Performance of Non-Parametric Regression Estimators in Presence of Skewed Distribution: An Application to Determinants of Poverty in selected Districts of Punjab

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CERTIFICATE

This is to certify that this thesis entitled: **"Performance of Non-Parametric Regression Estimators in Presence of Skewed Distribution: An application to Determinants of poverty in selected districts of Punjab"** submitted by Mr. Usama Sattar is accepted in its present form by the Department of Econometrics and Statistics, Pakistan Institute of Development Economics (PIDE), Islamabad as satisfying the requirements for partial fulfillment of the degree in **Master of Philosophy in Econometrics**.

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TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

ABSTRACT

Classical linear regression model has very nice statistical properties subject to validity of certain assumptions. However, in real life these assumptions often fail to hold, and the OLS does not possess its nice properties. Sometimes, the OLS gives very misleading results when the assumptions do not hold. The Non-Parametric methods are robust to such assumptions.

However, there are lots of Non-Parametric methods that can be applied to real data that does not exhibit the classical assumptions, and one has to choose between these estimators. Unfortunately, existing literature does not provide clear guidance on how to choose between these estimators. This study compares five non-parametric regression methods on the basis of their performance in real data. For the real data, the underlying data generating process is not known, Therefore, the size and power cannot be utilized. We use the forecast performance as a measure of performance of estimator. we have taken data of determinants of poverty from PSLM (Pakistan Social and Living Standard Measurement) for ten districts of Punjab. These kinds of data usually violate the standard OLS assumptions and such type of data need to treat using non-parametric Regression methods. Forecast Mean Square Error (FMSE) and Residual Sum of Square (RSS) are computed to check the performance of non-parametric regression estimators. We analyze Non-Parametric methods separately for highly and moderately skewed data. In presence of highly skewed data, we observe Theil-Sen and Least absolute deviation estimators perform better for highly skewed data and Quantile regression, M-estimator and least trimmed square estimator perform poorly for this kind of data. On the other hand, the M-estimator and least trimmed square estimator are very better non-parametric estimators for Moderately skewed data. While the Theil-Sen and LAD

estimator shows very poor performance in Moderately skewed distribution. We can also say that the Quantile Regression is not bad for this type of analysis.

CHAPTER 1

INTRODUCTION

Nonparametric regression is considered as an important data analysis tool. Nonparametric methods are used when some assumptions regarding classical regression analysis are untenable or when sample size is very small. Nonparametric regression analysis gives more efficient results as compared to parametric regression methods in case where data do not follow standard distributional pattern [Ohlson and Kim (2015)]. Nonparametric regression methods have a variety of estimators i.e. Quantile regression, least absolute deviation, M-estimator, Theil-Sen estimator, Trimmed Least Square estimator, Kernel estimator, Additive spline, Neural network, local linear kernel, random forests estimates, Nearest neighbor, regression trees, Penalized smoothing splines etc.

In case, if data in not following normality, it is recommended to use nonparametric methods. However, there are many nonparametric methods and there is no clarity on how to choose between these Nonparametric methods. There is very little known about relative merits of these methods and the question that how to choose between these estimators is still un-answered. The objective of the study is to find relative performance of nonparametric regression methods by assessing their performance on real data. Whereas, in Monte Carlo simulation, the experiments condition on some implicit specification and the design of data generating process supports the implicit assumptions. But for the real data series, implicit assumptions/arbitrary specification decisions are often unjustifiable and sometimes incompatible with data [Rehman (2011)]. We want to do comparison of five non-parametric regression estimators to facilitate the researcher to point out an efficient and best performance estimator in presence of skewed distribution.

The performance of Non-Parametric methods is tested for the case of determinants of poverty. The determinants of poverty data are used in estimated models that usually possess skewed distribution and therefore violate Standard model assumption. Therefore, the performance of Non-Parametric methods for this kind of data can help us in selection of appropriate estimation method.

There is another problem to testing the performance of Non-Parametric estimators for real data. For real data, the true data generating process is never known. Therefore, one cannot find the efficiency or unbiasedness of an estimator to judge performance on real data. However, the Forecast performance can be used to judge the relative performance of the Non-Parametric methods.

The comparison among Non-parametric regression estimators is made on the basis of forecast performance of these estimators on different models estimated on real data and the estimator with low forecast means square error shall be consider more efficient than all other estimators.

1.1 Objective of the Study

The objective of this study is to evaluate the performance of following Non-parametric methods: Quantile regression, Least absolute deviation estimator, Theil-sen estimator, M-estimator and Least trimmed square estimator for estimating and predicting poverty models.

1.2 Significance of Study

Nonparametric methods are routinely used as alternative of OLS type methods however there is now abundance of nonparametric methods with very little clarity about relative merits and demerits of these nonparametric methods. This study will provide a guide to choose between these methods and therefore will be helpful to the all researchers who intend to use nonparametric methods.

1.3 Outline of Thesis

The rest of this thesis is organized as follows,

Second chapter contains the review of the literature then third chapter would discuss the methodology and procedure that use to estimate the estimators. Moreover, fourth chapter address data description, results and analysis of the study. At last in chapter five we would describe conclusion and recommendations for the study.

CHAPTER 2

LITERATURE REVIEW

Non-Parametric regression methods are developed to besiege some restriction of classical parametric methods. Ordinary Least square methods have a vital role in estimation process if properties of OLS method fulfilled. Unfortunately, those properties are not met in many real-life cases. Non-Parametric methods emerged as solution to this problem. Parametric methods like OLS/GLS have explicit elegant formula and therefore are very easy to compute. On the other hand, Non-Parametric methods usually have cumbersome formula which are to be solved by numerical methods. Because of this reason, the development of Non-Parametric methods has been relatively slow. But with the advancement of computational technology, the Non-Parametric methods are now easily implementable. This has led to new wave of interest in the Non-Parametric methods. Non-parametric regression has attained more concentration since 1960s and is an active area of interest till today. Studies like Nadia and Mohammad (2013) and Kan-Kilinc and Alpu (2015) had evaluated the performance of Non-parametric regression estimators using Monte Carlo simulations. In current study, we want to assess the performance of five Non-parametric regression estimators in presence of real data taken from PSLM (Pakistan social and living standard measure).

The Literature Review is arranged as follows:

The first section of literature review describes the development of non-parametric methods, second section tells the comparison between parametric and non-parametric methods and last section discusses the comparison between non-parametric methods.

2.1 Development of Non-Parametric Regression Estimators

This section covers all relevant literature about selected non-parametric methods. These non-parametric methods include Least Absolute Deviation method, Quantile regression, Theil-Sen estimator, M-estimator and Least Trimmed Square estimator.

2.1.1 Least Absolute Deviation Method

Least absolute deviation method is a substitute of least square method used for the estimation of regression parameters in linear regression line. It minimizes the sum of absolute errors rather than minimizing the sum of square of residuals. The method of least absolute deviation is more robust than Least square method in presence of skewed data. Least absolute deviation method is also known as L_1 estimation method. The estimator takes the following form.

$$\sum_{i=1}^{n} |\varepsilon| \qquad \text{OR} \qquad \text{LAD} = \min_{\beta} \sum_{i=1}^{n} |\text{Yi} - \beta\text{Xi}| \qquad (2.1)$$

Bassett and Koenker (1978) have developed the asymptotic theory and large sample properties of least absolute deviation (LAD) method. They concluded that with specified mean and variance the sampling distribution of LAD estimator will be asymptotically normal. Armstrong, *et al.* (1980) have developed the linear programming with efficient solution of L_1 (least absolute deviation) and L_∞ (Chebychev estimation) using computer codes. Armstrong and Kung (1981) have investigated the algorithms of least absolute deviation as an alternate of least square to solve the problem of best subset of regressors.

Xiuqing and Jinde (2005) have investigated the asymptotic properties of LAD estimator i.e. consistency and normality for nonlinear regression models with randomly censored data. Simulations study concludes that in presence of censored data the LAD estimator is more robust than LS estimator. Ciuperca (2011) has analyzed the asymptotic properties of LAD method in non-linear parametric model using Monte Carlo simulation experiment and concluded that LAD estimator is more efficient than LS estimator in presence of outliers.

Feng *et al*. (2012) have used L_1 method and local linear technique for approximate functional coefficient in partially linear regression model. The validity of procedure was checked through simulations. Ogundele *et al*. (2016) have proposed least absolute deviation estimator in linear regression model as is more factual than existing method. His method is similar to the method given by Birkes and Dodge (1993).

2.1.2 Quantile Regression

Quantile regression approach is proposed by Koenker and Bassett (1978). They have suggested that Quantile regression is an efficient approach for analyzing how covariates have impact on the scale, location and shape of a response distribution, and elaborated quantile regression as an enhancement of least square estimation method of conditional mean models to conditional median functions. (Bassett and Koenker; 1986 and Koenker and Bassett; 1982) have utilized Quantile regression technique as proposed method which does not rely on parametric assumptions about the shape of error distribution. They have noted strong consistency of the quantile regression. They also advocated some robust methods which emphasis on analyzing of conditional central tendency and suggested that regularity conditions on error distribution are not necessary for estimating the conditional distributions of response variable. Quantile regression also has the ability to handle heterogeneous effects. In presence of censoring, Powell (1986) has extended Quantile regression model for censored data. When the observations on the dependent variable are censored then this model consistently estimates the conditional quantile. He has also discussed how various quantile estimators enhance efficiency when residuals are $i.i.d$, and investigated how to find difference of coefficients using test of homoskedasticity. The name of median regression as robust regression in skewed distribution was given by Hallock and Koenker (2001).

Karlsson (2007) has reviewed the study of Koenkar and Basset (1978) and has used quantile regression estimator on nonlinear longitudinal data by utilizing logistic growth model when errors follow AR (1) model. Comparison between Quantile regression and least square regression estimator was also made and noted that how Quantile regression gives more accurate results than mean regression (OLS). Jalali and Babanezhad (2011) have examined the Quantile regression and its efficiency by approximating the effect of age on satisfaction score. They have concluded that when the distribution of explanatory variable is highly skewed and have outliers then OLS method is not an appropriate and Quantile regression is a good choice.

2.1.3 Theil-Sen Estimator

The concept of Theil-Sen estimator is given by Henri (1950) and Sen (1968). The Theil-Sen slope was first studied by H.Theil and extended by P.K.Sen so this estimator became Theil-Sen estimator. This method is more robust then least square method in the presence of non-normal and heteroscedastic data. For estimating the linear trend, it has become the most popular nonparametric technique.

Peng et al. (2008) analyzed asymptotic distribution of the Theil-Sen estimator in linear regression model with random distributions and found that when error distribution is discontinuous then Theil-Sen estimator is super-efficient, on the other hand if distribution is continuous then asymptotic distribution of Theil-Sen estimator may or may not be normal. These results were conclude based on small simulation study.

2.1.4 M-Estimator

Huber (1973) has proposed M-estimator studied its asymptotic properties. The M-estimator is the simplification of Maximum Likelihood estimator (MLE). The aim of M-estimation is to minimize increasing function of errors and it is robust in presence of outliers, however, the break down point (BP) of this estimator is 1/ or 0%. Huber (1981) has described the properties of LS method like asymptotic normality and consistency and discerns that least square estimator will not perform better in presence of outliers in data. Huber extrapolates that only single outliers can have large effect on estimator performance and when errors are heavy tailed then OLS is not more efficient. Due to lack of robustness of least square estimator, Huber has identified the function ρ which minimizes the sum of less rapidly increasing function of residuals rather than minimized the sum of square of residuals.

$$\sum_{i=1}^{n} \rho\left(\frac{Yi-\beta o-\beta 1Xi}{s}\right) \tag{2.2}$$

Equation 2.2 identifies the function Huber has suggested and hence the resulting estimator is M-estimator (Huber 1973; and Huber 1981). He and Wang (1995) have investigated the algorithm for M estimator which covers both robust M-estimator and S estimator. Muthukrishnan and Myilsamy (2010) have evaluated the performance of M-estimator and OLS in regression model using simulation study in R software. M-estimator results were same as the results of least square in presence of normal data and when there are outliers in data, the least square principal is not able to give accurate results while the M-estimator is not influenced by outliers.

La Vecchia (2015) has investigated asymptotic technique for M-estimator and their Constancy in presence of outliers. He has suggested that estimation term Ψ and its derivative term has important role in estimation process. For this purpose, he has used different techniques which conclude that the term Ψ and its derivative's term remain stable in presence of extreme values.

2.1.5 Least Trimmed Square (LTS)

Least trimmed square (LTS) is proposed by Rousseeuw and Yohai (1984) as an alternative to the clasical least square estimator (OLS). LTS estimator has high breakdown point i.e. 50%. Least trimmed square (LTS) is a robust statistical technique that minimizes k subset out of n (total no of samples) sum of squares of residuals and is defined as:

$$\text{Min} \sum_{i=1}^{k} r^2_{(i)} \tag{2.3}$$

Where $r^2_{(i)}$ is arranged in ascending order, showing the i^{th} ordered square of errors i.e. $r^2_{(1)} \leq r^2_{(2)} \leq r^2_{(3)} \ldots \leq r^2_{(n)}$ and k = ((n/2)+1). At k=n, this estimator results same as ordinary least square (OLS) that has 0% breakdown point. The main difference between least square regression and Least trimmed square is that in LTS estimator the largest squared error is not used while remaining (n-k) values having not effect on estimator performance are used. Leroy and Rousseeuw (1987) have said that when k is around n/2 then best robustness features will be attained. They also investigate the performance of LTS versus OLS on real life data sets and conclude that the LTS line is good fit as compare to OLS line. Čížek and Víšek (2000) have also given their opinion on same thing as they said that only single value can badly effect on OLS performance. So, they take artificial data set with ten values in which only single value is an outlier.

