COMPARISON OF MACHINE LEARNING AND TIME SERIES MODELS FOR ECONOMIC GROWTH FORECASTING: EMPIRICAL EVIDENCE FROM PAKISTAN

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2020

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CERTIFICATE

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IN THE NAME OFALLAH

The Most Beneficent, The Most Merciful

"To Allah belongs whatever is in the heavens and whatever is in the earth. Whether you show what is within yourselves or conceal it, Allah will bring you to account for it. Then He will forgive whom He wills and punish whom He wills, and Allah is over all things competent."

(Al-Baqarah, 2:284)

Author's Declaration

I Ali Asgher hereby state that my M.Phil thesis titled "Comparison of Machine Learning and Time series Models for Economic Growth Forecasting: Empirical Evidence from Pakistan" is my own work and has not been submitted previously by me for taking any degree from this University "Pakistan Institute of Development Economics Islamabad" or anywhere else in the country/world.

At any time if my statement is found to be incorrect even after my Graduation the University has the right to withdraw my M.Phil degree.

Date: ______________ *Ali Asgher*

Dedication

Every challenging work needs self-efforts as well as guidance of elders especially those who were very close to our heart.

My humble effort I dedicate to my sweet and loving parents, brothers and sisters, whose affection, love, encouragement and prays of day and night make me able to get such success and honor.

Along with all hard working and respected Teachers.

ACKNOWLEDGEMENT

A major research work like this is never the work of anyone alone. As a matter of fact, people tend to forget those who are behind their achievements and have stood for them whenever they have needed assistance. Realizing the fact, it will be right to say that without their cooperation this effort may have ended up in disaster. Completion of this thesis was possible with the support of several people. First of all, thanks to Allah Almighty for His blessings upon me during this thesis and my whole life. I would like to express my gratitude to my supervisor Dr. Hafsa Hina, whose supervision and support helped me to progress smoothly even having many difficulties involved in this thesis.

I am thankful to my parents, my brothers, and my sisters whose prayers make difficulties of life into easiness. They always pray for my success; may God live them long. My special gratitude goes to my mother for their unconditional support.

I am thankful to my friend Mr. Faridoon Khan Marwat for helping me in my thesis estimations and guiding me whenever I needed his help. I am also thankful to my classmate/friends Saira Rasool and Shazia Anwar for supporting me at every difficult time. Last, but not least, I am glad to thank the Institution, PIDE, which provided me the platform and opportunity for learning.

Ali Asgher

ABSTRACT

The aim of this study is to compare machine learning and time series models for economic growth forecasting. The economic growth forecasting was analyzed based on export of goods and services, import of goods and services, trade openness, exchange rate, inflation, unemployment, remittances inflows, gross fixed capital formation and foreign direct investment. Machine learning (SVR and ANN) and time series (ARDL, AR, RW) model were used to forecast economic growth. To compare the forecasting performance of machine learning and time series models, root mean square error (RMSE), mean absolute error (MAE) and mean absolute percentage error (MAPE) were used. Based on empirical evidence from Pakistan, using quarterly time series data from 1981 to 2019, the key findings of this study are that all the models perform well but ARDL model forecast economic growth more accurately than all other machine learning and time series models. Given the results, ARDL model can be applied effectively in the applications of economic growth forecasting.

Keyword: Economic growth, ANN, SVR, AR, ARDL, RW, Forecasting

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LIST OF ABBREVIATIONS

ADF	Augmented Dickey-Fuller
ADL-MIDAS	Autoregressive Distributed Lag Mixed Data Sampling
AIC	Akaike Information Criteria
ANN	Artificial Neural Network
APE	Aggregate Prediction Error
AR	Autoregressive
ARDL	Autoregressive Distributed Lag
ARFIMA	Autoregressive Fractionally Integrated Moving Average
ARIMA	Autoregressive integrated Moving Average
BP	Back Propagation
CPI	Consumer Price Index
DOLS	Dynamic Ordinary Least Square
DW	Durbin Watson
ECM	Error Correction Mechanism
ELM	Extreme Learning Machine
ETS	Exponential Smoothing
EU	European Union
FDI	Foreign Direct Investment
FMOLS	Fully Modified Ordinary Least Square
GA	Genetic Algorithm
GDP	Gross Domestic Product

GNP	Gross National Product
HHI	Hirschman-Herfindahl Index
IFS	International Financial Statistics
IPM	Industrial Production Manufacturing Index
KNN	K-Nearest Neighbor
LM	Langrage Multiplier
LMS	Least Median of Square
LQD	Least Quartile Differences
LTS	Least Trimmed Square
MAD	Mean Absolute Deviation
MAE	Mean Absolute Error
MAPE	Mean Absolute Percentage Error
ME	Mean Error
ML	Machine Learning
MLR	Multiple Linear Regression
MSE	Mean Square error
OLS	Ordinary Least Square
PAM	Partial Adjustment Model
PCE	Personal Consumption Expenditures
PMG	Pooled Mean Group
PP	Phillips-Perron
R&D	Research and Development

RMSE	Root Mean Square Error
RW	Random Walk
SLFNs	Single Hidden Layer Feedforward Networks
SSE	Sum of Square of Errors
SVM	Support Vector Machine
SVR	Support Vector Regression
TO	Trade Openness
TS	Trade in services
USA	United States of America
VAR	Vector Autoregressive Regression
VECM	Vector Error Correction Mechanism
WDI	World Development Indicator

CHAPTER 1

INTRODUCTION

1.1 Background of the Study

Economic growth is a crucial topic in the field of economics particularly the economic growth forecasting, which has fascinated economists since a long time as it captures the direction of a country's economic progress. Gross domestic product (GDP) per capita is an imperative indicator to interrogate the growth of the country's economy. Among the variables of economic growth, Gross domestic product (GDP) per capita is an imperative indicator which is used to explain the economic growth of a country. The economic growth in the entire world has significantly varied in the long run, as some economies tend to grow faster than others. Though, the economists have predicted that the economies having a slower growth rate will eventually convert to fast growing economies. Whereas, only a few Asian countries, especially East Asian countries, have attained very fast economic growth and catching up with the rich countries. It is important to study these primary goals of each nation to achieve high and sustainable economic growth.

The economic growth studied through many factors, as Sokolov-Mladenovic et al. (2016) designates more importance to the trade parameters while Tümer and Akkuş (2018) gives more importance to the research and development (R&D) expenditures, patents per capita, number of papers per capita and education level. Miri and Maddah (2018) examine the economic growth using age structure of population, exports, imports, government expenditure, and gross domestic capital, and Kordanuli, *et al* (2017) forecast the economic growth using final expenditure on consumption of the general government in percentage, gross fixed capital formation in percentage, fertility rate and the industry in percentage form. Ali et al (2019) examine the economic growth of Pakistan through financial inclusion index, investment, inflation and trade openness. This study predicts economic growth through trade factors, remittances, exchange rate, unemployment, gross capital formation FDI inflows, and inflation. All these factors have different impacts on the economic growth depending on the economic conditions of the country. Though the determinants of growth are not same in all countries and differ from one country to another.

The nexus between trade factors and the economic growth remains as a significant problem in the policy making as well as in theory. This problem acquires more consideration in the previous a few decades at large scale dissimilarities in the performance of economic growth among different countries, exclusively among the developing countries to increase international trade combinations. So, trade is studied as a significant determinant of economic growth, the integration of trade factors enables more effective utilization of resources.

Economists have been concerned with economic growth for several years. This theme undertakes a significant place in the economic system, as economic growth has become a much important subject for the researchers since the development of the modern growth theories. As per the existing studies international trade plays a significant role in the advancement of both developed and developing countries. Based on 19th-century trade design, the traditional trade theories describe trade as an appliance of growth. Nonetheless, as the limitations reversed later, economists emphasized the aspect of trade as the ‘chambermaid of growth’. Traditional theories of trade had forecasted the consequential accomplishments from the trade side.

Economic growth is not sustainable in Pakistan because it faced many problems like political instability, lower foreign direct investment, terrorism, unemployment, trade deficit and inflation which make the country far behind from its other South Asian countries like India and Bangladesh. If we look at the gross domestic product (GDP) annual growth rate of Pakistan in 2018, it was 5.8% whereas the GDP growth in Bangladesh and India in 2018 was 7.9% and 6.8% respectively (Source: The World Bank). Like other developing countries, the focus of Pakistan's policymakers is to achieve high sustainable economic growth. However, to gain and retain high-level economic growth needs to be aware of the factors of economic growth and how policies influence growth.

There are studies (Kordanuli *et al* 2016 and Milačić *et al* 2016) examine the performance of machine learning (ML) models nearly convenient to forecast with the artificial neural network (ANN) algorithms. Sharda and Patil (1992) have been making comparison ARIMA to neural networks based on the data of M-competition. Hill et al. (1996) also used the time series data M-competition and make a comparison between traditional timer series models and neural networks. Whereas Wanson and White (1995) had practiced the evaluation of the nine United States macroeconomic series. Alon et al. (2001) had analyzed Box-Jenkins ARIMA winter's exponential smoothing and multivariate regression with neural networks, on the retail sales data. The conclusions of these papers show slightly different results, but mostly neural networks influence more to better perform classical linear models.

It is compared disparate models to analyze the accuracy and efficiency of different machine learning and the traditional model of time series. Most of the literature is focused on the comparison of artificial neural networks (ANN) with its two learning algorithms: extreme learning machine and BP (Milačić *et al* 2016). The growth of the economy relies on trade factors that were examined in the article (Sokolov-Mladenovic et al, 2016), and the results of the forecast show that gross domestic growth (GDP) forecasted accurately and efficiently by utilizing extreme learning machine (ELM).

Most of the literature is focused on the comparison of artificial neural networks (ANN) with its two learning algorithms: extreme learning machine and BP like Milačić *et al* (2016), Sokolov-Mladenovic et al (2016), Milačić *et al* (2017), Cogoljević *et al* (2018) and Stevanović, *et al* (2018). Beyca, et al (2019) compare the artificial neural network (ANN), support vector regression (SVR), and multiple linear regression (MLR) for forecasting the natural gas consumption in Istanbul Province. Ülke *et al* (2016) compare the different time series (ARDL, AR, VAR, RW), and machine learning (ANN, KNN, SVM) models to forecast the USA inflation which is our focused study. Based on their results that machine learning provide accurate forecasting results in seven conditions and time series models were better in nine conditions. Moreover, multivariate models give better results in fourteen conditions and univariate models are better in only two conditions.

1.2 Objectives of the Study

Objectives of this study given below

- To forecast economic growth through differently used traditional time series and Machine Learning models
- To make the comparison among Machine Learning and traditional time series models uses as a basis to forecast of the time series data
- To make comparison among univariate and multivariate models

1.3 Significance of the Study

The conclusions of the study to be conducive to the favor of society taking into consideration that trade parameters provide a significant character in the economic growth rate. Many studies estimate the economic growth, but the forecasting side is very rare at nation level. Moreover, this study compares the forecasting performance of different time series (Autoregressive, Random walk, and Autoregressive distributed lag models) and machine learning (ANN and SVR) models and as well as it compares univariate model and multivariate models. This comparison of different models ensures us which model performs better for economic growth of Pakistan and had minimum chance of error. This comparison is relying on the performance of forecasting of traditional time series and machine learning models based on their forecasted performance. The field of this study may be very helpful for researchers to make an evaluation of differently utilized traditional models of time series and models of machine learning which is not explored by many of another researcher. Thus, a new theory on the performance of these models may be arrived.

1.4 Contribution of the Study

The significant participation of that paper is to be evaluation of performance for forecasting by utilizing two univariate (AR and RW), one multivariate (ARDL) traditional time series modelling and two machine learning (SVR and ANN) models for economic growth perdition based on Pakistan data for trade factors, unemployment, exchange rate, FDI inflows, remittances, gross capital formation and inflation. The performance of prediction of all models evaluated through MAE and MAPE, and RMSE. The lowest value of error describes that model is more accurately predicted the economic growth. These errors check out the performance of traditional time series and machine learning models as well as univariate models, who forecast more accurately.

1.5 Organization of the Study

This study is organized into 5 different chapters. First chapter provides a brief introduction about the study. In this chapter, the background to the study, objectives of the study, significance of the study, contribution of the study and organization of the study is covered. Chapter two provides a review of the related study. The chapter contains the previous studies related international studies and national studies on economic growth and related models. The last section of chapter two contains summary and literature gap of the study. Chapter three is based on the data description and methodology. It describes the about the data and variables utilized in that study. Moreover, it contains detailed overview of econometric methodology applied in this study. The last section of this study describes about comparing criteria's for forecasting economic growth of Pakistan. Chapter four describes about the empirical analysis results. Which has graphical description of dependent indicator, descriptive statistics, unit root test results. After that this chapter has analysis description of all models used in this study with graphical visualization. Last section of fourth chapter compares forecasting performance of different traditional time series models and machine learning models on the basis of different errors and graphically. Chapter five is the conclusion of this study. In which summary of the conclusion and recommendations are described.

CHAPTER 2

REVIEW OF THE LITERATURE

2.1 Introduction

Before proceeding with this study, it is crucial to have an extensive concept of the current evolution in the previous study based on economic growth, trade, and different models' performance. It is needed to examine the previous study to determine the gaps and add up to the structure to fill this gap. A huge body of previous studies forecast the economic growth with different factors at international level but at national level this study is weak. In that chapter, we have examined the previous studies related to the objectives of this dissertation. Section 2.2 deals with the studies of the international studies. The next section 2.3 deals with national studies, 2.4 section provides the comparison study and last section describes the summary of the literature and gap.

