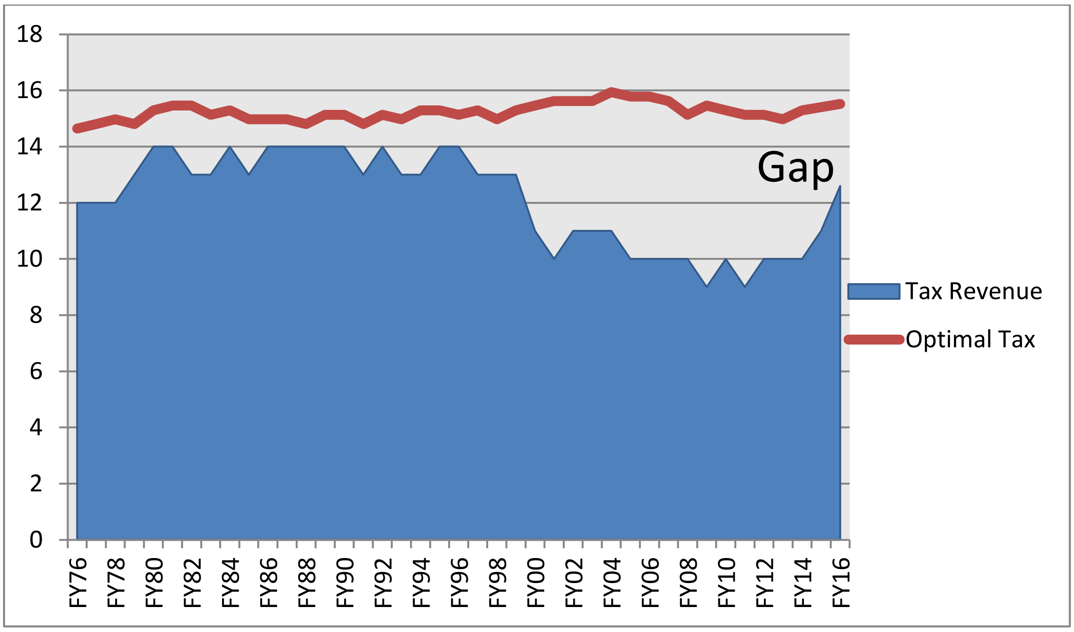
Applying OLS estimation to the unbalanced approach and substituting annual fiscal deficit in the equation yields optimal level of taxation for every year which can be explained graphically in the following lines. A glance at the graph reveals that a rise in deficit causes a trough in the optimal level of taxation and a decrease in the budgetary deficit causes a rise in optimal taxation. Actually, it is so because share of private sector decreases with increase in deficit and thus growth-optimizing size is negatively affected. A glance at equation 3.14 reveals that share of private sector is adversely affected by the tax rate as well as the budgetary deficit. At this point, it must be borne in mind that a budgetary deficit today inevitably leads to higher taxation tomorrow. Thus, a budgetary deficit adversely affects share of private sector not only directly but also indirectly through higher taxation in coming years.

Figure 4.1 Optimal level of taxation (1976-2016)



Having worked out optimal level of taxation, one needs to compare it with actual tax-to-GDP ratio in Pakistan. It makes an interesting analysis as the optimal level lends a yardstick to gauge the performance of the tax machinery. Though there has been hue and cry about the inefficiency of Pakistani tax machinery but the touchstone has always been imposed from the above. The optimal tax rate, however, is a standard which is derived from historical data of the economy and is actually growth optimizing level. Thus, a comparison between the actual tax rate and the optimal tax level cannot only evaluate our tax effort but can also show the gap between the two. Figure 4.2 efficiently sheds light on this dimension of the analysis. As is visible in the figure, the optimal tax level has been above the actual tax-to-GDP ratio and the gap has only widened over the years. The gap underlines the need to improve the tax effort. The result is in line with many studies conducted internationally as well as at national level which call for improvement in tax-to-GDP ratio.

Figure 4.2 : Tax Revenue, Optimal Tax and the Gap



CHAPTER V

CONCLUSION AND POLICY RECOMMENDATIONS

Pakistan is a developing country with a huge population. It is facing multiple challenges on economic front. An analysis of its economic problems quickly leads one to budgetary deficit and heavy external debt. The gist of problem is inability to live within the means and living beyond means has caused innumerable problems. The solution is very simple: either improve your means or learn to live within those. But it is more easily said than done. There are compelling social safety needs which are already being met with bare minimum allocations and any tinkering with those can potentially worsen the plight of millions of impoverished people. Thus, only option is to improve the means: taxation receipts need to be augmented substantially. Pakistan's tax-to-GDP ratio is one of the lowest in the world. Until and unless, there is a dramatic improvement on this front, not much can be done.

The study also suggests poor tax effort on Pakistan's part. There is a wide gap between the optimal level of taxation and the actual level of taxation. Another matter of concern is that the gap between the two is increasing with time. The conclusion is obvious: Pakistan needs to seriously improve its tax effort. All the untaxed areas need to be targeted. Tax exemptions need to be done away with. So far Pakistan taxation system has mainly relied on indirect taxation. This trend needs to be discouraged. The tax-to-GDP ratio cannot be increased as long as the structure and pattern of taxation is overhauled with greater emphasis on progressive taxes. Therefore, following decisions, if taken, may improve the situation drastically:

1. It is generally believed that Pakistan has a huge underground economy. There is a dire need to bring it into tax net. According to some estimates, the volume

of underground economy is around forty billion dollars. Still some others estimate it to be as high as thirty to forty percent of the total economy.

2. While setting tax targets, the budgetary deficit must be taken into account because high taxes coupled with high fiscal deficit do not leave much for the private sector and economic growth is adversely affected.

5.1 Limitations

The study has two major limitations:

1. The study’s inability to incorporate underground economy which is a sizable portion of total economy is a serious limitation. Therefore, any effort along these lines will be a major contribution to the existing literature. It will not only add to the existing knowledge but also yield more practical and relevant results.
2. The study mainly focuses on quantitative analysis based on macro data and thus overlooks inter-sectoral contributions to tax receipts. Thus, a study shedding light on qualitative dimension detailing sectoral contributions will be a handy contribution.

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