Hössjer (1995) has demonstrated an algorithm for evaluating the Least Trimmed squares estimator in simple regression model. After Hössjer, six different algorithms for LTS estimator were proposed. Firstly, by Rousseeuw and Driessen (1999), then by Zaman *et al*. (2001), Agulló (2001), Bai (2003), Rousseeuw and Driessen (2006) and Satman (2012). Agulló (2001) has suggested two algorithms for estimation of LTS. These algorithms were applied on simulated and real data and conclude that these algorithms are very fast. Whereas Zaman *et al.* (2001) proposed a method based on Rousseeuw and Zomeren (1990). On different economics, models they utilized were based on high breakdown robust regression and concluded that by eliminating some outliers having large impact on regression would result better. Bai (2003) has suggested an algorithm of LTS estimator which can be evaluated as a function of the residuals. Further for increased sample size this algorithm converges to real LTS results. In 2006 Rousseeuw and Van Driessen introduced a new algorithm labeled as FAST LTS. For larger sample size FAST LTS algorithm provides more efficient and fast results as compared to existing algorithm of LTS. Satman (2012) has proposed a new and amended algorithm of Rousseeuw and Driessen (2006) for computation of LTS estimators in large sample size. R package is used for simulations and its results shows smaller Mean square errors, biases and variance of LTS estimator significantly so algorithm perform better for very large data sets.

Giloni and Padberg (2002) has evaluated the performance of two estimators with high BP (breakdown point) such as LTS and least median square (LMS) on the perspective of optimization. They have derived the properties of objective function for design exact solution of LTS algorithm. Cizek (2004) has derived the significant asymptotic properties of Least trimmed square estimator (LTS) involving normality, variance and β mixing condition on independent variable.

Willems and Aelst (2005) have proposed an alternative bootstrap method for LTS estimator which is very simple and robust. Simulations results demonstrate that this method performs better than classical bootstrap method.

2.3 Comparison Between Parametric and Non-Parametric Regression Estimator

There are several studies that compare the Parametric methods with Non-Parametric methods. Lawrence and shier (1981) made comparison between least square and least absolute deviation and concluded that LAD is better than least square estimation method. Dietz (1987) has compared mean square errors of several estimators of slope, intercept and mean response in simple linear regression and concluded that the mean square error of least square regression method is smaller than the mean square error of other competitors in presence of normal errors. When errors are non-normal then the mean square error of least square regression is larger than the mean square error of other slope estimators and intercepts. Dietz (1989) has analyzed different estimators of slope, intercept and mean response in simple linear regression on the basis of bias, efficiency and mean square errors and concluded that the intercept estimators based on Theil-Sen estimator and Theil-Sen slope estimator are most robust, efficient and easy to calculate than least square estimator and spirited in term of mean square error with different slope estimators. The median of residual based on Theil-Sen estimator is better when errors form heavy tailed distribution.

Lind *et al.* (1992) have analyzed the performance of some estimators like least square, least absolute deviation and M-estimator when error term follows skewed distribution. The LAD estimator perform better when percentage of observations in one tail is not more and gives good basic point for the M- estimator. McDonald and White (1993) have compared LS method, LAD method, partially adaptive estimators and some other robust methods explored by Huber. Sample size 50 was used with disturbance term following standard normal, lognormal, bimodal mixture of normal and contaminated normal. When errors were non-normal, the adapted procedure was better than all other procedures. It is concluded that in some cases these can better perform then least square due to 50-80% diminution in standard errors.

Min and Kim (2004) made a comparison between parametric OLS and nonparametric regression based on Quantile method via Monte Carlo simulations and concluded that Quantile regression is more robust when model is nonlinear and errors are not normal. Sangun *et al.* (2006) made comparison between OLS and LAD method and concluded that LAD estimation gives more accurate results than OLS method in presence of outliers in distribution. The value of coefficient of determination and significant test was used to make comparison. Dielman (2009) has made a comparison between LS regression and LAD method using Monte Carlo simulation when errors follow asymmetric distribution and concluded that least absolute deviation is more efficient than least square method. Quantile regression is more efficient than OLS and very useful tool for analyzing non-normal distribution (Bancayrin-Baguio *et al.*; 2009, Jalali and Babanezhad; 2011).

Alma (2011) has declared the deportment of extreme values in linear regression and made a comparison of some robust estimators i.e. S-estimator, M-estimator, MM estimator and Least trimmed square estimator with OLS via simulations. They concluded that S and M-estimators are more efficient in presence of outliers than LTS and MM estimators. The OLS perform poor in this kind of data.

Wilcox (2012) has suggested that efficiency of OLS estimator is very poor as compared to other estimators when error term follows heteroscedastic distributions. The standard errors of ordinary least square are very small when errors follow normal and homoscedastic distribution. Thanoon (2015) made a comparison between least square method and least absolute deviation method. They concluded that the least absolute deviation is more efficient than least square method in approximation of coefficients of regression in different cases when errors follow normal and abnormal distributions.

Ohlson and Kim (2015) discussed that OLS faces two main problems first is presence of outliers and second is heteroscedasticity. Theil-Sen estimator can easily face these types of problems. They also made a comparison between Theil-Sen and ordinary least square estimator. To evaluate the performance of these two estimators they focused on two methods first stability of coefficient and value of coefficient of determination, results show that Theil-Sen estimator perform better than OLS estimator.

2.4 Comparison Between Non-Parametric Methods

There are some studies that compare the Non-Parametric regression methods. Mutan (2004) investigated some robust and nonparametric regression techniques for simple linear regression model when disturbance term was generated from generalized logistic distribution and made comparison of nonparametric regression estimators i.e. modified maximum likelihood (MML), Least trimmed squares (LTS), Winsorized least square, Least absolute deviation (LAD), Theil and weighted Theil through simulations study. The performance of these estimators was calculated using variance, mean, bias, mean square error (MSE) and relative mean square error (RMSE) and concluded that Weighted Theil method and Winsorized least squares showed best results because these MSE values was decreases from 1%-20% and 1-14% respectively. As sample size increases, Theil-Sen estimator became more efficient. The value of RMSE of LAD and LTS was negative. Mount *et al*. (2014) have noted that LTS estimator is the associate with Least median square estimator (LMS) which minimizes the median of square residuals and Least trimmed absolute which minimizes the sum of small percent of absolute errors. LTS estimator is more efficient than LMS estimator. They proposed a new algorithm, approximate and exact for analyzing least trimmed estimator and demonstrated the results for both algorithm exact and approximate LTA and LTS estimator.

Nadia and Mohammad (2013) have assessed the performance of the LAD method, M-estimator, LTS estimator and Theil-Sen estimator with Ordinary least square estimator when there are outliers in data distribution. The capability of estimators was analyzed based on the value of their mean square error and noted that the LAD method gives efficient results than all other nonparametric estimators. In presence of outliers the least square regression estimators showed poor performance than all other competitors.

Kan-Kilinç and Alpu (2015) have evaluated the performance of robust biased estimator in presence of two problems such as multicollinearity and outliers in (x, and in x-y direction) via R-Package. They used the algorithm of LTS estimator proposed by Rousseeuw and Driessen (2006). Khan et al. (2016) evaluated the performance, in simple and multiple regressions, of some estimators i.e. LTS estimator, LTA estimator and M estimator using simulations. Simulations were made according to different scenario in the presence of outliers for evaluating the performance of each method. All methods perform better according to their capacity of break down point. They concluded that when h=n (n is the sample size), LTS perform better than LTA estimator for standardize errors and LTA is better than LTS in existence of Laplace error.

2.5 Gap in Literature

There exist comparisons of Non-Parametric methods in the literature e.g. Nadia and Muhammad (2013) and Kan-Kilinç and Alpu (2015). However, the comparisons are based on Monte-Carlo simulation. In the Monte-Carlo simulations, the data is generated with specific well-known properties. The real-life data may not be following same conditions [A Rehman (2011)]. The performance of estimators on real data allows us to the estimators beyond set of pre-decided conditions. Therefore, in this study we made a comparison of non-parametric regression methods. The comparison is based on their forecast performance on real data. Forecast mean square error (FMSE) and Residual sum of square (RSS) are computed for this purpose. Whereas, 80% data are used for estimation and remaining 20% for forecasting.

CHAPTER 3

METHODOLOGY

This chapter explains the procedure of comparing non-parametric regression estimators. In chapter 2 we have discussed the review of literature about nonparametric regression estimator, this chapter we discussed the methodology those methods and explain the models.

3.1 Procedure

The aim of this study is to compare the performance of Non-Parametric methods for the real data. For this purpose, we apply the Non-Parametric regression methods to estimate models for determinants of poverty. The estimation of poverty models involves data on determinants of poverty, which usually violates standard OLS assumptions and such data sets need to treated using Non-Parametric methods. There is a large variety of models for determinant of poverty and we have chosen number of models for the underling phenomena. The variety of models allows us to compare the Non-Parametric methods under the condition of correctly specified models and poorly specified models.

The large data set of PSLM (Pakistan Social and Living Standard Measurement) had utilized for that purpose and 80% observations will be used for estimation while remaining 20% shall be used for forecasting. We would take data sets of data of Determinants of Poverty for Ten Districts of Punjab.

The algorithm for the comparing is described in figure 3.1.

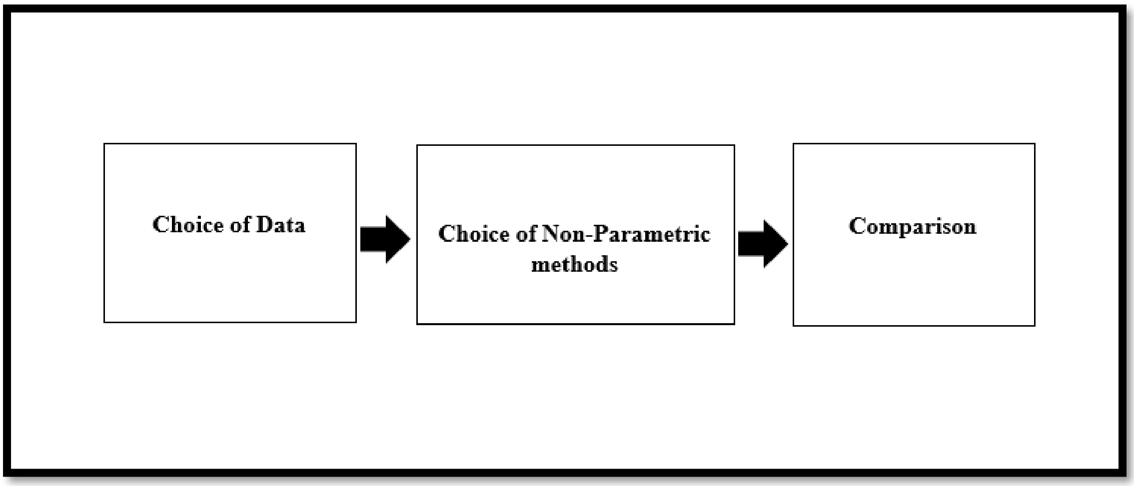

Figure. 3.1: Comparison Procedure

3.1.1 Choice of Data

We have data on the determinants of poverty for 10 districts of Punjab. For each district we have large number of observations obtained from the PSLM. For every district, 80% of all observations are used for estimating the Models. The estimators are used to forecast poverty for remaining 20% of the data. The observations are selected randomly for the forecast group and estimation group.

3.1.2 Choice of Non-Parametric Methods

We have five Non-Parametric methods which are mentioned in section 3.2. There are a lot of non-parametric methods from those we have selected five Non-parametric methods. These methods are declared best from earlier simulations studies. For instance, the studies by Sangun et al (2006) and Thanoon (2015) declare that LAD estimator is efficient than OLS method. Whereas, the study by Muthukrishnan and Myilsamy (2010) proposed that M-estimator is better choice as compare to OLS method in presence of Outliers. Min and Kim (2004) and Jalali and Babanezhad (2011) suggested that Quantile regression is efficient than OLS in presence of outliers. Ohlson and Kim (2015) recommend that Theil-Sen estimator efficient non-parametric regression estimator as compare to OLS in presence of outliers and heteroscedasticity. Čížek and Víšek (2000) demonstrate that LTS estimator in more efficient than OLS method in existence of outliers in data. In current study we have evaluated their forecast performance on real data.

3.1.3 Comparison

As discussed in section 3.1.1 we have data on determinants of poverty for ten districts of Punjab, 80% of data used for estimation and remaining 20% used for forecasting. Residual sum of square (RSS) and FMSE (Forecast Mean Square Error) are used for this purpose. An estimator with lower values of FMSE (Forecast Mean Square Error) and RSS (Residual Sum of Square) consider to be best the estimator. We have estimated Non-Parametric methods by using these different software, i.e. Eviews, MATLAB and R-Software.

3.2 The Methods to be Compared

We are comparing five nonparametric methods whose detail is as follows

3.2.1 Least Absolute Deviation Method

Least absolute deviation minimizes sum of absolute residuals

$$\sum_{i=1}^{n} |\varepsilon| \qquad \text{OR} \qquad \text{LAD} = \min \sum_{i=1}^{n} |Yi - \beta Xi| \tag{3.1}$$

We used the algorithm proposed by Birkes and Dodge (1993) in his book. They developed this algorithm for the simple linear regression model. This algorithm starts with one of the data points denoted by (x_0, y_0) tries to find best line passing through it. This point is given as below

$$\frac{Y_i - Y_O}{X_i - X_O} \tag{3.2}$$

Where data became $(Y_1 - Y_O)/(X_I - X_O) \leq (Y_2 - Y_O)/(X_2 - X_O) \leq \cdots \leq (Y_n - Y_O)/(X_n - X_O)$

$$T = \sum_{i=1}^{n} |X_i - X_O| \tag{3.3}$$

$$|X_1 - X_0| + \ldots\ldots\ldots\ldots + |X_{k-1} - X_0| < \tfrac{1}{2} T \tag{3.4}$$

$$|X_1 - X_0| + \ldots\ldots\ldots\ldots + |X_{k-1} - X_0| + |X_k - X_0| > \tfrac{1}{2} T \tag{3.5}$$

The slop of Least absolute deviation method is written as

$$B_1^{LAD} = \frac{Y_k - Y_o}{X_k - X_o} \tag{3.6}$$

and intercept constant of LAD is

$$B_0 = Y_0 - B_1^{LAD} X_0 \tag{3.7}$$

3.2.2 Theil-Sen Estimator

Theil–Sen estimator is a nonparametric technique is an alternative of least square method. The concept of Theil-Sen estimator is given by Henri and Sen in 1950 and 1968 respectively. The Theil-Sen slop was first studied by H.Theil and prolonged by P.K.Sen so this estimator will became Theil-Sen estimator. For computing Theil-Sen estimator all x (independent variable) are arranged in ascending order. Theil-Sen slop estimate is calculated by comparing each data pair to all other in a pair wise manner. This method is computed by this below formula.

$$(F_{ij}) = \frac{\Delta Y}{\Delta X} = \frac{Y_j - Y_i}{X_i - X_i} \quad ;\ x_i \neq x_j,\ 1 \leq i < j \leq n \tag{3.8}$$

Slop coefficient $\beta^{Theil} = (F_{ij})$ $\quad ;1 \leq i < j \leq n$ (3.9)

Intercept of Theil-Sen estimator

$$\beta_0 = \text{median}\ (Y_i) - \beta^{Theil}\ \text{median}\ (X_i) \tag{3.10}$$

3.2.3 Quantile Regression:

The quantile regression models the relationship between x (independent variable) and conditional quantile of y (dependent variable) rather than just the conditional mean of y. Quantile regression gives more comprehensive picture of effect of predictor variable on predictand variable. Quantile regression minimize

$$\sum_i q\,|\varepsilon i| + \sum_i (1-q)\,|\varepsilon i| \qquad (0<q<1) \tag{3.11}$$

q is stand for 1st Quantile (0.25)

Slope coefficient Formula:

$$Q(\beta_q) = \min \sum_q |y_i - \beta x_i| + \sum (1-q)\,|y_i - \beta x_i| \tag{3.12}$$

3.2.4 M-Estimator

The aim of M-estimation is to minimized increasing function of errors.

$$\sum_{i=1}^{n} \rho(\epsilon i/s) \tag{3.13}$$

Where *s* is the estimate of scale and can be evaluate by using this formula.

$$s = \frac{\text{median}|\, \epsilon i - median\,(\epsilon i)\,|}{0.6745} \tag{3.14}$$

And where

$$\sum_{i=1}^{n} \rho(\epsilon i/s) = \sum_{i=1}^{n} \rho\left[\frac{Yi - \beta o - \beta 1 Xi}{s}\right] \tag{3.15}$$

$$= \sum_{i=1}^{n} \rho\left[\frac{\epsilon i(\beta)}{s}\right] = \sum_{i=1}^{n} \rho(\mu) \tag{3.16}$$

And μ = is known as standardized errors.