2.2 International Study

Sharda and Patil (1992) analyzed the prediction performance between Box-Jenkins and neural network models. Time series data of M-competition was used to evaluating the performance of many techniques. They collect 1001 series from which only 111series were investigated in M-competition. They used Box-Jenkins method and neural network models for forecasting the data. The performance of models compared by using the values of root means square error (RMSE). The outcomes of the study indicate that both the models Box-Jenkins and neural network performing very well but Box-Jenkins forecasting for short term somewhat better than neural network model.

Alon *et al* (2001) forecasted the aggregate retail sales for comparing traditional methods and artificial neural network (ANN). For prediction, they used monthly retail sales data. They used traditional methods (Box-Jenkins ARIMA model winters exponential smoothing, and multiple regression) and ANN. The performance of all the models utilized in the study evaluated by MAPE. The outcomes of this study indicated that ANN performed very well than other traditional methods.

Du Preez and Witt (2003) analyzed the tourism demand to compare univariate and multivariate forecasting. They were utilized monthly data starting from the period of January 1980 to December 1994 for four European Countries United Kingdom, France, Germany, and Italy. For estimation univariate models ARIMA and univariate state-space and multivariate state space modeling utilized. The performance of all forecasting models investigated through aggregate prediction error (APE), RMSE, MAE and also by using ME. The estimation of tourism demands from four European countries showed that Autoregressive Integrated Moving Average (ARIMA) perform accurately than other univariate and multivariate modeling.

Kavaklioglu, (2011) forecasted the electricity consumption of Turkey. He used the data for the period from 1975 to 2006 and predicted electricity consumption until 2026. For the prediction of electricity consumption, he used socio-economic variables those are imports, exports, population, and gross national product (GNP) as input variables. Electricity consumption predicted by utilizing support vector machine (SVR). The predicted results show that electricity consumption of Turkey will not rise drastically till 2014 but after that, there is a steady upward trend.

Hamuda *et al* (2013) analyzed the factors of investment in Tunisia. They utilized time series annually data starting from 1961 to 2011. The output variable was gross capital formation and input variables were gross domestic product (GDP), monetary aggregate and trade openness. To investigate this study, they applied ARDL method. The results indicating that GDP had long run but insignificant impact on investment and in short run it is unambiguous. Monetary base and investment are bounded together in long-term.

Caleb *et al* (2014) investigated the nexus among economic growth and international trade factors for Zimbabwe. They used annual data starting from the year 1975-2005. In their study, the gross domestic product (GDP) was an output indicator and input variables were share of export in GDP, share of import in GDP, trade openness, investment, inflation, government budget deficit shares in GDP and government expenditures. They used Engle-Granger cointegration method with the procedure of Ordinary Least Square (OLS). The outcomes showed, GDP and its regressors had a long-term nexus.

Bozkurt (2015) investigate the nexus between economic growth and research and development (R&D) expenditure. He utilized annually data from 1998 to 2013. He used two indicators; research and development expenditure (R&D) and GDP per capita for economic growth the estimation of this paper was based on Johansen co-integration and vector error correction models. The findings of the paper show that there was a causal nexus from economic growth to R&D. as economic growth increases R&D also increases.

Kaytez *et al* (2015) forecasted the consumption of electricity. They utilized time-series data starting 1970 to 2009. For analyzing population, install capacity, total electricity generation, and total subscribership were used as input variables and electricity consumption as the explained indicator. The techniques Least Square Support Vector

Machine (LS-SVM) and Artificial Neural Network (ANNs) and Multiple Linear Regression (MLR) were used. Forecasting results were compared by using sum square error (SSE), means absolute percentage error (MAPE), maximum error (MaxError), and means square error (MSE). The estimation outcomes indicate that the least square support vector machines (LS-SVMs) performed effectively and accurately than the other two models ANN and MLR.

Blanco *et al* (2016) investigated the effect of research and development expenditure on productivity and economic growth. They took data from 1963 to 2007 in the USA at the state level. To investigation the variables, they used economic growth and total factor productivity as output variables and R&D, labor, physical and human capital and labor and capital shares as input variables. They estimated by using Dynamic Ordinary Least Square (DOLS) and Pooled mean group (PMG) methods. The estimation states that research and development affected both total factor productivity and economic growth in the long term at the state level. But the effect of research and development were based on human capital and development.

Mladenović, *et al* (2016) work on the administration and evaluation of the carbon dioxide emission, economic growth, and thermal comfort in the countries of European Union (EU). For the estimation of thermal comfort, they used air temperature, vapor pressure, wind speed, global radiation, and physiological equivalent temperature as input variables. For the estimation of CO_2 emission, they used rural population growth, rural population, population growth, urban population, and urban population growth as input variables and CO_2 ejection from gaseous fuel consumption, carbon dioxide ($CO_{2)}$ ejection from liquid fuel consumption and CO_2 from solid fuel consumption as output parameters. The out parameters of CO_2 emission used as input parameters for the prediction of real gross domestic growth (GDP) rate. They used support vector machine-firefly optimization algorithm (SVM-FFA), artificial neural network (ANN) and genetic algorithm (GA) for estimation. The accuracy of all these techniques compared with the RMSE, R^2, and the Pearson correlation coefficient (r). The outcomes of that paper specify that thermal comfort forecasted through SVM-FFA more efficiently than ANN and GA results. For the estimation of CO_2 emission SVM-FFA provides more accurate results than ANN and GA. In the end, the prediction results of GDP based on CO_2 emission give more accurate results than the assessed value of ANN and GA. The SVM-FFA algorithm could be utilized effectively in thermal comfort, CO_2 emission and GDP estimation and applications respectively.

Sokolov-Mladenovic *et al* (2016) forecast the economic growth through artificial neural network (ANN). The data was taken from 28 European Union (EU) countries. For this investigation, they used economic growth as output parameter and trade in services (sum of services imports and exports divided by GDP, imports of goods and services, the export of goods and services, trade (the sum of export and import of goods and services measured as a shard of GDP), and merchandise trade as a GDP share (sum of merchandise export and import divided by GDP) as input parameters. Results were forecasted by using the ANN with the help of BP and ELM learning algorithms. The performance of these models was predicted by using root means square error (RMSE) and the coefficient of determination (R^2). The results of this study indicated that the ANN with ELM perform better in the replication of GDP growth rate forecasting. The extreme learning machine

(ELM) learning algorithm could be able to be utilized effectively in the estimation and application of GDP.

Ülke *et al* (2016) analyzed the machine learning and traditional time series models for predicting the inflation to check who forecast better. For that study, they utilized time series monthly data starting from the period of January 1984 to December 2013 for the USA**.** They divided this data into four separate horizons in the study (3 months horizon, 6 months horizon, 9 months horizon and 12 months horizon). For inflation forecast, they used four monthly price indexes. Those variables were Consumer Price Index for all items (CPI), Consumer Price Index eliminating food and energy (Core-CPI), Personal Consumption Expenditure deflator for all items (PCE), and Personal Consumption Expenditure deflator eliminating food and energy (Core-PCE). For inflation forecasting, they include six economic actions including civil rate of unemployment, the index of industrial production, the real personal consumption expenditure, employee on the nonfarm payrolls, the term spread, and housing starts. For comparison among traditional time series and machine learning modelling, they used four-time series models (two univariate and two multivariate), Autoregressive model (AR), Random Walk, Autoregressive distributed lag model (ARDL), and vector autoregressive regression model (VAR) and three machine learning model, K-nearest neighbor model (KNN), artificial neural network model (ANN), and support vector regression model (SVM). They check forecasting performance with the use of root means square error (RMSE). The forecasting outcomes showing that traditional models of time series provide accurate results in nine conditions and machine learning techniques give better outcomes in the seven different conditions. Furthermore, univariate models were more accurate in just two conditions and multivariate models were accurate in fourteen conditions.

Kordanuli, *et al* (2017) analyzed and developed the artificial neural network (ANN) to forecast the Hirschman-Herfindahl Index (HHI) and the gross domestic product (GDP). For the prediction of GDP four inputs final expenditure on consumption of the general government in percentage, gross fixed capital formation in percentage, fertility rate and the industry in percentage form. For the prediction of the Hirschman-Herfindahl Index (HHI) based on three inputs the total number of companies, the number of workers and bonus per capita. The artificial neural network was developed and applied with extreme learning machine (ELM) and back-propagation (BP) learning algorithms. The extreme learning machine (ELM) was established for algorithms with the single hidden layer feedforward networks (SLFNs) which has some benefits by comparing with traditional learning algorithms such as ELM and BP learning algorithm. The performance was compared with the coefficient of determination (R^2) and the root means square error (RMSE). The prediction of GDP by applying ANN with ELM and BP learning algorithm indicates that the ELM performed better than the BP learning algorithm. The prediction of HHI also showing the same outcomes.

Milačić *et al* (2017) analyzed the economic growth by utilizing ANN with the learning algorithms ELM and BP. For forecasting the economic growth, four inputs were used. The input variable was agriculture which had fishing, hunting, forestry, cultivation of crops and livestock production value-added in the percentage of GDP. The other input variable was manufacturing which had value-added of net output in value-added in the % of GDP. The third input variable was an industry that had value-added in mining, manufacturing, construction, water gas and electricity. Forth input variable was services which had percentage value added in transport, wholesale and retail trade, government financial and professional and personal services like education real estate and health care services. For the investigation of economic growth, they employed ANN with the use of two learning algorithms ELM and BP. The performance of the learning algorithms evaluated with the help of RMSE and R^2 and Pearson coefficient (r) indicators. The results indicate that three statistical indicators demonstrated that two learning approaches for ANN, ELM model performed better than back-propagation model. For GDP estimations ELM algorithm applied effectively.

Cogoljević *et al* (2018) predicting the connection between economic growth and energy resources through a machine learning approach. Economic growth predicted by using following input parameters, surrogate and nuclear energy as a percentage of total energy use, fossil fuel energy as a % of total energy adoption and volatile renewables and waste as a percent of total energy necessity. For prediction, they estimate ANN with the application of ELM and BP learning algorithms methods. The performance of prediction for both methods was evaluated by using the R^2 and RMSE. The outcomes of this analysis showed that the extreme learning machine (ELM) learning algorithm had better predictive than back-propagation (BP) learning algorithms.

Khamis *et al* (2018) forecasted the economic growth by utilizing a prosperous VAR model. For the investigation of this study, they used monthly data from January 1998 to January 2016. The circulation of currency, rate of exchange, reserve money, and external reserve that were input variables. The robust VAR model was used for forecasting economic growth. This study used two techniques that were filtering by least trimmed squares (LTS), least quartile differences (LQD), and least median of squares (LMS). These techniques were used to disclosure of an outlier from time-series data by a robust approach. The performance of prediction of the models was checked through the MAPE. The result of that study indicates that the robust VAR LQD technique had the least error measurement, pursued by VAR LMS, VAR Median, VAR Mean, and VAR LTS. So, the VAR LQD model was to forecast the economic growth more accurately than other techniques.

Miri and Maddah (2018) analyzed the impact of the age structure of the population on economic growth. They utilized time series annual data starting from 1987 to 2017 for Iran. They took gross domestic product (GDP) growth per capita as explained variable and age structure of the population, exports, imports, government consumption expenditures and gross domestic capital as independent variables. The age structure of the population divided into three groups: the first group was 0 to 14 years; the second group was 15 to 64 years and the third group were older than 64 years. For estimation autoregressive distributed lag model (ARDL) was used. The outcomes indicated that a rise in the population group age 15 to 64 in both long term and short term had a positively effecting on the economic growth but a raise in the population of older than 65 years had negatively effecting the economic growth for long term. A rise in population of under age 15 had a negatively impacted on the economic growth in the short term but it could have had a positive impact on economic growth in the long term.

Okoroafor *et al* (2018) analyzed the causal nexus between the economic growth and inflation and also measure the threshold and forecasting of inflation in Nigeria. They utilized time series annual data starting from 1961 to 2016. Three indicators inflation rate, GDP growth rate, and the real GDP weighted broad money supply was used. For the estimation purpose, they used the Granger causality test, Vector Autoregressive (VAR), ARIMA and ARDL models. The causality outcomes providing that inflation doesn't cause the economic growth and the economic growth doesn't cause inflation. The threshold results depict that the inflation rate and the real GDP weighted broad money supply impacted the GDP growth rate negatively in the short term though the real GDP weighted broad money supply was statistically insignificant. In the long term respective the inflation rate and the real GDP weighted broad money supply impacted the GDP growth rate negatively and the real GDP weighted broad money supply was statistically insignificant. The forecasting results of inflation defining that VAR (1) predicted the rate of inflation more accurately for Nigeria.

Stevanović, *et al* (2018) estimated the GDP based on the consumption of electricity. The data was taken from the European Union (EU) countries for estimation. The electricity consumption was estimated occupying on distinctive sources, those were coal, nuclear sources, and renewable. The output indicator was the gross domestic product. The estimation was based on ANN with the ELM and BP learning algorithms. The evaluation of both learning algorithms was tested through the value of R^2 and RMSE and Pearson coefficient (r). The outcome of this study indicated that the extreme learning training algorithm had the highest precision for the investigation of the GDP rate based on the consumption of electricity. The back-propagation algorithm had the lowest precision for the estimation of GDP rate based on the consumption of electricity.

Tian *et al* (2018) analyze the nexus between the economic growth and water consumption in China. They utilized time-series annual data starting from 2003 to 2017 for this investigation. They used the gross domestic product (GDP) as the variable of measuring the economic growth and the consumption of agricultural water, the consumption of industrial water, living water consumption and total water consumption as variables of water consumption. They used Vector Auto Regression (VAR) model to analyzed to a dynamic correlation between water resources consumption and economic growth and forecast the water stress situation in China. The results of this study show that water consumption had a co-integration nexus with the economic growth. The cumulative value of gross domestic product (GDP) response to total water consumption per unit was negative but the cumulative value to total water consumption response to gross domestic product (GDP) was positive. All of this shows that GDP carries about a raise in the total water consumption and the decrease of water resources had an adhesive effect on GDP growth.

Tümer and Akkuş (2018) investigated the gross domestic product by using non-economic variables. For investigation of this study they use panel data of 13 different countries for the time period of 1996 to 2015. GDP per capita forecasted by using level of education, the number of the papers per capita, researchers per employed R&D expenditure (% of GDP) and the number of patents per capita as an independent variable. The study investigated with artificial neural network (ANN) and performance of prediction of ANN checked with RMSE, R^2 and logarithmic transformation variable (e) was utilized. The outcomes of the study indicate that BP learning algorithm gives a better optimal network based on the R factor. The results concluded that ANN could be utilized as effective tool for GDP per capita investigation.

Beyca, *et al* (2019) forecasted the consumption of natural gas. Monthly data for the consumption of natural gas in Istanbul province was used from 2004 to 2015. The consumption of natural gas was forecasted by temperature forecast, gas consumption of past months and various time features like seasons and months of the year. The data was divided into 2 parts testing and training data. Training type data utilized to predict the model and testing data set employed to measure the performance of models. The forecasting was based on three machine learning techniques including ANN, SVR, and MLR. The performance of all three models was compared by MAPE and MSE. The outcomes of the study demonstrated that support vector regression (SVR) forecasted better than artificial neural network (ANN) and multiple linear regression (MLR) which has the lowest value of both the means absolute % error (MAPE) and mean square error (MSE). Yet all models performed better and had error less than 5% for estimating the values of real consumption.

2.3 National Study

Iqbal *et al* (2010) investigating the causality nexus among the economic growth, international trade (import and export) and FDI for Pakistan. They were utilized time series quarterly data starting from the period of 1998-2009. They employed cointegration test vector error correction mechanism model (VECM) for causality test.

Malik *et al* (2010) estimated the external debt and economic growth. They use time-series data starting from the period of 1972 to 2005 for Pakistan. They used the GDP as an output variable and external debt and debt servicing as input variables. The nexus between these variables checked through applying the usual OLS modelling. The findings of the study indicate that foreign debt and debt services have negatively and significant effect on GDP growth. As these two external debt and debt servicing increasing, there will be less chance to raise in economic growth.

Chani *et al* (2011) investigated the character of inflation and economic growth for justifying thy pervasiveness of poverty. They were utilized time series annually data starting from 1972 to 2008. They utilized poverty as output variable and input variables were GDP, investment, trade openness and inflation. They were applied ARDL bound test for investigation. The findings of this study indicated that economic growth, trade openness, poverty, investment and inflation had a long run nexus. Empirically inflation had a positive and economic growth had a negative effect on poverty. The trade openness impact on poverty was not significant.

Hussain and Malik (2011) analyzed the nexus between the economic growth and inflation in Pakistan. They had been used annually data starting from 1960 to 2006. Estimation was conducted by using threshold model, error correction mechanism (ECM) and Granger causality test. The outcomes of that study revealed the positive nexuses between inflation and economic growth and this relation is a uni-directional, which meanings that inflation caused economic growth, but economic growth doesn't cause inflation. The outcomes of ECM indicated that in short run inflation is not near to equilibrium. The threshold model suggested that inflation is above from which it lowered the economic growth. Pakistan remained its inflation in single value because higher rate of economic growth may raise the rate of inflation in the case of Pakistan.

Klasra (2011) analyzed the FDI, economic growth and trade openness in case of Turkey and Pakistan. They had to employed time series annual data starting from the time of 1975 to 2004. They employed GDP as output variable and input variables are FDI, export, and trade openness. They investigated the study by utilizing ARDL modelling. The findings of that study showing that exports and trade openness had bi-directional causal nexus for

Pakistan and exports and FDI for Turkey. The results of long run nexus supporting the trade openness and growth relationship in Pakistan and growth driven export hypothesis in Turkey.

Abbas (2012) analyzed the causal relation between the export and GDP for Pakistan. They utilized data for the period of 1975-2010. To investigate the long termed and short termed causality, they used Johansen cointegration test and Granger causality test. The findings of the study indicated that there was a short run and long run uni-directional causality. There should be needed to build domestically production units and make investment friendly environment.

Sultana *et al* (2013) forecast the economic growth and inflation for Pakistan by applying time series methods. They used monthly time series data from July 2008 to June 2013. They forecasted economic growth and inflation by using decomposition method and ARIMA model. They compared their performance by using mean absolute deviation (MAD) and sum of square of errors (SSE). They founded that ARIMA model provide better results than decomposition method to forecasting inflation and economic growth.

Ali and Abdullah (2015) investigate the nexus and effect of trade openness on the economic growth for Pakistan. They had to utilized time series data for investigation for the period of 1980-2010. They utilized real gross domestic as output variable and gross fixed capital formation, financial development, gross fixed capital formation, and secondary education enrollment. They applied Johanson cointegration test and Granger causality test and vector error correction model for short term nexus. The findings showed that GDP growth and trade openness had short run positive nexus of the country and trade liberalization had a negative effect on economic growth for the country. This is due to raw material exports rather final goods.

Gokmenoglu *et al* (2015) estimated the relation among the economic growth and financial development and international trade in Pakistan. For investigation, they had utilized time series annual data starting from the period of 1967 to 2013. The output variable was economic growth and input variables were import, export, queasy money, domestic credit to private banks and domestic credit to private sector. To analyzing the nexus between variables, they were utilized cointegration test and Granger Causality method. The outcomes of the study showed that there was a long-term nexus among financial development, economic growth, and international trade. The causality test results founded that a change in financial development anticipate a change in economic growth and change in financial development, economic growth anticipate change in imports. The findings showed that economic growth will be stabled if government should bolster the financial development.

Jebran *et al* (2018) estimate the effects of terms of trade on the economic growth in case of Pakistan. They utilized time-series annual data starting from 1980 to 2013. They used GDP per capita as an output indicator and gross capital formation, total labor, income term of trade and net barter terms of trade as input variables. They utilized ARDL bound model for the estimation of economic growth in both the long-term and the short-term. The outcomes of that study indicating that labor impacted economic growth positively in the long-term and the short-term. The capital had a significant and positively effected in the long run but an insignificant impact on economic growth in the short run. Terms of trade had significant negatively affected the economic growth in the long-term and short term.

The study suggests that there is a need to make economic policies regarding terms of trade because it is a factor of economic growth.

Ali *et al* (2019) investigated the effect of financial inclusion on the economic growth for Pakistan. In this paper, they had utilized time-series data starting from 1985 to 2017. They used annual real GDP growth as an output variable and financial inclusion index, investment, inflation, and trade openness as independent variables. They applied ARDL bound test for estimation. The outcomes of this paper showing that financial inclusion positively associated to economic growth in the long-term and the short-term. There is urgency to focus on financial literacy in the rural and female populations of the country.

Talib and Fan (2019) investigated the long-term and short-term nexus between energy consumption, manufacturing output and economic growth in the case of Pakistan. They utilized times-series data from 1981 to 2014. They employed GDP per capita as an output indicator and manufacturing output and energy usage per capita as an input variable. They applied autoregressive distributed lag (ARDL) bound approach for testing the long term and short-term nexus between these indicators and Granger causality. The findings of this study show that manufacturing output and energy consumption had positive long-run nexus with economic growth. The outcomes of Granger causality indicate that economic growth and manufacturing output had unidirectional causal nexus whereas, manufacturing output and energy consumption had bidirectional causality.

Awan and Qasim (2020) investigated the effect of external debt on the economic growth of Pakistan. They took time-series data from 1980 to 2017 and include GDP as output variable and external debt exports, imports, population growth rate per annum, gross capital formation, and debt services as input variables. To examine the nexus between variables, they used ARDL and Error Correction Models. The findings of this study indicate that debt services and volume of imports and external debt and population growth rate had a negatively affected on GDP on the other hand exports and employed labor force participation and gross capital formation had a positive impact on GDP. So, Pakistan needs to decrease external debt and increase resources through taxes and exports and productivity.

2.4 Comparison Study

Comparison of different forecasting procedures based on both time series and machine learning models to univariate and multivariate models. Aliha, et al (2019) compare the forecasting performance of multivariate models, autoregressive distributed lag model (ARDL), vector error correction model (VECM), dynamic ordinary least square (DOLS) and fully modified ordinary least square (FMOLS) with univariate models, autoregressive integrated moving average (ARIMA) and exponential smoothing (ETS). The authors concluded that FMOLS is the model with the most predictive power for both short and long terms. Estiko and Wahyuddin (2019) compares the forecasting performance of Neural network (NN) to ARIMA for Indonesia's inflation. They concluded that the NN model outperforms the ARIMA model in forecasting inflation. Salisu, et al (2018) compare the autoregressive distributed lag mixed data sampling (ADL-MIDAS) approach with AR(1), autoregressive fractionally integrated moving average (ARFIMA), ARDL and ARIMA models. They found that ADL-MIDAS was outperform than all other models for both in-sample and out-sample forecast. Nosier and Beram (2020) forecast the Covid-19 Infections and deaths horizon in Egypt and compare ARIMA, dynamic ARDL and ARIMA-ARCH models. They found that ARIMA is best to forecast daily cases, whereas dynamic ARDL is best to forecast daily deaths.

Adom and Dekoe (2012) forecast the electrical energy consumption requirements in Ghana by 2020. In which study compare the ARDL and Partial adjustment model (PAM). The evaluation of the forecasting ability of both models indicated that the ARDL dynamic model possessed good forecasting ability that the Partial adjustment dynamic model. Ulke et al (2016) compare time series and machine learning models for inflation forecasting. They found that time series models were better in nine conditions and machine learning models were better in seven conditions as well as univariate models better in two conditions and multivariate better in fourteen conditions. Jalerajsbi et al (2012) forecast the factors affecting bread waste using ARDL and ANN and compares their performance. The findings show that ANN-ARDL multi-layer perceptron model with hyperbolic tangent transfer function perform well for forecasting the amount of bread waste.

2.5 Summary and Literature Gap

Like the above-mentioned studies on growth and different forecasting models, some economists have tested to investigate the economic growth through utilizing machine learning models and comparing theme with time series models. As Ülke *et al* (2016) forecast inflation of the USA through time series and machine learning models and compare their forecasting performance. Mostly economist Milačić *et al* (2017), Sokolov-Mladenović *et al* (2016), Cogoljević *et al* (2018), Stevanović, *et al* (2018) and Kordanuli, *et al* (2017) forecast the economic growth through ANN with two learning algorithms BP ELM. In Pakistan, mostly economists estimate economic growth through time series models like ARDL bound test, ARIMA models. As Ali et al (2019), Awan and Qasim (2020) and Talib and Fan (2019) investigate economic growth by applying ARDL bound test and Sultana *et al* (2013) forecast the economic growth and inflation by using decomposition method and ARIMA model. The gap to this study that the forecasting side of economic growth at national level is very rare and as well there is no study comparing machine learning and time series models based on forecasting performance. This study forecasted the economic growth with trade indicators, exchange rate, unemployment rate, FDI inflows, remittances, gross capital formation and inflation through Machine learning (SVR and ANN) and time series (RW, AR, and ARDL) models. After that the performance of prediction of all these machine learning and traditional time series models as well as univariate models evaluated by using RMSE, MAE and MAPE.

CHAPTER 3

DATA & METHODOLOGY

3.1 Introduction

This chapter describes a methodological review for forecasting the economic growth. The first section provides a detailed description of data and variables. The next section 3.3 gives a brief description about each model used for forecasting the economic growth in this study. In which an overview of all model (ARDL, AR, RW, SVR and ANN) given. The next section 3.4 provides the overview of all comparison criteria's those gives performance evaluation of forecasting results of all machine learning and traditional time series and as well as univariate and multivariate models.

3.2 Description of data

To forecast the economic growth, we identify the prediction of economic according to Sokolov-Mladenovic et al (2016). We utilize the Pakistan's Quarterly data taken from International Financial Statistics (IFS) and World Development Indicator (WDI) spanning from January 1981 until December 2019. The data spilt into two parts training data from first Quarter 1981 to fourth Quarter 2014 used for forecasting purpose and testing data from first Quarter 2015 to fourth Quarter 2019 for forecasting performance evaluation. Because of Quarterly data for gross domestic product (GDP) per capita growth unavailable, we used industrial production, manufacturing index as proxy variables for GDP per capita growth which is an output variable in our study. The trade indicators and some other indicators were chosen as the input indicators. The export of goods and services, the import of goods and services, trade openness, exchange rate, FDI, unemployment rate, remittances inflows, gross capital formation and inflation are input variables.