Differentiating equation (4.4) with respect to β and making partial derivative to zero, so resulting equation we get can be written as

$$\sum_{i=1}^{n} \Psi[\frac{\epsilon \boldsymbol{i}(\boldsymbol{\beta})}{s}] X_i = \mathbf{0} \quad (3.17)$$

From above equation Ψ is the derivative of ρ.

For solving (4.5) equation define the weight function W(x) = if x≠ 0 and W(x) = Ψ (0)

And if X =0. Let W_i= W (μ_i) (3.18)

$$\sum W_i (Y_i-\beta_o-\beta_1X_i) = 0 \quad (3.19)$$

$$\sum W_i (Y_i-\beta_o-\beta_1X_i) X_i = 0 \quad (3.20)$$

3.2.5 Least Trimmed Estimator

Least trimmed square (LTS) is a robust statistical technique that minimize k subset out of n (total no of samples) square of residuals and is define as:

$$\text{Min} \sum_{i=1}^{k} r^2{}_{(i)}$$

Where $r^2{}_{(i)}$ are arrange in ascending order, showed the i^{th} order square of errors $r^2{}_{(1)} \leq r^2{}_{(2)} \leq r^2{}_{(3)} \ldots \leq r^2{}_{(n)}$ and where k = [(n/2)+1] and when k=n then this estimator's results same as ordinary least square (OLS) which has 0% breakdown point.

Residual Sum of Square:

$$\text{RSS} = \sum_{i=1}^{n}(Y_i - (\alpha + \beta X_i)^2)$$

RSS is stand for Residual Sum of Square. Whereas, Y_i is the i^{th} value of variable te be predicted, X_i is the i^{th} value of explanatory variables. While α is the estimated values of constant term a and β is the estimated value of slope coefficient b.

Forecast Mean Square Error:

$$\textbf{FMSE} = \sqrt{\frac{1}{n-K}\sum_{i=1}^{n}(Y-\hat{Y})^2}$$

FMSE is stand for Forecast Mean Square Error, where the n shows number of observation and k demonstrates no of parameters. While, $\hat{y}$ is the estimated value of predictor.

3.3 Models

We evaluate performance of non-parametric methods for determinants of poverty. We are not concern with the exact determinants of poverty. We gave the same determinants of poverty for each selected model and determinants are not changing with the methodology for the choice of methods. Therefore, for the same model the performance of non-parametric regression estimators is similar. There are lots of models for poverty, among these models we have selected 3 models for the current study. In fact, selecting an appropriate model has its own complexities and needs complicated set of procedures. The models we have chosen are not guaranteed to be the best models. However, all non-parametric methods are estimated for the same model which makes comparison reasonable because the poor model to be compared with poor and better model to be compared with better.

Model No.1:

This model is proposed by Chaudhry et al (2009). This model assumes the Per Capita income as a function of house hold head education level, room in house, female to male ratio, child dependency ratio, age of house hold head and participation ratio.

$$PCI = \beta_0 + \beta_1 SHH + \beta_2 HHEDU + \beta_3 RIH + \beta_4 FMR + \beta_5 CDER + \beta_6 AGEHH + \beta_7 PARR + \mu$$

Dependent variable: PCI = per capita Income

Explanatory variables: SHH = Size of household, HHEDU = Household head education level, RIH = Room in house FMR = Female-male ratio, CDER = Child dependency ration, AGEHH = Age of household head PARR Participation Rate and μ is error term

Model No.2:

This model is proposed by Megersa (2015). This model assumes the Per Capita income as a function of participation ratio size, house hold size, age of house hold head, year of schooling of family head, female to male ratio.

$$PCI = \beta_0 + \beta_1 PARR + \beta_2 HHSIZE + \beta_3 Age + \beta_4 YSFH + \beta_5 FMR + \mu$$

Dependent variable: PCI = per capita Income

Explanatory variables:

PARR= Participation Ratio, HHSize = Household Size, AGE = Age of house hold head

YSFH Years of schooling of the family head, FMR = Female to male ratio. and μ is error term

Model No.3:

This model is proposed by Malik (1996). This model assumes the Per Capita income as a function of male to female ratio, education, dependency ratio, participation ratio and house hold size.

$$PCI = \beta_0 + \beta_1 MFR + \beta_2 EDU + \beta_3 PARR + \beta_4 HHS + \mu$$

Dependent variable: PCI per capita Income

Explanatory variables:

MFR = Male -Female Ratio, EDU = Education, PAR =Participation Ratio, HHS Household Size, and μ is error term

3.4 Data and Sample Size

The comparison is based on the forecast performance of the estimators in real-life data. We have selected three models for the determinants of poverty for this purpose. The three models are mentioned in above section. The models shall be estimated on the data of determinants of poverty taken from PSLM (Pakistan Social and Living Standard Measure) 2014-15 data and 80% of the used for estimation and other 20% data is used for forecasting. Where the sample size of each district is varied. The data sets of these variables i.e. income per capita, household size, dependency ratio, participation rate, male-female ratio, age of household, household head education level, female-male ratio (worker), dependency rate, child dependency ratio, age of household head, gender of house hold. and rooms in house would be used for evaluating the forecast performance of non-parametric regression estimators.

The performance of estimators will be evaluated based on the value of forecast error. That estimator which has low value of forecast error shall be consider the best estimator.

Sample Size for each Districts:

Districts	**Sample Size**	**Districts**	**Sample Size**
Sargodha	383	Gujranwala	400
Faisalabad	660	Hafazabad	356
Chiniot	279	Okara	182
Jhang	640	Sahiwal	221
Toba Tek Singh	510	Pak Pattan	239

3.5 Implication and Generalizability

We have estimated the poverty models from two aspects while dependent variable is in its Raw form and secondly with dependent variables is in Log form. In presence of dependent variable with Raw form then data is highly skewed and tells its performance. On the other hand, when data is in log transformation then it is moderately skewed data and state its own performance.

CHAPTER 4

DATA DESCRIPTION RESULTS AND ANALYSIS

In this chapter in section 4.1 we will discuss about descriptive statistics, and section 4.2 will be about the estimation of results. As discussed in chapter 3, the Non-Parametric regression methods are compared on the basis of their forecast performance. We have estimated three models of determinants of poverty by using two kinds of data.

(i) Where the data is taken in raw form.

(ii) With log transformation of dependent variable.

These two transformations will allow us to evaluate the performance for highly Skewed data and moderately skewed data.

4.1 Descriptive Statistics

Before analyzing the data, it is important to explore the descriptive statistics of the data. So that we can assess how close the data is to the standard OLS assumptions. As discussed earlier, this allows us to analyze the performance of Non-Parametric methods for various levels of deviations from the standard models. The descriptive statistics for two districts are mentioned below in Table 4.1.1 and Table 4.1.2 whereas the results for remaining districts are given in appendix.

Table 4.1.1: Descriptive Statistics of the data series for Sargodha

Variables	Mean	Median	Mode	Standard deviation	Test for skewness	P-value	Test for excess kurtosis	P-value
LPCI	3.22	3.25	2.95	0.38	0.022	0.87	1.0147	0.00
PCI	2543.17	1777.78	888.89	4084.23	9.171	0.00	109.04	0.00
AGE	44.12	42.00	45.00	11.68	0.424	0.00	-0.211	0.44
EDU	8.00	8.00	10.00	2.96	0.418	0.00	0.592	0.03
HHSIZE	9.67	9.00	9.00	1.71	6.112	0.00	48.218	0.00
FMR	1.31	1.00	1.00	1.10	2.762	0.00	10.838	0.00
PARR	6.30	6.00	6.00	1.66	0.174	0.20	0.848	0.00
CDR	3.13	3.00	3.00	1.74	1.462	0.00	4.878	0.00
RIH	2.75	2.00	2.00	1.64	1.446	0.00	2.918	0.00
MFR	1.24	1.00	1.00	0.92	1.9949	0.00	5.4068	0.00

Results of Table 4.1.1, shows the descriptive statistics of variables. To find out the normality of the variables we will apply test for skewness and test for kurtosis. The test for skewness has the null hypothesis as the series is symmetric. From table 4.1.1 we can see that p-value for all variables i.e. PCI (Per capita Income), EDU (education), HHS (house hold size), CDR (child dependency ratio), RIH (room in house) and MFR (male to female ratio) is zero up to two decimals which are less than 5%. Therefore, this implies strong rejection of normality and positively skewed because the value of test for skewness has positive sign.

The test for Excess kurtosis having null hypothesis, the value of Excess kurtosis which is equal to zero (Series is Mesokurtic). In table 4.1.1 we have realized that p-value for all variables i.e. LPCI, PCI, MFR, EDU, HHS, CDR, RIH and FMR are less than 5%. This indicates all these variables are not Mesokurtic because we can reject the null hypothesis (Excess kurtosis is equal to zero). Therefore, all these variables are markedly different than a normal distribution. Similarly, same results are found from following Table 4.1.2, for District Faisalabad. However, the skewness for log of per capita income (LPCI) for the two districts is not much different from zero. This allows us to evaluate the performance of Non-Parametric methods.

Table 4.1.2: District Faisalabad

Variables	Mean	Median	Mode	Standard deviation	Test for skewness	P-value	Test for excess kurtosis	P-value
LPCI	3.29	3.33	3.00	0.41	-1.132	0.00	6.667	0.00
PCI	2921.26	2114.29	1000.00	3411.82	6.167	0.00	67.399	0.00
AGE	43.12	42.00	38.00	10.33	0.671	0.00	0.720	0.00
EDU	8.45	8.00	10.00	2.99	0.0781	0.46	-0.728	0.00
HHSIZE	9.41	9.00	9.00	1.07	5.835	0.00	48.970	0.00
FMR	1.25	1.00	1.00	0.95	2.159	0.00	5.699	0.00
PARR	6.24	6.00	6.00	1.57	-0.178	0.09	0.667	0.00
CDR	3.00	3.00	3.00	1.58	0.840	0.00	0.757	0.00
RIH	2.42	2.00	2.00	1.31	1.029	0.00	0.769	0.00
MFR	1.26	1.00	1.00	0.95	2.0702	0.00	5.504	0.00

For this kind of data, the Non-Parametric Methods are expected to work better. It can also be noted that skewness and Kurtosis for PCI has reduced after taking log transform.

4.2 Estimation Results

Non-Parametric regression methods had compared in terms of their forecast performance for different determinants of poverty models estimated on real data. The large data set of PSLM (Pakistan social and living standard measure) 2014-15, used for this justification and 80% of data was utilized for estimation while rest of 20% utilized for forecasting. We had taken data sets of determinants of poverty for ten districts of Punjab, where the sample size of each district was varied. Many studies have stated that in presence of skewed distribution OLS does not be able to provide accurate results while the Non-Parametric regression estimators performs better in this case. In present study, we want to evaluate the forecast performance of Non-Parametric regression estimators in presence of skewed distribution that's why we have selected five Non-Parametric regression estimators for this purpose. We have estimated three models discussed in chapter 3, section 3.3 for all ten districts. This makes a total of 150 Regression with 30 Regression for each Non-Parametric method. For simplicity, the regression results are capture in appendix.

The following methodology mentioned in section 3.2 of chapter 3. These results of RSS and FMSE for Non-parametric methods are given in table's 4.2.1- 4.2.6. These results were obtained from three models and five non-parametric regression estimators by using two kinds of data:

(i) Data is in its raw form

(ii) Data with log transformation of dependent variable.