Table 3.1: Definitions and source of variables

Variable	Definition	Source of data
Industrial production, manufacturing, Index	It is an index that shows growth in different industrial groups (electricity, mining, and manufacturing etc.) of the economy in a specified period.	IFS
Export of goods and services	Exports are goods and services which produced in one country and sold to another country	IFS
Import of goods and services	Imports are foreign goods and services which are bought by resident country	IFS
Trade Openness	Trade openness is the ratio between the sum of export of goods and services and import of goods and services and GDP	IFS
Exchange Rate	The price of a country's currency alternative the currency of other country. (US Dollar per Domestic Currency)	IFS
Inflation	Inflation is the increase in average price level of goods and services over some period of time.	IFS
Unemployment	A person that is finding a job but unable to find a work.	WDI
Remittances Inflows	Earnings those are transferred to the resident country from an international country.	WDI
Gross Fixed Capital Formation	It is essentially net investment.	WDI
Foreign Direct Investment	An investment invested by an individual/firm in a country into business resident of other country	WDI

Some of indicators (FDI, gross fixed capital formation, unemployment and remittances inflows) have converted into Quarterly data, annual data taken from World Development Indicator (WDI).

3.3 Econometric Methodology

In study, we are comparing the performance of forecasting of extensively utilized traditional time series modelling with the models of machine learning substitutes for the economic growth of Pakistan. We applied two univariate models (autoregressive and random walk) and one multivariate (autoregressive distributed lag) traditional time series models and two machine learning models (support vector regression and artificial neural networks) for the data spanning (training data) January, 1981 (first Quarter) to December, 2014(fourth Quarter). Moreover, we explore a simulated within sample performance of forecasting (testing data) for the period of January 2015 (first Quarter) to December 2019 (fourth Quarter). We will be employing five different models (AR, RW, ARDL, SVR, ANN) to forecast growth rate. The performance of the forecasting models evaluated through RMSE, MAPE and MAE. The description of each model given below.

3.3.1 Random Walk model

The model random walk (RW) is the simplest and important model in the forecasting of time series data. Random walk model supposes that during every period the variables take a random step far from its preceding value, and the steps are independently and identically distributed in size (iid).

The RW model is a general prediction model and represents as below

$$IPM_t = \alpha_1 IPM_{t-1} + \varepsilon_t \tag{1}$$

Where IPM_t industrial production manufacturing index, α_1 is a parameter and ε_t is an error term which is white noise. RW predicting that the value at time "t" equals to the last predicted values plus a stochastic ingredient those are white noise, which meanings $\varepsilon_t$is independently and identically distributed with zero mean and variance "σ^2".

3.3.2 Autoregressive (AR) model

The AR model forecasts the values by using previous values. The AR model identifies that the output variable depending on its lagged values and error terms. Previous studies showed that predicting economic indicators beneficial from the Autoregressive model to compare with more recently developed models. In our study lags of the model selected on the basis of Akaike information criteria (AIC). For lags selection different model with different lags applied and choose that one which has lowest AIC in the model. Following equation estimated for the autoregressive (AR) model.

$$IPM_t = C + \sum_{i=1}^{p} \beta_i \, IPM_{t-i} + \varepsilon_t \tag{2}$$

Autoregressive model of order p where IPM is industrial production manufacturing index, and β_1 ………………., β_p are parameters of the model c is the constant and ε_t is an error term. The selected autoregressive (AR) model written as:

$$IPM_{\boldsymbol{t}} = C + \beta_1 IPM_{t-1} + \beta_2 IPM_{t-2} + \beta_3 IPM_{t-3} + \beta_4 IPM_{t-4} + \varepsilon_t \tag{3}$$

3.3.3 Autoregressive distributed lag (ARDL) model

ARDL model was founded by Pesaran and Pesaran (1998), and Pesaran et al. (2001). That type of methodology becomes more famous because of its a few advantages as it's a model of single equation cointegration measurements and as well as some more cointegration measurements as Engle and Granger (1987) applicable for just two indicators and Johansen and Juselius (1990) can be applicable on just series also which have the same order of integration but for more than two indicators and accurate for extensive size data-set. ARDL model is applicable regardless of the exploratory indicators those have same integration order or not (Hamuda et al. 2013). It is also applicable for finite sample data. If the indicators integrated order is I (2), then the procedures of ARDL does not make any sense.

The ARDL methodology is an opportunity model to examine the co-integration that empower us to use integrated order I (0) and I (1) indicators together. A general ARDL can be written as

$$IPM_t = C + \sum_{k=1}^{l} A_k\, IPM_{t-k} + \sum_{k=0}^{m} B_k\, X_{t-k} + \sum_{k=0}^{n} C_k M_{t-k} + \sum_{k=0}^{o} D_k\, ER_{t-k} + \sum_{k=0}^{p} E_k\, TO_{t-k} + \sum_{k=0}^{q} F_k\, INF_{t-k} + \sum_{k=0}^{r} F_k\, UNEM_{t-k} + \sum_{k=0}^{s} F_k\, REM_{t-k} + \sum_{k=0}^{t} F_k\, GCF_{t-k} + \sum_{k=0}^{u} F_k\, FDI_{t-k} + \varepsilon_t \quad (4)$$

Where, IPM is industrial production manufacturing index, ER is exchange rate, X show export of goods and services, M is import of goods and services, TO is trade openness, INF is inflation UNEM is unemployment rate, REM is remittances, GCF is gross capital formation and FDI is foreign direct investment inflows. Economic growth is analyzed by lag values of output indicator and other independent indicators and their lag values for each indicator are observed by ARDL bond test.

3.3.4 Artificial neural network (ANN) model

ANN is the popular machine learning technique that is mostly used in economic and financial estimation. ANN is a multi-layer fully associated with neural nets. ANN subsist of different input layers, multiple hidden layers and on a given output layer. The nodes of one layer relate to next layer nodes. We can make the neural network deep by raising the hidden layers. ANN works like the human brain (ANN is an experiment to simulating the network of all neurons that makes a human brain that the computer make able to understand things and make valuable judgments as such human manner). A given node in neural network takes the weighted sum of its input layers and it passes from a non-linear activation function. This gives output of the node, after that this output node becomes input node for other layers of node. The action starts from left side to right side, and the final output is obtained by carrying out this action for all other nodes. ANN learn from data and gives output in the form of classification and prediction as neurons of our nervous system learn from past data. ANN is a non-linear model that shows complex nexus between the output and inputs. ANN model can be represented as follow:

$$IPM_t = \beta + w_1.X_t + w_2.M_t + w_3.ER_t + w_4.TO_t + w_5.INF_t + w_6.GCF_t + w_7.FDI_t + w_8.unemp_t + w_9.rem_t + \varepsilon_t \quad (5)$$

Where, IPM is industrial production manufacturing index, X_t is export of goods and services, M_t is import of goods and services, ER_t is exchange rate, TO_t is trade openness, INF_t is inflation, GCF_t is gross capital formation, FDI_t is foreign direct investment, $unemp_t$ is unemployment and rem_t is remittances inflows. Furthermore, w_1, w_2,…………, w_9 are the weights that show the strength of a particular node and β is a bias value that allows shifting the activation function up or down. In this equation all the products summed, fed to an activation function (convert an input signal of a node to an output signal) to generate output. The implementation of ANN tested with different hidden layers and neurons. We used 1 to 8 hidden layers with 10 and 20 neurons. From that it is noticed that the best network is with 4 layer and 10 neuron combination which gives lower values of errors. We tried it differently and choose which that model provides best performance i.e. lower the value of RMSE, MAPE and MAE.

3.3.5 Support Vector Regression (SVR)

SVR is a machine learning technique utilized for forecasting in economic and financial time series. SVR works the same as the principles of support vector machine (SVM) works. We can also say that SVR is the adopted form of SVM at that time when the output indicator is numerical rather than categorical. Support Vector Machine (SVM) is used in classification issues. But support vector machine (SVM) in not used in regression because it is not well documented. However, for regression purposes SVR is utilized. The much benefit of utilizing SVR is that it is a non-parametric technique that says that the model for the output from SVR doesn’t depend on the distribution of endogenous and exogenous variables. The SVR technique based on the kernel functions for the construction of the model. The function of kernel is to take the data as an input and convert it into the appropriate form. Linear, Polynomial, Sigmoid and radial basis are mostly used kernel functions. There are also some other kernel functions, but these are mostly used kernels. We will utilize these above-mentioned kernel functions and used in our experiment which provide better results. While the application of SVR technique needs to use appropriate kernel functions. Kernel functions converts the non-linear data spaces into linear data spaces. Thus, the construction of SVR model as follows:

$$y_t = w . x_t + \beta \tag{6}$$

Where, y_t is our output parameter (industrial production manufacturing index), w is a weighted vector, x_t is a vector with input variables (export of goods and services, import of goods and services, exchange rate, trade openness, unemployment, remittance inflows, FDI, gross capital formation and inflation) and β is a constant. The principles of SVR are as follow: i) Preparation of a pattern matrix, ii) Selecting of a Kernel function iii) Parameter selection iv) Execution of training algorithms v) Prediction of unseen data.

In our implementation of support vector regression (SVR), we tried four different kernels (“linear”, “radial basis function”, “polynomial”, “sigmoid”) and the best one from all these kernels is linear kernel function on the basis of low values of RMSE, MAPE and MAE and it is used in our forecasting of economic growth.

3.4 Comparison Criteria

The performance of prediction of all the models evaluated by using root means square error (RMSE), the mean absolute percentage error (MAPE), and mean absolute error (MAE). In our case we analyze the forecasting performance of autoregressive distributed lag model (ARDL), Autoregressive model (AR), random walk model (RW), ANN and SVR Which model has lowest values of RMSE, MAE and MAPE gives better forecasting performance as compare to other models. The formulas for these all comparing criteria’s as given below.

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(Y_t - \hat{Y}_t\right)^2} \tag{7}$$

$$MAE = \sum_{i=1}^{n}\left|Y_t - \hat{Y}_t\right| \tag{8}$$

$$MAPE = \frac{1}{n}\sum_{t=1}^{n}\left|\frac{Y_t - \hat{Y}_t}{Y_t}\right| \tag{9}$$

Here Y_t is the original value for the given period of time t, $\hat{Y}_t$ is the fitted forecasted value for the time t, and n is the total number of fitted points.

CHAPTER 4

EPPIRICAL ANALYSIS

4.1 Introduction

This fourth chapter of the study presents the outcomes of the thesis and comparable discussions of the forecasting the economic growth for Pakistan. That study is established upon quantitative analysis. The results have been obtained by applying AR model, Random Walk (RW) model, autoregressive distributed lag (ARDL), Artificial Neural Network (ANN) and Support Vector Regression (SVR). Forecasting economic growth by using industrial production manufacturing index (output variable) as proxy variable of economic growth due to unavailability of quarterly data for GDP per capita growth with export of goods and services, import of goods and services, exchange rate, trade openness, unemployment, FDI, remittances inflows, gross fixed capital formation and inflation (input variables). The study carried out on the forecasting of economic growth for Pakistan. The forecasting performance of all models compare by using the root mean square error (RMSE), the mean absolute error (MAE) and the mean absolute percentage error (MAPE). The lower values of these errors indicating the better performance of a model.

This chapter contains the detail discussion on forecasting performance of economic growth. First section 4.2 consists on the graphical visualization of the output indicator, the next section 4.3 based on the result for descriptive part which featured some of the weighty points respecting the statistical properties of the indicators of interest and is pursued by the section of analysis and results. The next section 4.4 gives the outcomes of unit root test for stationarity measures by utilizing ADF test and Phillips-Perron test. Once section 4.5

describes the results of Autoregressive model, and the next section 4.6 carries the results of random walk model. The next section 4.7 consists of the results of ARDL model after that next sections 4.8 and 4.9 based on the results of ANN and SVR, respectively. The last section 4.10 compares the forecasting performance of all models.

4.2 Graphical Description

Graphical description used to see the pattern of the main variable (industrial production manufacturing index). Time plot has been used to describe the industrial production manufacturing index (IPM). The line plots of the variable provided in Figure 4.1 before taking first difference and Figure 4.2 after taking first difference to check the pattern of the variable either it is stationary or not.

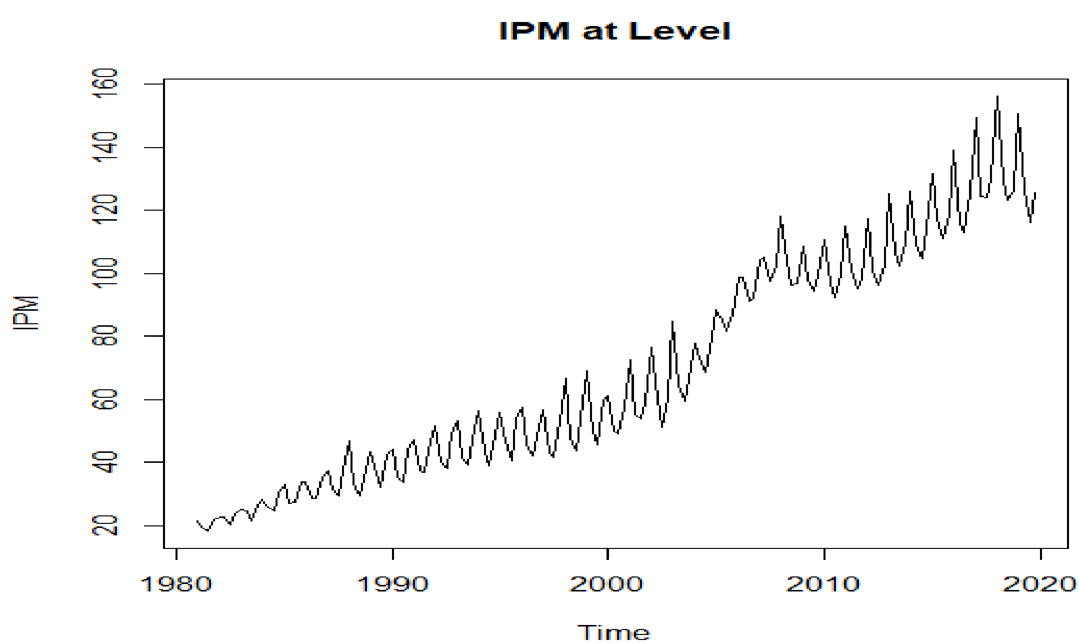

Figure 4.1: Industrial production manufacturing index at level

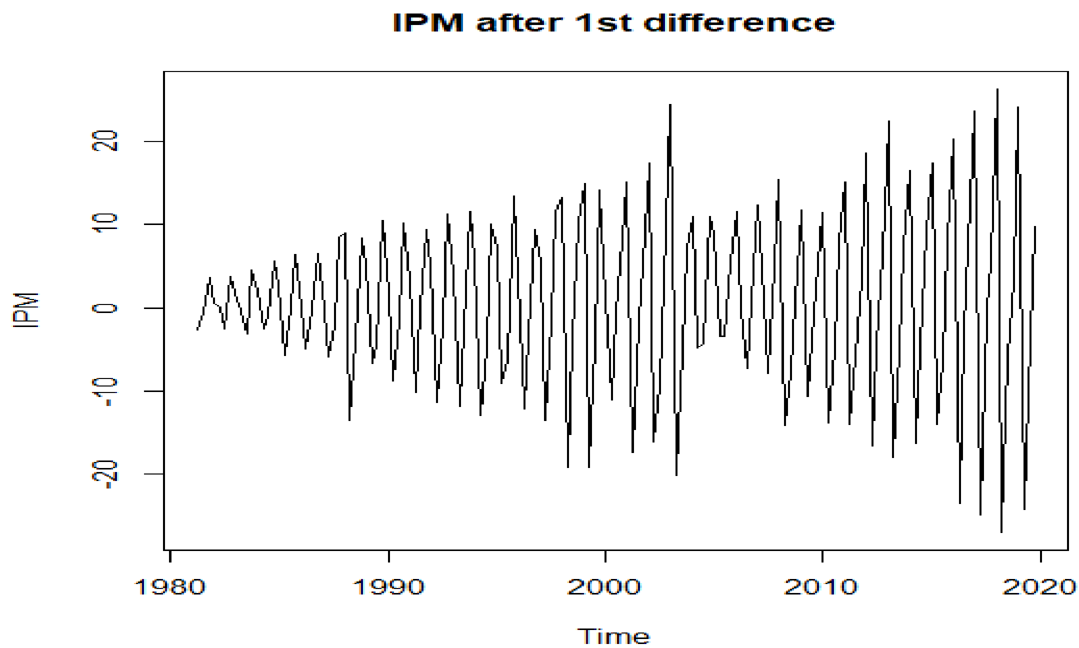

Figure 4.2: Industrial production manufacturing index after 1st difference

We can see from above plots of IPM in Figure 4.1 that show the series is non-stationary at level and Figure 4.2 show that series is stationary after taking 1st differences of industrial production manufacturing index (IPM). Furthermore, ADF and PP statistics used on all other series as well to exhibit the stationarity and non-stationarity.

4.3 Summary of the Descriptive Statistics

The main indicators of interest in this study are industrial production manufacturing index (IPM), export of goods and services (Export), import of goods and services (Import), Trade Openness (TO), Exchange rate (ER), Inflation, foreign direct investment (FDI), gross capital formation (GCF), Remittances inflows and Unemployment. Table 4.1 presents the summary of statistics on these indicators.

Table 4.1: Descriptive statistics of variables

Descriptive Statistics								
Variables	**Mean**	**Median**	**Maximum**	**Minimum**	**Std. dev.**	**Skewness**	**Kurtosis**	**Obs.**
IPM	69.7265	57.7126	156.1274	18.4571	35.9581	0.4182	1.9167	156
Export	3.06E+09	2.26E+09	7.08E+09	4.48E+08	1.96E+09	0.4126	1.7426	156
Import	5.46E+09	2.89E+09	1.65E+10	1.18E+09	4.41E+09	0.7658	2.0661	156
TO	1.10E+08	1.01E+08	1.83E+08	57789196	32809962	0.5040	2.1959	156
ER	0.0307	0.0190	0.1010	0.0063	0.0240	1.1999	3.5491	156
Inflation	7.6240	7.4109	21.9280	1.6239	3.6880	0.8377	4.3008	156
FDI	3.10E+08	1.66E+08	1.49E+09	6762107.82	3.54E+08	1.6981	5.4199	156
GCF	4.85E+09	3.02E+09	1.34E+10	1.19E+09	3.52E+09	0.8553	2.4324	156
Remittances	1.61E+09	6.64E+08	5.67E+09	2.45E+08	1.69E+09	1.2159	2.9569	156
Unemployment	0.9510	0.9947	2.0342	0.0075	0.5538	0.0672	2.0874	156

Source: Author’s own calculations

Table 4.1 descriptive statistics of indicators showed some key aspects of the indicators. All the indicators have same number of observations. The mean average values of the data and median about the most middle value of the ascending or descending order of the data. The standard deviation describes us about the deviation of the series from its mean for its analysis. The skewness and kurtosis show whether the series follows the normal distribution or not. Positive skewness shows that most of the observations lies to the right of its mean value while negatively skewness means that most of the observations lie to the left of its mean value. Similarly, kurtosis show about the peakness of the data that is whether the series is leptokurtic, platykurtic or mesokurtic (normal). As the value of kurtosis is equal to 3 than it means data is mesokurtic (normal) and the value of kurtosis greater than 3 show that data is leptokurtic and if the value of kurtosis is less than 3 than data is platykurtic.

In our case, skewness of all variables found to be positive. Kurtosis values of industrial production manufacturing index, export of goods and services, import of goods and services, trade openness, gross capital formation (GCF), Unemployment and Remittances inflows indicating platykurtic because of values which are less than 3, whereas inflation, exchange rate (ER) and FDI are leptokurtic which value has less than 3.

After analyzing the descriptive statistics, we have taken log of export of goods and services, import of goods and services, trade in services and trade openness. Before analysis of our data it is needed to identify the presence of non-stationarity, if yes, then the order of integration of all variables will need to be undertaking. The stationarity of all variables can be checked by unit root. To check unit root process Augmented Dicky-Fuller (ADF) test and Phillips-Perron test are used.

4.4 Unit root tests

Stationarity is the imperative in the analysis of econometric for the data type of time series. A white noise series has zero mean value and constant variance, and covariance is also zero (Charemza and Deadman, 1992). The series is non-stationary if it is not holding any property of white noise. This type of non-stationary is one of the antagonistic problems at the time when we are concerns with the time series estimations. Augment Dickey-Fuller (ADF) test is frequently utilized unit root test. In this we used ADF as well as Phillips-Perron (PP) test.

Unit root test outcomes in table 4.2 below based on two tests Augmented Dicky-Fuller test and Phillips-Perron test. The results show for ADF unit root test depicted that industrial production manufacturing index (IPM), Export of goods and services (Export), import of goods and services (Import), gross capital formation (GCF), unemployment and remittances inflows are non-stationary with and without trend at level. The outcomes of Phillips-Perron test show that industrial production manufacturing index (IPM) and Export of goods and services (Export) are non-stationary without trend, but both are stationary with trend at level with 1% level of significance. Furthermore, the outcomes of PP test for import of goods and services depict that it is non-stationary without trend but stationary with trend at level with 10% level of significance.

The outcomes of PP test depict that gross capital formation (GCF), Unemployment, and remittances inflows are non-stationary with and without trend at level. The outcomes of ADF test shows that trade openness (TO) is non-stationary without trend at level but stationary with trend at level with 1% level of significance.

Table 4.2: Unit root tests of level and transformed series

Variables	Unit Root Test							
	Level				Transformed			
	A: intercept		B: trend and intercept		A: intercept		B: trend and intercept	
	ADF	PP	ADF	PP	ADF	PP	ADF	PP
IPM	0.21	-1.09	-1.81	-9.11***	-6.62***	-25.50***	-6.63***	-25.47***
Export	-1.69	-0.77	-1.99	-6.74***	-6.85***	-26.25***	-6.97***	-26.37***
Import	-0.64	-0.48	-2.58	-3.21*	-5.34***	-18.39***	-5.31***	-18.34***
TO	-1.23	-4.21***	-3.32*	-8.22***	-6.05***	-23.96***	-6.03***	-24.23***
ER	-6.27***	-8.87***	-5.02***	-4.93***	-	-	-	-
Inflation	-3.23**	-2.81*	-3.23*	-2.79	-4.66***	-7.85***	-4.63***	-7.96***
FDI	-2.71*	-1.20	-3.53**	-2.36	-3.54***	-5.29***	-3.71**	-5.25***
GCF	-1.38	-0.93	-2.41	-2.01	-4.22***	-5.16***	-4.20***	-5.19***
Unemployment	-2.08	-1.81	-2.15	-1.84	-5.17***	-7.06***	-5.15***	-7.03***
Remittances	-0.05	0.21	-2.20	-1.29	-3.00**	-6.25***	-3.39*	-6.03***

* Describes the level of significance at 10%
**Describes the level of significance at 5%
*** Describes the level of significance at 1%

The Phillips-Perron test values show that trade openness (TO) is stationary without trend at level with 10% level of significance and stationary with trend at level with 1% level of significance. The ADF test depicted that inflation is stationary with and without trend at level with 10% and 5% level of significance, respectively. The Phillips-Perron test depicted that inflation is stationary without trend at level with 10% level of significance, but it is non-stationary with trend at level.

The outcomes of ADF test for foreign direct investment (FDI) depict that it is stationary without and with trend at level with 10% and 5% level of significance, respectively. The PP test outcomes for FDI depict that it is non-stationary with and without trend at level. The outcomes of both ADF and PP test for exchange rate (ER) depict that it is stationary with and without trend at level with 1% level of significance.

The outcomes of both ADF test and PP test depicted that industrial production manufacturing index, export of goods and services, import of goods and services, trade openness, inflation, gross capital formation, and unemployment are stationary with and without trend at 1% level of significance after taking first difference. The outcomes of ADF test for foreign direct investment after taking first difference depict that it is stationary without trend at 1% level of significance and with trend it is stationary at 5% level of significance. The outcomes of PP test for FDI after taking first difference depict that it is stationary with and without trend at 1% level of significance. The outcomes of ADF test for remittances inflows after taking first difference depict that it is stationary without trend at 5% level of significance and with trend it is stationary at 10% level of significance. The outcomes of PP test for remittances inflows after taking first difference depict that it is stationary with and without trend at 1% level of significance.

4.5 Analysis through Autoregressive Model

The first step is to identify the AR part form autocorrelations function (ACF) and partial autocorrelation function (PACF) plots which are showed the significant lags of the model. Figures 4.3 and 4.4 given below show ACF and PACF plots, respectively.