Table 4.2.1: The Residuals Sum of Square (RSS) and Forecast Mean Square Error (FMSE) for Model.1 with PCI (Per Capita Income) as dependent variable

	Methods									
DISTRICTS	**Quantile Regression**		**M-Estimator**		**Theil-Sen Estimator**		**LTS**		**LAD**	
	RSS	**FMSE**	**RSS**	**FMSE**	**RSS**	**FMSE**	**RSS**	**FMSE**	**RSS**	**FMSE**
Sargodha	215633693	1962	213395847	1952	**207456959**	**1924**	217150164	1969	213082701	1950
Faisalabad	260680920	1606	260549277	1606	264334318	1617	261272782	1608	250223018	1573
Chiniot	333025250	2885	331545840	2879	330292245	2873	335380289	2895	331687044	2879
Jhang	295443109	1693	283926688	1660	296268674	1695	282419242	1655	289393392	1676
Toba-Tek Singh	272956084	1920	256247854	1860	271454711	1915	263635505	1887	271518689	1915
Gujranwala	205507649	1850	197148880	1812	1709866032	5338	188471127	1772	200855302	1829
Hafazabad	87864425	1263	89802810	1277	88588121	1269	90924352	1285	85212355	1244
Okara	21175216	959	20207886	937	18392509	894	20524282	944	19142819	912
Sahiwal	106854666	1887	107844896	1896	125333508	2043	108188683	1899	104155920	1863
Pak Pattan	48947802	1182	47845175.91	1169	48025102	1171	48730759	1179	48008985	1171

The results of Table 4.2.1, shows that Theil-Sen and LAD estimators compete with each other on the basis of their forecast performance. Therefore, the value of RSS (Residual sum of square) and FMSE (Forecast mean square errors) of Theil-Sen estimator is low and minimum at districts Sargodha, Chiniot, and Okara. Similarly, the value of RSS and FMSE of LAD estimator is also minimum at three districts, district Faisalabad, Hafzabad and district Sahiwal. Whereas, the M-estimator and LTS estimator values of RSS and FMSE are minimum at district Toba Tek Singh, district Pak Pattan, Jhang district and Gujranwala respectively. In table 4.2.1 the only Quantile regression shows very poor results as compare to all other estimators in analysis because its values of RSS and FMSE are not minimal at any number of districts. Therefore, as we observe from table we can say that Quantile regression is not a suitable non-parametric regression estimator in presence of highly skewed data.

Table 4.2.2: The Residuals Sum of Square (RSS) and Forecast Mean Square Error (FMSE) for Model.2 with PCI as Dependent Variable:

DISTRICTS	Quantile Regression		M-Estimator		Theil-Sen Estimator		LTS		LAD	
	RSS	FMSE	RSS	FMSE	RSS	FMSE	RSS	FMSE	RSS	FMSE
Sargodha	3696447138	5232	3697987287	5233	3648955820	5198	3701643709	5236	3675488044	5217
Faisalabad	275594509	1478	267448942	1456	249171817	1406	286547356	1508	260816837	1438
Chiniot	307652358	2773	305076481	2761	312772990	2796	309353023	2780	303415639	2754
Jhang	309954405	1593	297923645	1562	304266614	1579	297341055	1561	305753028	1583
Toba-Tek Singh	298963814	1773	281893332	1722	310938283	1809	290790545	1749	298550640	1772
Gujranwala	219898207	1723	226113856	1748	228257693	1756	215628170	1707	213207728	1697
Hafazabad	114422921	1326	106927302	1282	101874580	1251	108215393	1290	105327781	1272
Okara	20010385	816	22058696	857	21347602	843	22104945	858	20577884	828
Sahiwal	110735464	1707	111680296	1714	108228863	1687	111083353	1709	107500485	1681
Pak Pattan	44184664	1025	43054865	1012	52265014	1115	44282847	1026	44907430	1034

Results from Table 4.2.2, indicated that the value of RSS and FMSE of Theil-Sen estimator are minimum at three districts i.e. Sargodha, Faisalabad and Hafizabad. The values of RSS and FMSE for the LAD estimator are low for three districts Chiniot, Gujranwala and district Sahiwal. M-estimator's value of RSS and FMSE was found minimum at two districts at District Toba Tek Singh and district Pak Pattan. The other two estimators Quantile regression and LTS estimator gave very poor results than other estimators. Their value of RSS and FMSE are minimal only at district Okara and Jhang respectively.

The results of model no.2 in table 4.2.2 show that the Theil-Sen estimator and Least Absolute deviation method perform better than other estimators, because their value of residual sum of square and forecast mean square errors are mostly time minimal at maximum number of districts than other estimators.

Table 4.2.3: The Residuals Sum of Square (RSS) and Forecast Mean Square Error (FMSE) for Model.3 with PCI as Dependent Variable

	Methods									
DISTRICTS	**Quantile Regression**		**M-Estimator**		**Theil-Sen Estimator**		**LTS**		**LAD**	
	RSS	**FMSE**	**RSS**	**FMSE**	**RSS**	**FMSE**	**RSS**	**FMSE**	**RSS**	**FMSE**
Sargodha	396900626	2315	364857079	2220	356783954	2195	387974892	2289	359680880	2204
Faisalabad	540052239	2062	345923495	1650	252422146	1409	324729188	1599	261651059	1435
Chiniot	345550331	2628	347577817	2636	350042330	2645	353772244	2659	348642299	2640
Jhang	306289075	1578	329736804	1637	311080547	1590	334704538	1649	310472720	1588
Toba-Tek Singh	317142222	1808	298115743	1753	317886107	1810	632055118	2552	297958154	1752
Gujranwala	182334317	1559	169045108	1501	219056927	1709	169871969	1504	216116093	1697
Hafazabad	113893099	1303	112336886	1294	101346661	1229	109372994	1277	103526600	1243
Okara	21764250	812	20389627	786	21005190	797	21099887	799	19330381	765
Sahiwal	105810278	1626	107564313	1639	108155636	1644	109676892	1655	108930272	1650
Pak Pattan	55588406	1136	59906077	1180	49446328	1072	57678321	1158	44359764	1015

Table 4.2.3, indicated that the value of RSS and FMSE of Theil-Sen estimator are minimum at three districts i.e. Sargodha, Faisalabad and Hafazabad. While the LAD estimator’s value of RSS and FMSE are minimal at three districts Toba Tek Singh, Okara and district Sahiwal. Quantile regression also performs better because their value of RSS and FMSE are minimal at three districts Chiniot, Jhang and Sahiwal. Where M-estimator is not much better, their value of RSS and FMSE are minimum at two districts at district Chiniot and Gujranwala. Least Trimmed Square estimator showed very poor performance because its values of RSS and FMSE are not minimal at any district.

Results from all districts showed that the Theil-Sen estimator, Least Absolute deviation method and Quantile regression performed better than other estimators because their values of Residual sum of square and forecast mean square errors are mostly minimum than other estimators.

Overall results from tables 4.2.1, 4.2.2 and 4.2.3 indicates that the Theil-Sen estimator and Least Absolute Deviation method are those Non-Parametric regression estimators which perform much better as compare to Quantile regression, M-estimator and Least trimmed square estimator in presence of highly skewed data. These estimators are very useful Non-Parametric regression estimators as compare to Quantile regression, M-estimator and Least Trimmed estimator for highly skewed data, because their values of RSS and FMSE were mostly found minimum at maximum number of districts. Therefore, we recommend these two estimators to the researcher for getting more accurate results in highly skewed distribution.

(ii). Data with log Transformation of Dependent Variable.

Table 4.2.4: The Residuals Sum of Square (RSS) and Forecast Mean Square Error (FMSE) for Model.1 with LPCI (Log of Per Capita Income) as dependent variable:

	Methods									
DISTRICTS	**Quantile Regression**		**M-Estimator**		**Theil-Sen Estimator**		**LTS**		**LAD**	
	RSS	**FMSE**	**RSS**	**FMSE**	**RSS**	**FMSE**	**RSS**	**FMSE**	**RSS**	**FMSE**
Sargodha	10.48	0.43	10.27	0.43	11.15	0.45	10.25	0.43	10.52	0.43
Faisalabad	15.15	0.39	76.76	0.87	16.25	0.40	14.13	0.37	14.40	0.38
Chiniot	8.54	0.46	8.93	0.47	9.40	0.48	8.76	0.47	8.85	0.47
Jhang	10.06	0.31	9.99	0.31	11.02	0.33	9.98	0.31	10.12	0.31
Toba-Tek Singh	11.56	0.40	11.33	0.39	11.81	0.40	11.19	0.39	11.35	0.39
Gujranwala	13.71	0.48	13.12	0.47	14.98	0.50	13.36	0.47	13.55	0.48
Hafazabad	8.28	0.39	8.19	0.39	8.45	0.39	8.33	0.39	8.45	0.39
Okara	2.76	0.35	2.70	0.34	2.72	0.34	2.78	0.35	2.68	0.34
Sahiwal	7.00	0.48	7.11	0.49	10.59	0.59	6.55	0.47	7.09	0.49
Pak Pattan	3.17	0.30	3.14	0.30	3.64	0.32	3.13	0.30	3.07	0.30

In Table 4.2.4 results from Model no.1 the performance of Least Trimmed square estimator is much better than other estimators in presence of log transform data. Its values of RSS and FMSE are mostly decline at maximum numbers of districts than other estimators. From table 4.2.4 LTS values of RSS and FMSE are minimum at districts Sargodha, Faisalabad, Jhang and district Toba Tek Singh. While M-estimator performance is small poor as compare to LTS estimators. Its value of RSS and FMSE is low at only two districts Gujranwala and district Hafizabad. The others two estimator Quantile regression and LAD estimator's performance is very low as compare to LTS estimator because their value of RSS and FMSE are minimum at two districts Chiniot and Okara respectively. The remaining estimator Theil-Sen which perform very well in presence of highly skewed data but here in log transform data its value of RSS and FMSE are highest than all other estimators at each district. Its shows that, Theil-Sen estimator does not perform well in the moderately skewed data.

Table 4.2.5: The Residuals Sum of Square (RSS) and Forecast Mean Square Error (FMSE) for Model.2 with LPCI as Dependent Variable:

	Methods									
DISTRICTS	**Quantile Regression**		**M-Estimator**		**Theil-Sen Estimator**		**LTS**		**LAD**	
	RSS	**FMSE**	**RSS**	**FMSE**	**RSS**	**FMSE**	**RSS**	**FMSE**	**RSS**	**FMSE**
Sargodha	24.7	0.43	24.03	0.42	24.31	0.42	24.54	0.43	24.11	0.42
Faisalabad	16.64	0.36	15.67	0.35	16.20	0.3	15.7	0.3	15.81	0.3
Chiniot	8.61	0.46	8.68	0.47	10.16	0.50	8.65	0.47	8.46	0.46
Jhang	10.91	0.30	10.73	0.30	11.56	0.31	10.85	0.30	10.86	0.30
Toba-Tek Singh	14.40	0.39	14.52	0.39	14.75	0.39	14.31	0.39	14.54	0.39
Gujranwala	17.24	0.48	16.04	0.47	18.55	0.50	16.08	0.47	17.27	0.48
Hafazabad	9.42	0.38	12.03	0.43	9.09	0.37	9.49	0.38	9.30	0.38
Okara	4.94	0.41	5.11	0.41	5.47	0.43	5.06	0.41	5.13	0.41
Sahiwal	6.54	0.41	6.58	0.42	6.91	0.43	6.66	0.42	6.63	0.42
Pak Pattan	3.34	0.28	3.20	0.28	3.55	0.29	3.16	0.27	3.17	0.27

In table 4.2.5 results of model no.2 indicates that M-estimator perform better than all other estimators because values of RSS and FMSE are smallest than other estimators' values. Its value of RSS and FMSE are low at districts Sargodha, Faisalabad, Jhang and districts Gujranwala. The Quantile Regression values of RSS and FMSE are minimal two districts Okara, Sahiwal. While the LTS estimator values are small at districts Toba Tek Singh and district Pak Pattan. The other remaining two estimators Theil-Sen estimator and LAD estimator perform very poor in second model with moderately skewed data. The value of RSS and FMSE of Theil-Sen and LAD estimators are lowest at only one, one district Hafizabad and district Chiniot respectively.

Table 4.2.6: The Residuals Sum of Square (RSS) and Forecast Mean Square Error (FMSE) for Model.3 with LPCI as Dependent Variable

	Methods									
DISTRICTS	Quantile Regression		M-Estimator		Theil-Sen Estimator		LTS		LAD	
	RSS	FMSE	RSS	FMSE	RSS	FMSE	RSS	FMSE	RSS	FMSE
Sargodha	12.97	0.42	12.73	0.41	13.02	0.42	13.03	0.42	13.00	0.42
Faisalabad	16.00	0.35	15.53	0.35	16.10	0.36	18.80	0.38	15.65	0.35
Chiniot	9.21	0.43	9.54	0.44	10.51	0.46	11.30	0.48	9.44	0.43
Jhang	11.24	0.30	11.08	0.30	11.71	0.31	11.13	0.30	11.20	0.30
Toba-Tek Singh	14.45	0.39	15.43	0.40	15.78	0.40	14.55	0.39	14.62	0.39
Gujranwala	16.98	0.48	16.49	0.47	17.69	0.49	10.84	0.38	17.05	0.48
Hafazabad	9.38	0.37	9.04	0.37	9.11	0.37	10.01	0.39	9.24	0.37
Okara	4.94	0.39	4.94	0.39	5.45	0.41	4.07	0.35	4.93	0.39
Sahiwal	6.48	0.40	6.50	0.40	6.59	0.41	5.12	0.36	6.59	0.41
Pak Pattan	2.97	0.26	3.03	0.27	3.20	0.27	3.95	0.30	3.02	0.26

In Table 4.2.6 the M-estimator performs similar as in table 4.2.5 because its values of RSS and FMSE are low at four districts i.e. Sargodha, Faisalabad, Jhang and district Hafizabad. In model 3rd Quantile regression also perform well because its values of RSS and FMSE are minimal at three districts Toba Tek Singh, Sahiwal, and district Pak Pattan. The value of RSS and FMSE of LTS estimator is minimum at only one district Gujranwala. While other two remaining estimators Theil-Sen and LAD estimator are very poor same as at 2nd model because its value of RSS and FMSE are not minimal at any districts.

Overall, results from tables 4.2.4, 4.2.5 and 4.2.6 indicated that the M-estimator and Least Trimmed Square estimator very suitable Non-Parametric estimator for moderately skewed data. Therefore, their values of RSS and FMSE are mostly low at maximum number of districts.

For the three Models and 10 Districts we had optimal performance of M-Estimator and LTS (Least Trimmed Square) estimator for case of log transformation of dependent variable. We see that M-Estimator and LTS estimator had optimal performance only for moderately skewed data. Whereas, Theil-Sen estimator and LAD estimator had optimal performance for highly skewed data.