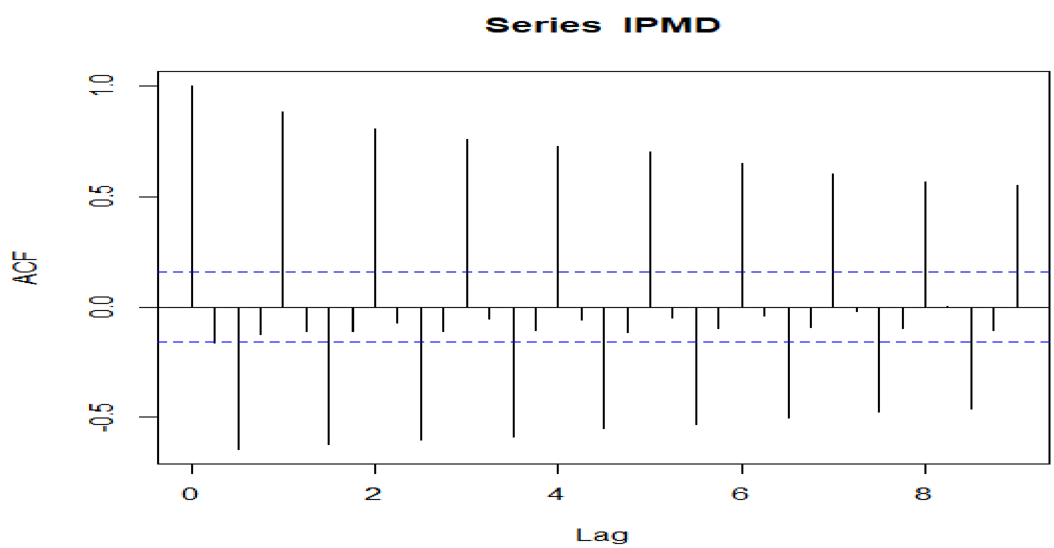

Figure 4.3: ACF for Industrial production manufacturing index

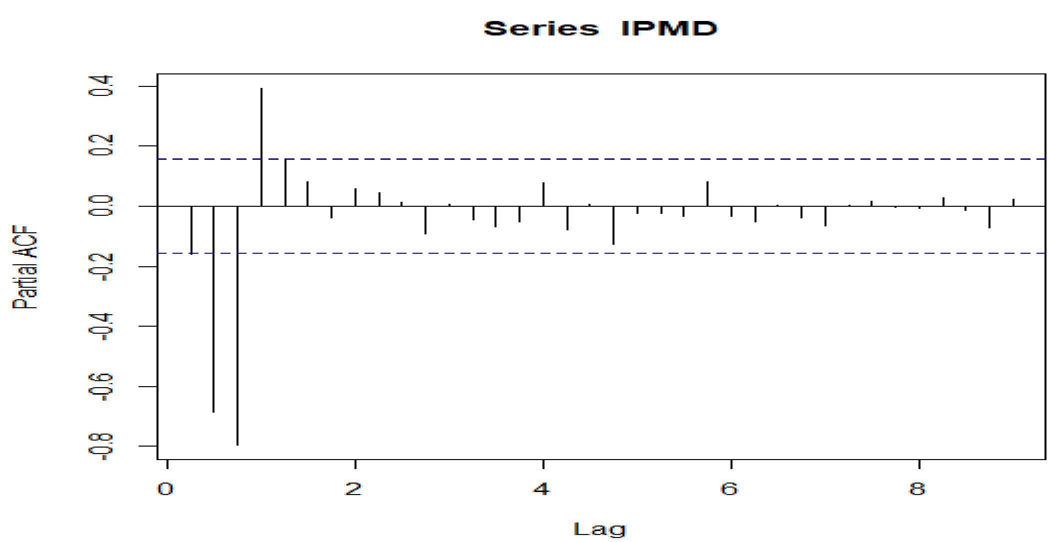

Figure 4.4: PACF for Industrial production manufacturing index

The ACF and PACF of differenced IPM series up to 36 lags are shown in above Figures 4.3 and 4.4, respectively. The above ACF follows geometric decay and PACF shown significant till 4^{th} lag which means that it becomes as ARIMA (4,0,0) or AR (4) model. The next step based on the analysis of AR model for industrial production manufacturing index (IPM). The outcomes of the estimated model are given below in Table 4.3.

Table 4.3: Autoregressive model results

Lags	Coefficients	Standard errors	t-statistics
AR (1)	-0.3013	0.0692	-4.3540
AR (2)	-0.3612	0.0693	-5.2083
AR (3)	-0.2778	0.0695	-3.9971
AR (4)	0.5864	0.0693	8.4617
σ2 = 13.29, Log Likelihood = -368.77, AIC = 747.54			

The outcomes showing that the sigma square is the value of constant variance which model assumed and the probability of the data we have observed shown by the value of log likelihood and AIC identify the performance of selecting the best possible model among others based on its minimum value.

After the analysis of the model diagnostic checking is made and the statistics of the Box-Ljung test given below in the Table 4.4

Table 4.4: Box-Ljung Test

Box-Ljung Test		
Chi-square	**Lags**	**P-value**
17.462	12	0.1330

The above table 4.4 provides the outcomes of Ljung Box test which indicates that p-value is greater than 0.05 meaning that we accepted the null hypothesis i.e. there is no autocorrelation among the residuals.

Table 4.5: Actual versus predicted values of AR model

Quarter	Actual Values	Predicted Values	Error	AE
2015 Q1	131.4693	127.2292	4.2401	4.2401
2015 Q2	117.5173	111.6620	5.8553	5.8553
2015 Q3	110.8445	106.2945	4.55	4.55
2015 Q4	119.0209	115.3024	3.7185	3.7185
2016 Q1	139.2967	126.5509	12.7458	12.7458
2016 Q2	115.7595	112.2715	3.488	3.488
2016 Q3	112.933	106.8607	6.0723	6.0723
2016 Q4	125.6264	115.8053	9.8211	9.8211
2017 Q1	149.2554	125.6278	23.6276	23.6276
2017 Q2	124.4719	112.5681	11.9038	11.9038
2017 Q3	124.2116	107.2970	16.9146	16.9146
2017 Q4	129.8739	116.1181	13.7558	13.7558
2018 Q1	156.1273	124.7524	31.3749	31.3749
2018 Q2	129.1818	112.7715	16.4103	16.4103
2018 Q3	123.3141	107.7209	15.5932	15.5932
2018 Q4	126.5968	116.3435	10.2533	10.2533
2019 Q1	150.7379	123.9615	26.7764	26.7764
2019 Q2	126.4733	112.9299	13.5434	13.5434
2019 Q3	115.9913	108.1447	7.8466	7.8466
2019 Q4	125.7455	116.5106	9.2349	9.2349

Table 4.5 be made up of actual and predicted values with their errors, which describes that AR model performs slightly better in forecasting. After that we look over through graph of actual and predicted values which is given as in Figure 4.5.

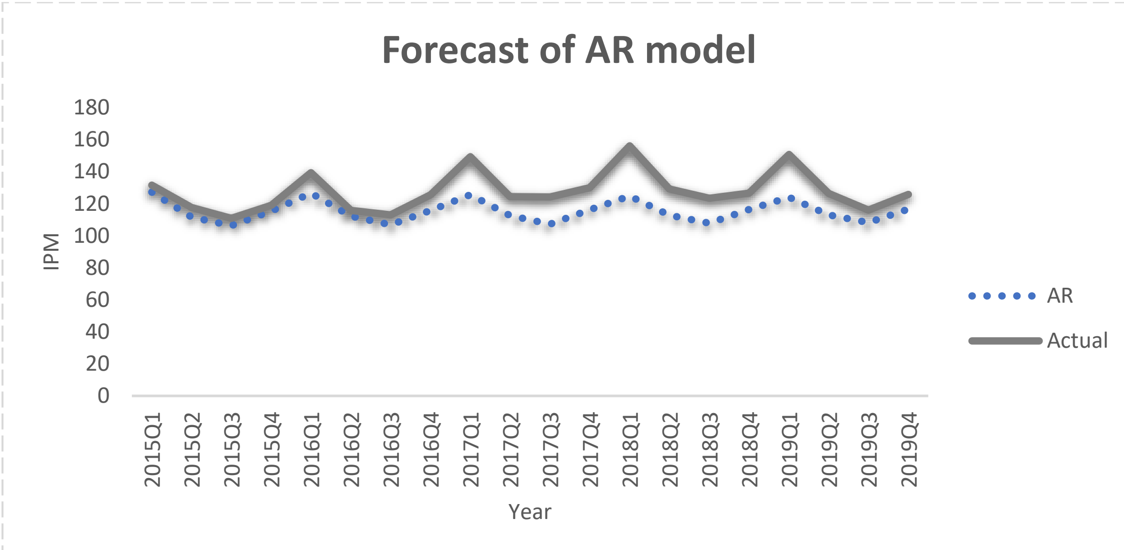

Figure 4.5: Graph for Actual versus Predicted of AR model

Figure 4.5 provides the graphical comparison of predicted values of AR model and actual values. Whereas dashed line showing the predicted values of AR model and solid line showing the actual values. The graph indicates that prediction with AR model trying its best to capturing the actual values.

4.6 Analysis through Random walk model

The analysis of Random walk model for economic growth forecasting provides a somehow satisfactory result. The prediction through RW model provides constant predicted values of the given model. Table 4.6 provide the predicted values from 2015 first Quarter to 2019 fourth Quarter which compares with actual values. Furthermore, it provides error in the model and absolute errors.

Table 4.6: Actual versus predicted values of RW model

Quarter	Actual Values	Predicted Values	Error	AE
2015 Q1	131.4693	114.0998	17.3695	17.3695
2015 Q2	117.5173	114.0998	3.4175	3.4175
2015 Q3	110.8445	114.0998	-3.2553	3.2553
2015 Q4	119.0209	114.0998	4.9211	4.9211
2016 Q1	139.2967	114.0998	25.1969	25.1969
2016 Q2	115.7595	114.0998	1.6597	1.6597
2016 Q3	112.933	114.0998	-1.1668	1.1668
2016 Q4	125.6264	114.0998	11.5266	11.5266
2017 Q1	149.2554	114.0998	35.1556	35.1556
2017 Q2	124.4719	114.0998	10.3721	10.3721
2017 Q3	124.2116	114.0998	10.1118	10.1118
2017 Q4	129.8739	114.0998	15.7741	15.7741
2018 Q1	156.1273	114.0998	42.0275	42.0275
2018 Q2	129.1818	114.0998	15.082	15.082
2018 Q3	123.3141	114.0998	9.2143	9.2143
2018 Q4	126.5968	114.0998	12.497	12.497
2019 Q1	150.7379	114.0998	36.6381	36.6381
2019 Q2	126.4733	114.0998	12.3735	12.3735
2019 Q3	115.9913	114.0998	1.8915	1.8915
2019 Q4	125.7455	114.0998	11.6457	11.6457

Table 4.6 holds the predicted and actual values for industrial production manufacturing index with their corresponding errors. The graphical visualization gives in the Figure 4.6 to analyze the gap among predicted and actual values.

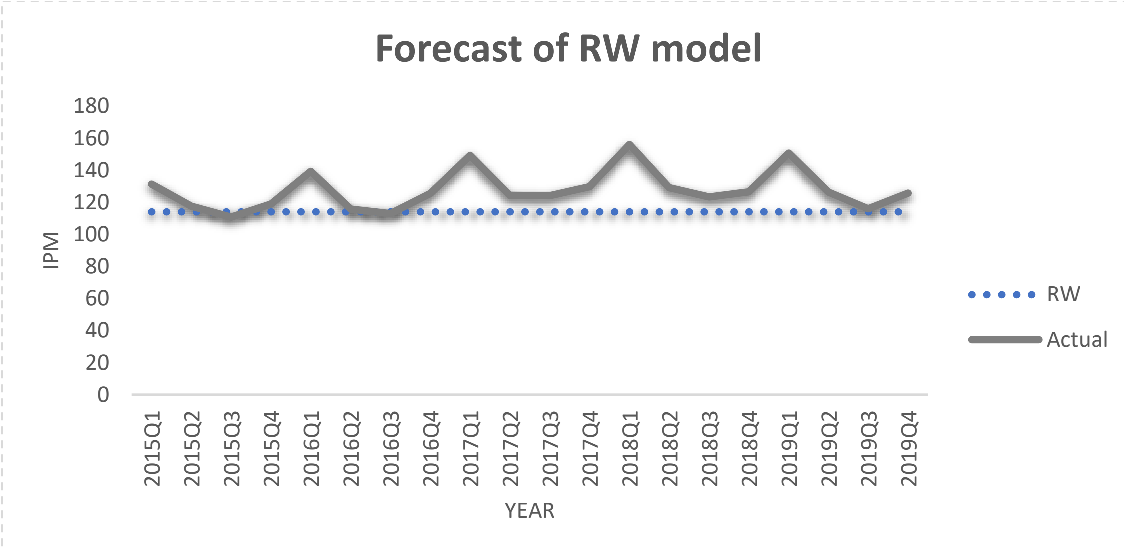

Figure 4.6: Graph for Actual versus Predicted of RW model

Figure 4.6 provides the graphical comparison of predicted values of RW model and actual values for industrial production manufacturing index (IPM). Whereas dashed line showing the predicted values of RW model and solid line showing the actual values of the given data. The sloid line of RW model prediction provides constant forecasted values. The graph indicates that prediction with RW model trying its best to capturing the actual values.

4.7 Analysis through ARDL Bound test Model

According to Augmented Dickey-Fuller test of stationarity of variables showed that some of the indicators are integrated of order one and some of them are integrated of order zero. There is no variable integrated of order two and the dependent variable also integrated of order one, which meets the assumption of ARDL bound test. In this situation economic growth has been investigated by using ARDL model. The findings of ARDL bound test model are given below:

4.7.1 Cointegration results using Bounds Test

For long run nexus following null and alternative hypothesis observed.

H_0: No long run relationships exits

H_1: long run relationships exit

Table 4.7: Bounds Test

Bounds Test				
F-statistic	**Significance**	**Lower Bound**	**Upper Bound**	**Remark**
14.2211	1%	2.65	3.97	Cointegrated

The outcomes of bounds test showing that the calculated value is greater than upper bound, meaning that we rejected the null hypothesis that there is no long run relationship exists and we accept the alternative hypothesis that there is long run relationship exist. The outcomes of cointegration given in table 4.8.