Table 4.3: Optimal RSS and FMSE for Non-parametric estimators ten districts for three models

Estimators / Types of data	Highly Skewed data	Moderately skewed data
Quantile regression	4	6
M-estimator	5	10
LTS estimator	3	10
Theil-Sen estimator	9	1
LAD estimator	9	3

CHAPTER 5

SUMMARY, CONCLUSION AND RECOMMENDATIONS

5.1 Summary

In this study, we have discussed five Non-Parametric regression estimators i.e. Quantile regression, LAD estimator, LTS estimator, Theil-Sen estimator and M-estimator. We want to assess the Forecast performance of these Non-Parametric methods on real life relationship. For this intent we have taken data of determinants of poverty from PSLM (Pakistan Social and Living Standard Measurement) of ten districts of Punjab. These types of data usually violate the standard OLS assumptions and such kind of data needs to be treated using Non-Parametric Regression methods. We have selected three models of determinants of poverty for estimation the Non-Parametric methods.

Two types data used for estimations and Forecasting

I. Data in raw form

II. Data with Log transform of dependent variable

To evaluate the performance of Non-Parametric methods the Residual Sum of square (RSS) and Forecast Mean Square Errors (FMSR) are computed. The 80% of data was used for estimation and other 20% of data is used for Forecasting.

In first case when data is in its Raw form, the results from Tables 4.2.1, 4.2.2 and 4.2.3 show that the Theil-Sen estimator and Least Absolute Deviation (LAD) methods gives optimal results because their values of RSS and FMSE are generally lowest at most districts.

The Quantile Regression, M-estimator and LTS estimators show very poor performance whereas, the M-estimator and LTS estimator have provided very good performance only for moderately skewed data as in Tables 4.2.4, 4.2.5 and 4.2.6. The Theil-Sen and LAD estimator are not suitable for this kind of data.

In case of highly skewed data the two estimators like Theil-Sen and LAD estimator perform well. In moderately skewed data, M-estimator and LTS estimator perform better than all other estimators. Their values of RSS and FMSE are minimum at ten outcomes. In absence of knowledge about skewness, M-estimator is better. It shows moderate performance for both kind of skewness than other estimators. From table 4.3, M-estimator's values of RSS and FMSE are minimum for 5 numbers of outcomes in highly skewed data and for moderately skewed data its values are minimum for 10 outcomes.

5.2 Conclusion

In case of highly skewed data when data is in its raw form the Theil-Sen estimator and LAD estimator gave optimal performance as compare to other non-parametric regression estimators. Their values of RSS and FMSE are generally lowest at most districts.

In case of moderately skewed data when our dependent variable is in log transform the M-estimator and LTS estimator perform better than all other estimators. Their values of RSS and FMSE are minimum at maximum numbers of districts.

In table 4.3 from section 4.2 the detailed analysis, we conclude that among non-parametric regression methods; Quantile regression is not useful for highly and moderately skewed data because this estimator shows poor performance in both kinds of data. We also note that Theil-Sen estimator and LAD estimator are not suitable for moderately skewed data and LTS estimator is not appropriate for highly skewed data in the class of non-parametric regression estimators.

5.3 Recommendations

1. More estimators can be included for further research.
2. The focus of this study is on skewness. One can analyze the performance for other violations of OLS assumptions like endogeneity.
3. The exercise can be repeated for other data sets, so that it can be judge that whether or not the non-parametric methods maintain their properties for other data sets.

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APPENDIX

Descriptive Statistics of the data series:

Districts	Variables	Mean	Median	Mode	Standard deviation	Test for skewness	P-Vlaue	Test for Excess kurtosis	P-Value
District Sargodha	LPCI	3.22	3.25	2.95	0.38	0.022	0.871	1.0147	0.0002
	PCI	2543.17	1777.78	888.89	4084.23	9.171	0.00	109.04	0.00
	AGE	44.12	42.00	45.00	11.68	0.424	0.002	-0.211	0.44
	EDU	8.00	8.00	10.00	2.96	0.418	0.0022	0.592	0.0297
	HHSIZE	9.67	9.00	9.00	1.71	6.112	0.00	48.218	0.00
	FMR	1.31	1.00	1.00	1.10	2.762	0.00	10.838	0.00
	PARR	6.30	6.00	6.00	1.66	0.174	0.202	0.848	0.002
	CDR	3.13	3.00	3.00	1.74	1.462	0.00	4.878	0.00
	RIH	2.75	2.00	2.00	1.64	1.446	0.00	2.918	0.00
	MFR	1.242	1	1	0.917	1.99	0.00	5.406	0.00
	YSFH	8.11	8	10	3.01	0.364	0.004	0.332	0.18
District Faisalabad	LPCI	3.29	3.33	3.00	0.41	-1.132	0.00	6.667	0.00
	PCI	2921.26	2114.29	1000.00	3411.82	6.167	0.00	67.399	0.00
	AGE	43.12	42.00	38.00	10.33	0.671	0.00	0.720	0.00
	EDU	8.45	8.00	10.00	2.99	0.0781	0.456	-0.728	0.0005
	HHSIZE	9.41	9.00	9.00	1.07	5.835	0.00	48.970	0.00
	FMR	1.25	1.00	1.00	0.95	2.159	0.00	5.699	0.00
	PARR	6.24	6.00	6.00	1.57	-0.178	0.089	0.667	0.0014
	CDR	3.00	3.00	3.00	1.58	0.840	0.00	0.757	0.00
	RIH	2.42	2.00	2.00	1.31	1.029	0.00	0.769	0.00
	MFR	1.26	1.00	1	0.95	2.07	0.00	5.50	0.00
	YSFH	8.49	8	10	3.01	0.190	0.045	-0.219	0.248
District Chiniot	LPCI	3.26	3.30	2.95	0.39	-0.639	0.00	1.366	0.00
	PCI	2566.17	1988.89	888.89	2361.20	2.831	0.00	12.053	0.00
	AGE	43.00	41.50	43.00	12.45	0.781	0.00	0.654	0.036
	EDU	8.09	8.00	5.00	2.62	0.203	0.195	-0.673	0.031
	HHSIZE	9.53	9.00	9.00	0.86	3.9868	0.00	26.083	0.00
	FMR	1.13	1.00	1.00	0.83	2.628	0.00	11.729	0.00
	PARR	6.42	7.00	7.00	1.53	-0.458	0.003	0.038	0.901
	CDR	2.89	3.00	3.00	1.43	0.911	0.00	1.943	0.00
	RIH	2.23	2.00	2.00	1.26	2.017	0.00	6.485	0.00
	MFR	1.36	1	1	0.99	1.794	0.00	4.437	0.00
	YSFH	8.06	8	5	2.60	0.194	0.183	-0.72	0.013

Districts	Variables	Mean	Median	Mode	Standard deviation	Test for skewness	P-Vlaue	Test for Excess kurtosis	P-Value
District Jhang	LPCI	3.27	3.29	3.00	0.34	-0.111	0.282	0.322	0.119
	PCI	2517.96	1960.00	1000.00	2312.06	3.259	0.00	16.292	0.00
	AGE	43.74	42.00	42.00	12.32	0.421	0.00	-0.159	0.441
	EDU	7.65	8.00	5.00	3.17	0.258	0.013	-0.6105	0.003
	HHSIZE	9.81	9.00	9.00	1.55	4.712	0.00	29.702	0.00
	FMR	1.34	1.00	1.00	1.02	2.396	0.00	9.859	0.00
	PARR	6.23	6.00	7.00	1.76	-0.153	0.138	-0.0796	0.699
	CDR	3.31	3.00	2.00	1.86	1.088	0.00	2.318	0.00
	RIH	2.53	2.00	2.00	1.42	1.305	0.00	2.077	0.00
	MFR	1.36	1	1	0.99	2.27	0.00	7.89	0.00
	YSFH	7.67	8	5	3.17	0.243	0.01	-0.62	0.001

Districts	Variables	Mean	Median	Mode	Standard deviation	Test for skewness	P-Vlaue	Test for Excess kurtosis	P-Value
District Toba Tek Singh	LPCI	3.27	3.28	3.44	0.35	-0.385	0.001	0.237	0.0038
	PCI	2521.11	1905.56	2777.78	2063.77	2.174	0.00	7.123	0.00
	AGE	45.21	43	42	12.41	0.543	0.00	-0.3209	0.1809
	EDU	7.95	8	10	2.86	0.023	0.846	-0.25	0.279
	HHSIZE	9.72	9	9	1.25	3.763	0.00	18.018	0.00
	FMR	1.37	1	1	1.03	1.986	0.00	5.049	0.00
	PARR	6.28	6	7	1.57	-0.202	0.093	-0.41	0.087
	CDR	3.10	3	3	1.67	0.988	0.00	1.674	0.00
	RIH	2.73	2	2	1.57	1.354	0.00	2.184	0.00
	MFR	1.12	1	1	0.84	2.13	0.00	6.38	0.00
	YSFH	8	8	10	2.86	0.006	0.95	-0.26	0.23
District Gujranwala	LPCI	3.37	3.43	2.95	0.38	-0.299	0.023	0.227	0.387
	PCI	3337.29	2666.67	888.89	3038.67	2.339	0.00	7.501	0.00
	AGE	44.32	43.00	45.00	11.42	0.5799	0.00	0.34726	0.1867
	EDU	8.15	8.00	10.00	2.94	-0.101	0.442	-0.263	0.316
	HHSIZE	9.56	9.00	9.00	1.56	6.288	0.00	49.873	0.00
	FMR	1.21	1.00	1.00	0.89	2.168	0.00	6.502	0.00
	PARR	6.22	6.00	7.00	1.62	-0.049	0.709	0.857	0.001
	CDR	3.06	3.00	2.00	1.82	1.6188	0.00	5.8586	0.00
	RIH	2.70	2.00	2.00	1.53	1.522	0.00	4.0465	0.00
	MFR	1.25	1	1	0.89	1.962	0.00	5.852	0.00
	YSFH	8.20	8	10.00	2.97	-0.08	0.49	-0.35	0.15
District Hafazabad	LPCI	3.19	3.20	2.82	0.35	-0.289	0.035	-0.0768	0.779
	PCI	2097.06	1590.00	666.67	1746.14	2.314	0.00	8.766	0.00
	AGE	42.86	40.00	40.00	12.29	0.4512	0.001	-0.1956	0.476
	EDU	7.58	8.00	5.00	3.08	0.1395	0.3106	-0.266	0.3319
	HHSIZE	9.79	9.00	9.00	1.51	3.9032	0.00	17.38	0.00
	FMR	1.35	1.00	1.00	1.02	1.757	0.00	3.116	0.00

	PARR	6.41	6.00	7.00	1.58	-0.3309	0.016	0.0385	0.888
	CDR	3.09	3.00	3.00	1.73	1.1982	0.00	1.7605	0.00
	RIH	2.74	2.00	2.00	1.51	1.5282	0.00	3.4002	0.00
	MFR	1.20	1	1	0.91	2.53	0.00	9.692	0.00
	YSFH	7.51	8.00	5.00	3.08	0.246	0.06	-0.048	0.85
District Okara	**LPCI**	3.10	3.09	3.00	0.35	-2.5604	0.00	17.362	0.00
	PCI	1717.48	1222.22	1000.00	1724.95	3.699	0.00	17.846	0.00
	AGE	41.98	40.00	40.00	12.10	0.7854	0.00	0.119	0.760
	EDU	7.71	8.00	5.00	3.06	0.1942	0.322	-0.489	0.2096
	HHSIZE	9.50	9.00	9.00	0.77	3.127	0.00	16.807	0.00
	FMR	1.40	1.00	1.00	1.13	2.294	0.00	6.3555	0.00
	PARR	6.06	6.00	7.00	1.63	-0.3156	0.1075	-0.496	0.203
	CDR	3.20	3.00	3.00	1.62	0.5895	0.0026	-0.188	0.629
	RIH	2.13	2.00	1.00	1.12	0.965	0.00	0.6096	0.117
	MFR	1.14	1	1	0.87	1.88	0.00	4.076	0.00
	YSFH	7.64	8.00	5.00	3.08	0.15	0.40	-0.46	0.201

Districts	**Variables**	**Mean**	**Median**	**Mode**	**Standard deviation**	**Test for skewness**	**P-Vlaue**	**Test for Excess kurtosis**	**P-Value**
District Sahiwal	**LPCI**	3.20	3.22	3.12	0.37	-0.1049	0.5508	0.0882	0.8010
	PCI	2225.41	1666.67	1333.33	2136.54	2.9626	0.00	13.028	0.00
	AGE	44.01	43.00	45.00	11.71	0.4802	0.006	-0.3249	0.353
	EDU	8.20	9.00	10.00	2.90	-0.2204	0.2099	-0.5216	0.1361
	HHSIZE	9.59	9.00	9.00	1.07	5.1485	0.00	37.294	0.00
	FMR	1.40	1.00	1.00	1.10	2.025	0.00	5.2069	0.00
	PARR	6.35	7.00	8.00	1.68	-0.321	0.067	-0.448	0.200
	CDR	2.99	3.00	2.00	1.60	0.8348	0.00	0.5965	0.088
	RIH	2.59	2.00	2.00	1.32	1.1335	0.00	1.5041	0.00
	MFR	1.16	1	1	0.94	0.32	0.048	-0.41	0.19
	YSFH	8.26	9.00	10.00	2.88	-0.20	0.21	-0.52	0.11
District Pak Pattan	**LPCI**	3.20	3.19	3.00	0.33	0.6906	0.00	4.0237	0.00
	PCI	2526.93	1555.56	1000.00	7673.54	13.425	0.00	187.06	0.00
	AGE	40.37	39.00	32.00	10.36	0.6611	0.00	0.03579	0.914
	EDU	7.04	7.00	5.00	3.22	0.29756	0.0742	-0.345	0.298
	HHSIZE	9.62	9.00	9.00	1.17	5.0241	0.00	32.666	0.00
	FMR	1.20	1.00	1.00	0.91	2.549	0.00	9.89	0.00
	PARR	6.20	6.00	7.00	1.55	-0.126	0.451	-0.474	0.153
	CDR	3.21	3.00	2.00	1.56	0.671	0.00	0.3214	0.3328
	RIH	2.20	2.00	1.00	1.32	2.335	0.00	10.098	0.00
	MFR	1.25	1	1	0.81	2.185	0.00	6.465	0.00
	YSFH	6.96	6.00	5.00	3.25	1.49	0.00	2.659	0.00

Estimation Results of Regressions:

Pattern -1 Data with Raw form without Log of Dependent variable.

1. LTS ESTIMATOR:

District		Model no.1	Model No.2	Model.No.3	District		Model no.1	Model No.2	Model.No.3
Sargodha	Variables	Coefficients	Coefficients	Coefficients	Faisalabad	Variables	Coefficients	Coefficients	Coefficients
	INTERCEPT	1143.912	952.334	1274.873		INTERCEPT	-310.81	522.106	299.79
	AGE	3.288	-2.646			AGE	19.84	9.587	
	EDU	-25.377		-28.313		EDU	-10.81		-27.30
	HHS	233.300	155.892	84.110		HHS	398.56	326.254	315.72
	FMR	32.172	-34.859			FMR	-32.77	-89.287	
	PARR	-171.975	-40.281	-35.542		PARR	-262.54	-240.906	-230.77
	CDR	-140.109				CDR	-12.62		
	RIH	-4.444				RIH	-93.46		
	MFR			-80.241		MFR			64.91
	YSFH		-35.243			YSFH		-20.024	
Chiniot	INTERCEPT	-1460.977	-1303.575	-1246.432	Jhang	INTERCEPT	1157.716	803.901	900.882
	AGE	8.384	3.705			AGE	6.144	7.196	
	EDU	8.130		4.644		EDU	28.267		32.822
	HHS	471.444	509.209	477.658		HHS	-35.543	114.752	112.534
	FMR	-91.130	-83.559			FMR	-19.896	23.001	
	PARR	-200.672	-222.453	-230.342		PARR	24.573	-92.246	-93.206
	CDR	49.572				CDR	118.778		
	RIH	-9.865				RIH	76.690		
	MFR			123.825		MFR			-45.393
	YSFH		2.339			YSFH		34.333	
Toba Tek Singh	INTERCEPT	1597.304	2016.008	2394.541	Gujranwala	INTERCEPT	4015.58	3834.74	4442.78
	AGE	5.995	3.858			AGE	12.82	17.09	
	EDU	3.748		-10.015		EDU	-114.11		-86.71
	HHS	-19.375	86.828	80.237		HHS	-136.81	-80.15	-90.64
	FMR	90.261	114.711			FMR	318.71	233.77	
	PARR	-40.041	-166.989	-167.671		PARR	-35.96	-104.66	-112.87
	CDR	135.592				CDR	51.00		
	RIH	30.378				RIH	50.39		
	MFR			-126.765		MFR			-85.56
	YSFH		-14.365			YSFH		-83.76	
Hafazabad	INTERCEPT	939.846	1066.549	1515.648	Okara	INTERCEPT	1696.512	1044.622	1299.680
	AGE	2.344	-4.442			AGE	-5.211	4.991	
	EDU	-40.242		-36.734		EDU	-23.192		-22.227
	HHS	91.294	164.363	153.584		HHS	-58.177	68.103	37.233
	FMR	165.125	170.614			FMR	-14.571	2.756	
	PARR	-40.197	-98.833	-104.976		PARR	41.768	-69.966	-49.788
	CDR	60.537				CDR	85.775		
	RIH	6.324				RIH	57.848		
	MFR			-84.093		MFR			12.183
	YSFH		-41.686			YSFH		-14.338	

District		Model no.1	Model No.2	Model.No.3	District		Model no.1	Model No.2	Model.No.3
Sahiwal	**Variables**	**Coefficients**	**Coefficients**	**Coefficients**	Pak Patten	**Variables**	**Coefficients**	**Coefficients**	**Coefficients**
	INTERCEPT	-1101.313	-597.081	-1199.942		**INTERCEPT**	-628.953	-1873.748	-587.51
	AGE	15.547	12.489			**AGE**	14.923	4.546	
	EDU	11.022		1.352		**EDU**	-38.400		-19.73
	HHS	193.022	361.474	396.976		**HHS**	230.142	452.354	283.43
	FMR	-4.989	-33.154			**FMR**	-3.501	-9.262	
	PARR	-62.359	-227.693	-218.191		**PARR**	-63.966	-131.991	-134.13
	CDR	249.282				**CDR**	-10.192		
	RIH	17.617				**RIH**	76.490		
	MFR			89.153		**MFR**			37.38
	YSFH		-1.087			**YSFH**		-29.334	

2. LAD ESTIMATOR:

District		Model no.1	Model No.2	Model.No.3			Model no.1	Model No.2	Model.No.3
Sargodha	**Variables**	**Coefficients**	**Coefficients**	**Coefficients**	Faisalabad	**Variables**	**Coefficients**	**Coefficients**	**Coefficients**
	INTERCEPT	838.298	681.334	1308.9		**INTERCEPT**	246.32	1153.5	1324.9
	AGE	2.9583	-1.1659			**AGE**	27.3445	15.4	
	EDU	-36.0286		-32.4		**EDU**	-24.4033		-32
	HHS	244.6804	188.2604	164		**HHS**	314.6119	214.6	243.1
	FMR	196.0294	188.4929			**FMR**	-13.5223	-34.2	
	PARR	-173.1949	-74.1488	-101.6		**PARR**	-263.7408	-230	-206.6
	CDR	-70.2548				**CDR**	-45.6311		
	RIH	-19.6828				**RIH**	-90.501		
	MFR			-90.4		**MFR**			40.7
	YSFH		-50.3735			**YSFH**		-30.7	
Chiniot	**INTERCEPT**	483.8781	294.5975	577.5904	Jhang	**INTERCEPT**	1515.5	1304.3	1616.1
	AGE	8.4364	2.2462			**AGE**	5.7	5.4	
	EDU	13.036		10.0018		**EDU**	18		21.7
	HHS	176.1857	269.7042	243.3504		**HHS**	-31.5	100.1	105.9
	FMR	-38.0035	-55.7901			**FMR**	24.5	46.6	
	PARR	-100.8701	-158.511	-163.5913		**PARR**	-26.9	-127	-125.1
	CDR	33.1458				**CDR**	107.5		
	RIH	9.122				**RIH**	74.2		
	MFR			62.5042		**MFR**			-50.1
	YSFH		16.9519			**YSFH**		22.5	
Toba Tek Singh	**INTERCEPT**	1055.3	1321.6	1699.6	Gujranwala	**INTERCEPT**	4193.7	4055.5	5021.1
	AGE	8.4	4.2			**AGE**	15.6	19.5	
	EDU	8.8		5.1		**EDU**	-117.5		-98.4
	HHS	7.2	138.5	145.2		**HHS**	-173.6	-61.5	-30.8
	FMR	79	112			**FMR**	268	191.2	
	PARR	-46.3	-184.8	-180.6		**PARR**	-47.6	-163.1	-157.5
	CDR	141.3				**CDR**	106.4		
	RIH	37.1				**RIH**	100.9		
	MFR			-98		**MFR**			-135.7
	YSFH		5.2			**YSFH**		-95.3	

		Model no.1	Model No.2	Model.No.3	District		Model no.1	Model No.2	Model.No.3
Hafazabad	Variables	Coefficients	Coefficients	Coefficients	Okara	Variables	Coefficients	Coefficients	Coefficients
	INTERCEPT	1172.4	1280.1	1414.2		INTERCEPT	545.704	-12.7923	43.2312
	AGE	-0.5	-5.4			AGE	-1.8106	5.3064	
	EDU	-36.4		-26.6		EDU	4.7327		13.0653
	HHS	211.7	157.2	147.2		HHS	21.4908	158.1003	161.3352
	FMR	128.8	125.3			FMR	-5.175	-0.9098	
	PARR	-184.4	-119.4	-131.4		PARR	26.2686	-76.2278	-61.0624
	CDR	-69.7				CDR	110.1546		
	RIH	16.2				RIH	59.9667		
	MFR			-35.9		MFR			43.9222
	YSFH		-30.2			YSFH		14.9437	

Sahiwal	INTERCEPT	-1245.8	-410.32	-691.981	Pak Patten	INTERCEPT	-392.986	-417.522	-313.226
	AGE	15.2	7.3801			AGE	13.1249	11.1816	
	EDU	49.6		15.3311		EDU	-39.5036		-14.0428
	HHS	82.4	287.3675	339.7025		HHS	255.4249	242.4208	264.0621
	FMR	28	15.9906			FMR	-34.8811	-24.0645	
	PARR	39.8	-152.1097	-135.4757		PARR	-100.1556	-109.1526	-97.9969
	CDR	314.8				CDR	-56.5933		
	RIH	-18.2				RIH	87.4097		
	MFR			13.8012		MFR			35.5666
	YSFH		15.4666			YSFH		-13.192	

3. Theil-Sen Estimator:

		Model no.1	Model No.2	Model.No.3			Model no.1	Model No.2	Model.No.3
Sargodha	Variables	Coefficients	Coefficients	Coefficients	Faisalabad	Variables	Coefficients	Coefficients	Coefficients
	INTERCEPT	307.4264	690.879	1664.28728		INTERCEPT	735.941	2673.017	3266.8889
	AGE	8.3333	1.7094			AGE	29.012	14.2857	
	EDU	-37.037		-28.4722		EDU	-22.2222		-29.8889
	HHS	159.5659	174.6032	127.1368		HHS	101.1111	33.3333	27.7778
	FMR	177.7778	166.6668			FMR	-33.3333	-42.8571	
	PARR	-98.6111	-60.057	-85.1852		PARR	-186.6667	-188.8889	-188.8889
	CDR	138.8889				CDR	211.1111		
	RIH	**0**				RIH	27.7778		
	MFR			-206.6667		MFR			0
	YSFH		-46.2963			YSFH		-27.7778	

Chiniot	INTERCEPT	-560.423	1446.665	1694.44	Jhang	INTERCEPT	927.6812	1721.799	2051.39
	AGE	15.3846	4.3875			AGE	8.8889	4.7619	
	EDU	2.2222		0		EDU	16.6667		16.6667
	HHS	227.7778	166.6667	166.6667		HHS	62.4074	63.4921	60.4167
	FMR	30.4762	33.3333			FMR	15.9365	66.66667	
	PARR	-144.4444	-155.5556	-167		PARR	-100	-111.1111	-111.1111
	CDR	175				CDR	130.7692		
	RIH	140.7				RIH	75		
	MFR			0		MFR			-44.4444
	YSFH		3.0769			YSFH		18.5185	
Toba Tek	INTERCEPT	826.3197555	2888.235	3415.9264	Gujarawala	INTERCEPT	1667.4883	3021.821	4495.992
	AGE	8.3333	3.7037			AGE	21.4087	20.3704	
	EDU	11.4286		0		EDU	-88.8889		-87.3016
	HHS	72.2222	-22.7124	-29.6667		HHS	111.1111	33.3333	15.873
	FMR	109.333	175			FMR	304.3982	199.9998	
	PARR	-152.7778	-175	-168.545		PARR	-125	-144.444	-144.7222
	CDR	180.5556				CDR	125		
	RIH	87.2934				RIH	33.333		

	MFR			-144.4444		**MFR**			-166.6668
	YSFH		0			**YSFH**		-83.3333	

		Model no.1	Model No.2	Model.No.3			Model no.1	Model No.2	Model.No.3
	Variables	**Coefficients**	**Coefficients**	**Coefficients**		**Variables**	**Coefficients**	**Coefficients**	**Coefficients**
Hafazabad	**INTERCEPT**	916.018522	1797.997	1877.124	**Okara**	**INTERCEPT**	1093.126833	544.0139	725.185333
	AGE	3.7037	-3.1746			**AGE**	-2.9762	2.4691	
	EDU	-30		-22.2222		**EDU**	16.6667		18.5185
	HHS	90.6667	73.1209	73.1209		**HHS**	36.3636	100	90
	FMR	58.0556	86.6666			**FMR**	-23.5741	**0**	
	PARR	-77.7778	-100	-100		**PARR**	-44.4444	-52.7778	-50
	CDR	133.3333				**CDR**	55.5556		
	RIH	0				**RIH**	11.1111		
	MFR			-100		**MFR**			**0**
	YSFH		-22.2222			**YSFH**		20	
Sahiwal	**INTERCEPT**	-474.99951	-191.947	-281.667	**Pak Patten**	**INTERCEPT**	-2875.0561	-1137.33	-425.278
	AGE	11.1111	7.2299			**AGE**	23.8095	15.3846	
	EDU	25		13.3333		**EDU**	-17.2222		**0**
	HHS	233.3333	244.4444	277.7778		**HHS**	344.4444	238.1944	222.2222
	FMR	22.2222	24.1212			**FMR**	-31.746	**0**	
	PARR	-194.4444	-111.1111	-100		**PARR**	**0**	-48.8889	-50
	CDR	233.3333				**CDR**	69.4444		
	RIH	55.5556				**RIH**	158.8382		
	MFR			**0**		**MFR**			43.0556
	YSFH		10			**YSFH**		-5.5556	

4. QUANTILE REGRESSIONS:

Districts		Model no.1	Model No.2	Model.No.3	Districts		Model no.1	Model No.2	Model.No.3
	Variables	**Coefficients**	**Coefficients**	**Coefficients**		**Variables**	**Coefficients**	**Coefficients**	**Coefficients**
Sargodha	**INTERCEPT**	592.5514	-555.44	1058.64	**Faisalabad**	**INTERCEPT**	116.280	565.16	431.05
	AGE	18.10934	3.87			**AGE**	43.710	26.47	
	EDU	-6.627068		-1.84		**EDU**	6.342		-20.83
	HHS	69.15538	308.80	123.16		**HHS**	182.336	279.17	279.46
	FMR	82.8461	60.72			**FMR**	-27.941	-40.93	
	PARR	-50.39248	-86.39	-115.92		**PARR**	-236.367	-328.10	-317.71
	CDR	81.32896				**CDR**	73.537		
	RIH	-60.53468				**RIH**	-107.590		
	MFR			-7.65		**MFR**			51.08
	YSFH		-40.62			**YSFH**		-5.47	
Chiniot	**INTERCEPT**	-2179.97	-566.64	-1784.60	**Jhang**	**INTERCEPT**	1190.46	1214.04	1193.07
	AGE	9.08	0.36			**AGE**	3.51	-0.72	
	EDU	33.09		25.97		**EDU**	15.47		27.47
	HHS	430.62	410.82	494.50		**HHS**	-52.96	128.86	130.82
	FMR	-95.34	-30.06			**FMR**	31.67	54.79	