Tables 4.8: Cointegration Results

Cointegration				
Variable	**Coefficient**	**Std. Error**	**t-Statistic**	**Prob.***
D(IPM(-1))	-0.43838	0.089256	-4.91151	0.0001***
D(IPM(-2))	-0.64882	0.066765	-9.71795	0.0001***
D(IPM(-3))	-0.65428	0.0656	-9.97381	0.0001***
D(IPM(-4))	0.12682	0.038899	3.260209	0.0015***
D(EXPORT)	49.37272	3.193253	15.46158	0.0001***

D(EXPORT(-1))	-10.6657	3.611604	-2.95316	0.004***
D(EXPORT(-2))	7.598588	3.742328	2.030444	0.0451**
D(EXPORT(-3))	38.31118	4.360715	8.785526	0.0001***
D(IMPORT)	90.94328	4.834422	18.81161	0.0001***
D(IMPORT(-1))	-10.6418	6.982707	-1.52402	0.1308
D(IMPORT(-2))	4.613146	6.524827	0.707014	0.4813
D(IMPORT(-3))	62.8249	6.672869	9.414976	0.0001***
D(INFLATION)	-0.08961	0.091604	-0.97827	0.3304
D(INFLATION(-1))	0.192592	0.143303	1.343954	0.1821
D(INFLATION(-2))	-0.16344	0.14006	-1.16695	0.2461
D(INFLATION(-3))	0.168906	0.090627	1.863759	0.0654*
D(TO)	-138.519	5.651552	-24.51	0.0001***
D(TO(-1))	26.79136	9.023663	2.969012	0.0038***
D(TO(-2))	-8.62787	8.844858	-0.97547	0.3318
D(TO(-3))	-103.836	10.10803	-10.2727	0.0001***
D(ER)	76.75305	61.54986	1.247006	0.2154
D(FDI)	-0.10363	0.786077	-0.13184	0.8954
D(GCF)	18.38287	11.95918	1.537135	0.1275
D(GCF(-1))	2.979379	17.36314	0.171592	0.8641
D(GCF(-2))	21.02611	17.37914	1.209848	0.2293
D(GCF(-3))	-30.7525	10.75301	-2.8599	0.0052***
D(UNEMPLOYMENT)	-0.7782	0.456688	-1.704	0.0916*
D(REMITTANCES)	0.252868	1.144112	0.221017	0.8255
CointEq(-1)	-0.43478	0.09063	-4.79731	0.0001***

R-squared = 0.9987 Adj. R-squared = 0.9983

F-statistic = 2321.37 Prob(F-statistic) = 0.0001 Durbin-Watson stat = 1.7625

* Express the level of significance at 10%
**Express the level of significance at 5%
*** Express the level of significance at 1%

Table 4.9: Long run Coefficients

Variable	Coefficient	Std. Error	t-Statistic	Prob.*
EXPORT	28.24965	10.65999	2.650064	0.0094***
IMPORT	67.02709	18.96303	3.534619	0.0006***
INFLATION	-0.68614	0.159262	-4.30828	0.0001***
TO	-88.0269	21.13159	-4.16565	0.0001***
ER	176.5339	128.685	1.371829	0.1733
FDI	-0.23836	1.790518	-0.13312	0.8944
GCF	52.69652	16.47711	3.198165	0.0019***
UNEMPLOYMENT	-1.78987	1.168511	-1.53176	0.1289
REMITTANCES	0.581603	2.624561	0.2216	0.8251
C	-639.51	83.20427	-7.68602	0.0001***

* Express the level of significance at 10%
**Express the level of significance at 5%
*** Express the level of significance at 1%

Table 4.9 show the cointegration and long run results. The outcomes indicate that import of goods and services and export of goods and services both has positive and significant nexus with industrial production manufacturing index (economic growth) in long run and short run. Inflation has negative but insignificant nexus in short run and negative and significant nexus in the long run. Trade openness has significant but negative relationship with economic growth in both long run and short run. Exchange rate has positive and insignificant nexus with economic growth in long run and short run. FDI has negative and insignificant nexus with economic growth of Pakistan in long run and short run. Gross capital formation has positive and insignificant nexus with economic growth in short run and it has positive and significant nexus with economic growth in long run. Unemployment has negative and significant nexus with economic growth in short run and negative and insignificant in long run. Remittances has positive and insignificant nexus with economic growth in long run and short run.

The outcomes in Table 4.8 indicate that ECT significant and negative value. The value of ECT -0.4347 encourage toward adjustment process. Somehow 43% of the disequilibrium in the previous time is adjusted toward the long run equilibrium in the current time. The significance of ECT with negative sign ratify the convergence of long run equilibrium.

4.7.2 Serial Correlation Test

There is need to test the serial correlation before using the results of ARDL model, it is vitally important to checking the serial correlation. It is a situation in which residuals are correlated. If the correlation of residuals is existing than the investigation of the model is inconsistent and biased. Two approaches are normally used to test the serial correlation. The first one is Durbin Watson (DW) and second is Breusch Godfrey test or Langrage Multiplier (LM) test. Durbin Watson (DW) test has some limitations because it is not used for higher order serial correlation. So, LM test is one that check serial correlation for higher order. The null hypothesis is that there is no serial correlation. The results of LM test are given below:

Table 4.10: Breusch-Godfrey Serial Correlation LM Test

Breusch-Godfrey Serial Correlation LM Test			
F-statistic	0.9821	Prob. F(2,94)	0.3783
Obs*R-squared	2.6813	Prob. Chi-Square (2)	0.2617

Table 4.10 we can see that the p-value of LM test in 26.17% more than 5% which indicates that we can’t reject the null hypothesis. Which shows that there is no issue of serial

correlation. After that next table show the actual and predicted values by using ARDL and their errors.

Table 4.11: Actual versus predicted values of ARDL model

Quarter	Actual Values	Predicted Values	Error	AE
2015 Q1	131.4693	129.6996	1.769743	1.769743
2015 Q2	117.5173	115.4086	2.108719	2.108719
2015 Q3	110.8445	109.8146	1.029941	1.029941
2015 Q4	119.0209	117.3313	1.689612	1.689612
2016 Q1	139.2967	136.1616	3.13508	3.13508
2016 Q2	115.7595	117.8105	-2.05096	2.05096
2016 Q3	112.933	111.7951	1.137918	1.137918
2016 Q4	125.6264	124.4154	1.211018	1.211018
2017 Q1	149.2554	143.7749	5.480512	5.480512
2017 Q2	124.4719	122.0428	2.429146	2.429146
2017 Q3	124.2116	118.0923	6.119293	6.119293
2017 Q4	129.8739	128.7469	1.126965	1.126965
2018 Q1	156.1273	151.2617	4.865553	4.865553
2018 Q2	129.1818	126.9805	2.201349	2.201349
2018 Q3	123.3141	121.3141	2.000042	2.000042
2018 Q4	126.5968	126.0016	0.59516	0.59516
2019 Q1	150.7379	150.6971	0.04079	0.04079
2019 Q2	126.4733	125.9265	0.546778	0.546778
2019 Q3	115.9913	115.707	0.284252	0.284252
2019 Q4	125.7455	121.9555	3.790024	3.790024

Table 4.11 holds the predicted and actual values for industrial production manufacturing index with their corresponding errors. The graphical visualization gives into the Figure 4.7 to analyze the gap among predicted and actual values.

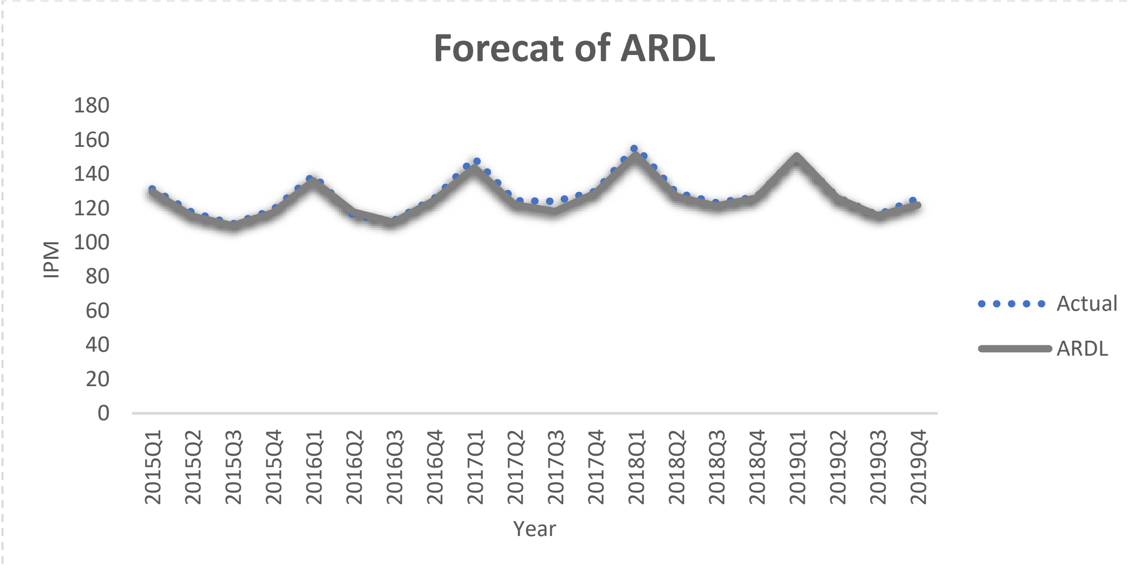

***Figure 4.7*:** Graph for Actual versus Predicted of ARDL model

Figure 4.7 provides the graphical comparison of predicted values of ARDL model and actual values for industrial production manufacturing index (IPM). Whereas dashed line showing the predicted values of ARDL model and solid line showing the actual values of the given data. The graph indicates that prediction with ARDL model gives best prediction of the model.

4.8 Analysis through ANN Model

The implementation of ANN tested with different hidden layers and neurons. We used 1 to 8 hidden layers with 10 and 20 neurons. From that it is noticed that the best network is with 4 layer and 10 neuron combination which gives lower values of errors. In the next step of prediction which is done by choosing best suitable model of 4 layer and 10 neurons. The predicted values from 2015 first Quarter to 2019 fourth Quarter given in Table 4.12 with their error and absolute errors.

Table 4.12: Actual versus predicted values of ANN model

Quarter	Actual Values	Predicted Values	Error	AE
2015 Q1	131.4693	145.7187	-14.2494	14.2494
2015 Q2	117.5173	143.4217	-25.9044	25.9044
2015 Q3	110.8445	142.7421	-31.8976	31.8976
2015 Q4	119.0209	144.3637	-25.3428	25.3428
2016 Q1	139.2967	147.446	-8.1493	8.1493
2016 Q2	115.7595	143.4283	-27.6688	27.6688
2016 Q3	112.933	143.4961	-30.5631	30.5631
2016 Q4	125.6264	145.1881	-19.5617	19.5617
2017 Q1	149.2554	148.7093	0.5461	0.5461
2017 Q2	124.4719	144.731	-20.2591	20.2591
2017 Q3	124.2116	144.9795	-20.7679	20.7679
2017 Q4	129.8739	145.5977	-15.7238	15.7238
2018 Q1	156.1273	149.4019	6.7254	6.7254
2018 Q2	129.1818	145.8139	-16.6321	16.6321
2018 Q3	123.3141	146.0897	-22.7756	22.7756
2018 Q4	126.5968	146.5605	-19.9637	19.9637
2019 Q1	150.7379	150.1541	0.5838	0.5838
2019 Q2	126.4733	146.4189	-19.9456	19.9456
2019 Q3	115.9913	144.7413	-28.75	28.75
2019 Q4	125.7455	145.6501	-19.9046	19.9046

Table 4.12 keeps the actual and predicted values with their errors. These values show that ANN predicted a little bit good on the basis of errors. The graphical visualizations of the actual and predicted values are provided in Figure 4.8 to examine the gap between predicted and actual values.

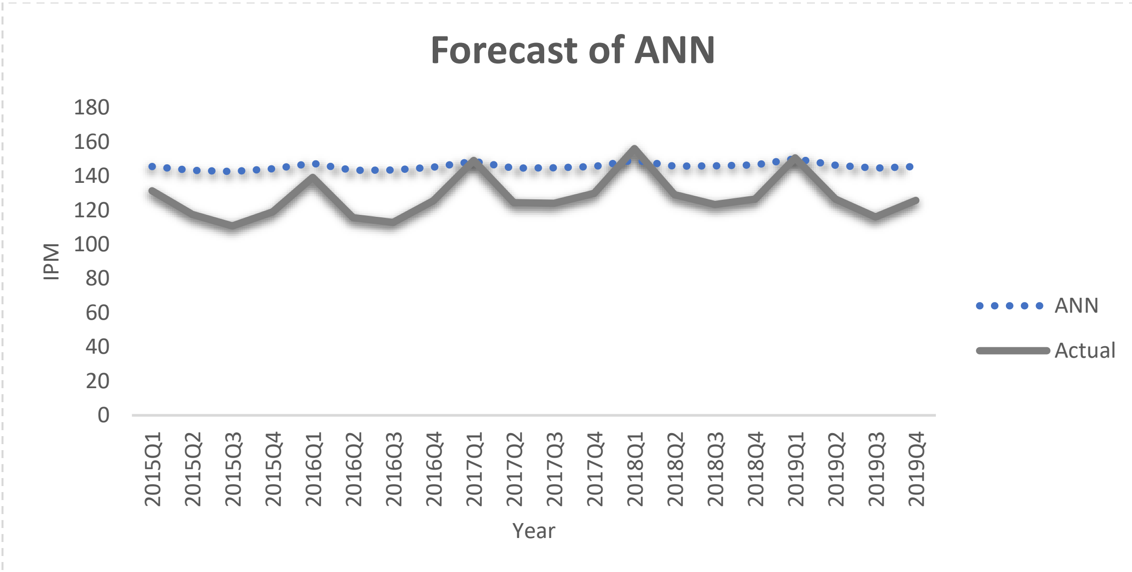

Figure 4.8: Graph for Actual versus Predicted of ANN model

Figure 4.8 provides the solid line for actual values and dashed line for predicted values. In can be easily seen from the figure that ANN not capturing the direction on economic growth and give average performance in prediction of economic growth.