	PARR	-79.36	-199.80	-191.19		**PARR**	20.22	-111.75	-105.39
	CDR	98.28				**CDR**	161.38		
	RIH	-65.02				**RIH**	131.48		
	MFR			145.39		**MFR**			-100.72
	YSFH		6.34			**YSFH**		20.55	
Toba Tek Singh	**INTERCEPT**	1181.732	919.64	1538.10	**Gujranwala**	**INTERCEPT**	4229.68	4511.09	4368.27
	AGE	4.082	5.25			**AGE**	17.15	18.25	
	EDU	1.645		3.98		**EDU**	-184.16		-116.58
	HHS	-82.076	173.20	184.38		**HHS**	-92.88	-55.55	-65.04
	FMR	123.032	166.62			**FMR**	468.32	252.26	
	PARR	26.990	-205.16	-229.64		**PARR**	-119.60	-191.99	-172.88
	CDR	249.342				**CDR**	12.21		
	RIH	97.835				**RIH**	158.49		
	MFR			-129.52		**MFR**			-1.68
	YSFH		16.71			**YSFH**		-131.87	
Hafazabad	**INTERCEPT**	-199.20	358.94	801.39	**Okara**	**INTERCEPT**	695.32	275.80	1094.38
	AGE	3.67	-3.47			**AGE**	-0.94	3.21	
	EDU	-49.28		-66.80		**EDU**	-12.85		-34.32
	HHS	446.00	253.78	246.51		**HHS**	3.67	136.03	91.61
	FMR	147.30	111.10			**FMR**	4.81	6.74	
	PARR	-266.66	-85.41	-131.82		**PARR**	33.16	-84.21	-95.93
	CDR	-209.50				**CDR**	142.81		
	RIH	-19.06				**RIH**	-0.52		
	MFR			10.19		**MFR**			35.75
	YSFH		-70.88			**YSFH**		21.17	

		Model no.1	**Model No.2**	**Model.No.3**			**Model no.1**	**Model No.2**	**Model.No.3**
Sahiwal	**Variables**	**Coefficients**	**Coefficients**	**Coefficients**	**Pak Patten**	**Variables**	**Coefficients**	**Coefficients**	**Coefficients**
	INTERCEPT	-583.35	-1106.03	-681.35		**INTERCEPT**	-1073.94	-855.65	-619.25
	AGE	18.01	7.05			**AGE**	13.28	11.25	
	EDU	71.93		14.74		**EDU**	-50.55		-16.59
	HHS	-49.07	374.64	292.64		**HHS**	463.15	317.17	324.32
	FMR	4.99	-131.92			**FMR**	9.81	97.57	
	PARR	56.35	-143.37	-151.12		**PARR**	-246.77	-165.64	-170.55
	CDR	369.98				**CDR**	-193.38		
	RIH	25.93				**RIH**	100.26		
	MFR			59.53		**MFR**			-59.51
	YSFH		20.73			**YSFH**		-16.34	

5. M-ESTIMATOR:

Districts		**Model no.1**	**Model No.2**	**Model.No.3**	**Districts**		**Model no.1**	**Model No.2**	**Model.No.3**
Sargodha	**Variables**	**Coefficients**	**Coefficients**	**Coefficients**	**Faisalabad**	**Variables**	**Coefficients**	**Coefficients**	**Coefficients**
	INTERCEPT	1178.34	955.60	1328.74		**INTERCEPT**	-423.94	567.65	379.92
	AGE	2.69	-2.61			**AGE**	23.69	13.83	
	EDU	-31.67		-28.52		**EDU**	-6.57		-24.33
	HHS	228.10	166.43	121.80		**HHS**	368.06	304.87	302.05
	FMR	51.06	-9.81			**FMR**	-15.44	-65.17	
	PARR	-179.84	-45.70	-66.90		**PARR**	-239.47	-247.85	-244.35

	CDR	-107.93				**CDR**	12.93		
	RIH	10.62				**RIH**	-112.43		
	MFR			-45.79		**MFR**			61.57
	YSFH		-45.87			**YSFH**		-20.24	
Chiniot	**INTERCEPT**	-251.58	-834.24	-603.06	**Jhang**	**INTERCEPT**	1386.05	1194.55	1236.98
	AGE	10.86	3.32			**AGE**	8.00	7.04	
	EDU	18.01		13.80		**EDU**	19.62		21.63
	HHS	137.86	422.14	390.02		**HHS**	-61.61	111.95	110.45
	FMR	-41.73	-59.54			**FMR**	-26.91	8.35	
	PARR	14.42	-184.36	-195.74		**PARR**	20.41	-119.38	-119.22
	CDR	183.61				**CDR**	151.10		
	RIH	-4.10				**RIH**	77.13		
	MFR			105.52		**MFR**			-30.04
	YSFH		19.74			**YSFH**		23.54	
Toba Tek Singh	**INTERCEPT**	1282.22	1512.56	1881.20	**Gujranwala**	**INTERCEPT**	3782.78	3692.31	4297.79
	AGE	7.24	4.16			**AGE**	16.69	22.94	
	EDU	4.25		-2.75		**EDU**	-103.87		-83.11
	HHS	35.67	147.78	142.90		**HHS**	-156.90	-71.70	-80.19
	FMR	66.77	107.02			**FMR**	325.74	246.22	
	PARR	-61.90	-185.53	-184.61		**PARR**	-22.55	-128.94	-135.92
	CDR	120.91				**CDR**	78.46		
	RIH	30.84				**RIH**	82.65		
	MFR			-136.04		**MFR**			-103.36
	YSFH		-2.48			**YSFH**		-81.10	
Districts	**INTERCEPT**	1028.25	1191.34	1549.15	**Districts**	**INTERCEPT**	1792.33	1269.78	1380.21
Hafazabad	**AGE**	1.90	-3.82		**Okara**	**AGE**	-3.17	3.35	
	EDU	-30.88		-26.39		**EDU**	-18.70		-6.61
	HHS	157.16	160.66	155.86		**HHS**	-53.72	36.51	18.82
	FMR	140.36	138.38			**FMR**	-2.16	-1.62	
	PARR	-111.43	-110.50	-118.39		**PARR**	3.87	-52.86	-46.86
	CDR	-2.07				**CDR**	68.31		
	RIH	-3.58				**RIH**	55.35		
	MFR			-49.65		**MFR**			12.83
	YSFH		-29.22			**YSFH**		-5.24	

Districts		**Model no.1**	**Model No.2**	**Model.No.3**	**Districts**		**Model no.1**	**Model No.2**	**Model.No.3**
Sahiwal	**Variables**	**Coefficients**	**Coefficients**	**Coefficients**	**Pak Patten**	**Variables**	**Coefficients**	**Coefficients**	**Coefficients**
	INTERCEPT	-992.58	-489.67	-1189.45		**INTERCEPT**	-677.50	-620.78	-617.18
	AGE	16.56	10.94			**AGE**	13.77	10.84	
	EDU	29.77		10.75		**EDU**	-40.65		-15.12
	HHS	101.88	340.98	376.55		**HHS**	279.34	281.40	272.97
	FMR	-41.38	-73.09			**FMR**	-15.39	-3.36	
	PARR	-1.08	-204.14	-193.20		**PARR**	-101.33	-120.51	-125.41
	CDR	322.15				**CDR**	-38.54		
	RIH	28.18				**RIH**	70.01		
	MFR			98.01		**MFR**			41.85
	YSFH		10.86			**YSFH**		-18.30	

2 .With log Transformation of Dependent variable:

1. M-ESTIMATOR:

Districts		Model no.1	Model No.2	Model.No.3	Districts		Model no.1	Model No.2	Model.No.3
Sargodha	Variables	Coefficients	Coefficients	Coefficients	Faisalabad	Variables	Coefficients	Coefficients	Coefficients
	INTERCEPT	3.06	3.013	3.180		INTERCEPT	2.10	3.119	3.195
	AGE	0.00092	-0.0004			AGE	0.01	0.004	
	EDU	-0.01		-0.010		EDU	-0.00291		-0.007
	HHS	0.06	0.044	0.035		HHS	0.068	0.043	0.051
	FMR	0.05	0.054			FMR	0.00179	-0.002	
	PARR	-0.05	-0.021	-0.027		PARR	-0.06	-0.055	-0.052
	CDR	-0.02				CDR	-0.01		
	RIH	-0.001982				RIH	-0.02		
	MFR			-0.023		MFR			0.011
	YSFH		-0.015			YSFH		-0.006	
Chiniot	INTERCEPT	3.062	3.070	3.102	Jhang	INTERCEPT	3.167	3.108	3.183
	AGE	0.002	0.001			AGE	0.002	0.002	
	EDU	0.003		0.003		EDU	0.005		0.006
	HHS	0.022	0.045	0.041		HHS	-0.007	0.026	0.028
	FMR	-0.018	-0.024			FMR	0.006	0.014	
	PARR	-0.017	-0.036	-0.037		PARR	-0.010	-0.035	-0.033
	CDR	0.011				CDR	0.025		
	RIH	0.003				RIH	0.017		
	MFR			0.017		MFR			-0.013
	YSFH		0.005			YSFH		0.007	
Toba.Tek Singh	INTERCEPT	3.087	3.171	3.268	Gujranwala	INTERCEPT	3.597	3.696	3.951
	AGE	0.002	0.001			AGE	0.003	0.004	
	EDU	0.002		0.008		EDU	-0.022		-0.020
	HHS	0.004	0.033	0.035		HHS	-0.022	-0.015	-0.014
	FMR	0.015	0.030			FMR	0.045	0.032	
	PARR	-0.015	-0.050	-0.049		PARR	-0.011	-0.034	-0.032
	CDR	0.031				CDR	0.017		
	RIH	0.007				RIH	0.017		
	MFR			-0.026		MFR			-0.027
	YSFH		0.001			YSFH		-0.019	
Hafazabad	INTERCEPT	3.114	3.161	3.194	Okara	INTERCEPT	2.892	2.712	2.729
	AGE	0.000257	-0.001			AGE	-0.00001	0.002	
	EDU	-0.010		-0.007		EDU	0.002		0.006
	HHS	0.048	0.035	0.032		HHS	0.006	0.046	0.050
	FMR	0.032	0.031			FMR	-0.003	-0.003	
	PARR	-0.045	-0.031	-0.034		PARR	0.001	-0.026	-0.021
	CDR	-0.016				CDR	0.035		
	RIH	0.0000737				RIH	0.021		
	MFR			-0.009		MFR			0.013
	YSFH		0.008			YSFH		0.01	

		Model no.1	Model No.2	Model.No.3			Model no.1	Model No.2	Model.No.3
	Variables	**Coefficients**	**Coefficients**	**Coefficients**		**Variables**	**Coefficients**	**Coefficients**	**Coefficients**
Sahiwal	**INTERCEPT**	2.518	2.614	2.525	**Pak Patten**	**INTERCEPT**	2.656	2.649	2.714
	AGE	0.004	0.002			**AGE**	0.003	0.003	
	EDU	0.016		0.007		**EDU**	-0.010		-0.003
	HHS	0.016	0.077	0.093		**HHS**	0.073	0.064	0.069
	FMR	0.005	-0.004			**FMR**	-0.003	0.001	
	PARR	0.007	-0.039	-0.035		**PARR**	-0.033	-0.031	-0.030
	CDR	0.078				**CDR**	-0.023		
	RIH	-0.004				**RIH**	0.020		
	MFR			0.001		**MFR**			0.008
	YSFH		0.007			**YSFH**		-0.003	

2. LTS ESTIMATOR:

		Model no.1	Model No.2	Model.No.3			Model no.1	Model No.2	Model.No.3
	Variables	**Coefficients**	**Coefficients**	**Coefficients**		**Variables**	**Coefficients**	**Coefficients**	**Coefficients**
Sargodha	**INTERCEPT**	3.102	3.086	3.148	**Faisalabad**	**INTERCEPT**	2.894	3.058	3.062
	AGE	0.000157	-0.001			**AGE**	0.006	0.004	
	EDU	-0.011		-0.008		**EDU**	0.000205		-0.004
	HHS	0.079	0.050	0.041		**HHS**	0.081	0.049	0.046
	FMR	0.023	0.006			**FMR**	0.002	-0.002	
	PARR	-0.071	-0.027	-0.029		**PARR**	-0.077	-0.057	-0.056
	CDR	-0.037				**CDR**	-0.028		
	RIH	0.005				**RIH**	-0.020		
	MFR			-0.019		**MFR**			0.007
	YSFH		-0.015			**YSFH**		-0.003	
Chiniot	**INTERCEPT**	2.923	2.877	2.812	**Jhang**	**INTERCEPT**	3.203	3.154	3.200
	AGE	0.00048	0.00016			**AGE**	0.003	0.002	
	EDU	-0.006		-0.040		**EDU**	0.004		0.006
	HHS	0.104	0.076	0.079		**HHS**	-0.001	0.022	0.020
	FMR	-0.027	-0.028			**FMR**	0.001	0.010	
	PARR	-0.068	-0.042	-0.041		**PARR**	-0.022	-0.036	-0.036
	CDR	-0.034				**CDR**	0.015		
	RIH	-0.004				**RIH**	0.010		
	MFR			0.026		**MFR**			-0.011
	YSFH		0.001			**YSFH**		0.007	
Toba Tek Singh	**INTERCEPT**	3.050	3.137	3.214	**Gujranwala**	**INTERCEPT**	3.193	3.406	3.517
	AGE	0.002	0.001			**AGE**	0.004	0.006	
	EDU	0.003		0.00019		**EDU**	-0.023		-0.018
	HHS	0.012	0.038	0.037		**HHS**	0.061	0.006	0.004
	FMR	-0.000359	0.022			**FMR**	0.053	0.034	
	PARR	-0.014	-0.046	-0.047		**PARR**	-0.061	-0.036	-0.037
	CDR	0.026				**CDR**	-0.029		
	RIH	0.003				**RIH**	0.022		
	MFR			-0.036		**MFR**			-0.021
	YSFH		-0.002			**YSFH**		-0.018	
Haf	**INTERCEPT**	3.077	3.129	3.164	**O**	**INTERCEPT**	2.939	2.733	2.752

	AGE	0.00041	-0.001	-0.001		**AGE**	0.002	0.003	
	EDU	-0.011		-0.008		**EDU**	0.004		0.007
	HHS	0.081	0.040	0.041		**HHS**	-0.031	0.041	0.037
	FMR	0.031	0.027			**FMR**	-0.006	-0.004	
	PARR	-0.070	-0.032	-0.036		**PARR**	0.032	-0.029	-0.027
	CDR	-0.042				**CDR**	0.060		
	RIH	-0.009				**RIH**	0.00029		
	MFR			0.002		**MFR**			0.007
	YSFH		-0.010			**YSFH**		0.008	

		Model no.1	**Model No.2**	**Model.No.3**			**Model no.1**	**Model No.2**	**Model.No.3**
	Variables	**Coefficients**	**Coefficients**	**Coefficients**		**Variables**	**Coefficients**	**Coefficients**	**Coefficients**
Sahiwal	**INTERCEPT**	2.697	2.728	2.435	**Pak Patten**	**INTERCEPT**	2.650	2.645	2.651
	AGE	0.004	0.005			**AGE**	0.003	0.003	
	EDU	0.015		0.007		**EDU**	-0.007		-0.001
	HHS	-0.017	0.059	0.088		**HHS**	0.092	0.069	0.062
	FMR	-0.021	0.004			**FMR**	-0.003	0.004	
	PARR	0.026	-0.059	-0.034		**PARR**	-0.052	-0.038	-0.037
	CDR	0.096				**CDR**	-0.039		
	RIH	0.004				**RIH**	0.015		
	MFR			0.019		**MFR**			0.012
	YSFH		0.009			**YSFH**		-0.003	

3. QUANTILE REGRESSION:

		Model no.1	**Model No.2**	**Model.No.3**			**Model no.1**	**Model No.2**	**Model.No.3**
	Variables	**Coefficients**	**Coefficients**	**Coefficients**		**Variables**	**Coefficients**	**Coefficients**	**Coefficients**
Sargodha	**INTERCEPT**	3.005	2.715	3.237	**Faisalabad**	**INTERCEPT**	2.848	3.039	3.099
	AGE	0.004	0.001			**AGE**	0.009	0.006	
	EDU	-0.001		-0.006		**EDU**	0.004		-0.007
	HHS	-0.003	0.072	0.027		**HHS**	0.029	0.050	0.066
	FMR	0.018	0.011			**FMR**	-0.016	-0.017	
	PARR	0.005	-0.022	-0.027		**PARR**	-0.039	-0.071	-0.058
	CDR	0.035				**CDR**	0.031		
	RIH	-0.016				**RIH**	-0.023		
	MFR			-0.009		**MFR**			0.010
	YSFH		-0.009			**YSFH**		0.0000112	
Chiniot	**INTERCEPT**	2.550	2.934	2.749	**Jhang**	**INTERCEPT**	3.129	3.140	3.167
	AGE	0.002	0.000			**AGE**	0.001	-0.00025	
	EDU	0.008		0.006		**EDU**	0.002		0.005
	HHS	0.035	0.061	0.076		**HHS**	-0.016	0.027	0.026
	FMR	-0.013	-0.001			**FMR**	0.008	0.013	
	PARR	0.020	-0.038	-0.041		**PARR**	0.006	-0.024	-0.023
	CDR	0.061				**CDR**	0.038		
	RIH	-0.011				**RIH**	0.027		
	MFR			0.029		**MFR**			-0.024
	YSFH		0.003			**YSFH**		0.004	
Toba Tek	**INTERCEPT**	3.119	3.068	3.253	**Gujranwala**	**INTERCEPT**	3.653	3.674	3.880
	AGE	0.001	0.002			**AGE**	0.003	0.005	
	EDU	0.002		-0.0001		**EDU**	-0.026		-0.020
	HHS	-0.022	0.037	0.036		**HHS**	-0.016	-0.013	-0.004
	FMR	0.025	0.029			**FMR**	0.065	0.034	

	PARR	0.009	-0.046	-0.048		**PARR**	-0.018	-0.035	-0.032
	CDR	0.056				**CDR**	0.007		
	RIH	0.019				**RIH**	0.019		
	MFR			-0.024		**MFR**			-0.040
	YSFH		0.003			**YSFH**		-0.021	
Hafazabad	**INTERCEPT**	3.010	2.968	3.098	**Okara**	**INTERCEPT**	3.015	2.867	3.040
	AGE	0.001	-0.00002			**AGE**	-0.001	0.001	
	EDU	-0.014		-0.015		**EDU**	-0.006		-0.004
	HHS	0.084	0.047	0.048		**HHS**	0.009	0.034	0.027
	FMR	0.029	0.023			**FMR**	0.001	0.008	
	PARR	-0.068	-0.020	-0.035		**PARR**	-0.004	-0.028	-0.023
	CDR	-0.045				**CDR**	0.030		
	RIH	0.001				**RIH**	0.007		
	MFR			0.004		**MFR**			0.010
	YSFH		-0.011			**YSFH**		0.006	

District		**Model no.1**	**Model No.2**	**Model.No.3**	District		**Model no.1**	**Model No.2**	**Model.No.3**
	Variables	**Coefficients**	**Coefficients**	**Coefficients**		**Variables**	**Coefficients**	**Coefficients**	**Coefficients**
Sahiwal	**INTERCEPT**	2.705	2.623	2.485	**Pak Patten**	**INTERCEPT**	2.504	2.572	2.786
	AGE	0.004	0.002			**AGE**	0.005	0.004	
	EDU	0.021		0.001		**EDU**	-0.012		-0.002
	HHS	-0.008	0.082	0.095		**HHS**	0.117	0.083	0.085
	FMR	-0.009	-0.034			**FMR**	0.008	0.031	
	PARR	0.005	-0.033	-0.027		**PARR**	-0.070	-0.051	-0.058
	CDR	0.075				**CDR**	-0.047		
	RIH	0.013				**RIH**	0.024		
	MFR			0.023		**MFR**			-0.030
	YSFH		0.004			**YSFH**		-0.006	

4. LAD ESTIMATOR:

District		**Model no.1**	**Model No.2**	**Model.No.3**	District		**Model no.1**	**Model No.2**	**Model.No.3**
	Variables	**Coefficients**	**Coefficients**	**Coefficients**		**Variables**	**Coefficients**	**Coefficients**	**Coefficients**
Sargodha	**INTERCEPT**	3.049	3.027	3.198	**Faisalabad**	**INTERCEPT**	2.941	3.151	3.221
	AGE	0.001	-0.0004			**AGE**	0.006	0.004	
	EDU	-0.012		-0.011		**EDU**	-0.004		-0.008
	HHS	0.060	0.044	0.037		**HHS**	0.055	0.043	0.049
	FMR	0.053	0.054			**FMR**	-0.005	-0.010	
	PARR	-0.048	-0.020	-0.027		**PARR**	-0.049	-0.056	-0.051
	CDR	-0.021				**CDR**	0.002		
	RIH	-0.004				**RIH**	-0.022		
	MFR			-0.025		**MFR**			0.015
	YSFH		-0.016			**YSFH**		-0.007	
Chiniot	**INTERCEPT**	3.004	3.010	3.052	**Jhang**	**INTERCEPT**	3.177	3.125	3.189
	AGE	0.002	0.001			**AGE**	0.002	0.001	
	EDU	0.005		0.004		**EDU**	0.005		0.006

	Variables	Model no.1	Model No.2	Model.No.3		Variables	Model no.1	Model No.2	Model.No.3
	HHS	0.032	0.050	0.046		**HHS**	-0.009	0.026	0.028
	FMR	-0.014	-0.019			**FMR**	0.005	0.012	
	PARR	-0.023	-0.038	-0.038		**PARR**	-0.006	-0.033	-0.032
	CDR	0.009				**CDR**	0.027		
	RIH	0.003				**RIH**	0.019		
	MFR			0.016		**MFR**			-0.013
	YSFH		0.006			**YSFH**		0.006	
Toba Tek Singh	**INTERCEPT**	3.122	3.187	3.310	**Gujranwala**	**INTERCEPT**	3.767	3.780	3.984
	AGE	0.002	0.00			**AGE**	0.003	0.004	
	EDU	0.003		0.002		**EDU**	-0.023		-0.020
	HHS	-0.001	0.02	0.029		**HHS**	-0.031	-0.020	-0.017
	FMR	0.017	0.032			**FMR**	0.047	0.034	
	PARR	-0.015	-0.050	-0.049		**PARR**	-0.018	-0.034	-0.032
	CDR	0.030				**CDR**	0.012		
	RIH	0.008				**RIH**	0.017		
	MFR			-0.026		**MFR**			-0.026
	YSFH		0.002			**YSFH**		-0.020	
Hafazabad	**INTERCEPT**	3.123	3.166	3.208	**Okara**	**INTERCEPT**	2.862	2.719	2.715
	AGE	0.0002	-0.002			**AGE**	-0.001	0.002	
	EDU	-0.010		-0.007		**EDU**	0.003		0.007
	HHS	0.047	0.037	0.032		**HHS**	0.006	0.045	0.049
	FMR	0.033	0.032			**FMR**	-0.001	-0.003	
	PARR	-0.043	-0.031	-0.034		**PARR**	0.008	-0.023	-0.019
	CDR	-0.015				**CDR**	0.036		
	RIH	0.001				**RIH**	0.018		
	MFR			-0.010		**MFR**			0.017
	YSFH		-0.009			**YSFH**		0.008	

Sahiwal		**Model no.1**	**Model No.2**	**Model.No.3**	**Pak Patten**		**Model no.1**	**Model No.2**	**Model.No.3**
	Variables	**Coefficients**	**Coefficients**	**Coefficients**		**Variables**	**Coefficients**	**Coefficients**	**Coefficients**
	INTERCEPT	2.514	2.59	2.513		**INTERCEPT**	2.724	2.701	2.735
	AGE	0.004	0.002			**AGE**	0.004	0.003	
	EDU	0.014		0.005		**EDU**	-0.011		-0.003
	HHS	0.029	0.08	0.096		**HHS**	0.078	0.062	0.069
	FMR	0.009	-0.0037			**FMR**	-0.008	-0.003	
	PARR	-0.002	-0.04	-0.035		**PARR**	-0.043	-0.033	-0.032
	CDR	0.068				**CDR**	-0.031		
	RIH	-0.006				**RIH**	0.021		
	MFR			0.003		**MFR**			0.010
	YSFH		0.01			**YSFH**		-0.003	

5. THEIL-SEN ESTIMATOR:

Sargodha		**Model no.1**	**Model No.2**	**Model.No.3**	**Faisalabad**		**Model no.1**	**Model No.2**	**Model.No.3**
	Variables	**Coefficients**	**Coefficients**	**Coefficients**		**Variables**	**Coefficients**	**Coefficients**	**Coefficients**
	INTERCEPT	2.890	3.002	3.252		**INTERCEPT**	3.008	3.434	3.625
	AGE	0.002	0.001			**AGE**	0.007	0.004	
	EDU	-0.011		-0.009		**EDU**	-0.006		-0.008

	HHS	0.040	0.044	0.033		**HHS**	0.024	0.012	0.009
	FMR	0.047	0.049			**FMR**	-0.006	-0.007	
	PARR	-0.026	-0.018	-0.025		**PARR**	-0.046	-0.050	-0.049
	CDR	0.037				**CDR**	0.051		
	RIH	0.000				**RIH**	0.006		
	MFR			-0.057		**MFR**			0.009
	YSFH		-0.014			**YSFH**		-0.008	
Chiniot	**INTERCEPT**	2.743	3.215	3.280	**Jhang**	**INTERCEPT**	3.043	3.240	3.324
	AGE	0.004	0.001			**AGE**	0.002	0.001	
	EDU	0.000		0.000		**EDU**	0.004		0.004
	HHS	0.048	0.034	0.035		**HHS**	0.015	0.015	0.015
	FMR	0.007	0.008			**FMR**	0.004	0.017	
	PARR	-0.034	-0.037	-0.041		**PARR**	-0.025	-0.029	-0.028
	CDR	0.041				**CDR**	0.032		
	RIH	0.036				**RIH**	0.019		
	MFR			0.000		**MFR**			-0.012
	YSFH		0.001			**YSFH**		0.005	
Toba Tek Singh	**INTERCEPT**	3.036	3.250	3.685	**Gujranwala**	**INTERCEPT**	3.264	3.500	3.789
	AGE	0.002	0			**AGE**	0.004	0.004	
	EDU	0.003		0.000		**EDU**	-0.017		-0.02
	HHS	0.016	0	-0.007		**HHS**	0.019	0.006	0.004
	FMR	0.028	0			**FMR**	0.053	0.038	
	PARR	-0.039	0	-0.046		**PARR**	-0.025	-0.029	-0.030
	CDR	0.043				**CDR**	0.024		
	RIH	0.021				**RIH**	0.006		
	MFR			-0.038		**MFR**			-0.035
	YSFH		**0**			**YSFH**		-0.017	
Hafazabad	**INTERCEPT**	3.063	3.353	3.313	**Okara**	**INTERCEPT**	2.987	2.849	2.937
	AGE	0.001	0			**AGE**	-0.001	0.001	
	EDU	-0.009		-0.007		**EDU**	0.006		0.007
	HHS	0.021	0	0.018		**HHS**	0.017	0.033	0.027
	FMR	0.015	0			**FMR**	0.000	0	
	PARR	-0.021	-0.0208	-0.028		**PARR**	-0.020	-0.017	-0.015
	CDR	0.033				**CDR**	0.025		
	RIH	0.000				**RIH**	0.009		
	MFR			-0.031		**MFR**			0.000
	YSFH		**0**			**YSFH**		0.008	

		Model no.1	**Model No.2**	**Model.No.3**			**Model no.1**	**Model No.2**	**Model.No.3**
	Variables	**Coefficients**	**Coefficients**	**Coefficients**		**Variables**	**Coefficients**	**Coefficients**	**Coefficients**
Sahiwal	**INTERCEPT**	2.591	2.612	2.625	**Pak Patten**	**INTERCEPT**	1.952	2.387	2.644
	AGE	0.003	0.002			**AGE**	0.007	0.005	
	EDU	0.007		0.004		**EDU**	-0.006		0.000
	HHS	0.068	0.077	0.082		**HHS**	0.095	0.071	0.063
	FMR	0.007	0.007			**FMR**	-0.009	0.000	
	PARR	-0.054	-0.031	-0.027		**PARR**	0.000	-0.016	-0.017
	CDR	0.066				**CDR**	0.021		
	RIH	0.016				**RIH**	0.046		
	MFR			0.000		**MFR**			0.015
	YSFH		0.003			**YSFH**		-0.002	