4.9 Analysis through SVR Model

The use of SVR model needs to be use appropriate kernel function. The kernel function converts the nonlinear data into linear data. In our study, the implementation of SVR tested with different kernel functions ("linear", "radial basis function (RBF)", "polynomial" and "sigmoid"). By implementing all these kernel function, it is noticed that the best kernel function for economic growth prediction is linear which gives lowest values of errors. In the next step of prediction which is done by choosing best suitable model with linear kernel function. The predicted values from 2015 first Quarter to 2019 fourth Quarter given in Table 4.13 with their error and absolute errors.

Table 4.13: Actual versus predicted values of SVR model

Quarter	Actual Values	Predicted Values	Error	AE
2015 Q1	131.4693	120.5006	10.9687	10.9687
2015 Q2	117.5173	116.2047	1.3126	1.3126
2015 Q3	110.8445	114.1289	-3.2844	3.2844
2015 Q4	119.0209	117.5477	1.4732	1.4732
2016 Q1	139.2967	124.2485	15.0482	15.0482
2016 Q2	115.7595	115.8223	-0.0628	0.0628
2016 Q3	112.933	114.8417	-1.9087	1.9087
2016 Q4	125.6264	120.2833	5.3431	5.3431
2017 Q1	149.2554	129.0774	20.178	20.178
2017 Q2	124.4719	120.6396	3.8323	3.8323
2017 Q3	124.2116	121.1505	3.0611	3.0611
2017 Q4	129.8739	123.3906	6.4833	6.4833
2018 Q1	156.1273	132.8426	23.2847	23.2847
2018 Q2	129.1818	123.4512	5.7306	5.7306
2018 Q3	123.3141	120.6257	2.6884	2.6884
2018 Q4	126.5968	121.1253	5.4715	5.4715
2019 Q1	150.7379	128.2494	22.4885	22.4885
2019 Q2	126.4733	118.6603	7.813	7.813
2019 Q3	115.9913	112.305	3.6863	3.6863
2019 Q4	125.7455	114.0882	11.6573	11.6573

Table 4.13 keeps the actual and predicted values with their errors. These values show that SVR predicted good on the basis of errors. The graphical visualizations of the actual and predicted values are provided in Figure 4.9 to examine the gap between predicted and actual values.

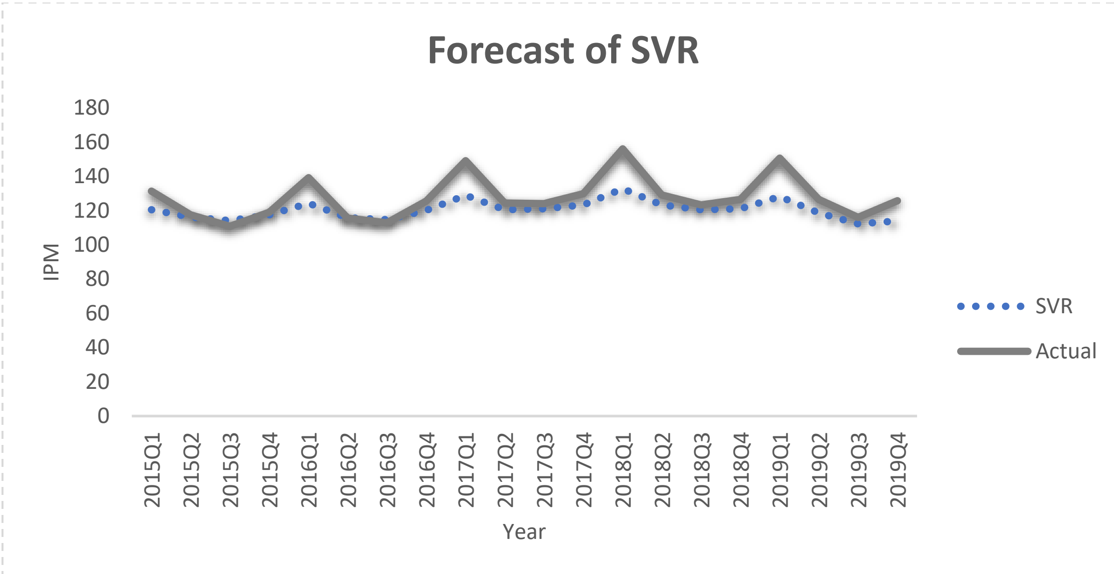

Figure 4.9: Graph for Actual versus Predicted of SVR model

Figure 4.8 provides the solid line for actual values and dashed line for predicted values. In can be easily seen from the figure that ANN capturing the direction on economic growth and gives good performance in prediction of economic growth.

4.10 Forecasting Performance

This study mainly compares the forecast performance of machine learning models and mostly used traditional time series models for the Pakistan economic growth forecasting. To forecasts the economic growth of Pakistan, we applied one multivariate model (ARDL) and two univariate models (AR and RW) and two machine learning models (SVR and ANN) for quarterly data spanning (training data) first quarter 1981 to fourth quarter 2014, and for examining the forecasting performance (testing data) from first quarter 2015 to fourth quarter 2019. We are applying five different methods (RW, AR, ARDL, SVR and ANN) to forecasting economic growth for Pakistan.

In this study, we determine the forecasting performance by mean absolute error (MAE), root mean square error (RMSE) and mean absolute percentage error (MAPE). The lowest value or these errors indicates the most accurate forecasting results. Table 4.5 presents the results for all models.

Table: 4.14: Forecasting Errors

Model	MAE	RMSE	MAPE
AR	14.5245	12.3863	10.6091
RW	14.0648	18.2813	10.3007
ARDL	**2.1806**	**2.7437**	**1.6766**
ANN	5.9474	6.4546	4.0920
SVR	7.7888	10.4769	5.6956

Note: Bold values indicate the lowest error value which is the most precise forecast results

Table 4.14 shows that the MAE value 2.18 for ARDL model is lower which indicating that ARDL performing more accurately comparing to other machine learning and time series models, and the RMSE value 2.74 for ARDL model is lower which also show that ARDL model perform accurately and the MAPE value 1.67 for ARDL model is lower which also show that ARDL model perform accurately than other machine learning and traditional time series models. The overall scenario shows that the performance of forecasting for economic growth by using ARDL model is more accurate than any other time series and machine learning models. On the other hand, we can also say that multivariate models perform more accurately for economic growth forecasting than univariate models (AR and RW). The results also show that the RW forecasting performance is worst for economic growth. Hence, the results show the best model for economic growth forecasting is ARDL.

We are also plot the predicted values combinedly for evaluating the forecasting performance of time series and machine learning models. This paper evaluating the performance of different machine learning and time series models with the aim to get best model of forecasting for economic growth. Figure 4.10 provides all the models graphical visualization.

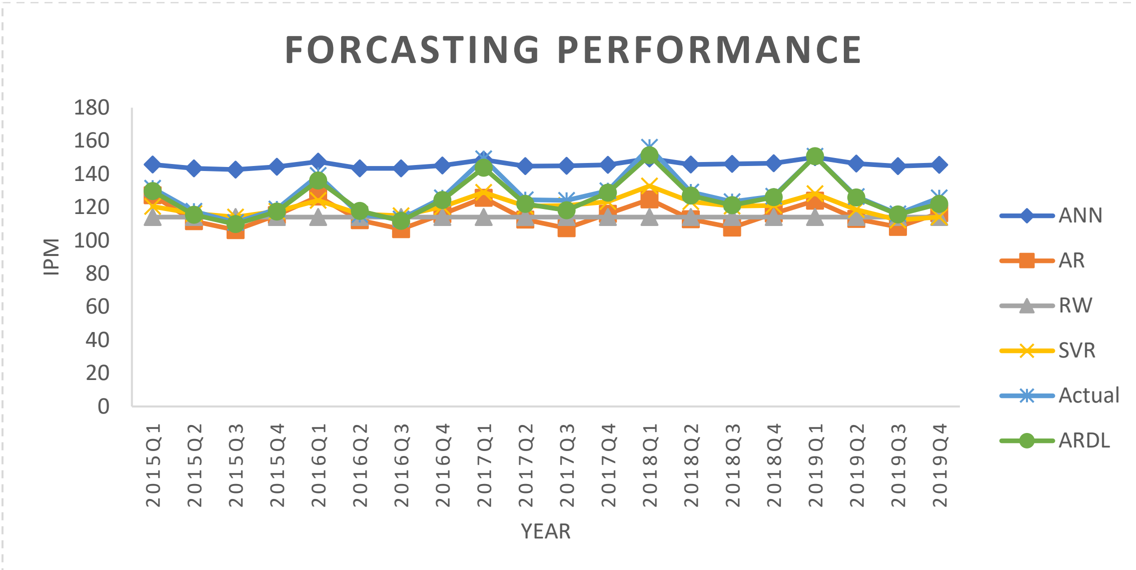

Figure 4.10: Forecasting performance of models

Fig. 4.10 shows the results of forecasting of economic growth. We can easily see from the above graphical visualization that ARDL model perform more accurately in the prediction of economic growth than all other time series and machine learning models as well as univariate models (AR and RW).

Consequently, the comparison of different time series and machine learning models as well as multivariate and univariate models concluded that the forecasting performance of the multivariate models are more accurate than the univariate models. Moreover, Time series model ARDL prevail against machine learning and other time series models in economic growth forecasting. The ARDL model outperformed for economic growth forecasting than all other time series (AR and RW) models and machine learning (ANN and SVR) models.

All these findings of comparison of different models based on the values of root mean square error (RMSE), mean absolute error (MAE) and mean absolute percentage error (MAPE). The lowest values of these errors show the best forecasting model of the study. Furthermore, these errors show that random walk perform worst than all other model in economic growth forecasting.

CHAPTER 5

CONCLUSION AND RECOMMENTATIONS

Conclusion

The study based on a comparison of machine learning and time series models and as well as multivariate and univariate time series models to forecast economic growth for Pakistan. Time series models are autoregressive distributed lag model (ARDL), autoregressive (AR) model and random walk (RW) model. Machine learning models are support vector regression (SVR) and ANN. The data set for this study consists of Quarterly data for the period of 1981 to 2019. The data spilt into two parts training data and testing data for performance evaluating purpose. Training data set consists of the period from first Quarter 1981 to fourth Quarter 2014 and testing data consists of the period from first Quarter 2015 to fourth Quarter 2019. The industrial production manufacturing index used proxy variable of economic growth as output indicator and input indicators are export of goods and services and import of goods and services, trade exchange rate, trade openness, FDI, remittances inflows, unemployment, gross fixed capital formation and inflation.

In this study, we applied five different machine learning and time series models and as well as univariate and multivariate models. The focus of this study is to compare the forecasting performance of these models for economic growth. Time series models are AR and RW model and autoregressive distributed lag model (ARDL). Machine learning models are artificial neural network (ANN) and SVR. The forecasting performance checked by using root mean square error (RMSE), mean absolute error (MAE), and mean absolute percentage error (MAPE). The lowest value of these errors shows the best forecasting performance of the model.

The results of this study indicate that the MAE and RMSE has lower error 2.06 and 2.588 respectively for ARDL model which indicating that ARDL performing more accurately comparing to other machine learning and time series models, and MAPE value 0.06 for ANN model is lower which show that ANN perform better than other machine learning and traditional time series models. The overall result describes that the forecasting performance of ARDL model is more accurate than other time series and machine learning models. On the other hand, we also find that multivariate models perform more accurately than univariate models (AR and RW). The results also show that the RW forecasting performance is worst for economic growth. Hence, the results show the best model for economic growth forecasting is ARDL.

Recommendations

The findings of the study show that industrial production, manufacturing index (IPM) has positive impact on export of goods and services, import of goods and services, trade in services and inflation. But IPM has a negative impact on trade in services. On the basis of results our policy recommendations are as:

Industrial production, manufacturing index has a big role for the measurement of the economy. High economic growth for a country can be achieved by increasing the production of industries. Government need to make a focus on industrial production, manufacturing index (IPM), because in the 21st century industries are the backbone for achieving high economic growth. IPM is the key for increasing the exports of a country.

Policy makers and Government needs to make polices helpful for industries to sustained economic growth. Higher the IPM will higher the exports of final goods for the country as well imports of raw materials. High economic growth may also increase the inflation in a country. So, the government has needed to keep inflation in one digit in Pakistan. At the end, if anyone wants to predict economic growth by using industrial production, manufacturing index than ARDL is best model for this. Researchers can use ARDL, multivariate model for forecasting purposes, because it provides more accurate results.

There are several gaps in our knowledge around public involvement in research that follow from our findings and would benefit from further research. This study can be expended by using time series data and divided it into four different horizons (3 Months, 6 Months, 9 Months and 12 Months) on which different time series and machine learning techniques compared. Moreover, many other time series models like ARIMAX, VAR, FMOLS and machine learning models like K-nearest neighbor (KNN), SVM, Naïve byes compared in further studies.

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